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Preprocessing of water distribution systems to assess connectivity and solvability in the presence of flow control devices

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ABSTRACT

Although mathematical modeling of the hydraulics and water quality of drinking water distribution networks is widely used in network planning and management existing solvers sometimes deliver no results or even wrong results if the connectivity of the system is not correctly maintained. In this paper two major causes for deficient network connectivity are considered.

In the first case, the network graph consists of several maximal connected components where some of them have no node with a fixed head source. Those deficient networks can result from errors in the reference data system (GIS) or during data transfer. In the second case, all the links and nodes of the network graph are connected. However, some links representing control devices have upper and/or lower bounds for the flows. Similar problems as in the first case can be observed if the topology of the network is reduced by removing links with active flow control devices from the graph resulting in a disconnected system. The problem is that the identification of control devices that are active (when an inequality constraint is fulfilled by equality) at a certain time step is not straight forward and depends on the actual hydraulic state of the distribution system.

In this paper two preprocessing steps of the hydraulic steady-state or extended period simulations are proposed to check the solvability of the mathematical problem with respect to the flow constraints. In the first step, the connectivity of the system is analyzed and network parts without a fixed head source are identified. In the second step, a Linear Program (LP) is formulated that includes the nodal continuity conditions plus additional inequality constraints that refer to the operation of the flow controlling devices. The optimal objective value of the LP indicates if for the original problem either a) a solution exists, b) does not exist or c) exists but has redundant control constraints.

INTRODUCTION

Mathematical modeling of the hydraulics and water quality of drinking water distribution networks is widely used in network planning and management. Nowadays, it is common that the network topology is derived from GIS-models and may include thousands of nodes and pipes. During model preparation a major issue is to check the connectivity of the system. Existing models deliver sometimes either no results or wrong results if the connectivity of the system is not correctly maintained.

For calculating the hydraulic behavior of the system in most practical applications demand driven analysis is chosen because of its simplicity. In this case, the water demand withdrawals by consumers are used as fixed boundary conditions of the numerical model. The hydraulic steady-state in the network can be formulated as a nonlinear minimization problem of the system content. To solve this formulation, an iterative Newton-Raphson based algorithm can be used.

However, in order to improve the efficiency modern distribution systems include an increasing number of control devices (remote or locally controlled). The objective is to improve controllability and efficiency of the system. The nature of these control devices is that they formulate upper and lower bounds for the flows and pressures at particular locations within the system. As a consequence the systems equations (including some non-linear equations) of the numerical model are extended by inequality constraints. The identification of control devices that are active (where the flow is at the set value) at a certain time step is not straight forward and depends on the actual hydraulic state of the distribution system.

In this paper two preprocessing steps of the hydraulic steady-state or extended period simulation are proposed to check the solvability of the mathematical problem with respect to flow constraints. In the first step, the connectivity of the system is analyzed and network parts without a fixed head source are identified. In the second step, a Linear Program (LP) is formulated that includes the nodal continuity conditions plus additional inequality constraints that refer to the operation of the flow controlling devices. The optimal objective value of the LP indicates if for the original problem either a) a solution exists, b) does not exist or c) exists but has redundant control constraints. Mathematically, redundancy of active constraints results in the singularity of the system of equations. The new approach identifies those constraints a priori and avoids the problem of a singularity from occurring.

An additional advantage of the proposed approach is that the flow distribution calculated by the LP can be used as starting point for the iterative Newton-Raphson method. Therefore an inner point of the polyhedral set that is defined by the flow constraints is calculated. The benefit from starting with such an interior point is that this point is not only primal but dual feasible as well. That means that all the Lagrangian multipliers of inactive inequality constraints are zero at the beginning of the iterative calculations. An example implementation of the new approach is shown

in this paper that uses the two open source software packages EPANET (Version 2.00.12, Rossman, 2000) and LpSolve (Berkelaar et al., 2004).

IMPACT OF DISCONNECTED NETWORK MODELS IN EPANET

The network in **Figure 1** can be used as an example to consider the impact of disconnected models in the solution process. Note that there are only fixed head sources at nodes 3 and 4. Pipe 1 does not have a fixed head source. Do you think the model shown in **Figure 1** will run successfully?

