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Real-Time Sky Color with Effect of Sun's Position

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Abstract— In the rendering of outdoor scenes in virtual environments, the sun's position, sky color, clouds, shadow, trees, grass etc play very important roles in making it realistic. In this paper Sky color and the sun's position are combined. Specific longitude, latitude, date and time are required parameters to calculate the exact position of the sun. The sun's position is calculated based on Julian dating; the sky's color is created by Perez modeling. A functional application is designed to show the position of the sun and then sky color in arbitrary location, date and time. It can be possible to use this application in commercial games for outdoor rendering and for teachers to teach some part of physics about earth orbit and effect of the sun on the sky and it can be used in building design.

Index Terms— Sky color, Sun's position, Real-time, Outdoor rendering

1 Introduction

In the rendering of outdoor scenes in virtual environments, the sun's position, sky color, clouds, shadow, trees, grass etc play important roles in making it appear realistic. The principle calculations of the sun's positions have been very well known for a long time. The ancient Egyptians were able many years ago to calculate the sun's position so. By digging a large hole inside one of the pyramids, just once a year, on the king's birthday, the sun could shine on the grave of their king [1]. The sun's position and the amount of sunshine have, historically, been a very attractive subject for most of researchers. For example, in 1958, Giover J. et al. worked on the principle amount of sunshine in a day. In 1990, Kambezidis et al. [2] provided several functions to calculate the sun's position by focusing on factors such as light refraction and right ascension. Many ideas and principle concepts can be found in Sayigh et al., published in 1997, Duffie et al. in 1980, Kreider et al. in 1981, Wieder in 1982, Iqbal. in 1983 and Muir in 1983 [3].

Numerous computer graphics researchers have tried to simulate the atmospheric effect on the sky. Many of them have simulated the light from the sky and the sun by considering the scattering and absorption of light in the earth's atmosphere. Early work in rendering and modeling the atmospheric effects was done, in 1982, by Blinn and Max in1986 [4]. Blinn propose a method of modeling Saturn's rings by using thin layers of clouds and dusty surface while Max introduced a single scattering model for light diffusion to generate haze in the atmosphere. In 1987, Klassen [5] tried to display sky color by taking into account spectral distribution due to particles in the atmosphere. However, this method has problems because the atmosphere is approximated as multiple layers of the plane-parallel atmosphere. But it is assume as uniform density. Thus, the method is different from the actual physical phenomenon. In 1991, Kaneda et al. [6] improved this method by approximating the actual physical phenomenon. He considered the spherical atmosphere with air density changing exponentially with altitude. This work had been extended using multiple scattering by Nishita et al. in 1996[7]. Most of the proposed methods can display a realistic sky color but have a time constraint in rendering. Tadamura et al. [8] combined both Kaneda's model and the CIE. They discussed the relationship between them and published their discussion in 1993.

Dobashi et al. in1997 [9], proposed a fast display method of sky color using basis functions. In the proposed method, cosine functions are used as basis function. The color of sky in the view direction of an arbitrary the sun position can be obtained from stored distributions and displayed quickly. The method is tested for natural scenes and architectural design. The problem with this method is not achieving the photo realistic target.

In1999, Preetham et al. [10] approached an analytic model for rendering the sky. The image generated is impressive. They present an inexpensive analytic sky model from Perez et al. [11] (Perez model) that approximates full spectrum daylight for various atmospheric conditions. At the same time, they also presented a model for aerial perspective, which enhanced the realism of outdoor rendering.

Sunar et al. [12] in 2001 created a sky dome simulate the effect of the sun's position on sky color using the Perez model. In 2007, Sheng Li et al [13] proposed unified volumes representation for light shaft and shadow, which is an efficient method of simulating natural light shafts and shadows with atmospheric scattering effect. In 2008, Halawani et al. [14] tried to produce illuminated 3D objects based upon the effects of interaction between the sunlight and sky and Sunkavalli et al. [15] proposed a model for temporal color changes and explores its use for the analysis of outdoor scenes from time-lapse video data.

