



## City Research Online

### City, University of London Institutional Repository

---

**Citation:** White, M. & Sayma, A. I. (2016). Improving the economy-of-scale of small organic rankine cycle systems through appropriate working fluid selection. *Applied Energy*, 183, pp. 1227-1239. doi: 10.1016/j.apenergy.2016.09.055

This is the accepted version of the paper.

This version of the publication may differ from the final published version.

---

**Permanent repository link:** <http://openaccess.city.ac.uk/16541/>

**Link to published version:** <http://dx.doi.org/10.1016/j.apenergy.2016.09.055>

**Copyright and reuse:** City Research Online aims to make research outputs of City, University of London available to a wider audience. Copyright and Moral Rights remain with the author(s) and/or copyright holders. URLs from City Research Online may be freely distributed and linked to.

---

City Research Online:

<http://openaccess.city.ac.uk/>

[publications@city.ac.uk](mailto:publications@city.ac.uk)

---

1 **IMPROVING THE ECONOMY-OF-SCALE OF SMALL ORGANIC RANKINE**  
2 **CYCLE SYSTEMS THROUGH APPROPRIATE WORKING FLUID SELECTION**

3  
4  
5 Martin White\* and Abdalnaser I Sayma

6  
7 School of Mathematics, Computer Science and Engineering  
8 City University London  
9 Northampton Square, London, EC1V 0HB, United Kingdom

10  
11 \* corresponding author, E-mail: [Martin.White.1@city.ac.uk](mailto:Martin.White.1@city.ac.uk)  
12  
13

14 **ABSTRACT**

15 A major challenge facing the widespread implementation of small and mini-scale organic  
16 Rankine cycles (ORCs) is the economy-of-scale. To overcome this challenge requires systems  
17 that can be manufactured in large volumes and then implemented into a wide variety of  
18 different applications where the heat source conditions may vary. Therefore, the aim of this  
19 paper is to investigate whether working fluid selection has a role in improving the current  
20 economy-of-scale by enabling the same system components to be used in multiple ORC  
21 systems. The performance map for a small-scale ORC radial turbine, obtained using CFD, is  
22 adapted to account for additional loss mechanisms not accounted for in the original CFD  
23 simulation, such as windage, volute and diffuser losses, before being non-dimensionalised  
24 using a modified similitude theory developed for subsonic ORC turbines. The updated  
25 performance map is then implemented into an ORC thermodynamic model. This model  
26 enables the construction of a single performance contour that displays the range of heat  
27 source conditions that can be accommodated by the existing turbine whilst using a particular  
28 working fluid. Constructing this performance map for a range of working fluids, this paper  
29 demonstrates that through selecting a suitable working fluid, the same turbine can efficiently  
30 utilise heat sources between 360 K and 400 K, with mass flow rates ranging between 0.5 kg/s  
31 and 2.75 kg/s respectively. This corresponds to using the same turbine in ORC applications  
32 where the heat available ranges between 50 and 380 kW<sub>th</sub>, with the resulting net power  
33 produced by the ORC system ranging between 2 kW and 30 kW. Further investigations also  
34 suggest that under these operating conditions the same working fluid pump could also be  
35 used; however, the required heat exchanger area is found to scale directly with increasing heat  
36 input. Overall, this paper demonstrates that through the optimal selection of the working fluid,  
37 the same turbomachinery components (i.e. pump and turbine) can be used in multiple ORC  
38 systems, which may offer an opportunity to improve on the current economy-of-scale.  
39  
40

## 41 NOMENCLATURE

$a$	Speed of sound, m/s
$A$	Area, m <sup>2</sup>
$A_r$	Diffuser area ratio
$c$	Velocity, m/s
$C_w$	Windage torque loss coefficient
$D$	Turbine rotor diameter, m
$g$	Acceleration due to gravity, m/s <sup>2</sup>
$h$	Enthalpy, J/kg
$H$	Pump head, m
$\dot{m}$	Mass flow rate, kg/s
$N$	Turbine rotational speed, rpm
$P$	Pressure, Pa
PP	Pinch point
PR	Pressure ratio
$q$	Thermal energy, J
$Q$	Volumetric flow rate, m <sup>3</sup> /s
$r$	Radius, m
Re	Reynolds number
$s$	Entropy, J/(kg K)
$T$	Temperature, K
$U$	Overall heat transfer coefficient, W/(m <sup>2</sup> K)
$W$	Work, J/s
$Y$	Total pressure loss coefficient
$\eta$	Efficiency, %
$\theta$	Diffuser divergence angle, °
$\mu$	Viscosity, Pa/s
$\rho$	Density, kg/m <sup>3</sup>
$\phi$	Pump flow coefficient
$\psi$	Pump head coefficient
$\omega$	Rotational speed, rad/s
$\omega_s$	Pump specific speed
$\Delta P_v$	Volute pressure drop
$\Delta T_{\log}$	Log mean temperature difference, K
$\Delta T_{sh}$	Amount of superheat, K

### Subscripts

*	Choked (sonic) flow conditions
0	Total conditions
1-5	Turbine locations
6	Pump inlet/condenser outlet
7	Pump outlet/evaporator inlet
8	Evaporator pinch point
c	Heat sink
d	Design point
h	Heat source
p	Pump
o	Organic fluid
s	Conditions after isentropic expansion
ts	Total-to-static
tt	Total-to-total
w	Windage

42

43

## 44 **1 INTRODUCTION**

45 The growing interest in organic Rankine cycles (ORC) can be attributed to its potential to  
46 effectively convert low temperature heat sources such as solar, geothermal, biomass and  
47 waste heat into mechanical power. However, low heat source temperatures imply low cycle  
48 thermal efficiencies, which places a greater pressure on the need to develop economically  
49 viable systems. Despite successful commercialisation for power outputs above a few hundred  
50 kilowatts, ORC technology has not been widely commercialised at the smaller-scale.  
51 However, a recent review [1] suggested that automotive waste heat recovery, combined heat  
52 and power, and concentrated solar power applications could be large potential markets for  
53 small-scale ORC systems. The authors of that paper also go on to say that the successful  
54 uptake of small-scale ORC systems can only be realised through the high volume production  
55 of modular systems, leading to lower system costs. To achieve this, it is necessary to widen  
56 the scope of existing systems by developing components that operate efficiently over a wide  
57 range of operating conditions, and with different working fluids. However, as stated in [2],  
58 many existing state-of-the-art ORC systems are designed for a nominal operating point and  
59 exhibit poor off-design. Clearly there is a need to develop new methods to understand and  
60 predict the design and off-design performance of ORC expanders, and also to investigate the  
61 impact of working fluid selection and replacement on the performance of both the expander  
62 and the whole ORC system.

63 The focus of many ORC studies within the literature has been thermodynamic  
64 modelling and optimisation. For clarification, the authors make a distinction here between  
65 design optimisation and cycle optimisation. In the former the aim is to optimise the design of  
66 the ORC system to deliver the best performance for the available heat source and heat sink. In  
67 this case the desired component efficiency can be specified during thermodynamic  
68 optimisation, and then during the component design phase the components are designed to  
69 achieve this performance. On the other hand, cycle optimisation concerns the case where pre-  
70 existing system components are available, and the cycle operating conditions are optimised to  
71 maximise performance. In this case, off-design components' models are critical since it is no  
72 longer suitable to assume constant expander efficiency. Many examples of design  
73 optimisation studies can be found within the literature, for example [3-5]. However, within  
74 the scope of this paper, cycle optimisation studies are more appropriate, where off-design  
75 models for the pump, evaporator, condenser and expander are implemented into  
76 thermodynamic models.

77 Even in the case of cycle optimisation, pump efficiency is often assumed constant. In  
78 [6] it was found that the pump could consume up to 15% of the power produced by the  
79 expander, demonstrating the large impact a change in pump efficiency can have on system



80 performance. The few authors that have considered pump performance have considered it  
81 within dynamic models [7,8]. These studies construct non-dimensional performance maps  
82 based on pump similitude theory, but this requires performance data that is particular to a  
83 given pump and not always available. The same authors have also constructed dynamic heat  
84 exchanger models, which apply a one-dimensional differential energy and mass balance to  
85 establish temperature distributions as a function of space and time. For steady-state models,  
86 heat exchanger performance is often obtained by establishing the effectiveness as a function  
87 of the heat exchanger geometry and flow conditions ( $\epsilon$ -NTU method), and this has been  
88 demonstrated for ORC systems in [9].

