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Throughput Enhancement of Restricted Access Window for Uniform Grouping Scheme in IEEE 802.11ah

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***Abstract*—IEEE 802.11ah has recently emerged as a promising standard for enabling massive machine-to-machine (M2M) communication. In order to support uplink data transmission from dense machine type clients (such as smart meters, IoT end nodes etc.), 802.11ah relies upon the restricted access window (RAW) based Medium Access Control (MAC) protocol. The underlying motivation behind this protocol is to reduce the contention for spectrum access among a large number of devices. The nodes contend with each other in their assigned RAW slot using Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA). In each RAW slot, the throughput depends upon the number of nodes. Current studies have suggested that the duration of each RAW slot should be the same in the entire RAW frame. However in this paper, we argue that the duration of each RAW slot should be chosen according to the size of the group. We present a model where a RAW frame is divided into two sub-frames and the duration of RAW slots in each sub-frame is chosen according to the size of the group. With the help of an analytical framework, we demonstrate that the throughput under our proposed scheme can be significantly enhanced when compared to a conventional implementation.**

I. Introduction

A. Motivation & Background

With the advent of the Internet of Things (IoT), there will be an exorbitant increase in the number of connected devices. According to recent predictions, around 100 billion connected devices will exist by the end of year 2020. With dense deployment of IoT and M2M nodes in large scale wireless networks, the existing contention based MAC protocols are expected to face performance degradation in terms of throughput. This can be attributed to both: (i) availability of finite spectrum; and (ii) inherent non-optimality of existing MAC protocols. To deal with the contention issue in such a large network of nodes, schemes such as increasing the size of the contention window have been shown to be impractical as they produce an extraordinary idle duration of time and thus reduce the overall efficiency [1]. One possible way to improve the throughput is by reducing the number of nodes contending for the channel access at any given time. This can be achieved by dividing the nodes into several groups and assigning each group to a different time interval. In this way, only the nodes within a group can contend with each other at a given time. This idea is adopted by IEEE 802.11ah Task Group in its latest draft [2] where a group of nodes is allowed to contend in a restricted interval called RAW slot. The collection of these RAW slots is called RAW frame and there can be one or more RAW frames within a Beacon Interval (BI) as shown in Fig. 1. In this way, IEEE 802.11ah can support up to 8,000 nodes by the use of prevalent Enhanced Distributed Channel Access (EDCA) protocols in each RAW slot. One similar approach to grouping is clustering, which exploits the space dimension of radio resource [3], [4], [5]. However cluster formation requires location information which may not be possible to obtain when the nodes are densely deployed.

Past studies have focused on the performance analysis of contention based channel access protocols. Bianchi presented a Discrete Time Markov Chain (DTMC) model to find the transmission probability of a single node and found the saturation throughput of IEEE 802.11 Distributed Coordination Function (DCF) [1]. After this, many studies have extended his model to consider various practical issues [6], [7]. The mean value analysis [8], [9] is another approach to find the probability of transmission and collision without using DTMC. All these approaches are for a network scenario when there is no grouping. The work presented in [10] and [11] provides an analytical model for finding the throughput when nodes are grouped together either uniformly or randomly but considers that the duration of each RAW slot is the same for each group. The throughput degrades significantly with an increase in the number of nodes contending in a single RAW slot [1].

B. Contribution

In a conventional model, the duration of each RAW slot is kept the same for groups of different sizes. However in order to enhance the throughput, we suggest that the duration of a RAW slot should be determined according to the size of group, i.e., the duration should be kept relatively smaller for a larger sized group and vice versa. In this paper, we consider the scenario of a uniform grouping scheme where the groups can have two different sizes and we propose a model where the entire RAW frame is divided into two sub-frames and each sub-frame consists of several RAW slots as shown in Fig. 2. The groups are organized on the basis of their sizes into sets and each set is assigned to a sub-frame. In our model, we choose the duration of a RAW slot in each sub-frame according to the size of groups, i.e., a larger sized group is assigned a relatively smaller RAW slot duration and vice versa. In this way, the

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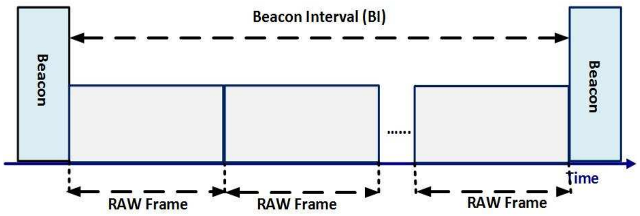

Fig. 1. Conventional structure of a Beacon Interval in IEEE 802.11ah.