The most important effect of the sun's position is on sky color. The position of the sun depends on location, date and time. Location depends on longitude and latitude but the effect is different on different days of the year.

2.1 Detect of Longitude and Latitude

Latitude is a distance from north to south of the equator. Longitude is the angular distance from east to west of the prime meridian of the Earth. Longitude is 180 degrees from east to

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west. Each 15 degrees represents one hour of time. For example, if you were to travel west at 15 degrees per hour, you would, one hour later, have travelled for an hour with no change in the actual time. The earth spins around the sun in specific orbit once year.
The angle between earth's orbit and the equator is 23.5 degrees at all times because the angle that the sun can be seen is different at different latitudes. Sunset and sunrise are produced by rotation of the earth. The first place that can see sunrise in each day is Japan [16]. This is the main reason that there are different days of the month, different seasonal months and different seasons of the year.

2.2 Dome Modeling

Before determining the sun's position, the sky must be modeled. To create the sky, virtual dome is a convenient tool. There are two ways to model the dome; using 3D modeling software such as 3D Max or Maya and using a mathematical function. Mathematical modeling is adopted for this real-time environment.

A dome is like a hemisphere in which the view point is located inside. To create a hemisphere using mathematical formulas the best formula to use is:

$$x^2 + y^2 + z^2 = r^2 \quad (1)$$

The above formula in angular system is
Where is the zenith and is the azimuth and

$$f(\theta,\varphi) = \cos^2\theta\cos^2\varphi + \sin^2\theta + \cos^2\varphi\sin^2\theta - r^2$$

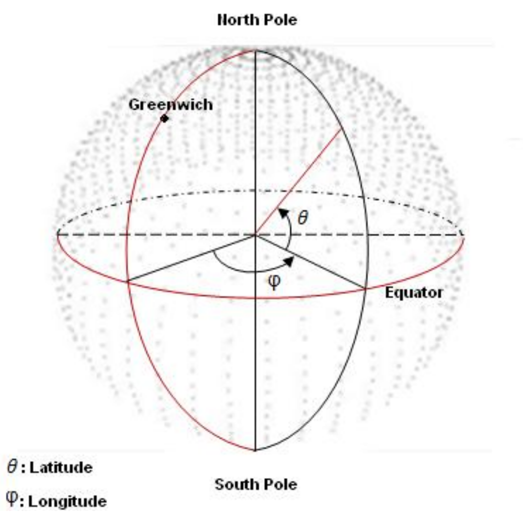

Fig 1: The zenithal and azimuthal angles on the hemisphere

Where θ is the zenith and φ is the azimuth and

$$0 \le \theta \le \frac{\pi}{2}$$
$$0 \le \varphi \le 2\pi$$

These ranges are used to create a specific part of the dome because in sky modeling, the north part of the sky dome is all that is needed.

3 Calculation of The sun's Position

Knowing zenith and azimuth are enough to calculate the position of the sun. To have zenith and azimuth, location, longitude, latitude, date and time are needed. Zenith is the angle that indicates the amount of sunrise while the azimuth is the angle that indicates the amount angle that the sun turns around the earth.
In 1983, Iqbal [17] proposed a formula to calculate the sun's position and in 1999, Preetham et al. [10] improved it. It is a

$$t = t_s + 0.17\sin\left(\frac{4\pi(j-80)}{373}\right) - 0.129\sin\left(\frac{2\pi(j-8)}{355}\right) + 12\frac{SM-L}{\pi}$$

common formula to calculate the position of the sun in physics.

where
t: Solar time
t_s: Standard time
J: Julian date
SM: Standard meridian
L: Longitude
The solar declination is calculated as the following formula:

$$\delta = 0.4093\sin\frac{2\pi(j-81)}{368}$$

δ : Solar declination
The time is calculated in decimal hours and degrees in radians.

