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## **Covariance Matrix Regularization for Banded Toeplitz Structure via Frobenius-Norm Discrepancy**

Xiangzhao Cui, Zhenyang Li, Jine Zhao, Defei Zhang and Jianxin Pan

Abstract In many practical applications, the structure of covariance matrix is often blurred due to random errors, making the estimation of covariance matrix very difficult particularly for high-dimensional data. In this article, we propose a regularization method for finding a possible banded Toeplitz structure for a given covariance matrix A (e.g., sample covariance matrix), which is usually an estimator of the unknown population covariance matrix  $\Sigma$ . We aim to find a matrix, say B, which is of banded Toeplitz structure, such that the Frobenius-norm discrepancy between Band A achieves the smallest in the whole class of banded Toeplitz structure matrices. As a result, the obtained Toeplitz structured matrix B recoveries the underlying structure behind  $\Sigma$ . Our simulation studies show that B is also more accurate than the sample covariance matrix A when estimating the covariance matrix  $\Sigma$  that has a banded Toeplitz structure. The studies also show that the proposed method works very well in regularization of covariance structure.

#### **1** Introduction

Estimation of covariance matrices is important in many application fields including spectroscopy, functional magnetic resonance imaging, text retrieval, gene array, climate study and imaging analysis. This problem has been widely researched in statistics. The traditional "Burg technique", which is to find the maximum likelihood estimator of a covariance matrix, needs to provide the sample covariance matrix and pre-specify the structure of the unknown population covariance matrix [3]. However, it is well known that the sample covariance matrix performs poorly for high-dimensional data, where the number of variables is bigger than the sample size

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[11]. Further, the underlying structure of the sample covariance matrix is usually blurred because of random noises from different sources.

For high-dimensional data, methods of covariance matrix regularization were developed in the literature, including hard-thresholding [9, 2], soft-thresholding and its various generalizations [18], and adaptive thresholding [4], among many others, but threshold estimators may have negative eigenvalues in finite samples. To overcome this difficulty, Rothman proposed a Lasso-type penalty based method to encourage sparsity and used a logarithmic barrier function to enforce the positive-definiteness [17]. [20] considered an alternating direction method to ensure the positive-definiteness of Lasso penalty covariance estimator. However, all such regularization approaches require a tuning parameter, and how to choose an appropriate tuning parameter remains a very challenging problem.

For covariance matrix estimation, it has recently attracted more attentions on regularizing an estimated covariance matrix so that the underlying structure becomes clear [14, 16]. Recently, the technique based on entropy loss function aiming to avoid the selection of tuning parameter has been proposed to regularize an estimated covariance matrix into one of the structured covariance matrices: MA(1), CS, AR(1) and banded Toeplitz [12]. However, the technique cannot handle a highdimensional data set, since the inverse of the sample covariance matrix does not exist. Very recently, the method of regularizing an estimated covariance matrix into the one with structures MA(1), CS, AR(1) and ARMA(1,1) via Frobenius-norm discrepancy was developed by [6, 7]. Although this method avoids the use of the inverse of the sample covariance matrix, it only considered the above four candidate covariance structures, that is, MA(1), CS, AR(1), ARMA(1,1). It is well known that banded Toeplitz covariance matrices are very common in practice, in particular, it is commonly used in time series. For example, in signal processes the covariance matrix of Gauss-Markov random process or cyclostationary process often displays a banded Toeplitz structure [13, 19, 5]. On the other hand, all the four structures, i.e., MA(1), CS, AR(1) and ARMA(1, 1), are a special case of banded Toeplitz structures.

In order to study a more general structure, in this paper we consider the regularization problem for covariance matrix with banded Toeplitz structures. The  $m \times m$ covariance matrix is of the form

$$B(\sigma^{2}, c_{1}, \cdots, c_{p}) = \sigma^{2} \begin{bmatrix} 1 & c_{1} & c_{2} & \cdots & c_{p} & 0 & \cdots & 0 \\ c_{1} & 1 & c_{1} & \ddots & \ddots & \ddots & \ddots & \vdots \\ c_{2} & c_{1} & 1 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & c_{p} \\ c_{p} & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & c_{p} & \cdots & c_{2} & c_{1} & 1 \end{bmatrix}$$
(1)

where  $\sigma^2 > 0, c_1, c_2, ..., c_p$  (m > p) are nonzero and all other off-diagonal elements are zero. Define  $q(t) = 1 + 2 \sum_{k=1}^{p} c_k \cos(kt)$ , then *B* is positive-definite if and only if  $q(t) \ge 0, q(t) \ne 0$ , for all  $t \in \mathbb{R}$  [15].

The outline of the paper is as follows. In Section 2, we transform the regularization problem into an optimization problem in numerical analysis and explore its general properties. In Section 3, we discuss the problem of finding the optimal structure from a class of banded Toeplitz structures, in the sense of minimizing the square of the Frobenius-norm discrepancy. In Section 4, we conduct simulation studies to measure the performance and effectiveness of the proposed approach. In Section 5, a brief discussion is provided.

#### 2 Problem of interest

In order to transform the regularization problem into an optimization problem, we suppose *A* is an available  $m \times m$  estimator of a covariance matrix, for example, the sample covariance matrix. Note that it may also be one of those obtained by using SQRT-Lasso [1], matrix-logarithm transformation [8] and thresholding principal orthogonal complements [10] among many others. Let  $\Omega$  be the set of all  $m \times m$  positive definite covariance matrix *A* and the set  $\Omega$  is defined by

$$D(A,\Omega) := \min_{B \in \Omega} L(A,B), \tag{2}$$

where L(A,B) is a measure of the distance between the two  $m \times m$  matrices A and B, defined by the square of the Frobenius-norm. In other words,

$$L(A,B) := tr((A-B)^{T}(A-B)).$$
(3)

Note that  $f(B) := L(A, B) = tr((A - B)^T (A - B))$  is a strictly convex function of *B* [7]. Also, the set  $\Omega$  is obviously convex. Since  $\Omega$  is the set of all positive definite matrices having a banded Toeplitz structure, the optimization problem in (2) is convex and so it has a unique solution.

The idea is that, in this set  $\Omega$ , the matrix *B* which satisfies the above equality can be considered as the regularized version of *A*. In other words, the matrix *B* with a banded Toeplitz structure is considered to have the underlying structure of  $\Omega$ .

We remark that the matrix A does not require a non-singularity, meaning that it applies to the case when the matrix A is singular, for example, A is the sample covariance matrix for high-dimensional data. In other words, even if the given covariance matrix A is singular, we can find a covariance matrix B which has a nonsingular banded Toeplitz structure, such that the F-norm discrepancy function L(A,B)achieves its smallest among the class of banded Toeplitz matrices.

### **3** Solution of problem

The matrix B in (1) can be rewritten as

$$B(\sigma^2, c_1, \cdots, c_p) = \sigma^2(I + \sum_{i=1}^p c_i T_i),$$

where  $T_i$  is a symmetric matrix with ones on the *i*th superdiagonal and subdiagonal and zeros elsewhere.

The discrepancy function in (3) is now

$$f(\sigma^2, c_1, \cdots, c_p) = \operatorname{tr}(AA^T) + \sigma^4(m + 2\sum_{i=1}^p (m-i)c_i^2) - 2\sigma^2(\operatorname{tr}(A) + \sum_{i=1}^p c_i \operatorname{tr}(AT_i)).$$

It follows that

$$\nabla f := \begin{bmatrix} \frac{\partial f}{\partial (\sigma^2)} \\ \frac{\partial f}{\partial c_1} \\ \vdots \\ \frac{\partial f}{\partial c_p} \end{bmatrix} = \begin{bmatrix} 2\sigma^2(m+2\sum_{i=1}^p (m-i)c_i^2) - 2(\operatorname{tr}(A) + \sum_{i=1}^p c_i \operatorname{tr}(AT_i)) \\ 4\sigma^4(m-1)c_1 - 2\sigma^2 \operatorname{tr}(AT_1) \\ \vdots \\ 4\sigma^4(m-p)c_p - 2\sigma^2 \operatorname{tr}(AT_p) \end{bmatrix},$$

and

$$\nabla^2 f := \begin{bmatrix} \frac{\partial^2 f}{\partial (\sigma^2)^2} & \frac{\partial^2 f}{\partial (\sigma^2) \partial c_1} & \cdots & \frac{\partial^2 f}{\partial (\sigma^2) \partial c_p} \\ \frac{\partial^2 f}{\partial c_1 \partial (\sigma^2)} & \frac{\partial^2 f}{\partial c_1^2} & \cdots & \frac{\partial^2 f}{\partial c_1 \partial c_p} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2 f}{\partial c_p \partial (\sigma^2)} & \frac{\partial^2 f}{\partial c_p \partial c_1} & \cdots & \frac{\partial^2 f}{\partial c_p^2} \end{bmatrix}$$

$$= \begin{bmatrix} 2m + 4\sum_{i=1}^p (m-i)c_i^2 & 8\sigma^2(m-1)c_1 - 2\operatorname{tr}(AT_1) & \cdots & 8\sigma^2(m-p)c_p - 2\operatorname{tr}(AT_p) \\ 8\sigma^2(m-1)c_1 - 2\operatorname{tr}(AT_1) & 4\sigma^4(m-1) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 8\sigma^2(m-p)c_p - 2\operatorname{tr}(AT_p) & 0 & \cdots & 4\sigma^4(m-p) \end{bmatrix} .$$

Hence the stationary points  $(\sigma^2, c_1, \cdots, c_p)$  must satisfy the following equations

Covariance Matrix Regularization

$$\int m\sigma^{2} + 2\sigma^{2} \sum_{i=1}^{p} (m-i)c_{i}^{2} - \operatorname{tr}(A) - \sum_{i=1}^{p} c_{i}\operatorname{tr}(AT_{i}) = 0$$

$$2\sigma^{2}(m-1)c_{1} - \operatorname{tr}(AT_{1}) = 0$$

$$\dots$$

$$2\sigma^{2}(m-p)c_{p} - \operatorname{tr}(AT_{p}) = 0.$$
(4)

Thus a unique stationary point is

$$\begin{cases} \sigma^2 = \frac{\operatorname{tr}(A)}{m} \\ c_1 = \frac{\operatorname{mtr}(AT_1)}{2(m-1)\operatorname{tr}(A)} \\ \dots \\ c_p = \frac{\operatorname{mtr}(AT_p)}{2(m-p)\operatorname{tr}(A)}. \end{cases}$$
(5)

Since

$$\nabla^2 f|_{(\sigma^2,c_1,\cdots,c_p)} := \begin{bmatrix} 2m + 4\sum_{i=1}^p (m-i)c_i^2 & 4\sigma^2(m-1)c_1 & \cdots & 4\sigma^2(m-p)c_p \\ 4\sigma^2(m-1)c_1 & 4\sigma^4(m-1) & \cdots & 0 \\ & \cdots & & \cdots & & \cdots \\ & 4\sigma^2(m-p)c_p & 0 & \cdots & 4\sigma^4(m-p) \end{bmatrix}$$

the  $k \times k$  leading principal minor of matrix  $\nabla^2 f|_{(\sigma^2, c_1, \dots, c_p)}$  is given by

$$W_{k} = \begin{cases} 2m + 4\sum_{i=1}^{p} (m-i)c_{i}^{2} & k = 1\\ (2m + 4\sum_{i=k}^{p} (m-i)c_{i}^{2})\prod_{i=1}^{k-1} 4\sigma^{4}(m-i) & k = 2, \cdots, p\\ 2m\prod_{i=1}^{p} 4\sigma^{4}(m-i) & k = p+1. \end{cases}$$

Recalling that  $\sigma^2 > 0$ ,  $p \le m-1$ ,  $c_1, c_2, \ldots, c_p$  are nonzero, we have  $W_k > 0$ . The Hessian matrix  $\nabla^2 f|_{(\sigma^2, c_1, \cdots, c_p)}$  is then positive definite, so that its stationary point is really a minimum point. This point, however, may be a local minima of f(B). It is well-known that if the function is convex then the local minimum is actually the global minimum, but it is difficult to check if f(B) is convex with respect to  $(c_1, \cdots, c_p)$  here due to its complexity. We suggest to compare the function values at all the local minimum so that the global minimum can be achieved.

We summarize the discussion above in the following theorem.

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**Theorem 1.** For a given estimator A of an  $m \times m$  covariance matrix, define f(B) := L(A,B) where B is a positive definite covariance matrix with a banded Toeplitz structure. Then the local or global minimum of f(B) can be achieved as the solution of (5).

#### 4 Simulation studies

To check the actual performance of the proposed regularization method, we carry out intensive simulation studies in this section. All the calculations are done using MATLAB R2012b, and we apply fzero and fsolve to find roots.

In the process of the simulation, we first generate an  $m \times m$  covariance matrix  $\Sigma$  which has one of the following structures: order-1 moving average structure (MA(1)), compound symmetry structure (CS), order-1 autoregressive structure (AR(1)), order-1 autoregressive moving average structure (ARMA(1,1)), cf. [6, 7], and a banded Toeplitz structure. We assume an *m* dimensional zero mean vector  $\mu = 0$ . We take a random sample of size *n* from multivariate normal distribution  $N_m(0, \Sigma)$  and then calculate the sample covariance matrix *A*. In the end, we find matrix *B* which has a banded Toeplitz structure mentioned above, achieving the minimum of the Frobenius-norm.

We need to state the following facts.

- 1. We only need to find the desired matrix among the ones which have the same structure as the correct covariance structure, because for the cases of MA(1), CS, AR(1) or ARMA(1,1) [6, 7] already showed that the matrix *B* that has the minimum F-norm remains the same structure as the true covariance matrix.
- 2. Since the function L(A,B) measures the absolute difference between matrices A and B, it can make the value of L(A,B) too large. To reduce the influence of the dimension of covariance matrix and its elements on L(A,B), we re-define the following function to replace L(A,B)

$$L^{*}(A,B) := \operatorname{tr}((A-B)^{T}(A-B))/\operatorname{tr}(A^{T}A),$$
(6)

which does not affect the finding of the optimal matrix *B*.

We will use the following notations:

- $\Sigma$ : the true covariance matrix being tested;
- *A*: the covariance matrix obtained from sample;
- *B*: the matrix which minimizes  $L^*(A, B)$  and has a structure of MA(1), *CS*, *AR*(1), *ARMA*(1,1) or banded Toeplitz;
- $L^*_{\Sigma,A}$ ,  $L^*_{A,B}$  and  $L^*_{\Sigma,B}$ : the difference functions  $L^*(\Sigma,A)$ ,  $L^*(A,B)$  and  $L^*(\Sigma,B)$ , respectively.

For  $\Sigma$  having MA(1), CS or AR(1) structure, we set the sample size n = 100, matrix dimension m = 10, 20, 50, 100, 200, 500, 1000,  $\sigma^2 = 0.5, 1, 2, 4$ , parameter c = 0.25, 0.5, 0.75. For  $\Sigma$  having ARMA(1, 1) structure, in addition to the above

6

values, we set r = 0.1, 0.35, 0.6 when c = 0.25; r = 0.2, 0.45, 0.75 when c = 0.5; and r = 0.25, 0.5, 0.8 when c = 0.75.

We apply the method proposed in [6, 7] to find the matrix *B* which has MA(1), *CS*, AR(1) or ARMA(1,1) structure and has the minimum F-norm difference. We also apply the method proposed in this paper for the case when the matrix has a banded Toeplitz structure with p = m - 1.

Table 1-12 gives the average results after 100 run simulations. In each of those tables, the first column provides the structure of the true covariance matrix  $\Sigma$ , the second column presents the F-norm difference between the sample covariance matrix *A* and  $\Sigma$ ; the third and forth columns give the F-norm differences between *B* and *A*,  $\Sigma$ , respectively, where *B* is obtained by using the same structure as  $\Sigma$ ; the fifth and sixth columns provide the F-norm differences between *B* and *A*,  $\Sigma$ , respectively, where *B* is obtained by using the same structure.

To save space, in Tables 1-12 we only report simulation results for n = 50, 100, 500 and 1000. Other results are very similar to these reported. From these tables, we can draw the following observations.

- 1. For the matrix *B* obtained by using either the same structure as  $\Sigma$  or a banded Toeplitz structure, the F-norm discrepancy between *B* and the real covariance matrix  $\Sigma$  is very small. It implies that the resulting covariance matrices have the same structure, and the minor difference is mainly due to random errors and computing errors.
- 2. There is no doubt that the matrix *B* obtained using a banded Toeplitz structure has a smaller F-norm difference than those using MA(1), *CS*, AR(1) or ARMA(1,1) structures. This is because all those four cases are special cases of banded Toeplitz structures. The bigger the definition area, the smaller the optimal function value. However, the difference in their F-norms is rather small, implying that the identified Toeplitz structure actually reduces to one of the special structures.
- 3. The F-norm discrepancy between B obtained from our proposed method and the true covariance matrix Σ is smaller than the F-norm discrepancy between A and Σ. It implies that the proposed method reduces random noises, so that a better estimator of the covariance matrix is obtained.

Since the real covariance matrix  $\Sigma$  is unknown in practice, the F-norm discrepancy between *B* and the sample covariance matrix *A* becomes very important in finding the underlying structure of  $\Sigma$ .

For  $\Sigma$  having a banded Toeplitz structure, apart from the above values of  $n, m, \sigma^2$  we actually randomly generate parameters  $c_1, c_2, \dots, c_p$  from uniform distribution U(0,1) but require the positive definiteness of covariance matrix  $\Sigma$ , where we choose p = m - 1. The results are presented in Table 13, where similarly each value is the average after repeating 100 times of simulations. The first column gives the dimension of the true covariance matrix  $\Sigma$ , the second column provides the F-norm discrepancy between the sample covariance matrix A and  $\Sigma$ , the rest columns present the F-norm discrepancy between  $A, \Sigma$  and the matrix B obtained using the specified structures indicated in the second line, respectively.

Form Table 13 we make conclusions below.

- 1. The matrix *B* having the minimum F-norm difference satisfies  $L^*(\Sigma, B) < L^*(\Sigma, A)$ , meaning that the estimator *B* obtained using the proposed regularization method removes the random noise in the sample covariance matrix *A*, so that a better estimator of the covariance matrix  $\Sigma$  is obtained.
- 2. The matrix *B* having the minimum F-norm difference and obtained by using a banded Toeplitz structure has a small value of  $L^*(\Sigma, B)$ . Hence, we can really find the structure of the true covariance matrix  $\Sigma$  through *B* which uses the information of the sample covariance matrix *A*.

It is worth noting that conclusions made in this section are valid for all the studied structures of  $\Sigma$  and all the settings of parameters  $n, m, \sigma^2, c, r, c_1, c_2, \dots, c_p$ . Thus, the proposed method is reliable in this sense. Especially, the proposed regularization method works very well even for high-dimensional covariance matrix, where we considered the cases for the sample size n = 100 and the dimension of the covariance matrix m = 100, 200, 500 and 1000, respectively.

#### **5** Discussion

For an estimator A of the covariance matrix  $\Sigma$ , we propose a new regularization method to find the matrix B that has a banded Toeplitz structure. By doing so, we not only find the structure of  $\Sigma$  but also a better estimator B in the sense of F-norm difference function. The method reveals the underlying covariance/correlation structure of the random process, and also reduces random noises.

We choose the sample covariance matrix A as the estimator of  $\Sigma$  in the simulations. Actually, we do not always have to choose the sample covariance matrix. Theoretically, the matrix A can be taken as any other reasonable estimators obtained using certain statistical method. Once matrix A is obtained, we can always do the regularization using the proposed method, so that the estimation can be improved in the sense of F-norm difference. Note that there is no strict condition imposed to the symmetric matrix A, which may be singular and/or high-dimensional. Also, the distribution of data which formulates the estimator A might be unknown. Hence the proposed method is more flexible in this sense.

Our simulation studies only considered the data having Gaussian distribution and the paper only studied the F-norm discrepancy. In practice, there are other distributions and other loss functions such as quadratic loss function. The study of such issues may be challenging but important. We will investigate this in our follow-up work.

