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Optical orbital angular momentum

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We present a brief introduction to the orbital angular momentum of light, the subject of our special issue and, in particular, to the developments in the thirteen years following the founding paper by Allen, Beijersbergen, Spreeuw and Woerdman. The papers by our invited authors serve to bring the field up to date and suggest where developments may take us next.

PACS numbers:

I. INTRODUCTION

This special issue is devoted to the subject of the angular momentum of light and, most especially to the orbital angular momentum about the axis of propagation. This field of research has brought together researchers from a wide range of backgrounds, all of whom refer to the topic by the acronym OAM. It is amusing to note, however, that there is not universal agreement as to what the 'O' stands for: some would have it be Orbital Angular Momentum, and others Optical Angular Momentum. There is no need to resolve this, indeed it is unlikely that even the three authors of this short paper would agree, hence the title of the paper. The point is that 'orbital' angular momentum is not unique to light and occurs for other fields, including electrons, neutrons, sound waves and atoms. Be it 'Orbital' or 'Optical' there is no doubt that the field we all agree to call OAM is a lively and growing one. It is twenty five years since the publication of the paper that gave rise to this activity [1] and it seems timely to take stock of what has been learnt.

During the process of bringing this special issue together we were saddened by the passing of our friend and mentor, Les Allen, who both was an author on the original paper and also introduced each of us to this field. This special issue, envisaged as a celebration, is therefore also an act of remembrance. We dedicate it to Les and to his memory.

At the end of the article is a short appreciation of Les, written and read by one of us (SMB) at a celebration of Les's life and achievements, attended by family, friends and the three authors of this article.

II. OPTICAL ANGULAR MOMENTUM

You have only to turn your face to the sun and feel the warmth to appreciate that light carries energy. This property follows directly from Maxwell's electromagnetism, as does the fact that light carries (linear) momentum; indeed Maxwell himself calculated the correct value of the radiation pressure due to solar radiation on the surface of the earth [2]. In the quantum theory of light, a plane wave of angular frequency ω and wavevector $\mathbf{k}$ has energy $\hbar\omega$ and momentum $\hbar\mathbf{k}$ carried by each photon. In addition to linear momentum, optical beams can also carry angular momentum which, in the quantum theory, is often expressed as the spin, $\pm\hbar\mathbf{k}/|\mathbf{k}|$, of the photon. As with the linear momentum, however, this spin angular momentum, and in particular its connection with circular polarisation, is also a feature of the classical Maxwell theory [3].

That the spin angular momentum of light cannot be all there is was realised very early in the development of the modern quantum theory. Darwin considered radiative processes and concluded, in particular that "For quadrupole emission the pure particle concept is a failure" [4]. By this he meant that the radiated photon must carry away not only spin but also orbital angular momentum, and he gave expressions for each of these components [4]. The orbital and spin parts of the angular momentum are, respectively,

$$\begin{aligned}\mathbf{L} &= \int \sum_j E_j(\mathbf{r}\times\boldsymbol{\nabla})A_j dV \\ \mathbf{S} &= \int \mathbf{E}\times\mathbf{A}dV, \end{aligned} \tag{1}$$

where $\mathbf{E}$ is the electric field and $\mathbf{A}$ is the *transverse* (that is, divergenceless) part of the vector potential. It has been suggested, for a variety of reasons, including gauge invariance and the lack of a rest-frame for the photon, that this separation of the angular momentum into spin and orbital parts is unphysical. We shall not rehearse these arguments here, but rather note that the resolution of this is that the spin and orbital parts are physically distinct and meaningful, but that neither is of itself a true angular momentum [5–7]. A more complete discussion of this subtlety is given in [8].

For much of the twentieth century, the study of the orbital angular momentum of light centred on the study of multipolar transitions. These form a key element in the study of the emission of gamma rays in nuclear decay [9] but are of less significance in the optical regime on account of the large difference in scale between optical wavelengths and the sizes of atoms and molecules [10]. This changed with the realisation that beams of intense laser light could be prepared with well-defined orbital angular momentum [1].

III. ORBITAL ANGULAR MOMENTUM

It is certainly possible to develop the theory of optical angular momentum directly from the forms given in (1), but let us take a less formal route, one that is closer in spirit to that followed in the original paper [1].

