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**PROBABILITY DISTRIBUTION FUNCTION BASED IRIS
RECOGNITION SYSTEM BOOSTED BY THE MEAN RULE**

Bachelor's thesis (12 ECTS)

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1 Introduction

The demand for an efficient and reliable means of identification has increased rapidly, mainly because of the events like 9/11 in the USA or 7/7 in London. Different institutions are willing to invest more and more of their financial resources to develop a method that is both comfortable for the end users and is able to identify them without any problems. Nevertheless, it is really important to keep the price as low as possible, because security is in most of the cases an extra cost for the companies.

The goals of this paper are the following:

- Introduce basic concepts of iris recognition for the reader to be able to gain more knowledge of the area.
- Introduce 3 different iris recognition algorithms developed so far.
- Propose an iris recognition algorithm that uses probability distribution functions (PDFs) of HSI and YCbCr colour spaces in the recognition phase.

The thesis consists of four chapters. The first one will give an overview of the history of iris recognition and present some examples of real-life applications of biometric identification using irises. The second chapter discusses the main concepts of iris recognition that are used in the coming parts. The third chapter introduces three different iris recognition algorithms for the reader to be able to draw parallels and compare these to the one introduced in the thesis. The final chapter proposes a new algorithm and introduces the simulation results.

2 Overview

2.1 History

The history of iris based person recognition is said to have begun in 1953 when F. H. Adler wrote in “Physiology of the Eye”: "In fact, the markings of the iris are so distinctive that it has been proposed to use photographs as a means of identification, instead of fingerprints." Adler in turn referenced to J. H. Doggart who claimed that permutations and combinations of irises are theoretically infinite. Based on the previous suggestions, Dr. Leonard Flom and Dr. Aran Safir patented the idea to use the iris as a way to recognize persons. However, they did not have an algorithm to implement the idea in real life. Therefore, they needed the assistance of J. G. Daugman, who in turn patented his method and published a paper called “High Confidence Visual Recognition of Persons by a Test of Statistical Independence” during the years 1993 and 1994. [1]

The proposed algorithm gained immediate popularity and a lot companies like IriScan, Iridian, Sarnoff, Sensor, LG-Iris, Panasonic, Oki, BI2, IrisGuard, Unisys, Sagem, Enschede, Securimetrics and L1 acquired the license and started to produce real life applications. [1]

Iris based person recognition has gained more and more popularity and Chandler *et al.* even considered it to be the most reliable mean of biometric identification out of seven most widespread methods. [2]

2.2 Application of Iris Recognition

Iris recognition is mostly used by border patrols and other institutions such as airports, banks, and employers where a quick and reliable mean to identify people is needed.

The most significant user of the method is currently Aadhaar that is operated by the Unique Identification Authority of India (UIDAI). They claim themselves to be the first ones in the world to offer state-controlled iris authentication system. Around 350 million people out of the total population of 1.2 billion were allowed to enrol in the programme by May 2013. According to their studies, it is possible to identify over 99% of the enrollees. [3]

In addition, the border patrol of the United Arab Emirates (the UAE) has been using iris based person recognition since September 2003. They set up iris recognition cameras at all border crossing points and deportation centres. By May 2013 they had been able to prevent almost 350,000 *personae non gratae* from entering the country who had previously been expelled from the country, but were still trying to return with a fake identity.[4]

Their system is load-balanced between several databases at various locations around the country and all the databases are always kept up to date. The system can make more than 650,000 comparisons per second. [5]

Because their system is “negative”, meaning that they will add the photos of the irises of the people whom they do not want to allow to enter the UAE, there is practically no way to determine the number of false negatives (people who should not have permission to enter the country, but still can for whatever reason) as those unwanted expellees, who still can cross the border, have no motivation to tell the authorities about their past. However, so far there have been no reports about failure to enrol a person to the system. [5]

The most well-known institution in Europe to use iris recognition is Schipol airport in Amsterdam. They offer the holders of a Privium Card to pass the security check by just showing their iris to a scanner. [6]

3 Background of iris recognition

3.1 Human eye

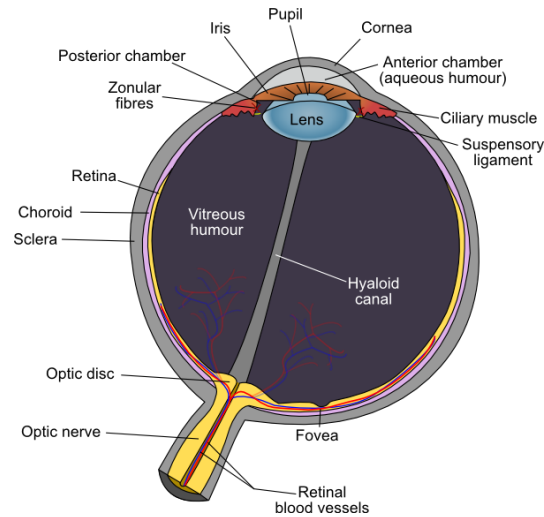


Figure 1. The construction of a human eye. [7]

Iris is a tissue that surrounds the pupil. Its main task is to control the size of the pupil i.e. how much light enters the eye [8]. However, only the development of its pattern and colour is regarded as needed information.

The development of the pattern of an iris is completely random and hence, the probability of having two different irises with the same pattern is virtually zero. Even twins do not have similar patterns and therefore, from the perspective of uniqueness, iris based person recognition is an ideal biometric identification tool. Furthermore, iris is the only internal organ visible to the outside world and it is practically impossible to permanently change its texture or colour without physically harming the eye. [8]

Also it is important to note that the pattern of the iris does not change during the human lifespan.

3.2 Colour spaces

3.2.1 RGB colour space

RGB (red, green, blue) colour space is one of the most well-known colour spaces [9]. The vast majority of the images are represented in RGB. It divides all the images into red, green and blue

spectral components. RGB is based on a traditional Cartesian coordinate system and all the primary colours are in the corners of the cube ($R=1,0,0$; $G=0,1,0$; $B=0,0,1$ when using normalized representation of the cube). [10]

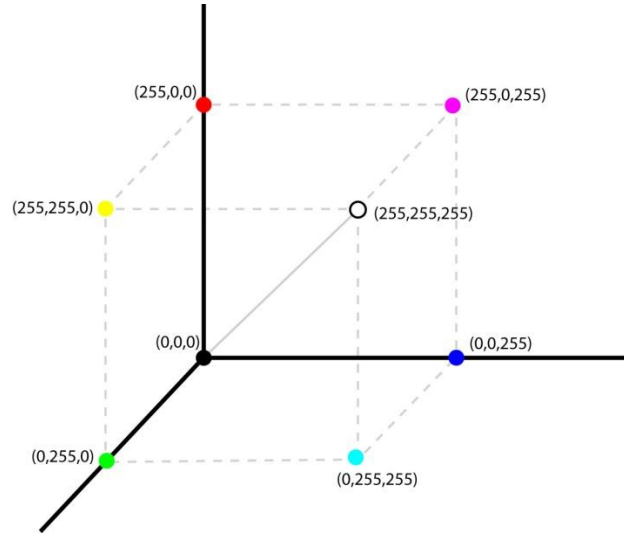


Figure 2. RGB cube in the Cartesian coordinate system [11]

Despite being widespread, RGB has a major drawback: a small change in illumination alters RGB coordinates significantly, because its intensity images combine both the tonal and illumination information [12].

