

**SEMIPARAMETRIC MODELS FOR  
NONLINEAR SPATIO-TEMPORAL  
DATA WITH APPLICATION TO THE  
UNITED STATES HOUSING PRICES  
INDEXES**

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# ABSTRACT

Modelling spatio-temporal data has received significant attention, recently and is widely applied in many disciplines such as economics, environmental and social sciences. In economics, housing price is a real example that indicates the importance of modeling such data. Estimating the movement in housing prices is an important but challenging problem due to the difficulties associated with spatio-temporal interactions. One main challenge is that there is no natural spatial ordering and thus it is not as straightforward as in time series analysis to transform data to be stationary across space. In addition, it is a challenge to model spatio-temporal data collected at irregularly spaced sampling locations due to the potentially large number of parameters. Moreover, the complexity of the dependence structure requires new effective statistical methods for modeling and analysis.

While nonparametric and semiparametric methods have been popular for nonlinear modeling of time series data in econometrics and statistics, they become increasingly challenging when extended to irregularly located spatio-temporal data with complex nonlinear structures. The literature on nonlinear spatio-temporal modeling is still rather rare except a few contributions recently done, for example, by (Lu et al., 2009), (Wikle and Hooten, 2010) and (Wikle and Holan, 2011). Therefore, the main aim of this thesis is to propose a class of semiparametric spatio-temporal autoregressive partially nonlinear regression models as a practical way to overcome these challenges. The main contributions of this thesis are summarised as follows:

(1) In Chapter 2, a class of semiparametric spatio-temporal autoregressive partially nonlinear regression models is proposed. The proposed models not only permit location-varying nonlinear relationships between the response variable and the covariates but also allow for the dependence structure to be nonstationary over space while alleviating the "curse of dimensionality" by using the popular idea of spatial weight matrix measuring the spatial interaction, which is assumed to be well defined as in spatial econometrics. Both the estimation and its finite- and large-sample properties for the proposed models are established.

(2) In order to more objectively let data decide the spatial interaction in the models, in Chapter 3, we propose a scheme of general data-driven models to estimate the spatial weights in the semiparametric spatio-temporal autoregressive partially nonlinear regression models by applying the adaptive lasso. Both estimation and its finite- and large-sample properties for this class of the general data-driven models are developed.

(3) An improved scheme for such data-driven models with spatial weight matrix is presented in Chapter 4. For this class of the improved data-driven models, we develop a computationally feasible method for estimation and thus enable our methodology to be applicable in practice. Asymptotic properties of our proposed estimates are established and comparisons are made, in theory and via simulations, between estimates before and after spatial smoothing.

(4) In empirical case studies, the proposed methodologies are applied to investigate the housing prices in relation to the mortgage rates in Chapter 2 and to the consumer price index in Chapter 3 and 4 for the 50 states and District of Columbia (DC) in the United States (U.S.). It is empirically found that such relationships could be of nonlinear features that help to improve the predictions and the third class of the improved data-driven models appears to work promisingly better.

# CONTENTS

|   |            |
|---|------------|
| <b>SIGNED STATEMENT</b>   | <b>i</b>   |
| <b>DECLARATION</b>  | <b>ii</b>  |
| <b>ACKNOWLEDGMENTS</b>  | <b>iii</b> |
| <b>ABSTRACT</b>   | <b>iv</b>  |
| <b>1 SPATIO-TEMPORAL MODELLING: A REVIEW</b>  | <b>2</b>   |
| 1.1 Background . . . . .  | 2          |
| 1.2 Literature review . . . . .   | 8          |
| 1.2.1 Spatio-temporal models . . . . .  | 8          |
| 1.2.2 Housing prices models . . . . .   | 18         |
| 1.3 Contributions . . . . .   | 19         |
| <b>2 SEMIPARAMETRIC MODELLING OF IRREGULARLY SITED<br/>    SPATIO-TEMPORAL DATA</b> | <b>23</b>  |
| 2.1 Introduction . . . . .  | 23         |
| 2.2 Model specification . . . . .   | 24         |
| 2.3 Spatial weight matrix . . . . .   | 27         |
| 2.3.1 Binary-based weight matrix . . . . .  | 27         |
| 2.3.2 Distance-based weight matrix . . . . .  | 30         |
| 2.3.3 General weight matrix . . . . .   | 31         |