For practical laser fields it often suffices to restrict ourselves to quasi-monochromatic fields with a rather well-defined direction of propagation. We take full account of diffraction, but if the beam is not too strongly focussed, then a description based on the paraxial approximation is justified [11]. It suffices to consider a beam of angular frequency ω propagating in the z direction and, for simplicity let us consider the beam to be linearly polarised, so as to avoid the complication of circular polarisation and spin angular momentum. The amplitude of the field, v, satisfies the Helmhotz equation

$$\nabla^2 v + k^2 v = 0, \tag{2}$$

where $k = \omega/c$ is the wavenumber. We obtain the paraxial wave equation by making the substitution

$$v = ue^{ikz} \tag{3}$$

so that our Helmhotz equation becomes

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) u + \frac{\partial^2}{\partial z^2} u + 2ik\frac{\partial}{\partial z} u = 0. \tag{4}$$

We make the paraxial approximation by noting that the variation of the amplitude u in the z direction is slow and this allows to drop the second term and so arrive at the paraxial wave equation [11]:

$$i\frac{\partial}{\partial z} u = -\frac{1}{2k}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) u, \tag{5}$$

which is formally equivalent to the Schrödinger equation in two dimensions, an equivalence that is both suggestive and useful.

A great number of solutions are known for the paraxial wave equation and many of these are routinely used to describe laser fields, including Gaussian beams and the higher-order Hermite-Gaussian modes. Of especial interest to us, however, are the Laguerre-Gaussian solutions. These are most conveniently expressed in cylindrical polar coordinates (ρ, ϕ, z) and take the form

$$\begin{aligned} u_{\ell,p} &= \sqrt{\frac{2p!}{\pi(p+|\ell|)!}} \frac{1}{w(z)} \left(\frac{\rho\sqrt{2}}{w(z)}\right)^{|\ell|} \exp\left(\frac{-\rho^2}{w^2(z)}\right) L_p^{|\ell|}\left(\frac{2\rho^2}{w^2(z)}\right) e^{i\ell\phi} \\ &\times \exp\left(-ik\frac{\rho^2 z}{2(z_R^2 + z^2)}\right) \exp\left[-i(2p + |\ell| + 1)\tan^{-1}(z/z_R)\right] \end{aligned} \tag{6}$$

where $L_p^{|\ell|}$ is an associated Laguerre polynomial, $z_R = kw^2(0)/2$ is the Rayleigh range (a measure of the tightness of the focus) and $w(z) = w(0)\sqrt{1 + z^2/z_R^2}$ is the beam width. The most significant part, for us, of this lengthy expression is the azimuthal phase dependence contained in the term $e^{i\ell\phi}$. This innocuous looking term indicates the existence of a phase vortex at $\rho = 0$; if we traverse a closed path around this axis then we a accumulate a phase of $2\pi\ell$ [12]. This azimuthal phase dependence is familiar from quantum theory in that a wavefunction with this is an eigenstate of the orbital angular momentum operator

$$\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi} \tag{7}$$

with eigenvalue $\ell\hbar$. The *analogy* with quantum theory is more than simply suggestive and turns out to be quantitively correct in that each photon in a Laguerre-Gaussian laser beam of the form (6) carries an orbital angular momentum of $\ell\hbar$ [1].

The quantity of angular momentum hinted at from the analogy with quantum theory follows, also, from electromagnetic theory. Perhaps the simplest way to see this is to note that in the eikonal approximation (which is a good one for paraxial beams) the Poynting vector has the form [13]

$$\mathbf{S} \propto \Im(v^* \boldsymbol{\nabla} v). \tag{8}$$

This expression embodies the idea that the phase fronts are locally plane in form and that the normal to these gives the local direction of propagation of the energy. The z and azimuthal components of this for our Laguerre-Gaussian beam are

$$\begin{aligned} S_z &\approx k|u|^2 \\ S_\phi &= \frac{\ell}{\rho}|u|^2. \end{aligned} \tag{9}$$

The remaining ρ component is associated with the spreading of the beam by diffraction. The momentum density is simply $\mathbf{g} = \mathbf{S}/c^2$ and it follows that the density of the z component of orbital angular momentum is

$$\ell_z = \rho g_\phi = \frac{\ell |u|^2}{c^2}. \tag{10}$$

The local energy density is $\mathcal{E} \approx c g_z$ so that

$$\frac{\ell_z}{\mathcal{E}} = \frac{\ell}{\omega}. \tag{11}$$

The energy of a single photon is $\hbar\omega$ and it follows, therefore, that the orbital angular momentum is $\ell\hbar$ per photon [1]. This simple treatment masks a number of interesting subtleties [14–16], but more sophisticated and rigorous treatments and experiments have verified the correctness of (11).

IV. THE EARLY YEARS: 1992–2005

The observation that laser beams could be engineered to carry a well-defined quantity of orbital angular momentum rapidly found wide application and these developments are described in early reviews of the field [17–23]. The third of these contains reprints of a selection of early papers. There is not the space here to do justice to these developments, but we hope that a few words will serve at least to indicate some of what has been achieved. The selection of topics and papers described is a subjective one and we hope that the expert reader will forgive us if we omit to mention a particular favourite; our intention is to give only a flavour of the work and its variety.