8

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$\sigma^2 = 0.50$			I	3	
$\sigma^2 = 0.5$	0	Same s	tructure	Toe	plitz
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
MA(1)	0.3144	0.3140	0.0004	0.3010	0.0134
CS	0.1303	0.1105	0.0198	0.1074	0.0229
AR(1)	0.3153	0.3145	0.0008	0.3012	0.0141
ARMA-r=0.1	0.3350	0.3342	0.0009	0.3214	0.0136
ARMA-r=0.35	0.2919	0.2908	0.0011	0.2780	0.0140
ARMA-r=0.6	0.2304	0.2292	0.0012	0.2147	0.0157
			F	3	
$\sigma^2 = 1$		Same s	tructure	Toe	plitz
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
MA(1)	0.3134	0.3128	0.0005	0.2998	0.0136
CS	0.1191	0.1018	0.0173	0.0989	0.0202
AR(1)	0.3115	0.3107	0.0008	0.2976	0.0139
ARMA-r=0.1	0.3367	0.3359	0.0007	0.3234	0.0133
ARMA-r=0.35	0.2923	0.2911	0.0012	0.2781	0.0142
$\Delta RM\Delta_r=0.6$	0 2279	0.2266	0.0013	0 2120	0.0150
AIGMA-1=0.0	0.2277	0.2266 0.0013 0.2129 0		0.0150	
	0.2279	0.2200	<u>10.0015</u>	0.2129 3	0.0150
$\sigma^2 = 2$	0.2279	Same s	tructure	0.2129 3 Toe	plitz
$\frac{\sigma^2 = 2}{\Sigma}$	$L^*_{\Sigma,A}$	Same s $L_{A,B}^*$	$\frac{1}{\frac{1}{\sum_{k=1}^{\infty}}}$	$\frac{10.2129}{100}$	$\frac{\text{plitz}}{L_{\Sigma,B}^*}$
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$	$L^*_{\Sigma,A}$ 0.3162	Same s $L_{A,B}^*$ 0.3156	$\frac{1}{1}$ tructure $\frac{L_{\Sigma,B}^*}{0.0005}$	$ \begin{array}{c} Interpretation of the second secon$	$\frac{L_{\Sigma,B}^*}{0.0138}$
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$	$\frac{L_{\Sigma,A}^{*}}{0.3162}$ 0.1185	Same s $L_{A,B}^*$ 0.3156 0.1036	$\frac{1}{\frac{L_{\Sigma,B}^{*}}{0.0005}}$	$[0.2129]{3} \\ \hline Toe \\ L^*_{A,B} \\ 0.3023 \\ 0.1007 \\ [10pt]$	$ \begin{array}{c} \text{plitz} \\ \underline{L_{\Sigma,B}^{*}} \\ 0.0138 \\ 0.0178 \end{array} $
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$ $\frac{\sigma^2 = 2}{CS}$ $\frac{MA(1)}{CS}$ $AR(1)$	$\frac{L^*_{\Sigma,A}}{0.3162}$ 0.3162 0.3137	$\begin{array}{c} \text{Same s} \\ \hline \\ L_{A,B}^{*} \\ 0.3156 \\ 0.1036 \\ 0.3131 \end{array}$	$\frac{L_{\Sigma,B}^{*}}{0.0005}$	$\begin{array}{c} \hline 0.2129\\ \hline 0.2129\\ \hline \\ 3\\ \hline \\ L^*_{A,B}\\ \hline 0.3023\\ \hline 0.1007\\ \hline 0.3006 \end{array}$	$ \begin{array}{r} \begin{array}{c} \text{plitz} \\ \hline L_{\Sigma,B}^{*} \\ \hline 0.0138 \\ \hline 0.0178 \\ \hline 0.0131 \end{array} $
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$ $\frac{AR(1)}{CS}$ $\frac{AR(1)}{ARMA-r=0.1}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.3162 \\ 0.1185 \\ 0.3137 \\ 0.3354 \end{array}$	$\begin{array}{c} \text{Same s} \\ \hline \\ \text{Same s} \\ \hline \\ L^*_{A,B} \\ 0.3156 \\ \hline 0.1036 \\ \hline 0.3131 \\ \hline 0.3345 \end{array}$	$\begin{array}{r} \hline & \\ \hline \\ \hline$	$\begin{array}{c} \hline 0.2129\\ \hline 0.2129\\ \hline 0.2129\\ \hline 0.2129\\ \hline 0.200\\ \hline 0.3023\\ \hline 0.1007\\ \hline 0.3006\\ \hline 0.3216\\ \hline \end{array}$	$\begin{array}{c} \text{plitz} \\ \hline L_{\Sigma,B}^{*} \\ \hline 0.0138 \\ \hline 0.0138 \\ \hline 0.0131 \\ \hline 0.0138 \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{MA(1)}{CS}$ $\frac{AR(1)}{ARMA-r=0.1}$ $ARMA-r=0.35$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.3162 \\ 0.3137 \\ 0.3354 \\ 0.2919 \end{array}$	$\begin{array}{c} \text{Same s} \\ \hline \\ \text{Same s} \\ \hline \\ L^*_{A,B} \\ 0.3156 \\ 0.3131 \\ 0.3345 \\ 0.2908 \end{array}$	$\begin{array}{c} \hline 0.0013 \\ \hline H \\ tructure \\ \hline L_{\Sigma,B}^* \\ \hline 0.0005 \\ \hline 0.0149 \\ \hline 0.0006 \\ \hline 0.0010 \\ \hline 0.0011 \\ \hline \end{array}$	$\begin{array}{c} \hline 0.2129\\ \hline 0.2129\\ \hline 0.2129\\ \hline 0.212\\ \hline 0.212\\ \hline 0.3023\\ \hline 0.1007\\ \hline 0.3006\\ \hline 0.3216\\ \hline 0.2781\\ \hline \end{array}$	$\begin{array}{c} \text{plitz} \\ \hline L_{\Sigma,B}^{*} \\ \hline 0.0138 \\ \hline 0.0131 \\ \hline 0.0138 \\ \hline 0.0138 \\ \hline 0.0138 \\ \hline 0.0138 \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta AA(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta RMA-r=0.1}{\Delta RMA-r=0.6}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.3162 \\ 0.1185 \\ 0.3137 \\ 0.3354 \\ 0.2919 \\ 0.2297 \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.3156 \\ 0.3131 \\ 0.3345 \\ 0.2908 \\ 0.2283 \end{array}$	$\begin{array}{c} \text{I} \\ $	$\begin{array}{c} \text{(0.2129)}\\ \hline \text{(0.2129)}\\ \hline \text{(0.2112)}\\ \hline \text{(0.2110)}\\ \hline \\text{(0.2110)}\\ \hline \\text{(0.2110)}\\ \hline \\text{(0.2110)}\\ \hline \\text{(0.2110)}\\ \hline \\text{(0.2110)}\\ \hline \ \text{(0.2110)}\\ \hline \ \ \text{(0.2110)}\\ \hline \ \ \\text{(0.2110)}\\ \hline \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $	$\begin{array}{c} \text{plitz}\\ \hline L_{\Sigma,B}^{*}\\ 0.0138\\ \hline 0.0138\\ \hline 0.0131\\ \hline 0.0138\\ \hline 0.0138\\ \hline 0.0157\\ \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta AR(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta RMA-r=0.1}{\Delta RMA-r=0.6}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.3162 \\ 0.1185 \\ 0.3137 \\ 0.3354 \\ 0.2919 \\ 0.2297 \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.3156 \\ 0.1036 \\ 0.3131 \\ 0.3345 \\ 0.2908 \\ 0.2283 \\ \end{array}$	$\begin{array}{c} 0.0013\\ \hline \text{H}\\ \text{tructure}\\ \hline L_{\Sigma,B}^*\\ 0.0005\\ \hline 0.0149\\ \hline 0.0006\\ \hline 0.0010\\ \hline 0.0011\\ \hline 0.0014\\ \hline \text{H}\\ \end{array}$	$\begin{array}{c} 0.2129\\ \hline 0.2129\\ \hline 0.2129\\ \hline 0.2129\\ \hline 0.3023\\ \hline 0.3023\\ \hline 0.3006\\ \hline 0.3216\\ \hline 0.2781\\ \hline 0.2140\\ \hline \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0138 \\ \hline 0.0178 \\ 0.0131 \\ \hline 0.0138 \\ 0.0138 \\ \hline 0.0138 \\ \hline 0.0157 \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta RMA-r=0.1}{\Delta RMA-r=0.6}$ $\frac{\Delta CS}{\Delta RMA-r=0.6}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.3162 \\ 0.1185 \\ 0.3137 \\ 0.3354 \\ 0.2919 \\ 0.2297 \end{array}$	$\begin{array}{c} \text{Same s} \\ \text{Same s} \\ L_{A,B}^{*} \\ 0.3156 \\ 0.1036 \\ 0.3131 \\ 0.3345 \\ 0.2908 \\ 0.2283 \\ \hline \\ \text{Same s} \end{array}$	$\begin{array}{c} 0.0013\\\hline \text{H}\\ \text{tructure}\\ L_{\Sigma,B}^{*}\\\hline 0.0005\\\hline 0.0149\\\hline 0.0006\\\hline 0.0010\\\hline 0.0011\\\hline 0.0014\\\hline \text{H}\\ \text{tructure}\\ \end{array}$	$\begin{array}{c} (0.2129)\\ (0.2129)\\ (0.2129)\\ (0.2129)\\ (0.2129)\\ (0.3006)\\ (0.3216)\\ (0.3216)\\ (0.2781)\\ (0.2140)\\$	$\begin{array}{c} 0.0130\\ \hline 0.0130\\ \hline L_{\Sigma,B}^{*}\\ \hline 0.0138\\ \hline 0.0131\\ \hline 0.0138\\ \hline 0.0138\\ \hline 0.0138\\ \hline 0.0157\\ \hline \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta RMA-r=0.1}{\Delta RMA-r=0.6}$ $\frac{\sigma^2 = 4}{\Sigma}$	$\begin{array}{c} L^{*}_{\Sigma,A} \\ 0.3162 \\ 0.1185 \\ 0.3137 \\ 0.3354 \\ 0.2919 \\ 0.2297 \\ \\ L^{*}_{\Sigma,A} \end{array}$	$\begin{array}{c} \text{Same s} \\ \text{Same s} \\ L_{A,B}^{*} \\ 0.3156 \\ 0.1036 \\ 0.3131 \\ 0.3345 \\ 0.2908 \\ 0.2283 \\ \hline \\ \text{Same s} \\ L_{A,B}^{*} \end{array}$	$\begin{array}{c} 0.0013\\\hline \text{H}\\ \text{tructure}\\ L_{\Sigma,B}^{*}\\\hline 0.0005\\\hline 0.0149\\\hline 0.0006\\\hline 0.0010\\\hline 0.0011\\\hline 0.0014\\\hline \text{H}\\ \text{tructure}\\ L_{\Sigma,B}^{*}\\ \end{array}$	$\begin{array}{c} (0.2129)\\ (0.2129)\\ (0.2129)\\ (0.2129)\\ (0.2129)\\ (0.3006)\\ (0.3006)\\ (0.3216)\\ (0.3216)\\ (0.2781)\\ (0.2140)\\$	$\begin{array}{c} 0.0130\\ \hline 0.0130\\ \hline L_{\Sigma,B}^{*}\\ \hline 0.0138\\ \hline 0.0131\\ \hline 0.0138\\ \hline 0.0138\\ \hline 0.0138\\ \hline 0.0157\\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \\ L_{\Sigma,B}^{*}\\ \hline \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta RMA-r=0.1}{\Delta RMA-r=0.6}$ $\frac{\sigma^2 = 4}{\Sigma}$ $\frac{\Delta L}{\Delta RMA(1)}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.3162 \\ 0.1185 \\ 0.3137 \\ 0.3354 \\ 0.2919 \\ 0.2297 \\ \hline \\ L^*_{\Sigma,A} \\ 0.3155 \end{array}$	$\begin{array}{c} \text{Same s} \\ \text{Same s} \\ L^*_{A,B} \\ 0.3156 \\ 0.1036 \\ 0.3131 \\ 0.3345 \\ 0.2908 \\ 0.2283 \\ \hline \\ \text{Same s} \\ L^*_{A,B} \\ 0.3149 \\ \end{array}$	$\begin{array}{c} 0.0013\\\hline \text{I}\\ \hline \text{I}\\ \text{tructure}\\ L^*_{\Sigma,B}\\ 0.0005\\\hline 0.0149\\\hline 0.0006\\\hline 0.0010\\\hline 0.0011\\\hline 0.0014\\\hline \text{I}\\ \text{tructure}\\ L^*_{\Sigma,B}\\\hline 0.0005\\\hline \end{array}$	$\begin{array}{c} 0.2129\\ \hline \text{Toep}\\ L^*_{A,B}\\ 0.3023\\ 0.1007\\ 0.3006\\ 0.3216\\ 0.2781\\ 0.2140\\ \textbf{3}\\ \hline \text{Toep}\\ L^*_{A,B}\\ 0.3013 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ \hline 0.0138 \\ \hline 0.0138 \\ \hline 0.0131 \\ \hline 0.0138 \\ \hline 0.0138 \\ \hline 0.0138 \\ \hline 0.0157 \\ \hline \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{\Delta R(1)}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta RMA-r=0.35}{\Delta RMA-r=0.6}$ $\frac{\sigma^2 = 4}{\Sigma}$ $\frac{\Delta L}{\Delta CS}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.3162 \\ 0.1185 \\ 0.3137 \\ 0.3354 \\ 0.2919 \\ 0.2297 \\ \hline \\ L^*_{\Sigma,A} \\ 0.3155 \\ 0.1233 \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.3156 \\ 0.1036 \\ 0.3131 \\ 0.3345 \\ 0.2908 \\ 0.2283 \\ \hline \\ \text{Same s} \\ L_{A,B}^{*} \\ 0.3149 \\ 0.1064 \\ \end{array}$	$\begin{array}{c} 0.0013\\\hline \text{I}\\ \hline \text{I}\\ \text{tructure}\\ L_{\Sigma,B}^{*}\\ 0.0005\\\hline 0.0149\\\hline 0.0006\\\hline 0.0010\\\hline 0.0011\\\hline 0.0014\\\hline \text{I}\\ \text{tructure}\\ L_{\Sigma,B}^{*}\\\hline 0.0005\\\hline 0.00170\\\hline \end{array}$	$\begin{array}{c} \text{(0.2129)}\\ \hline \text{Toep}\\ L_{A,B}^*\\ 0.3023\\ 0.1007\\ 0.3006\\ 0.3216\\ 0.2781\\ 0.2140\\ \textbf{3}\\ \hline \text{Toep}\\ L_{A,B}^*\\ 0.3013\\ 0.1035\\ \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ \hline 0.0138 \\ \hline 0.0138 \\ \hline 0.0131 \\ \hline 0.0138 \\ \hline 0.0138 \\ \hline 0.0138 \\ \hline 0.0138 \\ \hline 0.0157 \\ \hline \\ \hline \\ plitz \\ L_{\Sigma,B}^{*} \\ \hline 0.0142 \\ \hline 0.0199 \\ \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{\Delta R(1)}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta RMA-r=0.35}{\Delta RMA-r=0.6}$ $\frac{\sigma^2 = 4}{\Sigma}$ $\frac{\Delta CS}{\Delta R(1)}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.3162 \\ 0.1185 \\ 0.3137 \\ 0.3354 \\ 0.2919 \\ 0.2297 \\ \hline \\ L^*_{\Sigma,A} \\ 0.3155 \\ 0.1233 \\ 0.3129 \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.3156 \\ 0.1036 \\ 0.3131 \\ 0.3345 \\ 0.2908 \\ 0.2283 \\ \hline \\ \text{Same s} \\ L_{A,B}^{*} \\ 0.3149 \\ 0.1064 \\ 0.3120 \\ \end{array}$	$\begin{array}{c} \hline 0.0013\\ \hline 0.0013\\ \hline I\\ \hline I\\ \hline I\\ \hline L_{\Sigma,B}\\ \hline 0.0005\\ \hline 0.0149\\ \hline 0.0006\\ \hline 0.0010\\ \hline 0.0011\\ \hline 0.0014\\ \hline I\\ \hline I\\ \hline 0.0014\\ \hline I\\ \hline I\\ \hline L_{\Sigma,B}\\ \hline 0.0005\\ \hline 0.0170\\ \hline 0.0009 \end{array}$	$\begin{array}{c} \text{(0.2129)}\\ \hline \text{Toep}\\ L_{A,B}^*\\ 0.3023\\ 0.1007\\ 0.3006\\ 0.3216\\ 0.2781\\ 0.2140\\ \textbf{3}\\ \hline \text{Toep}\\ L_{A,B}^*\\ 0.3013\\ 0.1035\\ 0.2986\\ \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ \hline 0.0138 \\ \hline 0.0138 \\ \hline 0.0131 \\ \hline 0.0138 \\ \hline 0.0131 \\ \hline 0.0138 \\ \hline 0.0138 \\ \hline 0.0138 \\ \hline 0.0157 \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \hline \\ \hline \\$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$ MA(1) CS AR(1) CS AR(1) ARMA-r=0.1	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.3162 \\ 0.1185 \\ 0.3137 \\ 0.3354 \\ 0.2919 \\ 0.2297 \\ \hline \\ L^*_{\Sigma,A} \\ 0.3155 \\ 0.1233 \\ 0.3129 \\ 0.3345 \\ \hline \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^* \\ 0.3156 \\ 0.1036 \\ 0.3131 \\ 0.3345 \\ 0.2908 \\ 0.2283 \\ \hline \\ \text{Same s} \\ L_{A,B}^* \\ 0.3149 \\ 0.1064 \\ 0.3120 \\ 0.3336 \end{array}$	$\begin{array}{c} 0.0013\\\hline \text{I}\\ \hline \text{I}\\ \text{tructure}\\ \hline L_{\Sigma,B}^{*}\\ 0.0005\\\hline 0.0149\\\hline 0.0006\\\hline 0.0010\\\hline 0.0011\\\hline 0.0014\\\hline \hline \text{I}\\ \text{tructure}\\ \hline L_{\Sigma,B}^{*}\\\hline 0.0005\\\hline 0.0170\\\hline 0.0009\\\hline 0.0009\\\hline 0.0009\end{array}$	$\begin{array}{c} \text{(0.2129)}\\ \hline \text{Toep}\\ L_{A,B}^*\\ 0.3023\\ 0.1007\\ 0.3006\\ 0.3216\\ 0.2781\\ 0.2140\\ \textbf{3}\\ \hline \text{Toep}\\ L_{A,B}^*\\ 0.3013\\ 0.1035\\ 0.2986\\ 0.3205\\ \end{array}$	$\begin{array}{c} \text{o}, \text{o},$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$ MA(1) CS AR(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35	$\begin{array}{c} L_{\Sigma,A}^{*} \\ 0.3162 \\ 0.1185 \\ 0.3137 \\ 0.3354 \\ 0.2919 \\ 0.2297 \\ \hline \\ L_{\Sigma,A}^{*} \\ 0.3155 \\ 0.1233 \\ 0.3129 \\ 0.3345 \\ 0.2932 \\ \hline \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.3156 \\ 0.1036 \\ 0.3131 \\ 0.3345 \\ 0.2908 \\ 0.2283 \\ \hline \\ \text{Same s} \\ L_{A,B}^{*} \\ 0.3149 \\ 0.1064 \\ 0.3120 \\ 0.3336 \\ 0.2920 \\ \end{array}$	$\begin{array}{c} 0.0013\\\hline \text{I}\\ \hline \text{I}\\ \hline \text{I}\\ \text{tructure}\\ \hline L_{\Sigma,B}^*\\ 0.0005\\\hline 0.0149\\\hline 0.0006\\\hline 0.0010\\\hline 0.0011\\\hline 0.0014\\\hline \hline \text{I}\\ \hline \text{tructure}\\ \hline L_{\Sigma,B}^*\\\hline 0.0005\\\hline 0.0170\\\hline 0.0009\\\hline 0.0009\\\hline 0.00013\\\hline \end{array}$	$\begin{array}{c} \text{(0.2129)}\\ \hline \text{Toe}\\ L^*_{A,B}\\ 0.3023\\ 0.1007\\ 0.3006\\ 0.3216\\ 0.2781\\ 0.2140\\ \textbf{3}\\ \hline \text{Toe}\\ L^*_{A,B}\\ 0.3013\\ 0.1035\\ 0.2986\\ 0.3205\\ 0.2786\\ \end{array}$	$\begin{array}{c} \text{0.0130}\\ \hline \text{0.0130}\\ \hline L_{\Sigma,B}^*\\ \hline 0.0138\\ \hline 0.0131\\ \hline 0.0138\\ \hline 0.0138\\ \hline 0.0138\\ \hline 0.0138\\ \hline 0.0157\\ \hline \\ \hline$

**Table 1** Simulation results with m = 50; c = 0.25.

**Table 2** Simulation results with m = 50; c = 0.5.

			I	3	
$\sigma^2 = 0.5$	0	Same s	tructure	Toe	plitz
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
MA(1)	0.2584	0.2577	0.0006	0.2438	0.0146
CS	0.0500	0.0284	0.0216	0.0278	0.0222
AR(1)	0.2399	0.2383	0.0016	0.2247	0.0152
ARMA-r=0.2	0.3184	0.3173	0.0012	0.3053	0.0131
ARMA-r=0.45	0.2539	0.2524	0.0016	0.2397	0.0143
ARMA-r=0.75	0.1772	0.1749	0.0023	0.1607	0.0165
			I	3	
$\sigma^2 = 1$	ĺ	Same s	tructure	Toe	plitz
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
MA(1)	0.2590	0.2582	0.0007	0.2436	0.0154
CS	0.0528	0.0284	0.0243	0.0278	0.0250
AR(1)	0.2406	0.2390	0.0016	0.2257	0.0149
ARMA-r=0.2	0.3187	0.3176	0.0012	0.3056	0.0132
ARMA-r=0.45	0.2553	0.2538	0.0016	0.2402	0.0151
ARMA-r=0.75	0.1785	0.1765	0.0020	0.1628	0.0157
2			E	3	
$\sigma^2 = 2$		Same s	H tructure	3 Toe	plitz
$\frac{\sigma^2 = 2}{\Sigma}$	$L^*_{\Sigma,A}$	Same s $L^*_{A,B}$	$\frac{\text{tructure}}{L_{\Sigma,B}^*}$	Toe $L_{A,B}^*$	$\frac{\text{plitz}}{L^*_{\Sigma,B}}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1)	$\frac{L_{\Sigma,A}^{*}}{0.2591}$	Same s $L_{A,B}^*$ 0.2585	$\frac{L_{\Sigma,B}^{*}}{0.0006}$	Toej $L_{A,B}^{*}$ 0.2440	$\frac{\text{plitz}}{L_{\Sigma,B}^*}$ 0.0151
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$	$rac{L^*_{\Sigma,A}}{0.2591}$ 0.0430	Same s $L_{A,B}^*$ 0.2585 0.0282	$ \frac{L_{\Sigma,B}^{*}}{0.0006} $	$     Toe     L^*_{A,B}     0.2440     0.0277 $	plitz $L_{\Sigma,B}^{*}$ 0.0151 0.0153
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1)	$L^*_{\Sigma,A}$ 0.2591 0.0430 0.2395	Same s $L_{A,B}^*$ 0.2585 0.0282 0.2375	$     \frac{F_{\Sigma,B}}{L_{\Sigma,B}^*}     0.0006     0.0148     0.0020     $		$     plitz     L^*_{\Sigma,B}     0.0151     0.0153     0.0153     0.0153 $
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.2	$L^*_{\Sigma,A}$ 0.2591 0.0430 0.2395 0.3185	Same s $L^*_{A,B}$ 0.2585 0.0282 0.2375 0.3173	$\begin{tabular}{c} F \\ \hline tructure \\ \hline L^*_{\Sigma,B} \\ \hline 0.0006 \\ \hline 0.0148 \\ \hline 0.0020 \\ \hline 0.0012 \end{tabular}$	$\begin{array}{c} \text{Toe} \\ \hline L_{A,B}^{*} \\ 0.2440 \\ 0.0277 \\ 0.2242 \\ 0.3045 \end{array}$	$\begin{array}{c} \text{plitz} \\ \hline L_{\Sigma,B}^{*} \\ 0.0151 \\ 0.0153 \\ \hline 0.0153 \\ 0.0140 \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45	$L^*_{\Sigma,A}$ 0.2591 0.0430 0.2395 0.3185 0.2534	Same s $L^*_{A,B}$ 0.2585 0.0282 0.2375 0.3173 0.2512	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	$\begin{array}{c} {} \\ \hline Toe \\ L^*_{A,B} \\ 0.2440 \\ 0.0277 \\ 0.2242 \\ 0.3045 \\ 0.2386 \end{array}$	$\begin{array}{c} \text{plitz}\\ \hline L_{\Sigma,B}^{*}\\ \hline 0.0151\\ \hline 0.0153\\ \hline 0.0153\\ \hline 0.0140\\ \hline 0.0148\\ \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75	$L^*_{\Sigma,A}$ 0.2591 0.0430 0.2395 0.3185 0.2534 0.1772	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.2585 \\ 0.0282 \\ 0.2375 \\ 0.3173 \\ 0.2512 \\ 0.1746 \end{array}$	$\begin{tabular}{ c c c c c } \hline F \\ \hline tructure \\ \hline L^*_{\Sigma,B} \\ \hline 0.0006 \\ \hline 0.0148 \\ \hline 0.0020 \\ \hline 0.0012 \\ \hline 0.0022 \\ \hline 0.0026 \\ \hline \end{tabular}$	$\begin{array}{c} {} \\ \hline Toe \\ L^*_{A,B} \\ 0.2440 \\ 0.0277 \\ 0.2242 \\ 0.3045 \\ 0.2386 \\ 0.1605 \end{array}$	$\begin{array}{c} \text{plitz}\\ \hline L_{\Sigma,B}^{*}\\ \hline 0.0151\\ \hline 0.0153\\ \hline 0.0153\\ \hline 0.0140\\ \hline 0.0148\\ \hline 0.0167\\ \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.2591 \\ 0.0430 \\ 0.2395 \\ 0.3185 \\ 0.2534 \\ 0.1772 \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.2585 \\ 0.0282 \\ 0.2375 \\ 0.3173 \\ 0.2512 \\ 0.1746 \\ \end{array}$	$\begin{tabular}{c} \hline H \\ \hline tructure \\ \hline L^*_{\Sigma,B} \\ \hline 0.0006 \\ \hline 0.0148 \\ \hline 0.0020 \\ \hline 0.0012 \\ \hline 0.0022 \\ \hline 0.0026 \\ \hline H \end{tabular}$	3 Toej <i>L<sup>*</sup><sub>A,B</sub></i> 0.2440 0.0277 0.2242 0.3045 0.2386 0.1605 3	$\begin{array}{c} \text{plitz}\\ \hline L_{\Sigma,B}^{*}\\ \hline 0.0151\\ \hline 0.0153\\ \hline 0.0153\\ \hline 0.0140\\ \hline 0.0148\\ \hline 0.0167\\ \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\sigma^{2} = 4$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.2591 \\ 0.0430 \\ 0.2395 \\ 0.3185 \\ 0.2534 \\ 0.1772 \end{array}$	Same s $L_{A,B}^*$ 0.2585           0.0282           0.2375           0.3173           0.2512           0.1746           Same s	$\begin{tabular}{c} F \\ \hline F \\ F$	$\begin{array}{c} & \\ & \\ \hline Toe_{j} \\ L^{*}_{A,B} \\ 0.2440 \\ 0.0277 \\ 0.2242 \\ 0.3045 \\ 0.2386 \\ 0.1605 \\ 3 \\ \hline Toe_{j} \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ \hline 0.0151 \\ 0.0153 \\ \hline 0.0153 \\ \hline 0.0140 \\ 0.0148 \\ \hline 0.0167 \\ \hline \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\sigma^2 = 4$ $\Sigma$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.2591 \\ 0.0430 \\ 0.2395 \\ 0.3185 \\ 0.2534 \\ 0.1772 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ \hline Toe \\ L^*_{A,B} \\ 0.2440 \\ 0.0277 \\ 0.2242 \\ 0.3045 \\ 0.2386 \\ 0.1605 \\ \end{array} \\ \begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0151 \\ 0.0153 \\ 0.0153 \\ 0.0140 \\ 0.0148 \\ 0.0167 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\sigma^2 = 4$ $\Sigma$ MA(1)	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.2591 \\ 0.0430 \\ 0.2395 \\ 0.3185 \\ 0.2534 \\ 0.1772 \\ \\ \hline \\ L^*_{\Sigma,A} \\ 0.2593 \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c c c } \hline & & & & & & & & & & & & & & & & & & $	$\begin{array}{c} & \\ & \\ \hline Toee \\ L^*_{A,B} \\ 0.2440 \\ 0.0277 \\ 0.2242 \\ 0.3045 \\ 0.2386 \\ 0.1605 \\ 3 \\ \hline \hline Toee \\ L^*_{A,B} \\ 0.2449 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0151 \\ 0.0153 \\ 0.0140 \\ 0.0148 \\ 0.0167 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0144 \\ \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\frac{\sigma^2 = 4}{\Sigma}$ MA(1) CS	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.2591 \\ 0.0430 \\ 0.2395 \\ 0.3185 \\ 0.2534 \\ 0.1772 \\ \\ \hline \\ L^*_{\Sigma,A} \\ 0.2593 \\ 0.0451 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c } \hline F \\ \hline tructure \\ \hline $L^*_{\Sigma,B}$ \\ \hline $0.0006$ \\ \hline $0.0020$ \\ \hline $0.0022$ \\ \hline $0.0026$ \\ \hline $0.0026$ \\ \hline $F$ \\ \hline $tructure $L^*_{\Sigma,B}$ \\ \hline $0.0006$ \\ \hline $0.0167$ \\ \hline \end{tabular}$	$\begin{array}{c} & \\ & \\ \hline Toe \\ L^*_{A,B} \\ 0.2440 \\ 0.0277 \\ 0.2242 \\ 0.3045 \\ 0.2386 \\ 0.1605 \\ \end{array} \\ \hline \hline \\ & \\ \hline \\$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0151 \\ 0.0153 \\ 0.0140 \\ 0.0148 \\ 0.0167 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0144 \\ 0.0173 \\ \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\frac{\sigma^2 = 4}{\Sigma}$ MA(1) CS AR(1)	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.2591 \\ 0.0430 \\ 0.2395 \\ 0.3185 \\ 0.2534 \\ 0.1772 \\ \hline \\ L^*_{\Sigma,A} \\ 0.2593 \\ 0.0451 \\ 0.2426 \\ \hline \end{array}$	Same s $L_{A,B}^*$ 0.2585           0.0282           0.2375           0.3173           0.2512           0.1746           Same s $L_{A,B}^*$ 0.2587           0.02587           0.0284           0.2409	$\begin{tabular}{ c c c c c } \hline F \\ \hline tructure \\ \hline $L^*_{\Sigma,B}$ \\ \hline 0.0006 \\ \hline 0.0128 \\ \hline 0.0020 \\ \hline 0.0022 \\ \hline 0.0026 \\ \hline \hline F \\ tructure \\ \hline $L^*_{\Sigma,B}$ \\ \hline 0.0006 \\ \hline 0.0167 \\ \hline 0.0017 \\ \hline \end{tabular}$	$\begin{array}{c} & \\ & \\ \hline Toer \\ L^*_{A,B} \\ 0.2440 \\ 0.0277 \\ 0.2242 \\ 0.3045 \\ 0.2386 \\ 0.1605 \\ \hline 0.2386 \\ 0.1605 \\ \hline \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^* \\ 0.0151 \\ 0.0153 \\ 0.0153 \\ 0.0148 \\ 0.0167 \\ \hline \\ plitz \\ L_{\Sigma,B}^* \\ 0.0144 \\ 0.0173 \\ 0.0153 \\ \hline \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.45 $\sigma^2 = 4$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.2591 \\ 0.0430 \\ 0.2395 \\ 0.3185 \\ 0.2534 \\ 0.1772 \\ \hline \\ L^*_{\Sigma,A} \\ 0.2593 \\ 0.0451 \\ 0.2426 \\ 0.3195 \\ \hline \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c } \hline F \\ \hline tructure \\ \hline $L^*_{\Sigma,B}$ \\ \hline 0.0006 \\ \hline 0.0148 \\ \hline 0.0020 \\ \hline 0.0022 \\ \hline 0.0026 \\ \hline $F$ \\ \hline tructure \\ \hline $L^*_{\Sigma,B}$ \\ \hline 0.0006 \\ \hline 0.0167 \\ \hline 0.0017 \\ \hline 0.0013 \\ \hline \end{tabular}$	$\begin{array}{c} & \\ & \\ \hline Toer \\ L^*_{A,B} \\ 0.2440 \\ 0.0277 \\ 0.2242 \\ 0.3045 \\ 0.2386 \\ 0.1605 \\ \hline \\ 0.2386 \\ 0.1605 \\ \hline \\ 0.2386 \\ 0.2449 \\ 0.2449 \\ 0.0278 \\ 0.2273 \\ 0.3053 \\ \hline \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0151 \\ 0.0153 \\ 0.0153 \\ 0.0140 \\ 0.0148 \\ 0.0167 \\ \hline \\ \begin{array}{c} L_{\Sigma,B}^{*} \\ 0.0144 \\ 0.0173 \\ 0.0153 \\ \hline \\ 0.0142 \\ \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.45 $\sigma^2 = 4$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.2	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.2591 \\ 0.0430 \\ 0.2395 \\ 0.3185 \\ 0.2534 \\ 0.1772 \\ \hline \\ L^*_{\Sigma,A} \\ 0.2593 \\ 0.0451 \\ 0.2426 \\ 0.3195 \\ 0.2544 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c } \hline F \\ \hline tructure \\ \hline L^*_{\Sigma,B} \\ \hline 0.0006 \\ \hline 0.0148 \\ \hline 0.0020 \\ \hline 0.0022 \\ \hline 0.0026 \\ \hline 0.0026 \\ \hline F \\ \hline tructure \\ \hline L^*_{\Sigma,B} \\ \hline 0.0006 \\ \hline 0.0167 \\ \hline 0.0017 \\ \hline 0.0013 \\ \hline 0.0020 \\ \hline \end{tabular}$	$\begin{array}{c} & \\ & \\ \hline Toer \\ L^*_{A,B} \\ 0.2440 \\ 0.0277 \\ 0.2242 \\ 0.3045 \\ 0.2386 \\ 0.1605 \\ \hline \\ 0.2386 \\ 0.1605 \\ \hline \\ 0.2386 \\ 0.2449 \\ 0.0278 \\ 0.2449 \\ 0.0278 \\ 0.2273 \\ 0.3053 \\ 0.2392 \\ \hline \end{array}$	$\begin{array}{c} \\ plitz \\ L_{\Sigma,B}^{*} \\ 0.0151 \\ 0.0153 \\ 0.0153 \\ 0.0140 \\ 0.0140 \\ 0.0148 \\ 0.0167 \\ \hline \\ plitz \\ L_{\Sigma,B}^{*} \\ 0.0144 \\ 0.0173 \\ 0.0153 \\ 0.0142 \\ 0.0151 \\ \hline \end{array}$

