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CRANFIELD

THE DESIGN OF A MULTI-CELL BOX IN PURE BENDING FOR MINIMUM WEIGHT

by

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The Design of a Multi-Cell Box in Pure Bending for Minimum Weight

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SUMMARY

The optimum skin thickness, web thickness and web pitch to be used for a multi-cell box of given depth under a given bending load are obtained by two different methods, resulting in a graph where the optimum geometry is plotted against the structural index for a given material.

List of Contents

LIST OF SYMBOLS

b_s = width of skin panels

b_w = depth of box

E = Young's Modulus

E_s = Secant Modulus $= \frac{\sigma}{\epsilon}$

I = moment of inertia of each cell

K = a constant for determining the buckling stress σ_{cr} defined in Eq.3.

K_s = a constant used in Ref.2 for determining the buckling stress σ_{cr}, defined in Eq.3a.

m = bending moment per unit chordwise length applied on section

M = bending moment on each cell $= mb_s$

r_b = ratio of depth/width of each cell $= \frac{b_w}{b_s}$

r_t = ratio of web thickness/skin thickness $= \frac{t_w}{t_s}$

t_e = equivalent skin thickness as given by Eq.6.

t_s = skin thickness

t_w = web thickness

W = weight of each cell (per unit length, spanwise)
$= \rho(2b_s t_s + b_w t_w)$

w = $\frac{W}{b_s}$

ϵ = strain

μ = Poisson's ratio

ρ = density of the material

η = a plastic correction factor used in Ref.2. (See Eq.3a).

LIST OF SYMBOLS (cont)

σ = stress

σ_{cr} = buckling stress of skin panels

ϕ = $\dfrac{Kr_b^2}{(1+\frac{1}{6} r_b r_t)^2}$ (Eq. 8)

ψ = $\dfrac{1+\frac{1}{2} r_b r_t}{1+\frac{1}{6} r_b r_t}$ (Eq. 9)

THE DESIGN OF A MULTI-CELL BOX IN PURE BENDING FOR MINIMUM WEIGHT

1. Introduction

The aerodynamic demand for a low thickness/chord ratio wing in high speed flight, leads naturally to the thick-skin multi-cell box construction. The relative merits of this type of wing structure against the other types have been discussed in various papers such as Ref.1. This note is concerned only with the problem of finding the lightest section geometry to carry a given bending load. To this end, Ref.2 gives a method by which a certain skin thickness has to be first chosen, and then proceeds to find the web thickness and web pitch to be used to give the lightest combination. Several such calculations are necessary before the weight associated with each can be compared and the minimum found.

This note shows a process at the end of which the optimum geometry can be plotted against the loading. It is then only a matter of reading the curves to pick out the optimum geometry once the magnitude of the loadings is known.

The materials chosen to illustrate the method are light alloys. They will be acceptable if the speed of the aircraft or missile is not too high, so that the effect of kinetic heating is not appreciable. There is no reason why the same procedure should not be applied to other materials such as steel or titanium.

2. Assumptions

The assumptions made in the following analysis are:-

2A. The design criterion for the wing is one of pure bending. Other criteria such as torsional and shear stiffnesses are adequately covered.

2B. The section is idealised as rectangular with its depth b_w pre-fixed by aerodynamic consideration.

2C. Both the skins and the webs are fully effective in taking bending, and the stress is distributed according to engineers' theory. The effect of the angles that make the skin to web joint is neglected.

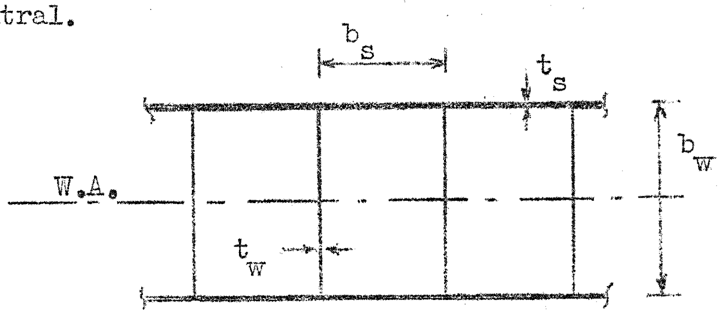
2D. The width of the box is sufficiently large in comparison with the depth for the panel buckling characteristics to be assumed to be the same as that of a box of infinite width - This assumption is made to facilitate the use of Fig.1 for finding the buckling stress.

2E. The same material is used for the skin and the web.

2F. The top and bottom skins are of the same thickness and therefore the nautral axis is central.

3. <u>Analysis</u>

3A. Stress in skin.



Let the bending moment applied on each cell be M, and the bending moment per unit length (chordwise) be m.

$$\therefore \quad M = mb_s \, .$$

Thenfollowing the assumptions 2C and 2F, the compression stress in the skin is given by

$$\sigma = \frac{M}{I}\left(\frac{b_w}{2}\right)$$

The moment of inertia of each cell

$$I = \frac{1}{12} b_w^3 t_w + 2b_s t_s \left(\frac{b_w}{2}\right)^2$$

$$= \frac{1}{2} b_w^2 b_s t_s \left(1 + \frac{1}{6} r_b r_t\right) \, . \qquad \ldots\ldots(1)$$

where $\quad r_b = \dfrac{b_w}{b_s}$,

and $\quad r_t = \dfrac{t_w}{t_s}$.

Hence $\quad \sigma = \dfrac{mb_s b_w}{2I} = \dfrac{m}{b_w t_s\left(1 + \frac{1}{6} r_b r_t\right)} \, . \qquad \ldots\ldots(2)$

3B Buckling Stress of Skin panels.

The buckling stress σ_{cr} for the skin panels is usually given as

$$\sigma_{cr} = KE_s \left(\frac{t_s}{b_s}\right)^2 , \qquad ....(3)$$

where E_s is the secant modulus $(= \frac{\sigma}{\epsilon})$ usually used in connection with this type of buckling (Ref.3), and K is a constant dependent on the panel geometry. The best source of information for the value of K is to be found in Ref.2, where the critical stress is given as

$$\sigma_{cr} = \frac{K_s \pi^2 \eta E}{12(1-\mu^2)} \left(\frac{t_s}{b_s}\right)^2 . \qquad ....(3a)$$

The value ηE in this formula corresponds approximately to the secant modulus E_s. Hence by comparing eq.3 with eq.3a,

$$K = \frac{\pi^2 K_s}{12(1-\mu^2)} . \qquad ....(4)$$

Fig.1 reproduces the curves that give the values of K_s from Ref.2. These values are meant for boxes of infinite width, but evidence given by Ref.4 shows that they are in very good agreement with test results for finite boxes with not less than three cells. Hence assumption 2D is justified.

