Particle Swarm Optimization: Theoretical analysis, modifications, and applications to constrained optimization problems

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Doctor of Philosophy (Ph.D.)

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I dedicate this thesis to my lovely wife, Maryam, and my adorable parents, Shahrokh and Mahvash.

Contents

Abstract

This is a PhD thesis by publication. It includes five journal papers, three of them already published, and two submitted for publication in a very high quality international journal in the field of Evolutionary Computation (one of which – as an invited paper). Further, the thesis contains also four conference papers presented and published in the top peer reviewed conferences, as well as one peer reviewed chapter book.

In this thesis we studied an optimization algorithm called Particle Swarm Optimization (PSO) from theoretical and application point of views. The main focus of the theoretical analysis of the algorithm was towards understanding and addressing its limitations that were related to transformations of the search space, convergence to quality solutions, and stability. Through analysis of the algorithm under transformations of the search space we proposed a modification to the original PSO so that a stable performance was guaranteed when the search space was transformed, i.e., rotated, scaled, and translated.

We also studied the ability of the original PSO in locating optimum solutions (local and global optima). Our study showed that this algorithm cannot guarantee to find a local optimum. We introduced a general formulation of topology for the original PSO and identified conditions so that it did not only guarantee local convergence but also transformation invariance. Further, we proposed a specific formulation, extracted from the general formulation, and experimentally confirmed the theoretical findings.

We investigated the most recent standard version of SPSO, called standard PSO 2011 (SPSO2011), from stability, convergence, and transformation sensitivity perspectives. Our investigations revealed essential differences between stability conditions for SPSO2011 with earlier PSO variants. We introduced stability conditions for SPSO2011 and analyzed the behavior of the algorithm before collapsing on its equilibrium. Also, we proved that this algorithm cannot guarantee to find a local optimum in the search space. We introduced sufficient condition for a general formulation that represents a large class of PSO variants so that convergence to a local optimum is guaranteed. We then modified SPSO2011 so that the mentioned sufficient condition was satisfied and convergence to local optimum was guaranteed.

Apart from theoretical analysis, we also studied performance of the algorithm when it was applied to continuous space constrained optimization problems (COPs). We introduced a new aspect in dealing with constrained optimization problems namely *locating disjoint feasible regions* in a search space. We developed a variant of PSO that was able to locate disjoint feasible regions in a search space. This track

of research can potentially be of interest of many other researchers in the field of optimization in future.

As it has been emphasized in many articles, the boundaries of feasible and infeasible regions in a COP can lead optimization algorithms to high quality solutions. We introduced a new approach to concentrate the search on the boundaries of the feasible and infeasible space. As a case study we used a variant of PSO and we confirmed that the results of the new approach are competitive with that of existing approaches in dealing with constraints.

We also proposed a coding scheme to map a discrete space constrained optimization problem, namely multi-dimensional knapsack problem, to a continuous space constrained optimization problem. Then, we investigated the ability of a variant of PSO to deal with the mapped version of this problem.

Statement of originality

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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I would like to express my heartfelt gratitude to Professor Zbigniew Michalewicz, my PhD advisor, whom I have enjoyed the opportunity to learn from not only how to perform research in a scientific era, but also how to think out-of-the-box. He taught me not only how to do research, but also how to find out why to do research. He helped me to find the answer of many questions I have always had in my life during the period of my PhD. I could not be prouder of my academic roots and hope that I can in turn pass on the research values and the dreams that he has given to me.

I would like to thank my PhD co-supervisors, Associate Professor Frank Neumann and Associate Professor Xiaodong Li, who were truly supportive. They have always cared about me and my future and provided great insights for me to pick my path in the best direction. I am so proud of working with you, thank you both.

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Statements of authorships

This thesis contains one chapter book (peer reviewed), five journal papers (indexed in ERA, ISI, and/or JSR), and four conference papers (all peer reviewed and ranked as A according to ERA 2012 report). I have provided a statement of authorship for each of these articles to certify that I was actively involved in the process of preparing each article.

The following is the list of all publications included in this thesis.

Bonyadi, M. R., Li, X., & Michalewicz, Z. A hybrid particle swarm with velocity mutation for constraint optimization problems. In *Genetic and Evolutionary Computation Conference, 2013* (pp. 1-8): ACM. doi:10.1145/2463372.2463378.

Bonyadi, M. R., Li, X., & Michalewicz, Z. (2014). A hybrid particle swarm with a time-adaptive topology for constrained optimization. *Swarm and Evolutionary Computation*, 18(1), 22-37.

Bonyadi, M. R., & Michalewicz, Z. A fast particle swarm optimization algorithm for the multidimensional knapsack problem. In *Congress on Evolutionary Computation, 2012* (pp. 1-8): IEEE.

Bonyadi, M. R., & Michalewicz, Z. (2014). A locally convergent rotationally invariant particle swarm optimization algorithm. *Swarm Intelligence, 8*(3), 159-198.

Bonyadi, M. R., & Michalewicz, Z. (2014). Locating potentially disjoint feasible regions of a search space with a particle swarm optimizer. In K. Deb, & R. Datta (Eds.), *Evolutionary Constrained Optimization*: Springer-Verlag.

Bonyadi, M. R., & Michalewicz, Z. On the edge of feasibility: a case study of the particle swarm optimizer. In *Congress on Evolutionary Computation, 2014* (pp. 3059-3066): IEEE

Bonyadi, M. R., & Michalewicz, Z. (2014). Particle swarm optimization for single objective continuous space problems: a review. *Evolutionary Computation (Under review)*.

Bonyadi, M. R., & Michalewicz, Z. SPSO2011 – Analysis of stability, local convergence, and rotation sensitivity. In *Genetic and Evolutionary Computation Conference, Vancouver, Canada, 2014* (pp. 9-16): ACM.

Bonyadi, M. R., & Michalewicz, Z. (2014). SPSO2011 – Analysis of stability, local convergence, and transformation sensitivity. *Evolutionary Computation, (Invited paper)*.

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