89         Arguably, the expander is the most critical component so this is the main focus within  
90 this paper. Particularly in small-scale systems it is not suitable to assume constant expander  
91 efficiency as the search for optimal cycle conditions may often move the expander  
92 performance away from the design point. Indeed, it has been highlighted that thermodynamic  
93 models are only accurate when expander performance is taken into account [10]. Performance  
94 maps can be used to model turbine performance, and these plot mass flow rate and turbine  
95 efficiency against pressure ratio and rotational speed. These maps are typically non-  
96 dimensionalised using similitude theory, which is well established for ideal gases [11]. Whilst  
97 similitude theory has been applied to ORC turbines as early as the 1980s [12], and has  
98 continued until more recently [13], these analyses focussed on turbine design rather than  
99 assessing off-design performance. Furthermore, these studies concerned axial, rather than  
100 radial turbines. For off-design, similitude has been applied to ORC turbines [14-17].  
101 However, these studies implemented a simplified similitude model that used ideal gas  
102 relationships that are not suitable for organic fluids. A recent study showed that these  
103 formulations cannot accurately predict turbine performance when using organic fluids [18].  
104 This agrees with recent work conducted by the authors [19]. However, the authors' work also  
105 proposed a modification to the similitude model, which accurately predicts ORC turbine  
106 performance during subsonic operation. It is worth noting that one-dimensional loss models  
107 could be used to assess turbine performance. These loss models have been applied to ORC  
108 turbines [20-22], however this is often for turbine design, rather than assessing off-design  
109 performance. Furthermore, these loss models are based on empirical data obtained for ideal  
110 gases, and have not been validated for organic fluids. However, if validated, these loss models  
111 could have a place in off-design modelling of ORC turbines.

112         Another important variable within an ORC system is the working fluid where  
113 working fluid selection remains an important research area. The key selection criteria for an  
114 optimal working fluid have been discussed and reiterated within many research papers [23-  
115 25]. Furthermore, there have been many working fluid studies where a number of working

116 fluid candidates have been evaluated for different applications, and this has also included  
117 considering different thermodynamic cycle configurations [26-27]. However, what is missing  
118 in most of these studies is a consideration of the impact that the working fluid has on the  
119 performance of the system components, both at design and off-design conditions. It should  
120 therefore be noted that the emphasis within this paper is to investigate this coupling between  
121 the working fluid and the turbine performance, rather than reiterating selection criteria and  
122 then repeating working fluid selection studies.

123 Previous work has led to the design of an ORC turbine [28], and the generation of the  
124 non-dimensional performance map using CFD. The focus of this paper is to combine this  
125 turbine performance map with thermodynamic cycle analysis in order to investigate the  
126 interaction between the selected working fluid and the turbine performance under different  
127 heat source conditions. Preliminary investigations have already been completed by the  
128 authors [29], and this paper extends this analysis by implementing the modified and more  
129 accurate similitude model, updating the turbine performance map to account for additional  
130 loss mechanisms not accounted for during the CFD simulation, whilst also including a  
131 consideration of how the pump and heat exchanger performance varies with different working  
132 fluids under different heat source conditions. The main novelty in this work is the ability  
133 establish the full range of heat source mass flow rates that could be accommodated using a  
134 particular turbine design and working fluid. This information is presented on a single contour  
135 plot, which can be used to evaluate the suitability of using that turbine and working fluid for a  
136 particular application. The main aim of this research is to then establish the range of heat  
137 sources that could be effectively converted into mechanical power using the same turbine  
138 design, and to demonstrate how the turbine can be matched to the available heat source by  
139 selecting the most suitable working fluid. Ultimately, this is envisioned as a useful first step  
140 towards improving the economy-of-scale of small ORC systems, since the same turbine can  
141 be manufactured in large volumes and then implemented within a range of different ORC  
142 systems designed for different heat source conditions. To the authors' knowledge, this study  
143 is the first to couple the modified similitude theory to an ORC thermodynamic model, and to  
144 then explore methods to improve the economy-of-scale of small-scale ORC systems.

145 After this introduction, the modified similitude theory is introduced in Section 2 and  
146 the performance map obtained using CFD is updated to account for additional loss  
147 mechanisms that were not accounted for during the CFD simulation. In Section 3 the turbine  
148 performance map is implemented into the cycle model whilst models for the pump and heat  
149 exchangers are described in Section 4. In Section 5, a case study is considered which  
150 produces an example of the performance contour plot, and then the model is run for a range of  
151 heat source temperatures and working fluids. For each working fluid and heat source

152 temperature the optimal operating point is established by evaluating the resulting contour plot,  
 153 and a range of potential applications are obtained. Then, in Section 6 the conclusions of this  
 154 research are outlined.

155

156

## 157 **2 TURBINE MODELLING**

158 Before discussing the turbine and system modelling in the next sections, it is necessary to  
 159 define the notation used throughout this paper. This is shown in Figure 1.

160

### 161 **2.1 Similitude theory**

162 The authors have investigated the application of similitude theory to ORC turbines, and this  
 163 led to a proposed modification to the existing model [19]. This modification is shown by  
 164 Equation (1), and uses the density and speed of sound at the choked stator throat, denoted  $\rho^*$   
 165 and  $a^*$  respectively, instead of the turbine total inlet conditions;  $\Delta h_s$  is the isentropic total-to-  
 166 total enthalpy drop across the turbine,  $N$  is the rotational speed,  $D$  is the rotor diameter,  $\eta_{tt}$  is  
 167 the turbine total-to-total isentropic efficiency,  $W$  is the power output and  $\dot{m}_o$  is the working  
 168 fluid mass flow rate. Although the ratio of specific heats is used in the conventional similitude  
 169 model, it has been neglected in Equation (1). For ideal gases  $\rho^*$  and  $a^*$  can be expressed using  
 170 the ideal gas law, such that the ratio of specific heats is contained within the other non-  
 171 dimensional groups. For a non-ideal gas, the ratio of specific heats has been removed as it is  
 172 assumed that the variation in gas composition is accounted for by using a suitable equation of  
 173 state to calculate  $\rho^*$ ,  $a^*$  and  $\Delta h_s$ .

174

$$\left[ \frac{\Delta h_s}{N^2 D^2}, \eta_{tt}, \frac{W}{\rho^* N^3 D^5} \right] = f \left( \frac{\dot{m}_o}{\rho^* N D^3}, \frac{ND}{a^*}, \frac{\rho^* N D^2}{\mu} \right) \quad (1)$$

175

176 Equation (1) can be simplified for a fixed turbine since the diameter cannot change.  
 177 Furthermore, the term on the far right of Equation (1) is the rotational Reynolds number, and  
 178 for ideal gas turbines this term is often neglected. The previous study suggested this term can  
 179 also be neglected for ORC turbines if the change in the Reynolds number is less than 200%  
 180 [19]. At higher deviations, Reynolds number effects may become more prevalent, which  
 181 might reduce turbine efficiency. Finally, the third term on the left hand side, the power  
 182 coefficient, has been omitted for simplicity since  $W$  can be derived once  $\dot{m}_o$ ,  $\eta_{tt}$  and  $\Delta h_s$  are  
 183 all known. This simplification leads to Equation (2).

184

$$\left[ \frac{\Delta h_s}{a^{*2}}, \eta_{tt} \right] = f \left( \frac{\dot{m}_o}{\rho^* a^*}, \frac{N}{a^*} \right) \quad (2)$$

185

186 Equation (2) shows that the reduced head coefficient ( $\Delta h_s/a^{*2}$ ) and turbine  
187 efficiency  $\eta_{tt}$  are both functions of the reduced flow coefficient ( $\dot{m}_o/\rho^* a^*$ ) and the reduced  
188 blade Mach number ( $N/a^*$ ). Therefore, non-dimensional performance maps can be  
189 constructed based on these four parameters. It has been found that for a radial turbine  
190 operating with R245fa, R123 and R1234yf working fluids, Equation (2) accurately predicts  
191 turbine performance to within 2% for all subsonic operating points, when compared to CFD  
192 simulations [19]. More recently, the similitude model has also been validated against unsteady  
193 CFD simulations for another radial turbine operating with these same working fluids in  
194 addition to R1234ze, pentane and isobutane [30]. In this case Equation (2) predicted the  
195 performance to within 1%. It should be noted that currently the authors have focused on radial  
196 turbines for small ORC systems. However, there should be no reason why Equation (2)  
197 cannot be used to model the performance of different types of turbines, namely axial turbines,  
198 but future research efforts should investigate this further. It should also be noted that there is  
199 also a need to confirm the suitability of Equation (2) experimentally.