TABLE I
LIST OF SYMBOLS

Description	Symbol
Node index	n_i
Group index	g_i
Size of a group	$\|g_i\|$
RAW Frame duration	$\mathcal{F}$
RAW slot duration	ϕ
Restricted Access Period	ℓ_r
Free Access Period (FAP)	ϕ'
Payload duration	ℓ_p
Total number of groups	K

overall throughput performance is shown to have improved in our proposed model when compared with the conventional model.

C. Organization

The organization of the rest of this paper is as follows: In Section II, we describe our proposed model and apply the throughput analysis to our model. Section III presents the performance analysis and discussion. At the end, conclusions and future work is discussed in Section IV.

II. Our Proposed Allocation Model

A. System Model

In this paper, we assume that there is a network with a total of N nodes indexed as $n_1,\ n_2,\ n_3, ..., n_N$. We assume all the nodes attempt to transmit their packets in the wireless medium towards the Access Point (AP). The nodes are considered to be in saturation mode, i.e., all nodes have data packets readily available in their buffers. We also assume that there are no hidden nodes in the network and the channel is in an ideal condition where there are no communication errors. These nodes are divided among K groups indexed as $g_1,\ g_2\ , ...g_K$. In a conventional model, each group g_k is assigned to a RAW slot k where $k \in \{1,\ 2,\ 3, ...\ K\}$ as shown in Fig. 3 [12]. The list of symbols which are frequently employed throughout this paper is given in table I.

B. Proposed Scheme

In our proposed model, we suppose that the nodes are distributed evenly among different groups according to the uniform grouping policy where a group g_k contains $\{n_k, n_{k+K}, n_{k+2K}, ...\}$ where $k \in \{1,\ 2, ...,\ K\}$ and K is the total number of groups.

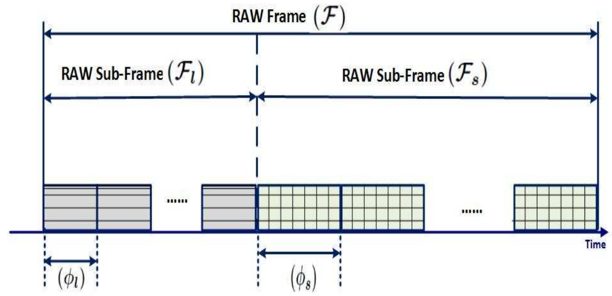

Fig. 2. Distribution of a RAW frame in the proposed model.

At a first step, we organize the groups on the basis of their size and make sets of equal-sized groups. So according to our proposed model, we have two sets of groups in a uniform grouping scheme. Let Ω_l be a set of groups where the size of each group is $\lfloor N/K \rfloor + 1$, and the cardinality of Ω_l is

$$|\Omega_l| = N\ mod\ \ K \tag{1}$$

where "$N\ \ mod\ \ K$" means "represent N modulo K". Similarly, let Ω_s denote a set of groups where each group has $\lfloor N/K \rfloor$ nodes and the cardinality of Ω_s is

$$|\Omega_s| = K - (N\ mod\ K) \tag{2}$$

where $K = |\Omega_l| + |\Omega_s|$.

Note that when N is an integer multiple of K, then $|\Omega_l| = 0$ and all the groups have equal number of nodes, and so $|\Omega_s| = K$.

At a second step, we divide a RAW frame $\mathcal{F}$ into two sub-frames denoted by $\mathcal{F}_l$ and $\mathcal{F}_s$. We then assign set Ω_l to the sub-frame $\mathcal{F}_l$ and set Ω_s to the sub-frame $\mathcal{F}_s$, where the sub-frame $\mathcal{F}_l$ contains $|\Omega_l|$ RAW slots and sub-frame $\mathcal{F}_s$ contains $|\Omega_s|$ RAW slots.

Let τ be the duration of one mini-slot and each RAW slot in sub-frame $\mathcal{F}_l$ consists of ϕ_l mini-slots. Similarly, each RAW slot in sub-frame $\mathcal{F}_s$ consists of ϕ_s mini-slots, and so the total number of mini-slots in the entire RAW frame is

$$\mathcal{F} = |\Omega_l|\, \phi_l + |\Omega_s|\, \phi_s. \tag{3}$$

Fig. 2 shows the distribution of a RAW frame according to our proposed model where values of ϕ_l and ϕ_s are chosen corresponding to $\lfloor N/K \rfloor + 1$ and $\lfloor N/K \rfloor$ nodes.