3C. Weight of the box.

The weight (per unit length spanwise) of each cell is

$$W = \rho(2b_s t_s + b_w t_w) .$$

Dividing this by the width of the cell b_s,

gives $$w = \frac{W}{b_s} = 2\rho t_s(1+\tfrac{1}{2}r_b r_t). \qquad ....(5)$$

Letting $$t_e = t_s(1+\tfrac{1}{2}r_b r_t) , \qquad ....(6)$$

(t_e = equivalent skin thickness)

Then $$w = 2\rho t_e . \qquad ....(5a)$$

4. Optimisation and Preparation of the Curves

The object of this investigation is to obtain the skin thickness t_s, web thickness t_w, and web spacing b_s for a box of given depth b_w, to resist an applied loading m so that the weight w is a minimum.

It is recognised that failure will soon occur after the compression stress σ in the skin reaches the buckling stress σ_{cr} given in eq. 3. Therefore σ_{cr} is used as the maximum permissible value for σ.

4A. First Approach.

(a) Squaring both sides of eq. 2 and substituting into it the expression for t_s^2 from eq. 3

$$\sigma^2 = \frac{\frac{m^2}{b_w^2}}{t_s^2(1+\frac{1}{6}r_b r_t)^2} = \frac{\left(\frac{m}{b_w}\right)^2 \frac{KE_s}{\sigma_{cr} b_s^2}}{\left(1+\frac{1}{6}r_b r_t\right)^2} \quad . \qquad \ldots\ldots(6)$$

When σ reaches the design stress σ_{cr},

$$\frac{\sigma^3}{E_s} = \sigma^2 t = \left(\frac{m}{b_w^2}\right)^2 \frac{Kr_b^2}{(1+\frac{1}{6}r_b r_t)^2} = \left(\frac{m}{b_w^2}\right)^2 \phi, \qquad \ldots(7)$$

$$\text{where } \phi = \frac{Kr_b^2}{(1+\frac{1}{6}r_b r_t)^2} \quad . \qquad \ldots\ldots(8)$$

Multiplying eqs. 2 and 5 together,

$$w\sigma = \frac{2\rho m(1+\frac{1}{2}r_b r_t)}{b_w(1+\frac{1}{6}r_b r_t)} \quad ,$$

from which

$$\frac{w}{2\rho\, b_w} = \frac{1}{\sigma}\left(\frac{m}{b_w^2}\right)\left(\frac{1+\frac{1}{2}\,r_b r_t}{1+\frac{1}{6}\,r_b r_t}\right) = \left(\frac{m}{b_w^2}\right)\frac{\psi}{\sigma} \quad , \qquad \ldots(9)$$

where $\psi = \dfrac{1+\frac{1}{2} r_b r_t}{1+\frac{1}{6} r_b r_t}$ . ...(10)

It can be seen that both ϕ and ψ are functions of r_b and r_t alone. For a minimum weight w, the parameter $\frac{w}{2\rho b_w}$ is a minimum and by eq.9 $\frac{\psi}{\sigma}$ should also be a minimum.

(b) Given a value of $\frac{m}{b_w^2}$, which can be called the structural index of this problem, we can obtain a value of $\sigma^2\epsilon$ from eq.7 corresponding to any given pair of r_b and r_t. The compression stress-strain curve of the material used (e.g. Fig.2) can be modified to give a curve of $\sigma^2\epsilon$ against σ (Fig.3) from which σ is obtained. The ratio $\frac{\psi}{\sigma}$ is calculated and then a graph using $\frac{\psi}{\sigma}$ as ordinate and r_b (or r_t) as abscissa with curves drawn for constant r_t (or r_b), can be plotted. A typical graph is shown in Fig.4. The minimum point of the envelope of all these curves gives the optimum ratio r_b and r_t for this particular value of structural index $\frac{m}{b_w^2}$.

Knowing now the optimum r_b and r_t, and the corresponding from eq.(7) and Fig.3, we can either obtain from eq.2

$$\frac{t_s}{b_w} = \frac{\frac{m}{b_w^2}}{\sigma(1+\frac{1}{6} r_b r_t)}, \qquad .....(2a)$$

or the ratio $\frac{t_s}{b_s}$ from eq.3. In any case the dimensions t_s, t_w, and b_s giving the minimum weight for the given m and b_w are now completely determined.

(c) Repeating the above procedure for a range of values of $\frac{m}{b_w^2}$ curves giving the optimum ratios of r_b, r_t and $\frac{t_s}{b_w}$ against $\frac{m}{b_w^2}$ can be prepared for any given material. (Fig.5). It is then only necessary to use these curves to find the optimum geometry when designing a box section of a given b_w under a given loading m.

(d) Appendix 1 shows a typical tabulation procedure employing the above method. The calculation is for a value $\frac{m}{b_w^2} = 1000$ lb/in^2 and the material used being DTD 687. The result of the entire calculation covering the whole range of $\frac{m}{b_w^2}$ is plotted in Fig.5.

4B. Second Approach.

This approach is a modification and extension of that used by Schuette and McCulloch in Ref.2.

a) Since w is directly proportional to t_e (see eq.5a), we can for optimum design make the parameter $\frac{m}{b_w t_e}$ a maximum instead of w a minimum. The significance of the parameter $\frac{m}{b_w t_e}$ is that it represents the stress on the skins of equivalent thickness t_e at a depth b_w apart.

Substituting t_e from eq.6, we have

$$\frac{m}{b_w t_e} = \frac{\left(\frac{m}{b_w t_s}\right)}{1+\frac{1}{2} r_b r_t} \quad . \qquad \text{.......(11)}$$

Compare this with the actual stress given in eq.2.

$$\sigma = \frac{\left(\frac{m}{b_w t_s}\right)}{1+\frac{1}{6} r_b r_t} \quad . \qquad \text{.......(2)}$$

We need also the ratio $\frac{t_s}{b_w}$, which can be written as

$$\frac{t_s}{b_w} = \frac{t_s}{b_s} \cdot \frac{b_s}{b_w} = \frac{1}{r_b} \cdot \frac{t_s}{b_s} \quad .$$

From eq.3 after equating σ_{cr} to σ,

$$\frac{t_s}{b_s} = \sqrt{\frac{\sigma}{K\epsilon_s}} = \sqrt{\frac{\epsilon}{K}} \quad .$$

Hence

$$\frac{t_s}{b_w} = \frac{1}{r_b} \sqrt{\frac{\epsilon}{K}} \quad . \qquad \text{.......(12)}$$

(b) Fix a value of $\frac{m}{b_w t_s}$. Calculate first of all the stress σ from eq.2 for any given pair of r_b and r_t. Then obtain the corresponding strain ϵ from the stress-strain curve of the material used (e.g. Fig.2), and hence $\frac{t_s}{b_w}$ from eq.12. Calculate also $\frac{m}{b_w t_e}$ from eq.11.