-2 0.50		В				
$\sigma^2 = 0.5$	0	Same s	tructure	Toe	plitz	
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	
CS	0.0393	0.0079	0.0314	0.0078	0.0315	
AR(1)	0.1356	0.1308	0.0049	0.1186	0.0170	
ARMA-r=0.25	0.2887	0.2866	0.0022	0.2751	0.0136	
ARMA-r=0.5	0.1994	0.1955	0.0039	0.1831	0.0163	
ARMA-r=0.8	0.1271	0.1219	0.0051	0.1094	0.0177	
			I	3		
$\sigma^2 = 1$		Same s	tructure	Toe	plitz	
$\Sigma$	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	
CS	0.0263	0.0076	0.0187	0.0075	0.0189	
AR(1)	0.1351	0.1302	0.0049	0.1162	0.0189	
ARMA-r=0.25	0.2862	0.2836	0.0027	0.2716	0.0146	
ARMA-r=0.5	0.2024	0.1988	0.0035	0.1869	0.0155	
ARMA-r=0.8	0.1268	0.1220	0.0049	0.1089	0.0179	
			I	3		
$\sigma^2 = 2$		Same s	tructure	Toe	plitz	
$\Sigma$	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	
CS	0.0260	0.0077	0.0182	0.0076	0.0184	
AR(1)	0.1350	0.1303	0.0047	0.1191	0.0159	
ARMA-r=0.25	0.2899	0.2879	0.0021	0.2760	0.0139	
ARMA-r=0.5	0.2038	0.2004	0.0034	0.1878	0.0160	
ARMA-r=0.8	0.1250	0.1201	0.0050	0.1077	0.0173	
			E	3		
$\sigma^2 = 4$		Same s	tructure	Toe	plitz	
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	
CS	0.0301	0.0078	0.0223	0.0076	0.0224	
AR(1)	0.1334	0.1283	0.0051	0.1168	0.0166	
ARMA-r=0.25	0.2888	0.2861	0.0026	0.2738	0.0149	
ARMA-r=0.5	0.2028	0.1987	0.0042	0.1874	0.0154	
ARMA-r=0.8	0.1251	0.1193	0.0059	0.1076	0.0176	

**Table 3** Simulation results with m = 50; c = 0.75.

**Table 4** Simulation results with m = 100; c = 0.25.

			I	3	
$\sigma^{2} = 0.5$	0	Same s	tructure	Toe	plitz
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
MA(1)	0.4758	0.4756	0.0002	0.4654	0.0105
CS	0.1342	0.1190	0.0152	0.1173	0.0169
AR(1)	0.4748	0.4745	0.0003	0.4643	0.0105
ARMA-r=0.1	0.4993	0.4990	0.0003	0.4891	0.0102
ARMA-r=0.35	0.4474	0.4469	0.0005	0.4366	0.0108
ARMA-r=0.6	0.3682	0.3676	0.0006	0.3559	0.0123
			ŀ	3	
$\sigma^2 = 1$	ĺ	Same s	tructure	Toe	plitz
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
MA(1)	0.4766	0.4764	0.0003	0.4662	0.0105
CS	0.1399	0.1204	0.0195	0.1187	0.0212
AR(1)	0.4743	0.4740	0.0003	0.4637	0.0106
ARMA-r=0.1	0.5001	0.4998	0.0003	0.4899	0.0102
ARMA-r=0.35	0.4481	0.4476	0.0005	0.4367	0.0113
ARMA-r=0.6	0.3676	0.3670	0.0007	0.3551	0.0125
ARMA-r=0.6 0.3676					
			F	3	
$\sigma^2 = 2$	I	Same s	H tructure	3 Toe	plitz
$\sigma^2 = 2$ $\Sigma$	$L^*_{\Sigma,A}$	Same s $L^*_{A,B}$	$\frac{\text{tructure}}{L_{\Sigma,B}^*}$	Toe $L_{A,B}^*$	$\frac{\text{plitz}}{L_{\Sigma,B}^*}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1)	$\frac{L^*_{\Sigma,A}}{0.4764}$	Same s $\frac{L_{A,B}^*}{0.4762}$	$\frac{L_{\Sigma,B}^*}{0.0002}$	Toe $L_{A,B}^{*}$ 0.4658	plitz $L^*_{\Sigma,B}$ 0.0106
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS	$L^*_{\Sigma,A}$ 0.4764 0.1352	Same s $L_{A,B}^*$ 0.4762 0.1174	$\frac{L_{\Sigma,B}^{*}}{0.0002}$	Toe $L_{A,B}^*$ 0.4658 0.1157	plitz $L_{\Sigma,B}^{*}$ 0.0106 0.0195
$\sigma^2 = 2$ $\Sigma$ MA(1) CS AR(1)	$L^*_{\Sigma,A}$ 0.4764 0.1352 0.4735	Same s $L^*_{A,B}$ 0.4762 0.1174 0.4731	$F_{\Sigma,B} = \frac{L_{\Sigma,B}^*}{0.0002}$	Toe $L_{A,B}^*$ 0.4658 0.1157 0.4623	$ \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ \hline 0.0106 \\ \hline 0.0195 \\ \hline 0.0111 \\ \end{array} $
$\sigma^2 = 2$ $\frac{\Sigma}{MA(1)}$ CS AR(1) ARMA-r=0.1	$L^*_{\Sigma,A}$ 0.4764 0.1352 0.4735 0.5003	Same s $L_{A,B}^*$ 0.4762 0.1174 0.4731 0.5000	$F_{\Sigma,B} = \frac{L_{\Sigma,B}^*}{0.0002}$ 0.0178 0.0004 0.0003	$\begin{array}{c} \text{Toe}\\ L_{A,B}^{*}\\ 0.4658\\ 0.1157\\ 0.4623\\ 0.4902 \end{array}$	$\begin{array}{c} \text{plitz} \\ \hline L_{\Sigma,B}^{*} \\ 0.0106 \\ \hline 0.0195 \\ \hline 0.0111 \\ \hline 0.0101 \end{array}$
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35	$L^*_{\Sigma,A}$ 0.4764 0.1352 0.4735 0.5003 0.4485	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$Fructure \\ L^*_{\Sigma,B} \\ 0.0002 \\ 0.0178 \\ 0.0004 \\ 0.0003 \\ 0.0005 \\ Fruction \\ Fructi$	Toe $L^*_{A,B}$ 0.4658 0.1157 0.4623 0.4902 0.4373	$\begin{array}{c} \text{plitz} \\ \hline L^*_{\Sigma,B} \\ 0.0106 \\ \hline 0.0195 \\ \hline 0.0111 \\ \hline 0.0101 \\ \hline 0.0112 \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6	$L^*_{\Sigma,A}$ 0.4764 0.1352 0.4735 0.5003 0.4485 0.3694	Same s $L_{A,B}^*$ 0.4762 0.1174 0.4731 0.5000 0.4480 0.3688	$\begin{tabular}{ c c c c c }\hline & & & & & & \\ \hline & & & & & & & \\ \hline & & & &$	Toe $L^*_{A,B}$ 0.4658 0.1157 0.4623 0.4902 0.4373 0.3569	$\begin{array}{c} \text{plitz} \\ \hline L^*_{\Sigma,B} \\ \hline 0.0106 \\ \hline 0.0195 \\ \hline 0.0111 \\ \hline 0.0101 \\ \hline 0.0112 \\ \hline 0.0125 \\ \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.4764 \\ 0.1352 \\ 0.4735 \\ 0.5003 \\ 0.4485 \\ 0.3694 \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.4762 \\ 0.1174 \\ 0.4731 \\ 0.5000 \\ 0.4480 \\ 0.3688 \end{array}$	$\begin{tabular}{c} \hline H \\ \hline tructure \\ $L^*_{\Sigma,B}$ \\ \hline 0.0002 \\ \hline 0.0178 \\ \hline 0.0004 \\ \hline 0.0003 \\ \hline 0.0005 \\ \hline 0.0006 \\ \hline H \end{tabular}$	Toe $L_{A,B}^*$ 0.4658 0.1157 0.4623 0.4902 0.4373 0.3569 3	$\begin{array}{c} \mbox{plitz} \\ \hline L^*_{\Sigma,B} \\ \hline 0.0106 \\ \hline 0.0195 \\ \hline 0.0101 \\ \hline 0.0101 \\ \hline 0.0112 \\ \hline 0.0125 \\ \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $\sigma^{2} = 4$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.4764 \\ 0.1352 \\ 0.4735 \\ 0.5003 \\ 0.4485 \\ 0.3694 \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.4762 \\ 0.1174 \\ 0.4731 \\ 0.5000 \\ 0.4480 \\ 0.3688 \\ \hline \\ \text{Same s} \end{array}$	$\begin{tabular}{c} $F_{\Sigma,B} \\ $100002 \\ $0.0002 \\ $0.0002 \\ $0.0004 \\ $0.0003 \\ $0.0005 \\ $0.0006 \\ $$10000 \\ $$10$	$\begin{array}{c} 3\\ \hline Toe \\ L^*_{A,B}\\ 0.4658\\ 0.1157\\ 0.4623\\ 0.4902\\ 0.4373\\ 0.3569\\ 3\\ \hline Toe \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ \hline 0.0106 \\ \hline 0.0195 \\ \hline 0.0111 \\ \hline 0.0101 \\ \hline 0.0112 \\ \hline 0.0125 \\ \hline \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$	$\begin{array}{c} L_{\Sigma,A}^{*}\\ 0.4764\\ 0.1352\\ 0.4735\\ 0.5003\\ 0.4485\\ 0.3694\\ \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.4762 \\ 0.1174 \\ 0.4731 \\ 0.5000 \\ 0.4480 \\ 0.3688 \\ \hline \\ \text{Same s} \\ L_{A,B}^{*} \end{array}$	$\begin{tabular}{c} \hline H \\ tructure \\ $L^*_{\Sigma,B}$ \\ 0.0002 \\ 0.0178 \\ 0.0004 \\ 0.0003 \\ 0.0005 \\ 0.0006 \\ \hline $H$ \\ tructure \\ $L^*_{\Sigma,B}$ \end{tabular}$	$\begin{array}{c} 3\\ \hline Toe \\ L^*_{A,B}\\ 0.4658\\ 0.1157\\ 0.4623\\ 0.4902\\ 0.4373\\ 0.3569\\ 3\\ \hline Toe \\ L^*_{A,B} \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0106 \\ 0.0195 \\ 0.0111 \\ 0.0101 \\ 0.0112 \\ 0.0125 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$ MA(1)	$\begin{array}{c} L_{\Sigma,A}^{*} \\ 0.4764 \\ 0.1352 \\ 0.4735 \\ 0.5003 \\ 0.4485 \\ 0.3694 \\ \hline \\ L_{\Sigma,A}^{*} \\ 0.4772 \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.4762 \\ 0.1174 \\ 0.4731 \\ 0.5000 \\ 0.4480 \\ 0.3688 \\ \hline \\ \text{Same s} \\ L_{A,B}^{*} \\ 0.4769 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ \hline Toe \\ & \\ L^*_{A,B} \\ 0.4658 \\ 0.1157 \\ 0.4623 \\ 0.4902 \\ 0.4373 \\ 0.3569 \\ 3 \\ \hline Toe \\ & \\ L^*_{A,B} \\ 0.4669 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0106 \\ 0.0195 \\ 0.0111 \\ 0.0101 \\ 0.0112 \\ 0.0125 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0103 \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$ MA(1) CS	$\begin{array}{c} L_{\Sigma,A}^{*}\\ 0.4764\\ 0.1352\\ 0.4735\\ 0.5003\\ 0.4485\\ 0.3694\\ \hline\\ L_{\Sigma,A}^{*}\\ 0.4772\\ 0.1360\\ \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.4762 \\ 0.1174 \\ 0.4731 \\ 0.5000 \\ 0.4480 \\ 0.3688 \\ \hline \\ \text{Same s} \\ L_{A,B}^{*} \\ 0.4769 \\ 0.1173 \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ \hline Toe \\ & \\ L_{A,B}^{*} \\ \hline 0.4658 \\ 0.1157 \\ 0.4623 \\ 0.4902 \\ 0.4373 \\ 0.3569 \\ \hline 0.3569 \\ \hline \\ B \\ \hline \\ \hline \\ C \\ L_{A,B}^{*} \\ \hline \\ 0.4669 \\ 0.1156 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0106 \\ 0.0195 \\ 0.0111 \\ 0.0101 \\ 0.0112 \\ 0.0125 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0103 \\ 0.0204 \\ \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$ MA(1) CS AR(1)	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.4764 \\ 0.1352 \\ 0.4735 \\ 0.5003 \\ 0.4485 \\ 0.3694 \\ \hline \\ L^*_{\Sigma,A} \\ 0.4772 \\ 0.1360 \\ 0.4753 \\ \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.4762 \\ 0.1174 \\ 0.4731 \\ 0.5000 \\ 0.4480 \\ 0.3688 \\ \hline \\ \text{Same s} \\ L_{A,B}^{*} \\ 0.4769 \\ 0.1173 \\ 0.4750 \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ \hline Toe \\ & \\ L^*_{A,B} \\ 0.4658 \\ 0.1157 \\ 0.4623 \\ 0.4902 \\ 0.4373 \\ 0.3569 \\ 3 \\ \hline \\ \hline \\ \hline \\ & \\ L^*_{A,B} \\ 0.4669 \\ 0.1156 \\ 0.4649 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0105 \\ 0.0195 \\ 0.0101 \\ 0.0101 \\ 0.0112 \\ 0.0125 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0103 \\ 0.0204 \\ \hline \\ 0.0104 \\ \hline \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1	$\begin{array}{c} L_{\Sigma,A}^{*} \\ 0.4764 \\ 0.1352 \\ 0.4735 \\ 0.5003 \\ 0.4485 \\ 0.3694 \\ \hline \\ L_{\Sigma,A}^{*} \\ 0.4772 \\ 0.1360 \\ 0.4753 \\ 0.5004 \\ \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.4762 \\ 0.1174 \\ 0.4731 \\ 0.5000 \\ 0.4480 \\ 0.3688 \\ \hline \\ \text{Same s} \\ L_{A,B}^{*} \\ 0.4769 \\ 0.1173 \\ 0.4750 \\ 0.5001 \\ \hline \end{array}$	$\begin{tabular}{ c c c c c c c } \hline F \\ \hline tructure & $$L^*_{\Sigma,B}$ \\ \hline 0.0002 & $$0.0002$ \\ \hline 0.0003 & $$0.0005$ \\ \hline 0.0005 & $$0.0006$ \\ \hline $$F$ \\ \hline tructure & $$L^*_{\Sigma,B}$ \\ \hline 0.0002 & $$0.0187$ \\ \hline 0.0003 & $$0.0003$ \\ \hline \hline 0.0003 & $$0.0003$ \\ \hline \end{tabular}$	$\begin{array}{c} & \\ & \\ \hline Toe \\ & \\ L^*_{A,B} \\ 0.4658 \\ 0.1157 \\ 0.4623 \\ 0.4902 \\ 0.4373 \\ 0.3569 \\ 0.3569 \\ \hline \\ & \\ \hline \\ \\ & \\ \hline \\ & \\ \hline \\ \\ & \\ \hline \\ & \\ \hline \\ \\ \\ \hline \\ \\ \\ \hline \\ \\ \hline \\ \\ \\ \hline \\ \\ \\ \hline \\ \\ \\ \hline \\ \hline \\ \\ \hline \\ \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \\ \hline \hline \\ \hline \hline \\ \hline \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \hline \\ $	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0106 \\ 0.0195 \\ 0.0111 \\ 0.0101 \\ 0.0102 \\ 0.0125 \\ \hline \\ \textbf{D}_{\Sigma,B} \\ 0.0103 \\ 0.0204 \\ \hline \\ 0.0104 \\ \hline \\ 0.0101 \\ \hline \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$ MA(1) CS AR(1) ARMA(1) ARMA-r=0.1 ARMA-r=0.35	$\begin{array}{c} L_{\Sigma,A}^{*} \\ 0.4764 \\ 0.1352 \\ 0.4735 \\ 0.5003 \\ 0.4485 \\ 0.3694 \\ \hline \\ L_{\Sigma,A}^{*} \\ 0.4772 \\ 0.1360 \\ 0.4753 \\ 0.5004 \\ 0.4480 \\ \hline \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.4762 \\ 0.1174 \\ 0.4762 \\ 0.1174 \\ 0.5000 \\ 0.4480 \\ 0.3688 \\ \hline \\ \text{Same s} \\ L_{A,B}^{*} \\ 0.4769 \\ 0.1173 \\ 0.4750 \\ 0.5001 \\ 0.4476 \end{array}$	$\begin{tabular}{ c c c c c }\hline & & & & & & & & \\ \hline & & & & & & & & \\ \hline & & & &$	$\begin{array}{c} & \\ & \\ \hline Toe \\ & \\ L^*_{A,B} \\ 0.4658 \\ 0.1157 \\ 0.4623 \\ 0.4902 \\ 0.4373 \\ 0.3569 \\ 0.3569 \\ \hline \\ & \\ \hline \\ \\ & \\ \hline \\ \\ & \\ \hline \\ \\ \hline \\ \\ & \\ \hline \\ \hline \\ \\ \hline \\ \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \\ \hline \\ \\ \hline \hline \\ \hline \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \hline \\ \hline \hline \hline \\ \hline \hline \\ \hline \hline \\ \hline \hline \hline \\ \hline \hline \hline \\ \hline \hline \hline \\ \hline \hline \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \hline \hline \hline \\ \hline \hline$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0106 \\ 0.0195 \\ 0.0111 \\ 0.0101 \\ 0.0101 \\ 0.0112 \\ 0.0125 \\ \hline \\ L_{\Sigma,B}^{*} \\ 0.0103 \\ 0.0204 \\ 0.0104 \\ \hline \\ 0.0106 \\ \hline \end{array}$

			E	3	
$\sigma^2 = 0.5$	0	Same s	tructure	Toe	plitz
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
MA(1)	0.4069	0.4067	0.0002	0.3949	0.0120
CS	0.0521	0.0296	0.0225	0.0293	0.0229
AR(1)	0.3832	0.3825	0.0007	0.3711	0.0121
ARMA-r=0.2	0.4793	0.4789	0.0004	0.4688	0.0105
ARMA-r=0.45	0.4000	0.3993	0.0008	0.3882	0.0118
ARMA-r=0.75	0.2945	0.2936	0.0009	0.2808	0.0136
			E	3	
$\sigma^2 = 1$		Same s	tructure	Toe	plitz
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
MA(1)	0.4074	0.4072	0.0002	0.3960	0.0114
CS	0.0459	0.0297	0.0162	0.0293	0.0165
AR(1)	0.3825	0.3817	0.0007	0.3698	0.0127
ARMA-r=0.2	0.4810	0.4805	0.0005	0.4705	0.0105
ARMA-r=0.45	0.4003	0.3997	0.0006	0.3878	0.0125
ARMA-r=0.75	0.2956	0.2945	0.0011	0.2814	0.0141
ARMA-r=0.75 0.2956					
			E	3	
$\sigma^2 = 2$		Same s	E tructure	3 Toe	plitz
$\frac{\sigma^2 = 2}{\Sigma}$	$L^*_{\Sigma,A}$	Same s $L^*_{A,B}$	$\frac{\text{tructure}}{L_{\Sigma,B}^*}$	Toe $L_{A,B}^*$	plitz $L^*_{\Sigma,B}$
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$	$L^*_{\Sigma,A}$ 0.4067	Same s $L_{A,B}^{*}$ 0.4064	$\frac{L_{\Sigma,B}^*}{0.0002}$	Toej $L_{A,B}^{*}$ 0.3942	plitz $L^*_{\Sigma,B}$ 0.0125
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$	$L^*_{\Sigma,A}$ 0.4067 0.0512	Same s $L_{A,B}^*$ 0.4064 0.0291		Toe $L_{A,B}^*$ 0.3942 0.0288	plitz $L_{\Sigma,B}^{*}$ 0.0125 0.0224
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1)	$L^*_{\Sigma,A}$ 0.4067 0.0512 0.3822	Same s $L_{A,B}^*$ 0.4064 0.0291 0.3816	$     E \\     E \\    $	Toe $L_{A,B}^*$ 0.3942 0.0288 0.3699	plitz $L_{\Sigma,B}^*$ 0.0125 0.0224 0.0123
	$L^*_{\Sigma,A}$ 0.4067 0.0512 0.3822 0.4804	Same s $L_{A,B}^*$ 0.4064 0.0291 0.3816 0.4799	$F_{\Sigma,B} = 0.0002 \\ 0.0221 \\ 0.0006 \\ 0.0005 \\ $	Toe $L_{A,B}^*$ 0.3942 0.0288 0.3699 0.4699	$\begin{array}{c} \text{plitz} \\ \hline L^*_{\Sigma,B} \\ 0.0125 \\ 0.0224 \\ \hline 0.0123 \\ 0.0105 \end{array}$
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45	$L^*_{\Sigma,A}$ 0.4067 0.0512 0.3822 0.4804 0.4006	Same s $L_{A,B}^*$ 0.4064 0.0291 0.3816 0.4799 0.4000	$F_{\Sigma,B} = 0.0002 \\ 0.0021 \\ 0.0006 \\ 0.0005 \\ 0.0007 \\ $	Toe $L_{A,B}^*$ 0.3942 0.0288 0.3699 0.4699 0.3887	plitz $L_{\Sigma,B}^*$ 0.0125 0.0224 0.0123 0.0105 0.0120
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75	$L^*_{\Sigma,A}$ 0.4067 0.0512 0.3822 0.4804 0.4006 0.2937	Same s $L^*_{A,B}$ 0.4064 0.0291 0.3816 0.4799 0.4000 0.2928	$\begin{tabular}{l} \hline F \\ tructure \\ $L^*_{\Sigma,B}$ \\ 0.0002 \\ 0.0221 \\ 0.0006 \\ 0.0005 \\ 0.0007 \\ 0.0009 \end{tabular}$	Toe $L_{A,B}^*$ 0.3942 0.0288 0.3699 0.4699 0.3887 0.2797	plitz $L_{\Sigma,B}^*$ 0.0125 0.0224 0.0123 0.0105 0.0120 0.0139
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta MA(1)}{CS}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.2}$ $\frac{\Delta RMA-r=0.45}{\Delta RMA-r=0.75}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.4067 \\ 0.0512 \\ 0.3822 \\ 0.4804 \\ 0.4006 \\ 0.2937 \end{array}$	Same s $L_{A,B}^*$ 0.4064 0.0291 0.3816 0.4799 0.4000 0.2928	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	$\begin{array}{c} & \\ \hline Toe \\ L^*_{A,B} \\ 0.3942 \\ 0.0288 \\ 0.3699 \\ 0.4699 \\ 0.3887 \\ 0.2797 \\ \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0125 \\ 0.0224 \\ 0.0123 \\ 0.0105 \\ 0.0120 \\ 0.0139 \\ \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{CS}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.2}$ $\frac{\Delta RMA-r=0.45}{\Delta RMA-r=0.75}$ $\sigma^2 = 4$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.4067 \\ 0.0512 \\ 0.3822 \\ 0.4804 \\ 0.4006 \\ 0.2937 \end{array}$	Same s $L^*_{A,B}$ 0.4064 0.0291 0.3816 0.4799 0.4000 0.2928 Same s	$F_{\Sigma,B} = \frac{L_{\Sigma,B}^*}{0.0002}$ 0.0021 0.0006 0.0005 0.0005 0.0007 0.0009 F_Tructure	$\begin{array}{c} 3\\ \hline Toe\\ L^*_{A,B}\\ 0.3942\\ 0.0288\\ 0.3699\\ 0.4699\\ 0.3887\\ 0.2797\\ \end{array}$	plitz $L_{\Sigma,B}^*$ 0.0125 0.0224 0.0123 0.0105 0.0120 0.0139 plitz
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{CS}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.2}$ $\frac{\Delta RMA-r=0.45}{\Delta RMA-r=0.75}$ $\frac{\sigma^2 = 4}{\Sigma}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.4067 \\ 0.0512 \\ 0.3822 \\ 0.4804 \\ 0.4006 \\ 0.2937 \\ \\ L^*_{\Sigma,A} \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.4064 \\ 0.0291 \\ 0.3816 \\ 0.4799 \\ 0.4000 \\ 0.2928 \\ \hline \\ \text{Same s} \\ L_{A,B}^{*} \end{array}$	$\begin{array}{c} & {\rm E} \\ tructure \\ L^*_{\Sigma,B} \\ 0.0002 \\ 0.0221 \\ 0.0006 \\ 0.0005 \\ 0.0007 \\ 0.0007 \\ \hline 0.0009 \\ \hline {\rm E} \\ tructure \\ L^*_{\Sigma,B} \end{array}$	$\begin{array}{c} 3\\ \hline Toe \\ L^*_{A,B}\\ 0.3942\\ 0.0288\\ 0.3699\\ 0.4699\\ 0.3887\\ 0.2797\\ \hline 0.2797\\ \hline 3\\ \hline Toe \\ L^*_{A,B} \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0125 \\ 0.0224 \\ 0.0105 \\ 0.0105 \\ 0.0120 \\ 0.0139 \\ \hline \\ 0.0139 \\ \hline \\ plitz \\ L_{\Sigma,B}^{*} \\ \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{\Delta R(1)}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.2}$ $\frac{\Delta RMA-r=0.25}{\Delta RMA-r=0.75}$ $\frac{\sigma^2 = 4}{\Sigma}$ $\frac{\Delta L}{\Delta R(1)}$	$\frac{L^*_{\Sigma,A}}{0.4067}$ 0.0512 0.3822 0.4804 0.4006 0.2937 $\frac{L^*_{\Sigma,A}}{0.4062}$	Same s $L^*_{A,B}$ 0.4064 0.0291 0.3816 0.4799 0.4000 0.2928 Same s $L^*_{A,B}$ 0.4059		$\begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0125 \\ 0.0224 \\ 0.0105 \\ 0.0105 \\ 0.0120 \\ 0.0139 \\ \hline \\ plitz \\ L_{\Sigma,B}^{*} \\ 0.0116 \\ \hline \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{\Delta R(1)}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.2}$ $\frac{\Delta RMA-r=0.2}{\Delta RMA-r=0.75}$ $\frac{\sigma^2 = 4}{\Sigma}$ $\frac{\Delta L}{\Delta RMA(1)}$ $\frac{\Delta L}{\Delta S}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.4067 \\ 0.0512 \\ 0.3822 \\ 0.4804 \\ 0.4006 \\ 0.2937 \\ \hline \\ L^*_{\Sigma,A} \\ 0.4062 \\ 0.0512 \\ \hline \end{array}$	Same s $L^*_{A,B}$ 0.4064 0.0291 0.3816 0.4799 0.4000 0.2928 Same s $L^*_{A,B}$ 0.4059 0.4059		$\begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0125 \\ 0.0224 \\ 0.0123 \\ 0.0105 \\ 0.0100 \\ 0.0139 \\ \hline \\ 0.0139 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0116 \\ 0.0231 \\ \hline \\ \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\frac{\sigma^2 = 4}{\Sigma}$ MA(1) CS AR(1) CS	$\frac{L^*_{\Sigma,A}}{0.4067}$ 0.0512 0.3822 0.4804 0.4006 0.2937 $\frac{L^*_{\Sigma,A}}{0.4062}$ 0.0512 0.3823	Same s $L^*_{A,B}$ 0.4064 0.0291 0.3816 0.4799 0.4000 0.2928 Same s $L^*_{A,B}$ 0.4059 0.0284 0.3818	$\begin{tabular}{ c c c c c c c } \hline F \\ tructure \\ $L^*_{\Sigma,B}$ \\ \hline 0.0002 \\ \hline 0.0005 \\ \hline 0.0005 \\ \hline 0.0007 \\ \hline 0.0009 \\ \hline F \\ tructure \\ $L^*_{\Sigma,B}$ \\ \hline 0.0003 \\ \hline 0.0228 \\ \hline 0.0005 \\ \hline $	Toe $L^*_{A,B}$ 0.3942 0.288 0.3699 0.4699 0.3887 0.2797 3 Toe $L^*_{A,B}$ 0.3946 0.0281 0.3701 0.4701	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0125 \\ 0.0224 \\ 0.0123 \\ 0.0105 \\ 0.0120 \\ 0.0139 \\ \hline \\ plitz \\ L_{\Sigma,B}^{*} \\ 0.0116 \\ 0.0231 \\ 0.0122 \\ 0.0122 \\ 0.0122 \\ \hline \\ 0.0122 \\ 0.0122 \\ \hline \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\sigma^2 = 4$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2	$\frac{L_{\Sigma,A}^*}{0.4067}$ 0.0512 0.3822 0.4804 0.4006 0.2937 $\frac{L_{\Sigma,A}^*}{0.4062}$ 0.0512 0.3823 0.4803 0.4031 0.4032 0.4032 0.4032 0.4032 0.4032 0.403	Same s $L^*_{A,B}$ 0.4064 0.2911 0.3816 0.4799 0.4000 0.2928 Same s $L^*_{A,B}$ 0.4059 0.0284 0.3818 0.4798	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	$\begin{array}{c} 3\\ \hline Toe\\ L^*_{A,B}\\ 0.3942\\ 0.0288\\ 0.3699\\ 0.4699\\ 0.3887\\ 0.2797\\ \hline 0.2797\\ \hline \\ B\\ \hline \\ 0.3946\\ 0.0281\\ 0.3701\\ 0.4695\\ \hline \\ 0.4695\\ \hline \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0125 \\ 0.0224 \\ 0.0123 \\ 0.0105 \\ 0.0120 \\ 0.0139 \\ \hline \\ plitz \\ L_{\Sigma,B}^{*} \\ 0.0116 \\ 0.0231 \\ 0.0122 \\ 0.0108 \\ \hline \\ 0.0108 \\ \hline \\ 0.0108 \\ \hline \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\sigma^2 = 4$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45	$\frac{L_{\Sigma,A}^*}{0.4067}$ 0.0512 0.3822 0.4804 0.4006 0.2937 $\frac{L_{\Sigma,A}^*}{0.4062}$ 0.0512 0.3823 0.4803 0.4011	Same s $L^*_{A,B}$ 0.4064 0.2911 0.3816 0.4799 0.4000 0.2928 Same s $L^*_{A,B}$ 0.4059 0.0284 0.3818 0.4798 0.4798		$\begin{array}{c} 3\\ \hline Toe\\ L^*_{A,B}\\ 0.3942\\ 0.0288\\ 0.3699\\ 0.4699\\ 0.3887\\ 0.2797\\ 3\\ \hline Toe\\ L^*_{A,B}\\ 0.3946\\ 0.0281\\ 0.3701\\ 0.4695\\ 0.3846\\ 0.281\\ 0.3701\\ 0.4695\\ 0.3846\\ 0.281\\ 0.3701\\ 0.4695\\ 0.384\\ 0.384\\ $	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0125 \\ 0.0224 \\ 0.0123 \\ 0.0105 \\ 0.0120 \\ 0.0120 \\ 0.0139 \\ \hline \\ plitz \\ L_{\Sigma,B}^{*} \\ 0.0116 \\ 0.0231 \\ 0.0122 \\ 0.0108 \\ 0.0126 \\ \hline \\ 0.0126 \\ \hline \end{array}$