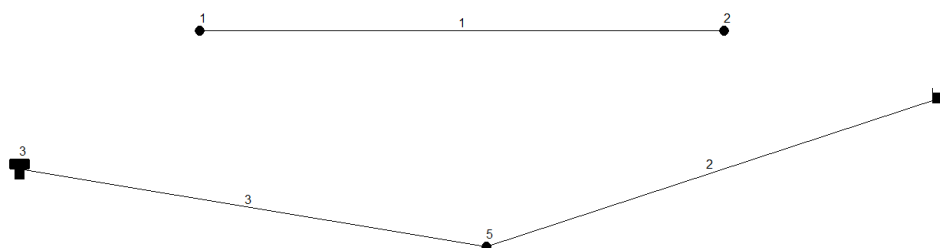


Figure 1: Example system with disconnected pipe

Well, it depends: EPANET is prepared to run two “separated networks” inside the same model, which is a great advantage in many scenarios but could be a source of trouble when the network contains disconnected subnets without a fixed head source and the user does not realize that this is occurring. The INP file of the model presented is shown in Figure 2 (without sections [TIMES] and [OPTIONS]).

```
[JUNCTIONS]
;ID          Elev          Demand          Pattern
1            0            0
2            0            0
5            0            -1
[RESERVOIRS]
;ID          Head          Pattern
4            16
[TANKS]
;ID  Elevation  InitLevel  MinLevel  MaxLevel  Diameter  MinVol  VolCurve
3    0          10         0         20        50        0
[PIPES]
;ID  Node1  Node2  Length  Diameter  Roughness  MinorLoss  Status
1    1      2      6       102.2000  0.1        0          Open ;
2    4      5     1000    200       0.01       0          Open ;
3    5      3     1000    200       0.01       0          Open ;
[COORDINATES]
;Node      X-Coord      Y-Coord
1          177.97      6000.00
2          4177.97  6000.00
5          2364.41  4355.93
4          5838.98  5491.53
3          -1194.92  4949.15
[END]
```

Figure 2: EPANET INP-Input data of example system of Figure 1

If you run the INP-File (see Figure 2) that defines the network in **Figure 1** in EPANET you will get results without problems. Thus EPANET determined that there were two separated “networks” to be analyzed independently. If we check the results for the disconnected pipe (Pipe 1) we confirm that there is no flow in the pipe and the pressure at its extreme nodes is set to zero. Nevertheless a big surprise results

if the diameter of the disconnected pipe is changed practically in an insignificant way; for example change the diameter from 102.2 mm (the current value) to 102.20001 mm. From an engineering point of view it is clear that the number is practically the same. After running it in EPANET the following warning window appears.

```
0:00:00:      System ill-conditioned at node 2
0:00:00      Reservoir 4 is closed
0:00:00      Tank 3 is closed at 10.00 m
System Error 110: cannot solve network hydraulic equations.
```

In this simple network it is really easy to identify visually that the situation being modeled with a disconnected pipe is unsatisfactory. In a network with a significant amount of pipes it could be really difficult to identify fully all disconnected pipes using just EPANET. The situation could be even worse when a disconnected pipe is superposed or very close to other pipes of the network. We could receive a warning window message without having any idea of what is happening exactly and where disconnected pipes or disconnected network sections (if any) could be located.

For the modeler the sensitivity of EPANET-results to very small changes in parameters is strange. A similar example can be reproduced making a change in the roughness of a disconnected pipe. This issue is really about the identification of the network topology rather than an issue with EPANET. Over time bigger and bigger models are analyzed using EPANET. Commonly network data is imported from a Geographic Information System, an AutoCad DXF file or some other resources. Tools for importing data are really useful but sometimes the importation process results in a model where some sections of the network (or just some pipes) are disconnected from the model.

The demonstrated behavior of EPANET can be explained by a more detailed investigation of the system of equations as well as the way they are solved in EPANET. For solution of the network hydraulics the Global Gradient Method (Todini and Pilati, 1988) is used that solves the continuity and pipe head loss equations at each iteration. For networks that contain disconnected pipes without a fixed head node the system matrix is singular. From Linear Algebra it is known that a system of equations with a singular coefficient matrix has no solution (if the rank of the extended coefficient matrix is larger than the rank of the coefficient matrix) or alternatively has an infinite number of solutions. In our case there is an infinite number of pressure heads for the nodes of the disconnected pipe. In other words due to the missing pressure definition of the single pipe there exists no unique solution.