200

## 201 **2.2 CFD turbine performance map**

202 The design specification for an ORC turbine is given in Table 1. For the specified inlet  
203 conditions and working fluid the turbine performance was evaluated over a range of pressure  
204 ratios and rotational speeds using CFD. The turbine design and CFD analysis is documented  
205 in [28]. After completing each CFD simulation the mass flow rate and isentropic efficiency  
206 were obtained and then scaled using Equation (2). The turbine performance maps were then  
207 obtained by curve fitting the CFD results, and these are shown in Figures 2 and 3.

208

## 209 **2.3 Loss models**

210 The CFD simulations used to construct Figures 2 and 3 were completed with periodic  
211 boundaries. Whilst this is necessary to reduce the computational expense of the simulations,  
212 this meant windage losses behind the rotor back face were not accounted for. Furthermore,  
213 these simulations did not consider the components upstream of the stator leading edge and  
214 downstream of the trailing edge, namely the volute and diffuser. Therefore, the performance  
215 maps should be updated to account for these additional losses before using them within  
216 further ORC studies. It should be noted that tip clearance was included within the CFD  
217 simulation and therefore tip clearance losses are already included.

218

219 2.3.1 Windage loss model

220 Within the clearance gap between the rotor back face and the rotor casing the circulation of  
221 fluid and the development of boundary layers on the rotor and casing walls results in a  
222 parasitic loss. As noted previously, the CFD simulation did not model this loss in an effort to  
223 reduce the simulation computational expense. Instead, a simple empirical model has been  
224 implemented for the sake of simplicity and cost. Of course, this empirical model was  
225 developed for ideal gases, so its validity for organic fluids should be confirmed through future  
226 computational and experimental studies.

227 This windage loss, expressed as an enthalpy loss  $\Delta h_w$ , is defined by Equation (3)  
228 where  $C_w$  is a torque loss coefficient,  $\rho_3$  is the density at the rotor inlet,  $\omega$  is the rotational  
229 speed in rad/s,  $r_3$  is the rotor inlet radius and  $\dot{m}_o$  is the working fluid mass flow rate.

230

$$\Delta h_w = \frac{\frac{1}{2} C_w \rho_3 \omega^3 r_3^5}{\dot{m}_o} \quad (3)$$

231

232 Four different flow regimes can occur, namely laminar and turbulent flow, both with  
233 merged and separated boundary layers respectively [31]. The flow within the clearance gap is  
234 laminar for  $Re < 10^5$  and turbulent for  $Re > 10^5$ , where  $Re$  is the rotational Reynolds number  
235 (Equation 4). The design point Reynolds number for the developed turbine is  $Re = 8.4 \times 10^6$ ,  
236 and therefore the flow is fully turbulent.

237

$$Re = \frac{\rho_3 \omega r_3^2}{\mu_3} \quad (4)$$

238

239 The ratio of the clearance gap  $\epsilon$ , to the rotor inlet radius establishes whether the  
240 boundary layers are merged or separated. Following from Dixon [32],  $\epsilon = 0.4\text{mm}$  was  
241 assumed which correlates to  $\epsilon/r = 0.012$ . This is sufficiently small to assume merged  
242 boundary layers. In this instance the torque loss coefficient is given by Equation (5), which is  
243 an empirical correlation based on experimental results and is described in Glassman [31].

244

$$C_w = \frac{0.0622}{\left(\frac{\epsilon}{r_3}\right)^{\frac{1}{4}} Re^{\frac{1}{4}}} \quad (5)$$

245

246 2.3.2 Diffuser design and performance analysis

247 It is often beneficial to install a diffuser downstream of the rotor to reclaim some of the  
248 kinetic energy contained within the flow. However, the design and CFD analysis completed

249 has not considered a diffuser, so it was necessary to design one. A simple straight-sided  
 250 conical diffuser was assumed, where the geometry is controlled by the area ratio  $A_r = A_5/A_4$ ,  
 251 and the diffuser divergence angle  $\theta$ .  $\theta$  is a critical parameter governing diffuser performance  
 252 and Aungier [33] suggested that optimal performance is obtained when  $2\theta = 11^\circ$ . Using this  
 253 value for  $\theta$ , a parametric study investigating a range of area ratios was conducted, and an  
 254 empirical diffuser performance model [33] was used to assess the diffuser performance. From  
 255 this study it was found that  $A_r = 2.5$  provided sufficient energy recovery, increasing the  
 256 isentropic total-to-static efficiency from 85.8% (no diffuser) to 88.1%. By comparison a  
 257 further increase to  $A_r = 4.0$  only resulted in a further increase of 0.3% to 88.4%.

258 It should be noted that the empirical diffuser performance model has not been  
 259 validated for organic fluids. However real gas effects are generally more prevalent at the  
 260 turbine inlet than at the outlet since the compressibility factor tends to reduce as the  
 261 temperature and pressure increases, and the operating conditions approach the critical point.

262

#### 263 **2.4 Updated turbine performance map**

264 Using the analysis discussed in Section 2.3, the CFD performance map was then updated. As  
 265 a starting point the turbine inlet conditions were set to the original design point ( $T_{01} = 350\text{K}$ ,  
 266  $P_{01} = 623.1\text{kPa}$ ). To account for losses upstream of the stator leading edge a total pressure  
 267 drop of  $\Delta P_v = 1\%$  was assumed within the volute, immediately supplying the conditions at the  
 268 stator inlet using a suitable equation of state. Within this paper REFPROP has been used,  
 269 which is a commercially available program containing state-of-the-art equations of state for a  
 270 wide variety of different fluids [34]. However, for the sake of generality, the calculation is  
 271 denoted with the notation ‘EoS’.

272

$$P_{02} = P_{01}(1 - \Delta P_v) \quad (6)$$

$$[T_{02}, s_{02}, \rho_{02}] = \text{EoS}(P_{02}, h_{01}, \text{fluid}) \quad (7)$$

273

274 Since the CFD performance map did not account for a volute, Figures 2 and 3 now  
 275 apply to these updated stator inlet conditions (location 2) instead of the design inlet conditions  
 276 (location 1). The choked conditions  $\rho^*$  and  $a^*$  are obtained by assuming an isentropic  
 277 expansion from the stator inlet to the throat. An array of head coefficients consisting of 100  
 278 elements ranging from 0 to 1.6 was then constructed, and each value was converted into the  
 279 isentropic total-to-total enthalpy drop from the stator inlet to the rotor outlet  $\Delta h_s$ . The size of  
 280 this array is not critical, as it only affects the resolution of the resulting contour plot. At each  
 281 head coefficient  $\dot{m}_o$ ,  $\eta_{tt}$  and  $\eta_{ts}$  were established at 50%, 80%, 100%, 120% and 150% of the  
 282 design reduced Mach number through interpolation of Figures 2 and 3. The total conditions at

283 the rotor outlet (location 4) then follow for each combination of head coefficient and reduced  
 284 blade Mach number. Here the subscript 's' refers to the conditions following an isentropic  
 285 expansion.

286

$$h_{04s} = h_{02} - \Delta h_s \quad (8)$$

$$P_{04} = \text{EoS}(h_{04s}, s_{02}, \text{fluid}) \quad (9)$$

$$h_{04} = h_{02} - \eta_{tt}(h_{02} - h_{04s}) \quad (10)$$

$$[T_{04}, s_{04}, \rho_{04}] = \text{EoS}(P_{04}, h_{04}, \text{fluid}) \quad (11)$$

287

288 Using the known value for  $\eta_{ts}$  the static conditions, and flow velocity  $c_4$ , at the rotor  
 289 outlet are obtained.

290

$$h_{4s} = h_{04} - \frac{h_{02} - h_{04}}{\eta_{ts}} \quad (12)$$

$$P_4 = \text{EoS}(h_{4s}, s_{02}, \text{fluid}) \quad (13)$$

$$[T_4, h_4, \rho_4] = \text{EoS}(P_4, s_{04}, \text{fluid}) \quad (14)$$

$$c_4 = \sqrt{2(h_{04} - h_4)} \quad (15)$$

291

292 With the rotor outlet conditions obtained, the diffuser performance model can then be  
 293 run using the defined diffuser geometry. This supplies the total and static conditions at the  
 294 diffuser outlet (location 5). The windage loss model is then run, and  $\eta_{tt}$  is reformulated as  
 295 follows.

296

$$h_{05s} = \text{EoS}(P_{05}, s_{01}, \text{fluid}) \quad (16)$$

$$\eta_{tt} = \frac{(h_{01} - h_{05}) - \Delta h_w}{h_{01} - h_{05s}} \quad (17)$$

297 The choked flow parameters,  $\rho^*$  and  $a^*$ , associated with the original turbine inlet  
 298 condition are then obtained, and the performance map is rescaled according to Equation (2).  
 299 The resulting performance maps are shown in Figures 4 and 5, where they are also compared  
 300 to the original CFD performance maps.