**Table 5** Simulation results with m = 100; c = 0.5.

#### Covariance Matrix Regularization

**Table 6** Simulation results with m = 100; c = 0.75.

			I	3		
$\sigma^{2} = 0.5$	0	Same s	tructure	Toe	plitz	
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	
CS	0.0265	0.0076	0.0190	0.0075	0.0190	
AR(1)	0.2307	0.2289	0.0018	0.2164	0.0143	
ARMA-r=0.25	0.4444	0.4436	0.0008	0.4337	0.0107	
ARMA-r=0.5	0.3265	0.3249	0.0016	0.3136	0.0128	
ARMA-r=0.8	0.2139	0.2119	0.0020	0.1990	0.0149	
			I	3		
$\sigma^2 = 1$		Same s	tructure	Toe	plitz	
$\Sigma$	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	
CS	0.0275	0.0078	0.0197	0.0078	0.0198	
AR(1)	0.2315	0.2294	0.0021	0.2176	0.0139	
ARMA-r=0.25	0.4445	0.4436	0.0009	0.4335	0.0110	
ARMA-r=0.5	0.3275	0.3259	0.0016	0.3139	0.0136	
ARMA-r=0.8	0.2137	0.2115	0.0022	0.1986	0.0150	
				B		
2 2			I	3		
$\sigma^2 = 2$		Same s	I tructure	3 Toe	plitz	
$\frac{\sigma^2 = 2}{\Sigma}$	$L^*_{\Sigma,A}$	Same s $L^*_{A,B}$	$\frac{1}{L^*_{\Sigma,B}}$	Toe $L_{A,B}^*$	plitz $L^*_{\Sigma,B}$	
$\frac{\sigma^2 = 2}{\frac{\Sigma}{CS}}$	$L^*_{\Sigma,A}$ 0.0384	Same s $L_{A,B}^*$ 0.0079	$\frac{L_{\Sigma,B}^*}{0.0305}$	Toe $L_{A,B}^{*}$ 0.0078	plitz $L^*_{\Sigma,B}$ 0.0306	
$\frac{\sigma^2 = 2}{\frac{\Sigma}{CS}}$	$rac{L^*_{\Sigma,A}}{0.0384}$ 0.2294	Same s $L_{A,B}^*$ 0.0079 0.2271	$\frac{L_{\Sigma,B}^*}{0.0305}$	Toe $L_{A,B}^*$ 0.0078 0.2138	plitz $L_{\Sigma,B}^{*}$ 0.0306 0.0156	
$\frac{\sigma^2 = 2}{\frac{\Sigma}{\text{CS}}}$ AR(1) ARMA-r=0.25	$L^*_{\Sigma,A}$ 0.0384 0.2294 0.4439	Same s $L_{A,B}^*$ 0.0079 0.2271 0.4428	$     Frac{L_{\Sigma,B}^{*}}{0.0305} \\     0.0023 \\     0.0011 $	Toej $L_{A,B}^*$ 0.0078 0.2138 0.4324	plitz $L_{\Sigma,B}^*$ 0.0306 0.0156 0.0115	
$\frac{\sigma^2 = 2}{\Sigma}$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5	$\frac{L^*_{\Sigma,A}}{0.0384}\\0.2294\\0.4439\\0.3253$	Same s $L^*_{A,B}$ 0.0079 0.2271 0.4428 0.3235	$\begin{tabular}{c} $F_{\Sigma,B} \\ \hline $0.0305$ \\ \hline $0.0023$ \\ \hline $0.0011$ \\ \hline $0.0018$ \\ \hline \end{tabular}$	$\begin{array}{c} \hline Toe \\ \hline L^*_{A,B} \\ 0.0078 \\ 0.2138 \\ 0.4324 \\ 0.3123 \end{array}$	$\begin{array}{c} \text{plitz} \\ \hline L^*_{\Sigma,B} \\ 0.0306 \\ \hline 0.0156 \\ \hline 0.0115 \\ \hline 0.0130 \end{array}$	
$\sigma^2 = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5 ARMA-r=0.8	$L^*_{\Sigma,A}$ 0.0384 0.2294 0.4439 0.3253 0.2151	Same s $L^*_{A,B}$ 0.0079 0.2271 0.4428 0.3235 0.2129	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	$\begin{array}{c} {} \\ \hline Toe \\ L^*_{A,B} \\ 0.0078 \\ 0.2138 \\ 0.4324 \\ 0.3123 \\ 0.2002 \end{array}$	$\begin{array}{c} \text{plitz}\\ \hline L_{\Sigma,B}^{*}\\ \hline 0.0306\\ \hline 0.0156\\ \hline 0.0115\\ \hline 0.0130\\ \hline 0.0149\\ \end{array}$	
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5 ARMA-r=0.8	$L^*_{\Sigma,A}$ 0.0384 0.2294 0.4439 0.3253 0.2151	Same s $L^*_{A,B}$ 0.0079 0.2271 0.4428 0.3235 0.2129	$\begin{tabular}{l} \hline H \\ tructure \\ $L^*_{\Sigma,B}$ \\ 0.0305 \\ 0.0023 \\ 0.0011 \\ 0.0018 \\ 0.0022 \\ H \end{tabular}$	$\begin{array}{c} \textbf{3} \\ \hline \textbf{Toe} \\ \textbf{L}_{A,B}^{*} \\ 0.0078 \\ 0.2138 \\ 0.4324 \\ 0.3123 \\ 0.2002 \\ \textbf{3} \end{array}$	plitz $L_{\Sigma,B}^*$ 0.0306 0.0156 0.0115 0.0130 0.0149	
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5 ARMA-r=0.8 $\sigma^{2} = 4$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.0384 \\ 0.2294 \\ 0.4439 \\ 0.3253 \\ 0.2151 \end{array}$	Same s $L^*_{A,B}$ 0.0079 0.2271 0.4428 0.3235 0.2129 Same s	$Fructure L_{\Sigma,B}^* = 0.0305 \\ 0.0023 \\ 0.0011 \\ 0.0018 \\ 0.0022 \\ Fructure$	$\begin{array}{c} \hline Toep\\ \hline L^*_{A,B}\\ 0.0078\\ 0.2138\\ 0.4324\\ 0.3123\\ 0.2002\\ \hline \end{array}$	plitz $L_{\Sigma,B}^*$ 0.0306 0.0156 0.0115 0.0130 0.0149 plitz	
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.8 $\sigma^{2} = 4$ $\Sigma$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.0384 \\ 0.2294 \\ 0.4439 \\ 0.3253 \\ 0.2151 \\ \\ L^*_{\Sigma,A} \end{array}$	$\begin{array}{c} \text{Same s} \\ L^*_{A,B} \\ 0.0079 \\ 0.2271 \\ 0.4428 \\ 0.3235 \\ 0.2129 \\ \hline \\ \text{Same s} \\ L^*_{A,B} \end{array}$		$\begin{array}{c} 3\\ \hline Toep\\ L^*_{A,B}\\ 0.0078\\ 0.2138\\ 0.4324\\ 0.3123\\ 0.2002\\ \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0306 \\ 0.0156 \\ 0.0115 \\ 0.0130 \\ 0.0149 \\ \end{array}$	
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5 ARMA-r=0.8 $\sigma^{2} = 4$ $\Sigma$ CS	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.0384 \\ 0.2294 \\ 0.4439 \\ 0.3253 \\ 0.2151 \\ \\ L^*_{\Sigma,A} \\ 0.0423 \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.0079 \\ 0.2271 \\ 0.4428 \\ 0.3235 \\ 0.2129 \\ \hline \\ \text{Same s} \\ L_{A,B}^{*} \\ 0.0080 \end{array}$		$\begin{array}{c} 3\\ \hline Toe \\ L^*_{A,B}\\ 0.0078\\ 0.2138\\ 0.4324\\ 0.3123\\ 0.2002\\ \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ \hline 0.0306 \\ 0.0156 \\ \hline 0.0115 \\ \hline 0.0130 \\ \hline 0.0149 \\ \hline \end{array}$	
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5 ARMA-r=0.8 $\sigma^{2} = 4$ $\Sigma$ CS AR(1)	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.0384 \\ 0.2294 \\ 0.4439 \\ 0.3253 \\ 0.2151 \\ \\ L^*_{\Sigma,A} \\ 0.0423 \\ 0.2291 \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^* \\ 0.0079 \\ 0.2271 \\ 0.4428 \\ 0.3235 \\ 0.2129 \\ \hline \\ \text{Same s} \\ L_{A,B}^* \\ 0.0080 \\ 0.2271 \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} \hline & \\ \hline & \\ L^{*}_{A,B} \\ 0.0078 \\ 0.2138 \\ 0.4324 \\ 0.3123 \\ 0.2002 \\ \hline \\ 0.2002 \\ \hline \\ $	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0306 \\ 0.0156 \\ 0.0115 \\ 0.0130 \\ 0.0149 \\ \hline \\ \hline \\ plitz \\ L_{\Sigma,B}^{*} \\ 0.0344 \\ 0.0157 \\ \end{array}$	
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.8 $\sigma^{2} = 4$ $\Sigma$ CS AR(1) ARMA-r=0.25	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.0384 \\ 0.2294 \\ 0.4439 \\ 0.3253 \\ 0.2151 \\ \hline \\ L^*_{\Sigma,A} \\ 0.0423 \\ 0.2291 \\ 0.4432 \\ \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^* \\ 0.0079 \\ 0.2271 \\ 0.4428 \\ 0.3235 \\ 0.2129 \\ \hline \\ \text{Same s} \\ L_{A,B}^* \\ 0.0080 \\ 0.2271 \\ 0.4421 \\ \hline \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} \hline & \\ & \\ \hline & \\ L^{*}_{A,B} \\ 0.0078 \\ 0.2138 \\ 0.4324 \\ 0.3123 \\ 0.2002 \\ \hline \\ 0.2002 \\ \hline \\ $	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0306 \\ 0.0156 \\ 0.0115 \\ 0.0130 \\ 0.0149 \\ \hline \\ \hline \\ plitz \\ L_{\Sigma,B}^{*} \\ 0.0344 \\ 0.0157 \\ 0.0112 \\ \end{array}$	
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.8 $\sigma^{2} = 4$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5	$\begin{matrix} L^*_{\Sigma,A} \\ 0.0384 \\ 0.2294 \\ 0.4439 \\ 0.3253 \\ 0.2151 \\ \end{matrix}$	$\begin{array}{c} \text{Same s} \\ L^*_{A,B} \\ 0.0079 \\ 0.2271 \\ 0.4428 \\ 0.3235 \\ 0.2129 \\ \hline \\ \text{Same s} \\ L^*_{A,B} \\ 0.0080 \\ 0.2271 \\ 0.4421 \\ 0.3248 \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} \hline Toe \\ L^*_{A,B} \\ 0.0078 \\ 0.2138 \\ 0.4324 \\ 0.3123 \\ 0.2002 \\ \hline \\ \hline \\ 0.2002 \\ \hline \\ \hline \\ \hline \\ \hline \\ 0.2002 \\ \hline \\ \hline \\ 0.2134 \\ 0.4320 \\ \hline \\ 0.3131 \\ \hline \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0306 \\ 0.0115 \\ 0.0115 \\ 0.0130 \\ 0.0149 \\ \end{array}$	

<b>-</b> <sup>2</sup> 0.50		В				
$\sigma^2 = 0.5$	0	Same s	tructure	Toe	plitz	
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	
MA(1)	0.8183	0.8183	0.0000	0.8147	0.0036	
CS	0.1483	0.1317	0.0166	0.1313	0.0170	
AR(1)	0.8169	0.8169	0.0000	0.8132	0.0037	
ARMA-r=0.1	0.8321	0.8321	0.0000	0.8287	0.0034	
ARMA-r=0.35	0.8006	0.8005	0.0000	0.7965	0.0040	
ARMA-r=0.6	0.7414	0.7413	0.0000	0.7361	0.0053	
			E	3		
$\sigma^2 = 1$		Same s	tructure	Toe	plitz	
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	
MA(1)	0.8182	0.8181	0.0000	0.8145	0.0037	
CS	0.1503	0.1310	0.0193	0.1306	0.0197	
AR(1)	0.8170	0.8170	0.0000	0.8134	0.0037	
ARMA-r=0.1	0.8321	0.8321	0.0000	0.8287	0.0034	
ARMA-r=0.35	0.8007	0.8007	0.0000	0.7967	0.0040	
ARMA-r=0.6	0.7417	0.7416	0.0000	0.7364	0.0052	
		7 0.7416 0.0000 0.7364				
			I	3		
$\sigma^2 = 2$		Same s	H tructure	3 Toe	plitz	
$\frac{\sigma^2 = 2}{\Sigma}$	$L^*_{\Sigma,A}$	Same s $L^*_{A,B}$	$\frac{\text{tructure}}{L_{\Sigma,B}^*}$	$\frac{\text{Toe}}{L_{A,B}^*}$	$\frac{\text{plitz}}{L_{\Sigma,B}^*}$	
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$	$\frac{L^*_{\Sigma,A}}{0.8182}$	Same s $L_{A,B}^*$ 0.8182	$\frac{L_{\Sigma,B}^*}{0.0000}$	Toe $L_{A,B}^{*}$ 0.8146	plitz $\frac{L_{\Sigma,B}^*}{0.0037}$	
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$	$\frac{L^*_{\Sigma,A}}{0.8182}$ 0.1511	Same s $L_{A,B}^*$ 0.8182 0.1322	$ \frac{L_{\Sigma,B}^{*}}{0.0000} $	Toe $L_{A,B}^*$ 0.8146 0.1318	plitz $L_{\Sigma,B}^{*}$ 0.0037 0.0193	
$ \frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}} \frac{\sigma^2}{CS} $ AR(1)	$L^*_{\Sigma,A}$ 0.8182 0.1511 0.8169	Same s $L^*_{A,B}$ 0.8182 0.1322 0.8169	$F_{\Sigma,B} = 0.00000 = 0.000000 = 0.000000 = 0.00000 = 0.00000 = 0.00000 = 0.00000 = 0.000000 = 0.000000 = 0.00000 = 0.00000 = 0.00000000$	Toe $L^*_{A,B}$ 0.8146 0.1318 0.8132	$ \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ \hline 0.0037 \\ \hline 0.0193 \\ \hline 0.0037 \\ \hline \end{array} $	
$ \frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}} $ $ \frac{CS}{AR(1)} $ $ ARMA-r=0.1 $	$L^*_{\Sigma,A}$ 0.8182 0.1511 0.8169 0.8319	Same s $L_{A,B}^*$ 0.8182 0.1322 0.8169 0.8319	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} {} \\ \hline Toe \\ L^*_{A,B} \\ 0.8146 \\ 0.1318 \\ 0.8132 \\ 0.8285 \end{array}$	$ \begin{array}{r} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0037 \\ 0.0193 \\ 0.0037 \\ 0.0034 \end{array} $	
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$ $\frac{MA(1)}{CS}$ $\frac{AR(1)}{ARMA-r=0.1}$ $ARMA-r=0.35$	$L^*_{\Sigma,A}$ 0.8182 0.1511 0.8169 0.8319 0.8009	Same s $L_{A,B}^*$ 0.8182 0.1322 0.8169 0.8319 0.8008	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	Toe $L^*_{A,B}$ 0.8146 0.1318 0.8132 0.8285 0.7969	$\begin{array}{c} \text{plitz}\\ \underline{L_{\Sigma,B}^{*}}\\ 0.0037\\ 0.0193\\ \hline 0.0037\\ 0.0034\\ \hline 0.0040\\ \end{array}$	
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$ $\frac{MA(1)}{CS}$ $\frac{AR(1)}{ARMA-r=0.1}$ $\frac{ARMA-r=0.35}{ARMA-r=0.6}$	$L^*_{\Sigma,A}$ 0.8182 0.1511 0.8169 0.8319 0.8009 0.7418	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c } \hline F \\ \hline tructure \\ \hline L^*_{\Sigma,B} \\ \hline 0.0000 \\ \hline 0.0189 \\ \hline 0.0000 \\ \hline 0.0000 \\ \hline 0.0000 \\ \hline 0.0001 \\ \hline \end{tabular}$	Toe $L^*_{A,B}$ 0.8146 0.1318 0.8132 0.8285 0.7969 0.7366	$\begin{array}{c} \text{plitz}\\ \overline{L_{\Sigma,B}^{*}}\\ 0.0037\\ 0.0193\\ 0.0037\\ 0.0034\\ 0.0040\\ 0.0053\\ \end{array}$	
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6	$L^*_{\Sigma,A}$ 0.8182 0.1511 0.8169 0.8319 0.8009 0.7418	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	$\begin{array}{c} & \\ & \\ \hline & \\ L^*_{A,B} \\ 0.8146 \\ 0.1318 \\ 0.8132 \\ 0.8285 \\ 0.7969 \\ 0.7366 \\ \end{array}$	$\begin{array}{c} \mbox{plitz} \\ \hline L^*_{\Sigma,B} \\ 0.0037 \\ 0.0193 \\ 0.0037 \\ 0.0034 \\ 0.0040 \\ 0.0053 \end{array}$	
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $ \sigma^2 = 4 $	$\frac{L^*_{\Sigma,A}}{0.8182}\\ 0.1511\\ 0.8169\\ 0.8319\\ 0.8009\\ 0.7418$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} {} \\ \hline Toe \\ L^*_{A,B} \\ 0.8146 \\ 0.1318 \\ 0.8132 \\ 0.8285 \\ 0.7969 \\ 0.7366 \\ 3 \\ \hline Toe \end{array}$	$\begin{array}{c} \label{eq:constraint} \\ \hline plitz \\ \hline L^*_{\Sigma,B} \\ \hline 0.0037 \\ \hline 0.0193 \\ \hline 0.0037 \\ \hline 0.0034 \\ \hline 0.0040 \\ \hline 0.0053 \\ \hline \end{array}$	
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta RMA-r=0.35}{\Delta RMA-r=0.6}$ $\frac{\sigma^2 = 4}{\Sigma}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8182 \\ 0.1511 \\ 0.8169 \\ 0.8319 \\ 0.8009 \\ 0.7418 \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} 3\\ \hline Toe \\ L^*_{A,B}\\ 0.8146\\ 0.1318\\ 0.8132\\ 0.8285\\ 0.7969\\ 0.7366\\ 3\\ \hline Toe \\ L^*_{A,B} \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0037 \\ 0.0193 \\ 0.0037 \\ 0.0034 \\ 0.0040 \\ 0.0053 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ \end{array}$	
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{CS}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta RMA-r=0.35}{\Delta RMA-r=0.6}$ $\frac{\sigma^2 = 4}{\Sigma}$ $\frac{\Delta L}{\Delta RMA(1)}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8182 \\ 0.1511 \\ 0.8169 \\ 0.8319 \\ 0.8009 \\ 0.7418 \\ \hline L^*_{\Sigma,A} \\ 0.8181 \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0037 \\ 0.0193 \\ 0.0037 \\ 0.0034 \\ 0.0040 \\ 0.0053 \\ \end{array}$	
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{CS}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta RMA-r=0.35}{\Delta RMA-r=0.6}$ $\frac{\sigma^2 = 4}{\Sigma}$ $\frac{\Delta L}{\Delta RMA(1)}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8182 \\ 0.1511 \\ 0.8169 \\ 0.8319 \\ 0.8009 \\ 0.7418 \\ \hline \\ L^*_{\Sigma,A} \\ 0.8181 \\ 0.1499 \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0037 \\ 0.0193 \\ 0.0037 \\ 0.0034 \\ 0.0040 \\ 0.0053 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0037 \\ 0.0182 \\ \end{array}$	
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{CS}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta RMA-r=0.35}{\Delta RMA-r=0.6}$ $\frac{\sigma^2 = 4}{\Sigma}$ $\frac{\Delta CS}{\Delta R(1)}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8182 \\ 0.1511 \\ 0.8169 \\ 0.8319 \\ 0.8009 \\ 0.7418 \\ \hline \\ L^*_{\Sigma,A} \\ 0.8181 \\ 0.1499 \\ 0.8171 \\ \hline \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c c c } \hline F \\ \hline tructure & $$L^*_{\Sigma,B}$ \\ \hline 0.0000 & $$0.0000$ \\ \hline 0.0000 & $$0.0000$ \\ \hline 0.0000 & $$0.0001$ \\ \hline F \\ tructure & $$L^*_{\Sigma,B}$ \\ \hline 0.0000 & $$0.0178$ \\ \hline 0.0000 & $$0.0000$ \\ \hline \hline \end{tabular}$	$\begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0037 \\ 0.0193 \\ 0.0037 \\ 0.0034 \\ 0.0040 \\ 0.0053 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0037 \\ 0.0182 \\ \hline \\ 0.0036 \\ \hline \end{array}$	
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta RMA-r=0.35}{\Delta RMA-r=0.6}$ $\frac{\sigma^2 = 4}{\Sigma}$ $\frac{\Delta R(1)}{\Delta RMA(1)}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$	$\frac{L^*_{\Sigma,A}}{0.8182}$ 0.1511 0.8169 0.8319 0.8009 0.7418 $\frac{L^*_{\Sigma,A}}{0.8181}$ 0.1499 0.8171 0.8321	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c }\hline & & & & & & \\ \hline & & & & & & \\ \hline & & & &$	$\begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} \label{eq:constraint} \\ plitz \\ \hline L^*_{\Sigma,B} \\ 0.0037 \\ \hline 0.0037 \\ 0.0034 \\ \hline 0.0040 \\ 0.0053 \\ \hline \\ plitz \\ \hline L^*_{\Sigma,B} \\ 0.0037 \\ \hline 0.0182 \\ \hline 0.0036 \\ \hline 0.0034 \\ \hline \end{array}$	
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta RMA-r=0.1}{\Delta RMA-r=0.6}$ $\frac{\sigma^2 = 4}{\Sigma}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.1}$	$\frac{L^*_{\Sigma,A}}{0.8182}\\ 0.1511\\ 0.8169\\ 0.8319\\ 0.8009\\ 0.7418\\ \hline \\ \frac{L^*_{\Sigma,A}}{0.8181}\\ 0.1499\\ 0.8171\\ 0.8321\\ 0.8007\\ \hline \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c }\hline & & & & & & \\ \hline tructure & & & & \\ \hline $L^*_{\Sigma,B}$ & & & & \\ \hline $0.0000$ & & & \\ \hline $0.0000$ & & \\ \hline \end{tabular}$	$\begin{array}{c} & \\ & \\ \hline Toe \\ & \\ L_{A,B}^{*} \\ \hline 0.8146 \\ 0.1318 \\ 0.8132 \\ 0.8285 \\ 0.7969 \\ 0.7366 \\ \hline 0.7366 \\ \hline 0.7366 \\ \hline 0.8144 \\ 0.1317 \\ \hline 0.8134 \\ 0.8287 \\ \hline 0.7967 \\ \hline \end{array}$	$\begin{array}{c} \label{eq:constraint} \\ plitz \\ \hline L^*_{\Sigma,B} \\ 0.0037 \\ \hline 0.0037 \\ 0.0034 \\ \hline 0.0040 \\ 0.0053 \\ \hline \\ \hline \\ L^*_{\Sigma,B} \\ 0.0037 \\ \hline \\ 0.0037 \\ 0.0182 \\ \hline \\ 0.0036 \\ \hline \\ 0.0034 \\ \hline \\ 0.0040 \\ \hline \end{array}$	