|       |   |    |
|-------|---|----|
| 2.4   | Estimation strategy . . . . .   | 32 |
| 2.4.1 | Time-series based estimation . . . . .                                  | 32 |
| 2.4.2 | Spatial smoothing estimation . . . . .                                  | 35 |
| 2.5   | Large-sample properties . . . . .                                       | 36 |
| 2.5.1 | Regularity conditions . . . . .   | 36 |
| 2.5.2 | Time-series estimators . . . . .  | 40 |
| 2.5.3 | Spatial smoothing estimators . . . . .                                  | 40 |
| 2.6   | Application: analysis of housing price indexes in the United States . . | 43 |
| 2.6.1 | Data and exploratory analysis . . . . .                                 | 44 |
| 2.6.2 | Model and data analysis . . . . .                                       | 49 |
| 2.6.3 | Prediction and interpretation . . . . .                                 | 56 |
| 2.7   | Simulation study . . . . .  | 61 |
| 2.8   | Conclusion . . . . .  | 65 |
| 2.9   | Appendix . . . . .  | 66 |
| 2.9.1 | Proof of Theorem (1) . . . . .  | 67 |
| 2.9.2 | Proof of Theorem(2) . . . . .   | 69 |
| 2.9.3 | Proof of Theorem(3) . . . . .   | 72 |
| 2.9.4 | Proof of Theorem (4) . . . . .  | 74 |
| 2.9.5 | The hat matrix $\mathbf{H}$ . . . . .                                   | 79 |

### **3 PENALIZATION BASED SEMIPARAMETRIC MODELLING OF SPATIO-TEMPORAL DATA 81**

|       |  |    |
|-------|--|----|
| 3.1   | Introduction into ideas of penalization based estimation and selection | 82 |
| 3.1.1 | Penalization methods . . . . .   | 83 |
| 3.2   | Application of lasso . . . . .   | 89 |
| 3.2.1 | Spatial weight matrix . . . . .  | 89 |
| 3.3   | Model specification and estimation strategy . . . . .                  | 91 |
| 3.3.1 | Time-series based estimation . . . . .                                 | 93 |
| 3.3.2 | Spatial smoothing estimation . . . . .                                 | 97 |



|       |   |     |
|-------|---|-----|
| 3.4   | Large-sample properties . . . . .                                       | 97  |
| 3.4.1 | Notation . . . . .  | 97  |
| 3.4.2 | Regularity conditions . . . . .   | 98  |
| 3.4.3 | Time-series estimators . . . . .  | 101 |
| 3.4.4 | Spatial smoothing estimators . . . . .                                  | 103 |
| 3.5   | Application: analysis of housing price indexes in the United States . . | 104 |
| 3.5.1 | Data and exploratory analysis . . . . .                                 | 104 |
| 3.5.2 | Model analysis . . . . .  | 110 |
| 3.5.3 | Prediction and interpretation . . . . .                                 | 115 |
| 3.6   | Simulation study . . . . .  | 117 |
| 3.7   | Conclusion . . . . .  | 118 |
| 3.8   | Appendix . . . . .  | 124 |
| 3.8.1 | Notation . . . . .  | 124 |
| 3.8.2 | Proof of Theorem 5 . . . . .  | 125 |
| 3.8.3 | Proof of Theorem 6 . . . . .  | 127 |
| 3.8.4 | Proof of Theorem 7 . . . . .  | 128 |
| 3.8.5 | Proof of Theorem 8 . . . . .  | 129 |
| 3.8.6 | Proof of Theorem 9 . . . . .  | 129 |
| 3.8.7 | Proof of Theorem 10 . . . . .   | 133 |
| 3.8.8 | The estimated values of the coefficients $\lambda_i^k(s_j)$ . . . . .   | 134 |