For example, taking a photo of a blue surface with and without a source of light yields two images, where pixel values differ significantly even though in real life the colour stayed the same. Illumination conditions change usually the most when between different samples of the same iris and therefore RGB colour space should be excluded from the algorithm that is introduced in the next chapter, because it might affect significantly our recognition rate.

In conclusion, the proposed iris recognition method would benefit from having a colour space where the tone is separated from the illumination information.

3.2.2 HSI colour space

In the search for a better colour space than RGB, one of the possible candidates is HSI that stands for hue, saturation and intensity. Some sources call HSI also HSV, where V means value, but both are theoretically and practically the same representations.

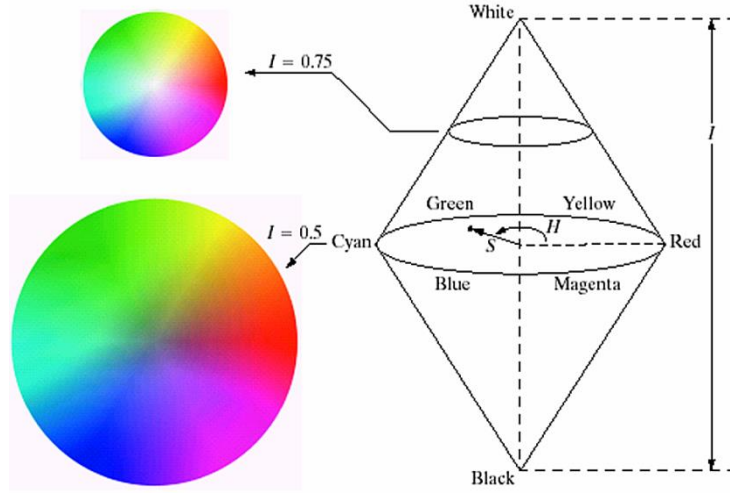


Figure 3. HSI (HSV) colour space.

HSI colour space splits colour-carrying and illumination information into three different components and this might be a more appropriate colour space for our algorithm than RGB.

Hue represents only the tonal information, meaning that it will tell whether the colour is red, green, blue or something in between. H can range from 0 to 360°. Saturation indicates how “pure” or saturated the colour is. The lower the saturation, the dimmer the pixel. S ranges from 0 to 1. These two channels carry the colour information. The last one, I, describes the illumination conditions. I can range from 0 to 1. When the value of I is 0, then the pixel is black, no matter what the other values are. If I is 1, then the pixel is the brightest possible and the final appearance depends on H and S values.

3.2.2.1 Converting RGB to HSI

Converting RGB to HSI is a straightforward process on the basis of the following equation (1).

Digging deeper into the conversion algorithm, the first step would be to normalize the RGB cube, meaning that all the RGB values would range from 0 to 1.

The easiest component to determine is I. It can be noticed from Figure 2 that there exists a diagonal line from black to white. It is called intensity axis. To get a value for I, it is needed to draw a plane perpendicular to the axis and containing the RGB pixel value into the cube. The intersection of the inserted plane and the intensity axis is the value for I. [10]

$$\begin{aligned}
r &= \frac{R}{R+G+B}; g = \frac{G}{R+G+B}; b = \frac{B}{R+G+B} \\
H &= \cos^{-1} \left\{ \frac{0.5*[(r-g)+(r-b)]}{\sqrt{(r-g)^2+(r-b)(g-b)}} \right\} \text{ if } b \leq g, h \in [0, \pi] \\
H &= 2\pi - \cos^{-1} \left\{ \frac{0.5*[(r-g)+(r-b)]}{\sqrt{(r-g)^2+(r-b)(g-b)}} \right\} \text{ if } b > g, h \in [\pi, 2\pi] \\
S &= 1 - 3 * \min(r, g, b) \quad I = \frac{R+G+B}{3*255}
\end{aligned} \tag{1}$$

Hue is determined by the triangle that consists of the intensity axis and a point that is on the edge or the vertex of the RGB cube. By rotating the triangle around the intensity axis, it is possible to alter the hue. Hence, the same H represents all the RGB values that lay on the defined triangle. By looking down to the RGB cube from the intensity axis vertices, it can be noticed that both primary and secondary colours are separated by 120°. This is illustrated by Figure 4. In most of the cases red is regarded as 0° hue and the value increases counter-clockwise from there. [10]

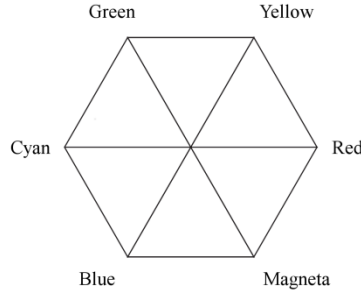


Figure 4. HSI H channel

Saturation S describes how far the pixel in RGB colour space is from the intensity axis.

3.2.1 YCbCr colour space

The YCbCr colour space is one of the most popular colour spaces used for video coding and image processing. It is a bit similar to HSI colour space: Y channel represents the luminance and Cb-Cr the tonal information. Being more specific, Cb and Cr represent blue-difference and red-difference chroma information. Figure 5 shows an image in both RGB and YCbCr colour space.

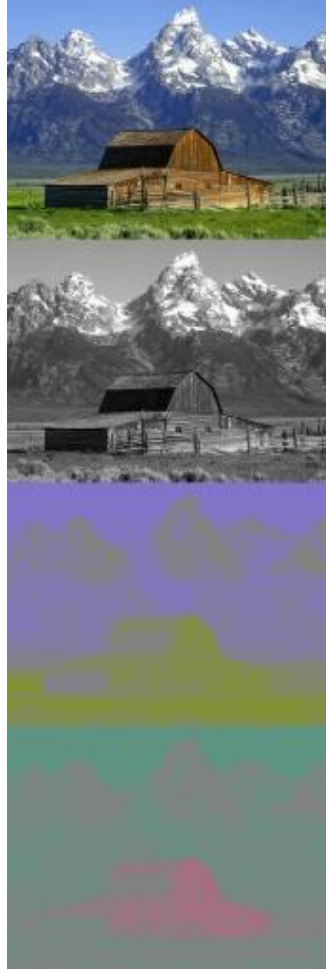


Figure 5. RGB image and YCbCr colour space components [13]

3.2.1.1 RGB to YCbCr

Conversion from RGB to YCbCr is a straightforward process by using the transformation matrix described by equation (2) for every pixel in the RGB space.

$$[Y \ Cb \ Cr] = [R \ G \ B] \begin{bmatrix} 0.299 & -0.168935 & 0.499813 \\ 0.587 & -0.331665 & -0.418531 \\ 0.114 & 0.50059 & -0.08182 \end{bmatrix} \quad (2)$$

3.3 Probability Distribution Function

Probability distribution function (PDF) is a function that describes the probability of occurrence of a specific value from all the possible values and it is used as a tool to describe the properties of an iris in the proposed algorithm described in chapter 5.