**Table 7** Simulation results with m = 500; c = 0.25.

**Table 8** Simulation results with m = 500; c = 0.5.

			E	3	
$\sigma^{2} = 0.5$	0	Same s	tructure	Toe	plitz
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
MA(1)	0.7716	0.7716	0.0000	0.7670	0.0047
CS	0.0493	0.0307	0.0186	0.0306	0.0187
AR(1)	0.7527	0.7526	0.0001	0.7476	0.0050
ARMA-r=0.2	0.8206	0.8206	0.0000	0.8170	0.0036
ARMA-r=0.45	0.7669	0.7668	0.0001	0.7622	0.0046
ARMA-r=0.75	0.6702	0.6701	0.0001	0.6637	0.0065
			F	3	
$\sigma^2 = 1$	ĺ	Same s	tructure	Toe	plitz
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
MA(1)	0.7719	0.7719	0.0000	0.7674	0.0046
CS	0.0596	0.0303	0.0293	0.0302	0.0294
AR(1)	0.7523	0.7523	0.0000	0.7473	0.0050
ARMA-r=0.2	0.8208	0.8208	0.0000	0.8172	0.0036
ARMA-r=0.45	0.7673	0.7672	0.0001	0.7626	0.0047
ARMA-r=0.75	0.6699	0.6699	0.0001	0.6633	0.0067
		1			
			E	3	
$\sigma^2 = 2$		Same s	E tructure	3 Toej	plitz
$\sigma^2 = 2$ $\Sigma$	$L^*_{\Sigma,A}$	Same s $L^*_{A,B}$	$\frac{\text{tructure}}{L_{\Sigma,B}^*}$	Toe $L_{A,B}^*$	plitz $L^*_{\Sigma,B}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1)	$\frac{L_{\Sigma,A}^{*}}{0.7716}$	Same s $\frac{L_{A,B}^*}{0.7715}$	$\frac{L_{\Sigma,B}^{*}}{0.0000}$	Toej $L_{A,B}^{*}$ 0.7670	plitz $L_{\Sigma,B}^*$ 0.0045
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$	$\frac{L^*_{\Sigma,A}}{0.7716}$ 0.0543	Same s $L_{A,B}^*$ 0.7715 0.0306	$     E \\     E \\    $	Toej $L_{A,B}^*$ 0.7670 0.0305	plitz $L_{\Sigma,B}^{*}$ 0.0045 0.0238
$\sigma^2 = 2$ $\Sigma$ MA(1) CS AR(1)	$L^*_{\Sigma,A}$ 0.7716 0.0543 0.7530	Same s $L^*_{A,B}$ 0.7715 0.0306 0.7529	$     E \\     Eructure \\     L_{\Sigma,B}^* \\     0.0000 \\     0.0237 \\     0.0001   $	Toej $L_{A,B}^*$ 0.7670 0.0305 0.7480	plitz $L_{\Sigma,B}^*$ 0.0045 0.0238 0.0050
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2	$L^*_{\Sigma,A}$ 0.7716 0.0543 0.7530 0.8204	Same s $L_{A,B}^*$ 0.7715 0.0306 0.7529 0.8204	$\frac{E}{L_{\Sigma,B}^*}$ 0.0000 0.0237 0.0001 0.0000	$\begin{array}{c} \text{Toe}\\ L_{A,B}^{*}\\ 0.7670\\ 0.0305\\ 0.7480\\ 0.8168 \end{array}$	$\begin{array}{c} \text{plitz} \\ \hline L^*_{\Sigma,B} \\ 0.0045 \\ \hline 0.0238 \\ \hline 0.0050 \\ \hline 0.0036 \end{array}$
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45	$L^*_{\Sigma,A}$ 0.7716 0.0543 0.7530 0.8204 0.7671	Same s $L^*_{A,B}$ 0.7715 0.0306 0.7529 0.8204 0.7670	$\begin{tabular}{l} \hline H \\ tructure \\ \hline L^*_{\Sigma,B} \\ \hline 0.0000 \\ \hline 0.0237 \\ \hline 0.0001 \\ \hline 0.0000 \\ \hline 0.0000 \end{tabular}$	Toej $L^*_{A,B}$ 0.7670 0.0305 0.7480 0.8168 0.7624	plitz $L_{\Sigma,B}^*$ 0.0045 0.0238 0.0050 0.0036 0.0047
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75	$L^*_{\Sigma,A}$ 0.7716 0.0543 0.7530 0.8204 0.7671 0.6705	Same s $L_{A,B}^*$ 0.7715 0.0306 0.7529 0.8204 0.7670 0.6704	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	Toej $L_{A,B}^*$ 0.7670 0.0305 0.7480 0.8168 0.7624 0.6639	plitz $L_{\Sigma,B}^*$ 0.0045 0.0238 0.0050 0.0036 0.0047 0.0066
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.7716 \\ 0.0543 \\ 0.7530 \\ 0.8204 \\ 0.7671 \\ 0.6705 \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.7715 \\ 0.0306 \\ 0.7529 \\ 0.8204 \\ 0.7670 \\ 0.6704 \\ \end{array}$	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	Toep $L_{A,B}^*$ 0.7670 0.0305 0.7480 0.8168 0.7624 0.6639 3	plitz $L_{\Sigma,B}^*$ 0.0045 0.0238 0.0050 0.0036 0.0047 0.0066
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\sigma^{2} = 4$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.7716 \\ 0.0543 \\ 0.7530 \\ 0.8204 \\ 0.7671 \\ 0.6705 \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.7715 \\ 0.0306 \\ 0.7529 \\ 0.8204 \\ 0.7670 \\ 0.6704 \\ \hline \\ \text{Same s} \end{array}$	$\begin{tabular}{c} $F$ tructure $$ $L_{\Sigma,B}^*$ 0.0000 $$ 0.0237 $$ 0.0001 $$ 0.0000 $$ 0.0000 $$ 0.0000 $$ 0.0000 $$ $$ 0.0001 $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ tructure $$ $$ $$ $$ $$ tructure $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$$	$\begin{array}{c} 3\\ \hline Toep\\ L^*_{A,B}\\ 0.7670\\ 0.0305\\ 0.7480\\ 0.8168\\ 0.7624\\ 0.6639\\ 3\\ \hline Toep \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0045 \\ 0.0238 \\ 0.0050 \\ 0.0036 \\ 0.0047 \\ 0.0066 \\ \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\sigma^2 = 4$ $\Sigma$	$\begin{array}{c} L_{\Sigma,A}^{*}\\ 0.7716\\ 0.0543\\ 0.7530\\ 0.8204\\ 0.7671\\ 0.6705\\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$		$\begin{array}{c} 3\\ \hline Toep\\ L^*_{A,B}\\ 0.7670\\ 0.0305\\ 0.7480\\ 0.8168\\ 0.7624\\ 0.6639\\ 3\\ \hline Toep\\ L^*_{A,B} \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0045 \\ 0.0238 \\ 0.0050 \\ 0.0036 \\ 0.0047 \\ 0.0066 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\sigma^{2} = 4$ $\Sigma$ MA(1)	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.7716 \\ 0.0543 \\ 0.7530 \\ 0.8204 \\ 0.7671 \\ 0.6705 \\ \hline \\ L^*_{\Sigma,A} \\ 0.7717 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \text{F}\\ \text{tructure}\\ & L_{\Sigma,B}^*\\ 0.0000\\ 0.0237\\ 0.0000\\ 0.0000\\ 0.0000\\ \hline 0.0000\\ \hline \\ & \text{F}\\ \text{tructure}\\ & L_{\Sigma,B}^*\\ \hline \\ 0.0000\\ \end{array}$	$\begin{array}{c} & \\ & \\ \hline Toej \\ L^*_{A,B} \\ 0.7670 \\ 0.0305 \\ 0.7480 \\ 0.305 \\ 0.7480 \\ 0.305 \\ 0.7624 \\ 0.6639 \\ 3 \\ \hline Toej \\ L^*_{A,B} \\ 0.7671 \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0045 \\ 0.0238 \\ 0.0050 \\ 0.0036 \\ 0.0047 \\ 0.0066 \\ \end{array}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\sigma^2 = 4$ $\Sigma$ MA(1) CS	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.7716 \\ 0.0543 \\ 0.7530 \\ 0.8204 \\ 0.7671 \\ 0.6705 \\ \hline \\ L^*_{\Sigma,A} \\ 0.7717 \\ 0.0547 \\ \end{array}$	$\begin{array}{c} \text{Same s} \\ L^*_{A,B} \\ 0.7715 \\ 0.0306 \\ 0.7529 \\ 0.8204 \\ 0.7670 \\ 0.6704 \\ \hline \\ \text{Same s} \\ L^*_{A,B} \\ 0.7717 \\ 0.0304 \\ \end{array}$	$\begin{array}{c} & \text{F}\\ \text{tructure}\\ & L_{\Sigma,B}^{*}\\ 0.0000\\ 0.0237\\ 0.0000\\ 0.0000\\ 0.0000\\ \hline 0.0000\\ \hline 0.0000\\ \hline \\ \text{tructure}\\ & L_{\Sigma,B}^{*}\\ \hline 0.0000\\ \hline 0.0244 \end{array}$	$\begin{array}{c} & \\ & \\ \hline Toej \\ L^*_{A,B} \\ 0.7670 \\ 0.0305 \\ 0.7480 \\ 0.8168 \\ 0.7624 \\ 0.6639 \\ 3 \\ \hline \\ \hline \\ \hline \\ L^*_{A,B} \\ 0.7671 \\ 0.0303 \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0045 \\ 0.0238 \\ 0.0050 \\ 0.0036 \\ 0.0047 \\ 0.0066 \\ \hline \\ 0.0047 \\ 0.0066 \\ \hline \\ \hline \\ L_{\Sigma,B}^{*} \\ 0.0045 \\ 0.0244 \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\sigma^{2} = 4$ $\Sigma$ MA(1) CS AR(1)	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.7716 \\ 0.0543 \\ 0.7530 \\ 0.8204 \\ 0.7671 \\ 0.6705 \\ \hline \\ L^*_{\Sigma,A} \\ 0.7717 \\ 0.0547 \\ 0.7529 \end{array}$	$\begin{array}{c} \text{Same s} \\ L^*_{A,B} \\ 0.7715 \\ 0.0306 \\ 0.7529 \\ 0.8204 \\ 0.7670 \\ 0.6704 \\ \hline \\ \text{Same s} \\ L^*_{A,B} \\ 0.7717 \\ 0.0304 \\ 0.7529 \end{array}$	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	$\begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0045 \\ 0.0038 \\ 0.0050 \\ 0.0036 \\ 0.0047 \\ 0.0066 \\ \hline \\ \hline \\ L_{\Sigma,B}^{*} \\ 0.0045 \\ 0.0244 \\ 0.0050 \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\sigma^{2} = 4$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.7716 \\ 0.0543 \\ 0.7530 \\ 0.8204 \\ 0.7671 \\ 0.6705 \\ \hline \\ L^*_{\Sigma,A} \\ 0.7717 \\ 0.0547 \\ 0.7529 \\ 0.8208 \\ \hline \end{array}$	$\begin{array}{c} \text{Same s} \\ L^*_{A,B} \\ 0.7715 \\ 0.0306 \\ 0.7529 \\ 0.8204 \\ 0.7670 \\ 0.6704 \\ \hline \\ \text{Same s} \\ L^*_{A,B} \\ 0.7717 \\ 0.0304 \\ 0.7529 \\ 0.8207 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0045 \\ 0.0038 \\ 0.0050 \\ 0.0036 \\ 0.0047 \\ 0.0066 \\ \hline \\ \hline \\ L_{\Sigma,B}^{*} \\ 0.0045 \\ 0.0244 \\ 0.0050 \\ \hline \\ 0.0036 \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\sigma^{2} = 4$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45	$\frac{L^*_{\Sigma,A}}{0.7716}$ 0.07716 0.0543 0.7530 0.8204 0.7671 0.6705 $\frac{L^*_{\Sigma,A}}{0.7717}$ 0.0547 0.7529 0.8208 0.7669	$\begin{array}{c} \text{Same s} \\ L^*_{A,B} \\ 0.7715 \\ 0.0306 \\ 0.7529 \\ 0.8204 \\ 0.7670 \\ 0.6704 \\ \hline \\ \text{Same s} \\ L^*_{A,B} \\ 0.7717 \\ 0.0304 \\ 0.7529 \\ 0.8207 \\ 0.7669 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0045 \\ 0.0038 \\ 0.0050 \\ 0.0036 \\ 0.0047 \\ 0.0066 \\ \hline \\ \hline \\ L_{\Sigma,B}^{*} \\ 0.0045 \\ 0.0244 \\ 0.0050 \\ \hline \\ 0.0036 \\ \hline \\ 0.0047 \\ \end{array}$

		В				
$\sigma^2 = 0.5$	0	Same	tructure	, Tea	alitz	
0 = 0.5	U I *			100		
2	$L_{\Sigma,A}$	$L_{A,B}$	$L_{\Sigma,B}$	$L_{A,B}$	$L_{\Sigma,B}$	
CS	0.0289	0.0081	0.0208	0.0081	0.0208	
AR(1)	0.5890	0.5888	0.0002	0.5808	0.0082	
ARMA-r=0.25	0.7978	0.7977	0.0001	0.7936	0.0041	
ARMA-r=0.5	0.7041	0.7040	0.0001	0.6983	0.0059	
ARMA-r=0.8	0.5662	0.5660	0.0002	0.5573	0.0089	
			E	3		
$\sigma^2 = 1$	ĺ	Same s	tructure	Toe	plitz	
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	
CS	0.0273	0.0081	0.0192	0.0081	0.0192	
AR(1)	0.5890	0.5887	0.0002	0.5808	0.0082	
ARMA-r=0.25	0.7976	0.7976	0.0001	0.7935	0.0041	
ARMA-r=0.5	0.7039	0.7038	0.0002	0.6979	0.0061	
ARMA-r=0.8	0.5652	0.5650	0.0002	0.5565	0.0087	
			I	3		
$\sigma^2 = 2$		Same s	tructure	Toe	plitz	
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	
CS	0.0258	0.0078	0.0180	0.0078	0.0180	
AR(1)	0.5875	0.5872	0.0002	0.5793	0.0082	
ARMA-r=0.25	0.7976	0.7976	0.0001	0.7936	0.0041	
ARMA-r=0.5	0.7036	0.7034	0.0002	0.6977	0.0059	
ARMA-r=0.8	0 5652	0 5640	0.0002	0 55(0	0.0004	
	0.5052	B		0.5568	0.0084	
	0.5052	0.3049	0.0003 E	0.5568 3	0.0084	
$\sigma^2 = 4$	0.5052	Same s	tructure	0.5568 3 Toej	plitz	
$\frac{\sigma^2 = 4}{\Sigma}$	$L^*_{\Sigma,A}$	Same si $L^*_{A,B}$	$\frac{0.0003}{\text{Fructure}}$	$\frac{0.5568}{\text{Toe}}$	$\frac{1}{L_{\Sigma,B}^*}$	
$\frac{\sigma^2 = 4}{\frac{\Sigma}{\text{CS}}}$	$L^*_{\Sigma,A}$ 0.0268	Same s $L_{A,B}^*$ 0.0080	$\frac{1}{1}$	$\begin{array}{c} 0.5568 \\ \hline \\ 3 \\ \hline \\ L^*_{A,B} \\ 0.0079 \end{array}$	$\frac{\text{plitz}}{L_{\Sigma,B}^*}$ 0.0188	
$\frac{\sigma^2 = 4}{\frac{\Sigma}{CS}}$	$L^*_{\Sigma,A}$ 0.0268 0.5887	Same si $L_{A,B}^*$ 0.0080 0.5885	$     \begin{array}{r} \hline 0.0003 \\ \hline H \\ tructure \\ \hline L^*_{\Sigma,B} \\ \hline 0.0188 \\ \hline 0.0002 \\ \end{array} $	$\begin{array}{c} 0.5568 \\ \hline \\ \hline \\ \hline \\ L^*_{A,B} \\ 0.0079 \\ \hline \\ 0.5804 \end{array}$		
$\frac{\sigma^2 = 4}{\frac{\Sigma}{CS}}$ AR(1) ARMA-r=0.25	$\frac{L_{\Sigma,A}^{*}}{0.0268}$ 0.5887 0.7975		$ \begin{array}{r} \hline 0.0003 \\ \hline \text{H} \\ \hline \text{tructure} \\ \hline L^*_{\Sigma,B} \\ \hline 0.0188 \\ \hline 0.0002 \\ \hline 0.0001 \\ \hline \end{array} $	$\begin{array}{c} 0.5568 \\ \hline \\ Toe \\ L^*_{A,B} \\ 0.0079 \\ 0.5804 \\ 0.7934 \end{array}$	$ \begin{array}{c} \text{plitz} \\ \underline{L_{\Sigma,B}^{*}} \\ 0.0188 \\ 0.0084 \\ 0.0041 \end{array} $	
$\frac{\sigma^2 = 4}{\Sigma}$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5	$L^*_{\Sigma,A}$ 0.0268 0.5887 0.7975 0.7040	$\begin{array}{c} \text{Same s} \\ \hline \\ L_{A,B}^{*} \\ 0.0080 \\ 0.5885 \\ 0.7974 \\ 0.7039 \end{array}$	$\begin{array}{c} 0.0003\\ \hline \text{H}\\ \text{tructure}\\ \hline L^*_{\Sigma,B}\\ 0.0188\\ 0.0002\\ \hline 0.0001\\ 0.0001 \end{array}$	$\begin{array}{c} 0.5568 \\ \hline \\ Toe \\ \hline \\ L^*_{A,B} \\ 0.0079 \\ \hline \\ 0.5804 \\ \hline \\ 0.7934 \\ \hline \\ 0.6981 \end{array}$	$\begin{array}{c} \text{plitz} \\ \hline L_{\Sigma,B}^{*} \\ 0.0188 \\ \hline 0.0084 \\ \hline 0.0041 \\ \hline 0.0059 \end{array}$	

**Table 9** Simulation results with m = 500; c = 0.75.

**Table 10** Simulation results with m = 1000; c = 0.25.

			E	3	
$\sigma^{2} = 0.5$	0	Same s	tructure	Toe	plitz
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
MA(1)	0.8999	0.8999	0.0000	0.8979	0.0020
CS	0.1571	0.1368	0.0203	0.1366	0.0205
AR(1)	0.8992	0.8992	0.0000	0.8972	0.0020
ARMA-r=0.1	0.9082	0.9082	0.0000	0.9064	0.0018
ARMA-r=0.35	0.8891	0.8891	0.0000	0.8869	0.0022
ARMA-r=0.6	0.8512	0.8512	0.0000	0.8482	0.0030
			F	3	
$\sigma^2 = 1$		Same s	tructure	Toe	plitz
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
MA(1)	0.8999	0.8999	0.0000	0.8979	0.0020
CS	0.1388	0.1268	0.0120	0.1266	0.0121
AR(1)	0.8992	0.8992	0.0000	0.8972	0.0020
ARMA-r=0.1	0.9083	0.9083	0.0000	0.9064	0.0019
ARMA-r=0.35	0.8893	0.8893	0.0000	0.8870	0.0023
ARMA-r=0.6	0.8515	0.8515	0.0000	0.8485	0.0030
2			E	3	
$\sigma^2 = 2$		Same s	E tructure	3 Toej	plitz
$\sigma^2 = 2$ $\Sigma$	$L^*_{\Sigma,A}$	Same s $L^*_{A,B}$	$\frac{L_{\Sigma,B}^{*}}{L_{\Sigma,B}^{*}}$	Toe $L^*_{A,B}$	plitz $L^*_{\Sigma,B}$
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1)	$L^*_{\Sigma,A}$ 0.8999	Same s $L_{A,B}^*$ 0.8999	$\frac{L_{\Sigma,B}^{*}}{0.0000}$	Toej $L_{A,B}^*$ 0.8979	plitz $L^*_{\Sigma,B}$ 0.0020
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$	$L^*_{\Sigma,A}$ 0.8999 0.1517	Same s $L_{A,B}^*$ 0.8999 0.1347	$\frac{L_{\Sigma,B}^{*}}{0.0000}$	Toej $L_{A,B}^*$ 0.8979 0.1345	plitz $L_{\Sigma,B}^{*}$ 0.0020 0.0172
$\sigma^2 = 2$ $\Sigma$ MA(1) CS AR(1)	$L^*_{\Sigma,A}$ 0.8999 0.1517 0.8993	Same s $L^*_{A,B}$ 0.8999 0.1347 0.8993	$\begin{tabular}{c} $F$ \\ tructure \\ $L^*_{\Sigma,B}$ \\ \hline 0.0000 \\ \hline 0.0170 \\ \hline 0.0000 \end{tabular}$	Toe $L_{A,B}^*$ 0.8979 0.1345 0.8973	plitz $L_{\Sigma,B}^{*}$ 0.0020 0.0172 0.0020
$\sigma^2 = 2$ $\frac{\Sigma}{MA(1)}$ CS AR(1) ARMA-r=0.1	$L^*_{\Sigma,A}$ 0.8999 0.1517 0.8993 0.9083	Same s $L_{A,B}^*$ 0.8999 0.1347 0.8993 0.9083	$\frac{E}{L_{\Sigma,B}^*}$ 0.0000 0.0170 0.0000 0.0000	$\begin{array}{c} \text{Toe}\\ L_{A,B}^{*}\\ 0.8979\\ 0.1345\\ 0.8973\\ 0.9064 \end{array}$	$\begin{array}{c} \text{plitz} \\ \hline L_{\Sigma,B}^{*} \\ 0.0020 \\ \hline 0.0172 \\ \hline 0.0020 \\ \hline 0.0019 \end{array}$
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35	$L^*_{\Sigma,A}$ 0.8999 0.1517 0.8993 0.9083 0.8892	Same s <i>L</i> <sup>*</sup> <sub><i>A,B</i></sub> 0.8999 0.1347 0.8993 0.9083 0.8892	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	Toej $L^*_{A,B}$ 0.8979 0.1345 0.8973 0.9064 0.8870	plitz $L^*_{\Sigma,B}$ 0.0020 0.0172 0.0020 0.0019 0.0022
$\sigma^2 = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6	$L^*_{\Sigma,A}$ 0.8999 0.1517 0.8993 0.9083 0.8892 0.8513	Same s $L_{A,B}^*$ 0.8999 0.1347 0.8993 0.9083 0.9083 0.8892 0.8513	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	Toej $L_{A,B}^*$ 0.8979 0.1345 0.8973 0.9064 0.8870 0.8483	plitz $L_{\Sigma,B}^*$ 0.0020 0.0172 0.0020 0.0019 0.0022 0.0030
$\sigma^2 = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6	$L^*_{\Sigma,A}$ 0.8999 0.1517 0.8993 0.9083 0.8892 0.8513	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.8999 \\ 0.1347 \\ 0.8993 \\ 0.9083 \\ 0.9083 \\ 0.8892 \\ 0.8513 \\ \end{array}$	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	Toej L <sup>*</sup> <sub>A,B</sub> 0.8979 0.1345 0.8973 0.9064 0.8870 0.8483 3	$\begin{array}{c} \text{plitz}\\ \overline{L_{\Sigma,B}^*}\\ 0.0020\\ 0.0172\\ 0.0020\\ 0.0019\\ 0.0022\\ 0.0030\\ \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $\sigma^{2} = 4$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8999 \\ 0.1517 \\ 0.8993 \\ 0.9083 \\ 0.8892 \\ 0.8513 \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.8999 \\ 0.1347 \\ 0.8993 \\ 0.9083 \\ 0.9083 \\ 0.8892 \\ 0.8513 \\ \hline \\ \text{Same s} \end{array}$	$\begin{tabular}{c} $F$ tructure $$ $L_{\Sigma,B}^*$ $$ $0.0000$ $$ $$ $0.0000$ $$ $$ $$ $0.0000$ $$ $$ $$ $$ $$ $0.0000$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$	$\begin{array}{c} 3\\ \hline Toep\\ L^*_{A,B}\\ 0.8979\\ 0.1345\\ 0.8973\\ 0.9064\\ 0.8870\\ 0.8483\\ 3\\ \hline Toep \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0020 \\ 0.0172 \\ 0.0020 \\ 0.0020 \\ 0.0019 \\ 0.0022 \\ 0.0030 \\ \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$	$\begin{array}{c} L_{\Sigma,A}^{*} \\ 0.8999 \\ 0.1517 \\ 0.8993 \\ 0.9083 \\ 0.8892 \\ 0.8513 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$		$\begin{array}{c} 3\\ \hline Toep\\ L^*_{A,B}\\ 0.8979\\ 0.1345\\ 0.8973\\ 0.9064\\ 0.8870\\ 0.8483\\ 3\\ \hline Toep\\ L^*_{A,B} \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ \hline 0.0020 \\ 0.0172 \\ \hline 0.0020 \\ \hline 0.0019 \\ \hline 0.0022 \\ \hline 0.0030 \\ \hline \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$ MA(1)	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8999 \\ 0.1517 \\ 0.8993 \\ 0.9083 \\ 0.8892 \\ 0.8513 \\ \hline \\ L^*_{\Sigma,A} \\ 0.8999 \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \text{F}\\ \text{tructure}\\ & L_{\Sigma,B}^{*}\\ 0.0000\\ 0.0170\\ 0.0000\\ 0.0000\\ 0.0000\\ \hline & \text{0.0000}\\ \hline & \text{E}\\ \text{tructure}\\ & L_{\Sigma,B}^{*}\\ \hline & 0.0000 \end{array}$	$\begin{array}{c} & \\ & \\ \hline Toej \\ L^*_{A,B} \\ 0.8979 \\ 0.1345 \\ 0.8973 \\ 0.9064 \\ 0.8870 \\ 0.8483 \\ 0.8483 \\ \hline Toej \\ L^*_{A,B} \\ 0.8979 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0020 \\ 0.0172 \\ 0.0020 \\ 0.0019 \\ 0.0022 \\ 0.0030 \\ \hline \\ 0.0030 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0020 \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$ MA(1) CS	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8999 \\ 0.1517 \\ 0.8993 \\ 0.9083 \\ 0.8892 \\ 0.8513 \\ \hline \\ L^*_{\Sigma,A} \\ 0.8999 \\ 0.1388 \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \text{F}\\ \text{tructure}\\ & L_{\Sigma,B}^{*}\\ 0.0000\\ 0.0170\\ 0.0000\\ 0.0000\\ 0.0000\\ \hline & \text{0.0000}\\ \hline & \text{E}\\ \text{tructure}\\ & L_{\Sigma,B}^{*}\\ \hline & 0.0000\\ \hline & 0.0120\\ \end{array}$	$\begin{array}{c} & \\ & \\ \hline Toej \\ L^*_{A,B} \\ 0.8979 \\ 0.1345 \\ 0.8973 \\ 0.9064 \\ 0.8870 \\ 0.8483 \\ \hline 0.8483 \\ \hline \\ \hline \\ L^*_{A,B} \\ 0.8979 \\ 0.1266 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0020 \\ 0.0172 \\ 0.0020 \\ 0.0019 \\ 0.0022 \\ 0.0030 \\ \hline \\ 0.0030 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0020 \\ 0.0121 \\ \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$ MA(1) CS AR(1)	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8999 \\ 0.1517 \\ 0.8993 \\ 0.9083 \\ 0.8892 \\ 0.8513 \\ \hline \\ L^*_{\Sigma,A} \\ 0.8999 \\ 0.1388 \\ 0.8992 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ \hline Toej \\ L^*_{A,B} \\ 0.8979 \\ 0.1345 \\ 0.8973 \\ 0.9064 \\ 0.8870 \\ 0.8483 \\ \hline 0.8483 \\ \hline \\ \hline \\ L^*_{A,B} \\ 0.8979 \\ 0.1266 \\ \hline \\ 0.8972 \\ \hline \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0020 \\ 0.0172 \\ 0.0020 \\ 0.0019 \\ 0.0022 \\ 0.0030 \\ \hline \\ 0.0020 \\ \hline \\ L_{\Sigma,B}^{*} \\ 0.0020 \\ \hline \\ 0.0121 \\ 0.0020 \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8999 \\ 0.1517 \\ 0.8993 \\ 0.9083 \\ 0.8892 \\ 0.8513 \\ \hline \\ L^*_{\Sigma,A} \\ 0.8999 \\ 0.1388 \\ 0.8992 \\ 0.9083 \\ \hline \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c c c } \hline F \\ tructure & $L^*_{\Sigma,B}$ \\ \hline 0.0000 & $0.0000$ \\ \hline 0.0000 & $0.0000$ \\ \hline 0.0000 & $0.0000$ \\ \hline F \\ tructure & $L^*_{\Sigma,B}$ \\ \hline 0.0000 & $0.0120$ \\ \hline 0.0000 & $0.0000$ \\ \hline \hline 0.0000 & $0.0000$ \\ \hline \end{tabular}$	$\begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0020 \\ 0.0172 \\ 0.0020 \\ 0.0019 \\ 0.0022 \\ 0.0030 \\ \hline \\ 0.0022 \\ 0.0030 \\ \hline \\ L_{\Sigma,B}^{*} \\ 0.0020 \\ 0.0121 \\ \hline \\ 0.0020 \\ \hline \\ 0.0019 \\ \hline \end{array}$
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.6 $\sigma^{2} = 4$ $\Sigma$ MA(1) CS AR(1) CS AR(1) ARMA-r=0.1 ARMA-r=0.35	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8999 \\ 0.1517 \\ 0.8993 \\ 0.9083 \\ 0.8892 \\ 0.8513 \\ \hline \\ L^*_{\Sigma,A} \\ 0.8999 \\ 0.1388 \\ 0.8992 \\ 0.9083 \\ 0.8893 \\ 0.8893 \\ \hline \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c }\hline & & & & & & \\ \hline tructure & & & & \\ \hline $L^*_{\Sigma,B}$ & & & & & \\ \hline $0.0000$ & & & & \\ \hline $0.0000$ & & \\ \hline $0.0000$ & & \\ \hline $0.0000$ & & \\ \hline \hline \end{tabular}$	$\begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0020 \\ 0.0172 \\ 0.0020 \\ 0.0019 \\ 0.0022 \\ 0.0030 \\ \hline \\ \hline \\ L_{\Sigma,B}^{*} \\ 0.0020 \\ 0.0121 \\ \hline \\ 0.0020 \\ 0.0019 \\ \hline \\ 0.0023 \\ \end{array}$