#### 4 IMPROVED PENALIZED SEMIPARAMETRIC MODELLING OF SPATIO-TEMPORAL DATA 148

|       |   |     |
|-------|---|-----|
| 4.1   | Introduction . . . . .                                | 148 |
| 4.2   | Model specification and estimation strategy . . . . . | 149 |
| 4.2.1 | Time-series based estimation . . . . .                | 151 |
| 4.2.2 | Spatial smoothing estimation . . . . .                | 153 |
| 4.3   | Large-sample properties . . . . .                     | 154 |
| 4.3.1 | Notation . . . . .                                    | 154 |

|          |   |            |
|----------|---|------------|
| 4.3.2    | Regularity conditions . . . . .   | 154        |
| 4.3.3    | Time-series estimators . . . . .  | 157        |
| 4.3.4    | Spatial smoothing estimators . . . . .                                  | 158        |
| 4.4      | Application: analysis of housing price indexes in the United States . . | 159        |
| 4.4.1    | Model analysis . . . . .  | 160        |
| 4.4.2    | Prediction and interpretation . . . . .                                 | 164        |
| 4.5      | Simulation study . . . . .  | 169        |
| 4.6      | Conclusion . . . . .  | 170        |
| 4.7      | Appendix . . . . .  | 177        |
| 4.7.1    | Notation . . . . .  | 177        |
| 4.7.2    | Proof of Theorem 11 . . . . .   | 177        |
| 4.7.3    | Proof of Theorem 12 . . . . .   | 181        |
| 4.7.4    | Proof of Theorem 13 . . . . .   | 182        |
| 4.7.5    | Proof of Theorem 14 . . . . .   | 182        |
| 4.7.6    | Proof of Theorem 15 . . . . .   | 183        |
| 4.7.7    | Proof of Theorem 16 . . . . .   | 183        |
| 4.7.8    | Proof of Theorem 17 . . . . .   | 184        |
| <b>5</b> | <b>CONCLUSION AND OUTLOOK</b>   | <b>185</b> |
| <b>6</b> | <b>APPENDIX</b>   | <b>188</b> |
|          | <b>BIBLIOGRAPHY</b>   | <b>242</b> |

# LIST OF TABLES

|  |     |
|--|-----|
| 2.6.1 MSPE for the first proposed spatio-temporal model . . . . .                            | 59  |
| 3.5.1 MSPE for the second proposed spatio-temporal model . . . . .                           | 117 |
| 3.8.1 Abbreviation of all American states . . . . .  | 134 |
| 4.4.1 MSPE for the third proposed spatio-temporal model . . . . .                            | 168 |
| 4.4.2 Comparison between the 3 proposed spatio-temporal models in terms<br>of MSPE . . . . . | 169 |

# LIST OF FIGURES

|  |    |
|--|----|
| 2.3.1 Linear contiguity. . . . .   | 27 |
| 2.3.2 Rook contiguity. . . . .   | 28 |
| 2.3.3 Bishop contiguity. . . . .   | 28 |
| 2.3.4 Queen contiguity. . . . .  | 29 |
| 2.3.5 Example of four locations allocated irregularly in grid. . . . .   | 29 |
| 2.3.6 Power distance weight matrix with $\alpha = 1$ . . . . .   | 31 |
| 2.3.7 Exponential distance weight matrix with $\alpha = 1$ . . . . .   | 31 |
| 2.6.1 Plot the quarterly HPI data for the 50 states and DC. . . . .  | 45 |
| 2.6.2 Plot the quarterly HPI and HPI geometric return for DC. . . . .  | 46 |
| 2.6.3 Spatial map plots of the HPI geometric return for the 50 states and<br>DC for the first 6 quarters . . . . .                                 | 47 |
| 2.6.4 Spatial map plots of the HPI geometric return for the 50 states and<br>DC for the second 6 quarters . . . . .                                | 48 |
| 2.6.5 Boxplots of the HPI geometric return series for the 50 states and DC   | 49 |
| 2.6.6 Map plots of the mean and standard deviation of the HPI geometric<br>return for the 50 states and DC . . . . .                               | 50 |
| 2.6.7 Plots of mortgage rate time series data . . . . .  | 51 |
| 2.6.8 Plot the estimated function $g$ without and with spatial smoothing for<br>the 50 states and DC. . . . .                                      | 52 |
| 2.6.9 Map plot of the parameters $\hat{\alpha}_i$ , $i = 1, 2, \dots, 5$ estimated without<br>spatial smoothing for the 50 states and DC . . . . . | 54 |