When a group is assigned to a RAW slot, the nodes in the group contend with each other according to the normal DCF procedure. According to this procedure, each node having a data packet for transmission monitors the channel until it is found idle for DCF Inter Frame Spacing (DIFS) mini-slots denoted by $\ell_d = (T_{DIFS}/\tau)$, where T_{DIFS} is the time duration of the DIFS. The node then enters a backoff stage and chooses a value randomly in the range of $(0,\ W_0 - 1)$ where W_0 is the size of intitial contention window. The node adopts a truncated binary exponential backoff scheme according to which the size of the contention window gets doubled on each successive collision until it reaches CW_{max} and it is then reset to W_0 when a packet is successfully transmitted. The backoff counter decrements by 1 after every mini-slot. When the value

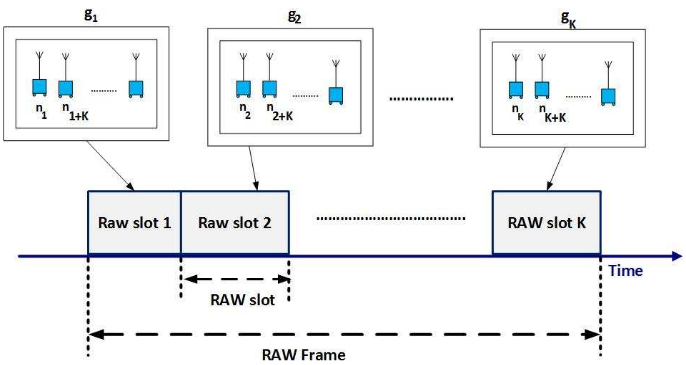

Fig. 3. Conventional slot assignment in the uniform grouping scheme.

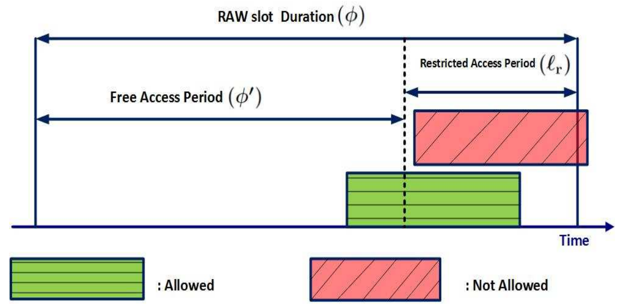

Fig. 4. RAW slot duration with a cross slot boundary.

of the counter reaches zero, the node starts its transmission. We assume that the size of a data packet is constant for each node. If T_{DATA} is the time duration for the transmission of payload data, then the number of mini-slots needed to transmit the payload is $\ell_p = (T_{DATA}/\tau)$. We also assume that any node in a group cannot cross its assigned RAW slot boundary during transmission of a data packet, i.e., nodes are restricted to start transmission during a Restricted Access Period which is given as $\ell_r = \ell_p + \ell_d$. If ϕ is the duration of one RAW slot, the nodes can start their transmission only during the Free Access Period (FAP) given as $\phi' = \phi - \ell_r$.

C. Throughput Analysis

In this section, we present the throughput analysis of an entire RAW frame for both the conventional approach as well as our proposed model.

Let $|g_k|$ be the number of nodes contending for the medium access in the k^{th} RAW slot. Assuming that each packet collides with a constant and independent probability p_c , the bidimensional process $\{s(t), b(t)\}$ can be modelled with a DTMC as shown in Fig. 5, where $s(t)$ denotes a stochastic process for the backoff stage at any time t and $b(t)$ represents the value of the backoff counter of a given node [1].

Then the probability of transmission of a single node (as given in Appendix A) is [1]

$$p_t = \frac{2(1-2p_c)}{(1-2p_c)(W_0+1) + p_c W_0 (1-(2p_c)^u)} \tag{4}$$

where u is the maximum number of retransmission attempts and W_0 is the size of the initial contention window. Here p_t depends upon the probability of a collision p_c. In each RAW slot where there are $|g_k|$ nodes and a node encounters a collision during transmission of its packet if at least one of remaining $|g_k| - 1$ nodes also start their transmission. Since each of the remaining stations also transmit with probability p_t, so we get

$$p_c = 1 - (1-p_t)^{|g_k|-1}. \tag{5}$$

Eq. (4) and Eq. (5) form a non-linear system having two unknowns which can be solved by the use of numerical methods. A node transmits successfully if exactly one node is transmitting out of $|g_k|$ nodes, and then using Eq. (4) and Eq. (5), the probability of success p_s is given as [1]

$$p_s(|g_k|) = \frac{|g_k|\, p_t(1-p_t)^{|g_k|-1}}{1-(1-p_t)^{|g_k|}} \ . \tag{6}$$

Now we find the saturation throughput, i.e., the throughput when all the nodes are in a saturation mode in a single RAW slot as in [11].

