Investigation and Modelling of Tremor Reducer with the Technology of Targeted Energy Transfer

Ding Zhao

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Thesis Summary

This thesis presents the study of a two-degree-of-freedom (2 DOF) nonlinear system consisting of two grounded linear oscillators coupled to two separate light weight nonlinear energy sinks of an essentially nonlinear stiffness. In this thesis, Targeted Energy Transfer (TET) and NES concept are introduced. Previous studies and research of Energy pumping and NES are presented. The characters in nonlinear energy pumping have been introduced at the start of the thesis. For the aim to design the application of a tremor reduction assessment device, the knowledge of tremor reduction has also been mentioned.

Two main parties have been presented in the research: dynamical theoretic method of nonlinear energy pumping study and experiments of nonlinear vibration reduction model. In this thesis, nonlinear energy sink (NES) has been studied and used as a core attachment for the research. A new theoretic method of nonlinear vibration reduction which with two NESs has been attached to a primary system has been designed and tested with the technology of targeted energy transfer. Series connection and parallel connection structure systems have been designed to run the tests. Genetic algorithm has been used and presented in the thesis for searching the fit components. One more experiment has been tested with the final components. The results have been compared to find out most efficiency structure and components for the theoretic model.

A tremor reduction experiment has been designed and presented in the thesis. The experiment is for designing an application for reducing human body tremor. By using the theoretic method earlier, the experiment has been designed and tested with a tremor reduction model. The experiment includes several tests, one single NES attached system and two NESs attached systems with different structures. The results of theoretic models and experiment models have been compared. The discussion has been made in the end. At the end of the thesis, some further work has been considered to designing the device of the tremor reduction.

Acknowledgements

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Chapter 1

Introduction

1.1 Tremor Reduction

Tremor is an unintentional movement of the muscle in the human body. The characteristics of pathological tremor (PT) are rhythmic shaking of body parts such as arms, hands, vocal cord, legs and head.[1] PT is caused by disorders in parts of the brain that controls the muscles and can be caused by Parkinson's disease (PD), essential tremor (ET), dystonia, stroke, multiple sclerosis, etc. PT affects over a million people in the UK, and it is estimated that 1 in 6 people suffer from tremor to a certain extent. Tremor may occur at any age but is most common in middle-aged and older persons. [2] People with tremor find it really uncomfortable to perform daily tasks as their body and mind are not performing simultaneously. Tremor is not life threatening. [3] The general characteristics of tremor are rhythmic shaking of body parts such as arms, hands, vocal cord, legs and head. Tremor mostly occurs in middle aged and elderly people in both genders equally. [4] Tremor is a common movement disorder that affects everyone in different levels, for some it is an insignificant nuisance and for others it is a major disorder that prevents them to perform regular task.

There are many medical solution and tremor assisting devices but they are not effective on all individual and are very expensive. [4] There is a big gap in the market for a tremor reduction device that will assist people to perform daily duties.

The aim of this project is to find a practical solution to reduce tremor mechanically allowing the patients to perform their daily duties by using the theory of nonlinear energy transfer. Initially research has to be conducted to find the cause of tremor and the solution that are available in the current situation.

The main objective is to design a feasible mechanical device to reduce tremor in those who are affected by it. The aesthetic and ergonomic factors should be considered while designing the product. The device should encourage the patient to socialise normally and should enable them to perform daily duties without any hustle.

1.2 Targeted Energy Transfer

Targeted energy transfer (TET) is becoming more and more interested in many different study areas. Targeted energy transfers (TETs, or Non-linear energy pumping) refers to some energy form is directed from a source (primary subsystem) to a receiver (non-linear attachment) in a one-way irreversible transfer. It is achieved by the fact of resonance and the energy transferred away from the nonlinear attachment along the intrinsic periodic solution branches. [5]

Many studies have been made of Targeted Energy Transfer (TET). The nonlinear vibration energy is absorbed from a linear oscillator (LO) to a nonlinear energy receiver [6]. In a targeted energy transfer with an NES, the essentially nonlinear stiffness of the NES enables the NES to resonate with any of the linearized modes of the linear oscillator (primary subsystem). Latter, the vibrational energy would be transferred through resonant modal and dissipated by the linear damper inside the NES.

Gendelman first observed the realization of TET in 2001 (shown as Figure 1.1, [7]). In his study, a primary system receives an input energy from outside, and the vibration energy is transferred from the primary system to the nonlinear attachment. In his result, the imparted energy to the primary system gets absorbed by the nonlinear attachment and then dissipated. The nonlinear attachment is named nonlinear energy sink (NES).

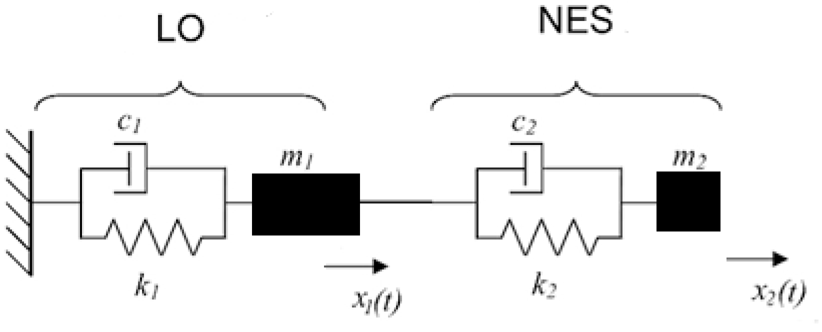

Figure 1. 1 Two-degree-of freedom nonlinear system consisting of a grounded linear oscillator coupled to a light mass by means of an essentially nonlinear stiffness.

Figure 1.1 Two-degree-of freedom nonlinear system consisting of a grounded linear oscillator coupled to a light mass by means of an essentially nonlinear stiffness.

There are different kinds of nonlinear attachment other than NES. A linear dynamic absorber (also called Tuned Mass Damper (TMD)) has been designed by Frahm[8] and Den Hartog[9] with the linear theories development. A TMD looks similar to an NES in configuration; however, they are totally different in nature. The device of TMD is simple and efficient, but is only effective when it is precisely tuned to the frequency of a vibration mode. Different from a TMD, an NES has no preferential resonant frequency, it may engage in resonance capture with any mode of the primary system. Thus, the NES is meaningful in both cases of SDOF structure and multi-degree-of-freedom (MDOF) primary structures [10]. As a basic mechanism for non-linear TET, many early studies have been made for nonlinear attachment [11–17]. Applications of NES to mechanical oscillators can also be found in references [19-24].In recently engineering problems, NESs have been applied to dissipate unwanted energy from a primary system.

1.3 Genetic Algorithm

In Chapter 4, Genetic algorithm has been presented in the research. It is a heuristic method of searching the optimal solutions by mimics the evolution of nature selection and survival. [25]The method started with a potential solution set of a certain problem. The population is joined by several gene encoded individual components. Each individual is actually a chromosome entity with features. The chromosome is the main carrier of genetic material, which is a plurality of sets of genes. Its internal representation (ie, genotype) is a gene combination. [26] It determines the external representation of individual shape. Thus, coding from phenotype to genotype mapping is needed to achieve. Since the gene encoding modelled work is complex, a simplified as binary coding is required. [27] After the first generation populations produced, in accordance with the principle of survival of the fittest, each generation evolution produces better approximate solution. In every generation, the individuals have been selected (selection) according to the fitness function of the individual domain (fitness) size. [28] By means of nature genetic operators combined crossover and mutation to produce a population representative of a new solution set. This process will lead the new generation population is better than the previous generation population as the nature evolution, which is more adapted to the environment. The optimal solutions have been picked up from the last generation population. [29]

1.4 Thesis Organisation

Targeted energy transfer has been further studied in this thesis. More than one nonlinear energy absorber has been applied on primary system to research the different TET inside the system. This thesis comprises five chapters, including two sections of original researches together with a chapter of literature review and a chapter of conclusions. The thesis has been organized in the following fashion:

- **Chapter 2 Literature Review**
  In Chapter 2, the published works related to the research of this thesis are reviewed. It includes: 1. Study of targeted energy transfer, which introduced the primary study of the principle of this research. 2. Nonlinear energy pumping. It is the core technique for investigation of targeted energy transfer. 3. Numerical methods of dynamics of NES system. Several related researched methods to the thesis have been reviewed. The numerical method of this research is based on these published works. 4. Tremor reduction device history. An application of tremor reduction has been designed and tested in Chapter 5. A review of different kinds of tremor reduction application has taken for future reference in the design process.

- **Chapter 3 Nonlinear Dynamical Investigation**
  This chapter presents a study of nonlinear dynamics of nonlinear energy pumping. From the reviews in Chapter 2, the characters of nonlinear energy

transfer have been used into the study. A system of two primary linear oscillators connected each other and both attached by a single NES has been studied. Frequency-Energy Plot (FEP) and impulsive orbits (IOs) have been introduced and presented in this study.

- **Chapter 4 Theoretical Nonlinear Energy Transfer**

  A theoretical nonlinear energy transfer method has developed in this chapter. The method is based on the knowledge of targeted energy transfer between the primary system and its attachment(s). In this chapter, the theoretical model of vibration amplitude reduction has been researched. Different from the methods in the literature, the model has been developed with more than one nonlinear energy attachments with different formations. Genetic algorithm has been used in this chapter for selecting the best components parameters. The theoretical methods have been tested by simulations with Matlab program. The results of the tests have been discussed. In the end of chapter, a conclusion has been made.

- **Chapter 5 Application Design and Experiment Work**

  In Chapter 5, a base-excitation experimental system is developed refers to the theory in Chapter 3. Nonlinear vibration reduction has been introduced at the start of the chapter. A potential application has been designed for the theory. The application has been tested with different settings. In the following section of this chapter, the experiment has been presented from the set up to the results. The experimental results have been simulated and discussed by comparing with the results from the theoretical method in Chapter 3.

- **Chapter 6 Conclusions and Future Work**

  The conclusions have been presented as well as the limitations that have been drawn or found during the works in this research. The limitation in the research has been discussed in this chapter. Further study and research has been suggested.

Aims and objectives

The aim of the project is to design a tremor reducer by using the technology of nonlinear energy transfer. The main Objectives of the research involved in this thesis may be summarised as follows:

1. Build mathematical nonlinear energy transfer models for numerical testing. The testing results to be used as reference data for the design of tremor reducer.

2. The modelling and simulation of the tremor reducer model. The numerical results are compared with the mathematical model results.

3. Apply the design to build up a tremor reducer and testing the tremor reduction with the design.

Chapter 2

Literature Review

2.1 Introduction of Nonlinear Energy Pumping

This thesis presented the research refers of two main parts: the dynamical investigation of nonlinear energy transfer, the study of the nonlinear vibration reduction. Also a wide range of topics cover the presented work, including the nonlinear energy pumping, applications of nonlinear energy sink and tremor assessment history. The related literatures related in these areas of research are reviewed in this chapter.

Firstly, the study of targeted energy transfer has taken place. It introduces the primary study of the principle of this presented research. Some studies of the nonlinear vibration energy is absorbed from a linear oscillator to a nonlinear energy recovered has been reviewed. In 2009, Gendelman presented a system of a nonlinear energy receiver attached to a linear oscillator in the book of Nonlinear Targeted Energy Transfer in Mechanical and Structural Systems. The targeted energy transfer inside this system has been reviewed. Tuned Mass Damper (TMD) has been studied in the end of this section. TMD is a linear energy absorber which is simple and efficiency.

Nonlinear energy pumping has been investigated. Nonlinear energy sink is designed to transform unwanted energy from a source to another. In this section, to understand the passive TET between the Primary system and the nonlinear attachment, different kinds of NES configurations are presented from previous studies. The study of inducing passive nonlinear NESs of Vakakls (2001) [22] introduced a complexification-averaging technique to acquire modulation equations for the slow-flow dynamics. The study shows that, For a MDOF chain impulsively loaded structure system with an NES attached to the primary subsystem, after some initial transients, a fast frequency identical motion dominated the response of the NES to the lower bound of propagation zone of the linear chain. By comparing with two-DOF chain system problem, the TET has been reduced. The reason is that, after some initial transients, in a in phase mode at lower frequency boundary conditions of the infinite linear chain, the semi-infinite chain in essence vibrates.

Several related researched methods to the thesis have been reviewed. The numerical method of this research is based on these published works. In this section, numerical study of the periodic orbits of the undamped system has been continued on the system of two linear oscillators and two NESs. The linear oscillators act as the primary systems, coupled to ungrounded attachments (NESs) through a pure cubic stiffness. In Lee et al. (2005) [23]'s study, possibility of passively and irreversibly transferring a significant part of the imparted energy to the nonlinear oscillator has been examined.

A review of different kinds of tremor assessment has taken for future designing. There are few organisations such as universities and medical research foundations that have produced few devices to reduce tremor, but due to the aesthetical and ergonomic issues these products were not well received by the patients. Some similar products could be found in the market these days, such as cochlear implant and microphone hearing aids. Different types of hearing aid products have been chosen for different levels of hearing lose. The following text will introduce the difference of the hearing aids, includes their advantages and disadvantages. A potential application of tremor reduction has been designed and tested in chapter4 based on the literature of this section.

2.2 Nonlinear Energy Attachments

In 2001 Vakakis and Gendelman studied, the primary system was attached by a ground NES using an essential nonlinearity ([24] the configuration is shown as Figure 2.1). Then the irreversible energy channelling from the primary structure to the attached NES was defined as TET [24] [25]. It is shown that, the damped vibrational system can only exhibit nonlinear beat phenomenon, but the conserved energy keep transforming between the primary system and NES with no TET occurred [24].

In many recent studies, nonlinear TET is further investigated to understand energy pumping in a two-Degree of Freedom (2 DOF) nonlinear coupled system [26]. At a fixed energy level, action-angle formulation is applied as a reduction method to achieve a single second-order ordinary differential equation. The periodic solutions are computed by the non-smooth temporal transformations (NSTTs [27]) of the action-angel formulation. According to the result of Vakais Rand's (2004) study of a dumped system, a 1:1 subharmoic orbits (only excited at suffidently high energies) of the underlying Hamiltonian system is mainly responsible for the TET phenomenon. Thus, a transient bridging orbit satisfying zero initial conditions must be impulsively excited [26].

Some studies have been taken of the degenerate bifurcation structure a system of coupled oscillators with an NES [28], the results of the studies observed 'degenerate' and ‘non-degenerate’ two types of bifurcations of periodic solutions. Former is at high energy, and the non-degenerate bifurcation is near the exact 1:1 internal resonances (IR). It is shown once the linear coupling stiffness approaches zero, the degeneracy occurs [28]. Gendelman et al. (2003) [25] explored that hitch reflects the high degeneracy of the underlying nonlinear Hamiltonian system composed of the undamped LO coupled to an undamped acttachment with pure cubic stiffness nonlinearity. The resonant dynamics of dumped system under condition of 1:1 IR has been discussed by Vakais Rand (2004). The existence of NNMs and elliptic orbits periodic motions has been explored. Over more, the results of the same work show that bifurcations may accompany the time evolution of the invariant manifold. Destruction of the resonance regime may be caused by passage of the invariant manifold through bifurcations, and essential gain in the energy dissipation rate. A loss of NES ability to dissipate the vibrational energy may happen if the damping coefficient is not chosen to ensure the possibility of bifurcation of the NNM invariant manifold.

Both impulsive periodic orbits and quasi-periodic orbits are analysed by apart considering low-, moderate-, and high-energy impulsive motions [29]. Interpretative approximations of impulsive periodic orbits, which are separated by corresponding uncountable infinities of quasi-periodic impulsive orbits (IOs), are performed. It is shown that the impulsive dynamics of the system is very complex due to its high degeneracy as it undergoes a codimension-3 bifurcation (indeed, the equations of motion for the ungrounded NES configuration can be transformed to those for a grounded NES configuration as in reference [26). Applications of active control to compensate for dissipation effects are addressed, keeping the localized motion preserved in the system as energy decreases [28].

Single- and multi-mode TET phenomena are investigated in a two-DOF primary structure linearly coupled to a grounded NES [30]. Single-mode energy pumping is caused by isolated resonance captures in neighbourhoods of only one of the linear modes of the primary structure and is dominated by the corresponding linearized

Eigen frequencies. Different from single-mode energy pumping, resonance capture cascades leading to multi-mode energy pumping that involve several linear modes, and pumping dynamics are divided into different frequency levels with each level being leaded by a different fast frequency next to an Eigen frequency of the linear system. By following the damped transitions close to branches of the underlying Hamiltonian system as energy reduces to damping dissipation, such resonance capture cascades can be certainly depicted in Frequency-Energy Plots (FEPs) [14]. Study of isolated resonance captures leading to single- or multi-mode passive TET with a NES has been taken, single- and multi-mode energy pumping phenomena are investigated in a two-DOF primary structure linearly coupled to a grounded NES [30].

