Nonlinear control for tracking and obstacle avoidance of a wheeled mobile robot with nonholonomic constraint

Hongjiu Yang, Xiaozhao Fan, Peng Shi, *Fellow, IEEE*, and Changchun Hua, *Member, IEEE*

***Abstract*—This paper presents a novel control scheme for some problems on tracking and obstacle avoidance of a wheeled mobile robot with nonholonomic constraint. An extended state observer is introduced to estimate unknown disturbances and velocity information of the wheeled mobile robot. A nonlinear controller is designed to achieve tracking target and obstacle avoidance in complex environments. Note that tracking errors converge to a residual set outside the obstacle detection region. Moreover, the obstacle avoidance is also guaranteed inside the obstacle detection region. Simulation results are given to verify the effectiveness and robustness of the proposed design scheme.**

***Index Terms*—Wheeled mobile robot, trajectory tracking, obstacle avoidance, nonholonomic constraint, extended state observer.**

I. Introduction

Various types of mobile robots will change our lives in the near further. Environment information is obtained by sensors in motion control for a mobile robot [1], [2]. As an important branch of mobile robots, wheeled mobile robots have better flexibility and larger work space than traditional industrial robots [3], [4]. Therefore, they are widely used in complex environments of military and civil occasions in recent years [5]. Some control problems on a nonholonomic wheeled mobile robot have been investigated via neural networks in [6]. Adaptive sliding mode control has been used to deal with the model uncertainty in wheeled mobile robots [7]. A radio frequency identification-based control method has also been proposed for a mobile robot [8], [9]. Moreover, wheeled mobile robots often encounter obstacles when working in complex environment [10]. Based on kinematical equations, some control problems have been studied on tracking and obstacle avoidance, please refer to [11], [12], [13], [14], and so on. Note that trajectory tracking and obstacle avoidance controllers are designed separately in most existing works, which easily lead to the low work efficiency and cause high frequency noise [15]. Furthermore, there is still a lot of space to improve the anti-interference ability of controllers in the current results.

Hongjiu Yang, Xiaozhao Fan and Changchun Hua are with the School of Electrical Engineering, Yanshan University, Qinhuangdao 066004, China. E-mails: yanghongjiu@ysu.edu.cn, yusfxzh@163.com, cch@ysu.edu.cn.

Peng Shi is with the School of Electrical and Electronic Engineering, The University of Adelaide, SA 5005, Australia; and College of Engineering and Science, Victoria University, Melbourne VIC 8001, Australia. E-mail: peng.shi@adelaide.edu.au,

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The technique of active disturbance rejection control is proposed by Jingqing Han in 1990s [16]. Nowadays, it becomes a very attractive methodology in the field of automation [17]. Extended state observer is one of an important component part in the active disturbance rejection control technique [18], [19]. As a control scheme, the extended state observer has been well studied and applied successfully in many researches [20]. The extended state observer is not dependent on the specific mathematical models of disturbances, and it also does not need to measure the effects of disturbances directly [21]. A non-smooth feedback function, which is inherently robust against plant variations, is used to reject the disturbances in the form of orders of magnitude [22]. All of these special feedback mechanism make the active disturbance rejection control technique have a particularly satisfactory performance [23], [24]. Hence, it is an interesting idea to study trajectory tracking and obstacle avoidance of a wheeled mobile robot via active disturbance rejection control, which motivates us to make an effort in this paper.

Notation: In the following, if not explicity stated, matrices are assumed to have compatible dimensions. The shorthand $\text{diag}\{C_1 \ \ C_2 \ \ \cdots \ \ C_n\}$ denotes a diagonal matrix. $\Re^{n\times m}$ denotes n-dimensional configuration space $\mathbf{C}$ with generalized coordinate $(q_1, \cdot\cdot\cdot, q_n)$ and subject to m constraints. Note that $\|\cdot\|$ represents the 2-norm of a vector. The piecewise continuous function $\mathbf{fal}(\cdot)$ is given as follows

$$\mathbf{fal}(\mathcal{E},\alpha,\delta)=\begin{cases}|\mathcal{E}|^{\alpha} & \|\mathcal{E}\|>\delta\\ \frac{\mathcal{E}}{\delta^{1-\alpha}} & \|\mathcal{E}\|\le\delta\end{cases} \tag{1}$$

where α and δ are constants, $\mathcal{E}$ is a variable. To relax notation, the $\mathbf{fal}(\cdot)$ is used in place of $\mathbf{fal}(\mathcal{E},\alpha,\delta)$ in this paper.

II. Preliminaries and Problem Statement

A. Obstacle avoidance problem description

Diagram description of obstacle avoidance problem for the mobile robot tracking the given target is depicted in Fig. 1, in which $[x_r \ y_r \ \theta_r]^T$ is the target position, and $[x_i \ y_i \ \theta_i]^T$ is the real-time position and orientation of the mobile robot.

Fig. 1 Mobile robot avoidance obstacle problem description.

In Fig. 1, assuming that there exist some obstacles between the target position and the current position.

B. Nonholonomic wheeled mobile robot model

The model of a wheeled mobile robot is shown in Fig. 2.