301 Figure 4 shows the variation in the reduced flow coefficient with the reduced head  
 302 coefficient and reduced blade Mach number. The behaviour shown in Figure 4 can be  
 303 explained by considering each additional loss that has now been modelled. Firstly, the  
 304 windage loss is a parasitic loss that absorbs a fraction of the total power produced by the  
 305 rotor. Therefore, it is not associated with a total pressure loss, so there is no effect on the

306 reduced head coefficient.

307 To consider the diffuser performance, the total pressure loss coefficient  $Y$  is  
308 introduced (Equation 18). This is defined as the ratio of the total pressure drop through the  
309 diffuser, to the difference between the total and static pressures at the diffuser outlet.

310

$$Y = \frac{P_{05} - P_{04}}{P_{05} - P_5} \quad (18)$$

311 Across the operating conditions considered  $Y$  ranged between 0.05 and 0.3.  
312 Furthermore, the flow leaves the diffuser with a low velocity, which implies a small  
313 difference between  $P_{05}$  and  $P_5$ . This implies a small total pressure drop within the diffuser,  
314 and a minimal change in the total-to-total isentropic enthalpy drop across the turbine. This  
315 will have a minimal effect on the reduced head coefficient. Therefore, the main shift seen in  
316 Figure 4 can be attributed to the 1% pressure drop applied upstream of the stator leading edge.  
317 This additional pressure drop increases the total-to-total pressure ratio across the whole  
318 turbine, and therefore increases the reduced head coefficient. Since the mass flow rate is  
319 unaffected, volute pressure drop simply shifts the constant blade Mach number lines to the  
320 right, as observed in Figure 4.

321 Figure 5 shows the variation in  $\eta_{tt}$  with the reduced head coefficient, and reduced  
322 blade Mach number. Considering first the diffuser, it has already been determined that there is  
323 a small total pressure drop within the diffuser, and a minimal change in total-to-total  
324 isentropic enthalpy drop. Furthermore, there is no energy transfer within the diffuser (i.e.  
325  $h_{04} = h_{05}$ ), so the change in  $\eta_{tt}$  is also minimal. Of course, if Figure 5 had plotted  $\eta_{ts}$ , a  
326 more significant shift would be observed since the purpose of the diffuser is to recover the  
327 kinetic energy and increase  $\eta_{ts}$ .

328 Using Equations (3) – (5) it can be shown that the windage loss is proportional to the  
329 rotational speed  $\omega$ , the meridional velocity at the rotor inlet  $c_{m3}$  and the fluid properties  $\rho_3$   
330 and  $\mu_3$  (Equation 19).

331

$$\Delta h_w \propto \frac{\omega^{\frac{11}{4}}}{c_{m3}(\rho_3\mu_3)^{\frac{1}{4}}} \quad (19)$$

332

333 Firstly, from Equation (19) it can be seen that windage loss increases with increasing  
334 rotational speed. This effect can be seen in Figure 5 where the constant reduced Mach number  
335 lines are increasingly shifted to the right with increasing speed. Secondly, Equation (19)  
336 implies that with increasing head coefficient, and therefore increasing mass flow rate, the  
337 windage loss will reduce. This is because a higher mass flow rate also implies a higher



338 meridional velocity at the rotor inlet. This effect is also shown in Figure 5, where the original  
339 and adapted reduced Mach number lines appear to converge with increasing head coefficient.

340 Finally, we can consider the effect of applying a 1% pressure drop in the volute. This  
341 additional loss increases the total-to-total isentropic enthalpy drop across the turbine.  
342 Therefore, since there is no energy transfer in the volute the total enthalpy drop across the  
343 turbine remains constant,  $\eta_{tt}$  must reduce. Furthermore, throughout this analysis  $\Delta P_v$  was kept  
344 constant, which means that at lower reduced head coefficients, which correspond to lower  
345 total-to-total pressure ratios, the volute total pressure loss is a higher fraction of the overall  
346 pressure drop across the turbine. This results in a more significant drop in efficiency at lower  
347 head coefficients, which further explains why the original and adapted reduced Mach number  
348 lines appear to converge at increasing head coefficients. It should be noted that in future  
349 studies it might be more beneficial to employ a more sophisticated volute performance model  
350 rather than applying a simple fixed value pressure drop.

351

### 352 **3 SYSTEM MODELLING**

353 A novel thermodynamic model has been developed which aims to establish the full range of  
354 heat source mass flow rates at a specified temperature that can be utilised using an existing  
355 turbine design, and present this information on a single contour plot. To obtain this contour  
356 plot, thermodynamic cycle analysis is coupled to the updated non-dimensional turbine  
357 performance curves (Figures 4 and 5). The result is a single contour plot that describes the  
358 performance of an ORC that utilises a particular heat source and operates with a specific  
359 turbine and working fluid. Ultimately, this plot can be used to determine the optimal heat  
360 source mass flow rates that can be effectively converted into useful power using this existing  
361 turbine. A simple subcritical ORC without a recuperator has been considered. Not only does  
362 this simplify the analysis, but it also reduces the overall cost of the system. Since the main  
363 focus is to investigate the interaction between turbine and cycle performance, additional  
364 aspects such as the required heat transfer areas, and pump performance are not considered,  
365 but instead are discussed later.

366 An ORC can be defined by the ORC condensation temperature  $T_6$ , the pressure ratio  
367 and the amount of superheat  $\Delta T_{sh}$ . If pressure drops within the pipes and heat exchangers are  
368 neglected, it is then simple to determine the working fluid properties at the pump inlet  
369 (location 6) and turbine inlet. For this analysis constant pump efficiency is assumed, from  
370 which the evaporator inlet conditions follow (location 7). The evaporator analysis is restricted  
371 to a simple energy balance when supplied with the evaporator pinch point  $PP_h$  (location 8).  
372 Since the aim of this analysis is to determine the optimal heat source mass flow rate, this  
373 parameter is unknown. However, the ratio of the working fluid mass flow rate  $\dot{m}_o$ , to the heat

374 source mass flow rate  $\dot{m}_h$ , is given by Equation (20), where the subscripts  $h_{hi}$  and  $h_{hp}$  refer to  
 375 the heat source enthalpy at the evaporator inlet and pinch point respectively.

376

$$\frac{\dot{m}_o}{\dot{m}_h} = \frac{h_{hi} - h_{hp}}{h_{o1} - h_8} \quad (20)$$

377

378 With the turbine inlet conditions defined (i.e.  $T_{01}, P_{01}$ ) the choked flow conditions  
 379 ( $a^*$  and  $\rho^*$ ) follow by assuming an isentropic expansion from the inlet to a Mach number of 1.  
 380 Furthermore, the turbine outlet pressure is defined by  $T_6$ , which in turn determines the  
 381 reduced head coefficient  $(h_{o1} - h_{o5s})/a^{*2}$ . Referring back to Figure 4, for a known reduced  
 382 head coefficient, there is a minimum and maximum flow coefficient that this turbine can  
 383 accommodate, which correspond to the maximum and minimum reduced blade Mach  
 384 numbers respectively. The minimum and maximum flow coefficients can be converted into  
 385 the physical mass flow rate limits for the turbine and an array of mass flow rates can be  
 386 constructed between these limits. For each value of  $\dot{m}_o$  interpolation of Figure 4 supplies the  
 387 reduced blade Mach number, whilst interpolation of Figure 5 supplies  $\eta_{tt}$ . This allows the  
 388 turbine outlet conditions to be obtained, whilst  $\dot{m}_h$  follows from Equation 20. A simple  
 389 energy balance within the condenser, assuming a condenser pinch point  $PP_c$ , provides the  
 390 cooling mass flow rate and completes the analysis. Ultimately, the result of this model is that  
 391 for specified  $T_6$ , PR,  $\Delta T_{sh}$  and  $PP_h$  values there is a range of  $\dot{m}_h$  values that can be converted  
 392 into power using this existing turbine.