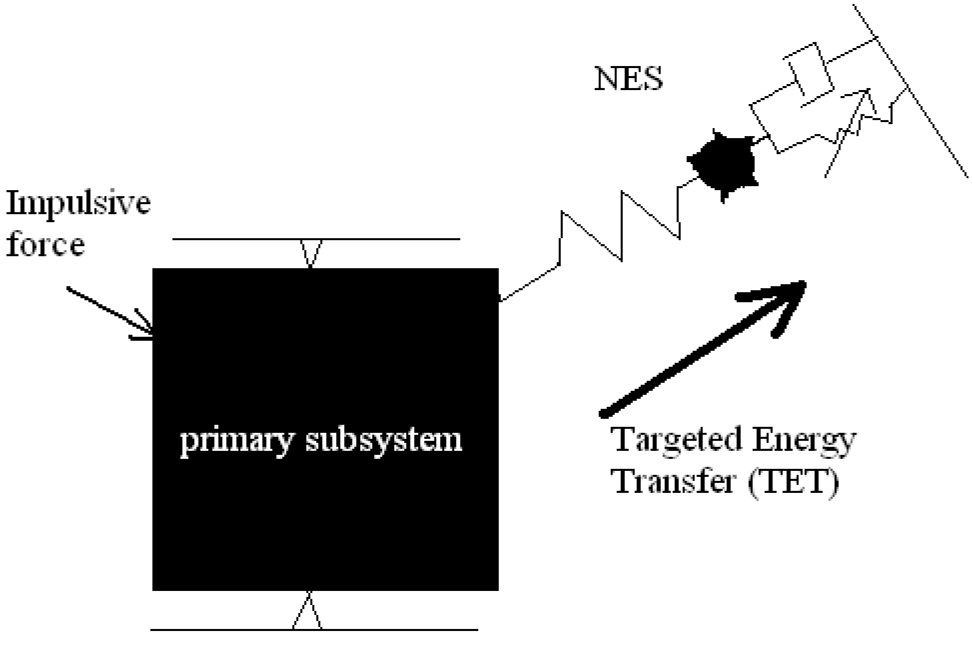

Figure 2. 1 Impulsively loaded primary structure weakly coupled to a grounded NES

In 2005, Gendelman et al. [24] introduced a lightweight and ungrounded NES configuration. This new configuration led to efficient nonlinear TET from the primary system to which it is attached [31]. Two mechanisms of ungrounded NES configuration are examined: (1) the resonant has no excitation in high-frequency level vibration of the NES; (2) The primary system and the NES share same resonance capture. According to Kerschen's study [28], there is no complete equivalence between the ungrounded and grounded NES configurations. Furthermore, by changing the variables, an ungrounded NES configuration is possible to transform to a grounded configuration refers to figure 2.2.

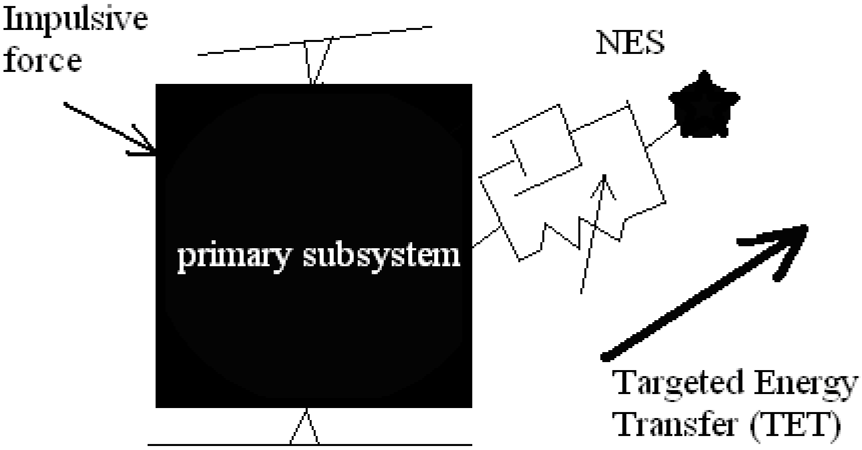

Figure 2. 2 Impulsively loaded primary structure weakly coupled to an ungrounded NES

Another study of ungrounded NES configuration is investigated in [31] [33]. The configuration is with nonlinear cubic stiffness non-linearity coupled to a primary structure. Compare with a grounded NES, the ungrounded configuration has less limitation with heavy mass of the non-linear attachment. Lee et al. (2005) [31] have used a shooting method to solve the bifurcation structure of symmetric and unsysmmetric periodic solutions of a undamped system on a FEP, the Nonlinear Boundary Value Problem (NLBVP, also called two point non-linear boundary value problem) defined though suitable NSTTs based on the two Eigen functions of a vibrio-impact (VI) problem [33]. By using the complexification-averaging technique in terms of mode localization, some valuable resonance branches are examined. The performance of the lightly damped system's TET can be found by superimposing wavelet transforms of the displacement of the primary system and the NES.

For in-phase and out-of-phase impulsive forcing, the transient dynamics of the lightly damped system is investigated. The relative displacement between the primary system and the NES has been wavelet transferred. The results of the transfer are superimposed on the FEP to display branch transitions as the decreasing of the total energy. Complex dynamics in a two-DOF primary structure coupled to an MDOF NES are investigated [34], where strong passive TET capacity is identified [35].

Vakakis (2004) has made a comparison of Transient resonance captures (TRCs), in the work, coupled to a grounded SDOF NES and to an ungrounded MDOF has been compared in finite linear chains with the chain-NES is reduced to a single nonlinear integro-differential equation [36]. similarities based on Jacobian elliptic functions [37] yielded an approximate set of two non-linear integro-differential modulation equations for amplitude and phase, and disturbance analysis as a 1:1 resonant manifold are performed. For the ungrounded MDOF NES, no detectable resonance capture cascades in the system, but coinstantaneous multimodal resonant interactions are found, which insinuated robust and wide relation of TET to different engineering problems. Gourdon et al. has done similar work [38]. In the work, Hilbert transform has been applied on the instantaneous frequencies of the primary structure and NES displacements. The nonlinear theory has been presented in Chapter 3 and Chapter 4 is based on these nonlinear energy investigations.

The two remarkable properties by using NES couples the primary system characterize as (1) TET can be achieved through both resonance capture cascades and single resonance captures by attached an NES to a primary system; (2) NES is able to engage in resonance with different mode of the primary system.

As a basic mechanism for non-linear TET, many early studies have been made for nonlinear attachment [11-18]. Applications of NES to mechanical oscillators can also be found in references [19-21].In recently engineering problems, NES is applied to dissipate unwanted energy from a primary system. In following sections, the previous studies in this covered area are reviewed.

2.3 History of tremor reduction application

In Chapter 5, one aim of this research is designing an application of vibration reduction. Some studies of the tremor reduction application have been reviewed for gathering knowledge and background to do the designing.

Wearable essential tremor suppression

The wearable essential tremor suppression is a simple device that terminates tremor by using dampers. [39]The daspots contains two syringes that act as a damper; the ends of the syringes are connected to the frame and finger loop rail therefore every time the patient tries to move their hands the syringes acts as a damper and reduce the movement hence reducing tremor. The damper absorbs the vibrations and reduces the movement. The stiffness of the Hinge joint depends on the density of the liquid in the syringe. The frame is securely attached to the wrist using an elastic bandage.

There are few flaws in the design which makes it a non commercial product. For example the bandage has to wrap around tightly around the wrist causing numbness and discomfort to the patients. [40]The product does not have an aesthetically pleasing look; the patient will feel ashamed to wear this product in public as it creates a lot of public attention. The patient cannot perform many tasks while wearing the device such as driving or cooking.

Wearable Orthosis for Tremor Assessment

For tremor control and reduction, the application called orthosis has been designed. The study of this device offered idea and knowledge to design the prototype application in Chapter 5.

The application is a flexible robotic exoskeleton which is able to monitor, diagnose on the subjects. The robotic exoskeleton is an active energy reduction system. Kinematic and kinetic sensors are used for detective the vibration of the tremor and measure the joint angular displacement, velocity and acceleration, in order to calculate the interaction force and energy between the limb and the device. [41] The dynamic force could be applied to users' upper limb for reducing the vibration. The force is generated by a pack of flat dc motors. The portability of this device has an advantage in similar applications. According to 10 users who tested this device, they all found it gave high appraisal of its non invasiveness. [42]

The WOTAS have two functions to tremor reduction, active function and passive function. Both the function generated dynamic forces to reduce the tremor. For active function, the device detects the energy between the upper limb and the WOTAS. [43] Certain dynamic force is generated for responding the energy. The dynamic force worked as an impact onto the users arm to reduce the amplitude of tremor. A passive function is available for WOTAS. In passive mode, the devices continually generating a constant dynamic force apply to users' upper limb to reduce the tremor. [44]

From this study, the passive function of WOTAS could be used for designing the research application. In stand of using dc motors to generate impact force, the technology of using nonlinear energy pumping will take the place.

There are many products that are available in the current market to ease daily activities performed by people with tremor. These products do not reduce tremor but it helps people with tremor to perform daily duties without any hustle.

The Swedish cutting board on the left contains a vice and spikes to grip the vegetables while cutting, this enables the user to use the knife safely. This cutting board is also designed to be used by people with one hand. The stainless steel spikes and the vice enable the user to clean the board easily. The vice also has a multipurpose use as it can also hold bottles while opening it. [45]

There is a tremor reduction mug with two handles and two separate lids. Two handles enables the user to hold the mug tightly while using it. One lid is with a spout and the other is an anti-spill with a hole and a straw. This product is perfect for people with tremor as it helps avoid spillage. [46]

The jug kettle tipper is a stand that holds the kettle in position while pouring the water out. This is a safe and easy way to pour the water out of the kettle. This product is also for people who suffer with tremor. [47]

A cutlery has been designed for tremor hands using. The cutlery are similar the normal cutlery but with a thicker handles which has a good grip. The knife, Folk and spoon are made of stainless steel, this enables the user to clean the product easily and it will not rust. The handles are made of polypropylene and have a flat side which gives a good grip. The cutlery can be adjusted to any angle the user is comfortable using. This product can also be used by patients with tremor. [48]

An assist pen works close to a normal pen but with a bigger base. This product assists people with tremor to write neatly. The enduring base enables the user to hold the pen at an angle to reduce shaky handwriting and the large surface area enables the patient with tremor to hold the pen at their will. [49]

2.4 Summary

In this chapter, several studies have been reviewed for further work. The literature of targeted energy transfer progressing has been studied. This offered a background of doing the research of the energy transfer inside the system in later work. The studies of nonlinear energy attachment have been reviewed. NES has been forced studied as the core attachment which in used in simulation of numerical method in Chapter 3 and the experiment in Chapter 5. The related numerical method of a NES attached to primary system has been reviewed for further development. In Chapter 3, the theoretic method is based on these studies. In the final section of this chapter, the exited tremor assessment has been reviewed. The reviews give the direction the design the experiment in Chapter 5 and the application for tremor reduction.

Chapter 3

Nonlinear Dynamical Investigation

3.1 Introduction of Numerical Methods of Dynamics of NES system

In this section, numerical study of the periodic orbits of the undamped system has been continued on the system of two linear oscillators and two NESs. The linear oscillators act as the primary systems, coupled to ungrounded attachments (NESs) through a pure cubic stiffness. In Lee et al. (2005) [23]'s study, possibility of passively and irreversibly transferring a significant part of the imparted energy to the nonlinear oscillator has been examined. The study is based on Vakakis' s study and it is on a two-degree-of-freedom system with two NESs linked to two linear mass separately as Figure 3.1 shown.

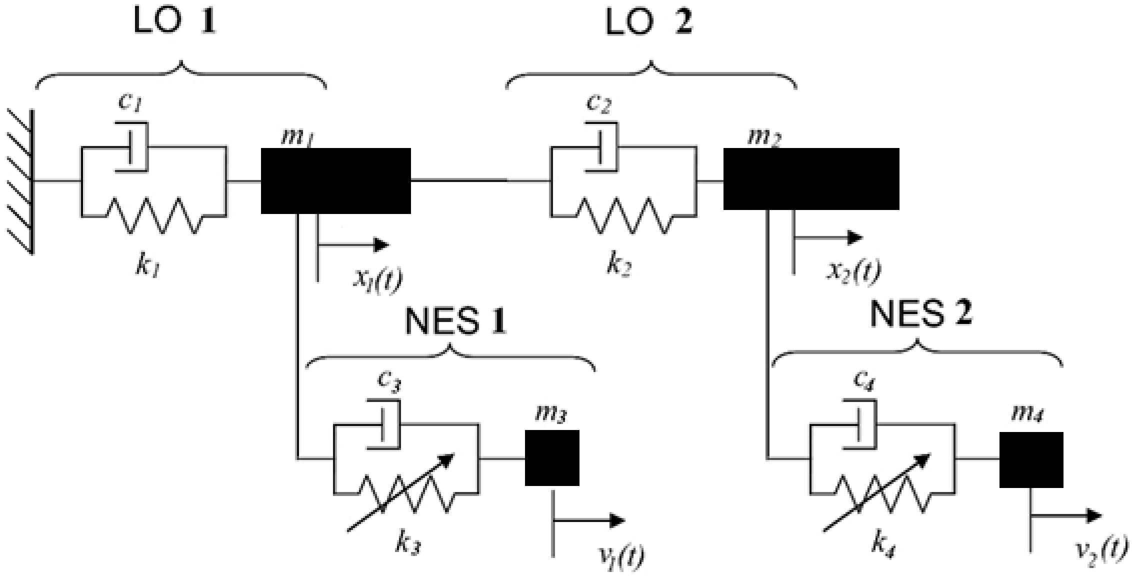

Figure 3. 1 Two-degree-of-freedom (2 DOF) nonlinear system consisting of two grounded linear oscillators coupled to two separate light weight nonlinear energy sinks

The equation of this structure system is given by

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + c_3(\dot{x}_1 - \dot{x}_3) + k_1x_1 + k_2(x_1 - x_2) + k_3(x_1 - x_3)^3 = 0$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + c_4(\dot{x}_2 - \dot{x}_4) + k_2(x_2 - x_1) + k_4(x_2 - x_4)^3 = 0$$

$$m_3\ddot{x}_3 + c_3(\dot{x}_3 - \dot{x}_1) + k_3(x_3 - x_1)^3 = 0$$

$$m_4\ddot{x}_4 + c_4(\dot{x}_4 - \dot{x}_2) + k_4(x_4 - x_2)^3 = 0$$

(1)

where x(t) and v(t) are the displacement of two LOs and of two NESs. The equations of system 2) can be rescaled as:

$$\ddot{x}_1 + \lambda_1\dot{x}_1 + \lambda_2(\dot{x}_1 - \dot{x}_2) + \lambda_3(\dot{x}_1 - \dot{x}_3) + {\omega_1}^2x_1 + {\omega_2}^2(x_1 - x_2) + c_1(x_1 - x_3)^3 = 0$$

$$\ddot{x}_2 + \lambda_2(\dot{x}_2 - \dot{x}_1) + \lambda_4(\dot{x}_2 - \dot{x}_4) + {\omega_2}^2(x_2 - x_1) + c_2(x_2 - x_4)^3 = 0$$

$$\varepsilon_1\ddot{x}_3 + \lambda_3(\dot{x}_3 - \dot{x}_1) + c_1(x_3 - x_1)^3 = 0$$

$$\varepsilon_2\ddot{x}_4 + \lambda_4(\dot{x}_4 - \dot{x}_2) + c_2(x_4 - x_2)^3 = 0$$

(2)

where

$\varepsilon_1 = \varepsilon_2 = \frac{m_3}{m_1} = \frac{m_4}{m_2},\ {\omega_1}^2 = \frac{k_1}{m_1}, {\omega_2}^2 = \frac{k_2}{m_1}, C_1 = \frac{k_3}{m_1}, C_2 = \frac{k_4}{m_1}, \lambda_1 = \frac{C_1}{m_1}, \lambda_2 = \frac{C_2}{m_1}, \lambda_3 = \frac{C_3}{m_1}, \lambda_4 = \frac{C_4}{m_1}$

In 1985, Pilipchunk first investigated the periodic orbits of the system which are numerically computed with the theory of non-smooth transformations [35, 36]. In this later research, the system has been attached to strongly nonlinear oscillators [37]. In this study, the method has been applied to the numerical and analytical study of periodic obits of nonlinear system energy transfer. Two non-smooth variables τ and e determines as

$$x_1(t) = e\left(\frac{t}{\alpha}\right) y_1\left(\tau\left(\frac{t}{\alpha}\right)\right),\quad x_2(t) = e\left(\frac{t}{\alpha}\right) y_2\left(\tau\left(\frac{t}{\alpha}\right)\right),$$

$$x_3(t) = e\left(\frac{t}{\alpha}\right) y_3\left(\tau\left(\frac{t}{\alpha}\right)\right), x_4(t) = e\left(\frac{t}{\alpha}\right) y_4\left(\tau\left(\frac{t}{\alpha}\right)\right),$$

(3)

Where $\alpha = T/4$ represents the (yet unknown) quarter-period. The non-smooth functions τ(u) and e(u) are defined according to the expressions:

$$\tau(u) = \frac{2}{\pi} sin^{-1}\left(\sin\left(\frac{\pi}{2\alpha}\right) u\right),\ e\ (u)\ =\ \tau'\ (u),$$

(4)

and are used to replace the independent time variable from the equations of motion (prime denotes differentiation with respect to the argument); their graphic depiction is given in Figure 5.

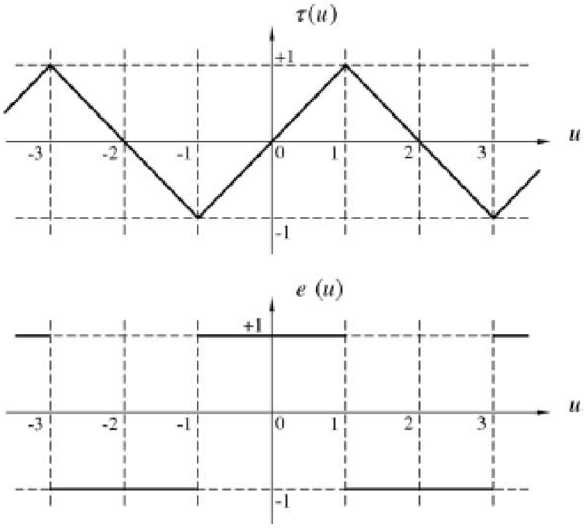

Figure 3. 2 The non-smooth functions τ(u) and e(u)

By set all values of λ to zero, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$, and substituting (2) into (1), the Pilipchunk non-smooth transformation method [37]has been imposed to eliminate singular terms of the resulting equations:

$$e^{'}(u) = \tau^{''}(u) = 2\sum_{k=-\infty}^{\infty}[\delta(u+1-4k) - \delta(u-1-4k)].$$

Non-smooth variable e here has been multiplied by the zero components of the transformed equations. It is formulated the following two-point nonlinear boundary value problem (NLBVP) in terms of the non-smooth variable τ, in the interval -1≤ τ ≤ 1:

$$y^{'}_{1} = y_5,$$

$$y^{'}_{2} = y_6,$$

$$y'_3 = y_7,$$

$$y'_4 = y_8,$$

$$y'_5 = -\omega_0{}^2\alpha^2 y_1 - \omega_0{}^2\alpha^2(y_1 - y_2) - C\alpha^2(y_1 - y_3)^3$$

$$y'_6 = -\omega_0{}^2\alpha^2(y_2 - y_1) - C\alpha^2(y_2 - y_4)^3$$

$$y'_7 = -\frac{C}{\varepsilon_1}\alpha^2(y_3 - y_1)^3$$

$$y'_8 = -\frac{C}{\varepsilon_2}\alpha^2(y_4 - y_2)^3 \qquad (5)$$

where primes represents difference with respect to the non-smooth variable τ, and a state formulation is used. The boundary conditions above result from the aforementioned smoothening conditions [37].