However, coming back to the example of Figure 1 the description given above does not explain why the system is sometimes successfully solved and after very slight changes the solution process terminates with the error message “system ill-conditioned ...”. From the explanation in the previous paragraph it is clear that the system of equations is always singular independent of the choice of the diameter of the disconnected pipe (Pipe 1). Consequently, the expected outcome would be an error message for all conditions. The different behavior can be understood by a review of the EPANET source code at the point where possible singularity is

checked. During the numeric factorization of the matrix (Cholesky factorization) it is checked to see if the matrix diagonal is smaller or equal zero. In this case the factorization is stopped and the calculation is terminated with the singularity error message.

Now, if the diagonal value in the matrix is not exactly zero but a very small number, say for example $1e-10$, the singularity is not detected since it is still larger than zero. Numerical calculations that include a number of multiplications and divisions are always subject to rounding errors. In our example it is a random process if the singularity of the system of equations is detected or not for this example of the disconnected pipe.

The risk of random behavior of the solver results in:

- Unexpected results: After successful calculation runs the program may stop with an ill-conditioned error message after minor changes in input data.
- Impact on post-calculation analysis: Often the network models are used for additional purposes like asset management, risk analysis, etc. In case of additional disconnected pipes resulting from GIS-errors the outcome of those analyses may have a detrimental impact.

NON-EXISTENCE OF FEASIBLE FLOW DISTRIBUTIONS FOR FLOW CONSTRAINT NETWORKS

In a previous paper (Deuerlein et al., 2008) it was shown that under some configurations of flow control devices such as check valves and flow control valves hydraulic solvers like EPANET fail to converge or result in infeasible solutions. There a number of different outcomes including unreasonable negative pressures or flows that contradict the constraints associated with the control devices. These problems can be often explained by disconnected network parts that appear during the iterative process if the links representing active flow control devices are removed from the network graph leading to the same singularity of the system of equations as explained in the section above. In fact, the heuristics implemented in EPANET replace each active flow control device by a pair of nodal demands (the flow set value of the device) at the initial and end node of the link.

PROPOSED MODIFICATIONS TO EPANET

For detection of disconnected network parts as well as identification of infeasible flow constraints the EPANET source code has been modified in two steps. The first step includes a connectivity check at the beginning of a hydraulic calculation. In the second step, Linear Programming is used for pre-calculation of the existence of a feasible solution in the presence of flow controlling devices.

Step I:

The source code of EPANET already includes the function `disconnected()` for checking the connectivity of a network. However, in the existing version 2.00.12 this function is called only if a singularity in the system of equations is detected. As

explained above for numerical reasons it is left to chance if disconnected parts that do not contradict the continuity equation are detected or not.

Therefore we propose to run a connectivity check on at the discretion of the modeler or before any hydraulic calculations are made in order to make sure that the network is set up properly. In our test implementation a scenario file is written in the case that disconnected pipes and nodes are detected. The scenario file can be viewed and this helps identifying the disconnected parts. Due to the simplicity of the connectivity checking algorithm we are not going into detail but focus on the second step.

Step II:

Previously Deuerlein et al. (2008) showed that a feasible or even strictly feasible starting point for the iterative calculation can be found by solution of the following linear minimization problem.

$$\min_{(\mathbf{u}, \xi) \in \Phi} \xi \quad \text{with} \quad \Phi = \left\{ (\mathbf{u}, \xi) \in \mathbf{R}^n \times \mathbf{R} : [\mathbf{G}\mathbf{u} - \mathbf{b}_1]_i \leq \xi, i = 1, \dots, m \wedge \mathbf{H}\mathbf{u} - \mathbf{b}_2 = \mathbf{0} \right\} \quad (1)$$

where \mathbf{b}_1 and \mathbf{b}_2 are vectors of the set values of flow control devices with inequality constraints and equality constraints, respectively. The unknowns are the loop flows \mathbf{u} and the parameter ξ whose value indicates if there exists a (strictly) feasible solution for the original system of linear inequalities or not. Matrix \mathbf{G} and matrix \mathbf{H} consist of the rows of the loop matrix \mathbf{C} that correspond to links with inequality flow constraints and equality flow constraints, respectively. The difficulty with the formulation above is that the loops of the network have to be known.