Varying r_b and r_t, $\frac{m}{b_w t_e}$ can be plotted against the corresponding values of $\frac{t_s}{b_w}$, and curves for constant r_t (and/or r_b) are drawn, (typical curves shown in Fig.6). The envelope of all the curves represents the maximum $\frac{m}{b_w t_s}$, and the point of contact of each r_t (or r_b) curve with the envelope, gives the corresponding value of $\frac{t_s}{b_w}$ to that r_t (or r_b).

Applying the same procedure for a range of values of $\frac{m}{b_w t_s}$, we can obtain two sets of curves: one giving maximum $\frac{m}{b_w t_e}$ against $\frac{t_s}{b_w}$ (Fig.7), the other plotting r_t (or r_b) against $\frac{t_s}{b_w}$ (Fig.8), both obtained from the envelope as shown in Fig.6.

From Fig.7, there is a value of $\frac{m}{b_w t_e}$ corresponding to any given pair of values of $\frac{t_s}{b_w}$ and $\frac{m}{b_w t_s}$, which when multiplied together give a value of $\frac{m}{b_w^2}$. Therefore Fig.7 can be modified into a set of curves giving $\frac{m}{b_w t_e}$ (which corresponds to minimum weight) obtainable for any structural index $\frac{m}{b_w^2}$, and the particular curve that is tangent to the envelope at this point gives the optimum ratio of $\frac{t_s}{b_w}$.

Fig.8 can similarly be modified into curves giving r_t (or r_b) as functions of $\frac{m}{b_w^2}$ and $\frac{t_s}{b_w}$ (Fig.10). Furthermore since the optimum $\frac{t_s}{b_w}$ for each $\frac{m}{b_w^2}$ is given from the envelope of Fig.9, the ratio r_t (or r_b) associated with the optimum can be obtained from Fig.8. The other ratio r_b (or r_t), if it has not been obtained from the plotting of graphs as described, can be calculated from eq.6. which gives

$$\frac{t_e}{t_s} = 1 + \frac{1}{2} r_b r_t , \qquad \ldots\ldots\ldots(6a)$$

where

$$\frac{t_e}{t_s} = \frac{\frac{m}{b_w}2}{\left(\frac{m}{b_w t_e}\right)\left(\frac{t_s}{b_w}\right)} .$$

The two ratios in the denominator are given of course by the envelope of Fig.9. This provides a means of checking should both r_b and r_t be obtained graphically.

The curves identical to Fig.5 giving the optimum ratios of $\frac{t_s}{b_w}$, r_t and r_b can now be prepared.

(c) Appendix 2 gives and illustration of the procedure described above. The material used there is DTD 546. The value of $\frac{m}{b_w t_s}$ used in Tables 8 A-D is 30,000 lbs/in^2 and the resulting plot of $\frac{m}{b_w t_e}$ against $\frac{t_s}{b_w}$ is shown in Fig.6. Similar plots can be made for other values of $\frac{m}{b_w t_s}$ which are not shown in detail. The optimum goemetry resulting from the calculation is shown in Fig.11 which can be seen to be similar to Fig.5 obtained in Appendix 1 for the material DTD 687.

5. Discussion

Each of the two approaches has its own advantages. If the object is to prepare a curve like Fig.5 which gives the optimum geometry for any structural index, then the first approach is more direct and the amount of work involved is comparatively less. Very often, when only one value of structural index is of interest to a particular design, then only one graph such as that shown in Fig.4 needs to be prepared for that particular value of $\frac{m}{b_w^2}$. This graph has the further advantage of showing rapidly, should the optimum r_b be impractical [this is often the case with a box of finite width because the number of cells (= width/b_s) must be an integer] and another value chosen, the correct value of r_t associated and the precentage weight increase as a result.

On the other hand, if the second approach is used and Fig.9 prepared, then the penalty in choosing a skin thickness t_s other than that of the optimum (this may be due to the sheet gauge limitation) is readily comparable from the values of $\frac{m}{b_w t_e}$ and Fig.8 or 10 will give readily the best ratio of r_t (and hence r_b) to be associated with such non-optimum skin thickness.

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7. Acknowledgement.

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APPENDIX I.

The following calculation is an illustration of the method described in section 4A. The material used for the calculation is DTD 687, the stress-strain curve of which is shown in Fig.2. and the $\sigma^2\epsilon$ against curve in Fig.3.

Table 1 below gives the values of K, calculated from

$$K = \frac{\pi^2 K_s}{12(1-\mu^2)}, \qquad \ldots\ldots\ldots(4)$$

with the Poisson's Ratio $\mu = 0.3$ and the values of K_s from Fig.1.

Tables 2 and 3 tabulate the values of ϕ and ψ respectively from the equations:

$$\phi = \frac{K r_b{}^2}{(1+\frac{1}{6} r_b r_t)^2}, \qquad \ldots\ldots\ldots(8)$$

and

$$\psi = \frac{1+\frac{1}{2} r_b r_t}{1+\frac{1}{6} r_b r_t}. \qquad \ldots\ldots\ldots(10)$$

For any given structural index $\frac{m}{b_w^2}$, the value of $\sigma^2\epsilon$ is given by

$$\sigma^2\epsilon = \left(\frac{m}{b_w{}^2}\right)^2 \phi, \qquad \ldots\ldots\ldots(7)$$

and Table 4 shows the values for a particular $\frac{m}{b_w^2} = 1000$ lb/in^2. It is in fact simply Table 2 multiplied by a constant $\left(\frac{m}{b_w^2}\right)^2 = 10^6$

The values of σ corresponding to those $\sigma^2\epsilon$ in Table 4 are obtained from Fig.3 and tabulated in Table 5. After which, the values of ψ in Table 3 can be divided by those of σ in Table 5 to obtain Table 6, which is plotted in Fig.4 with r_b as absissa and cross-plotted in Fig.4a with r_t as absissa (to facilitate easy reading of the ratios r_b and r_t corresponding to the minimum $\frac{\psi}{\sigma}$).