|        |  |     |
|--------|--|-----|
| 2.6.10 | Map plot of the parameters $\hat{\alpha}_i$ , $i = 1, 2, \dots, 5$ estimated with spatial smoothing for the 50 states and DC . . . . .     | 55  |
| 2.6.11 | Map plot of the parameters $\hat{\lambda}_i$ , $i = 1, 2, \dots, 5$ estimated without spatial smoothing for the 50 states and DC . . . . . | 57  |
| 2.6.12 | Map plot of the parameters $\hat{\lambda}_i$ , $i = 1, 2, \dots, 5$ estimated with spatial smoothing for the 50 states and DC . . . . .    | 58  |
| 2.6.13 | Real and predicted HPI for the last 10 observations without and with spatial smoothing for some selected states. . . . .                   | 60  |
| 2.7.1  | Boxplots of squared estimation error (SEE) for the estimation of $g(\cdot)$ without and with spatial smoothing. . . . .                    | 62  |
| 2.7.2  | Boxplots of squared estimation error (SEE) for the estimation of $\lambda_i$ without and with spatial smoothing. . . . .                   | 63  |
| 2.7.3  | Boxplots of squared estimation error (SEE) for the estimation of $\alpha_i$ without and with spatial smoothing. . . . .                    | 64  |
| 3.5.1  | Plot the monthly HPI data for the 50 states and DC. . . . .  | 105 |
| 3.5.2  | Map plots of the mean and standard deviation of the HPI geometric return for the 50 states and DC. . . . .                                 | 106 |
| 3.5.3  | Spatial map plots of the HPI geometric return for the 50 states and DC for the first 6 months of 1975 . . . . .                            | 107 |
| 3.5.4  | Spatial map plots of the HPI geometric return for the 50 states and DC for the second 6 months of 1975 . . . . .                           | 108 |
| 3.5.5  | Boxplots of the HPI geometric return series for the 50 states and DC   | 109 |
| 3.5.6  | Plots of CPI-U:rent time series data. . . . .  | 110 |
| 3.5.7  | Plot the estimated function $g$ without and with spatial smoothing for the 50 states and DC. . . . .                                       | 112 |
| 3.5.8  | Map plot of the parameters $\hat{\alpha}_i$ , $i = 1, 2, \dots, 6$ estimated without spatial smoothing for the 50 states and DC. . . . .   | 113 |

|  |     |
|--|-----|
| 3.5.9 Map plot of the parameters $\hat{\alpha}_i$ , $i = 1, 2, \dots, 6$ estimated with spatial smoothing for the 50 states and DC. . . . .  | 114 |
| 3.5.10 Real and predicted HPI for the last 50 observations without and with spatial smoothing for some selected states. . . . .  | 115 |
| 3.6.1 Boxplots of squared estimation error (SEE) for the estimation of $g(\cdot)$ without and with spatial smoothing. . . . .  | 119 |
| 3.6.2 Boxplots of squared estimation error (SEE) for the estimation of $\alpha_l(\mathbf{s})$ 's without and with spatial smoothing for $T = 500$ time points. . . . .                 | 120 |
| 3.6.3 Boxplots of squared estimation error (SEE) for the estimation of $\alpha_l(\mathbf{s})$ 's without and with spatial smoothing for $T = 700$ time points. . . . .                 | 121 |
| 3.6.4 Boxplots of squared estimation error (SEE) for the estimation of $\sum_{k=1}^N \lambda_i^k(\mathbf{s})$ 's without and with spatial smoothing for $T = 500$ time points. . . . . | 122 |
| 3.6.5 Boxplots of squared estimation error (SEE) for the estimation of $\sum_{k=1}^N \lambda_i^k(\mathbf{s})$ 's without and with spatial smoothing for $T = 700$ time points. . . . . | 123 |
| 3.8.1 Heat map plot of the spatial coefficients $\lambda_1^k(\mathbf{s}_j)$ for $j = k = 1, \dots, 51$ estimated without spatial smoothing. . . . .                                    | 136 |
| 3.8.2 Heat map plot of the spatial coefficients $\lambda_1^k(\mathbf{s}_j)$ for $j = k = 1, \dots, 51$ estimated with spatial smoothing. . . . .                                       | 137 |
| 3.8.3 Heat map plot of the spatial coefficients $\lambda_2^k(\mathbf{s}_j)$ for $j = k = 1, \dots, 51$ estimated without spatial smoothing. . . . .                                    | 138 |
| 3.8.4 Heat map plot of the spatial coefficients $\lambda_2^k(\mathbf{s}_j)$ for $j = k = 1, \dots, 51$ estimated with spatial smoothing. . . . .                                       | 139 |
| 3.8.5 Heat map plot of the spatial coefficients $\lambda_3^k(\mathbf{s}_j)$ for $j = k = 1, \dots, 51$ estimated without spatial smoothing. . . . .                                    | 140 |
| 3.8.6 Heat map plot of the spatial coefficients $\lambda_3^k(\mathbf{s}_j)$ for $j = k = 1, \dots, 51$ estimated with spatial smoothing. . . . .                                       | 141 |