In this paper, the method for calculation of an interior point of the feasible set is applied to the Global Gradient formulation of the network equations that is implemented in EPANET version 2.00.12. It was shown by Todini and Pilati (1988) who referred to an earlier paper of Collins et al. (1978) that the solution of the hydraulic network equations is equivalent to the minimization of the so called content function (Collins, 1978):

$$\begin{aligned} \min_{\mathbf{q} \in \mathbf{R}^m} C(\mathbf{q}) \\ \text{s.t.} \quad \mathbf{A}^T \mathbf{q} = \mathbf{Q} \end{aligned} \quad (2)$$

where $C(\mathbf{q})$ is the system content, m is the number of pipes, \mathbf{q} is the vector of pipe flows, \mathbf{A} is the $(m \times n)$ incidence matrix of pipes and demand nodes (number n) and \mathbf{Q} is the n -vector of nodal demands. The equality constraints consist of the continuity equations of the demand nodes. More details on the formulation of the Content minimization as an equivalent problem for the calculation of the hydraulic steady-state can be found in Collins et al. (1978), Todini and Pilati (1988) and Deuerlein et al. (2009). In this paper we are focusing on the flow constraints only.

Following the theorem of Lagrange, the constrained optimization problem can be transformed into an unconstrained one. The necessary conditions (Kuhn-Tucker conditions) are the well-known equations of the Global Gradient formulation where the Lagrangian Multipliers correspond to the unknown heads of the demand nodes.

So far, the system consists of network elements having resistance only (that is pipes). For general networks including control devices like FCVs or check valves additional inequality constraints have to be considered. For each link that represents such a control device an inequality condition can be added to the constraints of the content minimization problem. The extended version of the Content minimization problem is:

$$\begin{aligned}
& \min_{\mathbf{q} \in R^m} C(\mathbf{q}) \\
& s.t. \quad \mathbf{A}^T \mathbf{q} = \mathbf{Q} \\
& \quad \alpha_i q_i \leq b_i, \quad i \in I_{FC}
\end{aligned} \tag{3}$$

I_{FC} denotes the index set of links with inequality flow constraints, $\alpha_i \in \{-1,1\}$ stands for lower bounds or upper bounds (value b_i), respectively. In this case, the affine feasible set of the original problem is generalized to the polyhedral set described by the continuity equations and the lower and upper flow bounds imposed by the flow controlling devices.

The calculation of the hydraulic steady-state is equivalent to the solution of the constrained convex nonlinear optimization problem. There is a rich literature on the solution of this type of problem. Interior point methods are known for example as barrier methods where the inequality constrained problem is transferred to an unconstrained one by adding a barrier function to the objective. Interior point methods require a starting point in the relative interior of the feasible set. Therefore as for Linear Programming, the term Phase I methods is used. The so called Phase I includes the calculation of an interior point. In our case this method shall be used for determining if a solution exists and for finding a good starting point for the iterative calculation.

In hydraulic solvers like EPANET the iterative calculation starts with an arbitrary estimate of pipe flows - such as a velocity of 1 ft/s in all pipes. Inequality constraints are not considered at this time in EPANET and later treated by heuristic methods. For example, if the flow through a Check Valve is reversed during the iterative calculation the corresponding link is treated like a closed link in subsequent iteration steps. This method works well in most cases, however, in some cases it results in severe problems as explained above. Moreover it does not allow for a secure statement as to whether a feasible flow distribution exists at all. As a consequence it is highly desirable to know if the feasible set is empty or not. In addition it is an advantage to know as well if there exists an interior point that can be used as starting point for the iterative calculations. The main advantage of starting with such an interior point is that the state of the flow control devices is uniquely determined. All flow control devices are in an inactive state. This is important to avoid flipping of the status of flow control devices since the chosen values of the starting flow vector coincide with the valve states. In contrast, in the current heuristics it starts with the assumption of active state for all devices and the flows such that they could be in contradiction to the valve states.