The same procedure is then applied to all values of $\frac{m}{b_w}$ and a graph for $\frac{\psi}{\sigma}$ is plotted for every one. The optimum r_b and r_t are found from the minimum point of the envelope of the curves in each graph, and the corresponding $\frac{t_s}{b_w}$ calculated in Table 7. These optimum ratios are plotted against $\frac{m}{b_w}$ in Fig.5.

TABLE 1.

Values of K.

r_b \ r_t	.25	.40	.50	.60	.80	1.00
0.5	3.62	3.71	3.80	3.89	4.18	4.45
1.0	2.35	3.62	3.70	3.78	4.07	4.37
1.5	0.80	2.67	3.32	3.62	3.93	4.29
2.0	-	1.45	2.25	2.96	3.71	4.18
2.5	-	-	1.23	2.00	2.95	3.75
3.0	-	-	-	-	2.09	3.04

TABLE 2.

Values of ϕ

r_b \ r_t	.25	.40	.50	.60	.80	1.00
0.5	0.87	0.87	0.88	0.88	0.92	0.95
1.0	2.17	3.18	3.15	3.12	3.17	3.21
1.5	1.59	4.96	5.90	6.16	6.14	6.18
2.0	-	4.52	6.61	8.22	9.25	9.41
2.5	-	-	5.27	8.00	10.37	11.68
3.0	-	-	-	-	9.60	12.16

TABLE 3

Values of ψ

r_b \ r_t	.25	.40	.50	.60	.80	1.00
0.5	1.041	1.065	1.080	1.095	1.125	1.153
1.0	1.080	1.125	1.152	1.182	1.235	1.286
1.5	1.118	1.182	1.222	1.261	1.333	1.400
2.0	-	1.235	1.286	1.333	1.421	1.500
2.5	-	-	1.345	1.400	1.500	1.588
3.0	-	-	-	-	1.571	1.667

TABLE 4

Values of $\sigma^2 \epsilon$ (x 10^{-6})

$\left[\text{for } \frac{m}{b_w^2} = 1000 \text{ lb/in}^2\right]$

r_b \ r_t	.25	.40	.50	.60	.80	1.00
0.5	0.87	0.87	0.88	0.88	0.92	0.95
1.0	2.17	3.18	3.15	3.12	3.17	3.21
1.5	1.59	4.96	5.90	6.16	6.14	6.18
2.0	-	4.52	6.61	8.22	9.25	9.41
2.5	-	-	5.27	8.00	10.37	11.68
3.0	-	-	-	-	9.60	12.16

TABLE 5

Values of σ

(for $\frac{m}{b_w^2}$ = 1000 lb/in^2)

r_b \ r_t	.25	.40	.50	.60	.80	1.00
0.5	20,500	20,500	20,600	20,600	20,900	21,200
1.0	27,900	31,700	31,600	31,500	31,600	31,800
1.5	25,100	36,700	38,900	39,500	39,450	39,500
2.0	-	35,600	40,400	43,500	45,200	45,500
2.5	-	-	37,500	43,100	47,000	48,900
3.0	-	-	-	-	45,800	49,500

TABLE 6

Values of $\frac{\psi}{\sigma}$ (x 10^{-3})

[for $\frac{m}{b_w^2}$ = 1000 lb/in^2]

r_b \ r_t	.25	.40	.50	.60	.80	1.00
0.5	.0507	.0520	.0524	.0531	.0538	.0545
1.0	.0387	.0355	.0364	.0375	.0391	.0404
1.5	.0445	.0322	.0314	.0320	.0338	.0354
2.0	-	.0347	.0318	.0306	.0314	.0330
2.5	-	-	.0359	.0325	.0319	.0325
3.0	-	-	-	-	.0343	.0337

TABLE 7

Calculation of $\frac{t_s}{b_w}$ corresponding to the optimum ratios of r_b and r_t.

$\frac{m}{b_w^2}$	Optimum ratios r_b	r_t	K	ϕ	ψ	$\sigma^2\epsilon$ $(x10^{-6})$	σ	$\frac{t_s}{b_w}$
200	1.92	.63	3.25	8.30	1.335	0.33	14,900	.0112
400	1.89	.61	3.22	8.10	1.322	1.30	23,500	.0143
660	1.86	.60	3.18	7.83	1.314	2.82	30,400	.0167
800	1.83	.58	3.15	7.62	1.301	4.88	36,500	.0186
1000	1.81	.57	3.12	7.45	1.293	7.45	42,100	.0204
1200	1.78	.56	3.10	7.22	1.285	10.40	47,000	.0219
1400	1.76	.55	3.08	7.09	1.278	13.90	50,900	.0237
1600	1.73	.53	3.05	6.85	1.263	17.56	55,900	.0248
1800	1.70	.51	3.00	6.60	1.253	21.40	58,900	.0267
2000	1.67	.50	2.96	6.34	1.249	25.40	61,000	.0288

APPENDIX 2.

To illustrate the second approach to the problem of optimisation as discussed in Section 4B, we here choose the material DTD 546 as an example.

Table 8 shows the detail calculations for a chosen value of $\frac{m}{b_w t_s}$ = 30,000 lb/in^2. First of all, the stress σ is calculated from

$$\sigma = \frac{\left(\frac{m}{b_w t_s}\right)}{1+\frac{1}{6} r_b r_t} , \qquad \text{......(2)}$$

for all combinations of r_b and r_t and is tabulated in Table 8A. Then the corresponding strain is read from the stress-strain curve in Fig. 2 and tabulated (Table 8B). Next $\frac{\epsilon}{K}$ can be computed (with the value of K from Table 1, Appendix 1) and hence Table 8C which gives

$$\frac{t_s}{b_w} = \frac{1}{r_b} \frac{\epsilon}{K} . \qquad \text{..........(12)}$$

Calculate for all combinations of r_b and r_t

$$\frac{m}{b_w t_e} = \frac{\left(\frac{m}{b_w t_s}\right)}{1+\frac{1}{2} r_b r_t} , \qquad \text{......(11)}$$

which is tabulated in Table 8D.