|   |     |
|---|-----|
| 3.8.7 Heat map plot of the spatial coefficients $\lambda_4^k(\mathbf{s}_j)$ for $j = k = 1, \dots, 51$<br>estimated without spatial smoothing. . . . .  | 142 |
| 3.8.8 Heat map plot of the spatial coefficients $\lambda_4^k(\mathbf{s}_j)$ for $j = k = 1, \dots, 51$<br>estimated with spatial smoothing. . . . .     | 143 |
| 3.8.9 Heat map plot of the spatial coefficients $\lambda_5^k(\mathbf{s}_j)$ for $j = k = 1, \dots, 51$<br>estimated without spatial smoothing. . . . .  | 144 |
| 3.8.10 Heat map plot of the spatial coefficients $\lambda_5^k(\mathbf{s}_j)$ for $j = k = 1, \dots, 51$<br>estimated with spatial smoothing. . . . .    | 145 |
| 3.8.11 Heat map plot of the spatial coefficients $\lambda_6^k(\mathbf{s}_j)$ for $j = k = 1, \dots, 51$<br>estimated without spatial smoothing. . . . . | 146 |
| 3.8.12 Heat map plot of the spatial coefficients $\lambda_6^k(\mathbf{s}_j)$ for $j = k = 1, \dots, 51$<br>estimated with spatial smoothing. . . . .    | 147 |
| 4.4.1 Plot the estimated function $g$ without and with spatial smoothing for<br>the 50 states and DC. . . . .   | 161 |
| 4.4.2 Map plot of the parameters $\hat{\alpha}_i$ , $i = 1, 2, \dots, 6$ estimated without<br>spatial smoothing for the 50 states and DC. . . . .       | 162 |
| 4.4.3 Map plot of the parameters $\hat{\alpha}_i$ , $i = 1, 2, \dots, 6$ estimated with spatial<br>smoothing for the 50 states and DC. . . . .          | 163 |
| 4.4.4 Map plot of the parameters $\hat{\lambda}_i$ , $i = 1, 2, \dots, 6$ estimated without<br>spatial smoothing for the 50 states and DC. . . . .      | 165 |
| 4.4.5 Map plot of the parameters $\hat{\lambda}_i$ , $i = 1, 2, \dots, 6$ estimated with spatial<br>smoothing for the 50 states and DC. . . . .         | 166 |
| 4.4.6 Heat map plot of the spatial weight matrix estimated without spatial<br>smoothing. . . . .  | 167 |
| 4.4.7 Real and predicted HPI for the last 50 observations without and with<br>spatial smoothing for some selected states. . . . .                       | 168 |
| 4.5.1 Boxplots of squared estimation error (SEE) for the estimation of $g(\cdot)$<br>without and with spatial smoothing. . . . .                        | 171 |

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|       |   |     |
|-------|---|-----|
| 4.5.2 | Boxplots of squared estimation error (SEE) for the estimation of $\alpha_l(\mathbf{s})$ 's without and with spatial smoothing for $T = 500$ time points. .  | 172 |
| 4.5.3 | Boxplots of squared estimation error (SEE) for the estimation of $\alpha_l(\mathbf{s})$ 's without and with spatial smoothing $T = 700$ time points. . . .  | 173 |
| 4.5.4 | Boxplots of squared estimation error (SEE) for the estimation of $\lambda_i(\mathbf{s})$ 's without and with spatial smoothing for $T = 500$ time points. . | 174 |
| 4.5.5 | Boxplots of squared estimation error (SEE) for the estimation of $\lambda_i(\mathbf{s})$ 's without and with spatial smoothing for $T = 700$ time points. . | 175 |