Let ψ_i be a random variable representing the number of backoff slots a node n_i waits before starting its transmission. Then we assume that it follows a geometric distribution, where

$$Pr\{\psi_i = j\} = p_t(1-p_t)^{j-1}, \quad (j \geq 1) \tag{7}$$

where j is the number of backoff slots.

Proposition 1. *Let $\triangle_{b,m}$ be a random variable representing the number of backoff slots between two consecutive transmissions and it is the minimum of value of backoff of all $|g_k|$ nodes, i.e., $\triangle_{b,m} = min(\psi_1, \psi_2, ..., \psi_{|g_k|})$, then*

$$Pr\{\triangle_b = j|\, |g_k|\} = p'_t(1-p'_t)^{j-1}, \quad (j \geq 1) \tag{8}$$

where $p'_t = 1-(1-p_t)^{j-1}$and j is the number of backoff slots between two consecutive transmissions.

Proof: see Appendix B. ∎

Proposition 2. *Let $\triangle_{b,m}$ denote the minimum value of backoff slots among $|g_k|$ nodes before the start of m^{th} transmission and let Y_m be a random variable representing the total number of backoff slots before the start of m transmissions. If $Y_m = \sum_{m'=1}^{m} \triangle_{b,m'}$ then the CDF of Y_m is*

$$F_{Y_m}(z) = \sum_{z'=m}^{z} \binom{z'-1}{m-1} p'_t(1-p'_t)^{z'-m}. \tag{9}$$

Proof: See Appendix C. ∎

Let M be a random variable representing the number of transmissions initiated during the FAP of duration ϕ' mini-slots. There will be no transmissions initiated within the FAP, i.e., $(M = 0)$ when $\triangle_{b,1} > \phi'$ and

$$Pr\{M=0\} = Pr\{Y_1 > \phi'\} = 1 - F_{Y_1}(\phi'). \tag{10}$$

However, there will be at least one transmission initiated within FAP when $\triangle_{b,1} < \phi'$. Let T_m denote the number of