The first step of obtaining a PDF, is to generate a histogram of an image. A histogram describes the tonal distribution of an image by indicating how many pixels of a specific value there are and helps to understand the overall distribution of brightness and colours. It is described by equation (3).

$$H = [\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots, \varepsilon_x]$$

$$N = \sum_{i=1}^x \varepsilon_i \quad (3)$$

Variable x denotes here bin size, ε_n indicates the number of pixels in bin n . Hence, N is the total number of pixels in the image.

PDF is a normalized version of the histogram and instead of containing integers, it contains floating point numbers that indicate the proportion of the number of pixels in corresponding bins.

3.4 Kullback-Leibler Divergence

Kullback-Leibler divergence (KLD) is a tool to measure the similarity of two probability distribution functions (PDF) and it quantifies, how close a PDF is to a model function [14]. Vidyasagar defines it as relative entropy between two probability density functions $f(x)$ and $g(x)$ [15].

$$D_{KL}(P||Q) = \sum_i P(i) * \log \frac{P(i)}{Q(i)} \quad (4)$$

P in equation (4) is the PDF of an iris in the database and Q is the PDF of an input iris.

It is really important to notice that KLD does not expect histograms, but probability distribution functions as an input.

The value of KLD is always a non-negative integer and when two vectors P and Q are exactly the same, then the KLD value will be 0 and theoretically infinite for completely different vectors. [14]

As it can be noticed from equation (4), KLD is not symmetric, meaning that $D_{KL}(P||Q) \neq D_{KL}(Q||P)$.

3.5 Cross correlation

Cross correlation is probably one of the most well-known metric for defining the similarity between two arrays. It is defined by equation (5).

$$r(t) = \left| \frac{\sum_i [(x(i) - \mu_x) * (y(i - t) - \mu_y)]}{\sqrt{\sum_i (x(i) - \mu_x)^2} * \sqrt{\sum_i (y(i - t) - \mu_y)^2}} \right| \quad (5)$$

Cross-correlation is symmetric as the algorithm uses the absolute value of it. x denotes the input array and μ_x mean of the array x . The same applies for y as well: y is the input array, μ_y the mean of y and t the offset index, but this should be always 0, because shifting array y with respect to x means that equation (5) compares the wrong bins.

The more similar the arrays, the larger the value $r(t)$, reaching 1 for arrays that correlate perfectly and 0 for arrays that do not have absolutely any correlation between them.

3.6 Image entropy

Image entropy describes the ‘busyness’ of the image i.e. the statistical randomness of the patterns on it. Entropy is defined by equation (6).

$$Entropy = - \sum_i P_i * \log(P_i) \quad (6)$$

P_i indicates the probability of pixel value i . The value is taken from the PDF of an image.

3.7 Biometric system performance rating

3.7.1 False Acceptance Rate

False Acceptance Rate (FAR) is one of the most important attributes of a biometric recognition system. It describes the proportion of people who can gain access to the system, even though in reality they should not have been authorized. FAR is also regarded as type II error in statistics and it is also known as false positive.

FAR is really dangerous, especially for positive identification systems, where a match grants an access to a location or some special privilege. Therefore, banks or an institution that needs high-security identification would suffer tremendously even if only one false positive were to occur.

3.7.2 False Rejection Rate

False Recognition Rate (FRR) describes the proportion of people who have been denied access to a resource even though they have been granted permission, because the identification system is not able to recognize them. FRR is also regarded as type I error in statistics and occurrence of it is called false negative.

FRR is more tolerable than FAR in positive identification systems, because FRR just means that the person who should have access or special rights, is denied and should try once or twice more. However, very high FRR means that the biometric identification system is very inconvenient to use, because the user has to try to repeat the authentication process several times.

FRR, on the other hand, can be a great inconvenience in negative systems like the UAE border patrol one.

3.7.3 Equal Error Rate

Equal Error Rate (EER) is the rate at which FAR and FRR match. EER is considered to be one of the most important metric of a biometric identification system. The lower EER, the better the algorithm is considered to be. [16]

3.8 Majority Voting

Majority voting (MV) is probably the best known from politics, but it also serves the purpose of a convenient data fusion method.

Data fusion is the process of combining different information in order to make the right decisions or enhance a property of a subject under research.

Majority voting obtains decisions from different sources and looks for the most popular one.

Figure 6 indicates the simplest case of MV. Red has 3 votes and both green and blue 1 vote. Hence red is chosen as the final answer. In the case, where the best decisions get exactly the same amount of votes, some other metric like similarity rate has to be used to determine the result.

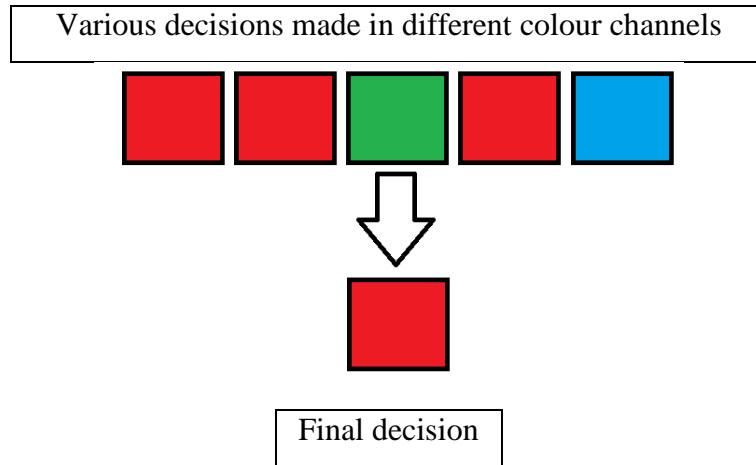


Figure 6. Illustration of majority voting

3.9 Advantages of iris recognition

Genotype is defined as the genetic information that is inheritable and phenotype is the manifestation of the organism that is both dependent on the genotype and the influences of the environment that surrounds the organism.

The patterns of the iris are the phenotypic features and the degrees-of-freedom, that indicates the statistically independent characteristics, reaches up to 266. The iris is regarded as a stable and protected internal organ that could be used for recognition in a convenient way. [17]

It offers the end-users a possibility to identify themselves contactless and the fact is also a clear advantage of the iris recognition.

3.10 Disadvantages of iris recognition

The most important drawback of iris recognition is the fact that the supporting infrastructure is rather expensive and thereby passwords or RFID-cards dominate the market at the moment. [18]

Another difficulty is iris detection itself, because the size of the iris can vary significantly and the right algorithms need to be implemented to remove as much noise as possible. For example, the size of the pupil increases significantly and as a result the area of the iris decreases, when a person has consumed alcoholic beverages. A research conducted by Arora *et al.* indicated that the identification might be problematic for around 20% of the cases [19]. Also there are known cases when some people have used eye-drops to enlarge the pupils on purpose [5].