3.2 Frequency-Energy Plots (FEPs)

Last section has presented that the two-DOF undamped system with two LOs and can be defined with NLBVP. In this section, the Frequency- Energy Plots (FEPs) has been used to analyze the system for the numerical study. FEP is a tool for analysis two-degree-of-freedom system. It gathers all the periodic orbits of the considered system. FEPs present the frequency indices (FIs) between the LOs and the NESs against the energy of the system. The horizontal and vertical axis of this plot corresponds to the amount of energy injected in the system and to the pulsation of the motion [23]. The branches of the FEPs are the collection of the periodic orbits possessing the same qualitative features, which perform the ratio of the indices multiplied by the driving frequency of the system on the branch.

Program of Nonlinear Normal Mode (NNM) Computation in MATLAB has been used for mapping the FEPs of the nonlinear normal modes in this section. The modes have been set up as the defining dynamical system, which is used to locate the branches of the mode by changing the step size of the program.

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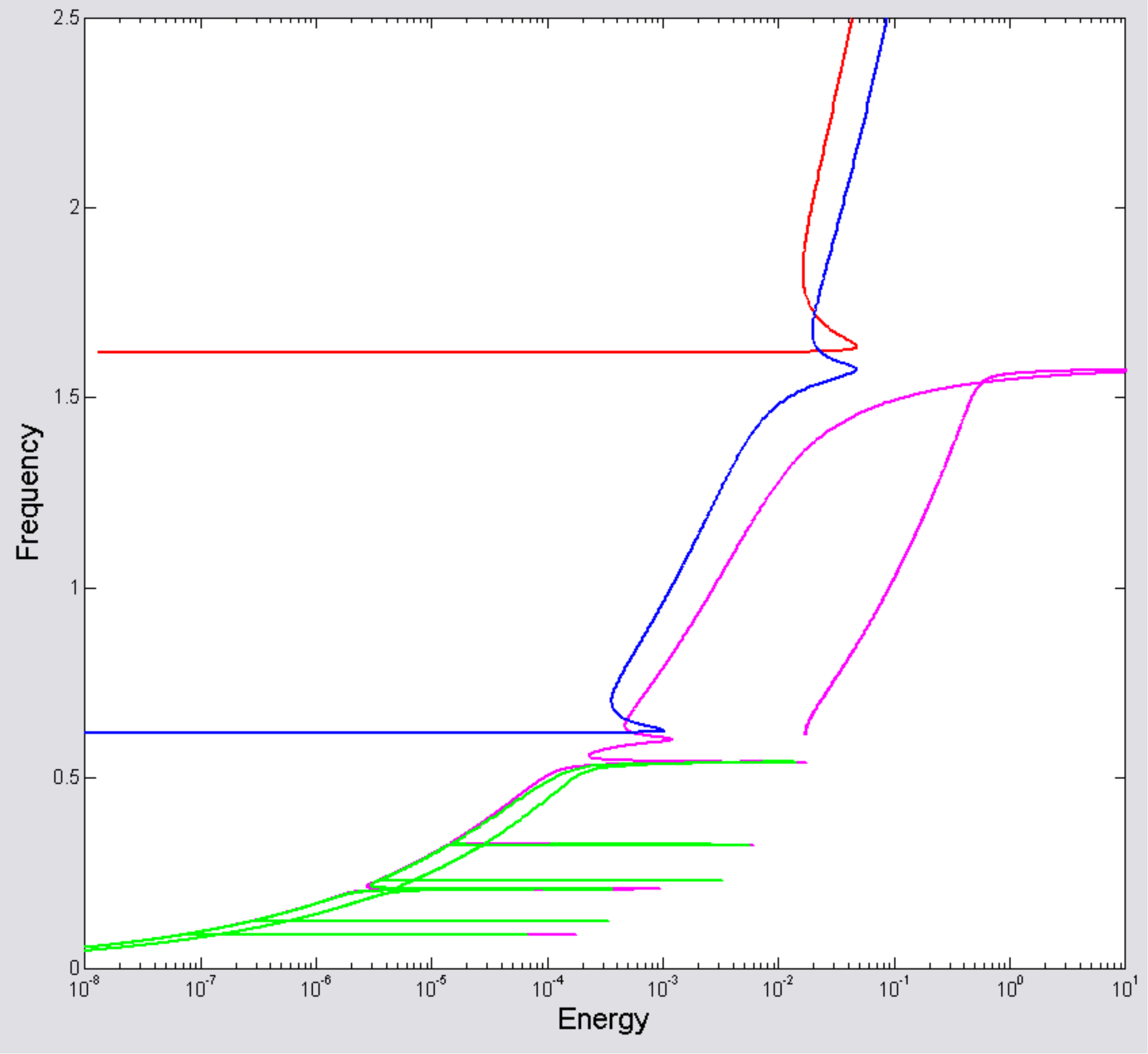


Figure 3. 3 FEP of Two Primary systems and two NESs separately

As shown in the figure 3.3, the branches of four modes of the system have been plotted. Branches have been differenced by colour: the branches of first mode, which is the first LO have been plotted in green. The branches in purple belong to second LO. The blue branches are the results of the NES which is attached to the first LO, and the red branches belong to the NES which is attached to the second LO.

Several key features of the FEP have been described: All the periodic orbits that characterize the system can be classified into two distinct families of solutions, the symmetric and the unsymmetrical ones, denoted by S and U, respectively. The symmetric solutions are defined as periodic orbits that satisfy the condition $x(t) = -x(t + \frac{T}{2})$ with x corresponding to the state vector and T to the period of the motion, whereas periodic orbits that do not satisfy this condition are classified as unsymmetrical solutions.

3.3 Impulsive Orbits (IOs)

In section 3.2, the study of the FEP of impulsive responses of a two-degree-of-freedom system with two NESs linked to two linear mass separately have been presented. In this section, the FEP is used to show and study the impulsive orbits of the two NESs linked to two linear mass system. It is considered, the impulsive orbits are the orbit relevant to initial conditions. Or it is an initial primary system attached with nonlinear attachment was applied an impulse [38].

It is important to study some periodic orbits of the families of NNMs.

The impulsive orbits that relevant to the condition of the undamped system suffice the initial condition: $x_1 = x_2 = \dot{x}_2 = 0$ and $\dot{x}_1 \neq 0$ t after that application of an impulse on the first DOF. It has been mentioned in [38] that the mechanisms triggers initiating passive energy pumping is the impulsive excitation of one of the stable impulsive orbits. Because the NES is able to engage in several infinity of n: m internal resonance with the primary system, the exits a countable infinity of periodic IOs can be relative prime integers with LO. The orbits are aligned along a smooth curve in the FEP.

We consider the system of coupled oscillators depicted in Figure 3.1, with governing equations of motion [38],

$$\ddot{y}_1 + y_1 + {\omega_0}^2(y_1 - y_2) + c_0(y_1 - y_3)^3 = 0$$

$$\ddot{y}_2 + {\omega_0}^2(y_2 - y_1) + c_0(y_2 - y_4)^3 = 0$$

$$\varepsilon_1\ddot{y}_3 + c_1(y_3 - y_1)^3 = 0$$

$$\varepsilon_2\ddot{y}_4 + c_2(y_4 - y_2)^3 = 0, 0 < \varepsilon << 1$$

(6)

where dots denote differentiation with respect to the time variable t. The system consists of two grounded primary systems connoted with two light NESs. An impulsive orbit of the system can be defined as the orbit corresponding to initial conditions [41]:

$$\ddot{y}_1(0) \neq 0, \qquad \ddot{y}_2(0) \neq 0, \qquad y_1(0) = y_2(0) = y_3(0) = y_4(0) = \dot{y}_3(0) = \dot{y}_4(0) = 0$$

or, to an impulse applied to one of the primary systems with the nonlinear attachments being initially at rest.

As a demonstrative study, the equations of the considered impulsively forced damped system [41] are as follows,

$$\ddot{y}_1 + 0.005\dot{y}_1 + 0.005(\dot{y}_1 - \dot{y}_2) + y_1 + (y_1 - y_3)^3 = Y_1\delta(t)$$

$$\ddot{y}_2 + 0.005\dot{y}_2 + 0.005(\dot{y}_2 - \dot{y}_1) + (y_2 - y_4)^3 = Y_2\delta(t)$$

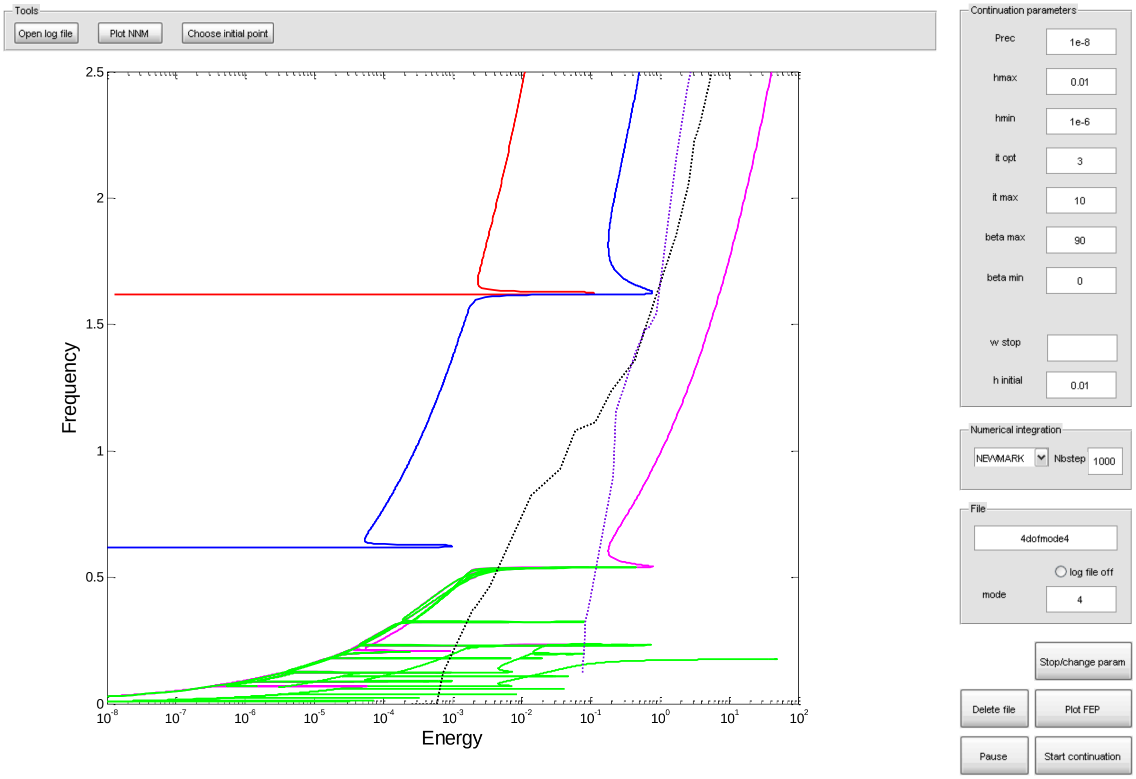
$$0.05\ddot{y}_3 + 0.005(\dot{y}_3 - \dot{y}_1) + (y_3 - y_1)^3 = 0$$

$$0.05\ddot{y}_4 + 0.005(\dot{y}_4 - \dot{y}_2) + (y_4 - y_2)^3 = 0$$

(7)

This system is composed of two impulsively forced damped linear oscillators with two NESs absorbing vibration energy from the primary system, in a one-way, irreversible transfer[42].

MATLAB has been used for computing the impulsive orbits. The steps of selecting IOs on FEP are firstly computation of the modal directions, then choice of impulsive orbit manifold of the system. After define both maximum and minimum energy (this is the logarithm of the minimum energy for which we are looking for IOs) [43] of the system, the energy levels for which periodic orbits are sought. At last the IOs are selected and plotted on the FEP plots which is done in last section.



Prec 1e-8
hmax 0.01
hmin 1e-6
it opt 3
it max 10
beta max 90
beta min 0
w stop
h initial 0.01
NEWMARK
Nbstep 1000
4dofmode4
○ log file off
mode 4

Figure 3. 4 IOs plot on FEP

Figure 3.4 presents the position of some impulsive orbits as well as an estimate of locus of all the existing impulsive orbits.

In Figure 3.5, we depict the high-energy unstable IO on branch U21 are initiated the motion corresponding to initial conditions Y=0.5.

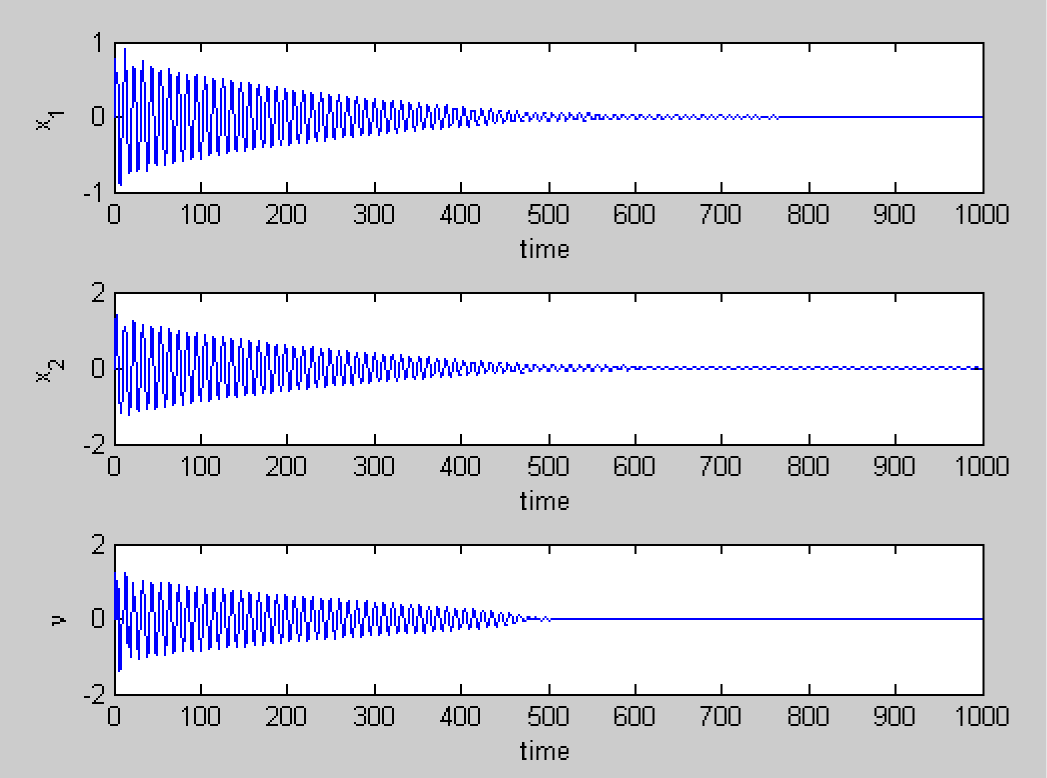


Figure 3. 5 Impulsive orbits at the high-energy regime.

Some further studies will be taken on in the future, which includes analysis of the how the energy level of the vibration affects IOs, the corresponding percentage of total instantaneous energies contained in the NESs in different energy levels, and wavelet transforms (WT)[44] to the periodic solutions of the underlying Hamiltonian system. It will be introduced in later section.

3.4 Conclusion

The study in this chapter is on a two-degree-of-freedom (2 DOF) nonlinear system consisting of two grounded linear oscillators coupled to two separate light weight nonlinear energy sinks of an essentially nonlinear stiffness. The basic study method followed Vakakis' work of a two-degree-of freedom nonlinear system consisting of a grounded linear oscillator coupled to a light mass. Literature studies of different kinds of NES configuration has been reviewed for further designing of the NES structure. Followed Vakakis' numerical study of periodic orbits of the undamped system, a new numerical study of our new structure system has been done. The report has present the first part of the numerical study of the dynamics of the mechanical structure system, a nonlinear boundary value problem (NLBVP) has been study. A continue study will be taken in the next step of study the numerical methods. The numerical study is the basic knowledge for analyse later studies of the system. FEP and mode shapes of the system present the responds of the system TET in the frequency domain. A FEP has been plotted with MATLAB for our two DOF system, but very few analyse have been done to the FEP. In later section of future plan, it is noticed we are interesting in the detailed plot of very region of the FEP. The mapping method of FEPs with MATLAB may not work. Vakakis ect have used a shooting method to plot FEPs, the method is under studying, and it will be used for later FEP plotting and analyses. We have studied the impulsive responses of our tow DOF system. We analyses the periodic and quasi-periodic dynamics of the undamped system and high-energy impulsive motions. Impulsive orbits have been studied and found. For use the IOs we have found, a time response and wavelet transforms of the response with initial conditions on different points of IOs will be studied, more detail of this part will be present in next section.