Finally zenith and azimuth can be calculated as follows:

$$\varphi_s = \tan^{-1}\left(\frac{-\cos\delta\sin\frac{\pi t}{12}}{\cos l\sin\delta - \sin l\cos\delta\cos\frac{\pi t}{12}}\right)$$
$$\theta_s = \frac{\pi}{2} - \sin^{-1}\left(\sin l\sin\delta - \cos l\cos\delta\cos\frac{\pi t}{12}\right)$$

Where
θ_s : Solar zenith
φ_s : Solar azimuth
L: Latitude
With calculation of zenith (θ_s)and azimuth (φ_s)the sun's position is obvious. To have the sun's position a Cartesian coordinate is needed. The point (x, y, and z) is the position of the sun and it is specific in every location, date and time.

4 SKY COLOR

4.1 What is color?

Traditionally, colors have been described in words, usually by allusion to common objects such as "red apple", "green spinach" or "blue sky". More precisely, color is communicated in the painting and dying industry by production of charts of sample colors. The numerical specification of color has a long history that began with the famous physics legend, Sir Isaac Newton. It was only in the twentieth century that numerical systems became important in industry.

The interaction of light, an object and the eye creates color. There must be a light to illuminate the object. As said in Poynton, color is the perceptual result in the visible region of the spectrum, having wavelengths in the region of 400 nm to 700 nm, incident upon the retina. The human retina has three kinds of color preceptor cone cells with peak sensitivities to 580 nm ("red"), 545 nm ("green") and 440 nm ("blue"). This is stated in the tri-stimulus theory of color perception in Apple (1996).

In 1931, The Commission International De l'Eclairage (CIE) [18] developed a device-independent color model that was based on human perception. It is known as the CIE XYZ model that defines three primaries known as X, Y and Z. The three primaries can be combined to match with any color that humans see which is related to the tri-stimulus theory of color perception. The topic of CIE color spaces will be discussed in more detail in the color spaces section.

Although there are approximately four billion colors that humans can see, color is normally divided into eleven basic categories. As Berlin and Kay said in 1969 [19], the eleven basic categories are white, black, red, green, yellow, blue, brown, purple, pink, orange and grey.

4.2 Light Scattering

The color perceived by our eyes is not the pure color of the object. The color had been scattered on its journey to our eye. Therefore, the color of the tree at the mountain is not the same as the tree in front of our eyes.

In computer graphics and old commercial games, the color of the sky was blue but sky color is not simply blue. During the course of a day, the color of the sky changes with the position of the sun. The sky color around the horizon and zenith is different almost all the time. When color of sky is blue, near the horizon it is close to white. At the start or at the end of the day, or at sunrise and sunset, the color of the sky becomes red, orange and white. The most important cause of these various colors is the scattering of aerosols and air molecules. Sky color is related to the suns direction, date, time and location of viewpoints. To calculate the sky color, several methods are proposed but all of them consider just single scattering. Multi scattering can produce high quality sky color for virtual environments [20].

The scattering of incoming sunlight is very important for the brightness and color of sunlight and skylight. Scattering is the process by which small particles suspended in a medium of a different index of refraction diffuse a portion of the incident radiation in all directions. Figure 2 illustrates scattered sunlight in atmospheric articles. Oxygen and nitrogen are two examples of air molecules, which are small in size. Thus, they are more effective at scattering shorter wavelengths of light (blue and violet). The selective scattering by air molecules is responsible for producing our blue skies on a clear sunny day. Most of the atmospheric particles can be assumed to be spherical and homogeneous. For this reason, Rayleigh and Mie scattering theories can be used to describe the scattering. In 1881, Lord Rayleigh [21] introduced a scattering theory for the scattering of light by the molecules of the air. It can be extended to the scattering of particles of up to about a tenth of the wavelength of the light. The scattering theory is known as Rayleigh scattering.

Mie scattering theory involves scattering particle sizes larger than a wavelength. The theory is suitable for particles of a diameter from at least twice the wavelength of the light source. Gustav Mie presented the Mie scattering theory in 1906 [22].

4.3 Color Space

To represent color information in terms of intensity values, a color space is modeled. The dimensions or components that represent intensity values are defined in 1D, 2D, 3D or 4D spaces. Each component of color is also referred to as a color channel.

There are several different color spaces available. This will give the appropriate working with whichever type of color data is best. It can be categories as grey spaces, RGB, CMYK, device-independent color spaces, named color spaces and heterogeneous HiFi color spaces.