Besides using different substances for trying to trick the system, there are also glasses that might be scratched or dirty or contact lenses that decorate the iris and as a result the recognition might be hindered. [5]

In conclusion, it is really difficult to proceed with the image acquisition without the willingness of the person being identified.

4 Current recognition algorithms

Current recognition algorithms can be theoretically divided into two groups: algorithms that use only texture information of an iris during the recognition phase and the ones that use colour information as well. The first ones are more widespread today, but there have been several research papers published about the methods that belong to the second group.

The algorithms using only the texture information use mostly near-infrared (NIR) cameras for image acquisition, because it is believed to reveal more information than a traditional camera. [20]

On the other hand, the traditional colour cameras are more widespread today and therefore a suitable algorithm using colour information would make the recognition process more robust and affordable. [21]

4.1 Daugman's algorithm

Daugman's algorithm that was published and patented during 1993 and 1994 was the first efficient algorithm for wide-spread use in the market and it has maintained its position up till now. The algorithm describes the whole process from extracting an iris from the initial image to finding the matching coefficient. [22]

The first major task of the method is to find the real iris of a person from the photo of an iris. It is regarded as the most difficult and yet most important task as the original photos include several kinds of noise like eyelids, eyelashes, sclera, reflections and other unwanted, but visible attributes caused by classes or lenses, for instance. In order to get a good result, an integro-differential operator described by equation (7) is applied to the image twice: during the first run an iris with the pupil is filtered out and the second run excludes the pupil from the final image that will be converted into a comparable data. [22]

$$\max_{(r,x_0,y_0)} = \left| G(r) * \frac{\partial}{\partial r} \oint_{r,x_0,y_0} \frac{I(x,y)}{2\pi r} ds \right| \quad (7)$$

$G_\sigma(r)$ – Gaussian or any other smoothing filter with the scale of σ ; $*$ - convolution; $I(x, y)$ – pixel intensity value at location (x, y) ; r – circle radius; x_0 and y_0 – center coordinates of the circle. [22]

Equation (7) acts as a circular edge detector where the centre of the circle is at point $I(x, y)$ and the radius is r . It looks for maximum contour integral derivative while increasing the radius iteratively. While looking for the outer boundaries of an iris, the contour integration is limited by two cones opposite to each other in the upper and lower parts of the eye, because the probability of having eyelids in those parts is very high. The second iteration includes only the upper 270° part, because there tends to be reflections on the lower 90° of the iris. As said previously, any kind of reflection is harmful to the recognition algorithm and might result in false rejection or false matches. [22]

The complexity of the algorithm is $O(M*N*R)$ where M is the height, N is the width of the image and R is the maximum allowed radius, because it has to go over all the pixels on the image to try to fit R different circles with different radiuses to a certain pixel.

The limbus of an eye (the connection point of the iris and the sclera) produces a positive figure after applying equation (7) on it. Hence, the operator can also be considered as an efficient way to detect whether an iris exists on the image or not. If the output of equation (7) is not large enough, then it can be assumed that iris does not exist in the image, it is too blurry or there is too much noise and it is impossible to detect it. [22]

In real-life applications Daugman suggests to take several images in a row during the identification of an iris and proceed with the process only and only if there have been several confirmations that an iris exists on the image. In addition, to preventing deceiving the system by showing a high resolution image to the camera or printing a pattern to a lens, it is recommended to keep track of increase and decrease of the diameter of the pupil. The phenomenon is called *hippus* or pupillary unrest. It gives the possibility to calculate *hippus* measure that is defined as “as the coefficient of variation (standard deviation divided by mean) for the fluctuating time series of these diameter ratios“. [22]

The traditional Cartesian coordinate space is converted into polar coordinate system that serves the purpose of maintaining reference to the same regions of iris regardless of the angle under

which the picture was taken, the zoom or the size of the pupil, etc. It assigns every point of the iris a new pair of dimensionless coordinates (r, θ) . The new coordinate system is not guaranteed to be concentric, because the pupil might not be exactly in the centre of the iris. [22]

The algorithm excludes the upper part and the lower 45° slice of iris from the analysis as the probability of having eyelids, –lashes or reflections in these regions is high and therefore it would influence the recognition rate in a negative way. [22]

After successfully filtering the iris and converting it from one coordinate system to the other, it is vital to generate appropriate function(s) in order to compare the irises in an efficient and reliable way. Daugman’s paper has opted for Gabor wavelet to encode the texture of the irises into 256-byte IrisCode. [22]

Gabor function, which is mother wavelet in Gabor wavelets, was proposed by Hungarian electrical engineer Dennis Gabor in 1946. It is used mainly in feature analysis of an image and therefore it is an ideal tool to encode the iris into a data that is efficiently comparable [23].

Daugman himself claimed that 2D Gabor filters were an ideal tool to represent the textures content and location the best way that is possible. [22]

Gabor filter in polar coordinate system takes the form described by equation (8).

$$G(r, \theta) = e^{-i\omega(\theta-\theta_0)} e^{\frac{-(r-r_0)^2}{\alpha^2}} e^{\frac{-(\theta-\theta_0)^2}{\beta^2}} \quad (8)$$

Parameters α, β, ω are the input parameters for the bandpass filter.

The real parts are truncated to eliminate the DC component and only the imaginary part is used to combine the IrisCode. [22]

In order to compare the irises, Daugman proposed to use Hamming distance that compares two vectors bit-by-bit defined by equation (9). [22]

$$H = \frac{1}{N} \sum_{i=1}^N A_i(XOR)B_i \quad (9)$$

A and B are the input arrays and XOR is 1 only if the bits of A and B are the same. Hamming distance is indicating how different the arrays are in %, when to multiply equation (9) by 100. N is the length of arrays A and B. [22]

The database iris most similar to the input iris is regarded as the matching one. [22]

Even though Daugman's algorithm is the most popular in the field, it is computationally complex.

4.2 Principal Component Analysis

Principal Component Analysis (PCA) is another method that allows efficiently decomposing the texture of an iris into a comparable vector that exposes the underlying patterns. It is also known as Karhunen-Loève transformation. The main principle of PCA is to assume that it is possible to use linear interpolation to characterize the information and get rid of redundant data. [14,24]

To understand the PCA in order to derive an iris recognition algorithm out of it, it is important to familiarize with the terms eigenvalue and eigenvector. Eigenvector is defined as “a nonzero vector that is mapped by a given linear transformation of a vector space onto a vector that is the product of a scalar multiplied by the original vector” [25] and eigenvalue as “a scalar associated with a given linear transformation of a vector space and having the property that there is some nonzero vector which when multiplied by the scalar is equal to the vector obtained by letting the transformation operate on the vector” [26].