393 Although cycle performance could be evaluated by the net power  $W_n$  or the cycle  
 394 thermal efficiency  $\eta_o$ , these evaluations do not give a clear indication of whether  
 395 implementing the existing turbine design is a feasible solution. Instead,  $W_n$  is compared to the  
 396 maximum net power that could be produced using the same heat source but with a turbine  
 397 operating at an optimal efficiency. For fixed values of  $T_6$ ,  $\Delta T_{sh}$ ,  $PP_h$ ,  $T_{hi}$  and  $\dot{m}_h$  there exists  
 398 an optimal pressure ratio at which optimal power can be produced. This optimum exists  
 399 because, whilst a higher pressure ratio increases the cycle efficiency, a higher pressure ratio  
 400 also leads to a higher evaporation temperature, and a smaller heat source temperature drop  
 401 and ORC mass flow rate. Since  $W_n$  is the product of the specific power and the mass flow  
 402 rate, there is a trade-off between maximising the cycle efficiency, and maximising the amount  
 403 of heat absorbed by the working fluid. This trade-off has been investigated in Figure 6 for a  
 404 range of heat source conditions, where the following assumptions have been made:  
 405  $T_6 = 313$  K,  $\Delta T_{sh} = 10$  K,  $PP_h = 15$  K,  $\eta_p = 70\%$  and  $\eta_{tt} = 85\%$ . The top graph considers a  
 406 range of heat source temperatures, all with a fixed  $\dot{m}_h$ , and clearly at higher heat source  
 407 temperatures, the optimal pressure ratio increases. The bottom graph shows that for a fixed

408  $T_{hi}$ , the optimal pressure ratio is independent of  $\dot{m}_h$ , and  $W_n$  increases linearly with increasing  
 409  $\dot{m}_h$ . Therefore, when supplied with  $T_{hi}$  and  $\dot{m}_h$  Figure 6 can be used to obtain the maximum  
 410 potential power that could be obtained for a turbine operating at  $\eta_{tt} = 85\%$ . Here 85% was  
 411 considered to be an achievable target at the design point. If  $W_n$  is greater than the maximum  
 412 potential power this is the result of the turbine operating at a higher efficiency than 85%.

413

#### 414 **4 OTHER SYSTEM COMPONENTS**

415 The motive behind the system model is to establish the range of heat source conditions that  
 416 can be converted into power using the existing turbine. By simplifying the pump and heat  
 417 exchanger analysis this stops the analysis being restricted by, for example, the pump  
 418 performance. Therefore, it is assumed that whilst the same turbine could be used within a  
 419 number of different systems, thus improving the economy-of-scale, alternative pumps and  
 420 heat exchangers may be required. However, after completing the analysis, it is interesting to  
 421 investigate the feasibility of also using the same pump and heat exchangers.

422

#### 423 **4.1 Pump modelling**

424 The pump can also be modelled using similitude laws. This is expressed by Equation (21),  
 425 where the pump head coefficient  $\psi = gH/(r\omega)^2$ , and pump efficiency  $\eta_p$ , are functions of  
 426 the flow coefficient  $\phi = Q/\omega r^3$ ;  $g$  is the acceleration due to gravity,  $H$  is the pump head,  $r$  is  
 427 the pump radius,  $\omega$  is the rotational speed, and  $Q$  is the volumetric flow rate.

428

$$\left[ \frac{gH}{(r\omega)^2}, \eta_p \right] = f \left( \frac{Q}{\omega r^3} \right) \quad (21)$$

429

430 Following from [35], the relationships between  $\psi$  and  $\phi$ , and  $\eta_p$  and  $\phi$ , can be  
 431 expressed using a simple quadratic expression of the form  $y = ax^2 + bx + c$ . Along with the  
 432 design point data (i.e.  $\phi_d, \psi_d, \eta_{p,d}$ ) the maximum head coefficient and maximum flow  
 433 coefficient are needed to determine the quadratic coefficients for each expression. These are  
 434 denoted as  $\psi_0$  and  $\phi_0$  respectively, and correspond to pump operation when  $Q = 0$  and  $H = 0$   
 435 respectively. At these operating points  $\eta_p = 0$ .

436 Before modelling pump performance, a pump design is required. Conveniently  $\psi$  and  
 437  $\phi$  can be combined to obtain pump specific speed  $\omega_s$  (Equation (22)). Karassik [36]  
 438 suggested that for a centrifugal pump  $\omega_s$  can be as low as 0.2 and for this value,  $\psi = 0.6$ . For  
 439 the ORC defined in Table 1, this corresponds to a design rotational speed of  $\omega_d = 5,300$  rpm  
 440 and a pump radius of  $r = 37.5$  mm. The design point efficiency is assumed to be  $\eta_{p,d} = 70\%$ .

441

$$\omega_s = \frac{\phi^{\frac{1}{2}}}{\psi^{\frac{3}{4}}} = \frac{\omega_d Q^{\frac{1}{2}}}{(gH)^{\frac{3}{4}}} \quad (22)$$

442

443 To construct the pump performance map, values for  $\psi_0$  and  $\phi_0$  are needed. A typical  
 444 value for  $\psi_0$  is 0.585 [36], whilst  $\phi_0$  is assumed to be  $2\phi_d$ . Whilst these are primitive  
 445 assumptions, this facilitates the construction of the pump performance map (see Figure 11),  
 446 which can be used during a preliminary assessment of pump performance following a change  
 447 in working fluid. Future efforts should establish the performance map for a specific ORC  
 448 pump.

449

## 450 4.2 Heat exchanger modelling

451 The required heat exchanger area  $A$  is given by Equation (23), where  $q$  is the heat transferred,  
 452  $\Delta T_{\log}$  is the log mean temperature difference, and  $U$  is the overall heat transfer coefficient.  
 453 Whilst  $q$  and  $\Delta T_{\log}$  follow from the cycle analysis completed in Section 3,  $U$  is dependent on  
 454 the heat exchanger geometry. Since the heat exchanger design is not a focus of this study  
 455 characteristic values for  $U$  have been estimated, as is typical during preliminary heat  
 456 exchanger sizing. For this analysis  $U = 50 \text{ W}/(\text{m}^2 \text{ K})$  is used during superheating and  
 457 precooling, whilst  $U = 1000 \text{ W}/(\text{m}^2 \text{ K})$  is used during preheating, evaporation and  
 458 condensation. These values are set according to [37].

459

$$A = \frac{q}{\Delta T_{\log} U} \quad (23)$$

460

461 With fixed  $U$  values, it is easy to deduce from Equation (23) that it is unlikely that the  
 462 same heat exchangers can be used within a range of different systems. Assuming that a  
 463 similar temperature profile is maintained (i.e.  $\Delta T_{\log}$ ), the required heat exchanger area should  
 464 scale directly with the heat input.

465

## 466 5 RESULTS AND DISCUSSION

467

### 468 5.1 R245fa case study

469 An initial case study demonstrates the thermodynamic model developed in Section 3. A heat  
 470 source of pressurised water ( $T_{\text{hi}} = 380 \text{ K}$ ,  $P_{\text{h}} = 400 \text{ kPa}$ ) has been defined and the ORC  
 471 working fluid has been kept as R245fa. The ORC parameters were fixed according to Table 2.  
 472 Both  $T_6$  and  $PP_c$  dictate the condenser area and the heat sink mass flow rate. The heat sink

473 temperature is  $T_c = 288$  K, whilst  $T_6 = 313$  K and  $PP_c = 10$  K corresponds to an approximate  
474 15 K temperature rise in the heat sink through the condenser. The value for  $PP_h$  has been  
475 estimated to be 15 K. Pinch points represent a trade-off between performance and cost and the  
476 values selected have been found to provide a reasonable balance. It has been widely shown  
477 that superheating is not necessary for organic fluids, but a small superheat of  $\Delta T_{sh} = 2$  K  
478 ensures full vaporisation at the turbine inlet. Since the pump performance is not considered at  
479 this stage  $\eta_p = 70\%$  is assumed.

480 The ORC model was then run over a range of pressure ratios, and a range of  
481  $\dot{m}_o$  values were established at each pressure ratio. At each combination of  $\dot{m}_o$  and PR,  $\dot{m}_h$  was  
482 determined allowing the maximum potential power to be obtained. The result of this analysis  
483 is a performance map that shows the variation of  $W_n$ , as a percentage of the maximum  
484 potential power, with PR and  $\dot{m}_o$  (Figure 7). The black lines, overlaid on the contour plot,  
485 indicate the resulting  $\dot{m}_h$  values in kg/s.

486 Figure 7 is useful since, for a specified heat source at  $T_{hi} = 380$  K, it is easy to assess  
487 the feasibility of using this turbine. For example, for  $\dot{m}_h = 1.0$  kg/s and pressure ratio of 2.2,  
488 the turbine efficiency is high and 100% of the maximum potential net power can be achieved.  
489 The optimal operating point corresponds to  $PR = 2.17$ ,  $\dot{m}_o = 0.60$  kg/s and  $\dot{m}_h = 0.91$  kg/s. At  
490 this operating condition the turbine operates at 88.7% of the design reduced rotational speed  
491 ( $N/a^*$ ), which is within feasible limits.