Now the values of $\frac{m}{b_w t_e}$ from Table 8D can be plotted against the corresponding values of $\frac{t_s}{b_w}$ from Table 8C, and curves of constant r_t are drawn (Fig. 6). An envelope can be drawn over the curves.

The same process is repeated for a range of values of $\frac{m}{b_w t_s}$. The resulting envelopes are shown in Fig. 7, from which readings of $\frac{m}{b_w t_e}$ at regular intervals of $\frac{t_s}{b_w}$ are tabulated in Table 9.

The envelope in Fig.6 represents the highest $\frac{m}{b_w t_e}$ (i.e. lightest weight) attainable for any $\frac{t_s}{b_w}$, and the r_t curve that is in contact with the envelope at this point gives the value of the corresponding r_t.

From Fig.6 read $\frac{t_s}{b_w}$ at the point of contact of each r_t curve with the envelope. The results for a range of $\frac{m}{b_w t_s}$ are tabulated in the manner of Table 10 and plotted with r_t as ordinate and $\frac{t_s}{b_w}$ as absissa in Fig.8.

From Fig.8, r_t can be read at regular intervals of $\frac{t_s}{b_w}$ and re-tabulated into Table 11.

Since $\frac{t_s}{b_w} \times \frac{m}{b_w t_s} = \frac{m}{b_w^2}$, the values of $\frac{t_s}{b_w}$ and $\frac{m}{b_w t_s}$ in Tables 9 and 11 can be multiplied together to obtain Table 12.

Now the values of $\frac{m}{b_w t_e}$ from Table 9, can be plotted against the corresponding values of $\frac{m}{b_w^2}$ from Table 12 for constant values of $\frac{t_s}{b_w}$. (Fig.9, A and B).

Similarly, corresponding values of r_t and $\frac{m}{b_w^2}$ from Tables 11 and 12 can be plotted for constant values of $\frac{t_s}{b_w}$ (Fig.10).

An envelope can be drawn to the curves of Fig.9, which represents the maximum efficiency attainable for a given structural index $\frac{m}{b_w^2}$. The particular $\frac{t_s}{b_w}$ curve that is in contact with the envelope at this point gives the value of $\frac{t_s}{b_w}$ associated with the optimum condition. The optimum $\frac{t_s}{b_w}$ thus obtained is plotted against $\frac{m}{b_w^2}$ in Fig.11.

Having obtained the optimum $\frac{t_s}{b_w}$ for a given $\frac{m}{b_w^2}$, the associated r_t can be found from Fig.10 and $\frac{m}{b_w t_e}$ from the envelope of Fig.9.

Then since

$$\frac{\left(\frac{m}{b_w^2}\right)}{\left(\frac{m}{b_w t_e}\right)\left(\frac{t_s}{b_w}\right)} = \frac{t_e}{t_s} \quad ;$$

and from eq.(6a)

$$\frac{t_e}{t_s} = 1 + \frac{1}{2} r_b r_t \; ,$$

then

$$r_b = \frac{2}{r_t}\left(\frac{t_e}{t_s} - 1\right) \; ,$$

and is calculated in Table 13.

The optimum ratios r_b and r_t are also plotted in Fig.11.

TABLE 8

Calculation of $\frac{m}{b_w t_e}$ and $\frac{t_s}{b_w}$ for $\frac{m}{b_w t_e} = 30{,}000$ lb/in^2.

(Material DTD 546)

(8A) Values of σ

r_b \ r_t	.25	.40	.50	.60	.80	1.00
0.5	29,400	29,050	28,800	28,600	28,150	27,700
1.0	28,800	28,150	27,700	27,300	26,460	25,700
1.5	28,200	27,300	26,700	26,100	25,000	24,000
2.0	-	26,460	25,700	25,000	23,700	22,500
2.5	-	-	24,830	24,000	22,500	21,200
3.0	-	-	-	-	21,440	20,000

(8B) Values of ϵ

r_b \ r_t	.25	.40	.50	.60	.80	1.00
0.5	.00307	.00303	.00300	.00298	.00294	.00289
1.0	.00300	.00294	.00289	.00285	.00276	.00268
1.5	.00294	.00285	.00279	.00272	.00261	.00250
2.0	-	.00276	.00268	.00261	.00248	.00235
2.5	-	-	.00259	.00250	.00235	.00221
3.0	-	-	-	-	.00224	.00209

(8C) Values of $\frac{t_s}{b_w}$

r_b \ r_t	.25	.40	.50	.60	.80	1.00
0.5	.0581	.0571	.0561	.0552	.0529	.0507
1.0	.0357	.0285	.0279	.0275	.0260	.0247
1.5	.0347	.0218	.0191	.0183	.0171	.0161
2.0	-	.0218	.0175	.0148	.0129	.0118
2.5	-	-	.0178	.0142	.0114	.0097
3.0	-	-	-	-	.0107	.0087

(8D) Values of $\frac{m}{b_w t_e}$

r_b \ r_t	.25	.40	.50	.60	.80	1.00
0.5	28,200	27,300	26,700	26,100	25,000	24,000
1.0	26,700	25,000	24,000	23,100	21,400	20,000
1.5	25,200	23,100	21,800	20,700	18,700	17,100
2.0	-	21,400	20,000	18,700	16,600	15,000
2.5	-	-	18,400	17,100	15,000	13,300
3.0	-	-	-	-	13,600	12,000

TABLE 9

Values of Maximum $\frac{m}{b_w t_e}$

$\frac{t_s}{b_w}$ \ $\frac{m}{b_w t_s}$	15,000	20,000	25,000	30,000	35,000	40,000	45,000	50,000	55,000
.006	6,000	6,800	7,800	-	9,300	-	10,800	-	11,600
.008	7,600	8,800	10,100	11,300	12,400	13,200	14,000	14,700	15,400
.010	8,900	10,600	12,200	13,900	15,200	16,300	17,400	18,200	19,200
.012	10,000	12,200	14,100	16,000	17,600	19,000	20,400	21,300	22,700
.014	11,000	13,600	15,800	18,000	19,800	21,500	23,000	24,400	25,900
.015	11,400	14,200	16,600	18,900	20,900	22,600	24,400	26,000	27,500
.016	11,700	14,700	17,300	19,700	21,800	23,700	25,600	27,200	28,900
.018	12,200	15,600	18,300	21,200	23,500	25,700	28,000	29,700	31,700
.020	12,600	16,200	19,400	22,300	25,000	27,400	30,000	32,000	34,200
.022	13,000	16,700	20,200	23,200	26,200	28,900	31,700	34,000	36,400
.024	13,300	17,100	20,800	24,000	27,300	30,100	33,200	35,600	38,300
.025	13,400	17,300	21,000	24,300	27,800	30,700	33,800	36,400	39,200
.030	14,000	18,100	22,000	25,700	29,500	32,800	36,400	39,400	42,600
.035	14,200	18,600	22,600	26,600	30,600	34,400	38,000	41,400	45,000
.040	14,500	18,900	23,100	27,200	31,400	35,400	39,300	42,900	46,700
.045	14,600	19,200	23,400	27,700	32,000	36,100	40,200	44,100	48,000
.050	14,700	19,300	23,700	28,100	23,400	36,700	40,900	45,000	49,111
.055	-	19,400	23,900	28,400	32,800	37,200	41,400	45,800	50,000
.060	-	-	24,000	28,700	33,000	37,600	41,900	46,400	50,800