			I	3		
$\sigma^2 = 0.5$	0	Same s	tructure	Toe	plitz	
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	
MA(1)	0.8709	0.8709	0.0000	0.8683	0.0026	
CS	0.0525	0.0308	0.0217	0.0308	0.0217	
AR(1)	0.8587	0.8586	0.0000	0.8558	0.0028	
ARMA-r=0.2	0.9014	0.9014	0.0000	0.8994	0.0020	
ARMA-r=0.45	0.8678	0.8678	0.0000	0.8651	0.0027	
ARMA-r=0.75	0.8022	0.8022	0.0000	0.7983	0.0039	
			I	3		
$\sigma^2 = 1$		Same s	tructure	Toe	plitz	
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	
MA(1)	0.8709	0.8709	0.0000	0.8683	0.0026	
CS	0.0550	0.0306	0.0244	0.0306	0.0245	
AR(1)	0.8586	0.8586	0.0000	0.8558	0.0029	
ARMA-r=0.2	0.9014	0.9014	0.0000	0.8994	0.0020	
ARMA-r=0.45	0.8680	0.8680	0.0000	0.8654	0.0026	
ARMA-r=0.75	0.8020	0.8020	0.0000	0.7980	0.0040	
		8020 0.8020 0.		B		
			F	3		
$\sigma^2 = 2$		Same s	I tructure	3 Toe	plitz	
$\frac{\sigma^2 = 2}{\Sigma}$	$L^*_{\Sigma,A}$	Same s $L^*_{A,B}$	$\frac{1}{L^*_{\Sigma,B}}$	Toe $L_{A,B}^*$	plitz $L^*_{\Sigma,B}$	
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$	$L^*_{\Sigma,A}$ 0.8709	Same s $L_{A,B}^*$ 0.8709	$\frac{L_{\Sigma,B}^{*}}{0.0000}$	Toe $L_{A,B}^{*}$ 0.8683	$\frac{\frac{L_{\Sigma,B}^{*}}{0.0026}}{0.0026}$	
$\frac{\sigma^2 = 2}{\frac{\Sigma}{MA(1)}}$	$L^*_{\Sigma,A}$ 0.8709 0.0525	Same s $L_{A,B}^*$ 0.8709 0.0302	$\frac{E_{\Sigma,B}}{0.0000}$	Toe $L_{A,B}^*$ 0.8683 0.0301	plitz $L_{\Sigma,B}^{*}$ 0.0026 0.0224	
$     \frac{\sigma^2 = 2}{\Sigma}     \frac{\Sigma}{MA(1)}     CS     AR(1)     $	$L^*_{\Sigma,A}$ 0.8709 0.0525 0.8586	Same s $L_{A,B}^*$ 0.8709 0.0302 0.8586	$F_{\Sigma,B} = 0.0000 = 0.0223 = 0.0000$	Toe $L_{A,B}^*$ 0.8683 0.0301 0.8557	plitz $L_{\Sigma,B}^*$ 0.0026 0.0224 0.0029	
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.2	$L^*_{\Sigma,A}$ 0.8709 0.0525 0.8586 0.9013	Same s $L_{A,B}^*$ 0.8709 0.0302 0.8586 0.9013	$F_{\Sigma,B} = \frac{L_{\Sigma,B}^*}{0.0000} \\ 0.0223 \\ 0.0000 \\ 0.00$	$\begin{array}{c} \text{Toe}\\ L_{A,B}^{*}\\ 0.8683\\ 0.0301\\ 0.8557\\ 0.8993 \end{array}$	$\begin{array}{c} \text{plitz} \\ \hline L_{\Sigma,B}^{*} \\ 0.0026 \\ \hline 0.0224 \\ \hline 0.0029 \\ \hline 0.0020 \end{array}$	
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45	$L^*_{\Sigma,A}$ 0.8709 0.0525 0.8586 0.9013 0.8678	Same s $L_{A,B}^*$ 0.8709 0.0302 0.8586 0.9013 0.8678	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	Toe $L^*_{A,B}$ 0.8683 0.0301 0.8557 0.8993 0.8652	$\begin{array}{c} \text{plitz} \\ \hline L^*_{\Sigma,B} \\ \hline 0.0026 \\ \hline 0.0224 \\ \hline 0.0029 \\ \hline 0.0020 \\ \hline 0.0026 \end{array}$	
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75	$L^*_{\Sigma,A}$ 0.8709 0.0525 0.8586 0.9013 0.8678 0.8018	$\begin{array}{c} \text{Same s} \\ L^*_{A,B} \\ 0.8709 \\ 0.0302 \\ 0.8586 \\ 0.9013 \\ 0.8678 \\ 0.8018 \end{array}$	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	Toe $L_{A,B}^*$ 0.8683 0.0301 0.8557 0.8993 0.8652 0.7978	plitz $L_{\Sigma,B}^*$ 0.0026 0.0224 0.0029 0.0020 0.0026 0.0026	
$ \frac{\sigma^2 = 2}{\Sigma} $ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75	$L^*_{\Sigma,A}$ 0.8709 0.0525 0.8586 0.9013 0.8678 0.8018	$\begin{array}{c} \text{Same s} \\ L_{A,B}^{*} \\ 0.8709 \\ 0.0302 \\ 0.8586 \\ 0.9013 \\ 0.8678 \\ 0.8018 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	Toe $L_{A,B}^*$ 0.8683 0.0301 0.8557 0.8993 0.8652 0.7978 3	$\begin{array}{c} \text{plitz}\\ L_{\Sigma,B}^{*}\\ \hline 0.0026\\ \hline 0.0224\\ \hline 0.0029\\ \hline 0.0020\\ \hline 0.0026\\ \hline 0.0040\\ \end{array}$	
$\sigma^{2} = 2$ $\Sigma$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\sigma^{2} = 4$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8709 \\ 0.0525 \\ 0.8586 \\ 0.9013 \\ 0.8678 \\ 0.8018 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{c} F \\ tructure \\ $L^*_{\Sigma,B}$ \\ 0.0000 \\ 0.0223 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ \hline $F$ \\ tructure \\ \end{tabular}$	$\begin{array}{c c} & & & \\ & & & \\ \hline & & & \\ L^*_{A,B} \\ 0.8683 \\ 0.0301 \\ 0.8557 \\ 0.8993 \\ 0.8652 \\ 0.7978 \\ \hline & & \\ 3 \\ \hline & & \\ \hline & & \\ Toe \end{array}$	$\begin{array}{c} \label{eq:linear_product} plitz \\ L_{\Sigma,B}^{*} \\ 0.0026 \\ 0.0224 \\ 0.0029 \\ 0.0020 \\ 0.0020 \\ 0.0040 \\ \hline 0.0040 \\ \hline \end{array}$	
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{CS}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.2}$ $\frac{\Delta RMA-r=0.45}{\Delta RMA-r=0.75}$ $\frac{\sigma^2 = 4}{\Sigma}$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8709 \\ 0.0525 \\ 0.8586 \\ 0.9013 \\ 0.8678 \\ 0.8018 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0026 \\ 0.0224 \\ 0.0029 \\ 0.0020 \\ 0.0026 \\ 0.0040 \\ \hline 0.0040 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \end{array}$	
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{CS}$ $\frac{\Delta R(1)}{ARMA-r=0.2}$ $\frac{\Delta RMA-r=0.45}{ARMA-r=0.75}$ $\frac{\sigma^2 = 4}{\Sigma}$ $\frac{\Delta L}{MA(1)}$	$\begin{matrix} L^*_{\Sigma,A} \\ 0.8709 \\ 0.0525 \\ 0.8586 \\ 0.9013 \\ 0.8678 \\ 0.8018 \\ \begin{matrix} L^*_{\Sigma,A} \\ 0.8709 \end{matrix}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ Toe \\ & \\ L_{A,B}^{*} \\ 0.8683 \\ 0.0301 \\ 0.8557 \\ 0.8993 \\ 0.8652 \\ 0.7978 \\ 3 \\ \hline \\ & \\ Toe \\ & \\ L_{A,B}^{*} \\ 0.8683 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0026 \\ 0.0224 \\ 0.0029 \\ 0.0020 \\ 0.0026 \\ 0.0040 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0026 \\ \hline \end{array}$	
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{\Delta R(1)}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.2}$ $\frac{\Delta RMA-r=0.25}{\Delta RMA-r=0.75}$ $\frac{\sigma^2 = 4}{\Sigma}$ $\frac{\Delta L}{\Delta RMA(1)}$ $CS$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8709 \\ 0.0525 \\ 0.8586 \\ 0.9013 \\ 0.8678 \\ 0.8018 \\ \hline \\ L^*_{\Sigma,A} \\ 0.8709 \\ 0.0555 \end{array}$	$\begin{array}{c} \text{Same s} \\ L^*_{A,B} \\ 0.8709 \\ 0.0302 \\ 0.8586 \\ 0.9013 \\ 0.8678 \\ 0.8018 \\ \hline \\ \text{Same s} \\ L^*_{A,B} \\ 0.8709 \\ 0.0309 \\ \hline \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	Toe L <sup>*</sup> <sub>A,B</sub> 0.8683 0.0301 0.8557 0.8993 0.8652 0.7978 3 Toe L <sup>*</sup> <sub>A,B</sub> 0.8683 0.309	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0026 \\ 0.0224 \\ 0.0029 \\ 0.0020 \\ 0.0026 \\ 0.0040 \\ \hline \\ 0.0040 \\ \hline \\ L_{\Sigma,B}^{*} \\ 0.0026 \\ 0.0245 \\ \hline \end{array}$	
$\frac{\sigma^2 = 2}{\Sigma}$ $\frac{\Delta R(1)}{\Delta R(1)}$ $\frac{\Delta R(1)}{\Delta RMA-r=0.2}$ $\frac{\Delta RMA-r=0.2}{\Delta RMA-r=0.75}$ $\frac{\sigma^2 = 4}{\Sigma}$ $\frac{\Delta CS}{\Delta R(1)}$	$\begin{matrix} L^*_{\Sigma,A} \\ 0.8709 \\ 0.0525 \\ 0.8586 \\ 0.9013 \\ 0.8678 \\ 0.8018 \\ \begin{matrix} L^*_{\Sigma,A} \\ 0.8709 \\ 0.0555 \\ 0.8585 \end{matrix}$	$\begin{array}{c} \text{Same s} \\ L^*_{A,B} \\ 0.8709 \\ 0.0302 \\ 0.8586 \\ 0.9013 \\ 0.8678 \\ 0.8018 \\ \hline \\ \text{Same s} \\ L^*_{A,B} \\ 0.8709 \\ 0.0309 \\ 0.0309 \\ 0.8585 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ \hline Toe \\ & \\ L^*_{A,B} \\ 0.8683 \\ 0.0301 \\ 0.8557 \\ 0.8993 \\ 0.8652 \\ 0.7978 \\ \hline 3 \\ \hline \\ \hline \\ & \\ L^*_{A,B} \\ 0.8683 \\ 0.0309 \\ 0.8557 \\ \hline \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0026 \\ 0.0224 \\ 0.0029 \\ 0.0020 \\ 0.0026 \\ 0.0040 \\ \hline \\ L_{\Sigma,B}^{*} \\ 0.0026 \\ 0.0026 \\ 0.0245 \\ 0.0028 \\ \hline \end{array}$	
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\frac{\sigma^2 = 4}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8709 \\ 0.0525 \\ 0.8586 \\ 0.9013 \\ 0.8678 \\ 0.8018 \\ \hline \\ L^*_{\Sigma,A} \\ 0.8709 \\ 0.0555 \\ 0.8585 \\ 0.9013 \\ \end{array}$	$\begin{array}{c} \text{Same s} \\ L^*_{A,B} \\ 0.8709 \\ 0.0302 \\ 0.8586 \\ 0.9013 \\ 0.8678 \\ 0.8018 \\ \hline \\ \text{Same s} \\ L^*_{A,B} \\ 0.8709 \\ 0.0309 \\ 0.0309 \\ 0.8585 \\ 0.9013 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ \hline Toe \\ & \\ L^*_{A,B} \\ 0.8683 \\ 0.0301 \\ 0.8557 \\ 0.8993 \\ 0.8652 \\ 0.7978 \\ \hline \\ 0.8652 \\ 0.7978 \\ \hline \\ 0.8683 \\ 0.0309 \\ 0.8557 \\ 0.8993 \\ \hline \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0026 \\ 0.0224 \\ 0.0029 \\ 0.0020 \\ 0.0026 \\ 0.0040 \\ \hline \\ L_{\Sigma,B}^{*} \\ 0.0026 \\ 0.0026 \\ 0.0245 \\ 0.0028 \\ 0.0020 \\ \hline \end{array}$	
$\frac{\sigma^2 = 2}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45 ARMA-r=0.75 $\frac{\sigma^2 = 4}{\Sigma}$ MA(1) CS AR(1) ARMA-r=0.2 ARMA-r=0.45	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.8709 \\ 0.0525 \\ 0.8586 \\ 0.9013 \\ 0.8678 \\ 0.8018 \\ \hline \\ L^*_{\Sigma,A} \\ 0.8709 \\ 0.0555 \\ 0.8585 \\ 0.9013 \\ 0.8679 \\ \hline \end{array}$	$\begin{array}{c} \text{Same s} \\ L^*_{A,B} \\ 0.8709 \\ 0.0302 \\ 0.8586 \\ 0.9013 \\ 0.8678 \\ 0.8018 \\ \hline \\ \text{Same s} \\ L^*_{A,B} \\ 0.8709 \\ 0.0309 \\ 0.0309 \\ 0.8585 \\ 0.9013 \\ 0.8679 \\ \hline \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} & \\ & \\ \hline Toe \\ & \\ L^*_{A,B} \\ 0.8683 \\ 0.0301 \\ 0.8557 \\ 0.8993 \\ 0.8652 \\ 0.7978 \\ \hline \\ 0.8652 \\ 0.7978 \\ \hline \\ 0.8683 \\ 0.0309 \\ 0.8557 \\ 0.8993 \\ 0.8652 \\ \hline \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0026 \\ 0.0224 \\ 0.0029 \\ 0.0020 \\ 0.0026 \\ 0.0040 \\ \hline \\ L_{\Sigma,B}^{*} \\ 0.0026 \\ 0.0026 \\ 0.0024 \\ \hline \\ 0.0028 \\ 0.0020 \\ 0.0027 \\ \hline \end{array}$	

**Table 11** Simulation results with m = 1000; c = 0.5.

**Table 12** Simulation results with m = 1000; c = 0.75.

			E	3	
$\sigma^{2} = 0.5$	0	Same s	tructure	Toe	plitz
Σ	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
CS	0.0412	0.0082	0.0330	0.0082	0.0330
AR(1)	0.7400	0.7399	0.0001	0.7349	0.0051
ARMA-r=0.25	0.8870	0.8869	0.0000	0.8846	0.0023
ARMA-r=0.5	0.8242	0.8242	0.0000	0.8209	0.0033
ARMA-r=0.8	0.7206	0.7205	0.0001	0.7151	0.0055
			F	3	
$\sigma^2 = 1$		Same s	tructure	Toe	plitz
$\Sigma$	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
CS	0.0268	0.0080	0.0187	0.0080	0.0187
AR(1)	0.7396	0.7396	0.0001	0.7341	0.0055
ARMA-r=0.25	0.8874	0.8874	0.0000	0.8851	0.0022
ARMA-r=0.5	0.8265	0.8264	0.0000	0.8231	0.0034
ARMA-r=0.8	0.7206	0.7205	0.0001	0.7147	0.0058
		0.7205 0.0001 0.7147			
2			E	3	
$\sigma^2 = 2$		Same s	E tructure	3 Toej	plitz
$\frac{\sigma^2 = 2}{\Sigma}$	$L^*_{\Sigma,A}$	Same s $L^*_{A,B}$	tructure $L^*_{\Sigma,B}$	Toe $L_{A,B}^*$	plitz $L^*_{\Sigma,B}$
$\sigma^2 = 2$ $\Sigma$ CS	$\frac{L_{\Sigma,A}^{*}}{0.0359}$	Same s $L_{A,B}^*$ 0.0079	$\frac{L_{\Sigma,B}^*}{0.0279}$	Toej $L_{A,B}^{*}$ 0.0079	plitz $L^*_{\Sigma,B}$ 0.0279
$\frac{\sigma^2 = 2}{\frac{\Sigma}{CS}}$	$L^*_{\Sigma,A}$ 0.0359 0.7413	Same s $L_{A,B}^*$ 0.0079 0.7412	$     E \\     tructure \\     L^*_{\Sigma,B} \\     0.0279 \\     0.0001 $	$     Toe      L^*_{A,B}     0.0079     0.7356 $	plitz $L_{\Sigma,B}^{*}$ 0.0279 0.0057
$\frac{\sigma^2 = 2}{\Sigma}$ CS AR(1) ARMA-r=0.25	$\frac{L^*_{\Sigma,A}}{0.0359}\\0.7413\\0.8872$	Same s $L_{A,B}^*$ 0.0079 0.7412 0.8872	E E E E E E E E E E E E E E E E E	Toep $L_{A,B}^*$ 0.0079 0.7356 0.8850	plitz $L_{\Sigma,B}^*$ 0.0279 0.0057 0.0022
$ \frac{\sigma^2 = 2}{\Sigma} $ CS AR(1) ARMA-r=0.25 ARMA-r=0.5	$L^*_{\Sigma,A}$ 0.0359 0.7413 0.8872 0.8249	Same s $L^*_{A,B}$ 0.0079 0.7412 0.8872 0.8249	$\begin{tabular}{l} \hline $E$ tructure $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$$	Toej $L_{A,B}^*$ 0.0079 0.7356 0.8850 0.8216	plitz $L_{\Sigma,B}^*$ 0.0279 0.0057 0.0022 0.0033
$\sigma^2 = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5 ARMA-r=0.8	$L^*_{\Sigma,A}$ 0.0359 0.7413 0.8872 0.8249 0.7216	Same s $L^*_{A,B}$ 0.0079 0.7412 0.8872 0.8249 0.7215	$F_{\Sigma,B} = 0.0279 \\ 0.0001 \\ 0.0000 \\ $	$\begin{array}{c} {} & {} \\ \hline Toel \\ L^*_{A,B} \\ 0.0079 \\ 0.7356 \\ 0.8850 \\ 0.8216 \\ 0.7164 \end{array}$	plitz $L_{\Sigma,B}^*$ 0.0279 0.0057 0.0022 0.0033 0.0052
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5 ARMA-r=0.8	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.0359 \\ 0.7413 \\ 0.8872 \\ 0.8249 \\ 0.7216 \end{array}$	Same s $L_{A,B}^*$ 0.0079 0.7412 0.8872 0.8249 0.7215	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	$\begin{array}{c} \hline Toep \\ \hline L^*_{A,B} \\ 0.0079 \\ 0.7356 \\ 0.8850 \\ 0.8216 \\ 0.7164 \\ \hline \end{array}$	plitz $L_{\Sigma,B}^*$ 0.0279 0.0057 0.0022 0.0033 0.0052
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.8 $\sigma^{2} = 4$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.0359 \\ 0.7413 \\ 0.8872 \\ 0.8249 \\ 0.7216 \end{array}$	Same s $L^*_{A,B}$ 0.0079 0.7412 0.8872 0.8249 0.7215 Same s		$\begin{array}{c} \hline Toep\\ L^*_{A,B}\\ 0.0079\\ 0.7356\\ 0.8850\\ 0.8216\\ 0.7164\\ \hline \end{array}$	plitz $L_{\Sigma,B}^{*}$ 0.0279 0.0057 0.0022 0.0033 0.0052 plitz
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5 ARMA-r=0.8 $\sigma^{2} = 4$ $\Sigma$	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.0359 \\ 0.7413 \\ 0.8872 \\ 0.8249 \\ 0.7216 \\ \\ L^*_{\Sigma,A} \end{array}$	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$		$\begin{array}{c} 3\\ Toep\\ L^*_{A,B}\\ 0.0079\\ 0.7356\\ 0.8850\\ 0.8216\\ 0.7164\\ \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0279 \\ 0.0057 \\ 0.0022 \\ 0.0033 \\ 0.0052 \\ \end{array}$
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5 ARMA-r=0.8 $\sigma^{2} = 4$ $\Sigma$ CS	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.0359 \\ 0.7413 \\ 0.8872 \\ 0.8249 \\ 0.7216 \\ \hline \\ L^*_{\Sigma,A} \\ 0.0292 \end{array}$	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$		$\begin{array}{c} & \\ & \\ & \\ Toej \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ $	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0279 \\ 0.0057 \\ 0.0022 \\ 0.0033 \\ 0.0052 \\ \hline 0.0052 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0210 \\ \hline \end{array}$
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5 ARMA-r=0.8 $\sigma^{2} = 4$ $\Sigma$ CS AR(1)	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.0359 \\ 0.7413 \\ 0.8872 \\ 0.8249 \\ 0.7216 \\ \hline \\ L^*_{\Sigma,A} \\ 0.0292 \\ 0.7411 \\ \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^* \\ 0.0079 \\ 0.7412 \\ 0.8872 \\ 0.8249 \\ 0.7215 \\ \hline \\ \text{Same s} \\ L_{A,B}^* \\ 0.0082 \\ 0.7411 \\ \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} 3\\ \hline Toe \\ L^*_{A,B}\\ 0.0079\\ 0.7356\\ 0.8850\\ 0.8216\\ 0.7164\\ 3\\ \hline Toe \\ L^*_{A,B}\\ 0.0082\\ 0.7360\\ \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0279 \\ 0.0057 \\ 0.0022 \\ 0.0033 \\ 0.0052 \\ \end{array}$
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.8 $\sigma^{2} = 4$ $\Sigma$ CS AR(1) ARMA-r=0.25	$\begin{array}{c} L^*_{\Sigma,A} \\ 0.0359 \\ 0.7413 \\ 0.8872 \\ 0.8249 \\ 0.7216 \\ \hline \\ L^*_{\Sigma,A} \\ 0.0292 \\ 0.7411 \\ 0.8874 \\ \end{array}$	$\begin{array}{c} \text{Same s} \\ L_{A,B}^* \\ 0.0079 \\ 0.7412 \\ 0.8872 \\ 0.8249 \\ 0.7215 \\ \hline \\ \text{Same s} \\ L_{A,B}^* \\ 0.0082 \\ 0.7411 \\ 0.8874 \\ \end{array}$	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	$\begin{array}{c} 3\\ \hline Toe \\ L^*_{A,B}\\ 0.0079\\ 0.7356\\ 0.8850\\ 0.8216\\ 0.7164\\ 3\\ \hline Toe \\ L^*_{A,B}\\ 0.0082\\ 0.7360\\ 0.8850\\ \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0279 \\ 0.0057 \\ 0.0022 \\ 0.0033 \\ 0.0052 \\ \hline \\ plitz \\ L_{\Sigma,B}^{*} \\ 0.0210 \\ 0.0052 \\ 0.0024 \\ \end{array}$
$\sigma^{2} = 2$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.8 $\sigma^{2} = 4$ $\Sigma$ CS AR(1) ARMA-r=0.25 ARMA-r=0.5	$\begin{matrix} L^*_{\Sigma,A} \\ 0.0359 \\ 0.7413 \\ 0.8872 \\ 0.8249 \\ 0.7216 \\ \end{matrix}$	$\begin{array}{c} \text{Same s} \\ L^*_{A,B} \\ 0.0079 \\ 0.7412 \\ 0.8872 \\ 0.8249 \\ 0.7215 \\ \hline \\ \text{Same s} \\ L^*_{A,B} \\ 0.0082 \\ 0.7411 \\ 0.8874 \\ 0.8237 \\ \hline \end{array}$	$\begin{tabular}{ c c c c c } \hline F \\ tructure \\ $L^*_{\Sigma,B}$ \\ \hline 0.0279 \\ \hline 0.0000 \\ \hline 0.0000 \\ \hline 0.0000 \\ \hline F \\ tructure \\ $L^*_{\Sigma,B}$ \\ \hline 0.0210 \\ \hline 0.0001 \\ \hline 0.0000 \\ \hline 0.0001 \\ \hline \end{tabular}$	$\begin{array}{c} 3\\ \hline Toe \\ L^*_{A,B}\\ 0.0079\\ 0.7356\\ 0.8850\\ 0.8216\\ 0.7164\\ \end{array}\\ \hline Toe \\ L^*_{A,B}\\ 0.0082\\ 0.7360\\ 0.8850\\ 0.8203\\ \end{array}$	$\begin{array}{c} \begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0279 \\ 0.0057 \\ 0.0022 \\ 0.0033 \\ 0.0052 \\ \end{array}$