492 As  $\dot{m}_h$  moves away from this optimal point, the ORC performance deteriorates  
493 leading to a lower percentage of the maximum power being produced. However, it is found  
494 that for this heat source at 380 K, this existing turbine, operating with R245fa, can effectively  
495 operate with pressure ratios between 1.75 and 2.75. This corresponds to heat source mass  
496 flow rates between 0.5 kg/s and 1.75 kg/s, whilst  $N/a^*$  remains between 80% and 110% of  
497 the design value. Within these limits  $W_n$  should remain above 90% of the maximum potential  
498 power. At alternative heat source conditions an alternative turbine design may offer improved  
499 performance, and further analysis would be required to establish whether the improved  
500 performance would outweigh the increased costs of developing an alternative design.

501

## 502 **5.2 Alternative working fluids**

503 The analysis discussed in Section 5.1 can now be repeated for different heat source  
504 temperatures and working fluids. Reiterating that working fluid selection criteria is not a  
505 focus of this paper, 15 typical ORC working fluids have been arbitrarily selected. The heat  
506 source temperatures were then selected as 360 K, 380 K and 400 K. It is expected that below  
507 360 K the cycle thermal efficiency would reduce which would lead to uneconomical systems.  
508 On the other hand, higher temperature heat sources above 400 K could result in higher

509 pressure ratios across the turbine, and likely lead to supersonic flow within the turbine. Under  
510 these conditions it is likely that an alternative turbine design with a supersonic stator would be  
511 required. Hence at this stage it can already be hypothesised that the advantage of running the  
512 same turbine with different working fluids will be that the same turbine can be used for  
513 different heat source mass flow rates, but at similar operating temperatures.

514 For these studies the heat sink conditions,  $T_6$ ,  $\eta_p$ ,  $\Delta T_{sh}$ ,  $PP_h$  and  $PP_c$  were all fixed  
515 according to Table 2. For each combination of working fluid and heat source temperature the  
516 performance contour plot was obtained (i.e. Figure 7), allowing the optimal operating point to  
517 be obtained. Figure 8 displays the results in terms of the optimal  $\dot{m}_h$  and  $W_n$  values for each  
518 working fluid. The top-right plot in Figure 8 shows a summary all of the results, with each  
519 marker representing the result obtained for a particular working fluid at the respective heat  
520 source temperature. The remaining plots expand on these results by showing which working  
521 fluid each marker represents.

522 It is clear that a large spread of heat sources can be effectively utilised by this turbine.  
523 For example, for  $T_{hi} = 400$  K this turbine can convert heat sources between 0.5 kg/s and  
524 1.65 kg/s, with  $W_n$  ranging between 7.9 kW and 30.2 kW, by simply changing the working  
525 fluid. Furthermore, across all of the operating points it was found that the optimal point is  
526 consistently close to 100% of the maximum potential power, thus corresponding to turbine  
527 isentropic efficiencies close to 85%. This confirms that at the corresponding heat source  
528 conditions, the ORC is operating at an optimal pressure ratio that corresponds to the optimal  
529 head coefficient. In other words, it would be unlikely that an alternative turbine would offer  
530 much improvement on the turbine, and cycle, performance.

531 The optimal operating point for each working fluid and heat source have been plotted  
532 onto the turbine performance maps in Figure 9. This is useful to see how close to the design  
533 point the turbine is operating for each combination of working fluid and heat source  
534 temperature. Ultimately it is observed that as the heat source increases and the pressure ratio,  
535 and therefore reduced head coefficient increases, the reduced rotational speed is increased to  
536 ensure that the turbine efficiency remains close to the maximum. This ensures the turbine  
537 operates close to its design point and therefore operates efficiently over the range of  
538 conditions considered. Furthermore, for the range of heat source temperatures considered, the  
539 reduced rotational speed remains between 82% and 116% of the original design, confirming  
540 feasible turbine operation. Figure 9 also validates the selection of  $T_{hi} = 360$  K and  $T_{hi} =$   
541 400 K as the limits of operation for this turbine. For lower heat source temperatures optimal  
542 operating points would shift to the left leading to lower reduced rotational speeds, and low  
543 turbine efficiencies. A similar scenario can be seen for increasing head coefficients, which  
544 correspond to higher heat source temperatures. Hence this confirms that the same turbine

545 cannot be used with significantly different heat source temperatures, but can be used across a  
546 wide range of heat source mass flow rates.

547 The resulting cycle efficiencies  $\eta_o$  are shown in Figure 10.  $\eta_o$  increases with  
548 increasing heat source temperature, however there is only a small variation in  $\eta_o$  amongst the  
549 different working fluids. This is largely due to the optimal pressure ratio for a given heat  
550 source temperature being independent of the working fluid mass flow rate. It is arguable that  
551 at  $T_{hi} = 360$  K,  $\eta_o$  is too low to develop an economically feasible system.

552 Overall, Figure 8 suggests that the same turbine can be utilised within a number of  
553 different ORC applications with different heat source mass flow rates by selecting a suitable  
554 working fluid to match the available heat source. For example, for a heat source of 1.0 kg/s at  
555 380 K, R245fa could be selected as the working fluid and power generated would be around  
556 8 kW. However, for a heat source of around 1.75 kg/s at 400 K, R1234ze or isobutane could  
557 be selected and the power generated would increase to 30 kW. In Figure 11, the thermal input  
558 that each operating point corresponds to is also shown. This clearly shows that for a 360 K  
559 heat source that has between 50 and 200 kW<sub>th</sub> of heat available, the same turbine can be used  
560 if the working fluid is matched to the heat available. Similarly, a heat source temperature of  
561 380 K corresponds to heat inputs ranging between around 70 and 270 kW<sub>th</sub>, whilst a heat  
562 source of 400 K corresponds to values between 100 and 380 kW<sub>th</sub>. Hence, Figure 11 gives a  
563 clear indication of the range of potential applications that this turbine could be utilised within.  
564 Ultimately, this allows the same turbine to be manufactured in large volumes, thus facilitating  
565 an improvement in the economy-of-scale, and an improvement in the economic feasibility of  
566 implementing such a system.

567 Before progressing, it is important to discuss possible limitations to implementing the  
568 same turbine within a number of different systems. Firstly, the results in Figure 8 were  
569 obtained by varying only the pressure ratio. Therefore, the effects of  $T_6$ ,  $\Delta T_{sh}$ ,  $PP_h$  and  $PP_c$   
570 were not considered. Therefore, it could be argued that the same turbine and working fluid  
571 could be used in different ORC systems by optimising these cycle parameters rather than  
572 changing the working fluid. However, whilst this might be true for fluids with similar  
573 performance, (i.e. they lie close to each other in Figure 8), it is unlikely that this would be  
574 possible when  $\dot{m}_h$  changes significantly (i.e. from 0.5 kg/s to 1.5 kg/s). Secondly, additional  
575 factors, such as the bearing system and generator, are not taken into consideration during this  
576 study, and this may limit the feasibility of using the same turbine assembly across a wide  
577 range of power outputs. However, in these instances, even if modifications to the mechanical  
578 design are required, the costs associated with the aerodynamic design and manufacture of the  
579 stator and rotor assembly can still be avoided. Finally, within this study a wide range of  
580 working fluids were considered, which in reality may not be suitable due to availability, cost

581 and legislative restrictions. Nonetheless, this work may be a novel contribution to the ORC  
582 community, demonstrating how non-dimensional turbine maps can be implemented within  
583 cycle analysis studies, and ultimately how the economy-of-scale of small-scale ORC systems  
584 could be improved.

585

### 586 **5.3 Pump and heat exchanger performance**

587 Having established the possibility of implementing the turbine within a number of different  
588 ORC configurations, the performance of the pump and heat exchanger performance can now  
589 be investigated. For each working fluid, at each heat source temperature, the optimal  $\dot{m}_o$  and  
590 PR values are already known, which supplies the pump volumetric flow rate and the pump  
591 head. Using the pump performance map this provides the required rotational speed  $\omega$  and  
592 pump efficiency  $\eta_p$ . Figure 12 displays the results of this analysis plotted onto the pump  
593 performance map for the pump design discussed in Section 4.1. Here  $\phi$  and  $\psi$  have been  
594 normalised by the design values (i.e.  $\phi_d, \psi_d$ ). It is clear that for all the operating points  
595 considered  $\phi$  remains between  $0.6\phi_d$  and  $1.5\phi_d$ , which corresponds to values of  $0.6\psi_d$  and  
596  $1.1\psi_d$  respectively. Under these conditions, the pump operates far enough away from the  
597 shut-off head, and run-out flow rate that  $\eta_p$  remains above 50%.