TABLE 10

Values of $\frac{t_s}{b_w}$ corresponding with r_t

r_t \ $\frac{m}{b_w t_s}$	15,000	20,000	25,000	30,000	35,000	40,000	45,000	50,000	55,000
.25	.0270	.0320	.0350	.0380	.0410	.0450	.0475	.0490	.0520
.40	.0160	.0175	.0210	.0225	.0240	.0260	.0280	.0305	.0310
.50	.0135	.0155	.0175	.0190	.0200	.0220	.0240	.0250	.0260
.60	.0110	.0122	.0140	.0150	.0165	.0170	.0190	.0195	.0205
.80	.0085	.0100	.0110	.0120	.0130	.0140	.0150	.0160	.0165
1.00	.0065	.0080	.0085	.0095	.0100	.0110	.0110	.0120	.0130

TABLE 11

Retabulation of Table 10 from Fig. 8
Values of r_t corresponding with t_s/b_w

$\frac{t_s}{b_w}$ \ $\frac{m}{b_w t_s}$	15,000	20,000	25,000	30,000	35,000	40,000	45,000	50,000	55,000
.008	.830	.970	-	-	-	-	-	-	-
.010	.675	.780	.880	.950	-	-	-	-	-
.012	.555	.640	.725	.800	.860	.920	.970	-	-
.014	.465	.540	.610	.680	.730	.790	.840	.885	.930
.016	.400	.470	.530	.585	.635	.690	.740	.785	.820
.018	.360	.410	.465	.515	.555	.605	.660	.700	.755
.020	.320	.375	.420	.460	.495	.540	.580	.625	.660
.025	.260	.305	.340	.370	.390	.420	.455	.490	.520
.030	.230	.260	.290	.310	.330	.350	.375	.405	.430
.035	-	-	.250	.270	.290	.305	.325	.350	.370
.040	-	-	-	.235	.255	.270	.285	.310	.330
.045	-	-	-	-	-	.240	.255	.275	.290

TABLE 12

Values of $\frac{m}{b_w^2}$.

$\frac{m}{b_w t_s}$ / $\frac{t_s}{b_w}$	15,000	20,000	25,000	30,000	35,000	40,000	45,000	50,000	55,000
.006	90	120	150	180	210	240	270	300	300
.008	120	160	200	240	280	320	360	400	440
.010	150	200	250	300	350	400	450	500	550
.012	180	240	300	360	420	480	540	600	660
.014	210	280	350	420	490	560	630	700	770
.015	225	300	375	450	525	600	675	750	825
.016	240	320	400	480	560	640	720	800	880
.018	270	360	450	540	630	720	810	900	990
.020	300	400	500	600	700	800	900	1000	1100
.022	330	440	550	660	770	880	990	1100	1210
.024	360	480	600	720	840	960	1080	1200	1320
.025	375	500	625	750	875	1000	1125	1250	1375
.030	450	600	750	900	1050	1200	1350	1500	1650
.035	525	700	875	1050	1225	1400	1575	1750	1925
.040	600	800	1000	1200	1400	1600	1800	2000	2200
.045	675	900	1125	1350	1575	1800	2025	2250	2475
.050	750	1000	1250	1500	1750	2000	2250	2500	2750
.055	825	1100	1375	1650	1925	2200	2475	2750	3025
.060	900	1200	1500	1800	2100	2400	2700	3000	3300

TABLE 13

Calculation of the optimum ratio r_b

$\frac{m}{b_w}$	$\frac{t_s}{b_w}$	r_t	$\frac{m}{b_w t_e}$	$\frac{t_e}{t_s}$	r_b
200	.0110	.68	10,900	1.665	1.96
400	.0140	.66	17,500	1.631	1.91
600	.0163	.65	22,900	1.609	1.87
800	.0183	.63	27,800	1.572	1.82
1000	.0201	.62	32,100	1.550	1.77
1200	.0218	.60	36,200	1.520	1.73
1400	.0233	.59	40,300	1.490	1.66
1600	.0250	.57	43,800	1.460	1.62
1800	.0265	.56	47,200	1.439	1.57
2000	.0280	.55	50,100	1.421	1.53

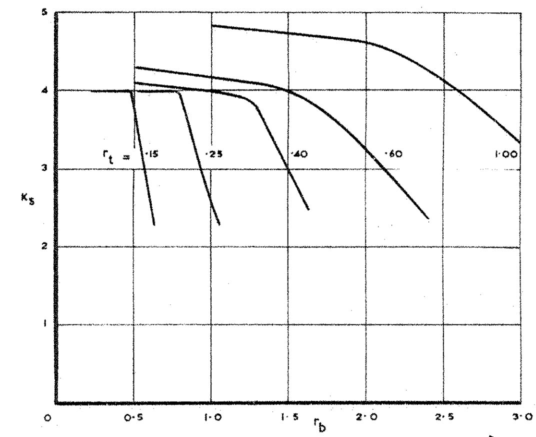

FIG. 1. VALUES OF K_s (FROM FIG. 2. N.A.C.A. TN 1323)