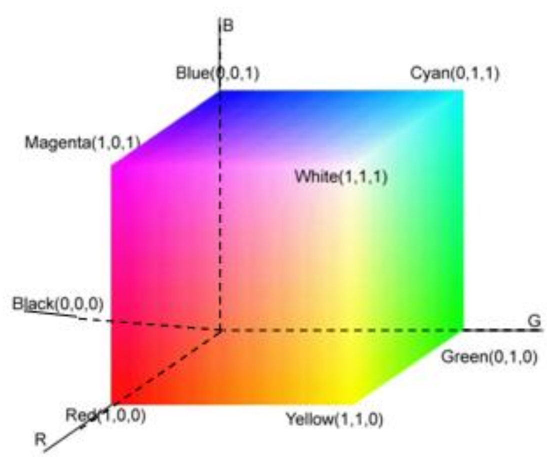

Fig 2: The RGB color space

4.4 CIE XYZ Spaces

The XYZ space allows the color to be expressed as a mixture of the three tri-stimulus values X, Y and Z. The CIE standard allows a color to be classified as a numeric triple (X, Y, Z). All the CIE-based color spaces are derived from the fundamental XYZ space.

CIE XYZ space accepts all colors perceivable by human beings and it is based on experimentally determined color matching functions. Thus, it is a device-independent color space.

All visible light can be shown as a positive combination of X, Y and Z. Therefore, Y component almost associates to the apparent lightness of a color. In general, the mixture of X, Y and Z components that describe a color can be expressed as percentages.

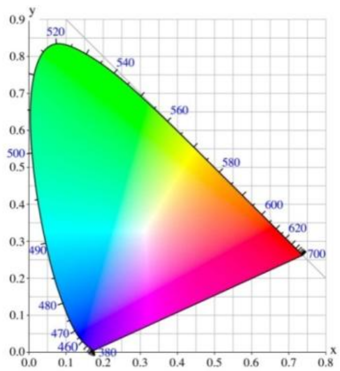

Fig 3: CIE color space for xy chromaticity's.

4.5 CIE Yxy Spaces

Yxy space is another space that determines the XYZ values in terms of x and y chromaticity co-ordinates. To convert XYZ space into Yxy co-ordinates the following formulas are used:

$$Y = Y$$
$$x = \frac{X}{X+Y+Z}$$
$$y = \frac{Y}{X+Y+Z}$$

For the other way around, to convert from Yxy co-ordinates to XYZ, the following formulas can be used:

$$X = \frac{x}{y}Y$$
$$Y = Y$$
$$Z = \frac{1-x-y}{y}Y$$

The Z tri-stimulus value is not visible by itself and is combined with the new co-ordinates. The layout of color in the x and y plane of Yxy space is shown in Figure 4. This diagram is well known as the wing-shaped CIE chromaticity diagram, which is extensively used in color science.

In another situation, to convert between the device independence CIE based color space to a device dependent RGB color space in computer graphics, a color transformation matrix is essential. It is useful for mapping the CIE XYZ values to RGB monitor values. Nevertheless, this conversion is not easy. It is because the trans- formation matrix is dependent upon the behavior of the particular phosphor in a specific monitor. For that reason, creating a general transformation matrix to obtain an accurate color conversion is not possible. Sometimes a CIE XYZ value will give a negative value when converted to a RGB color space, but RGB values cannot be any negative. The negative value is out of the RGB gamut that defines no color. Therefore, a color matching process is needed. Hence, some colors that are described in the CIE color space cannot be described in the RGB color space.