According to the studies up to know, it is shown the responses of two-degree-of-freedom nonlinear system consisting of two grounded linear oscillators coupled to two separate light weight nonlinear energy sinks of an essentially nonlinear stiffness is more complicated than the two-degree of freedom nonlinear system consisting a single oscillator and a single NES. The study is currently continuing on, a detail future plan has been presented in next section.

Chapter 4

Theoretical Nonlinear Energy Transfer

4.1 Introduction of Numerical Model

Applications of nonlinear energy pumping become a significant structure in the area of targeted energy transfer. In this chapter, a numerical method of LO attached with two different NESs refers to the literatures in Chapter 2 has been investigated. Theoretical model of tremor reduction is based on the numerical method of one NES attached on a primary system. For further investigation, the model has been designed for solving the tremor vibration of human body. In section 4.2, a basic tremor vibration model has been built up. By programming with MATLAB, some parameters have been chosen for solving the vibration reduction. Later in the section, two NESs have been applied onto the primary system. The models have been studied in two different structures: Two NESs series connected to the primary system, two NESs parallel connected to the primary system. In the following section, the results have been taken and compared. Two different structure models have been investigated from the comparing results. Genetic algorithm has been used to choose the components for better results. Genetic algorithm has been studied for creating the MATLAB program. The tests of two different structure models have been run with the new components parameters. The results then have been gathered and discussed. In the final discussion, Frequency- Energy plots have been used as a tool for investigating the energy transfer inside the model.

4.2 Theoretical model of Tremor Reduction

In order to design the application device, a theoretical model has been made for the early stage studying. In the theoretical model, the NES has been connected to the primary system. Assume there is a continued internal force of the arm F=10sin14πt. The system is shown as Figure 4.1

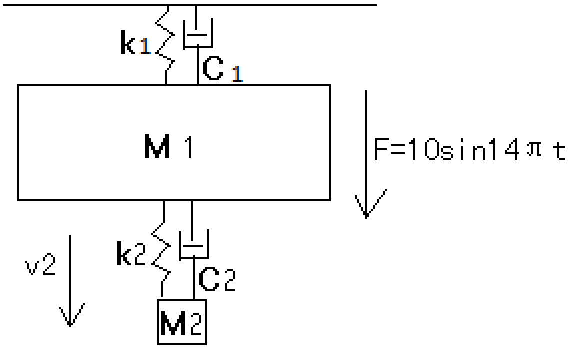


Figure 4. 1 One NES Attached System

The equation the system can be defined as

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_1x_1 + k_2(x_1 - x_2)^3 - F = 0$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1)^3 = 0$$

By using the ode45 codes of Matlab to solve this equation with

m1=1; m2=0.1; C1=0; C2=0.11; k1=1; k2=1;

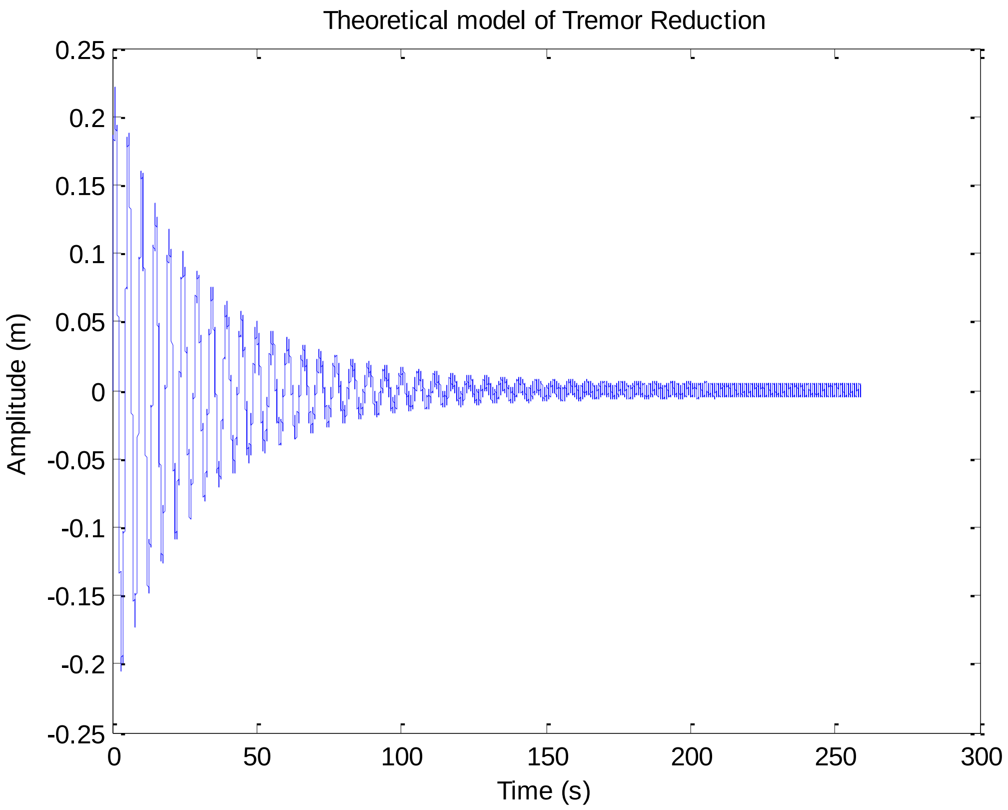


Figure 4. 2 Theoretical Model of Tremor Reduction

Figure 4.2 Result shows the tremor can be reduced by time. At time 63.3216s, the tremor amplitude reduced 1 / 4 of original amplitude.

The later stage of studying the theoretical model, it is found that, by using two connected energy sinks on primary system, the results of tremor reduction are shown more efficiency.

The energy pump has been designed in two different ways: two NESs are series connected to the primary system; Two NESs is parallel connected to primary system.

The equation of the series NESs connection system is:

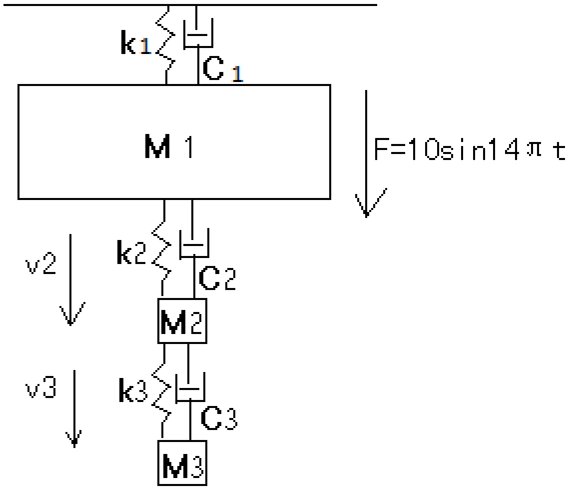


Figure 4. 3 Two NES Series Attached System

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_1x_1 + k_2(x_1 - x_2)^3 - F = 0$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + c_3(\dot{x}_1 - \dot{x}_3) + k_2(x_2 - x_1)^3 + k_3(x_1 - x_3)^3 = 0$$

$$m_3\ddot{x}_3 + c_3(\dot{x}_3 - \dot{x}_1) + k_3(x_3 - x_1)^3 = 0$$

By using the ode45 codes of Matlab to solve this equation with

m1=1; m2=0.1; m3=0.1; C1=0; C2=0.11; C3=0.5; k1=1; k2=1; k3=1;

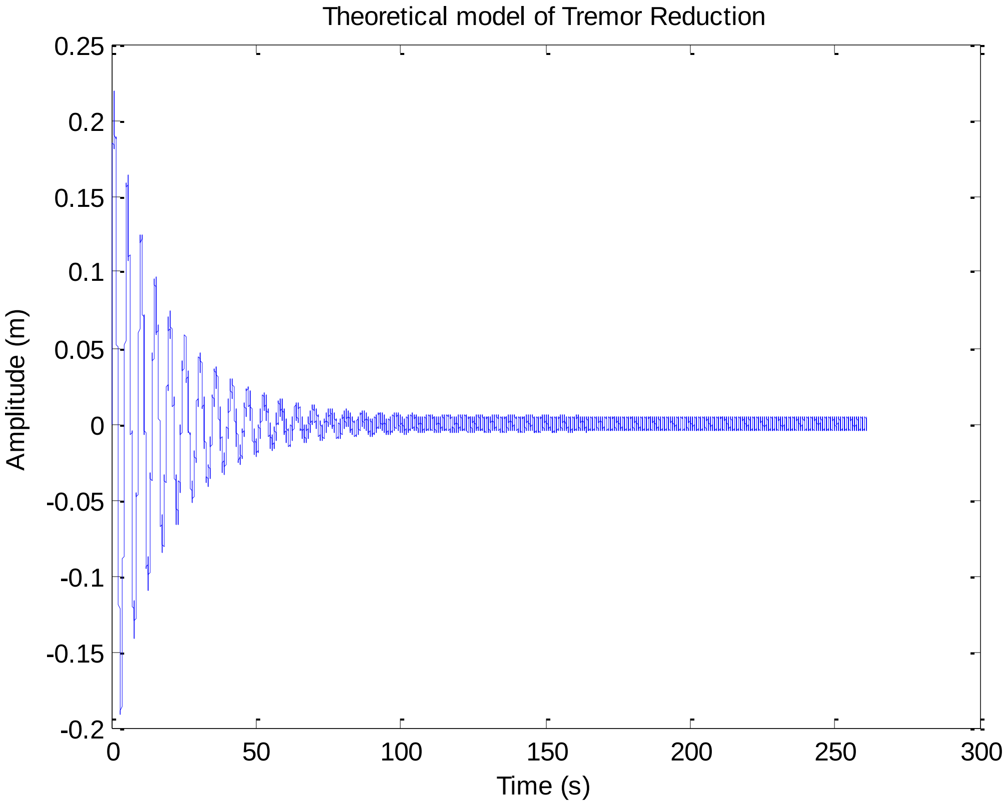


Figure 4. 4 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.1; C1=0; C2=0.11; C3=0.5; k1=1; k2=1; k3=1

The result shows the tremor can be reduced by time. At time 37.1748s, the tremor amplitude reduced 1 / 4 of original amplitude.

Other results – changing damping coefficient

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.9; C3=0.012; k1=1; k2=1.0022; k3=0.9936;

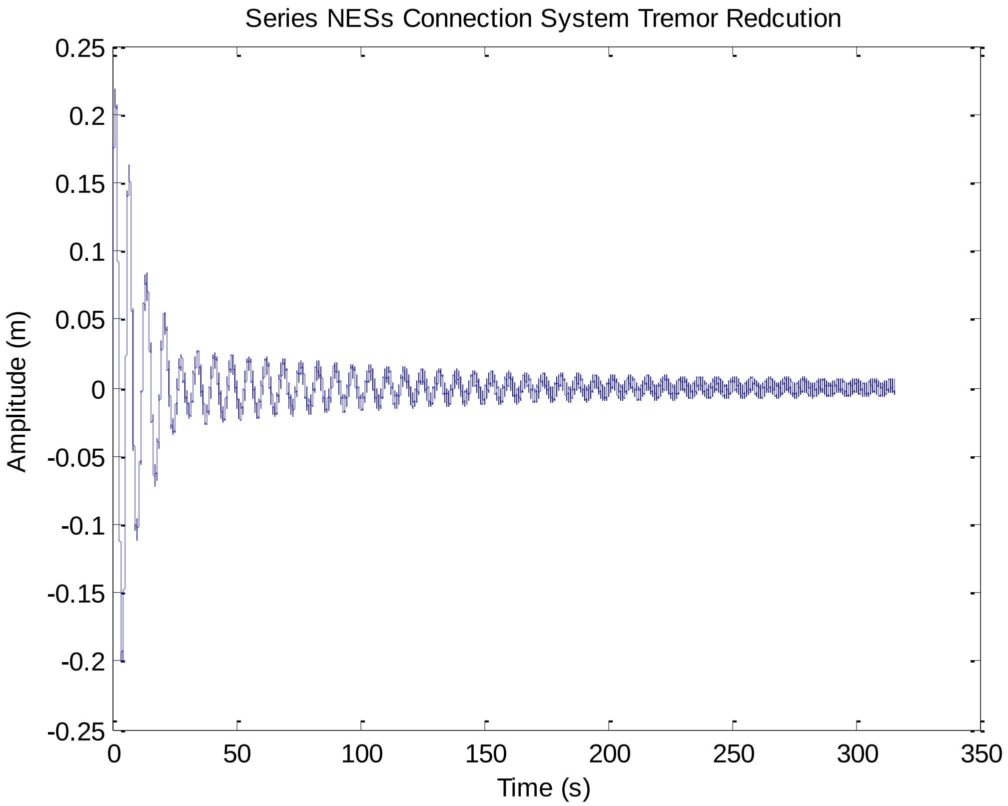


Figure 4. 5 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.9; C3=0.012; k1=1; k2=1.0022; k3=0.9936

Time to reduce the tremor amplitude to a 0.05m limit 0.05m is 22.9699s.

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.891; C3=0.02; k1=1; k2=1.0022; k3=0.9936;

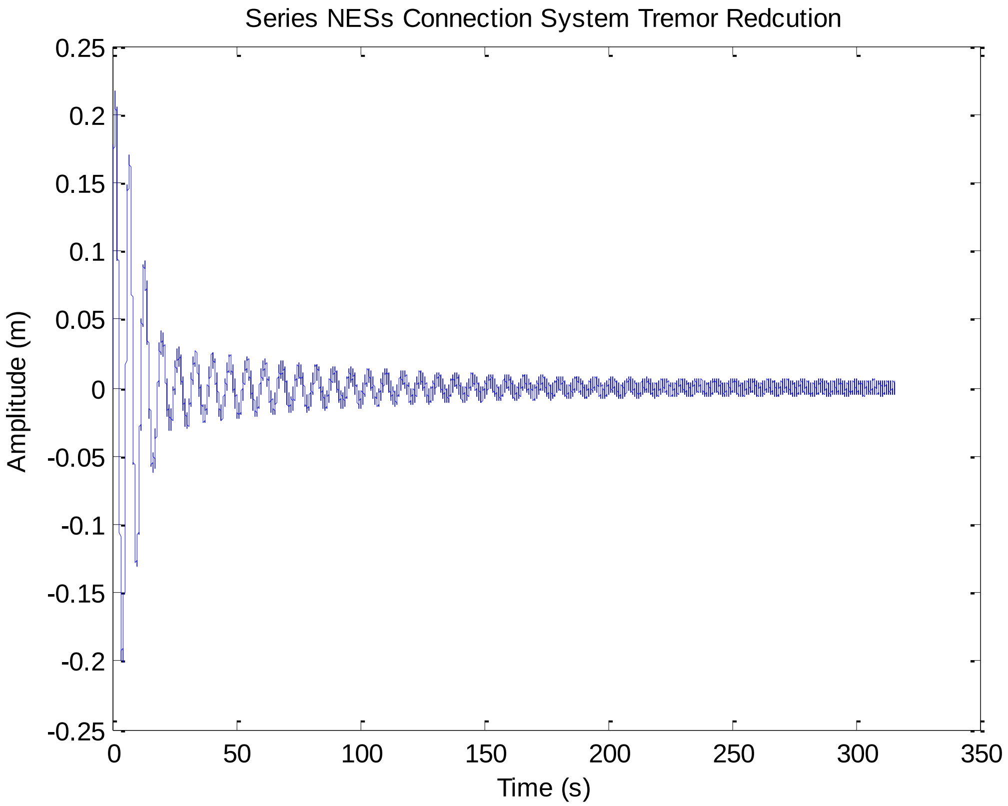


Figure 4. 6 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.891; C3=0.02; k1=1; k2=1.0022; k3=0.9936

Time to reduce the tremor amplitude to a 0.05m limit 0.05m is 18.9067s.

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.95; C3=0.008; k1=1; k2=1.0022; k3=0.9936;

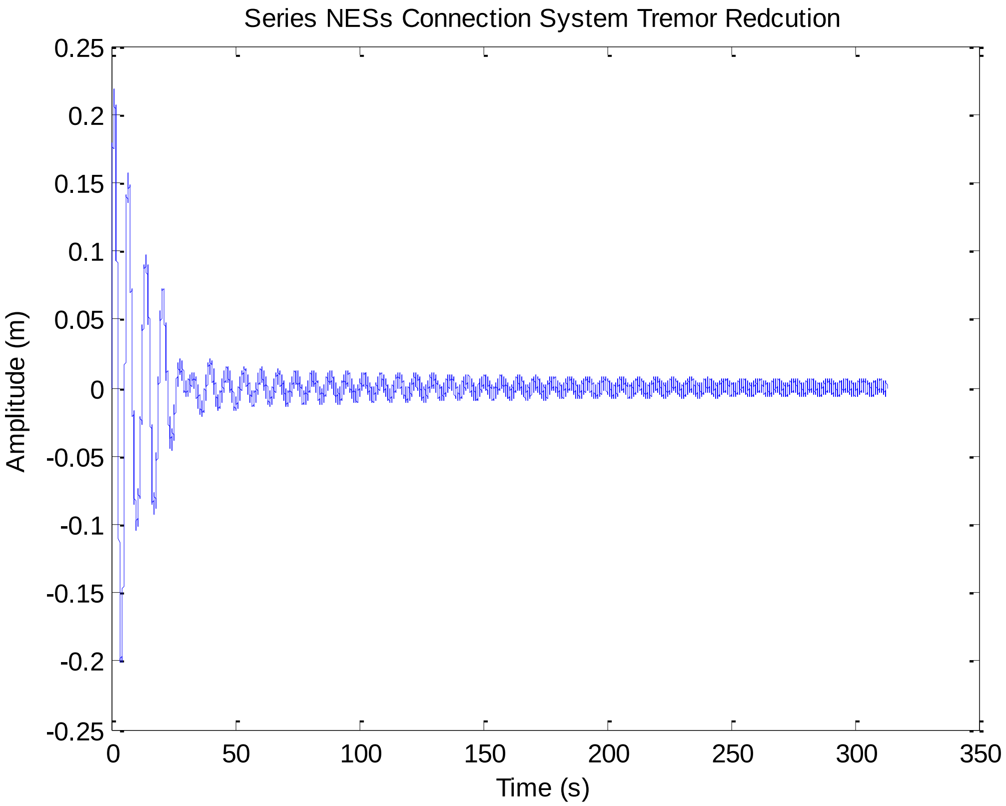


Figure 4. 7 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.95; C3=0.008; k1=1; k2=1.0022; k3=0.9936

Time to reduce the tremor amplitude to a 0.05m limit 0.05m is 23.4101s.