PCA encoding is done in two different modes: the training mode and the classification mode. During the training mode the training irises (irises in the database) are used to set up the eigenspace by using PCA. The classification phase includes mapping the test irises (the input irises) to the created eigenspace and finding appropriate classifiers to them. [24]

The first step in the training phase is to find the average of all the irises in the training set. This means that when the training set includes irises $A_1, A_2, A_3, \dots, A_M$, then the average is defined by equation (10). [24]

$$\Psi = \frac{1}{M} \sum_{n=1}^M \Theta_n \quad (10)$$

After obtaining the mean training iris, it is possible to calculate the difference for every iris in the training set. The process is described by equation (11). [24]

Equation (12) is used for the next step: calculating the covariance matrix.

$$\Phi_i = \Theta_i - \Psi \quad (11)$$

$$C = \frac{1}{M} \sum_{n=1}^M \Phi_n \Phi_n^T = A A^T \quad (12)$$

However, there is a major problem that needs a solution: the dimensions of the covariance matrix are $N^2 \times N^2$ (for 500x500 samples it is 250 000 x 250 000 matrix), where N is the width and height of the image. Calculating the eigenvectors and eigenvalues for the matrix for that large is computationally a very complex task that needs simplifying. [24]

$$A^T A v_i = \mu v_i \quad (13)$$

$$A A^T A v_i = \mu A v_i \quad (14)$$

Considering equation (13), it is possible to notice that v_i is the eigenvector of $A^T A$. Multiplying equation (13) with A results in equation (14). Therefore the new eigenvector for $C = A^T A$ is $A v_i$ and the eigenvalue is μ . [24]

Proceeding with the process, after equations (13) and (14) it is needed to construct a new matrix with size M x M, where M is the number of irises in the training set in order to make further calculations less time-consuming, because usually the number of pixels N is notably larger than M and this means that $M \ll N^2$. Hence, the next step is to construct a MxM matrix L which is $A^T A$, and $L_{mn} = \Phi_m^T \Phi_n$ and identify M eigenvectors v_i of L that help to determine the linear combinations for transforming the ordinary training irises into the eigenirises U. [24]

$$U_I = \sum_{k=1}^M v_{Ik} * \Phi_k, I=1, \dots, M \quad (15)$$

$$w_k = U_k^T (\Gamma - \Psi) \quad (16)$$

$$\Omega^T = [w_1 w_2 w_3 \dots w_M] \quad (17)$$

The input irises (also defined as training irises Γ) are transformed into comparable eigenirises by using equations (16) and (17). Equation (16) calculates the weights that can be used to form projection vectors as described in equation (17). [24]

The last step is to find the match for all the test irises from the set of training irises. There are several more or less appropriate methods that are suitable for finding the similarity rate for the

irises. The PCA algorithm described is taking advantage of the traditional Euclidean distance. The smaller the distance between a test and a training iris, the more similar they are. Hence, the target is to find the minimum distance between the projection of the input and database iris. The operation is described by equation (18), where Ω_k is the vector describing iris k. [24]

$$\varepsilon_k = ||\Omega - \Omega_k|| \quad (18)$$

4.3 Iris Recognition System Using Combined Histogram Statistics

Anbarjafari et al. introduced an iris recognition algorithm that uses colour information in addition to the texture of the iris during the recognition phase. [27]

Their idea was to convert all the irises from RGB to HSI colour space, separate the channels and generate histograms out of them and use them as a data to compare the irises.

To find a match for an input iris from the database, the method used maximum cross-correlation that is defined by equation (5).

The array that has the highest cross-correlation was chosen as the match for a specific channel. For data fusion, the proposed method used majority voting, meaning that all the decision made by H, S and I are observed and the class that gets the most votes out of the three is chosen as the matching iris.

The recognition rate of the of the method using UPOL database [28] is 98% using 1/3 of the irises as the database of the system (number of training) and the rest 2/3 as an input to the system and 100% using 2/3 of the irises as the database and 1/3 as an input with EER of 0%. The results of the proposed method are described in Table 1.

Table 1. Performance of recognition system using combined histogram statistics

# of training	PCA	Recognition rate using Majority Voting
1	60 %	98.91 %
2	70 %	100 %

5 PDF Based Iris Recognition

The main purpose of the thesis is to introduce a new iris recognition method that takes advantage of both the texture and the colour information on the iris like the algorithm proposed by Anbarjafari *et al.*

The problem with the Daugman's algorithm is that it is patented and hence, the users have to pay for using the method. In addition, it is computationally complex. From the latter perspective, the algorithm introduced by Anbarjafari *et al.* is significantly better. However, the algorithm proposed in the current thesis is more robust and more likely to perform better on larger databases.

The method consists of the following steps:

1. colour space conversion;
2. probability distribution generation;
3. iris comparison using the mean rule;
4. iris detection;

In order to test the performance of the algorithm, the UPOL iris database [28] was chosen. The database provides 3 samples of 64 persons using both left and right iris. The irises are RGB images with the size of 768 x 576 and bit depth 24, meaning that every channel has 8 bits and the values can vary from 0 to 255. An example can be seen on Figure 7.

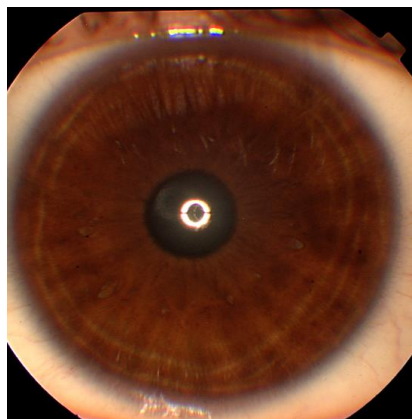


Figure 7. Cropped original input iris [28]

5.1 Iris detection and iris segmentation

One of the most important steps of iris recognition process is to identify and filter the iris as well as possible. The failure to do so will lead to incorrect input data and therefore it will definitely lower the recognition rate. Nevertheless, an efficient iris recognition algorithm that is able to detect the correct shape of the iris, pupil and remove any excessive noise that might hinder to the process of recognition, enhances the performance of the algorithm significantly.

There are several ways to detect the iris: Daugman's operator mentioned in chapter 4.1, Hough transform that is implemented in popular image processing libraries like OpenCV.

Regarding the algorithm proposed in the thesis, it is not important to deal with the detection process itself and therefore we have developed a binary mask that helps us to filter the irises in a convenient way and to be sure that we have enough irises with minimum noise as an input for the comparison process.

It has to be noted, however, that the mask is suitable only for the UPOL database and implementing the algorithm using some other set of irises for testing, a new binary mask has to be developed or some other circle detector has to be applied.