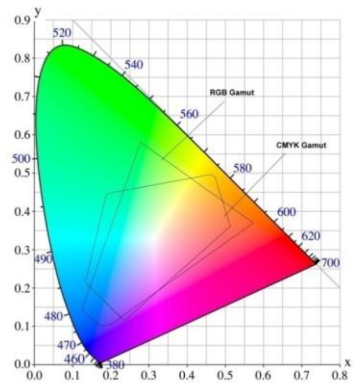

Fig 4: Color Gamut for two devices expressed Yxy

5 PEREZ SKY MODEL

The Perez model is convenient method to illuminate arbitrary point of the sky dome respect to the sun's position. The Perez model uses CIE standard and it can be used for a wide range of atmosphere with different conditions. Luminance of point can be calculated by using the following formula:

$$L(\theta_p, \gamma_p) = (1 + Ae^{\frac{B}{\cos\theta_p}})(1 + Ce^{D\gamma_p} + E\cos^2\gamma_p)$$

$$\gamma_p = \cos^{-1}(\sin\theta_s \sin\theta_p \cos(\varphi_p - \varphi_s) + \cos\theta_s \cos\theta_p)$$

Where

A: Darkening or brightening of the horizon
B: Luminance gradient near the horizon
C: Relative intensity of circumsolar region
D: Width of the circumsolar region
E: Relative backscattered light received at the earth surface

6 SKYLIGHT DISTRIBUTION COEFFICIENTS AND ZENITH VALUE

The zenith values and skylight distribution coefficient are specified in Preetham et al., 1999 [10]. The calculation can be shown by the matrices below. The values of zenith are the functions of turbidity (T) and the sun's position, while a different T value will give a different distribution coefficient.

Distribution coefficients for luminance, Y distribution function:

$$\begin{pmatrix} A_Y \\ B_Y \\ C_Y \\ D_Y \\ E_Y \end{pmatrix} = \begin{pmatrix} 0.1787 & -1.4630 \\ -0.3554 & 0.4275 \\ -0.0227 & 5.3251 \\ 0.1206 & -2.5771 \\ -0.0670 & 0.3703 \end{pmatrix} \begin{pmatrix} T \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} A_x \\ B_x \\ C_x \\ D_x \\ E_x \end{pmatrix} = \begin{pmatrix} -0.0193 & -0.2592 \\ -0.0665 & 0.0008 \\ -0.0004 & 0.2125 \\ -0.0641 & -0.8989 \\ -0.0033 & 0.0452 \end{pmatrix} \begin{pmatrix} T \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} A_y \\ B_y \\ C_y \\ D_y \\ E_y \end{pmatrix} = \begin{pmatrix} -0.0167 & -0.2608 \\ -0.0950 & 0.0092 \\ -0.0079 & 0.2102 \\ -0.0441 & -1.6537 \\ -0.0109 & 0.0529 \end{pmatrix} \begin{pmatrix} T \\ 1 \end{pmatrix}$$

Absolute value of zenith luminance:

$$Yz=(4.0453T-4.9710)\tan\left(\left(\frac{4}{9}-\frac{T}{120}\right)(\pi-2\theta_s)\right)-0.2155T+2.4192$$

Zenith x, xz and zenith y, yz are given by following matrix:

$$x_z = \begin{pmatrix} T^2 & T & 1 \end{pmatrix} \begin{pmatrix} 1.00166 & -0.00375 & 0.00209 & 0 \\ -0.02903 & 0.06377 & -0.03202 & 0.00394 \\ 0.11693 & -0.21196 & 0.06052 & 0.25886 \end{pmatrix} \begin{pmatrix} \theta_s^3 \\ \theta_s^2 \\ \theta_s \\ 1 \end{pmatrix}$$

$$y_z = \begin{pmatrix} T^2 & T & 1 \end{pmatrix} \begin{pmatrix} 0.00275 & -0.00610 & 0.00317 & 0 \\ -0.04214 & 0.08970 & -0.04153 & 0.00516 \\ 0.15346 & -0.26756 & 0.06670 & 0.26688 \end{pmatrix} \begin{pmatrix} \theta_s^3 \\ \theta_s^2 \\ \theta_s \\ 1 \end{pmatrix}$$

7 Result and Evaluation

Sky color and the sun's position in real-time computer games can make a game as realistic as possible. To keep the real position of the sun in a virtual environment, a substantial amount of precision is needed. Solar energy is free and a blessing from God, but optimized usage of this blessing needs a mastermind. On the other hand, in building design and architecture, possible recognition of which direction is best to build a building in specific location.