C1=0; C2=0.9; C3=0.012	22.97
C1=0; C2=0.891; C3=0.02	18.907
C1=0; C2=0.95; C3=0.008	23.41

Other results – changing mass of NESs

m1=1; m2=0.5; m3=0.0746; C1=0; C2=0.891; C3=0.012; k1=1; k2=0.90; k3=0.71;

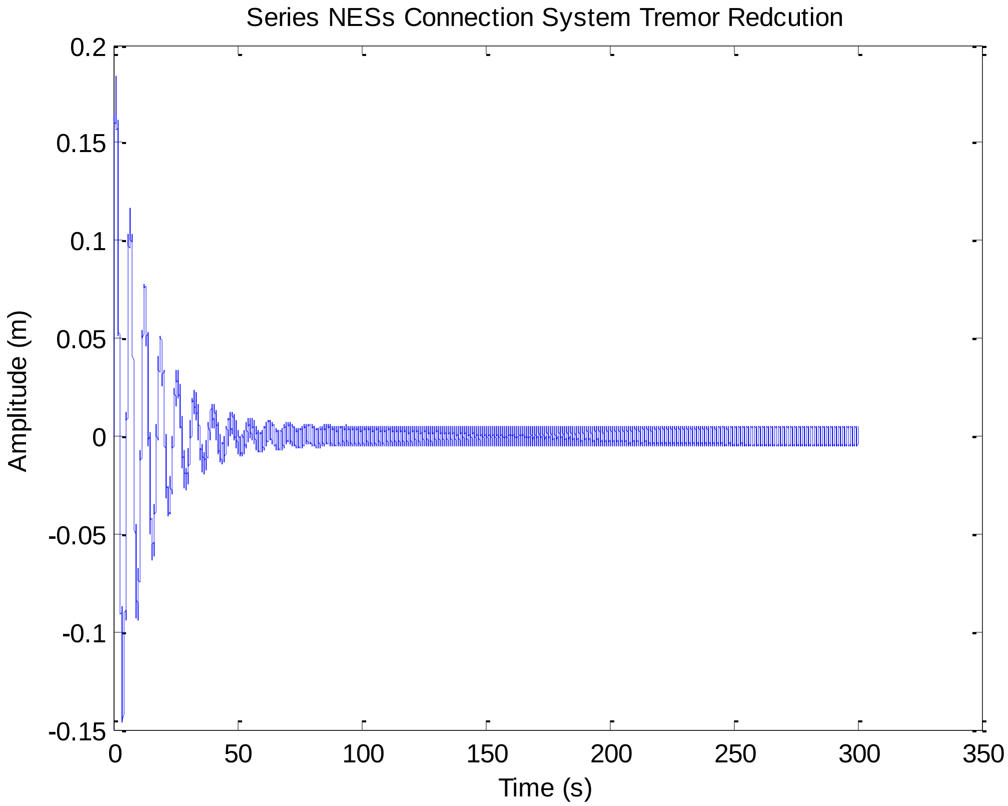


Figure 4. 8 Tremor Reduction with parameters of m1=1; m2=0.5; m3=0.0746; C1=0; C2=0.891; C3=0.012; k1=1; k2=0.90; k3=0.71

Time to reduce the tremor amplitude to a 0.05m limit 0.05m is 24.1046s.

m1=1; m2=0.1; m3=0.5; C1=0; C2=0.891; C3=0.012; k1=1; k2=0.90; k3=0.71;

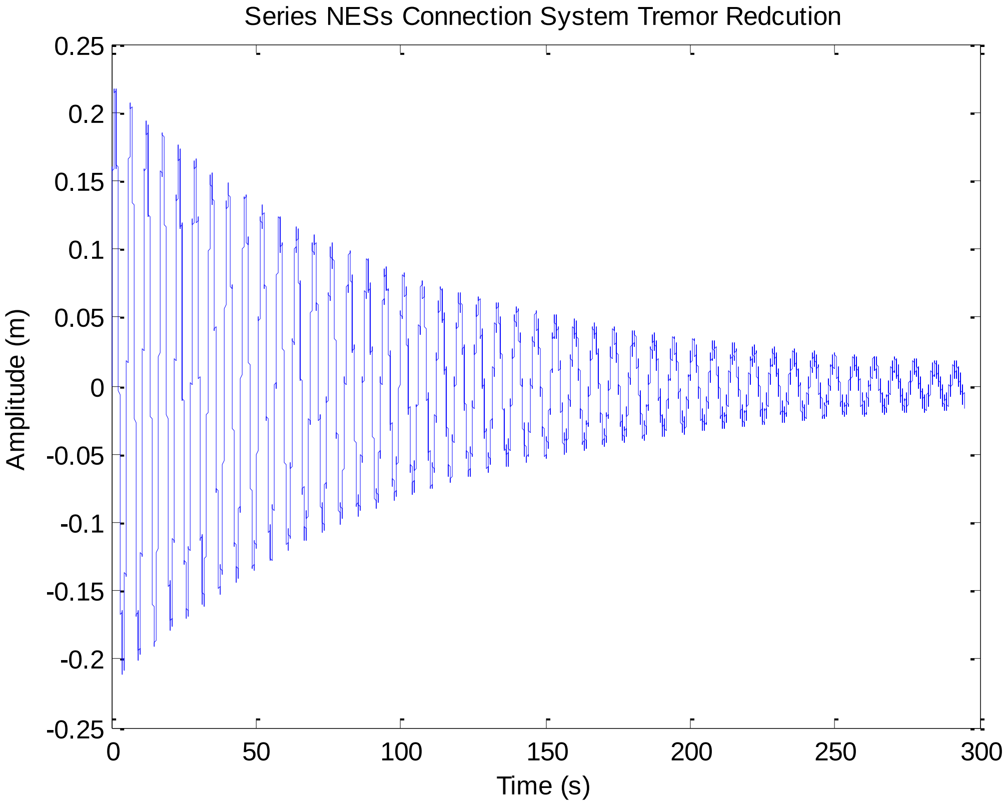


Figure 4. 9 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.5; C1=0; C2=0.891; C3=0.012; k1=1; k2=0.90; k3=0.71

Time to reduce the tremor amplitude to a 0.05m limit 0.05m is 213.1046s.

m1=1; m2=0.5; m3=0.5; C1=0; C2=0.891; C3=0.012; k1=1; k2=0.90; k3=0.71;

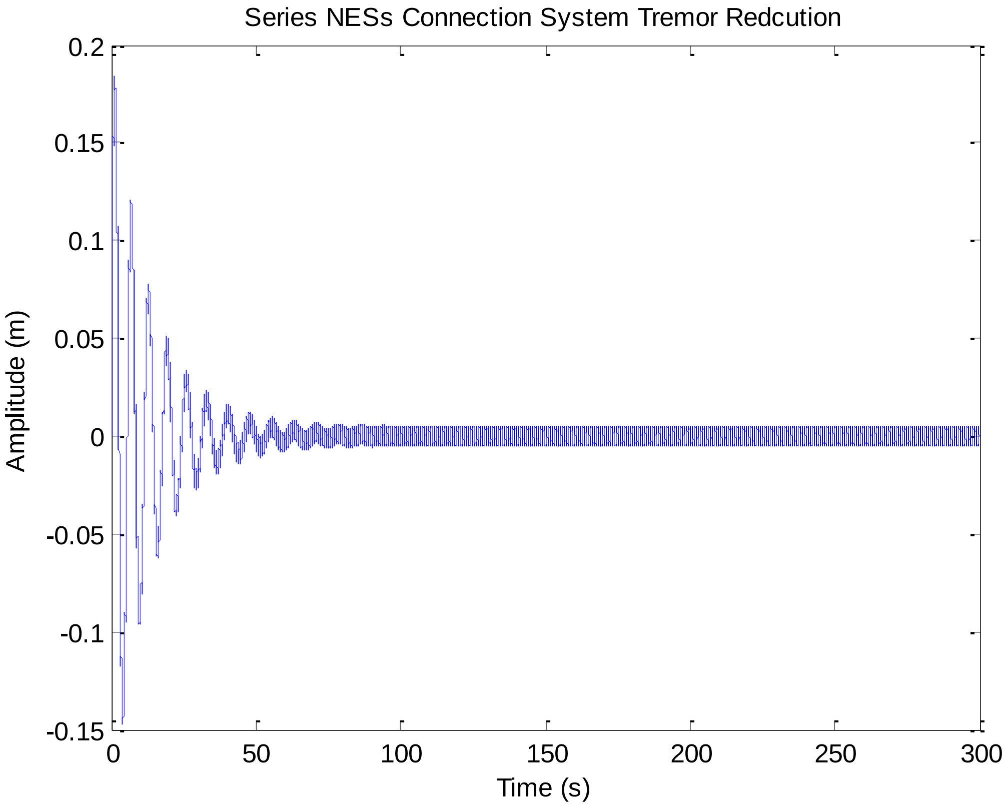


Figure 4. 10 Tremor Reduction with parameters of m1=1; m2=0.5; m3=0.5; C1=0; C2=0.891; C3=0.012; k1=1; k2=0.90; k3=0.71

Time to reduce the tremor amplitude to a 0.05m limit 0.05m is 24.2534s.

m1=1; m2=0.5; m3=0.0746	24.105
m1=1; m2=0.1; m3=0.5	213.1
m1=1; m2=0.5; m3=0.5	24.253

Other results – changing stiffness of the springs

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.891; C3=0.012; k1=1; k2=1; k3=0.71;

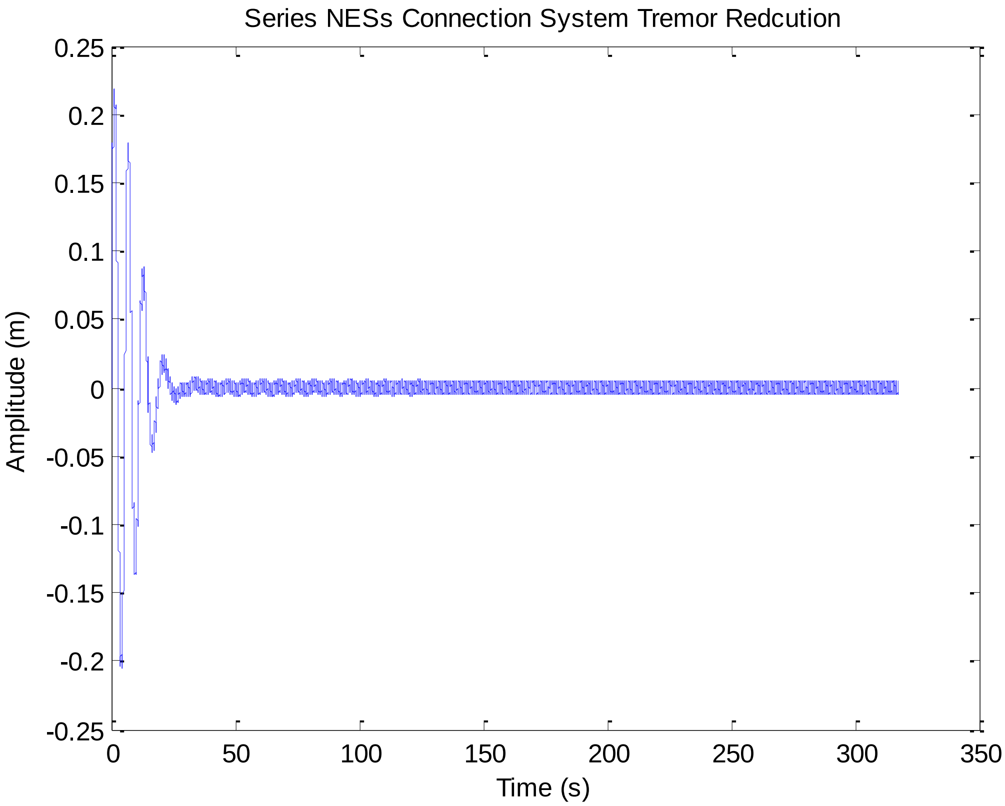


Figure 4. 11 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.891; C3=0.012; k1=1; k2=1; k3=0.71

Time to reduce the tremor amplitude to a 0.05m limit 0.05m is 15.5564s.

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.891; C3=0.012; k1=1; k2=0.90; k3=0.90

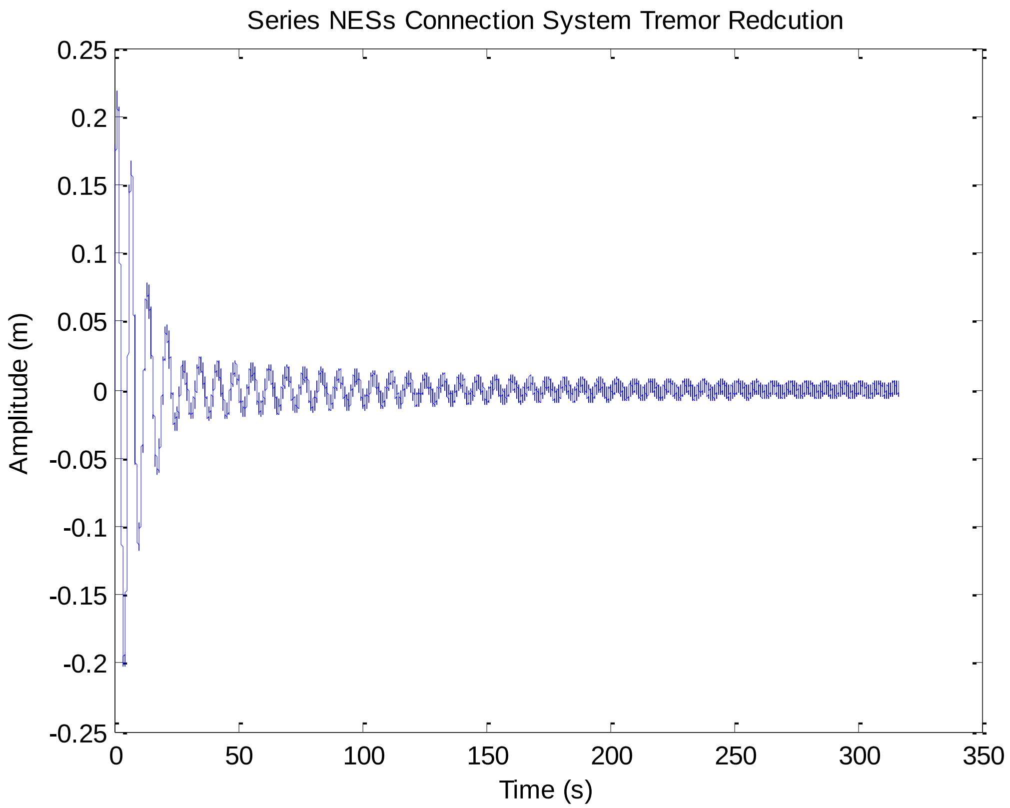


Figure 4. 12 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.891; C3=0.012; k1=1; k2=0.90; k3=0.90

Time to reduce the tremor amplitude to a 0.05m limit 0.05m is 19.6195s m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.891; C3=0.012; k1=1; k2=1; k3=0.6;

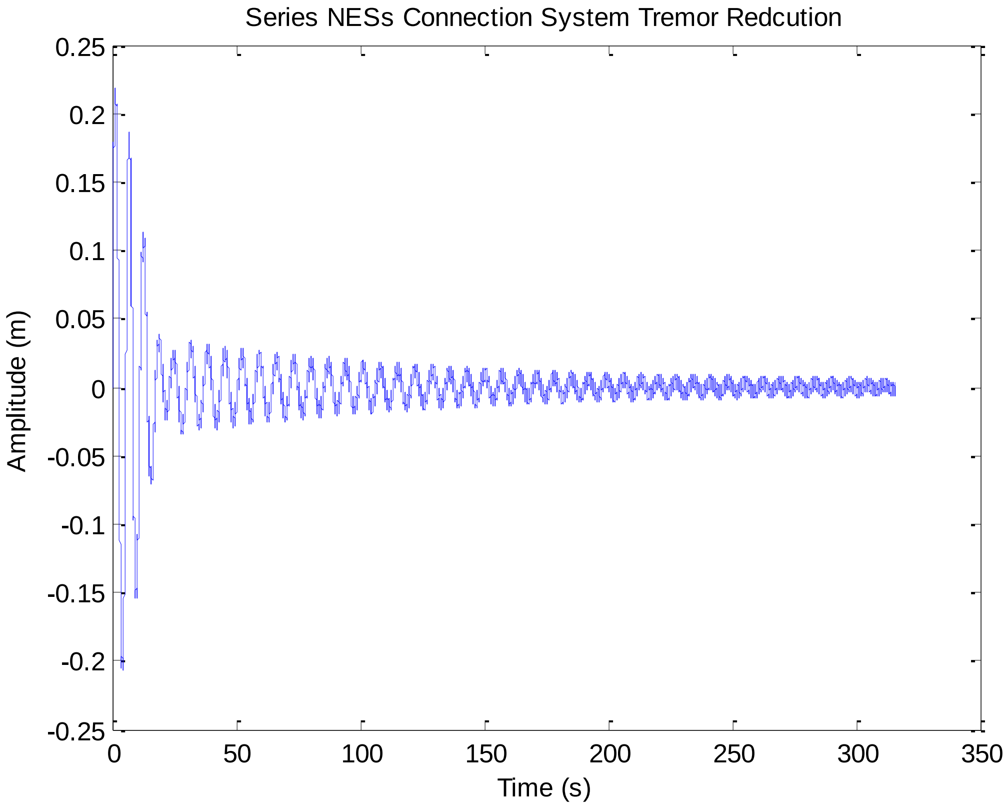


Figure 4. 13 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.891; C3=0.012; k1=1; k2=1; k3=0.6