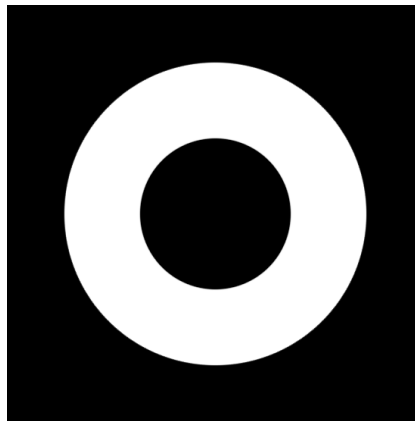


Figure 8. Binary mask

As seen from Figure 8, the mask is an image consisting of pixel values 0 (black) and 255 (white) in RGB colour space with 24bit depth. The size of the mask is 510 x 510.

In order to obtain the filtered iris, the first step is to crop the original iris by 129 pixels from both left and right side of an iris image in order to obtain an iris that is 510 pixels. The same operation

has to be applied to the top and bottom part, but the cutting range is 33 pixels. The procedure is described by equations (19) and (20).

$$widthToRemove = \frac{768 - 510}{2} \quad (19)$$

$$heightToRemove = \frac{576 - 510}{2} \quad (20)$$

The next task is to apply Boolean AND to the cropped iris and the input filter that results in a filtered output, where only the iris is visible and the rest of the image is just an area full of black pixels.

Figure 8 displays two output irises. The left one can be considered as an ideal, but the right one is not perfect, because it has a small part of pupil visible. However, the occurrence of it is not fatal for the algorithm and it still able to perform quite well.

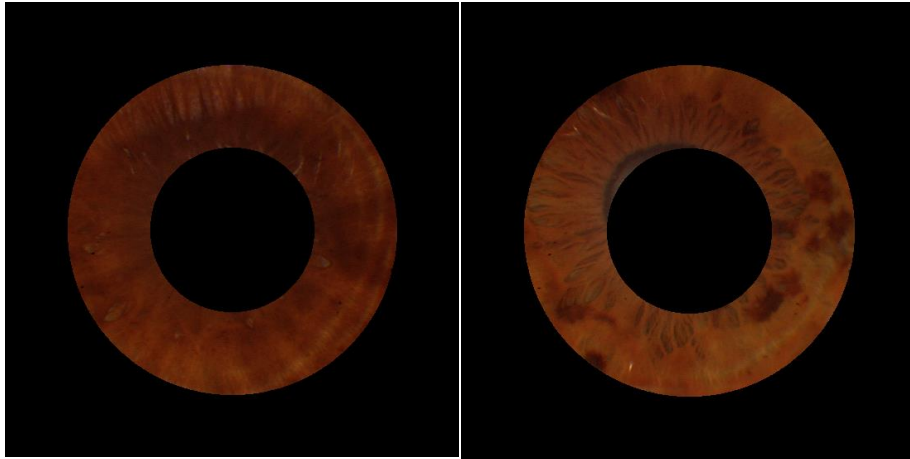


Figure 9. An example of the output irises (a) left (b) right

5.2 Colour space conversion

The filtered irises are in RGB colour space, but this reduces the recognition rates significantly, because of the possible change in illumination of the sample irises. The problem is described in more depth in chapter 3.2.1.

Therefore, it is needed to convert the RGB images into HSV and YCbCr colour spaces to boost the recognition rates. This can be done using equations (1) and (2), respectively. However two of the most popular image processing frameworks/languages OpenCV and Matlab have functions

that will do the task (for example, Matlab has methods `rgb2hsv` and `rgb2ycbcr`, OpenCV `cv::cvtColor(input_rgb_image, output_hsv_image, CV_BGR2HSV)`).

After processing the irises, only the filtered irises in HSI and YCbCr are needed to proceed with the recognition process.

5.3 Probability distribution function generation

The next step is to generate reliable and distinctive data out of both texture and colour information of the irises that could be efficiently used for iris recognition. Probability distribution function (PDF) is used in the current thesis just like Daugman used 256-byte IrisCode in his proposed algorithm.

The attributes of probability distribution function are described in chapter 3.3.

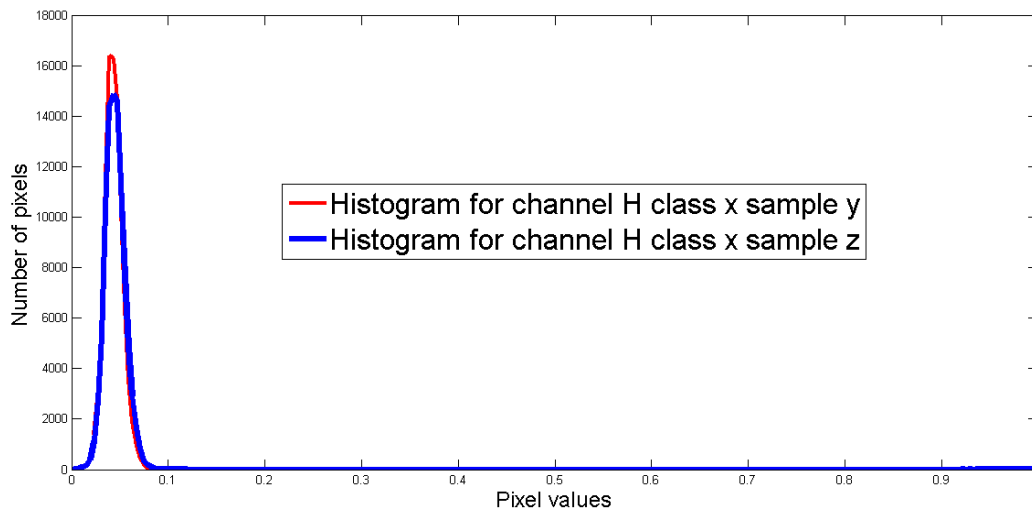


Figure 10. Histograms for channel H of two irises of the same class

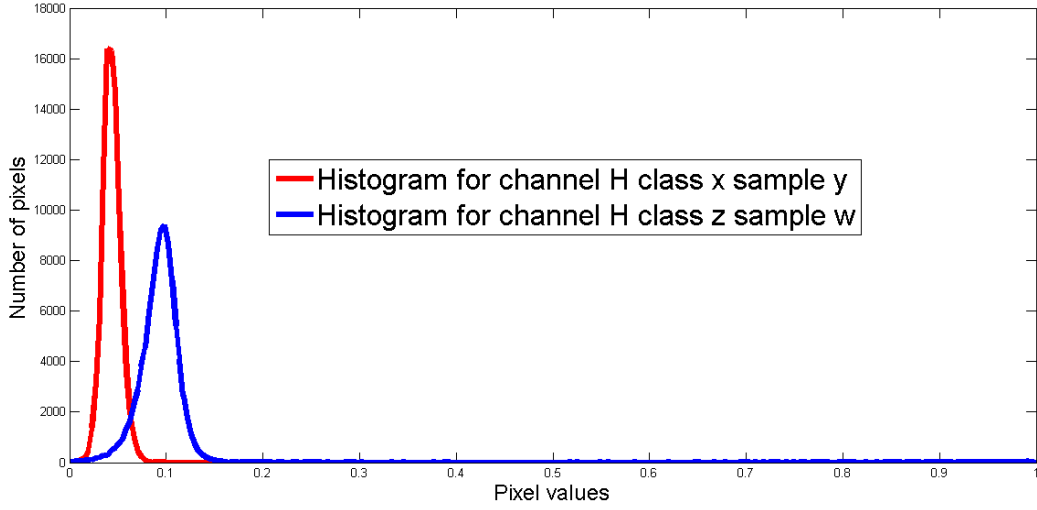


Figure 11. Histograms for channel H of two irises of different classes

The algorithm requires having the PDFs of H, S, Y, Cb and Cr. As explained earlier, RGB is not needed, just like channel I. The main reason for omitting the latter is the fact that I contains theoretically the same information as Y and hence using both of them for would enhance the “voting power” of illumination. As illumination is the factor that varies usually the most between different samples, it would reduce the recognition rate.