598 Figure 13 displays the  $\omega$  for each case and clearly, as  $T_{hi}$  and  $\dot{m}_h$  increase,  $\omega$   
599 increases. The maximum rotational speed is around 14,000 rpm, which with  $r_d = 37.5$  mm,  
600 corresponds to a maximum pump impeller tip speed of 55 m/s. The maximum allowable tip  
601 speed is governed by the mechanical design, and the prevention of cavitation within the  
602 pump. However, a typical maximum is around 50 m/s. Therefore, at this maximum rotational  
603 speed, the pump may be operating at the limit of feasible operation.

604 Overall, this analysis suggests that it would be possible to use the same pump within  
605 the majority of operating points shown in Figure 8, and under these conditions  $\eta_p$  would  
606 remain between 50% and 70%. Further analysis is required to establish the impact of this  
607 reduction in  $\eta_p$  on the whole system. More detailed research is also required for the design  
608 and analysis of ORC pumps to obtain more accurate performance maps, and to validate the  
609 use of similitude theory to ORC pumps. Nonetheless, the analysis presented here is believed  
610 to be an important first step.

611 The required head transfer areas for the evaporator and condenser for each working  
612 fluid and heat source combination have been calculated and are presented in Figures 14 and  
613 15. Ultimately these results confirm that it is not feasible to use the same heat exchanger  
614 across a range of different operating conditions. As discussed previously, it was expected that  
615 the required heat transfer area would directly scale with increasing heat input  $q$ . Furthermore,  
616 since  $q = W_n/\eta_o$ , and Figure 10 has already shown that  $\eta_o$  is independent of  $T_{hi}$ , this means



617 that the required evaporator heat transfer area directly scales with  $W_n$ , and therefore  $\dot{m}_n$ . This  
618 relationship is clearly observed in Figure 14.

619

## 620 **6 CONCLUSIONS**

621 To improve the economy-of-scale of small ORC systems, it may be necessary to implement  
622 the same system components into a range of different applications. This paper has  
623 investigated improvements in this area by combining component performance models with  
624 thermodynamic cycle analysis. First a turbine performance map, obtained using CFD, was  
625 adjusted to account for additional loss mechanisms, before being non-dimensionalised using a  
626 modified similitude theory. A novel thermodynamic model was then constructed, and a case  
627 study was considered. This study showed that for a given heat source temperature and  
628 working fluid there exists an optimal heat source mass flow rate that can be efficiently  
629 converted into power using the existing turbine design. Repeating this analysis for different  
630 heat source temperatures and working fluids has demonstrated the possibility of utilising the  
631 same turbine for a range of different heat source flow rates. In particular, this study  
632 demonstrated that through selecting a suitable working fluid the existing turbine could  
633 convert heat sources ranging from 360 K and 400 K, with mass flow rates between 0.5 kg/s  
634 and 2.75 kg/s, into power outputs between 2 kW and 30 kW without compromising on turbine  
635 performance. Whilst the required heat exchanger areas were found to scale directly with  
636 increasing heat input, the possibility of also using the same pump within a number of different  
637 applications was also demonstrated. Therefore, this study has demonstrated the possibility of  
638 using the same pump and turbine within a number of different ORC systems. This is expected  
639 to potential to improve the economy-of-scale of small ORC systems, allowing the same  
640 components to be manufactured in large volumes and then implemented within different  
641 applications, thus reducing costs and facilitating a move towards more economically viable  
642 ORC systems. Further efforts should investigate whether these findings are equally applicable  
643 to higher temperature ORCs, which are expected to introduce more uncertainties into the  
644 modelling process. Firstly, these systems will require alternative working fluids that are  
645 operated closer to their critical point and exhibit more extreme real gas behaviour.  
646 Furthermore, due to the low speed of sound supersonic turbines may be required, which will  
647 also require the modified similitude model to be investigated for supersonic flows. Finally,  
648 more effort is needed to validate both numerically and experimentally the use of similitude  
649 theory, and give due consideration to its validity to other types of turbines and ORC pumps.

650

## 651 **7 ACKNOWLEDGMENTS**

652 The authors would like to thank the UK Engineering and Physical Sciences Research Council  
653 for funding this research.

654

## 655 **8 REFERENCES**

656 [1] Colonna, P., Casati, E., Trapp, C., Mathijssen, T., Larjola, J., Turunen-Saaresti, T., and  
657 Uusitalo, A., 2015, “Organic Rankine cycle power systems: from the concept to current  
658 technology, applications and an outlook for the future”, *Journal of Engineering for Gas*  
659 *Turbines and Power*, **137**(10).

660 [2] Quoilin, S., van den Broek, M., Declaye, S., Dewallef, P., and Lemort, V., 2013, “  
661 Techno-economic survey of organic Rankine cycle (ORC) systems”, *Renewable and*  
662 *Sustainable Energy Reviews*, **22**, pp. 168-186.

663 [3] Hettiarachchi, M. H., Golubovic, M., Worek, W. M., and Ikegami, Y., 2007, “The optimal  
664 evaporatorion temperature and working fluids for subcritical organic Rankine cycle”, *Energy*,  
665 **38**(1), pp. 1198-1706.

666 [4] Sauret, E., and Rowlands, A. S., 2011, “Candidate radial-inflow turbines and high density  
667 working fluids for geothermal power systems”, *Energy*, **36**(7), pp. 4460-4467.

668 [5] Peirobon, L., Nguyen, T. V., and Larsen, U., Haglind, F., and Elmegaard, B., 2013,  
669 “Multi-objective optimization of organic Rankine cycles for waste heat recovery: Application  
670 in an offshore platform”, *Energy*, **58**(1), pp. 538-529.

671 [6] Borsukiewicz-Gozdur, A., 2013, “Pumping work in the organic Rankine cycle”, *Applied*  
672 *Thermal Engineering*, **51**(1), pp. 781-786.

673 [7] Wei, D., Lu, X., Lu, Z., and Gu, J., 2008, “Dynamic modelling and simulation of an  
674 Organic Rankine Cycle (ORC) system for waste heat recovery”, *Applied Thermal*  
675 *Engineering*, **28**(10), pp. 1216-1224.

676 [8] Vaja, I., and Gambarotta, A., 2010, “Dynamic model of an Organic Ranine Cycle system.  
677 Part I – Mathematical description of main components”, *ECOS Vol. III: Cycles & Building*,  
678 14-17<sup>th</sup> June, Lausanne, Switzerland.

679 [9] Sun, J., and Li, W., 2011, “Operation optimization of an organic Rankine cycle (ORC)  
680 heat recovery plant”, *Applied Thermal Engineering*, **31**(11-12), pp. 2032-2041.

681 [10] Clemente, S., Micheli, D., Reini, M., and Taccani, R., 2012, “Energy efficiency analysis  
682 of Organic Rankine Cycles with scroll expanders for cogenerative applications”, *Applied*  
683 *Thermal Energy*, **97**(1), pp. 792-801.

684 [11] Moustapha, H., Zelesky, M. F., Baines, N. C., and Japiske, D., 2003, *Axial and radial*  
685 *turbines*, Concepts NREC, Inc.

- 686 [12] Angelino, G., Gaia, M., and Macchi, E., 1984, "A review of Italian activity in the field of  
687 Organic Rankine Cycles", Proceedings of the International VDI-Seminar on ORC-HP-  
688 Technology, 10-12<sup>th</sup> September, Dusseldorf, Germany.
- 689 [13] Astolfi, M., and Macchi, E., 2015, "Efficiency correlations for axial-flow turbines  
690 working with non-conventional fluids", 3<sup>rd</sup> International Seminar on ORC Power Systems",  
691 12-14<sup>th</sup> October, Brussels, Belgium.
- 692 [14] Li, J., Pei, G., Li, Y., Wang, D., and Ji, J., 2013, "Examination of the expander leaving  
693 loss in variable organic Rankine cycle operation", Energy Conversion and Management,  
694 **65**(1), pp. 106-116.
- 695 [15] Calise, F., Capuozzo, C., Carotenuto, A., and Vanoli, L., 2013, "Thermoeconomic  
696 analysis and off-design performance of an organic Rankine cycle power by medium  
697 temperature heat sources", Solar Energy, **103**(1), pp. 595-609.
- 698 [16] Manente, G., Toffolo, A., Lazzaretto, A., and Paci, M., 2013, "An Organic Rankine  
699 Cycle off-design model for the search of the optimal control strategy", Energy, **58**(1), pp. 97-  
700 106.
- 701 [17] Cameretti, M. C., Ferrara, F., Gimelli, A., and Tuccillo, R., 2015, "Employing micro-  
702 turbine components in integrated Solar-MGT-ORC power plants", ASME Turbo Expo 2015,  
703 15-19<sup>th</sup> June, Montreal, Canada
- 704 [18] Wong, C. S., and Krumdieck, S., 2015, "Scaling of gas turbine from air to refrigerants  
705 for organic Rankine cycles (ORC) using similarity concept", ASME-ORC 2015, 12-14<sup>th</sup>  
706 October, Brussels, Belgium.
- 707 [19] White, M., and Sayma, A. I., 2015, "The application of similitude theory for the  
708 performance prediction of radial turbines within small-scale low-temperature organic Rankine  
709 cycles", Journal of Engineering for Gas Turbines and Power, **137**(12).
- 710 [20] Hu, D., Li, S., Zheng, Y., Wang, J., and Dai, Y., 2015, "Preliminary design and off-  
711 design performance analysis of an Organic Rankine cycle for geothermal sources", Energy  
712 Conversion and Management, **96**(1), pp. 175-187.
- 713 [21] Fiaschi, D., Manfrida, G., and Maraschiello, F., 2015, "Design and performance  
714 prediction of radial ORC turboexpanders", Applied Energy, **138**(1), pp. 517-532.
- 715 [22] Rahbar, K., Mahmoud, S., Al-Dadah, R. K., and Moazami, F., 2015, "Parametric  
716 analysis and optimization of a small-scale radial turbine for organic Rankine cycle", Energy,  
717 **83**(1), pp. 696-711.
- 718 [23] Badr, O., Probert, S. D., and O'Callaghan, P. W., 1985, "Selecting a working fluid for a  
719 Rankine-cycle engine", Applied Energy, 21, pp. 1-42.