Time to reduce the tremor amplitude to a 0.05m limit 0.05m is 18.3370s

k1=1; k2=1; k3=0.71	15.5564
k1=1; k2=0.90; k3=0.90	19.6195
k1=1; k2=1; k3=0.6	18.3370

The equation of the parallel NESs connection system is:

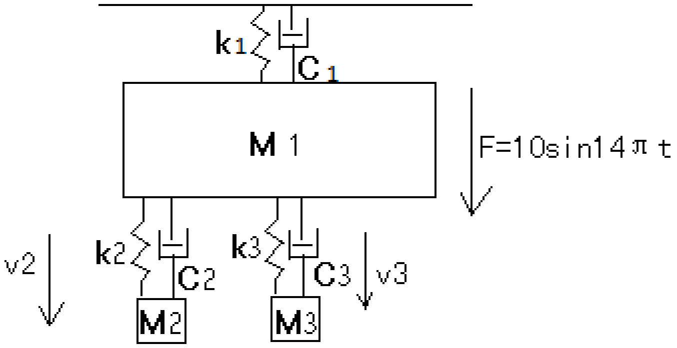


Figure 4. 14 Two NES Parallel Attached Sytem

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + c_3(\dot{x}_1 - \dot{x}_3) + k_1x_1 + k_2(x_1 - x_2)^3 + k_3(x_1 - x_3)^3 - F = 0$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1)^3 = 0$$

$$m_3\ddot{x}_3 + c_3(\dot{x}_3 - \dot{x}_1) + k_3(x_3 - x_1)^3 = 0$$

By using the ode45 codes of Matlab to solve this equation with m1=1; m2=0.1; m3=0.1; C1=0; C2=0.11; C3=0.5; k1=1; k2=1; k3=1;

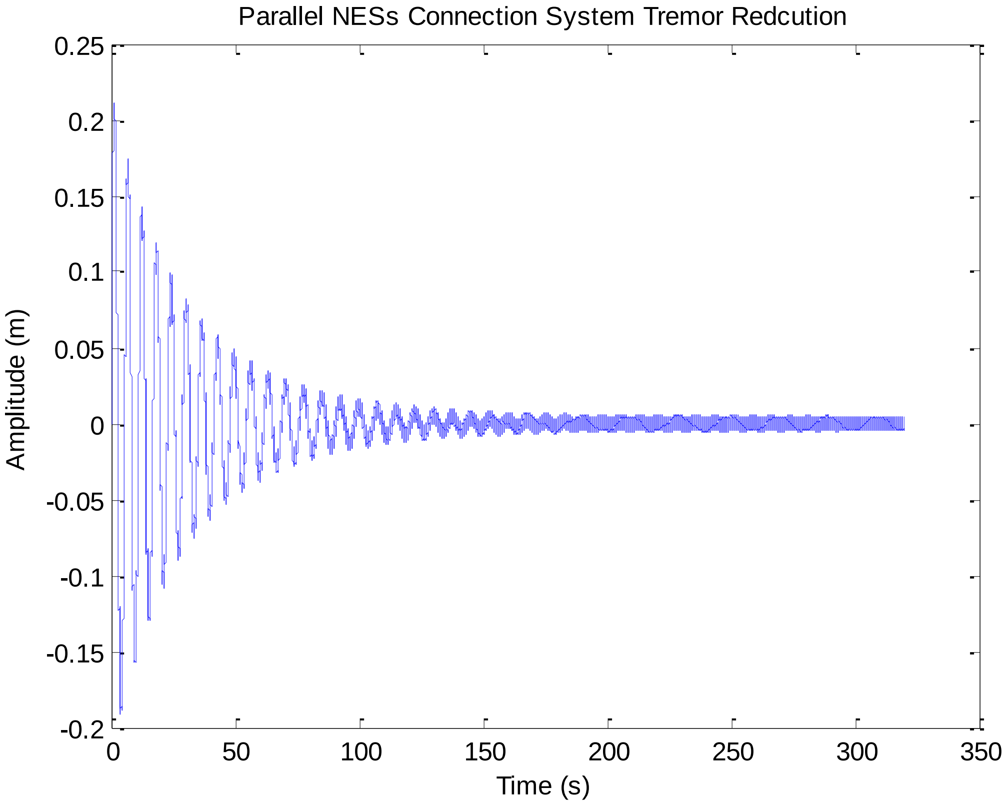


Figure 4. 15 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.1; C1=0; C2=0.11; C3=0.5; k1=1; k2=1; k3=1

The result shows the tremor can be reduced by time. At time 52.7493s, the tremor amplitude reduced 1 / 4 of original amplitude.

From the similar settings, the results show that, tremor amplitude has further reduction efficiency by using two connected non-linear energy sinks. But the components setting was given roughly, in order to do further studying of the model, genetic algorithm has been used to choose the components in the next stage.

Other results – changing damping coefficient

m1=1; m2=0.1; m3=0.0746; C1=0; C2=1; C3=1; k1=1; k2=1.0022; k3=0.9936;

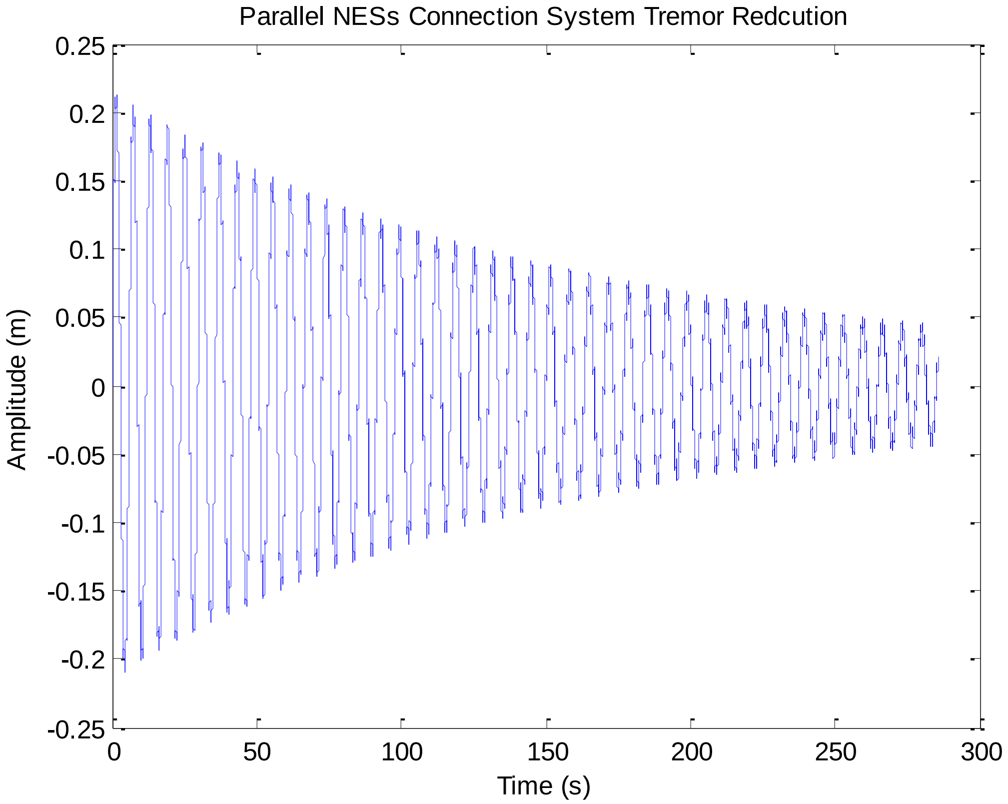


Figure 4. 16 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=1; C3=1; k1=1; k2=1.0022; k3=0.9936

Time to reduce the tremor amplitude to a 0.05m limit 0.05 m is 293.9624s.

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0; C3=0; k1=1; k2=1.0022; k3=0.9936;

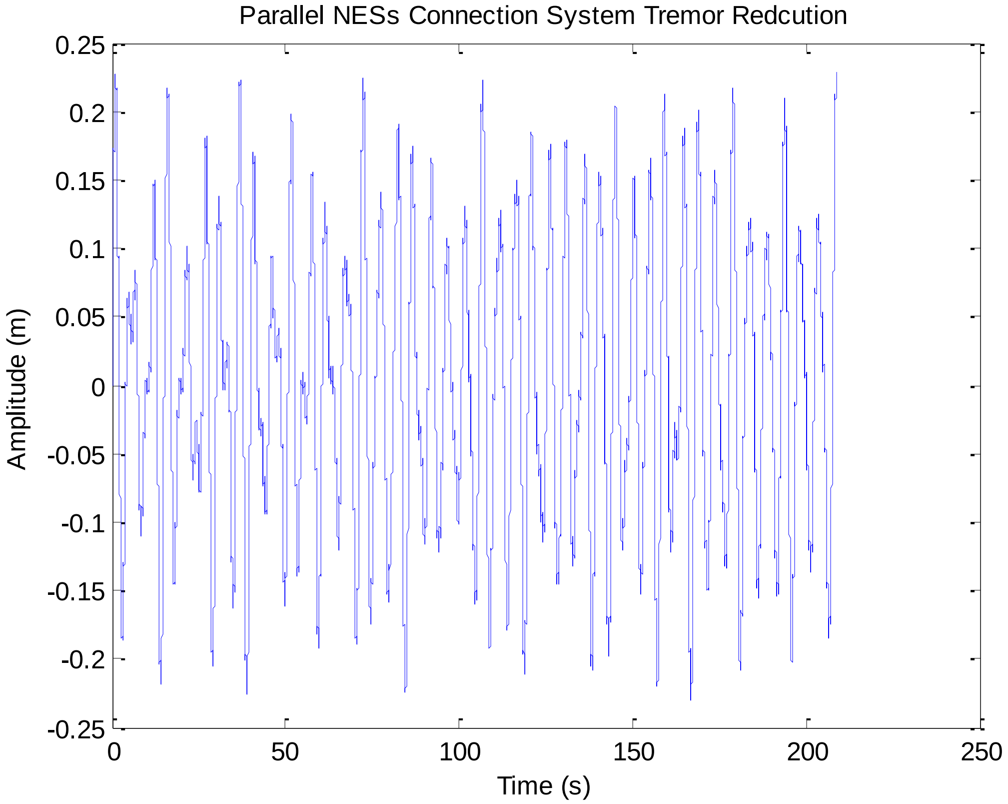


Figure 4. 17 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=0; C3=0; k1=1; k2=1.0022; k3=0.9936

The amplitude will never match the goal.

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.5; C3=0.5; k1=1; k2=1.0022; k3=0.9936;

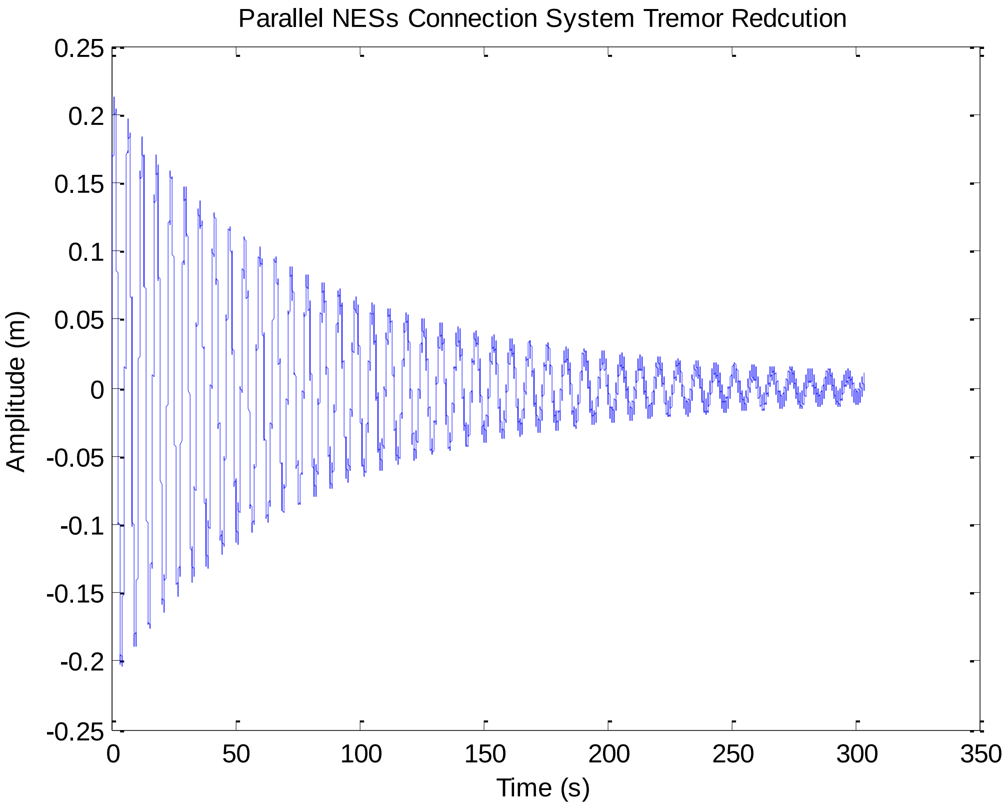


Figure 4. 18 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.5; C3=0.5; k1=1; k2=1.0022; k3=0.9936

Time to reduce the tremor amplitude to a 0.05m limit 0.05 m is 177.8209s.

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.1; C3=0.1; k1=1; k2=1.0022; k3=0.9936;

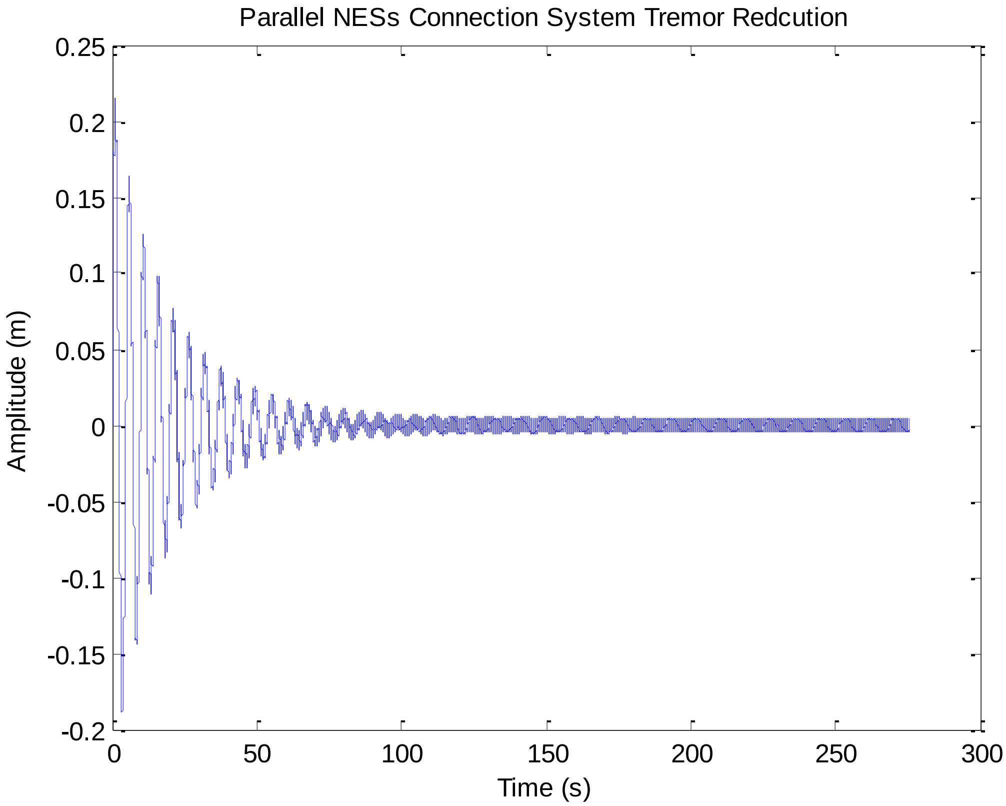


Figure 4. 19 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.1; C3=0.1; k1=1; k2=1.0022; k3=0.9936

Time to reduce the tremor amplitude to a 0.05m limit 0.05 m is 38.3280s.