		1				I	3				
$\sigma^{2} = 0.50$		MA(1)		CS		AR(1)		ARMA(1,1)		Toeplitz	
	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
m=10	0.0775	0.2049	0.1431	0.1262	0.0695	0.1202	0.0624	0.0820	0.0281	0.0581	0.0194
m=20	0.1329	0.2685	0.1523	0.2177	0.0990	0.1837	0.0691	0.1336	0.0181	0.1154	0.0175
m=50	0.2693	0.3637	0.0960	0.3856	0.1248	0.3192	0.0533	0.2871	0.0271	0.2541	0.0152
m=100	0.4876	0.5106	0.0237	0.5015	0.0166	0.5087	0.0218	0.5128	0.0251	0.4774	0.0102
m=200	0.6569	0.6703	0.0136	0.6668	0.0108	0.6695	0.0129	0.6722	0.0155	0.6500	0.0069
m=500	0.8319	0.8352	0.0034	0.8336	0.0019	0.8352	0.0033	0.8355	0.0037	0.8305	0.0014
m=1000	0.9091	0.9100	0.0009	0.9095	0.0005	0.9100	0.0009	0.9101	0.0009	0.9090	0.0001
		В									
$\sigma^2 = 1$		MA(1)		CS		AR(1)		ARMA(1,1)		Toeplitz	
	$L^*_{\Sigma,A}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$
m=10	0.0852	0.1848	0.1169	0.1011	0.0413	0.1365	0.0768	0.1246	0.0711	0.0635	0.0218
m=20	0.1564	0.2366	0.0827	0.1801	0.0323	0.2253	0.0741	0.2622	0.1067	0.1400	0.0164
m=50	0.3097	0.3684	0.0618	0.3398	0.0338	0.3611	0.0553	0.3852	0.0780	0.2959	0.0138
m=100	0.4943	0.5120	0.0176	0.5122	0.0194	0.5115	0.0171	0.5152	0.0209	0.4843	0.0100
m=200	0.6598	0.6701	0.0102	0.6672	0.0077	0.6696	0.0098	0.6715	0.0115	0.6528	0.0069
m=500	0.8320	0.8346	0.0027	0.8338	0.0020	0.8345	0.0026	0.8352	0.0033	0.8306	0.0014
m = 1000	0.9089	0 9099	0.0010	0 9096	0.0007	0 9099	0.0010	0 9101	0.0012	0 9087	0.0002
m=1000	0.2002	0.7077	0.0010	0.2020	0.0007	0.7077	0.0010	0.7101	0.0012	0.2007	0.0002
	0.2002	0.5077	0.0010	0.2020	0.0007	I	3	0.9101	0.0012	012007	0.0002
$\sigma^2 = \sigma^2$	= 2	MA	A(1)	C	s	I	B.(1)	ARM	A(1,1)	Toe	plitz
$\frac{\sigma^2}{\sigma^2}$	$= 2$ $L_{\Sigma,A}^*$	$\frac{MA}{L_{A,B}^*}$	$\frac{A(1)}{L_{\Sigma,B}^*}$	$L^*_{A,B}$	$L^*_{\Sigma,B}$	$\frac{1}{L_{A,B}^*}$	$\frac{1}{L_{\Sigma,B}^*}$	ARM $L^*_{A,B}$	$\frac{A(1,1)}{L_{\Sigma,B}^*}$	Toe $L_{A,B}^*$	plitz $L^*_{\Sigma,B}$
$\frac{\sigma^2}{m=10}$	$= 2  L_{\Sigma,A}^{*}  0.0908$	$\frac{MA}{L_{A,B}^*} = 0.1445$	$\frac{A(1)}{L_{\Sigma,B}^*}$ 0.0683	$\frac{C}{L_{A,B}^{*}}$ 0.0956	$\frac{L_{\Sigma,B}^*}{0.0230}$	I =      I =     I	$\frac{L_{\Sigma,B}^{*}}{0.0557}$	ARM $L^*_{A,B}$ 0.1563	$\frac{A(1,1)}{L_{\Sigma,B}^*}$	Toe $L_{A,B}^*$ 0.0714	plitz $L^*_{\Sigma,B}$ 0.0194
$\frac{\sigma^2}{m=10}$	$= 2$ $L_{\Sigma,A}^{*}$ 0.0908 0.1488	$\frac{MA}{L_{A,B}^*}$ 0.1445 0.2081	$\frac{L^{*}_{\Sigma,B}}{0.0683}$	$C = \frac{L_{A,B}^*}{0.0956} = 0.2195$	$L_{\Sigma,B}^*$ 0.0230 0.0818	$[1,3,0,3,0] I \\ AR \\ L^*_{A,B} \\ 0.1282 \\ 0.1741$	$\frac{1}{L_{\Sigma,B}^*}$ 0.0557 0.0333	ARM $L^*_{A,B}$ 0.1563 0.1522	$\frac{A(1,1)}{L_{\Sigma,B}^*}$ 0.0817 0.0153	$\frac{\text{Toe}}{L_{A,B}^*}$ 0.0714 0.1418	plitz $L^*_{\Sigma,B}$ 0.0194 0.0070
$\frac{\sigma^2}{m=10}$ $\frac{m=10}{m=20}$ $m=50$	= 2 $= 2$ $0.0908$ $0.1488$ $0.3208$	$\begin{array}{c} MA \\ \hline \\ L^*_{A,B} \\ 0.1445 \\ 0.2081 \\ 0.3557 \end{array}$	$\begin{array}{r} \hline A(1) \\ \hline L^*_{\Sigma,B} \\ \hline 0.0683 \\ \hline 0.0651 \\ \hline 0.0375 \end{array}$	$\begin{array}{c} C\\ L^*_{A,B}\\ 0.0956\\ 0.2195\\ 0.3399 \end{array}$	$\frac{L_{\Sigma,B}^{*}}{0.0230}$ 0.0230 0.0818 0.0235	$\begin{array}{r} 1 \\ \hline AR \\ L^*_{A,B} \\ 0.1282 \\ 0.1741 \\ 0.3521 \end{array}$	$\begin{array}{c} \hline 0.0013\\ \hline 0.0013\\ \hline 0.0013\\ \hline 0.0013\\ \hline 0.0000\\ \hline 0.00000\\ \hline 0.0000\\ \hline 0.000\\ \hline 0.000\\ \hline 0.000\\ \hline 0.000\\ \hline 0.0000\\ \hline 0.000\\ \hline$	$\frac{\text{ARM}}{L_{A,B}^*}$ 0.1563 0.1522 0.3622	$\frac{A(1,1)}{L_{\Sigma,B}^*}$ 0.0817 0.0153 0.0427	$\begin{array}{c} \text{Toe} \\ L_{A,B}^{*} \\ 0.0714 \\ 0.1418 \\ 0.3068 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^* \\ \hline 0.0194 \\ \hline 0.0070 \\ \hline 0.0140 \end{array}$
$ \frac{\sigma^{2}}{m=10} $ $ \frac{m=10}{m=20} $ $ \frac{m=50}{m=100} $	= 2 $= 2$ $0.0908$ $0.1488$ $0.3208$ $0.4923$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} A(1)\\ \hline L_{\Sigma,B}^{*}\\ \hline 0.0683\\ \hline 0.0651\\ \hline 0.0375\\ \hline 0.0193\\ \end{array}$	$\begin{array}{c} C\\ L_{A,B}^{*}\\ 0.0956\\ 0.2195\\ 0.3399\\ 0.5094 \end{array}$	$\begin{array}{c} \overline{L_{\Sigma,B}^{*}} \\ 0.0230 \\ 0.0818 \\ 0.0235 \\ 0.0189 \end{array}$	$\begin{array}{r} \text{I}\\ \text{AR}\\ L_{A,B}^{*}\\ 0.1282\\ 0.1741\\ 0.3521\\ 0.5110\\ \end{array}$	$\begin{array}{c} \hline 0.0010\\ \hline 3\\ \hline .(1)\\ \hline L_{\Sigma,B}^{*}\\ \hline 0.0557\\ \hline 0.0333\\ \hline 0.0342\\ \hline 0.0188 \end{array}$	$\begin{array}{r} \text{ARM} \\ L_{A,B}^{*} \\ 0.1563 \\ 0.1522 \\ 0.3622 \\ 0.5152 \end{array}$	$\begin{array}{c} \text{A(1,1)}\\ \hline L_{\Sigma,B}^{*}\\ \hline 0.0817\\ \hline 0.0153\\ \hline 0.0427\\ \hline 0.0229 \end{array}$	$\begin{array}{c} \hline \text{Toe} \\ L_{A,B}^{*} \\ 0.0714 \\ 0.1418 \\ 0.3068 \\ 0.4820 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^* \\ 0.0194 \\ 0.0070 \\ 0.0140 \\ 0.0103 \end{array}$
	= 2 $= 2$ $0.0908$ $0.1488$ $0.3208$ $0.4923$ $0.6631$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} A(1)\\ \hline L_{\Sigma,B}^{*}\\ 0.0683\\ \hline 0.0651\\ \hline 0.0375\\ \hline 0.0193\\ \hline 0.0096\\ \end{array}$	$\begin{array}{c} C\\ L_{A,B}^{*}\\ 0.0956\\ 0.2195\\ 0.3399\\ 0.5094\\ 0.6759 \end{array}$	$\frac{L_{\Sigma,B}^{*}}{0.0230}$ 0.0230 0.0818 0.0235 0.0189 0.0135	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} (1)\\ \hline L_{\Sigma,B}^{*}\\ 0.0557\\ \hline 0.0333\\ \hline 0.0342\\ \hline 0.0188\\ \hline 0.0096\\ \end{array}$	$\begin{array}{c} \text{ARM} \\ L_{A,B}^{*} \\ 0.1563 \\ 0.1522 \\ 0.3622 \\ 0.5152 \\ 0.6733 \end{array}$	$\begin{array}{c} \text{A}(1,1)\\ \hline L^*_{\Sigma,B}\\ 0.0817\\ \hline 0.0153\\ \hline 0.0427\\ \hline 0.0229\\ \hline 0.0103\\ \end{array}$	$\begin{array}{c} \text{Toe} \\ L_{A,B}^{*} \\ 0.0714 \\ 0.1418 \\ 0.3068 \\ 0.4820 \\ 0.6563 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^* \\ 0.0194 \\ 0.0070 \\ 0.0140 \\ 0.0103 \\ 0.0068 \end{array}$
$ \frac{\sigma^2}{m=10} = \frac{1000}{m=200} $ $ \frac{m=100}{m=200} = \frac{1000}{m=500} $	= 2 $= 2$ $0.0908$ $0.1488$ $0.3208$ $0.4923$ $0.6631$ $0.8310$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} A(1)\\ \hline L_{\Sigma,B}^{*}\\ \hline 0.0683\\ \hline 0.0651\\ \hline 0.0375\\ \hline 0.0193\\ \hline 0.0096\\ \hline 0.0043\\ \end{array}$	$\begin{array}{c} C\\ L^*_{A,B}\\ 0.0956\\ 0.2195\\ 0.3399\\ 0.5094\\ 0.6759\\ 0.8337\end{array}$	$\begin{array}{c} \underline{S}\\ \underline{L}_{\Sigma,B}^{*}\\ 0.0230\\ 0.0818\\ 0.0235\\ 0.0189\\ 0.0135\\ 0.0029 \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} (1)\\ \hline L_{\Sigma,B}^{*}\\ \hline 0.0557\\ \hline 0.0333\\ \hline 0.0342\\ \hline 0.0188\\ \hline 0.0096\\ \hline 0.0043\\ \end{array}$	$\begin{array}{c} \text{ARM} \\ L^*_{A,B} \\ 0.1563 \\ 0.1522 \\ 0.3622 \\ 0.5152 \\ 0.6733 \\ 0.8357 \end{array}$	$\begin{array}{c} \text{A(1,1)}\\ \hline L_{\Sigma,B}^{*}\\ \hline 0.0817\\ \hline 0.0153\\ \hline 0.0427\\ \hline 0.0229\\ \hline 0.0103\\ \hline 0.0049\\ \end{array}$	$\begin{array}{c} \text{Toe} \\ L_{A,B}^{*} \\ 0.0714 \\ 0.1418 \\ 0.3068 \\ 0.4820 \\ 0.6563 \\ 0.8296 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^* \\ 0.0194 \\ 0.0070 \\ 0.0140 \\ 0.0103 \\ 0.0068 \\ 0.0014 \end{array}$
$ \frac{\sigma^2}{m=10} = \frac{1000}{m=200} $ $ \frac{m=100}{m=200} = \frac{1000}{m=1000} $	= 2 $= 2$ $0.0908$ $0.1488$ $0.3208$ $0.4923$ $0.6631$ $0.8310$ $0.9088$	$\begin{array}{c} MA\\ L^*_{A,B}\\ 0.1445\\ 0.2081\\ 0.3557\\ 0.5115\\ 0.6727\\ 0.8352\\ 0.9099\end{array}$	$\begin{array}{c} A(1)\\ L_{\Sigma,B}^{*}\\ 0.0683\\ 0.0651\\ 0.0375\\ 0.0193\\ 0.0096\\ 0.0043\\ 0.0010\\ \end{array}$	$\begin{array}{c} C\\ L^*_{A,B}\\ 0.0956\\ 0.2195\\ 0.3399\\ 0.5094\\ 0.6759\\ 0.8337\\ 0.9095 \end{array}$	$\frac{L_{\Sigma,B}^*}{0.0230}$ $\frac{0.0230}{0.0818}$ $\frac{0.0235}{0.0189}$ $\frac{0.0135}{0.0029}$ $0.0008$	$\begin{array}{c} \text{I} \text{I} \text{I} \text{I} \text{I} \text{I} \text{I} I$	$\begin{array}{c} (1)\\ L_{\Sigma,B}^{*}\\ \hline 0.0557\\ 0.0333\\ 0.0342\\ \hline 0.0188\\ 0.0096\\ \hline 0.0043\\ \hline 0.0010\\ \end{array}$	$\begin{array}{c} \text{ARM} \\ L^*_{A,B} \\ 0.1563 \\ 0.1522 \\ 0.3622 \\ 0.5152 \\ 0.6733 \\ 0.8357 \\ 0.9101 \end{array}$	$\begin{array}{c} \text{A}(1,1) \\ L_{\Sigma,B}^{*} \\ 0.0817 \\ 0.0153 \\ 0.0427 \\ 0.0229 \\ 0.0103 \\ 0.0049 \\ 0.0012 \end{array}$	$\begin{array}{c} \text{Toe} \\ L_{A,B}^* \\ 0.0714 \\ 0.1418 \\ 0.3068 \\ 0.4820 \\ 0.6563 \\ 0.8296 \\ 0.9084 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^* \\ 0.0194 \\ 0.0070 \\ 0.0140 \\ 0.0103 \\ 0.0068 \\ 0.0014 \\ 0.0004 \end{array}$
$ \frac{\sigma^2}{m=10} = \frac{100}{m=20} $ $ \frac{m=10}{m=50} = \frac{100}{m=200} $ $ \frac{m=500}{m=1000} = \frac{1000}{m=10000} $	$= 2$ $L_{\Sigma,A}^{*}$ 0.0908 0.1488 0.3208 0.4923 0.6631 0.8310 0.9088	$\begin{array}{c} MA\\ L^*_{A,B}\\ 0.1445\\ 0.2081\\ 0.3557\\ 0.5115\\ 0.6727\\ 0.8352\\ 0.9099\\ \end{array}$	$\begin{array}{c} A(1) \\ L_{\Sigma,B}^{*} \\ 0.0683 \\ 0.0651 \\ 0.0375 \\ 0.0193 \\ 0.0096 \\ 0.0043 \\ 0.0010 \end{array}$	$\begin{array}{c} C\\ L^*_{A,B}\\ 0.0956\\ 0.2195\\ 0.3399\\ 0.5094\\ 0.6759\\ 0.8337\\ 0.9095\\ \end{array}$	$\frac{L_{\Sigma,B}^*}{0.0230}$ $\frac{0.0230}{0.0818}$ $\frac{0.0235}{0.0135}$ $\frac{0.0029}{0.0008}$	$\begin{array}{c} \text{I} \text{I} \text{I} \text{I} \text{I} \text{I} \text{I} I$	$\begin{array}{c} \hline 0.0010 \\ \hline 3 \\ \hline (.1) \\ \hline L^*_{\Sigma,B} \\ \hline 0.0557 \\ \hline 0.0333 \\ \hline 0.0342 \\ \hline 0.0188 \\ \hline 0.0096 \\ \hline 0.0043 \\ \hline 0.0010 \\ \hline 3 \end{array}$	$\begin{array}{c} \text{ARM.}\\ L_{A,B}^{*}\\ 0.1563\\ 0.1522\\ 0.3622\\ 0.5152\\ 0.6733\\ 0.8357\\ 0.9101\\ \end{array}$	$\begin{array}{c} A(1,1)\\ L^*_{\Sigma,B}\\ 0.0817\\ 0.0153\\ 0.0427\\ 0.0229\\ 0.0103\\ 0.0049\\ 0.0012\\ \end{array}$	$\begin{tabular}{c} \hline Toe \\ \hline $L^*_{A,B}$ \\ 0.0714 \\ 0.1418 \\ 0.3068 \\ 0.4820 \\ 0.6563 \\ 0.8296 \\ 0.9084 \end{tabular}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0194 \\ 0.0070 \\ 0.0140 \\ 0.0103 \\ 0.0068 \\ 0.0014 \\ 0.0004 \end{array}$
$\begin{array}{c} m = 1000 \\ \sigma^2 = \\ \hline m = 10 \\ m = 20 \\ m = 500 \\ \hline m = 1000 \\ m = 1000 \\ \hline m = 1000 \\ \hline m = 1000 \\ \hline \sigma^2 = \end{array}$	$= 2$ $L_{\Sigma,A}^*$ 0.0908 0.1488 0.3208 0.4923 0.6631 0.8310 0.9088 $= 4$	MA L <sup>*</sup> <sub>A,B</sub> 0.1445 0.2081 0.3557 0.5115 0.6727 0.8352 0.9099 MA	$\begin{array}{c} A(1) \\ L_{\Sigma,B}^{*} \\ 0.0683 \\ 0.0651 \\ 0.0375 \\ 0.0193 \\ 0.0096 \\ 0.0043 \\ 0.0010 \\ A(1) \end{array}$	$\begin{array}{c} C\\ L^*_{A,B}\\ 0.0956\\ 0.2195\\ 0.3399\\ 0.5094\\ 0.6759\\ 0.8337\\ 0.9095\\ \hline \end{array}$	$\frac{L_{\Sigma,B}^{*}}{0.0230}$ $\frac{1}{0.0230}$ $\frac{1}{0.0235}$ $\frac{1}{0.0235}$ $\frac{1}{0.0029}$ $\frac{1}{0.0008}$ $\frac{1}{0.0008}$	$\begin{array}{c} \text{I}, \text{I},$	$\begin{array}{c} (.(1) \\ \hline L_{\Sigma,B}^* \\ 0.0557 \\ \hline 0.0333 \\ \hline 0.0342 \\ \hline 0.0188 \\ \hline 0.0043 \\ \hline 0.0010 \\ \hline 3 \\ .(1) \end{array}$	$\begin{array}{c} \text{ARM.}\\ L_{A,B}^{*}\\ 0.1563\\ 0.1522\\ 0.3622\\ 0.5152\\ 0.6733\\ 0.8357\\ 0.9101\\ \hline\\ \text{ARM.} \end{array}$	$\begin{array}{c} A(1,1)\\ \hline L^*_{\Sigma,B}\\ 0.0817\\ \hline 0.0153\\ \hline 0.0427\\ \hline 0.0229\\ \hline 0.0103\\ \hline 0.0049\\ \hline 0.0012\\ \hline A(1,1) \end{array}$	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0194 \\ 0.0070 \\ 0.0140 \\ 0.0103 \\ 0.0068 \\ 0.0014 \\ 0.0004 \\ \end{array}$
$\begin{array}{c} \sigma^2 = \\ \hline \sigma^2 = \\ \hline m=100 \\ m=20 \\ m=50 \\ m=1000 \\ m=500 \\ m=1000 \\ \hline m=1000 \\ m=1000 \\ \hline \sigma^2 = \\ \hline \sigma^2 = \\ m=1000 \\ \hline \sigma^2 = \\ m=1000 \\ \sigma^2 = \\ m=1000 \\ \hline \sigma^2 = \\ m=100 \\ m=10$	$= 2$ $= 2$ $L_{\Sigma,A}^{*}$ 0.0908 0.1488 0.3208 0.4923 0.6631 0.8310 0.9088 $= 4$ $L_{\Sigma,A}^{*}$	$\begin{array}{c} MA \\ L^*_{A,B} \\ 0.1445 \\ 0.2081 \\ 0.3557 \\ 0.5115 \\ 0.6727 \\ 0.8352 \\ 0.9099 \\ \hline \\ MA \\ L^*_{A,B} \end{array}$	$\begin{array}{c} A(1) \\ L_{\Sigma,B}^{*} \\ 0.0683 \\ 0.0651 \\ 0.0375 \\ 0.0193 \\ 0.0096 \\ 0.0043 \\ 0.0010 \\ A(1) \\ L_{\Sigma,B}^{*} \end{array}$	$\begin{array}{c} C\\ L_{A,B}^{*}\\ 0.0956\\ 0.2195\\ 0.3399\\ 0.5094\\ 0.6759\\ 0.8337\\ 0.9095\\ \hline C\\ L_{A,B}^{*} \end{array}$	$\begin{array}{c} S\\ \hline L_{\Sigma,B}^{*}\\ 0.0230\\ 0.0818\\ 0.0235\\ 0.0189\\ 0.0135\\ 0.0029\\ 0.0008\\ \hline S\\ L_{\Sigma,B}^{*}\\ \end{array}$	$\begin{array}{c} \text{I} \text{I} \text{I} \text{I} \text{I} \text{I} \text{I} I$	$\begin{array}{c} \hline 0.0010 \\ \hline 0.0010 \\ \hline B \\ \hline (.1) \\ \hline L_{\Sigma,B}^* \\ \hline 0.0557 \\ \hline 0.0333 \\ \hline 0.0557 \\ \hline 0.0333 \\ \hline 0.0342 \\ \hline 0.0108 \\ \hline 0.0043 \\ \hline 0.0010 \\ \hline B \\ \hline (.1) \\ \hline L_{\Sigma,B}^* \\ \hline \end{array}$	$\begin{array}{c} \text{ARM.}\\ \text{ARM.}\\ L_{A,B}^{*}\\ 0.1563\\ 0.1522\\ 0.3622\\ 0.5152\\ 0.6733\\ 0.8357\\ 0.9101\\ \hline\\ \text{ARM.}\\ L_{A,B}^{*} \end{array}$	$\begin{array}{c} A(1,1)\\ L^*_{\Sigma,B}\\ 0.0817\\ 0.0153\\ 0.0427\\ 0.0229\\ 0.0103\\ 0.0049\\ 0.0012\\ \hline A(1,1)\\ L^*_{\Sigma,B} \end{array}$	$\begin{array}{c} \text{Toe} \\ L_{A,B}^{*} \\ 0.0714 \\ 0.1418 \\ 0.3068 \\ 0.4820 \\ 0.6563 \\ 0.8296 \\ 0.9084 \\ \hline \\ \text{Toe} \\ L_{A,B}^{*} \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^{*} \\ 0.0194 \\ 0.0070 \\ 0.0140 \\ 0.0103 \\ 0.0068 \\ 0.0014 \\ 0.0004 \\ \end{array}$
$\begin{array}{c} \sigma^{2} = \\ \hline m = 100 \\ \hline m = 20 \\ \hline m = 50 \\ \hline m = 100 \\ \hline m = 500 \\ \hline m = 1000 \\ \hline \sigma^{2} = \\ \hline m = 10 \\ \hline \end{array}$	$= 2$ $L_{\Sigma,A}^{*}$ 0.0908 0.1488 0.3208 0.4923 0.6631 0.8310 0.9088 $= 4$ $L_{\Sigma,A}^{*}$ 0.0664	$\begin{array}{c} MA \\ L^*_{A,B} \\ 0.1445 \\ 0.2081 \\ 0.3557 \\ 0.5115 \\ 0.6727 \\ 0.8352 \\ 0.9099 \\ \hline \\ MA \\ L^*_{A,B} \\ 0.2742 \end{array}$	$\begin{array}{c} \Lambda(1) \\ L_{\Sigma,B}^{*} \\ 0.0683 \\ 0.0651 \\ 0.0375 \\ 0.0193 \\ 0.0096 \\ 0.0043 \\ 0.0010 \\ \hline \Lambda(1) \\ L_{\Sigma,B}^{*} \\ 0.2150 \end{array}$	$\begin{array}{c} C\\ L_{A,B}^{*}\\ 0.0956\\ 0.2195\\ 0.3399\\ 0.5094\\ 0.6759\\ 0.8337\\ 0.9095\\ \hline \\ C\\ L_{A,B}^{*}\\ 0.1093\\ \end{array}$	$\begin{array}{c} S\\ L_{\Sigma,B}^{*}\\ 0.0230\\ 0.0818\\ 0.0235\\ 0.0189\\ 0.0135\\ 0.0029\\ 0.0008\\ S\\ L_{\Sigma,B}^{*}\\ 0.0656\\ \end{array}$	$\begin{array}{c} \text{I} \text{I} \text{I} \text{I} \text{I} \text{I} \text{I} I$	$\begin{array}{c} 0.0010\\ \hline 3\\ \hline (.1)\\ \hline L^*_{\Sigma,B}\\ \hline 0.0557\\ \hline 0.0333\\ \hline 0.0342\\ \hline 0.0188\\ \hline 0.0096\\ \hline 0.0043\\ \hline 0.0010\\ \hline 3\\ \hline (.1)\\ \hline L^*_{\Sigma,B}\\ \hline 0.0828\\ \end{array}$	$\begin{array}{c} \text{ARM.}\\ \text{ARM.}\\ L_{A,B}^{*}\\ 0.1563\\ 0.1522\\ 0.3622\\ 0.5152\\ 0.6733\\ 0.8357\\ 0.9101\\ \hline\\ \text{ARM.}\\ L_{A,B}^{*}\\ 0.0684\\ \end{array}$	$\begin{array}{c} A(1,1)\\ L^*_{\Sigma,B}\\ 0.0817\\ 0.0153\\ 0.0427\\ 0.0229\\ 0.0103\\ 0.0049\\ 0.0012\\ \hline A(1,1)\\ L^*_{\Sigma,B}\\ 0.0268 \end{array}$	$\begin{array}{c} \text{Toe} \\ L_{A,B}^{*} \\ 0.0714 \\ 0.1418 \\ 0.3068 \\ 0.4820 \\ 0.6563 \\ 0.8296 \\ 0.9084 \\ \hline \\ \hline \\ \text{Toe} \\ L_{A,B}^{*} \\ 0.0492 \end{array}$	$\begin{array}{c} \text{Plitz} \\ L_{\Sigma,B}^{*} \\ 0.0194 \\ 0.0070 \\ 0.0140 \\ 0.0103 \\ 0.0068 \\ 0.0014 \\ 0.0004 \\ \end{array}$