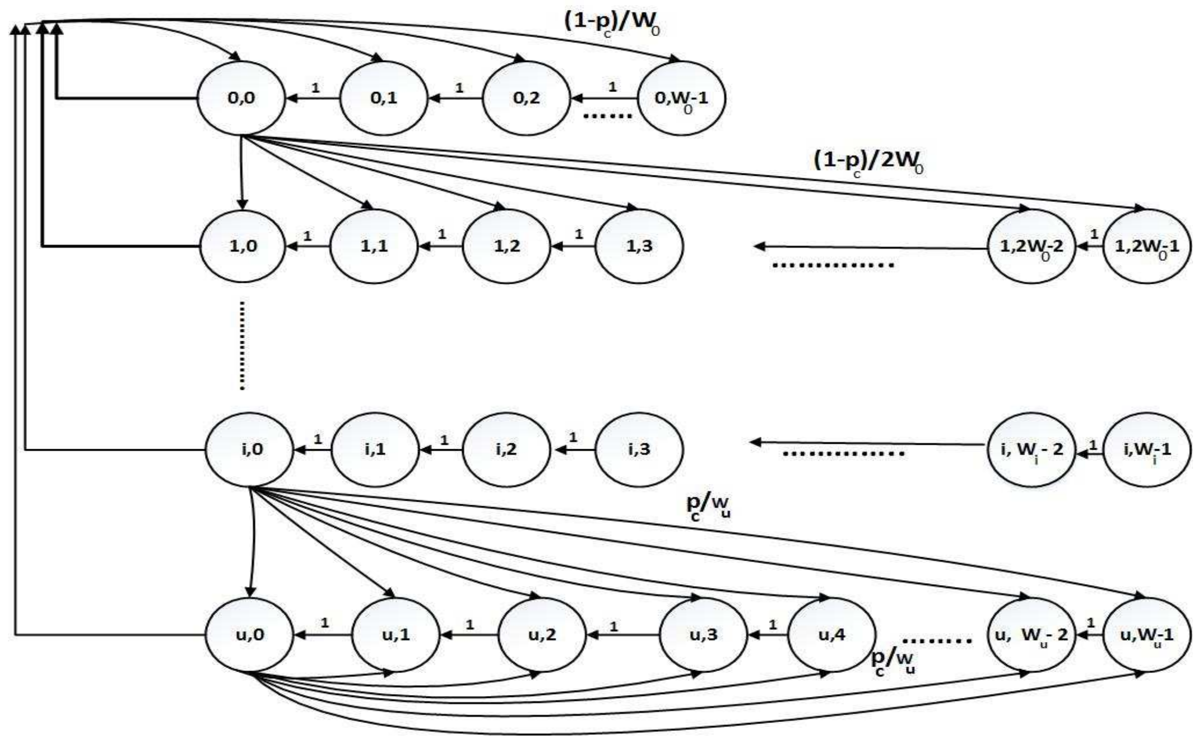

Fig. 5. Bianchi DTMC with one step transition probabilities [1].

mini-slots for transmission of both m data packets and the DIFS and is given as $T_m = m(\ell_p + \ell_d)$. There will be exactly m transmissions initiated within a FAP when $\sum_{m'=1}^{m} \triangle_{b,m} \leq (\phi' - T_{m-1} - \ell_d - 1)$ and $\sum_{m'=1}^{m+1} \triangle_{b,m} > (\phi' - T_m - \ell_d - 1)$. Then the probability that there will be exactly m transmissions initiated within the FAP ϕ' is

$$Pr\{M = m\} = F_{Y_m}(\phi' - T_{m-1} - \ell_d - 1) - F(\phi' - T_m - \ell_d - 1). \tag{11}$$

Let $M_U(\phi)$ denote the maximum number of transmissions that can be initiated within a RAW slot of duration ϕ mini-slots and thus can be written as

$$M_U(\phi) = \left\lfloor \frac{\phi'}{(1 + \ell'_p)} \right\rfloor + I_{\{\phi' > [\ell_d + 1 + \left\lfloor \frac{\phi'}{1+\ell'_p}(\ell'_p + 1) \right\rfloor]\}} \tag{12}$$

where $I_{\{x>0\}}$ is an indicator function which is equal to 1 when $x > 0$ is true and zero otherwise. The mean number of transmissions within one RAW slot of duration ϕ mini-slots where $|g_k|$ nodes are contending for the medium access is

$$E_M(\phi, |g_k|) = \sum_{m'=0}^{M_U(\phi')} m' \; Pr\{M = m'\}. \tag{13}$$

Then the saturation throughput of the k^{th} RAW slot is

$$\Gamma_k(\phi, |g_k|) = \frac{\ell_p}{\phi} \; p_s(|g_k|) \; E_M(\phi, |g_k|). \tag{14}$$

In a conventional model where all the K RAW slots are assumed to have an equal number of nodes and the duration of each RAW slot ϕ is kept the same, then the mean number of transmissions for the entire RAW frame is $KE_M(\phi, |g_k|)$. Therefore, the throughput of an entire RAW frame in a conventional model is

$$\Gamma_{\mathcal{F}c}(K\phi, |g_k|) = \frac{K\ell_p}{\phi} \; p_s(|g_k|) \; E_M(\phi, |g_k|). \tag{15}$$

According to our proposed model, the RAW frame is divided into two sub-frames $\mathcal{F}_l$ and $\mathcal{F}_s$. Let ϕ_l be the duration of each of the $|\Omega_l|$ RAW slots in sub-frame $\mathcal{F}_l$. Then the throughput in sub-frame $\mathcal{F}_l$ is

$$\Gamma_{\mathcal{F}_l} = \frac{|\Omega_l| \, \ell_p}{\phi_l} \; p_s(\lfloor N/K \rfloor + 1) \; E_M(\phi_l, \lfloor N/K \rfloor + 1). \tag{16}$$

Similarly, the throughput in sub-frame $\mathcal{F}_s$ consisting of the $|\Omega_s|$ RAW slots, each of duration ϕ_s, is

$$\Gamma_{\mathcal{F}_s} = \frac{|\Omega_s| \, \ell_p}{\phi_s} \; p_s(\lfloor N/K \rfloor) \; E_M(\phi_s, \lfloor N/K \rfloor). \tag{17}$$

Then, the net throughput in the entire RAW frame in our proposed model denoted by $\Gamma_{\mathcal{F}p}$ is

$$\Gamma_{\mathcal{F}p} = \Gamma_{\mathcal{F}_l} + \Gamma_{\mathcal{F}_s} \tag{18}$$

where the values of ϕ_s and ϕ_l are chosen by the AP according to the size of the group, i.e., the AP chooses $\phi_s > \phi_l$.