C1=0; C2=1; C3=1	293.9624
C1=0; C2=0; C3=0	N/A
C1=0; C2=0.5; C3=0.5	177.8209
C1=0; C2=0.1; C3=0.1	38.3280

Other results – changing mass of NESs

m1=1; m2=0.05; m3=0.05; C1=0; C2=0.017; C3=0.012; k1=1; k2=1.0022; k3=0.9936;

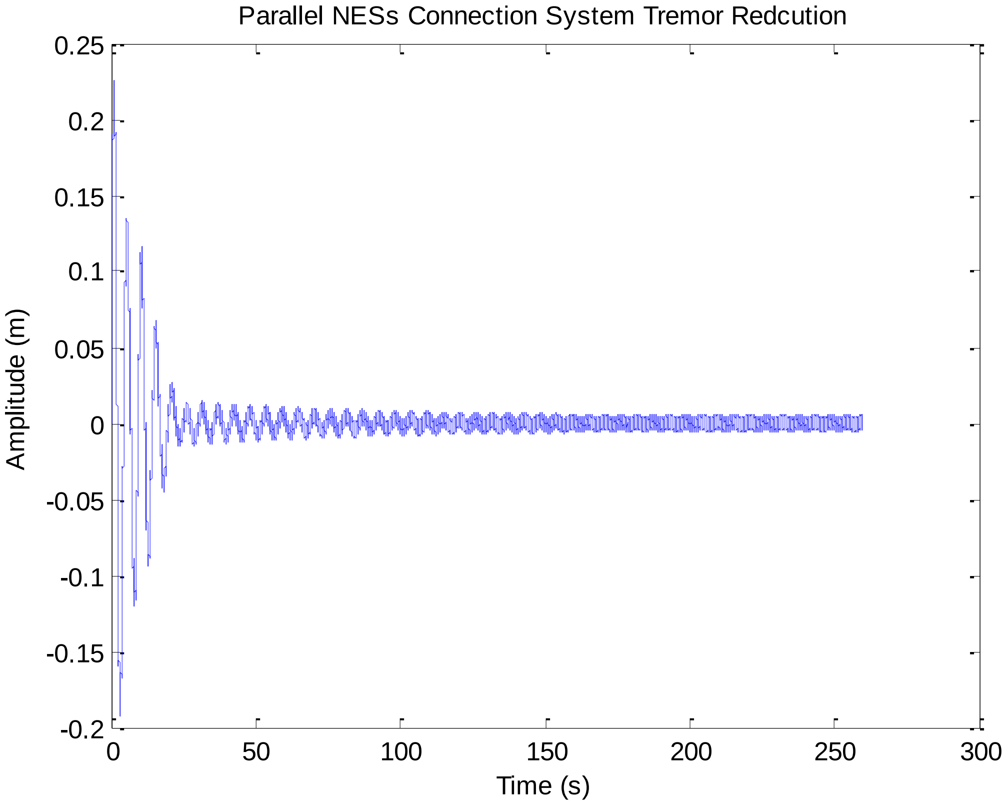


Figure 4. 20 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.1; C3=0.1; k1=1; k2=1.0022; k3=0.9936

Time to reduce the tremor amplitude to a 0.05m limit 0.05 m is 22.9790s.

m1=1; m2=0.1; m3=0.1; C1=0; C2=0.017; C3=0.012; k1=1; k2=1.0022; k3=0.9936;

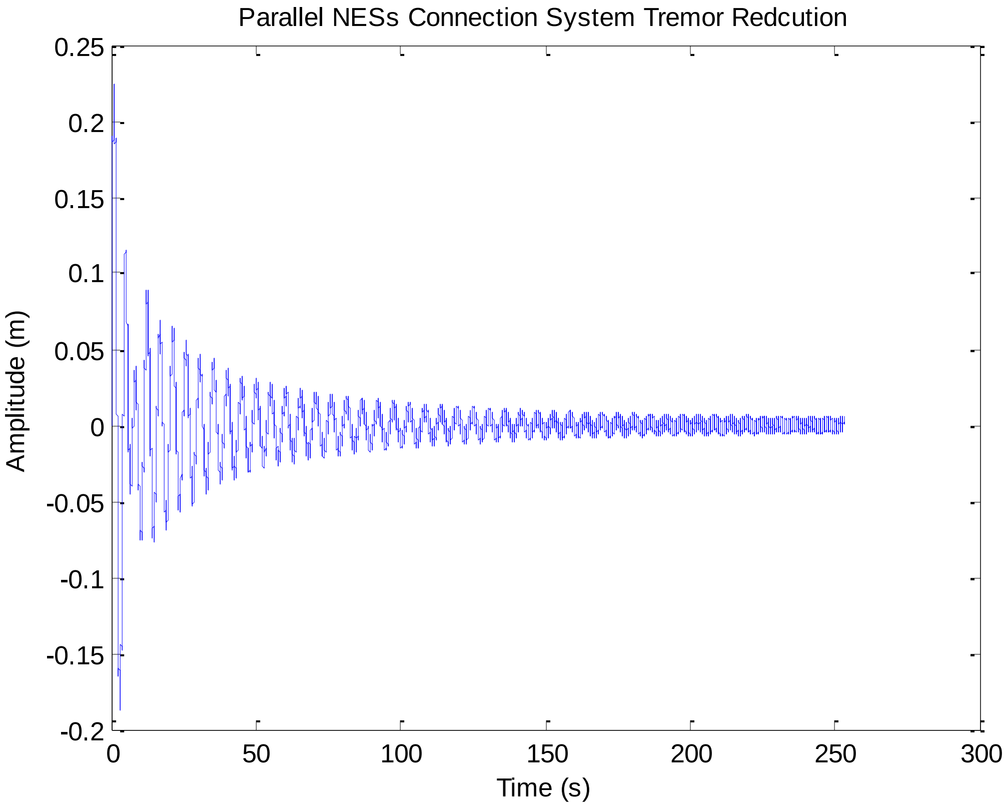


Figure 4. 21 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.1; C1=0; C2=0.017; C3=0.012; k1=1; k2=1.0022; k3=0.9936

Time to reduce the tremor amplitude to a 0.05m limit 0.05 m is 39.3204s.

m1=1; m2=0.15; m3=0.15; C1=0; C2=0.017; C3=0.012; k1=1; k2=1.0022; k3=0.9936;

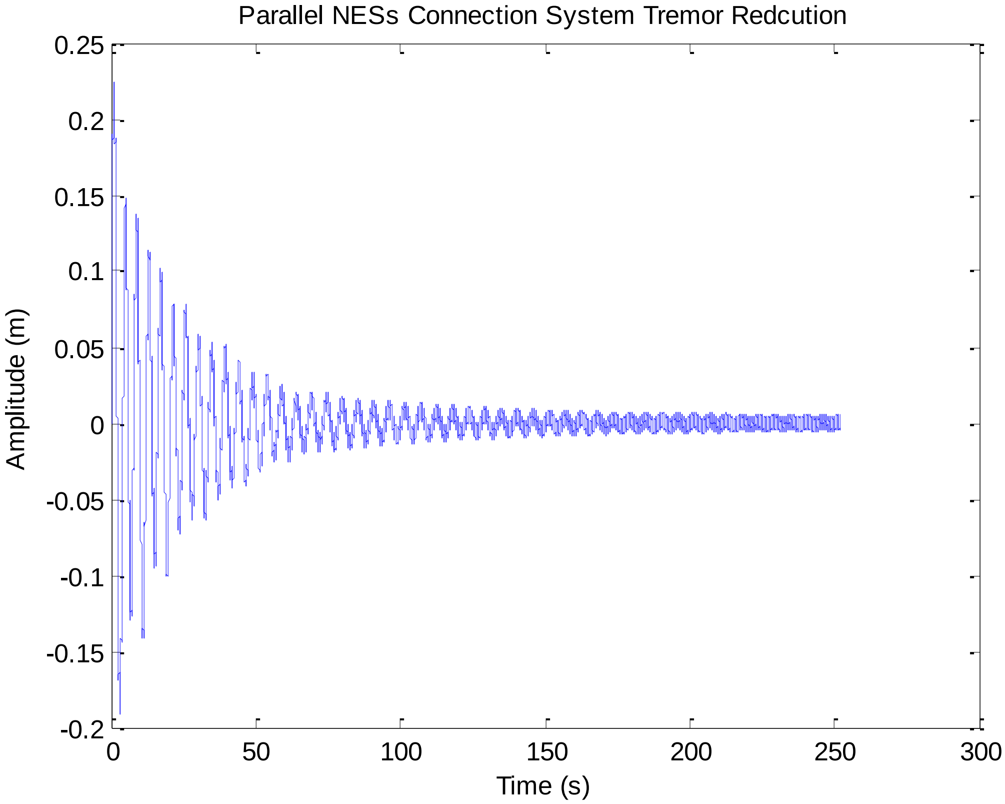


Figure 4. 22 Tremor Reduction with parameters of m1=1; m2=0.15; m3=0.15; C1=0; C2=0.017; C3=0.012; k1=1; k2=1.0022; k3=0.9936

Time to reduce the tremor amplitude to a 0.05m limit 0.05 m is 55.5378s.

m1=1; m2=0.05; m3=0.05	22.9790
m1=1; m2=0.1; m3=0.1	39.3204
m1=1; m2=0.15; m3=0.15	55.5378

Other results – changing stiffness of the springs

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.017; C3=0.012; k1=1; k2=0.5; k3=0.5;

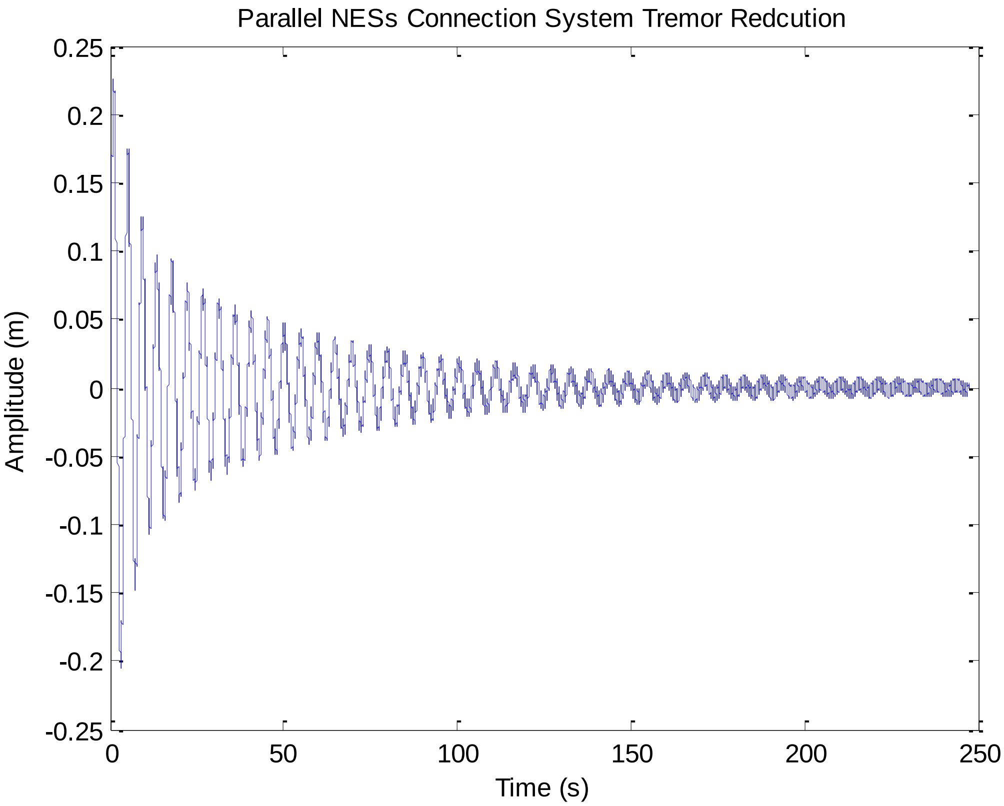


Figure 4. 23 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.017; C3=0.012; k1=1; k2=0.5; k3=0.5

Time to reduce the tremor amplitude to a 0.05m limit 0.05 m is 63.3926s.

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.017; C3=0.012; k1=1; k2=0.15; k3=0.15;

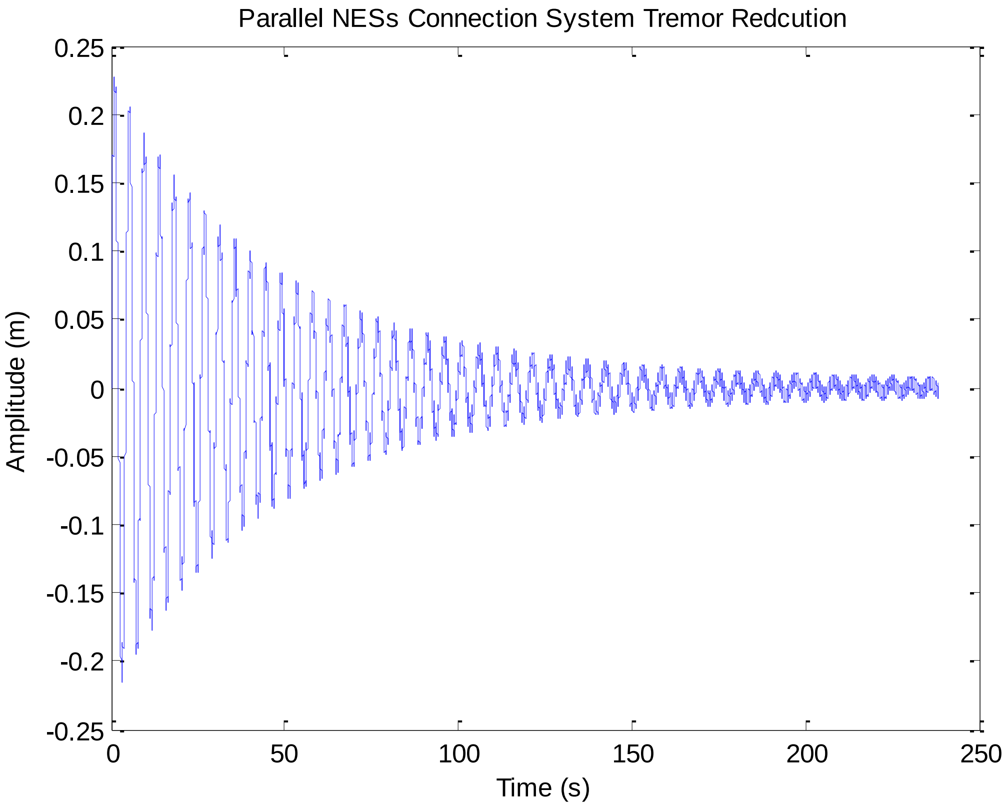


Figure 4. 24 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.017; C3=0.012; k1=1; k2=0.15; k3=0.15

Time to reduce the tremor amplitude to a 0.05m limit 0.05 m is 108.6717s.

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.017; C3=0.012; k1=1; k2=0.1; k3=0.05;

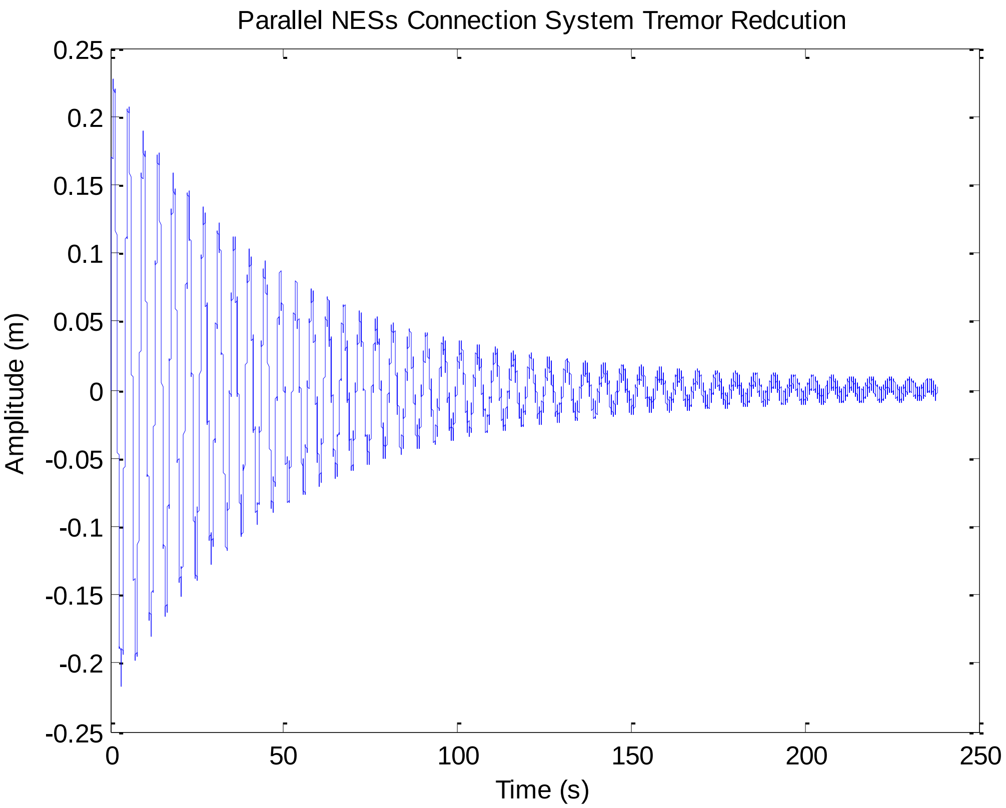


Figure 4. 25 Tremor Reduction with parameters of m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.017; C3=0.012; k1=1; k2=0.1; k3=0.05

Time to reduce the tremor amplitude to a 0.05m limit 0.05 m is 111.8874s.

k1=1; k2=0.5; k3=0.5	63.3926
k1=1; k2=0.15; k3=0.15	108.6717
k1=1; k2=0.1; k3=0.05	111.8874

4.3 Using Genetic Algorithm to Choose Components and Results of the Model

The results in section 4.2 presented different parameters of the masses of the NESs, and the coefficient of damping affects the efficiency of tremor reduction. In order to find the fittest solution of the system, genetic algorithm has been used.

A genetic algorithm (GA) is a search algorithm that mimics the process of natural selection. The algorithm is based on the principles of biological organisms “survival of the fittest.”It is considered as an “intelligent” search heuristic to generate needed solutions to optimisation problems. [76]In 1970s, John Henry Holland originally developed this algorithm. [77] This heuristic presents the individuals fitter to the environment, the better surviving chance they have. Rest individuals will be eliminated. In the terms of this work, the efficiency of vibration reduction means the scores of the system. The highest score system would survive and its component parameters would become the targets to find.[78]

In the genetic algorithm of this research, different sets of three the masses, three parameters of dumping coefficient and three parameters of spring stiffness are considered as population of candidate solutions.