$\begin{array}{c} \sigma^2 = \\ \hline m=100 \\ m=20 \\ m=50 \\ m=100 \\ m=500 \\ m=1000 \\ \sigma^2 = \\ \hline m=10 \\ m=20 \\ \hline \end{array}$	$= 2$ $= 2$ $L_{\Sigma,A}^{*}$ 0.0908 0.1488 0.3208 0.4923 0.6631 0.8310 0.9088 $= 4$ $L_{\Sigma,A}^{*}$ 0.0664 0.1540	$\begin{array}{c} MA \\ L^*_{A,B} \\ 0.1445 \\ 0.2081 \\ 0.3557 \\ 0.5115 \\ 0.6727 \\ 0.8352 \\ 0.9099 \\ \hline \\ MA \\ L^*_{A,B} \\ 0.2742 \\ 0.2462 \\ \end{array}$	$\begin{array}{c} \text{A}(1) \\ L_{\Sigma,B}^{*} \\ 0.0683 \\ 0.0651 \\ 0.0375 \\ 0.0193 \\ 0.0096 \\ 0.0043 \\ 0.0010 \\ \hline \\ \text{A}(1) \\ L_{\Sigma,B}^{*} \\ 0.2150 \\ 0.0960 \\ \hline \end{array}$	$\begin{array}{c} C\\ C\\ L^*_{A,B}\\ 0.0956\\ 0.2195\\ 0.3399\\ 0.5094\\ 0.6759\\ 0.8337\\ 0.9095\\ \hline C\\ L^*_{A,B}\\ 0.1093\\ 0.1886\\ \end{array}$	$\begin{array}{c} L_{\Sigma,B}^{*}\\ 0.0230\\ 0.0230\\ 0.0235\\ 0.0189\\ 0.0135\\ 0.0029\\ 0.0008\\ \hline \\ S\\ L_{\Sigma,B}^{*}\\ 0.0656\\ 0.0462 \end{array}$	$\begin{array}{c} \text{I}, \text{I},$	$\begin{array}{c} \text{(1)}\\ \text{(1)}\\ L_{\Sigma,B}^{*}\\ 0.0557\\ 0.0333\\ 0.0342\\ 0.0188\\ 0.0096\\ 0.0043\\ 0.0010\\ \textbf{3}\\ (1)\\ L_{\Sigma,B}^{*}\\ 0.0828\\ 0.0765\\ \end{array}$	$\begin{array}{c} \text{ARM.}\\ L^*_{A,B}\\ 0.1563\\ 0.1522\\ 0.3622\\ 0.5152\\ 0.6733\\ 0.8357\\ 0.9101\\ \hline\\ \text{ARM.}\\ L^*_{A,B}\\ 0.0684\\ 0.2807\\ \end{array}$	$\begin{array}{c} A(1,1)\\ L^*_{\Sigma,B}\\ 0.0817\\ 0.0153\\ 0.0427\\ 0.0229\\ 0.0103\\ 0.0049\\ 0.0012\\ \hline A(1,1)\\ L^*_{\Sigma,B}\\ 0.0268\\ 0.1302\\ \end{array}$	$\begin{array}{c} \text{Toe} \\ L^*_{A,B} \\ 0.0714 \\ 0.1418 \\ 0.3068 \\ 0.4820 \\ 0.6563 \\ 0.8296 \\ 0.9084 \\ \hline \\ \text{Toe} \\ L^*_{A,B} \\ 0.0492 \\ 0.1368 \end{array}$	$\begin{array}{c} \text{Plitz} \\ L_{\Sigma,B}^{*} \\ 0.0194 \\ 0.0070 \\ 0.0140 \\ 0.00103 \\ 0.0068 \\ 0.0014 \\ 0.0004 \\ \end{array}$
$\begin{array}{c} \sigma^2 = \\ \hline m=100 \\ m=20 \\ m=200 \\ m=100 \\ m=500 \\ m=1000 \\ \hline m=100 \\ m=20 \\ m=50 \\ \hline \end{array}$	$= 2$ $= 2$ $L_{\Sigma,A}^{*}$ 0.0908 0.1488 0.3208 0.4923 0.6631 0.8310 0.9088 $= 4$ $L_{\Sigma,A}^{*}$ 0.0664 0.1540 0.3098	MA           L <sup>*</sup> <sub>A,B</sub> 0.1445           0.2081           0.3557           0.5115           0.6727           0.8352           0.9099           MA           L <sup>*</sup> <sub>A,B</sub> 0.2742           0.2462           0.3743	$\begin{array}{c} \Lambda(1) \\ L_{\Sigma,B}^{*} \\ 0.0683 \\ 0.0651 \\ 0.0375 \\ 0.0193 \\ 0.0096 \\ 0.0043 \\ 0.0010 \\ \Lambda(1) \\ L_{\Sigma,B}^{*} \\ 0.2150 \\ 0.0960 \\ 0.0653 \end{array}$	$\begin{array}{c} C\\ L^*_{A,B}\\ 0.0956\\ 0.2195\\ 0.3399\\ 0.5094\\ 0.6759\\ 0.8337\\ 0.9095\\ \hline\\ C\\ L^*_{A,B}\\ 0.1093\\ 0.1886\\ 0.3392\\ \end{array}$	$\begin{array}{c} L_{\Sigma,B}^{*}\\ 0.0230\\ 0.0230\\ 0.0235\\ 0.0235\\ 0.0189\\ 0.0135\\ 0.0029\\ 0.0008\\ \hline \\ S\\ L_{\Sigma,B}^{*}\\ 0.0656\\ 0.0462\\ 0.0346\\ \end{array}$	$\begin{array}{c} \text{I}, \text{I},$	$\begin{array}{c} (.1) \\ L_{\Sigma,B}^{*} \\ (.1) \\ L_{\Sigma,B}^{*} \\ (.1) \\ 0.0557 \\ 0.0333 \\ 0.0342 \\ 0.0188 \\ 0.0096 \\ 0.0043 \\ 0.0010 \\ \hline 0.0059 \\ \hline 0.0599 \\ \hline \end{array}$	$\begin{array}{c} \text{ARM.}\\ L^*_{A,B}\\ 0.1563\\ 0.1522\\ 0.3622\\ 0.5152\\ 0.6733\\ 0.8357\\ 0.9101\\ \hline\\ \text{ARM.}\\ L^*_{A,B}\\ 0.0684\\ 0.2807\\ \hline\\ 0.3915\\ \end{array}$	$\begin{array}{c} \text{A}(1,1) \\ L_{\Sigma,B}^{*} \\ \hline 0.0817 \\ 0.0153 \\ 0.0427 \\ 0.0229 \\ 0.0103 \\ 0.0049 \\ 0.0012 \\ \hline \text{A}(1,1) \\ L_{\Sigma,B}^{*} \\ 0.0268 \\ 0.1302 \\ \hline 0.0831 \\ \end{array}$	$\begin{array}{c} \text{Toe} \\ L^*_{A,B} \\ 0.0714 \\ 0.1418 \\ 0.3068 \\ 0.4820 \\ 0.6563 \\ 0.8296 \\ 0.9084 \\ \hline \\ \hline \\ \text{Toe} \\ L^*_{A,B} \\ 0.0492 \\ 0.1368 \\ 0.2962 \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^* \\ 0.0194 \\ 0.0070 \\ 0.0140 \\ 0.0103 \\ 0.0068 \\ 0.0014 \\ 0.0004 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^* \\ 0.0172 \\ 0.0172 \\ 0.0172 \\ 0.0137 \\ \hline \end{array}$
$\begin{array}{c} \sigma^2 = \\ \hline m = 100 \\ m = 20 \\ m = 500 \\ m = 1000 \\ m = 1000 \\ \hline \sigma^2 = \\ \hline m = 10 \\ m = 20 \\ m = 50 \\ m = 100 \\ \hline \end{array}$	$= 2$ $L_{\Sigma,A}^*$ 0.0908 0.1488 0.3208 0.4923 0.6631 0.8310 0.9088 $= 4$ $L_{\Sigma,A}^*$ 0.0664 0.1540 0.3098 0.4889	MA           L <sup>*</sup> <sub>A,B</sub> 0.1445           0.2081           0.3557           0.5115           0.6727           0.8352           0.9099           MA           L <sup>*</sup> <sub>A,B</sub> 0.2742           0.2462           0.3743           0.5151	$\begin{array}{c} A(1) \\ L_{\Sigma,B}^{*} \\ 0.0683 \\ 0.0651 \\ 0.0375 \\ 0.0193 \\ 0.0096 \\ 0.0043 \\ 0.0010 \\ \hline A(1) \\ L_{\Sigma,B}^{*} \\ 0.2150 \\ 0.0960 \\ 0.0653 \\ 0.0291 \\ \end{array}$	$\begin{array}{c} C\\ L^*_{A,B}\\ 0.0956\\ 0.2195\\ 0.3399\\ 0.5094\\ 0.6759\\ 0.8337\\ 0.9095\\ \hline C\\ C\\ L^*_{A,B}\\ 0.1093\\ 0.1886\\ 0.3392\\ 0.4994\\ \end{array}$	$\begin{array}{c} S\\ \hline L^*_{\Sigma,B}\\ \hline 0.0230\\ \hline 0.0230\\ \hline 0.0235\\ \hline 0.0135\\ \hline 0.0135\\ \hline 0.0029\\ \hline 0.0008\\ \hline S\\ \hline L^*_{\Sigma,B}\\ \hline 0.0656\\ \hline 0.0462\\ \hline 0.0346\\ \hline 0.0122\\ \end{array}$	$\begin{array}{c} I.595\\ I.5\\ I.5\\ I.5\\ I.5\\ I.5\\ I.5\\ I.5\\ I.$	$\begin{array}{c} 3\\ \hline \\ (1)\\ \hline \\ L^*_{\Sigma,B}\\ \hline \\ 0.0557\\ \hline \\ 0.0333\\ \hline \\ 0.0342\\ \hline \\ 0.0342\\ \hline \\ 0.0188\\ \hline \\ 0.0096\\ \hline \\ 0.0043\\ \hline \\ 0.0010\\ \hline \\ 3\\ \hline \\ (1)\\ \hline \\ L^*_{\Sigma,B}\\ \hline \\ 0.0828\\ \hline \\ 0.0765\\ \hline \\ 0.0599\\ \hline \\ 0.0284\\ \end{array}$	$\begin{array}{c} \text{ARM} \\ L^*_{A,B} \\ 0.1563 \\ 0.1522 \\ 0.3622 \\ 0.5152 \\ 0.6733 \\ 0.8357 \\ 0.9101 \\ \hline \\ \text{ARM} \\ L^*_{A,B} \\ 0.0684 \\ 0.2807 \\ 0.3915 \\ 0.5189 \\ \hline \end{array}$	$\begin{array}{c} \text{A}(1,1) \\ L_{\Sigma,B}^{*} \\ 0.0817 \\ 0.0153 \\ 0.0427 \\ 0.0229 \\ 0.0103 \\ 0.0049 \\ 0.0012 \\ \hline \text{A}(1,1) \\ L_{\Sigma,B}^{*} \\ 0.0268 \\ 0.1302 \\ 0.0831 \\ 0.0329 \\ \hline \end{array}$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^* \\ 0.0194 \\ 0.0070 \\ 0.0140 \\ 0.0103 \\ 0.0068 \\ 0.0014 \\ 0.0004 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^* \\ 0.0172 \\ 0.0172 \\ 0.0177 \\ 0.0137 \\ 0.0107 \end{array}$
$\begin{array}{c} \sigma^2 := \\ \hline m=100 \\ m=200 \\ m=500 \\ m=1000 \\ m=1000 \\ \hline m=1000 \\ \sigma^2 := \\ \hline m=10 \\ m=20 \\ m=50 \\ m=100 \\ m=200 \\ \hline m=200 \\ m=200 \\ \hline m=100 \\ m$	$= 2$ $L_{\Sigma,A}^*$ 0.0908 0.1488 0.3208 0.4923 0.6631 0.8310 0.9088 $= 4$ $L_{\Sigma,A}^*$ 0.0664 0.1540 0.3098 0.4889 0.6636	$\begin{array}{c} M^{A}\\ L^{*}_{A,B}\\ 0.1445\\ 0.2081\\ 0.3557\\ 0.5115\\ 0.6727\\ 0.8352\\ 0.9099\\ \hline \\ M^{A}\\ L^{*}_{A,B}\\ 0.2742\\ 0.2462\\ 0.3743\\ 0.5151\\ 0.6728\\ \end{array}$	$\begin{array}{c} \Lambda(1) \\ L^*_{\Sigma,B} \\ 0.0683 \\ 0.0651 \\ 0.0375 \\ 0.0193 \\ 0.0010 \\ 0.00043 \\ 0.0010 \\ \Lambda(1) \\ L^*_{\Sigma,B} \\ 0.2150 \\ 0.0960 \\ 0.0653 \\ 0.0291 \\ 0.0095 \end{array}$	$\begin{array}{c} C\\ L^*_{A,B}\\ 0.0956\\ 0.2195\\ 0.3399\\ 0.5094\\ 0.6759\\ 0.8337\\ 0.9095\\ \hline \\ C\\ L^*_{A,B}\\ 0.1093\\ 0.1886\\ 0.3392\\ 0.4994\\ 0.6764\\ \end{array}$	$\begin{array}{c} S\\ S\\ L^*_{\Sigma,B}\\ 0.0230\\ 0.0818\\ 0.0235\\ 0.0135\\ 0.0135\\ 0.0029\\ 0.0008\\ \hline \\ S\\ L^*_{\Sigma,B}\\ 0.0656\\ 0.0462\\ 0.0346\\ 0.0122\\ 0.0133\\ \end{array}$	$\begin{array}{c} I.599\\ I.$	$\begin{array}{c} 3\\ \hline \\ (1)\\ \hline \\ L^*_{\Sigma,B}\\ \hline \\ 0.0557\\ \hline \\ 0.0333\\ \hline \\ 0.0342\\ \hline \\ 0.0188\\ \hline \\ 0.0096\\ \hline \\ 0.0043\\ \hline \\ 0.0010\\ \hline \\ \\ \hline \\ \\ \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	$\begin{array}{c} \text{ARM.}\\ L^*_{A,B}\\ 0.1563\\ 0.1522\\ 0.3622\\ 0.5152\\ 0.6733\\ 0.8357\\ 0.9101\\ \hline \\ \text{ARM.}\\ L^*_{A,B}\\ 0.0684\\ 0.2807\\ 0.3915\\ 0.5189\\ 0.6734\\ \end{array}$	$\begin{array}{c} A(1,1) \\ L^*_{\Sigma,B} \\ 0.0817 \\ 0.0153 \\ 0.0427 \\ 0.0229 \\ 0.0103 \\ 0.0049 \\ 0.0012 \\ \hline A(1,1) \\ L^*_{\Sigma,B} \\ 0.0268 \\ 0.1302 \\ 0.1302 \\ 0.0831 \\ 0.0329 \\ 0.0102 \\ \hline \end{array}$	$\begin{array}{c} \hline \text{Toe} \\ L^*_{A,B} \\ 0.0714 \\ 0.1418 \\ 0.3068 \\ 0.4820 \\ 0.6563 \\ 0.8296 \\ 0.9084 \\ \hline \text{Toe} \\ L^*_{A,B} \\ 0.0492 \\ 0.1368 \\ 0.2962 \\ 0.4783 \\ 0.6569 \\ \end{array}$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^* \\ 0.0194 \\ 0.0070 \\ 0.0140 \\ 0.0103 \\ 0.0004 \\ 0.0004 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^* \\ 0.0172 \\ 0.0172 \\ 0.0172 \\ 0.0137 \\ 0.0107 \\ 0.0067 \\ \hline \end{array}$
$\begin{array}{c} \sigma^2 :\\ \hline m=100\\ m=20\\ m=500\\ m=100\\ m=200\\ m=1000\\ \hline m=1000\\ m=100\\ m=20\\ m=500\\ m=100\\ m=200\\ m=500\\ \hline m=500\\ \end{array}$	$= 2$ $= 2$ $L_{\Sigma,A}^{*}$ 0.0908 0.1488 0.4923 0.6631 0.8310 0.9088 $= 4$ $L_{\Sigma,A}^{*}$ 0.0664 0.1540 0.3098 0.46836 0.4889 0.6636 0.6636 0.8319	$\begin{array}{c} MA \\ L^*_{A,B} \\ 0.1445 \\ 0.2081 \\ 0.3557 \\ 0.5115 \\ 0.6727 \\ 0.8352 \\ 0.9099 \\ \hline \\ MA \\ L^*_{A,B} \\ 0.2742 \\ 0.2462 \\ 0.3743 \\ 0.5151 \\ 0.6728 \\ 0.8350 \\ \hline \end{array}$	$\begin{array}{c} A(1) \\ L^*_{\Sigma,B} \\ 0.0683 \\ 0.0651 \\ 0.0375 \\ 0.0193 \\ 0.0096 \\ 0.0043 \\ 0.0010 \\ \hline \\ A(1) \\ L^*_{\Sigma,B} \\ 0.2150 \\ 0.0960 \\ 0.0653 \\ 0.0291 \\ 0.0095 \\ \hline \\ 0.0032 \\ \hline \end{array}$	$\begin{array}{c} C\\ L^*_{A,B}\\ 0.0956\\ 0.2195\\ 0.3399\\ 0.5094\\ 0.6759\\ 0.8337\\ 0.9095\\ \hline C\\ L^*_{A,B}\\ 0.1093\\ 0.1886\\ 0.3392\\ 0.4994\\ 0.6764\\ 0.8339\\ \end{array}$	$\begin{array}{c} \hline S\\ \hline \\ S\\ \hline \\ \\ L^{*}_{\Sigma,B}\\ \hline \\ 0.0230\\ \hline \\ 0.0230\\ \hline \\ 0.0135\\ \hline \\ 0.0135\\ \hline \\ 0.0029\\ \hline \\ 0.0008\\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \\ \hline \\ \\ \hline \\ \\ \\ \\ \hline \\ \\ \\ \hline \\ \\ \\ \\ \\ \hline \\$	$\begin{array}{c} I.599\\ I.$	$\begin{array}{c} 3\\ \hline \\ (1)\\ \hline \\ L^*_{\Sigma,B}\\ 0.0557\\ 0.0333\\ 0.0342\\ 0.0188\\ 0.0096\\ 0.0043\\ 0.0010\\ \hline \\ 0.00010\\ \hline \\ \hline \\ 0.00828\\ 0.0010\\ \hline \\ \hline \\ 0.0828\\ 0.0765\\ 0.0599\\ 0.0284\\ \hline \\ 0.0095\\ \hline \\ 0.0031\\ \hline \end{array}$	$\begin{array}{c} \text{ARM.} \\ L^*_{A,B} \\ 0.1563 \\ 0.1563 \\ 0.1522 \\ 0.3622 \\ 0.5152 \\ 0.6733 \\ 0.8357 \\ 0.9101 \\ \hline \\ \textbf{ARM.} \\ L^*_{A,B} \\ 0.0684 \\ 0.2807 \\ 0.3915 \\ 0.5189 \\ 0.5189 \\ 0.6734 \\ 0.8354 \\ \hline \end{array}$	$\begin{array}{c} \text{A}(1,1) \\ L^*_{\Sigma,B} \\ 0.0817 \\ 0.0153 \\ 0.0427 \\ 0.0229 \\ 0.0102 \\ 0.0012 \\ 0.0012 \\ \hline \text{A}(1,1) \\ L^*_{\Sigma,B} \\ 0.0268 \\ 0.1302 \\ 0.0831 \\ 0.0329 \\ 0.0102 \\ 0.0036 \end{array}$	$\begin{array}{c} \hline \text{Toe} \\ L^*_{A,B} \\ 0.0714 \\ 0.1418 \\ 0.3068 \\ 0.4820 \\ 0.6563 \\ 0.8296 \\ 0.9084 \\ \hline \\ \hline \\ 0.9084 \\ \hline \\ \hline \\ 0.9084 \\ \hline 0.9084 \\ \hline \\ 0.9084 \\ \hline 0.9084 \\ \hline \\ 0.9084 \\ \hline 0.$	$\begin{array}{c} \text{plitz} \\ L_{\Sigma,B}^* \\ 0.0194 \\ 0.0070 \\ 0.0140 \\ 0.00103 \\ 0.0068 \\ 0.0004 \\ 0.0004 \\ \hline \\ \text{plitz} \\ L_{\Sigma,B}^* \\ 0.0172 \\ 0.0172 \\ 0.0107 \\ 0.0107 \\ 0.0007 \\ 0.0007 \\ \hline \\ 0.00014 \\ \hline \end{array}$
$\begin{array}{c} \sigma^2 :\\ \hline m=10 \\ m=20 \\ m=500 \\ m=100 \\ m=200 \\ m=100 \\ m=100 \\ m=100 \\ m=200 \\ m=50 \\ m=200 \\ m=500 \\ m=1000 \\ m=100 \\ m=10 \\ m=$	$= 2$ $L_{\Sigma,A}^*$ 0.0908 0.1488 0.4923 0.6631 0.8310 0.9088 $= 4$ $L_{\Sigma,A}^*$ 0.0664 0.1540 0.3098 0.4889 0.6636 0.8319 0.9089	$\begin{array}{c} MA \\ L^*_{A,B} \\ 0.1445 \\ 0.2081 \\ 0.3557 \\ 0.5115 \\ 0.6727 \\ 0.8352 \\ 0.9099 \\ \hline \\ MA \\ L^*_{A,B} \\ 0.2742 \\ 0.2462 \\ 0.3743 \\ 0.5151 \\ 0.6728 \\ 0.8350 \\ 0.9100 \\ \end{array}$	$\begin{array}{c} \begin{array}{c} & (1) \\ L^{*}_{\Sigma,B} \\ 0.0683 \\ 0.0651 \\ 0.0375 \\ 0.0193 \\ 0.0096 \\ 0.0043 \\ 0.0010 \\ \hline \end{array}$	$\begin{array}{c} C\\ L^*_{A,B}\\ 0.0956\\ 0.2195\\ 0.3099\\ 0.5094\\ 0.6759\\ 0.8337\\ 0.9095\\ \hline C\\ L^*_{A,B}\\ 0.1093\\ 0.1886\\ 0.3392\\ 0.4994\\ 0.6764\\ 0.8339\\ 0.9095\\ \hline \end{array}$	$\begin{array}{c} S\\ S\\ L_{\Sigma,B}^{*}\\ 0.0230\\ 0.0818\\ 0.0235\\ 0.0189\\ 0.0135\\ 0.0029\\ 0.0008\\ \hline \\ S\\ L_{\Sigma,B}^{*}\\ 0.0656\\ 0.0462\\ 0.0346\\ 0.0123\\ 0.0012\\ 0.0034\\ \hline \\ 0.0123\\ 0.0021\\ \hline \\ 0.0007\\ \hline \end{array}$	$\begin{array}{c} \text{I}, \text{I},$	$\begin{array}{c} 3\\ \hline 3\\ \hline (1)\\ L^*_{\Sigma,B}\\ 0.0557\\ 0.0333\\ 0.0043\\ 0.0048\\ 0.0096\\ 0.0043\\ 0.0010\\ \hline 3\\ \hline (1)\\ L^*_{\Sigma,B}\\ 0.0828\\ 0.0765\\ 0.0828\\ 0.0765\\ 0.0599\\ 0.0284\\ 0.0095\\ 0.0091\\ \hline 0.0001\\ \hline 0.0001\\ \hline \end{array}$	$\begin{array}{c} \text{ARM.}\\ \textbf{L}^*_{A,B} \\ 0.1563 \\ 0.1522 \\ 0.3622 \\ 0.5152 \\ 0.6733 \\ 0.8357 \\ 0.9101 \\ \hline \textbf{ARM.} \\ \textbf{L}^*_{A,B} \\ 0.0684 \\ 0.2807 \\ 0.3915 \\ 0.5189 \\ 0.6734 \\ 0.8354 \\ 0.9101 \\ \hline \end{array}$	$\begin{array}{c} A(1,1)\\ L^*_{\Sigma,B}\\ 0.0817\\ 0.0153\\ 0.0429\\ 0.0103\\ 0.0049\\ 0.0012\\ 0.0012\\ \hline \\ A(1,1)\\ L^*_{\Sigma,B}\\ 0.0268\\ 0.1302\\ 0.0831\\ 0.0329\\ 0.0102\\ 0.0036\\ \hline \\ 0.0012\\ \hline \end{array}$	$\begin{array}{c} \hline \text{Toe}\\ \hline \text{Toe}\\ L^*_{A,B}\\ 0.0714\\ 0.1418\\ 0.3068\\ 0.3068\\ 0.3068\\ 0.4820\\ 0.6563\\ 0.8296\\ 0.9084\\ \hline \hline \text{Toe}\\ L^*_{A,B}\\ 0.0492\\ 0.1368\\ 0.2962\\ 0.4783\\ 0.6569\\ 0.8305\\ \hline 0.9085\\ \hline \end{array}$	$\begin{array}{c} 0.002\\ \hline \text{plitz}\\ L^*_{\Sigma,B}\\ 0.0194\\ 0.0070\\ 0.0140\\ 0.0103\\ 0.0068\\ 0.0014\\ 0.0004\\ \hline \text{plitz}\\ L^*_{\Sigma,B}\\ 0.0172\\ 0.0172\\ 0.0172\\ 0.0107\\ 0.0007\\ 0.0007\\ 0.0004\\ \hline \end{array}$

 Table 13 Simulation results with Toeplitz structure.