When deciding, whether to choose Y or I for the recognition, we decided to use Y, as and describe, it can be seen that Y performs a bit better than I.

Figures 10 and 11 indicate the possibility to use PDFs for iris recognition. The first graph is of the irises from the same class and the second one displays the histograms of irises from different classes. The figures show clearly that irises with the same texture and colour have histograms that are almost identical and irises with clearly different patterns and/or colour have completely distinct histograms. As a PDF is just a normalized histogram, the same stand for these as well.

Considering that the algorithm uses bin size of 256 for PDFs and every bin is a 32-bit float, then one PDF takes up about 8192 bits of space and 10 channels (5 for left and 5 for right) takes up about 81920 bits that is exactly 10 KiB.

5.4 Iris comparison

The comparison of the test (input) and training (database) iris is the key phase in the algorithm.

Comparing test class A and training class B requires to have PDFs for channels H, S, Y, Cb and Cr for both the left and right iris. The most important task is to calculate KLD values for the corresponding channels and combine the data obtained in order to minimize the error and maximize the correct decisions. The latter process is also known as data fusion.

There are several possible ways to combine the calculated KLD values.

- **Multiplication.** This means that only one false positive with KLD value is close to 0 reduces the total KLD value significantly. Therefore, it is an inappropriate way to combine the data in the current case.
- **Minimum.** This means that one false positive might define the matching iris and therefore is unsuitable for the algorithm.
- **Mean.** Mean sums all the KLD values and divides the sum by total number of channels (10 in the current case). This seems appropriate, because errors in one or two classes do not mean that the whole recognition process has failed.
- **Weighted mean.** Weighted mean multiplies all the KLD values with empirical coefficients and divides the sum by the sum of the coefficients. The method could enhance the performance of the algorithm even more by assigning appropriate multipliers for the channels that include more information (entropy is higher, for example) and have better recognition rate. However, calculating the exact coefficients is a task that requires a lot of testing.
- **Majority voting (MV).** Majority voting is a perfect data fusion method when the recognition rates by channels are very high. When the recognition rates are not that good, then the probability of getting the final decision wrong is quite high and hence, before deciding to use MV, there needs to be a clear understanding of the recognition rate of each channel.

The first two methods are not applicable for the algorithm using KLD, the fourth one needs more in-depth analysis, because incorrect coefficients reduce the recognition rate instead of enhancing it.

Therefore, the most convenient methods for data fusion are probably the mean rule that adds up all the KLD values of H, S, Y, Cb and Cr for the left and right irises and divides the sum by 10 and majority voting (MV) that takes the final decisions made by the 10 channels used.

5.5 Iris detection

The real process of iris recognition for a test iris includes finding the matching rate for all the irises in the training set and obtaining minimum of the calculated rates. The training iris corresponding to the minimum rate is considered as the matching one.

5.6 Experimental results

Table 2. Recognition rates of left irises by channels (%)

# of training	H	S	V	Y	Cb	Cr
1	64.06	75.00	46.88	46.88	62.50	60.16
2	78.13	87.50	65.63	65.63	78.13	75.00

Table 3. Recognition rates of right irises by channels (%)

# of training	H	S	V	Y	Cb	Cr
1	78.13	78.13	57.81	60.94	65.63	69.53
2	84.38	85.94	64.06	73.44	78.13	81.25

Table 4. Recognition rates using the mean rule by left and right iris using the mean rule (%)

# of training	Left	Right
1	88.28	93.75
2	92.19	98.44

Table 5. Recognition rates using the mean rule and both left and right iris (%)

# of training	Principal Component Analysis	Recognition rate for the mean rule	Recognition rate for majority voting
1	60	94.53	93.75
2	70	100.00	96.88

As explained previously, the UPOL database has 3 samples of both left and right irises from 64 classes (persons). In order to find the performance of the algorithm, the UPOL database is divided into 2 large sets: the training set and the test set. The classes in the test set need a match from the training set. Therefore, we need to iterate over the test set and find a match for every class from the training set.

Tables 2 - 5 have a column “# of training” that describes whether 1 or 2 sets out of the total 3 are in the training and the rest in the test. The situation, where 1/3 of the irises are training irises, should have lower recognition rate in the vast majority of the cases, because there are 128 classes that need to find a match from 64 classes. Having 2 in the training set means that there are 64 classes needing a matching class from 128 classes.

Tables 2 and 3 indicate that the recognition rate by channels for both left and right iris is quite low, ranging from 46.88% to 87%. A biometric identification system that makes 13 errors from 100 detections is definitely a very bad one.

Table 4 combines channels H, S, Y, Cb and Cr by using the mean rule defined in chapter 1.1, but keeping the left and right irises separated. It is clear that the recognition rate has improved significantly and the maximum is 98.44%. Using 5 channels for recognition compensates the errors made by one or more channels.

Table 5 shows the final results of the algorithm and the results are significantly better using the channels of both left and right iris by using both the mean rule and majority voting. The results indicate that the mean rule used produces slightly better results than majority voting. Thereby, the latter is dropped and the mean rule is used for data fusion.

There are 10 channels in total and the probability of compensating some incorrect KLD values by other channels is really high. There were not any false matches in the UPOL training database.

5.7 Error Estimation

A really important attribute of a biometric system is its equal error rate (EER). Even though KLD was used for obtaining the similarity rating between irises, it is not possible to define a stable threshold for all the databases as KLD has no upper limit and therefore a new limit has to be defined for every database. Hence, it would be wise to define the EER in the context of this thesis using cross-correlation as the metric, because it has both upper and lower boundaries (-1 and 1 or 0 and 1 when using the absolute value).

Figure 12 displays the FAR and FRR curves and EER. As it can be seen, the FAR is 4.7% for the cross-correlation value of 0.966.