The new idea (that laser beams could carry orbital angular momentum) was followed very quickly by both theoretical and experimental developments. In particular the use of cylindrical lenses to generate such beams was very quickly demonstrated experimentally [24]. Early theoretical developments included investigations of orbital angular momentum beyond the paraxial approximation [25, 26], the torque experienced by an atom in a field carrying orbital angular momentum [27] and the influence of angular momentum on the dynamics of solitons in a nonlinear medium [28].

Perhaps the most evocative of the early experimental studies using optical angular momentum were those in which this angular momentum was transferred to microscopic particles held in optical tweezers. Optical tweezers are tightly focussed beams of laser light, which trap transparent particles at the beam focus [29, 30]. The trapping force in these tweezers derives from the refraction of the light at the surface of the particle or, if the particle is absorbing, by the conservation of angular momentum in the absorption process. Rubinsztein-Dunlop and co-workers trapped microscopic particle in a Laguerre-Gaussian beam and the partial absorption of the light set the particle into rotation [31]. Subsequent studies were able to calibrate this torque by comparing the rotation induced by the orbital angular momentum to that caused by the spin angular momentum. This provided a direct demonstration that the orbital angular momentum is indeed $\ell\hbar$ per photon [32, 33]. For particles held off-axis in the bright annulus of the helically-phased beam the different natures of the spin and orbital angular momenta are revealed: the spin angular momentum causes the trapped particle to spin on its axis, but the orbital angular momentum causes the particle to orbit the vortex on the beam axis [34, 35]. Such experiments led directly to a resurgence of interest in optical tweezers where these and other exotic light beams could be used to manipulate optically confined micromachines.

The existence of quantised orbital angular momentum at the single-photon level was beautifully demonstrated by Zeilinger and co-workers who measured the angular momentum (or more precisely the phase vortices) for pairs of entangled photons generated in spontaneous parametric down-conversion [36]. This results from the conservation of angular momentum in the conversion of a single pump-beam photon to the signal and idler photons. The sum is fixed in this way but the orbital angular momenta of the individual photons is not, so the resulting superposition is an angular-momentum entangled state [37]. This advance has provided a new avenue for exploring entanglement in high-dimensional Hilbert spaces [38].

The high dimensionality of the orbital angular momentum state space contrasts with the fact that there are only two orthogonal polarisation or spin angular momentum states. This suggests the use of orbital angular momentum for information processing and communication tasks. Padgett and co-workers demonstrated the use of orbital angular momentum as a key component in a free-space communication link in which eight different orbital angular momentum beams could co-propagate along the same optical path and then be separated by the receiver [39]. This was followed by further studies of orbital angular momentum mediated communications, both in free space and in optical fibres and also in the optical region of the spectrum and at longer wavelengths.

Orbital angular momentum has an important influence in imaging, where shaping the point spread function of the imaging system leads to new possibilities in image processing, both in astronomy, as pioneered by Swatzlander and co-workers and in microscopy, as pioneered by Ritsch-Marte and her colleagues. The new possibilities offered by these techniques range from the identification of an object against a bright background [40] to omni-edge enhancement [41], the removal of up-down ambiguity in interferometry [42] and probing the properties of an object by separating the orbital angular momentum components of the reflected or transmitted light [43].

V. RECENT DEVELOPMENTS AND THIS SPECIAL ISSUE

The period following 2005 has seen an explosion of interest in OAM: the topic appears regularly in major optics journals and conferences, and there is a flourishing international conference series, ICOAM, dedicated to the topic. It would be hopeless to attempt to represent the full scope of this now flourishing activity but some attempt to capture this may be found in some more recent reviews [44–46].

This special issue is intended to convey both some of the excitement in the field of OAM and also to illustrate the wide and ever growing range of topics in which it plays a role. The choice of topics and also of invited contributors presented the editors with a difficult problem: it would have been possible, easily, to double the number of topics covered and triple the number of leading and renowned contributors. Space, however, did not allow this and so having made the necessary selection, we can only *apologise* to those whom we would liked to invite to contribute but were unable to do so.

The papers comprising this special issue are from world-leading researchers in the field of OAM and have been selected to provide both specialists and non-specialists an overview of the key ideas current in the field. We open with two papers describing some of the theoretical background and issues in the description of OAM [47, 48]. The generation of beams with OAM directly from lasers is the topic of the next paper [49]. Light carrying OAM has found a number of practical applications and we have papers describing two of these: the forces and torques induced by them [50] and the use of OAM in microscopy [51].