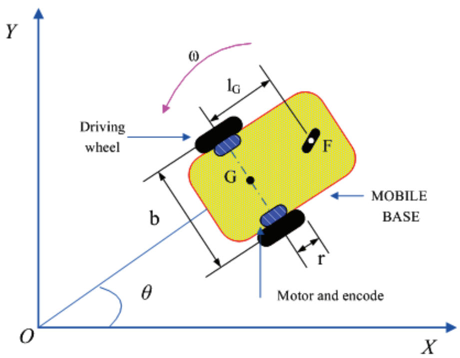


Fig. 2 The model of two-wheeled nonholonomic mobile robot.

In this paper, we consider a two-wheeled mobile robot which is described by the following nonlinear generalized dynamics system

$$\boldsymbol{C}(\mathbf{q})\ddot{\mathbf{q}} + \boldsymbol{B}_m(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \boldsymbol{F}(\dot{\mathbf{q}}) + \tau_d = \boldsymbol{E}(\mathbf{q})\tau - \boldsymbol{A}^T(\mathbf{q})\lambda \tag{2}$$

in which $\boldsymbol{C}(\mathbf{q}) \in \Re^{n\times n}$ is a symmetric positive definite inertia matrix, $\boldsymbol{B}_m(\mathbf{q},\dot{\mathbf{q}}) \in \Re^{n\times n}$ is the centripetal and coriolis matrix, $\boldsymbol{F}(\dot{\mathbf{q}}) \in \Re^{n\times 1}$ is the surface friction, τ_d is bounded unknown disturbances, $\boldsymbol{E}(\mathbf{q}) \in \Re^{n\times r}$ is the input transformation matrix, $\tau = [\tau_r\ \tau_l] \in \Re^{r\times 1}$ is the input vector, $\boldsymbol{A}(\mathbf{q}) \in \Re^{m\times n}$ is the matrix associated with constraints, and $\lambda \in \Re^{m\times 1}$ is the constraint forces vector. There exist some parameter relations of Fig. 2 to system (2), please refer to [6].

C. Structural properties of a mobile robot

In this subsection, a more appropriate dynamic model of the mobile robot than system (2) is obtained to achieve control purpose. The kinematic equation is given as follows

$$\dot{\mathbf{q}} = \boldsymbol{S}(\mathbf{q})\mathbf{v} \tag{3}$$

where $\mathbf{v} = [v\ w]^T$, $\mathbf{v}$ is bounded. $\boldsymbol{S}(\mathbf{q})$ is a Jacobian matrix. Taking the time derivative of (3), it is obtained that

$$\ddot{\mathbf{q}} = \mathbf{S}_q\dot{\mathbf{v}} + (\mathbf{S}_q \odot \dot{\mathbf{q}})\mathbf{v} \tag{4}$$

where $\mathbf{S}_q$ is the partial derivative of each component of Jacobian matrix $\mathbf{S}$ with respect to $\mathbf{q}$, $\odot$ is the multiplication of each component (which is a row vector) of the matrix $\mathbf{S}_q$ with $\dot{\mathbf{q}}$. Now substituting (4) in system (2), we rewrite system (2) as follows

$$\overline{\boldsymbol{C}}(\mathbf{q})\dot{\mathbf{v}} + \overline{\boldsymbol{B}}_m(\mathbf{q},\dot{\mathbf{q}})\mathbf{v} + \overline{\boldsymbol{F}} + \overline{\tau}_d = \overline{\tau} \tag{5}$$

in which $\overline{\boldsymbol{C}}(\mathbf{q}) \in \Re^{2\times 2}$ is the symmetric positive definite inertia matrix, $\overline{\boldsymbol{B}}_m(\mathbf{q},\dot{\mathbf{q}}) \in \Re^{2\times 2}$ is the centripetal and coriolis matrix, $\overline{\boldsymbol{F}} \in \Re^{2\times 1}$ is the surface friction which is assumed to be bounded, $\overline{\tau}_d$ denotes bounded unknown disturbances, $\overline{\tau} \in \Re^{2\times 1}$ is the input vector. Then, some fundamental properties regarding system (5) are summarized as follows.

Property 1: [6] $\dot{\overline{\boldsymbol{C}}}(\mathbf{q})$ and $\|\overline{\boldsymbol{B}}_m(\mathbf{q},\dot{\mathbf{q}})\|$ are bounded.

Property 2: [6] $\dot{\overline{\boldsymbol{C}}}(\mathbf{q}) - 2\overline{\boldsymbol{B}}_m(\mathbf{q},\dot{\mathbf{q}})$ is skew symmetric.

III. Dynamic Tracking and Avoidance Obstacle Control for Mobile Robot

A. Protential function for obstacle avoidance

From [10], the following potential function is introduced to solve the problems of obstacle avoidance for a wheeled mobile robot in this paper

$$V_{ob} = \left(\min\left\{0,\ (L_{rod}^2 - d_d^2)(L_{rod}^2 - d_s^2)^{-1}\right\}\right)^2 \tag{6}$$

where $d_d > d_s > 0$. Note that d_s and d_d are two radii of the avoidance and detection region, respectively. The avoidance obstacle and detection region of a wheeled mobile robot is shown in Fig. 3.