720 [24] Chen, H., Goswami, D. Y., and Stefanakos, E. K., 2010, “A review of thermodynamic  
721 cycles and working fluids for the conversion of low-grade heat”, *Renewable and Sustainable*  
722 *Energy Reviews*, 14(1), pp. 3059-3067.

723 [25] Bao, J. and Zhao, L., 2013, “A review of working fluid and expander selections for  
724 organic Rankine cycle”, *Renewable and Sustainable Energy Reviews*, 24, pp. 325-342.

725 [26] Saleh, B., Koglbauer, G., Wendland, M., and Fischer, J. (2007). Working fluids for low  
726 temperature organic Rankine cycles. *Energy*, 32(7), pp. 1210-1221.

727 [27] Li, G., 2016, “Organic Rankine cycle performance evaluation and thermoeconomic  
728 assessment with various applications part I: Energy and exergy performance evaluation”,  
729 *Renewable and Sustainable Energy Reviews*, 53, 477-499.

730 [28] White, M. and Sayma, A. I., 2015, “The one-dimensional meanline design of radial  
731 turbines for small scale low temperature organic Rankine cycles”, *ASME Turbo Expo 2015*,  
732 15-19<sup>th</sup> June, Montreal, Canada.

733 [29] White, M. and Sayma, A. I., 2015, “The impact of component performance on the  
734 overall cycle performance of small-scale low temperature organic Rankine cycles”, 9<sup>th</sup>  
735 *International Conference on Compressors and their Systems*, London, UK.

736 [30] White, M., and Sayma, A. I., 2016, “Investigating the effect of changing the working  
737 fluid on the three-dimensional flow within organic Rankine cycle turbine”, *ASME Turbo*  
738 *Expo 2016*, 13-7<sup>th</sup> June, Seoul, South Korea.

739 [31] Glassman, A. J., 1976, “Turbine design and application”, Technical report NASA SP-  
740 290, NASA, Cleveland, US.

741 [32] Dixon, S. L., and Hall, C. A., 2010, *Fluid mechanics and thermodynamics of*  
742 *turbomachinery*. Butterworth-Heinemann, UK, 6<sup>th</sup> Edition.

743 [33] Aungier, R. H., 2006, *Turbine aerodynamics: Axial-flow and radial in-flow turbine*  
744 *design and analysis*, ASME, New York, 1<sup>st</sup> Edition.

745 [34] Lemmon, E. W., Huber, M. L., and McLinden, M. O., 2013, *NIST Standard Reference*  
746 *Database 23: Reference Fluid Thermodynamic and Transport Properties-REFPROP*, Version  
747 9.1, National Institute of Standards and Technology, Standard Reference Data Program,  
748 Gaithersburg.

749 [35] van Putten, H., and Colonna, P., 2007, “Dynamic modelling of steam power cycles: Part  
750 II – Simulation of a small simple Rankine cycle system”, *Applied Thermal Engineering*,  
751 **27**(14-15), pp. 2566-2582.

752 [36] Karassik, I. J., Messina, J. P., Cooper, P., and Heald, C. C., 2008, *Pump handbook*,  
753 McGraw-Hill Professional, New York, US, 4<sup>th</sup> Edition.

754 [37] Roetzel, W., Spang, B., 2010, *VDI Heat Atlas: C3 Typical Values of Overall Heat*  
755 *Transfer Coefficients*, pp 75-78, Springer-Verlag Berlin Heidelberg, 2<sup>nd</sup> Edition.

756

757 **Figure 1.** Notation used to model the turbine and ORC system.  
758

759 **Figure 2.** Variation in the reduced flow coefficient at different reduced head coefficients and  
760 reduced blade Mach numbers, as predicted using CFD simulations.  
761

762 **Figure 3.** Variation in the turbine total-to-total efficiency at different reduced head  
763 coefficients and reduced blade Mach numbers, as predicted using CFD simulations.  
764

765 **Figure 4.** Updated turbine performance map showing the relationship between the reduced  
766 head coefficient and reduced flow coefficient for reduced Mach numbers ranging between  
767 50% and 150% of the design value.  
768

769 **Figure 5.** Updated turbine performance map showing the relationship between the reduced  
770 head coefficient and turbine efficiency for reduced Mach numbers ranging between 50% and  
771 150% of the design value.  
772

773 **Figure 6.** Variation in net power produced as a function of pressure ratio for different heat  
774 source conditions. Top: fixed heat source mass flow rate of 1.0kg/s; Bottom: fixed heat source  
775 temperature of 380K.  
776

777 **Figure 7.** Contour of the net power produced by an ORC operating with the candidate turbine  
778 as a percentage of the maximum potential power. Heat source of water at 380K, and R245fa  
779 as working fluid. The black lines indicate the heat source mass flow rate in kg/s, whilst the  
780 black dot represents the point of optimal operation.  
781

782 **Figure 8.** Cycle analysis results showing the heat source mass flow rates that can be  
783 accommodated by an ORC utilising the candidate turbine at each combination of heat source  
784 temperature and working fluid. Top left: summary of all results; top right: 360K; bottom left;  
785 380K; bottom right; 400K.  
786

787 **Figure 9.** Results from each combination of heat source temperature and working fluid  
788 overlaid onto the turbine performance map.  
789

790 **Figure 10.** Cycle analysis results showing variation in cycle at the three different heat source  
791 temperatures.  
792

793 **Figure 11.** Net work plotted against the thermal heat input into the ORC system for each heat

794 source temperature and working fluid. Each marker represents a particular working fluid.  
795

796 **Figure 12.** Non-dimensional pump performance map, overlaid with operating points for each  
797 heat source temperature.

798

799 **Figure 13.** Pump rotational speed for each heat source temperature and mass flow rate. Each  
800 marker represents a particular working fluid.

801

802 **Figure 14.** Required evaporator heat transfer area for each heat source temperature and mass  
803 flow rate. Each marker represents a particular working fluid.

804

805 **Figure 15.** Required condenser heat transfer area for each heat source temperature and mass  
806 flow rate. Each marker represents a particular working fluid.

807

808

809

**Table 1.** Design point specification for the ORC turbine.

Working fluid	-	R245fa	
ORC condensation temperature	$T_6$	313.0	K
Total inlet temperature	$T_{01}$	350.0	K
Total inlet pressure	$P_{01}$	623.1	kPa
Pressure ratio	PR	2.5	
Mass flow rate	$\dot{m}_o$	0.7	kg/s
Rotational speed	$N$	37,525	rpm
Rotor diameter	$D$	66.7	mm

810

811

812

813

**Table 2.** Fixed inputs for the R245fa case study.

814

Heat source fluid		water	
Heat source temperature	$T_{hi}$	380	K
Heat source pressure	$P_h$	400	kPa
Heat sink fluid		water	
Heat sink temperature	$T_c$	288	K
Heat sink pressure	$P_c$	101	kPa
Pump isentropic efficiency	$\eta_p$	70	%
ORC condensation pressure	$T_6$	313	K
Amount of superheat	$\Delta T_{sh}$	2	K
Evaporator pinch point	$PP_h$	15	K
Condenser pinch point	$PP_c$	10	K