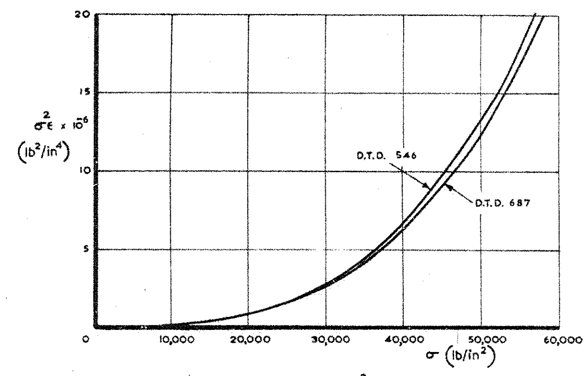

FIG. 3. CURVE OF $\sigma^2 \epsilon$ vs. σ

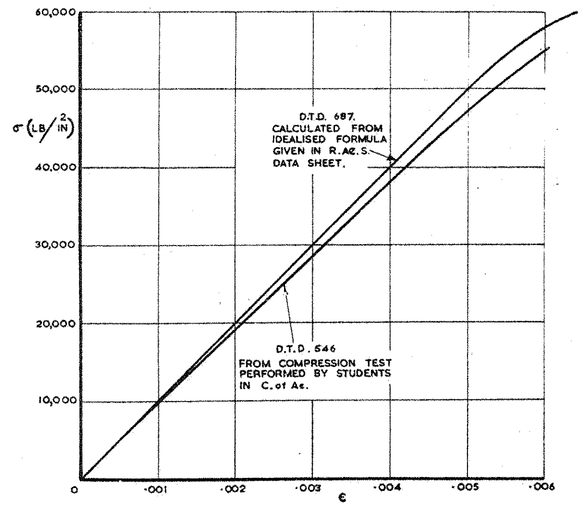

FIG. 2. STRESS-STRAIN CURVES.

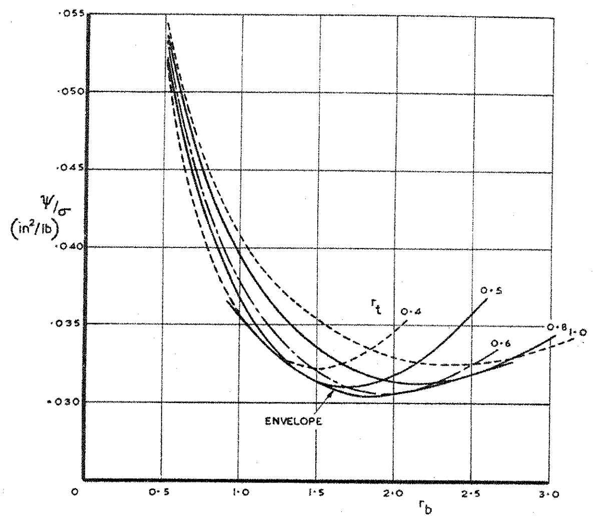

FIG. 4. VALUES OF Ψ/σ, $m/b_w^2 = 1000$ lb/in²
(ENVELOPE OF CURVES GIVES OPTIMUM $r_b = 1 \cdot 81$.)

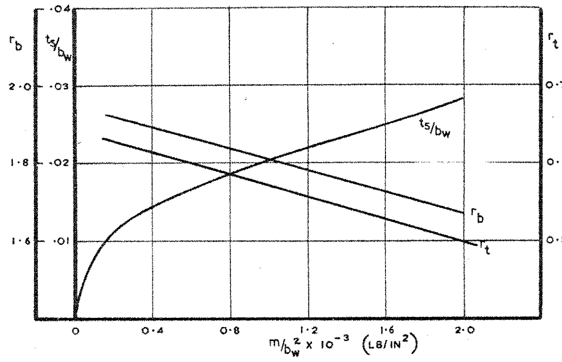

FIG. 5. OPTIMUM GEOMETRY FOR D.T.D. 687.

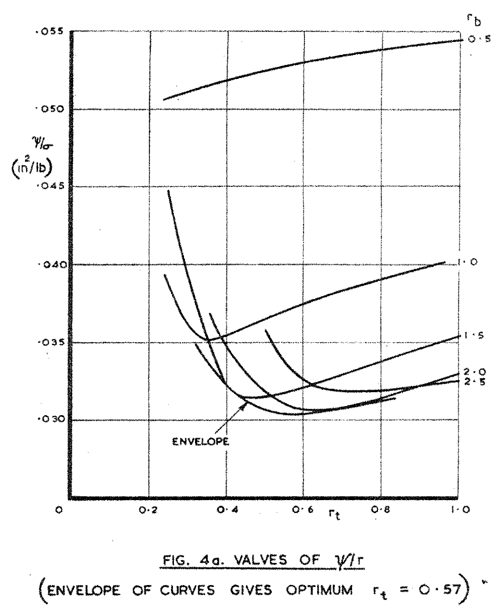

FIG. 4a. VALVES OF Ψ/r
(ENVELOPE OF CURVES GIVES OPTIMUM $r_t = 0 \cdot 57$)