For a period of one year, the amount of sunshine in the southern hemisphere is more than the northern hemisphere except for latitude less than1.5 degree. The sky's color changes with the position of the sun during the day and even at night. In nature, image quality varies every minute. The changes are not only in terms of the sun angle and intensity of radiation, which are different every time, but it is affected by changes in the color of light and contrast. At dawn, when the sun is not yet over the horizon, the sky along the horizon will be golden shiny and bright purple. When the sun can be seen over the horizon, the sky becomes yellow and then an attractive blue color.

The results Table 1 show the real and application generated data for sunrise and sunset in the first day and last day of each month in Universiti Teknologi Malaysia at latitude of 1.28 and longitude of 103.45 at different times of the day.

Table 1. The real and application generated data for sunrise and sunset time of Universiti Teknologi Malaysia

	January		February		March	
Sunrise real	07:07	07:17	07:17	07:15	07:15	07:06
Sunset real	19:10	19:20	19:20	19:20	19:20	19:13
Sunrise software	07:9	07:02	07:13	07:07	07:04	06:53
Sunset software	19:20	19:23	19:27	19:25	19:20	19:21
	April		May		June	
Sunrise real	07:05	06:57	06:57	06:57	06:57	07:02
Sunset real	19:13	19:07	19:07	19:09	19:09	19:15
Sunrise software	07:13	6:53	6:54	07:01	6:54	6:59
Sunset software	19:22	19:24	19:23	19:26	19:25	19:30
	July		August		September	
Sunrise real	07:02	07:06	07:06	07:01	07:01	06:52
Sunset real	19:15	19:17	19:17	19:10	19:09	18:58
Sunrise software	7:2	7:00	7:01	6:53	6:50	06:42
Sunset software	19:27	19:21	19:18	19:10	19:06	19:02
	October		November		December	
Sunrise real	06:52	06:47	06:47	06:52	06:53	07:07
Sunset real	18:58	18:51	18:50	18:55	18:55	19:09
Sunrise software	06:42	06:44	0:645	06:55	06:57	07:9
Sunset software	19:01	18:59	19:00	19:03	19:10	19:19

Fig 5: Real sky's color (UTM, 15 June, 2011)

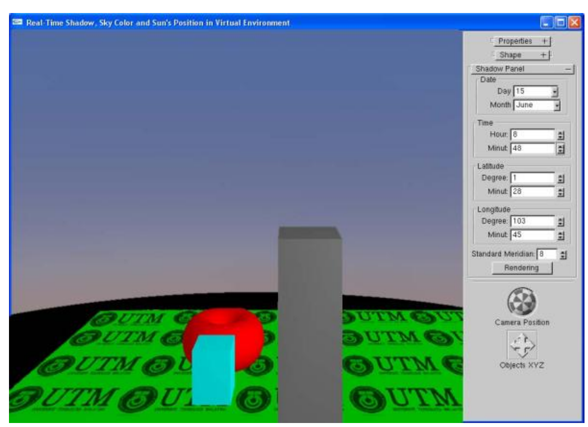

Fig 6: Result of application (UTM, 15 June, 2011)

8 Conclusion and Future Work

The sky color as determined by the sun's position, are the most important factors to consider when creating realistic outdoor scenes. The main target of this study was to develop a daylight sky's color with effect of sun's position. Thus, four objectives were determined in order to achieve the goal. First, the dome sky, which was modeled using mathematical functions, is a better way to represent the sky. The algorithm not only supports a triangular mesh, but also a quadratic mesh. The sun's position needs to be calculated by Juliann dating. Finally the color of the sky had to be carried out using Perez modeling which yields the result in CIE Yxy space.

A model that gives the sky's color according to the sun's position and the specific location, date and time has been developed. A software package of the real-time sky's color with effect of sun's position simulator was successfully constructed in C++ OpenGL. All the objectives were met in order to reach the main goal.

There are some extensions that can be made to this project. It is possible that the software could model not only the sky but also other natural phenomena such as cloud, landscape and the ocean. Shadow is another natural phenomenon that can make this package more complete and more realistic.

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