The propagation of OAM beams has been an important topic since the foundation of the topic [1] and recent developments have centred on using such beams for optical communications and the transfer of information. These aspects are represented by papers dealing with the use of OAM beams for free-space communication [52] and propagation through photonic crystal fibres [53].

OAM is a property both of intense laser beams in the classical domain but also of single photons. A dramatic demonstration of this is provided through the entanglement of OAM [54]. The OAM of light is manifest in the interaction between light and matter and we include two papers covering this aspect of the field: one describing the interaction with atoms [55] and one addressing the interaction with chiral objects, particularly chiral molecules [56].

The study of orbital angular momentum has had an impact beyond the field of optics and an issue devoted to OAM would not be complete without papers looking beyond optics. We have two discussing the existence of electron vortices and their angular momentum properties [57, 58].

The last word must go to Les Allen and we publish here his final thoughts on the topic that he did so much to develop and advance [59].

VI. ENVOI

This article does not have a conclusions section as its role is principally to serve as an introduction to the papers that are to follow. In place of the usual conclusion, we present a short piece on Les Allen and his work, written and read by one of us (SMB) at a celebration of Les's life.

Les Allen

FIG. 1: Les Allen, physicist, artist, mentor and friend, relaxing at home with one of his paintings "Against the Grain". Picture credit: F. Cameron Wilson, Penrith [60]

It is given to few scientists, very few, to found their own field of research. Les was one of these; his work on the angular momentum of light places him in this select band. What is all the more remarkable is that this contribution came in what was effectively Les's third career and although an undoubted highlight, it was by no means an isolated one.

The beginning of Les's academic career coincided with the invention of the first lasers and he appreciated immediately that this new device would be transformative. He became one of the first laser physicists and his experiments with early lasers, especially gas lasers, did much to establish the foundations of the field. Eager to transfer the knowledge he gained, Les wrote, in the 1960s, two of the earliest textbooks on lasers [61, 62] and then, with Joe Eberly, his text *Optical resonance and two-level atoms* [63]. Although now more than 40 years old, this remains required reading for quantum opticians and did much to establish the foundations of the field of laser spectroscopy.

In the mid 1980s, following the publication of a fourth book, this time on quantum optics (written with Peter Knight) [64], Les moved into academic administration, taking the role of Deputy Rector at the (then named) Polytechnic of East London. He did not turn his back on scientific research during this time, however, and by the beginning of the 1990s he had had enough and wanted to get back to doing physics. In his own words [59]: *I had spent six years in academic administration and did not wish to continue doing it, so was letting it be known to as many people as I could that I would like to return to physics. I met Han Woerdman at a conference in Hyderabad in January 1991. We had never met before, nor yet previously worked on a similar project. He said to me: 'I hear you want to get back into research. How many years do you need; one, two or three?'*

This characteristic generosity from Han led directly to Les's third, and in many ways, most remarkable career. Les and Han had no plan or programme of work, but were happy simply to see where their collaboration might lead. The result was the foundation of the field of optical angular momentum, established by a short paper written by them, together with Marco Beijersbergen and Robert Spreeuw [1]. This paper, as Les himself has said, was far from perfect, but the important thing was that it contained an intriguing idea and, like a fine work of art, stimulated the reader by the beauty of its conception rather than the rigour of its details. This idea, that laser beams could be made to carry orbital angular momentum (a sense of rotation about the direction of propagation), has turned out to be correct, useful and profound in that it has induced a fundamental change in the way in which we understand light.

Les was keen to share what he had learnt about optical angular momentum and it was his enthusiasm, and his knack of asking simple-sounding questions that had profound asnswers, that recruited many of us to work in the field, including Miles Padgett, Mohamed Babiker and me. The three if us are preparing a special issue of Philosophical Transactions of the Royal Society to celebrate 25 years of research into optical angular momentum. This will include articles from world-leading authorities on the subject from four continents, as well as Les's own and final contribution [59]. The special issue will be a celebration of what has been achieved as well as a look forward to what is to come. We propose, also, to dedicate this to Les and to his memory.

I would like to finish with a quotation from Sir Isaac Newton, perhaps the finest scientist of them all, which seems apt. Near the end of his life he said [65]: *I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the seashore, and diverting myself now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of the unknown lay all undiscovered before me.*

Les brought to his science the very same sense of child-like wonder and fun. Each scientific pebble that he turned over was greeted with joy and enthusiasm. His contributions have left an indelible mark on physics, as he has on those of us who had the pleasure and the privilege of working with him.

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