The objective is to determine if such an interior point exists and where it does exist to calculate an arbitrary interior point as the starting flow distribution for the network. For that reason the following linear optimization problem (LP) is considered:

$$\begin{aligned}
 & \min_{\xi \in \mathbb{R}} \xi \\
 \text{s.t.} \quad & \mathbf{A}^T \mathbf{q} = \mathbf{Q} \\
 & \alpha_i q_i - b_i \leq \xi, \quad i \in I_{FC}
 \end{aligned} \tag{4}$$

It can be shown that the LP defined by Eq. (4) is always strictly feasible (Boyd and Vandenberghe, 2004). Depending on the sign of the optimal value ξ^* it can be concluded if the original problem of minimizing the system content has a feasible solution or not. Three cases can be distinguished:

$\xi^* < 0$: The polyhedral set defined by the continuity equation and the inequality constraints of flow control devices is non-empty. Thus, \mathbf{q}^* is an interior point of the feasible set. This implies that all the control devices are in an inactive state.

$\xi^* > 0$: The polyhedral set defined by the continuity equation and the inequality constraints is empty. Thus, there is no feasible solution to the original problem.

$\xi^* = 0$: The polyhedral set defined by the continuity equation and the inequality constraints of flow control devices is non-empty but an interior point does not exist. In this case the corresponding flow vector \mathbf{q}^* is on the boundary of the feasible set. This implies that there are flow control devices that are always in an active state. These flow control devices can be identified. If the inequality is fulfilled by equality for some values of the optimal flow vector \mathbf{q}^* the corresponding device can never reach an inactive state.

In a modified version of EPANET developed in this research the open source software LpSolve (Berkelaar et al., 2004) has been linked to the hydraulic solver. After the connectivity check as explained above and in advance of the iterative hydraulic calculations the LP of Eq. (4) is solved and the calculated flows are used as initial values for the subsequent iterative procedure. Depending on the optimal value of the LP the calculation terminates with an error message or continues with the execution of the GGA.

EXAMPLE NETWORK

For illustration the simple system in Figure 3 is considered. It consists of a single source R and a single demand node (N5, $Q = 100$ L/s). The demand node is connected to the source node by two alternative paths each of them including a Flow Control Valve (V1 and V2). The system is symmetrical in terms of pipe characteristics. It is easy to see that if the sum of set flows of the valves is below the demand Q a feasible solution does not exist. If the sum of set flows equals Q we get redundant constraints and the solution is non-unique in terms of the nodal heads (N2, N4 and N5).

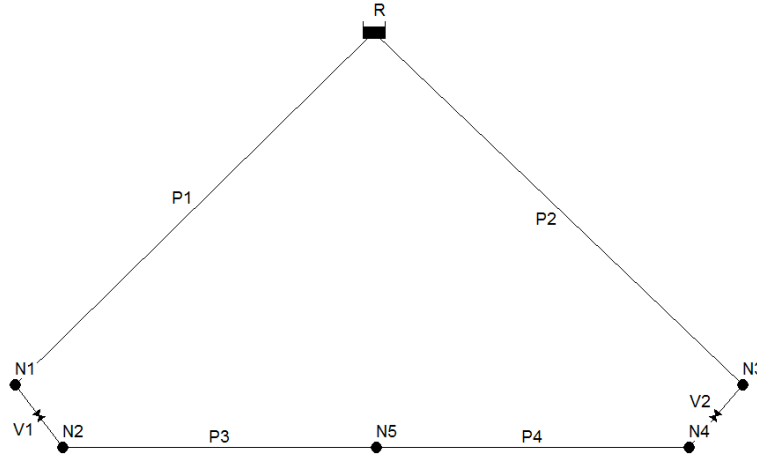


Figure 3: Example system with two FCVs

The LP for the example system is

$$\min_{\xi \in R} \xi \quad (5)$$

s.t.