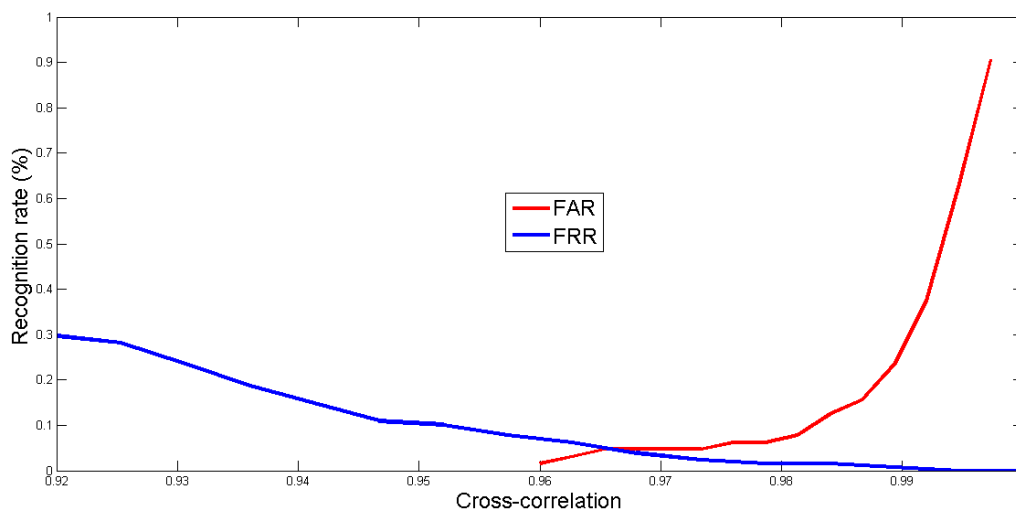


Figure 12. FAR and FRR curves

5.8 Proposed improvements

5.8.1 Weighted average

In order to enhance the recognition rate even more, an empirical weighted average could be used to calculate the mean rule. The idea is to develop the coefficients by using channels performance information in H and S or the entropy of the channels described in Table 6.

Table 6. Average entropy

H	S	I	Y	Cb	Cr
2.60	4.17	4.28	3.95	2.93	2.71

The procedure includes multiplying the KLD value with the corresponding coefficient and then dividing the KLD sum with the total sum of the coefficients.

When using KLD as a metric to calculate the similarity, one must not forget that the more similar two inputs are, the closer the value is to 0. Hence, multiplying the KLD value with the recognition rate or entropy of the corresponding channel would not work as emphasizing the channels that perform the best, but the opposite. The better the channel, the smaller the coefficient has to be and it would be wise to use the reciprocal of both the recognition rate and average entropy.

Using reciprocals of the recognition rates of the channels as the coefficients altered the recognition rate only 0.78% for 1 training as can be seen from Table 7.

Table 7. Recognition rates with weighted average using performances of the channels (%)

# of training	Recognition rate
1	95.31
2	100.00

The second option to try out is to use reciprocals of the average entropy as coefficients. The results are exactly the same as in i.e. average entropy did not affect the results.

Despite trying to find a coefficient for every channel to enhance the performance of the algorithm, the results were left basically unaffected. However, there is definitely a reason to proceed with looking for appropriate coefficients, because even though the algorithm achieved the recognition rate of 100%, EER could be lower than 4.7%.

6 Conclusion

In this thesis the basic concepts of iris recognition and a new algorithm using probability distribution functions were introduced.

The first section familiarized the reader with the most important facts needed to be able to grasp the idea behind different iris recognition algorithms.

The second part introduced some more or less popular algorithms. Daugman's proposed method is the most widely used one nowadays and most of the real-life applications take advantage of it. However, it is computationally rather complex. There is also the conventional principal component analysis (PCA) that creates eigenirises out of the initial database and a method proposed by Anbarjafari *et al.* that uses HSI colour space and majority voting to make the decision.

The third part of the thesis proposed a novel iris recognition algorithm based on the mean rule. The algorithm converts iris images from traditional RGB colour space to HSI and YCbCr and creates probability distribution functions (PDF) from channels H, S, Y, Cb and Cr for both left and right iris. Kullback-Leibler divergence is used as the metric to calculate the difference between the corresponding channels. The recognition process includes calculating KLD values for all the channels for left and right irises (i.e. there are 10 channels) and then using the mean rule to get an average of them. This means that probability of compensating errors made by some channels is quite high.

In order to test the algorithm, UPOL database was used. It includes three samples for both left and right iris for 64 people. The results are described in .

Even though the algorithm achieved 100% recognition rate for both left and right iris, there are theoretically several ways to enhance the performance even more like using weighted average while calculating the KLD value.

7 Acknowledgements

I would like to thank my supervisor Gholamreza Anbarjafari who gave me the ideas to implement and helped me out when I had some problems while working with the algorithm.

Also I would like to thank my family and friends for supporting me.

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8 Kokkuvõte

Kert Pjatkini bakalaureusetöö „Tõenäosus tihedusfunktsioonil ja keskmistamisel baseeruv iirisetuvastus“ tutvustab lugejale iirise põhise biomeetrilise identifitseerimise alustalasid ja uut algoritmi, mis kasutab HSI ja YcbCr värviruumide tõenäosus tihedusfunktsioone tuvastusfaasis.

Töö esimene osa selgitas lugejale põhilisi termineid, mida läheb vaja, mõistmaks erinevaid antud valdkonnas kasutusel olevaid meetodeid.

Töös käsitleti ka kolme algoritmi. Daugmani algoritm on tänapäeval üks kõige populaarsemaid algoritme selles vallas. Antud meetodi puuduseks suur arvutuskeerukus. Anjabafari *et al.* tutvustasid meetodit, mis kasutab HSI värviruumi ning genereerib kõikidest kanalitest histogrammid. Otsuse tegemiseks kasutatakse enamushääletamist. Kolmas tutvustatud meetod oli standardne peakomponentanalüüs.

Bakalaureusetöö peamine osa seisnes uue algoritmi tutvustamises, mis kasutab infoallikana nii HSI kui ka YCbCr värviruumi ning genereerib H, S, Y, Cb ja Cr kanalite tõenäosus tihedusfunktsioonid. Sarnasuse arvutamiseks arvutatakse Kullback-Lebleri divergentsi (KLD) näitajad nii vasaku kui ka parema silma jaoks ning saadud tulemused keskmistatakse. See tähendab, et kui üks kanal võis KLD arvutamisel olla ebakorrektnen, siis teised kanalid vähendasid antud vea mõju.

Väljapakutud meetodi testimiseks kasutati UPOLi andmebaasi, kus on 64 inimese nii paremast kui ka vasakust silmast kolmel eri ajahetkedel tehtud ülesvõtet. Antud allikas annab esialgse võimaluse hinnata algoritmi töökindlust.

Antud meetod suutis eksimatult tuvastada iiriseid, kui 2 ülesvõtet 3 oli treeninghulgas ehk andmebaasis ning ülejäänud kolmandikku käsitleti kui sisendit. Vastupidises olukorras oli tuvastusprotsent 95,31.

Viimase sammuna pakuti välja ka võimalusi algoritmi töökindluse suurendamiseks. Üks võimalus oli kasutada keskmise arvutamisel empiirilisi kaalusid. Paraku ei andnud väljapakutud meetod märkimisväärselt paremaid tulemusi.

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10 Appendices

10.1 Code

The code created for the thesis is provided with an extra DVD.

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