III. Performance Evaluation and Discussion

In this paper, simulations are performed according to the PHY and MAC layer parameters of IEEE 802.11 and its ammendment draft IEEE 802.11ah [2]. These parameters are listed in Table II. Due to the traffic model of an IoT network, we set the payload size to a small value, i.e., 512 bits. The data rate is set at 1 Mbps. Therefore, the duration of one packet

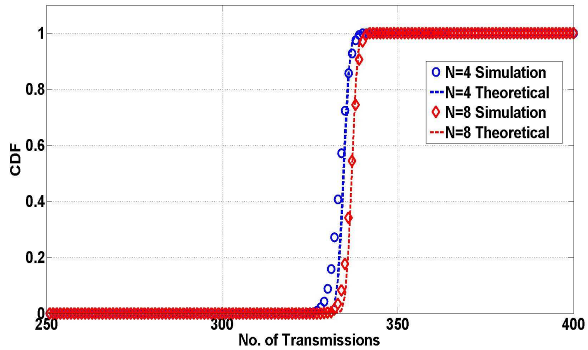

Fig. 6. CDF of number of transmissions within a RAW slot.

transmission including Short Inter Frame Spacing (SIFS) and the Acknowledgement (ACK) is kept at 1.1 ms.

In our simulations, we consider 64 groups and the size of each group is $|g_k| \in \{4, 8\}$ nodes and we set the RAW slot duration at 500 ms. To validate our model, we compare the commulative distribution function (CDF) of the number of transmissions in simulation with the analytical analysis in Section II. As Fig. 6 shows, the simulated and analytical results match well with each other.

TABLE II
LIST OF PARAMETERS USED IN SIMULATIONS

Parameter	Value
Mini-slot duration, τ	52 μsec
SIFS, T_{SIFS}	160 μsec
DIFS, T_{DIFS}	SIFS+2τ
MAC Header	272 bits
PHY Header	112 bits
ACK	112 bits
Initial contention window W_0	16
CW_{max}	1024

Fig. 7 shows the saturation throughput for different numbers of nodes according to the normal DCF CSMA/CA when nodes are divided into K groups with $K \in \{1, 2, 4\}$. It is evident that the saturation throughput degrades with an increase in the number of nodes per group. This is because contention among nodes causes collisions and failures which reduces throughput significantly. Therefore, the throughput can be improved by reducing the number of nodes per group.

Fig. 8 shows that the throughput fluctuates with the duration of each RAW slot as some of the mini-slots are wasted at the end of each RAW slot. This is because all the nodes refrain from starting to transmit in the restricted access period of the RAW slot. The figure also shows that the saturation throughput reaches its maximum possible value when the duration of RAW mini-slot is very large. This is owing to the fact that the restricted access period becomes negligible as compared to the FAP when the total RAW slot duration is very large. However, the duration of the RAW slot cannot be kept very large as it produces prolonged delays for nodes of the other groups. It can be observed that for a given number of nodes, the throughput becomes approximately equal to the maximum possible value at some smaller duration of the RAW slot. These are called the peak values of throughput as shown in Fig. 8 for each node and the AP can assign one of the peak values to a RAW slot.

Fig. 9 shows the throughput improvement using the proposed model as compared to the conventional one when a network of nodes is considered with 120 to 320 nodes. According to our proposed model, the AP chooses the peak values of a RAW slot according to the size of the group. The throughput is enhanced due to two reasons. First, the AP assigns a larger duration to better performing RAW slots where relatively lesser number of nodes contend for the medium. Second, the RAW slot interval is chosen from one of the peak values by the AP such that it minimizes the restricted access interval. Our simulation results prove that the throughput in our proposed model is enhanced when compared to the conventional method, as shown in Fig. 9.

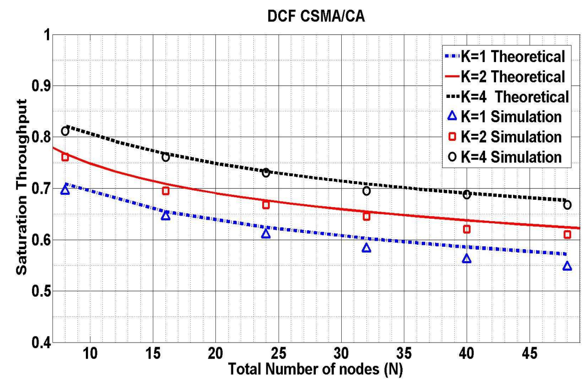

Fig. 7. Saturation throughput for basic CSMA/CA access with different number of groups.