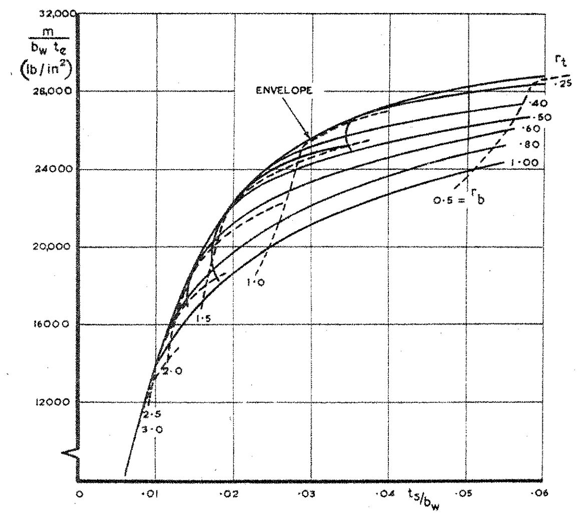

FIG. 6. $\frac{m}{b_w t_s} = 30{,}000$ lb/in² MATERIAL D.T.D. 546

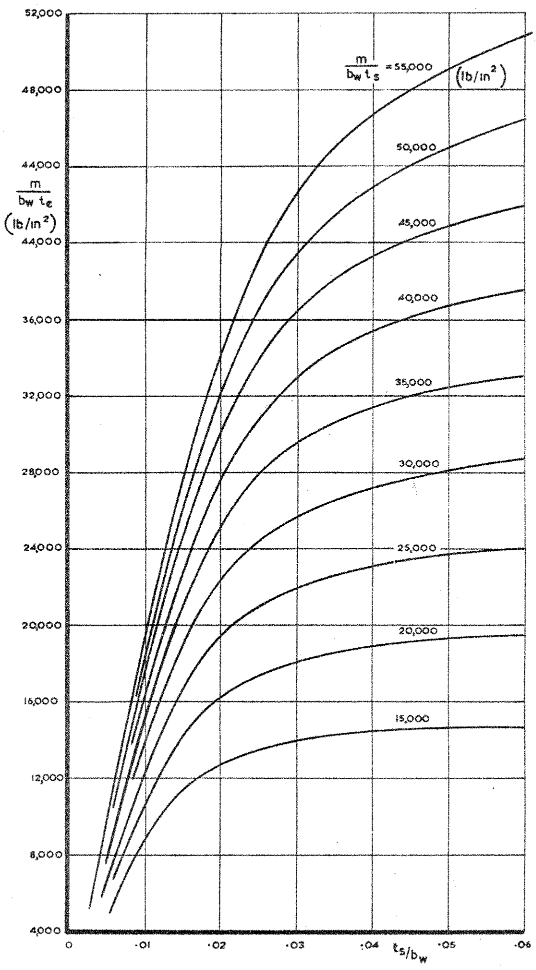

FIG. 7.

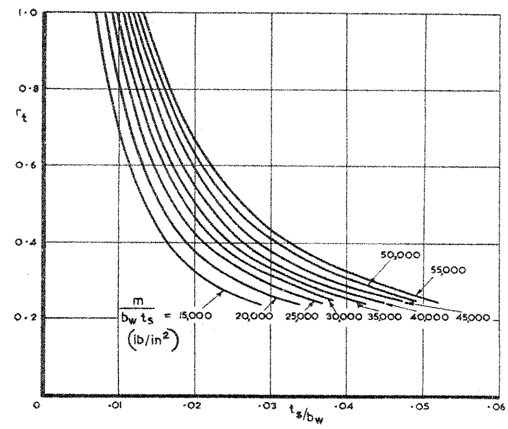

FIG. 8. VARIATION OF OPTIMUM r_t.

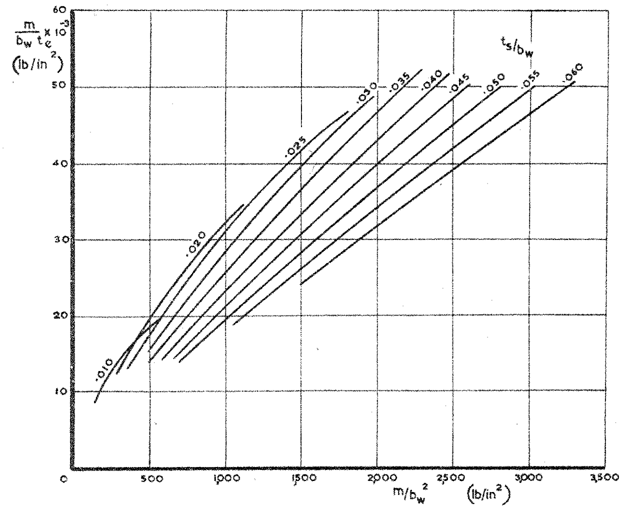

FIG. 9a. MINIMUM WEIGHT CURVES FOR LARGE VALVES OF $\frac{m}{b_w^2}$

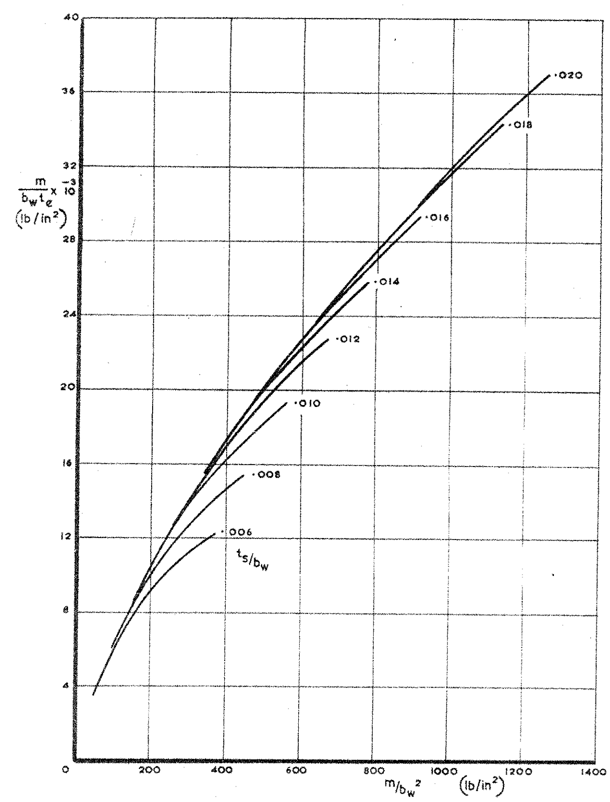

FIG. 9b. MINIMUM WEIGHT CURVES FOR SMALL VALUES OF $\frac{m}{b_w^2}$

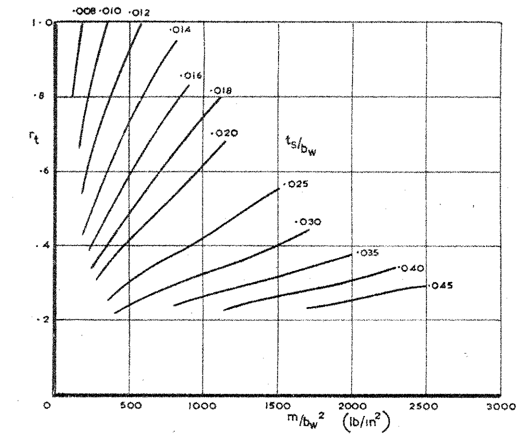

FIG. 10. OPTIMUM r_t FOR D.T.D. 546

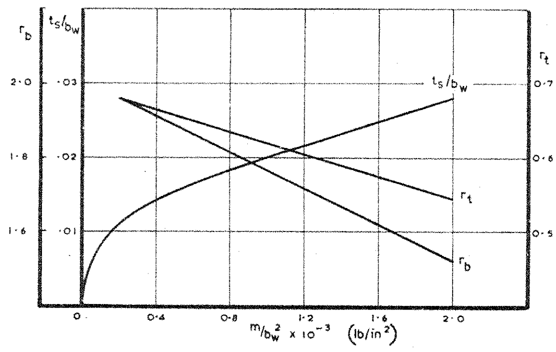

FIG. 11. OPTIMUM GEOMETRY FOR D.T.D. 546.