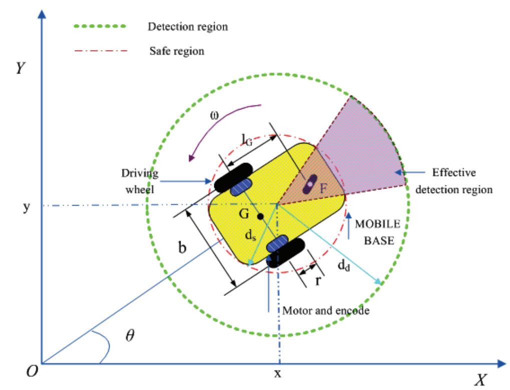


Fig. 3 Wheeled mobile robot with avoidance and detection region.

A reference trajectory is given as (x_r, y_r, θ_r) which is bounded. Define the tracking errors $e_x = x_i - x_r$, $e_y = y_i - y_r$ and $e_\theta = \theta_i - \theta_r$. Moreover, L_{rod} is the distance between the robot and the obstacle. Hence, we have the following exact mathematical expression

$$L_{rod} = \sqrt{(x_i - x_o)^2 + (y_i - y_o)^2} \tag{7}$$

where (x_o, y_o) denotes the obstacle position, and (x_i, y_i) denotes the real-time location position of a mobile robot.

Remark 1: The given trajectory is smooth, and satisfies $|e_\theta| \neq \frac{\pi}{2}$. The reference trajectory does not initiate sharp turns of $90°$ with respect to the current orientation of a mobile robot. We let a perturbed desired orientation $\bar{\theta}_r$ be instead of the desired orientation θ_r to solve singularity problem. In the case, there has $\bar{\theta}_r = \theta_r + \bar{\varepsilon}_\theta$, in which $\bar{\varepsilon}_\theta \neq 0$ is a small perturbation value.

B. Extended state observer design

From (5), we get

$$\dot{\mathbf{v}} = -\overline{\boldsymbol{C}}^{-1}(\mathbf{q})[\overline{\boldsymbol{B}}_m(\mathbf{q},\dot{\mathbf{q}})\mathbf{v} + \overline{\boldsymbol{F}} + \overline{\tau}_d] + \overline{\boldsymbol{C}}^{-1}(\mathbf{q})\overline{\tau} \tag{8}$$

Letting $\mathbf{x}_2 = -\overline{\boldsymbol{C}}^{-1}(\mathbf{q})[\overline{\boldsymbol{B}}_m(\mathbf{q},\dot{\mathbf{q}})\mathbf{x}_1 + \overline{\boldsymbol{F}} + \overline{\tau}_d]$, $\mathbf{x}_1 = \mathbf{v}$ and $\mathbf{u} = \overline{\boldsymbol{C}}^{-1}(\mathbf{q})\overline{\tau}$, one has that

$$\begin{cases} \dot{\mathbf{x}}_1 = \mathbf{x}_2 + \mathbf{u} \\ \dot{\mathbf{x}}_2 = \mathbf{h} \end{cases} \tag{9}$$

in which $\mathbf{h}$ is defined as the first-order derivative of extended state $\mathbf{x}_2$. An extended state observer for system (9) is structured by

$$\begin{cases} \dot{\hat{\mathbf{x}}}_1 = \hat{\mathbf{x}}_2 - \beta_{01}\mathbf{r}_1 + \mathbf{u} \\ \dot{\hat{\mathbf{x}}}_2 = -\beta_{02}\mathbf{fal} \end{cases} \tag{10}$$

in which $\hat{\mathbf{x}}_1$ and $\hat{\mathbf{x}}_2$ are two estimated values of $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively, and $\mathbf{r}_1 = \hat{\mathbf{x}}_1 - \mathbf{x}_1$ is extended state error. Parameters β_{01} and β_{02} are the regulable gain constants. According to (9) and (10), the error system is written as follows

$$\begin{cases} \dot{\mathbf{r}}_1 = \mathbf{r}_2 - \beta_{01}\mathbf{r}_1 \\ \dot{\mathbf{r}}_2 = -\beta_{02}\mathbf{fal} - \mathbf{h} \end{cases} \tag{11}$$

in which $\mathbf{r}_2 = \hat{\mathbf{x}}_2 - \mathbf{x}_2$ is extended state error.

Assumption 1: The first-order derivative of $\mathbf{x}_2$ is existed and bounded. That is, $\dot{\mathbf{x}}_2 = \mathbf{h}$ is bounded.

Remark 2: The variable $\mathbf{h}$ denotes change rate of external force, it is bounded in the wheeled mobile robot with non-holonomic constraint. Therefore, $\mathbf{h}$ is bounded in Assumption 1 as [21], [23], and practical.