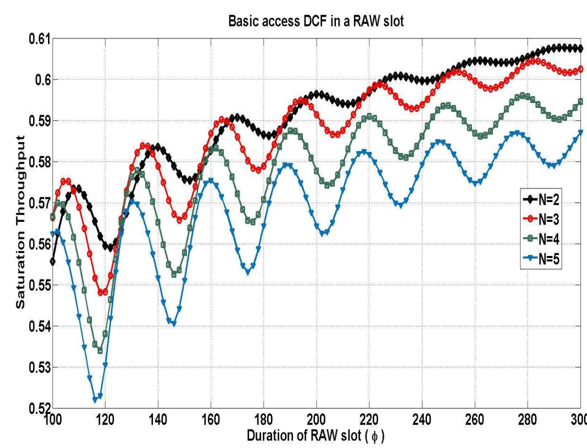

Fig. 8. Saturation throughput for different durations of a RAW slot.

IV. CONCLUSIONS AND FUTURE WORK

In this paper, we have proposed a novel approach to enhance the saturation throughput for a uniform grouping scheme in IEEE 802.11ah when transmission after the RAW slot boundary is not allowed. The analytical model is used to find the throughput of an entire RAW frame and the conventional RAW slot allocation scheme is compared with the two sub-frame

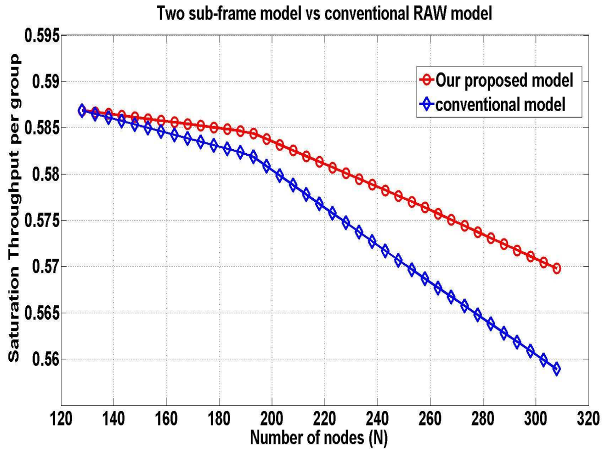

Fig. 9. Comparison of the proposed model with conventional model.

allocation model. The simulation results show that there is a significant improvement in throughput using our proposed two sub-frame allocation model as compared to the conventional one. This is because the duration of a RAW slot in the two sub-frame model is kept according to the size of group. The proposed model can also be used in a decentralized grouping scheme by splitting the RAW frame into multi sub-frames and allocating an optimal RAW slot duration of each sub-frame. The Throughput can be enhanced when better performing RAW slots are given more time duration as compared to others.

There are several issues for future research. A careful design of RAW slot duration for each group size is required to exploit the full advantage of the grouping schemes. While fulfilling the stringent latency requirements, it may not be practical to allocate a large value of RAW slot duration to each group. However, the trade-off between the size of the RAW slot duration and latency can be explored further to optimize all the performance metrics of a network. The joint optimization of duration of RAW slots, number of groups and size of each group, along with the trade-off between network throughput, latency and energy efficiency opens up various avenues for future research. The optimization issues become much more complicated when a heterogeneous payload, variable data rate and unsaturated traffic issues are considered.

Appendix A: Probability of Transmission for a Single Node

According to the Bianchi's DTMC model [1] as shown in Fig. 5, the backoff counter $b(t)$ decrements at the start of every mini-slot of duration τ until it reaches 0 and then

$$Pr\{s(t+\tau)=i; b(t+\tau)=k-1|s(t)=i; b(t)=k\}=1,$$
$$\forall\, k \in (1,\ W_0-1)\ , i \in (0,\ u).$$

The transmission begins when backoff counter reaches 0. When the transmission becomes successful, a new packet initilizes from the backoff stage 0, whose backoff counter is chosen uniformly in the range $(0,\ W_0-1)$ and so

$$Pr\{s(t+\tau)=0; b(t+\tau)=k|s(t)=i; b(t)=0\}=(1-p_c)/W_0,$$
$$\forall\, k \in (0,\ W_0-1),\ i \in (0,\ u).$$