$$q_{P2} - q_{V2} = 0; \quad q_{P1} - q_{V1} = 0; \quad q_{V1} - q_{P3} = 0; \quad q_{P3} + q_{P4} = Q; \quad q_{V2} - q_{P4} = 0;$$

$$q_{V2} - Q_{S,V2} \leq \xi; \quad q_{V1} - Q_{S,V1} \leq \xi; \quad -q_{V2} \leq \xi; \quad -q_{V1} \leq \xi;$$

Case 1: Feasible solution exists ($Q_{S,V1}=50$, $Q_{S,V2}=60$, $Q = 100$):

$$q_{P1} = q_{V1} = q_{P3} = 45, \quad q_{P2} = q_{V2} = q_{P4} = 55, \quad \xi = -5.$$

Case 2: Feasible solution does not exist ($Q_{S,V1}=50$, $Q_{S,V2}=50$, $Q = 110$):

$$q_{P1} = q_{V1} = q_{P3} = q_{P2} = q_{V2} = q_{P4} = 55, \quad \xi = +5.$$

Case 3: Feasible solution exists with redundant constraints ($Q_{S,V1}=50$, $Q_{S,V2}=50$, $Q = 100$):

$$q_{P1} = q_{V1} = q_{P3} = q_{P2} = q_{V2} = q_{P4} = 50, \quad \xi = 0.$$

In case 1 the situation is clear. A feasible solution exists and the number of active valves depends on the properties of the pipes. The modified and the original version of EPANET deliver the same results.

In case 2 where a feasible solution does not exist the results calculated by EPANET are wrong. The calculation is terminated with a warning message: “Negative pressures at time 0:00:00 hrs”. Both valve states are set to active and the flows are 55.0 L/s exceeding the set value of 50.0 L/s. In the modified version the calculation stops after the LP run with the following error message:

“ERROR 401: System infeasible at 0:00:00 hrs. Check flow constraints of links:
V2: Upper bound (flow setting) of FCV
V2: Upper bound (flow setting) of FCV
Contradictory flow constraints. Hydraulic calculation has not been carried out.”

In case 3 the active flow constraints together with the continuity equations form a redundant system of linear equations. The hydraulic steady-state is non-unique in terms of nodal heads. This can be explained by means of the Kuhn-Tucker-Conditions from Nonlinear Programming. A necessary condition for the existence of unique Lagrange Multipliers is that a suitable constraint qualification (CQ) holds. One such a CQ is the Linear Constraint Qualification (LICQ). This LICQ is violated in case 3 since the continuity equations form a redundant system with the FCV constraints. Imagine that the system in Figure 2 includes no control devices. The network has one loop and the flows are dependent on the properties of the pipes. After one flow control device is added the flow of the entire system is defined if the valve is active. Each additional FCV either contradicts the existing flow leading to infeasible states or is redundant. Now imagine that both valves are active with feasible set flows (case 3), then the heads of the nodes in between the valves are not uniquely defined since each minor headloss at valve 1 has a unique corresponding value at valve 2. In this case EPANET sets one of the valves to an open state. In the modified version this behavior is not changed. However, a warning message is created since the choice of the active valve is left to chance and could lead to misinterpretations.

CONCLUSION AND OUTLOOK

Problems of existing hydraulic simulation engines like EPANET with networks containing disconnected subgraphs have been discussed. A two-step preliminary analysis for the identification of disconnected parts and the investigation of the polyhedral set described by the continuity equations and inequality flow constraints resulting from flow control devices has been presented. The second step is based on the solution of a Linear Programming Problem whose optimal value indicates if either a) a feasible solution exists or b) does not exist or c) the solution is non-unique in terms of nodal heads.

For solution of the LP, the open source software LpSolve has been combined with EPANET. Its applicability was successfully tested for small example systems as well as large real world networks. However, the formulation of the LP including the continuity equations results in comparable long calculation times. The situation could be improved starting with a flow distribution of a spanning tree and formulation of the LP in the corresponding loop flows. One shortcoming of this method in comparison to the method proposed in this paper is that a more intensive topological analysis has to be carried out in advance that includes the identification of a spanning tree and the loops. Another approach could be to “help” the LP by providing a flow distribution that solves the continuity equations and can be calculated by the common EPANET solver within the first iteration.

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