C. Nonlinear controller design

The control objective is to design an appropriate controller, which assures a mobile robot accurately tracking the given trajectory with obstacles in complex environment. Let $\hat{\dot{\theta}}_r$ be $\hat{\dot{\theta}}_r = [E_x(t)\hat{\dot{E}}_y - E_y(t)\hat{\dot{E}}_x](E_x^2 + E_y^2)^{-1}$, where $\hat{\dot{E}}_x \approx [E_x(t+T) - E_x(t)]T^{-1}$, $\hat{\dot{E}}_y \approx [E_y(t+T) - E_y(t)]T^{-1}$, for some small $T > 0$. Note that both E_x and E_y are smooth. Let an auxiliary velocity control input $\mathbf{v}_r$ be given by

$$\mathbf{v}_r = \begin{bmatrix} -k_1\sqrt{E_x^2 + E_y^2}\cos e_\theta, & -k_2 e_\theta + \hat{\dot{\theta}}_r \end{bmatrix}^T \tag{12}$$

in which $k_1 > 0$ and $k_2 > 0$ are two adjustable parameters. Then, the auxiliary velocity error is obtained as follows

$$\mathbf{e}_v = \mathbf{v}_r - \mathbf{v} \tag{13}$$

Using (5) and differentiating (13), the mobile robot dynamics is rewritten as

$$\overline{\boldsymbol{C}}(\mathbf{q})\dot{\mathbf{e}}_v = -\overline{\boldsymbol{B}}_m(\mathbf{q}, \dot{\mathbf{q}})\mathbf{e}_v - \overline{\tau} + \mathbf{x}_2 \tag{14}$$

in which $\mathbf{x}_2 = \overline{\boldsymbol{C}}(\mathbf{q})\dot{\mathbf{v}}_r + \overline{\boldsymbol{B}}_m(\mathbf{q}, \dot{\mathbf{q}})\mathbf{v}_r - \overline{\boldsymbol{F}}(\mathbf{q}) + \overline{\tau}_d$. Hence, the actual controller is designed as

$$\overline{\tau} = \hat{\mathbf{x}}_2 + k_3\mathbf{fal}(\mathbf{e}_v, \alpha, \delta) \tag{15}$$

where k_3 is a sufficiently large positive constant.

IV. Effectiveness analysis of system controller

A. Convergence of second-order extended state observer

Theorem 1: Consider the error system given by (11) under Assumption 1. Choose the non-smooth function $\mathbf{fal}$ as in (1) which closes to the derivative of the system's nonlinearities. There exists appropriate positive matrices β_{01} and β_{02} which are defined in (10). By adjusting parameter β_{02} such that $\mathbf{r}_1 \simeq (\mathbf{h}\beta_{02}^{-1})^2$ holds, which shows that estimate error $\mathbf{r}_1$ is bounded. Based on (11), it is easy to know that $\mathbf{r}_2$ is also bounded. Then, the observation accuracy of extended state observer is guaranteed.

Proof 1: Consider the following Lyapunov function

$$V(\mathbf{r}_1, \mathbf{r}_2) = M|\mathbf{r}_1|^{\frac{3}{2}} - O\mathbf{r}_1\mathbf{r}_2 + N\mathbf{r}_2^2 \tag{16}$$

in which M, O and N are constants which are satisfied with

$$M > 0, O > 0, N > 0, O^2 - 4MN < 0 \tag{17}$$

Hence, it is obtained that (16) is positive. The partial derivatives of equation (16) with respect to $\mathbf{r}_1$ and $\mathbf{r}_2$ is given as follows $\frac{\partial V}{\partial \mathbf{r}_1} = \frac{3}{2}M|\mathbf{r}_1|^{\frac{1}{2}}sign(\mathbf{r}_1) - O\mathbf{r}_2$ and $\frac{\partial V}{\partial \mathbf{r}_2} = -O\mathbf{r}_1 + 2N\mathbf{r}_2$. One has that $\dot{V}(\mathbf{r}_1, \mathbf{r}_2) = \left(\frac{3}{2}M - 2N\beta_{02} + O\beta_{01}|\mathbf{r}_1|^{\frac{1}{2}}\right)|\mathbf{r}_1|^{-\frac{1}{4}}|\mathbf{r}_1|^{\frac{3}{4}}sign(\mathbf{r}_1)\mathbf{r}_2 - O\mathbf{r}_2^2 - \left(\frac{3}{2}M\beta_{01} - O\beta_{02}\right)|\mathbf{r}_1|^{2(\frac{3}{4})} + O\mathbf{h}\mathbf{r}_1 - 2N\mathbf{h}\mathbf{r}_2$ The above equation is regarded as the form of a quadratic function with variables $|\mathbf{r}_1|^{\frac{3}{4}}sign(\mathbf{r}_1)$ and $\mathbf{r}_2$. It is rewritten to the following form

$$\dot{V} = -p|\mathbf{r}_1|^{2(\frac{3}{4})} + q|\mathbf{r}_1|^{\frac{3}{4}}sign(\mathbf{r}_1)\mathbf{r}_2 - o\mathbf{r}_2^2 + \Theta \tag{18}$$