At any backoff stage i, when the backoff counter reaches 0 and the transmission is unsuccessful, the backoff counter chooses a value uniformly in the range $(0\ , W_{i+1}-1)$ because backoff stage becomes $i+1$ and so

$$Pr\{s(t+\tau)=i+1; b(t+\tau)=k|s(t)=i; b(t)=0\}=p_c/W_{i+1},$$
$$\forall\, k \in (0,\ W_{i+1}-1)\ , i \in (0,\ u).$$

If the packet faces consecutive failures and the backoff stage reaches at its maximum value u, then it does not increase any further and so

$$Pr\{s(t+\tau)=u; b(t+\tau)=k|s(t)=u; b(t)=0\}=p_c/W_u,$$
$$\forall\, k \in (0,\ W_u-1).$$

Let $\pi_{i,k}=\lim_{t\to\infty} Pr\{s(t)=i,\ b(t)=k\},\ k \in (0,\ W_i-1)\ , i \in (0,\ u)$ be the stationary distribution of DTMC. Then, we get

$$\pi_{i,k}=\frac{W_i-k}{W_i}\pi_{i,0} \qquad k \in (0,\ W_i-1)\ , i \in (0,\ u).$$

By imposing the normalization conditions of DTMC and simplifying, $\pi_{0,0}$ is obtained as

$$\pi_{0,0}=\frac{2(1-2p_c)(1-p_c)}{(1-2p_c)(W_0+1)+p_cW_0(1-(2p_c)^u)}.$$

Since the transmission is possible only when the backoff counter is equal to zero, irrespective of the backoff stage, then the transmission probability p_t is

$$p_t=\sum_{i=0}^{m}\pi_{i,0}=\frac{\pi_{0,0}}{1-p_c}.$$

which is simplied as [1]

$$p_t=\frac{2(1-2p_c)}{(1-2p_c)(W_0+1)+p_cW_0(1-(2p_c)^u)}. \tag{19}$$

Appendix B: Derivation of Probability Mass Function of Number of Backoff Slots

Let $\psi_1, \psi_2, ..., \psi_{|g_k|}$ be independent and identically distributed (i.i.d.) random variables representing the number of backoff slots for each node, each following a geometric distribution with probability p_t and let $\triangle_b$ =min$(\psi_1, \psi_2, ..., \psi_{|g_k|})$. Then we have $\triangle_b \geq j$ only when $\{\psi_1 \geq j\ \&\psi_2 \geq j\&\ ....\&\ \psi_{|g_k|} \geq j\}$.

Since $Pr\{\psi_i \geq j\}=(1-p_t)^{j-1}$, it follows that

$$Pr\{\Delta_b \geq j\}=(1-p_t)^{|g_k|(j-1)}. \tag{20}$$

Therefore,

$$Pr\{\Delta_b=j\}=Pr\{\Delta_b \geq j\}-Pr\{\Delta_b \geq j+1\} \tag{21}$$

$$=(1-p_t)^{|g_k|(j-1)}-(1-p_t)^{|g_k|j} \tag{22}$$

$$\implies p_t'(1-p_t')^{j-1} \tag{23}$$

where $p_t'=1-(1-p_t)^{|g_k|}$and j is the minimum value of backoff among $|g_k|$ nodes.

Appendix C: Derivation of Cumulative Distribution Function of Total Number of Backoff Slots

Let Y_m be a random variable such that $Y_m = \sum_{m'=1}^{m} \triangle_{b,m'}$, where $\triangle_{b,1}, \triangle_{b,2}, ..., \triangle_{b,m}$ are independent and identically distributed geometric random variables with parameter p_t' such that

$$Pr\{\triangle_{b,i} = j\} = p_t'(1-p_t')^{j-1}_{.} \tag{24}$$

Then Y_m is a negative binomial random variable with parameters m and p_t' such that

$$Pr\{Y_m = k\} = \binom{z'-1}{m-1} p_t'(1-p_t')^{z'-m}_{.} \tag{25}$$

Since the minimum value of each random variable $\triangle_{b,1}, \triangle_{b,2}, ..., , \triangle_{b,m}$ is 1, then the minimum value of Y_m is m. Therefore, the CDF of Y_m is

$$F_{Y_m}(z) = \sum_{z'=m}^{z} \binom{z'-1}{m-1} p_t'(1-p_t')^{z'-m}_{.} \tag{26}$$

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