in which p is a constant, q is a function of the variable $\mathbf{r}_1$, with $q = (\frac{3}{2}M - 2N\beta_{02} + O\beta_{01}|\mathbf{r}_1|^{\frac{1}{2}})|\mathbf{r}_1|^{-\frac{1}{4}}$, $p = (\frac{3}{2}M\beta_{01} - O\beta_{02})$ and $o = O$, Θ denotes the variable $O\mathbf{h}\mathbf{r}_1 - 2N\mathbf{h}\mathbf{r}_2$. The quadratic part of the equation (18) is negative if and only if $p > 0,\ q > 0,\ o > 0,\ q^2 - 4po < 0$. Let constants $a = O\beta_{01}$, $c = \frac{3}{2}M - 2N\beta_{02}$, $b = \sqrt{O\left(\frac{3}{2}M\beta_{01} - O\beta_{02}\right)}$ and variable $\phi = |\mathbf{r}_1|^{\frac{1}{4}}$. There exists $c + a\phi^2 < 2b\phi$ which holds for any ϕ, if and only if inequality $b^2 - ac > 0$ is satisfied. Inequality $q^2 - 4po < 0$ holds, if and only if ϕ in the quadratic equation $a\phi^2 - 2b\phi + c = 0$ is taken the value in the following set $\{\phi |\ \phi_1 < \phi < \phi_2\}$, where $\phi_1 = a^{-1}(b - \sqrt{b^2 - ac})$, $\phi_2 = a^{-1}(b + \sqrt{b^2 - ac})$ are two roots of $a\phi^2 - 2b\phi + c = 0$. Form inequality $b^2 - ac > 0$, we have

$$O\beta_{02}(2N\beta_{01} - O) > 0 \tag{19}$$

In order to obtain a large interval between the two roots ϕ_1 and ϕ_2 such that the smaller root approaches to zero, inequality (19) have to be large enough. Therefore, we need to take a large O, and choose N to make inequality $2N\beta_{01} - O$ be large. According to the above analysis, we have the conclusion that coefficient M, O and N of equation (16) are satisfied with $3M\beta_{01} > 2O\beta_{02}$, $3M > 4N\beta_{02}$ and $2N\beta_{01} > O$, where β_{01} and β_{02} are two given parameters. Giving a large $O > 0$, and closing $N > \frac{O}{2\beta_{01}}$, $M > \frac{4}{3}N\beta_{02}$, which also satisfy inequality (17), one has that $M > \frac{2O\beta_{02}}{3\beta_{01}}$ holds. Hence, equation (16) is positive and its derivative along the system trajectories is negative in a wide range of variables $\mathbf{r}_1$ and arbitrary $\mathbf{r}_2$. Therefore, the error system (11) is stable at its equilibrium point. Based on (18), we suppose $\dot{V} = \dot{V}_1 - \dot{V}_2$, where $\dot{V}_1 = -p|\mathbf{r}_1|^{2(\frac{3}{4})} + q|\mathbf{r}_1|^{\frac{3}{4}}sign(\mathbf{r}_1)\mathbf{r}_2 - o\mathbf{r}_2^2$, $\dot{V}_2 = -O\mathbf{h}\mathbf{r}_1 + 2N\mathbf{h}\mathbf{r}_2$. After selecting the coefficients M, O and N by the above-described manner, function (18) is positive in the region of the intersected upper portion of parabolic V_1 and flat plane V_2. Both the magnitude of $\mathbf{r}_1$ and the root of equation $p|\mathbf{r}_1|^{2(\frac{3}{4})} = O\mathbf{h}\mathbf{r}_1$ are in the same order, i.e., $\mathbf{r}_1 \simeq ((O\mathbf{h}p^{-1})^2$. Furthermore, it is known that p and $O\beta_{02}$ are in the same order from $p = (\frac{3}{2}M\beta_{01} - O\beta_{02})$. Hence, there exists $\mathbf{r}_1 \simeq (O\mathbf{h}p^{-1})^2 \simeq (\mathbf{h}\beta_{02}^{-1})^2$. The error $\mathbf{r}_1$ is bounded. Thus there exists a constant R_1 with $R_1 \leq 1$ such that $\mathbf{r}_1 \leq R_1 \leq 1$ by adjusting parameter β_{02}. There exists $\dot{\mathbf{r}}_1 = 0$ when the error system (11) is stable. Note that $\mathbf{r}_1$ and $\mathbf{r}_2$ are satisfied with $\mathbf{r}_2 = \beta_{01}\mathbf{r}_1$, so the estimation error $\mathbf{r}_2$ is also bounded. There also exists a known $R_{2\max}$ such that $\|\mathbf{r}_2\| \leq R_{2\max}$.

B. *Stability analysis of closed-loop system*

Theorem 2: Consider the system of a nonholonomic mobile robot (5), the extended state observer (10), the error system (11), the nonlinear controller $\bar{\tau}$ in (15), the non-smooth function **fal** as in (1) and Theorem 1. By adjusting parameters such that the following two inequalities

$$\|\mathbf{e}\| > \|\mathbf{d}\|\mathcal{K}_{\min}^{-1}(\mathcal{M}),\ \|e_\theta\| > \epsilon_\theta K_{2\min}^{-1},$$
$$\|\mathbf{e}_v\| > \max\left\{\delta^{1-\alpha}R_{2\max}K_{3\min}^{-1},\ R_{2\max}K_{3\min}^{-1}\right\}$$

hold. Then, both $\mathbf{e}$ and $\mathbf{e}_v$ converge in a residual set. That is, we have (i) the signals in the closed-loop system are bounded; (ii) the auxiliary velocity error is arbitrarily small value; (iii) the steady state tracking errors e_x, e_y and e_θ are uniformly ultimately bounded.

Proof 2: Consider the following Lyapunov function

$$V_3 = \frac{1}{2}(e_x^2 + e_y^2 + e_\theta^2) + \left(\min\left\{0, \frac{L_{rod}^2 - d_d^2}{L_{rod}^2 - d_s^2}\right\}\right)^2 + V_4$$

where $V_4 = \frac{1}{2}\mathbf{e}_v^T\overline{\mathbf{C}}(\mathbf{q})\mathbf{e}_v$. The derivative of the Lyapunov function V_3 is given by

$$\dot{V}_3 = e_x\dot{e}_x + e_y\dot{e}_y + e_\theta\dot{e}_\theta + \frac{\partial V_{ob}}{\partial x_i}\dot{x}_i + \frac{\partial V_{ob}}{\partial y_i}\dot{y}_i + \dot{V}_4 \quad (20)$$

Applying (14) and (15), taking the manipulate of derivative for V_4, one has that

$$\dot{V}_4 = \frac{1}{2}\mathbf{e}_v^T\left(\dot{\overline{\mathbf{C}}}(\mathbf{q}) - 2\overline{\mathbf{B}}_m(\mathbf{q},\dot{\mathbf{q}})\right)\mathbf{e}_v - \mathbf{e}_v^T k_3\mathbf{fal} + \mathbf{e}_v^T\mathbf{r}_2$$

By Property 2, the first term of the right-hand side in $\dot{V}_4$ is zero. Thus, there exists

$$\dot{V}_4 = -\mathbf{e}_v^T k_3\mathbf{fal} + \mathbf{e}_v^T\mathbf{r}_2 \quad (21)$$

Substituting (1) into equation (21), one has that

$$\begin{aligned} &\dot{V}_{4a} < -\|\mathbf{e}_v\|^2 K_{3\min}\delta^{\alpha-1} + \|\mathbf{e}_v\|R_{2\max} && \|\mathbf{e}_v\| \leq \delta \\ &\dot{V}_{4b} < -\|\mathbf{e}_v\|^2 K_{3\min} + \|\mathbf{e}_v\|R_{2\max} && \|\mathbf{e}_v\| > \delta \end{aligned}$$

in which $R_{2\max}$ is the maximum value of $\mathbf{r}_2$, $K_{3\min}$ is the minimum value of k_3. We have that $\dot{V}_{4a}$ is guaranteed negative as long as the following condition

$$\|\mathbf{e}_v\| > \delta^{1-\alpha}R_{2\max}K_{3\min}^{-1} \quad (22)$$

holds. Furthermore, it is obtained that $\dot{V}_{4b}$ is guaranteed negative as long as the following condition

$$\|\mathbf{e}_v\| > R_{2\max}K_{3\min}^{-1} \quad (23)$$

holds. Based on (22) and (23), we take

$$\|\mathbf{e}_v\| > \max\{\delta^{1-\alpha}R_{2\max}K_{3\min}^{-1},\ R_{2\max}K_{3\min}^{-1}\} \quad (24)$$

in this subsection.
Considering the error dynamics $\dot{e}_x, \dot{e}_y$ and $\dot{e}_\theta$, using the expressions $\cos(e_\theta) = E_x(E_x^2 + E_y^2)^{-1}$, $\sin(e_\theta) = E_y(E_x^2 + E_y^2)^{-1}$ and substituting (12) into the first five terms of equation (20), we have that

$$\dot{V}_5 \leq k_1\cos^2(e_\theta)(E_x^2 + E_y^2) - \|e_\theta\|(k_2\|e_\theta\| - \epsilon_\theta) - \zeta \quad (25)$$

in which $\zeta = e_x\dot{x}_r - e_y\dot{y}_r$. When the robot is outside the detection region, i.e., $L_{rod} > d_d$, we have $\frac{\partial V_{ob}}{\partial x_i} = \frac{\partial V_{ob}}{\partial y_i} = 0$ and the inequality (25) becomes

$$\dot{V}_5 = -\begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix}^T \mathcal{M}\begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} + \left\|\begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix}\right\|\left\|\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ -\epsilon_\theta \end{bmatrix}\right\|$$

for the reason of that $\cos^2(e_\theta)$ is not equal to zero. Hence, $\dot{V}_5 < 0$ holds if the following inequality holds

$$\|\mathbf{e}\| > \|\mathbf{d}\|\mathcal{K}_{\min}^{-1}(\mathcal{M}) \quad (26)$$

where $\mathbf{e} = [e_x \quad e_y \quad e_\theta]^T$, $\mathbf{d} = [\dot{x}_r \quad \dot{y}_r \quad \epsilon_\theta]$, and

$$\mathcal{M} = \begin{bmatrix} k_1\cos^2(e_\theta) & 0 & 0 \\ 0 & k_1\cos^2(e_\theta) & 0 \\ 0 & 0 & k_1 \end{bmatrix}$$

From the analysis above, we know that the tracking errors is bounded when the conditions (24) and (26) are satisfied. Thus the stability of the error dynamics (14) is guaranteed outside the detection region. When the robot is inside the detection region, i.e., $d_s \leq L_{rod} < d_d$, there exists $\dot{x}_r = \dot{y}_r = 0$ such that inequality (25) becomes $\dot{V}_5 \leq k_1\cos^2(e_\theta)(E_x^2 + E_y^2) - \|e_\theta\|(k_2\|e_\theta\| - \epsilon_\theta)$. We have that $\dot{V}_5$ is negative if and only if

$$\|e_\theta\| > \epsilon_\theta K_{2\min}^{-1} \quad (27)$$

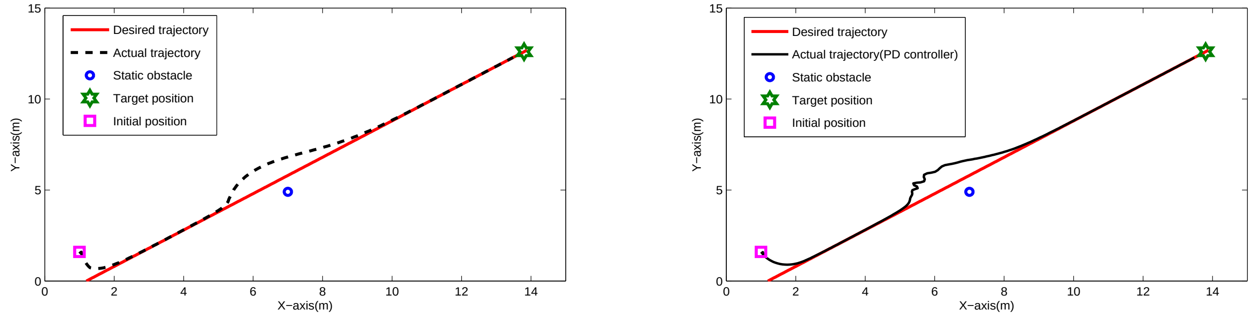
where $K_{2\min}$ is the minimum value of k_2. As shown in [10], if $\dot{V}$ is negative definite, then V is non-increasing inside the detection region. Since $\lim V_{ob} = \infty$ as $\|\mathbf{z} - \mathbf{z}_o\| \to r^+$, where $\mathbf{z} = [x_i \quad y_i]^T$ and $\mathbf{z}_o = [x_o \quad y_o]^T$. Collision avoidance is guaranteed inside the detection region when the conditions (24) and (27) are satisfied.

V. Simulation results

To illustrate the effectiveness of the designed control scheme, some simulation results are implemented based on the system (5), the extended state observer (10), and nonlinear controller (14). The physical parameters of wheeled robot in Fig. 2 are set as $m = 10.0$kg, $b = 0.22$m, $l_G = 0.17$m, $r = 0.05$m and $J_d = 0.75\text{kg} \times \text{m}^2$. The reference trajectory is a straight line with initial coordinates (1.2, 0.0) and orientation 45°, respectively. The desired velocity and angular are respective $v_r = 0.2$m/s, $w_r = 0.0$rad/s and the initial velocity and angular are respective $v_o = 0.35$m/s, $w_o = 0.2$rad/s. The initial coordinates and orientation of the vehicle are (1.0, 1.5) and 60°, respectively. The parameters of observer are chosen as $\beta_{01} = \text{diag}\{50, 50\}$ and $\beta_{02} = \text{diag}\{135, 135\}$.

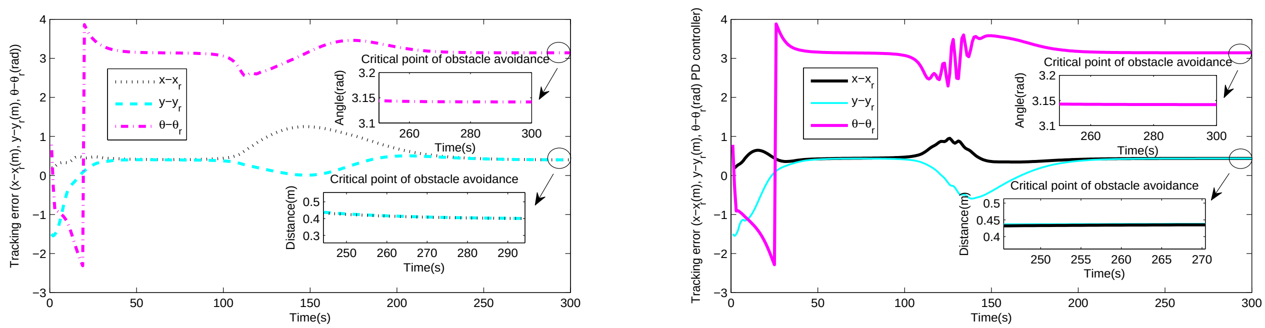
The position errors based on the proposed nonlinear controller and PD controller is shown in TABLE I.

Remark 3: In Fig. 5, the position, the shape and the size of the obstacle can be arbitrarily set through changing *m*, *n* and *r*



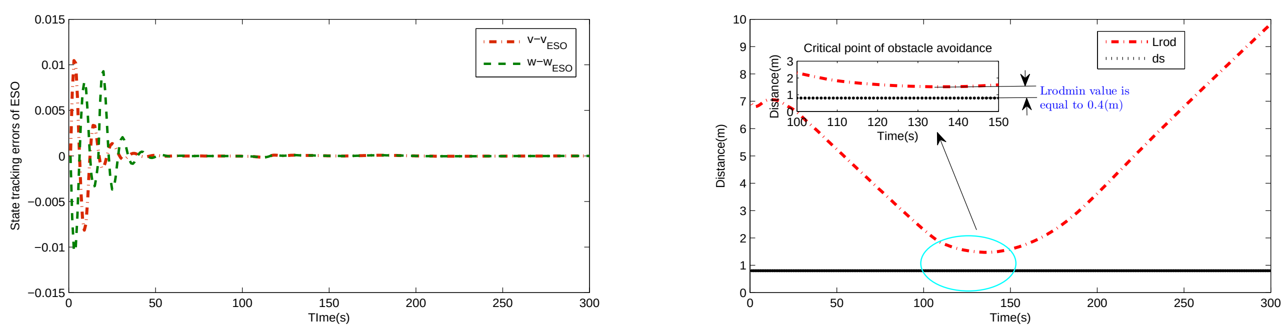
(a) Nonlinear controller (b) PD controller

Fig. 5 Tracking and obstacle avoidance results of the mobile robot



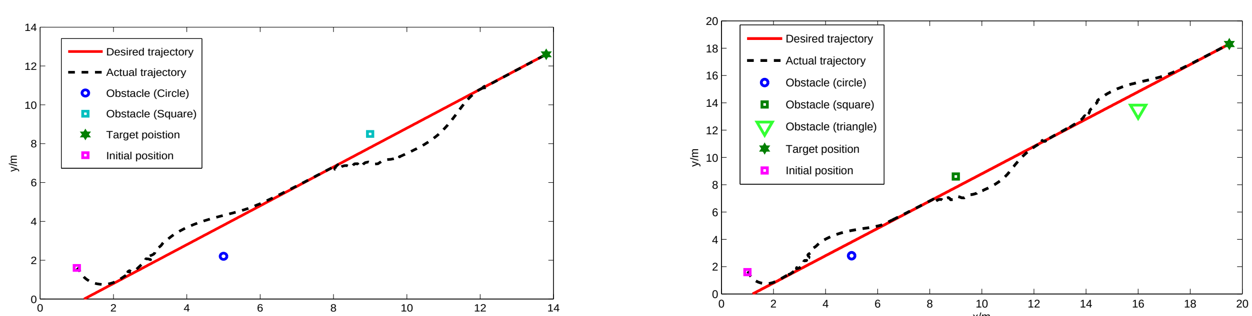
(c) Tracking errors based on nonlinear controller (d) Tracking errors based on PD controller

Fig. 6 Trajectory tracking errors of the mobile robot



(e) State tracking errors (f) The schematic diagram

Fig. 7 State tracking errors of ESO and the distance between the robot and the obstacle, and the avoidance region.



(g) Two obstacles with different shapes (h) Three obstacles with different shapes and sizes

Fig. 8 Tracking and obstacle avoidance results based on nonlinear controller in complex environment.

TABLE II
COMPARISONS OF THE OBTAINED RESULTS WITH OTHERS

References	Contributions
[11]	Adaptive control no considering external disturbances at the kinematic level
[12]	Model-reference control without external disturbances
[13]	An integrated method and path following no considering external disturbances
[14]	Binary logic controller and FLC without external disturbances
This paper	Nonlinear controller with considering external disturbances at the dynamic level

TABLE I
THE COMPARISON BETWEEN PD CONTROLLER AND NONLINEAR CONTROLLER.

	$e_x(m)$	$e_y(m)$	$e_\theta(rad)$	Jitter
PD	0.447	0.446	3.1400	Yes
This paper	0.400	0400	3.1400	No

according to the actual needs. In this paper, an extended state observer is introduced to estimate the unknown disturbances, which will be compensated in controller (15). The extended state observer has strong anti-interference ability [24], so the proposed control scheme in this paper has strong robustness. Some comparison results are given in TABLE II. Therefore, there exists some interesting content in this paper.

VI. CONCLUSION

In this paper, we have proposed a nonlinear controller for tracking and obstacle avoidance of a wheeled mobile robot with nonholonomic constraint. The proposed nonlinear control scheme for a wheeled mobile robot at the dynamics level has been designed without the assumption on the prefect velocity tracking. Simulation results demonstrate for the proposed scheme have been indeed feasible and effective.

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