The differential equation of the system is the fitness function. [79]The fitness of every individual in the population is calculated by solving the fitness function. And in each generation, the fitness of every individual in the population is evaluated.[79]

During each generation, a proportion of the sets of parameters (population) were selected to continue for the new generation. By solving the fitness function, individual solutions with high scores (fitness) were more likely to be survived and selected to breed the next generation. [80]The selected parameters were processed crossover and mutation to generate the next generation population of solutions. Some of the parameters have been crossover to another set of population, and some parameters have been changed randomly. [81]By repeating the generation process multiple times, once the solving results reached the most efficiency of the tremor reduction, the generation would be terminated.

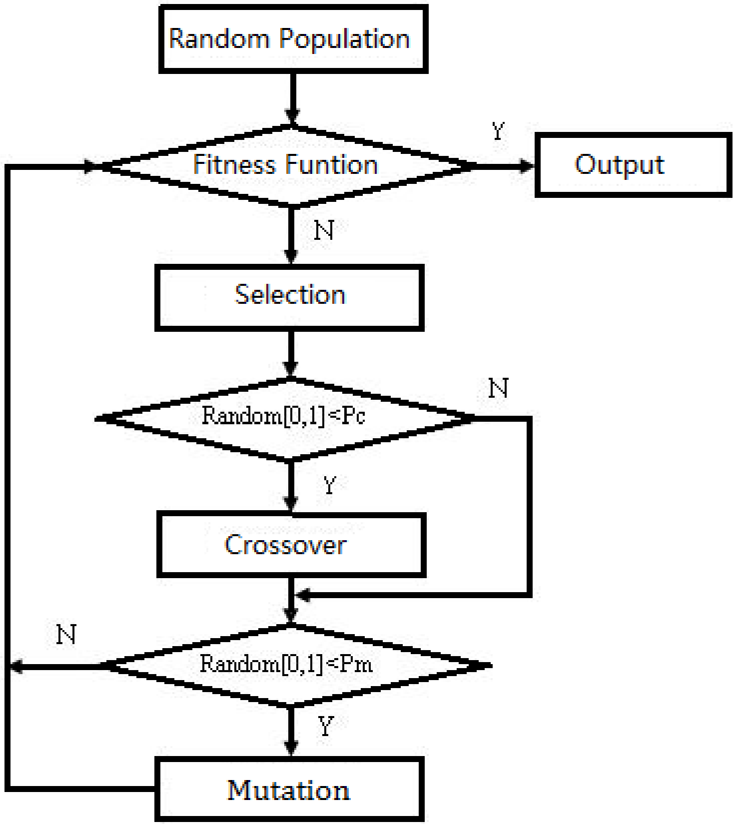


Figure 4. 26 Coding Genetic Algorithm

Best result of the series NESs connection system

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.891; C3=0.012; k1=1; k2=0.90; k3=0.71;

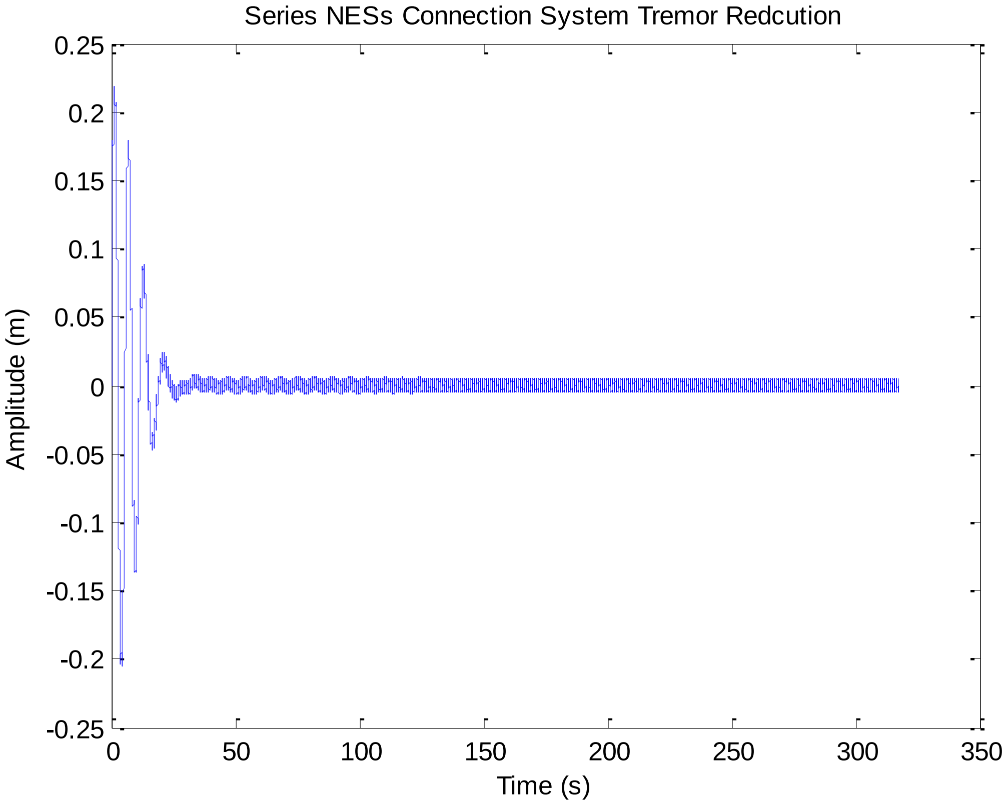


Figure 4. 27 Series NESs Connection System with Best Results

Time to reduce the tremor amplitude to a 0.05m limit 0.05m is 15.5559s.

Best result of the parallel NESs connection system

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.017; C3=0.012; k1=1; k2=1.0022; k3=0.9936;

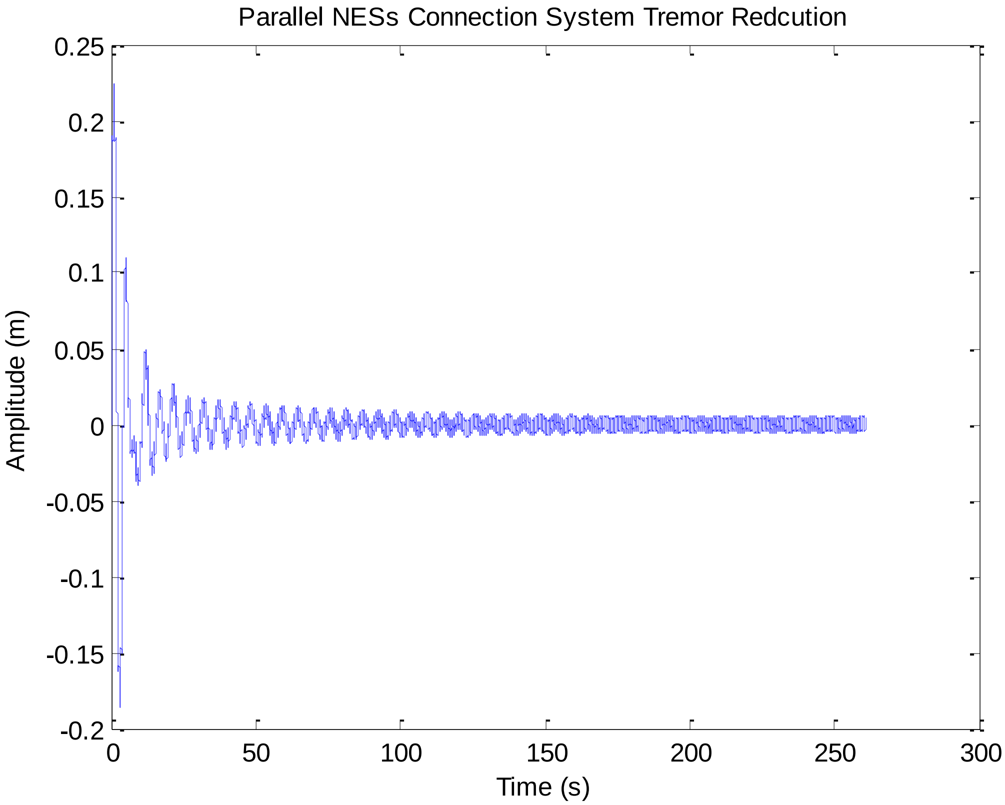


Figure 4. 28 Parallel NESs connection System with Best Results

Time to reduce the tremor amplitude to a 0.05m limit 0.05m is 8.5533s.

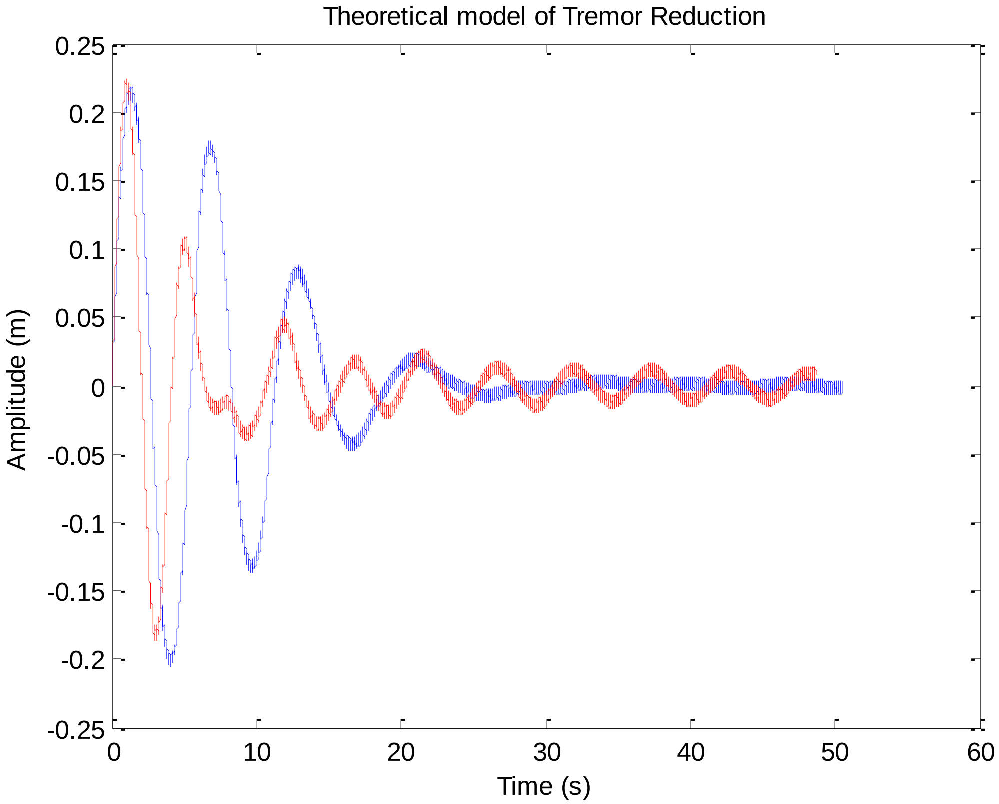


Figure 4. 29 Comparing Two Structure Systems Best Results

Blue curve is series structure; Red curve is parallel structure

By testing and changing the parameters of components other than the best results, the efficiency time to reduce the tremor changing randomly. For the best results, it is known the system of two NESs parallel connected to primary system has slightly shorter time than the system of two NESs series connected to primary system to reduce the tremor for certain amplitude. According the Fig. xx the parallel structure system has advantage on efficiency, and the series structure has the minimum amplitude after the reduction stabled.

Furthermore, a three NESs connected system has been studied. But by considered the weight of many NESs may become a burden for the patients. Also there is no huge change of the efficiency nether in amplitude or time of the tremor reduction, so the study has continued on two NESs systems.

4.4 Conclusion

This chapter presented a numerical method of nonlinear energy transfer with the system of NES(s) connected to a primary system. The method gives a theoretic background to design the experiment and application in Chapter 5. In this chapter, energy transfer between primary system and NES has been proposed. A further development of two NESs connected to primary system models has been made. The developed models were separated into two different structures. Both structures have been theoretic tested by MATLAB.

In the tests, it is found the different parameters components affect the tests results differently. By testing different sets of parameters system, it is found the system results changing very randomly. In order to discover the best match of parameters, Genetic algorithm has been used to choose the finest component parameters. More tests have been done with the new parameter components. The results have been compared and discussed in the last section of the chapter.

In this chapter, the numerical method of nonlinear energy transfer has been tested for later Tremor Reducer designing. The results of different structure of the nonlinear energy transfer will be applied to the design. It is interesting both parallel and series structures have their own feature in vibration reduced. Series structure reduces the amount of displacement more than parallel structure. In another hand, the parallel structure slightly more efficiency in the duration of reduction. By the different features of structures, the later design of the reducer maybe considering to the 'amount of reduction' or the 'efficiency reduction' these two different aims.

Chapter 5

Experimental Work

5.1 Introduction of Nonlinear Vibration Reduction and Application

In this study, nonlinear passive energy transfer is used to transfer the tremor energy from the human body in to an energy pump device, in order to reduce the tremor amplitude of the primary body. This experiment is for testing the energy absorbed by NESs.

According to the study of WOTAS in Chapter 2, tremor is the most common movement disorder in neurological practice. [50]Levodopa reduces tremor caused by certain parts of the brain but as a side effect it causes movement in other parts of the body such as gnawing, chewing, twisting tongue, head bobbing and twitching of shoulders, arm and legs. It also causes psychiatric instability such as anxiety, nervousness, restlessness, memory loss, lack of sleep due to nightmares and mental depression. [66] Propranolol has severe side effects that require serious medical care, the main side effects are allergic reaction to the skin, swelling of the mouth, face and lips, chest pain, sudden numbness, mood swings, sudden weigh gain, change of skin colour in arms and toes and shortness of breath. [67] Primidone also has severe side effects that affect people in different stages. The minor side effects are vomiting and loss of appetite this is because the patient's body is not used to the introduction of the new chemical. The severe side effects are clumsiness in walking, lack of sexual appetite, double vision, depression, behaviour change and suicidal thoughts. [68]These side effects do not occur on all patients, it depends on the patient's medical condition, health status and age.

By avoiding the side effects of medication curing tremor, many exoskeletons are designed for helping tremor patients' daily work and life. Such as WOTAS has been designed in this purpose, this is an exoskeleton device that contain mechatronic system that can detect, control and exchange information about the patient's disability such as tremor, fractured bone and arthritis, this device helps to evaluate the patient's disability stage, calculate the tremor frequency, tremor torque and power and gives out data is meaningful and understanding form. [69]

Another product of wearable essential tremor suppression is a simple device that terminates tremor by using dampers. The Despots contains two syringes that act as a damper; the ends of the syringes are connected to the frame and finger loop rail therefore every time the patient tries to move their hands the syringes acts as a damper and reduce the movement hence reducing tremor. The damper absorbs the vibrations and reduces the movement. The stiffness of the hinge joint depends on the density of the liquid in the syringe. [70]

In our study, the energy pump has been designed for the application of tremor reduction. Different from the exoskeleton devices, the application using the resonance to transform the unwanted nonlinear energy from the tremor hands of the patients to the energy absorbers (as also called nonlinear energy sink, NES).

In this chapter, a theoretical model of vibration reduction by using energy pump has been made. Genetic Algorithm has been used to choose the components of the model. Results are taken for showing the tremor has been reduced, and the results are compared for discuss.

5.2 Experiment set up

As shown in the photos below, in the experiment, the Tektronix TDS5034B Digital Phosphor Oscilloscope has been connected to a Brüel & Kjær Type 4506 accelerometer for recording the vibration amplitude of primary system.

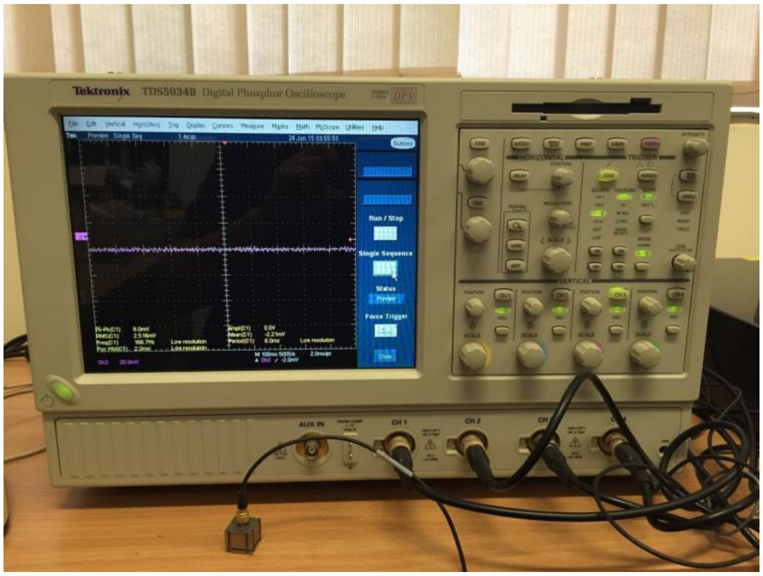


Figure 5. 1 TDS5034B Digital Phosphor Oscilloscope

The signal output devices are TTI TG550 Function Generator which is attached with Brüel & Kjær Power Amplifier Type 2719.

Figure 5. 2 Function Generator and Measurement Exciter

The function generator sends the signals into the amplifier. A Brüel & Kjær type 4808 measurement exciter creates the vibration reflects the signals from the amplifier.

Figure 5. 3 Equipped Testing

The direct wave form results have been gathered with the accelerometer and precented by the didtial phosphor oscilloscope. The results would be taken in the form of wave. Five sets of the results have been taken from this experiment.

A human hand model has been used as the primary system for the experiment. According to the results from Chapter 4, two NESs have been produced for passive reducing the nonlinear energy from the primary system.

Figure 5. 4 Testing Arm and Gear

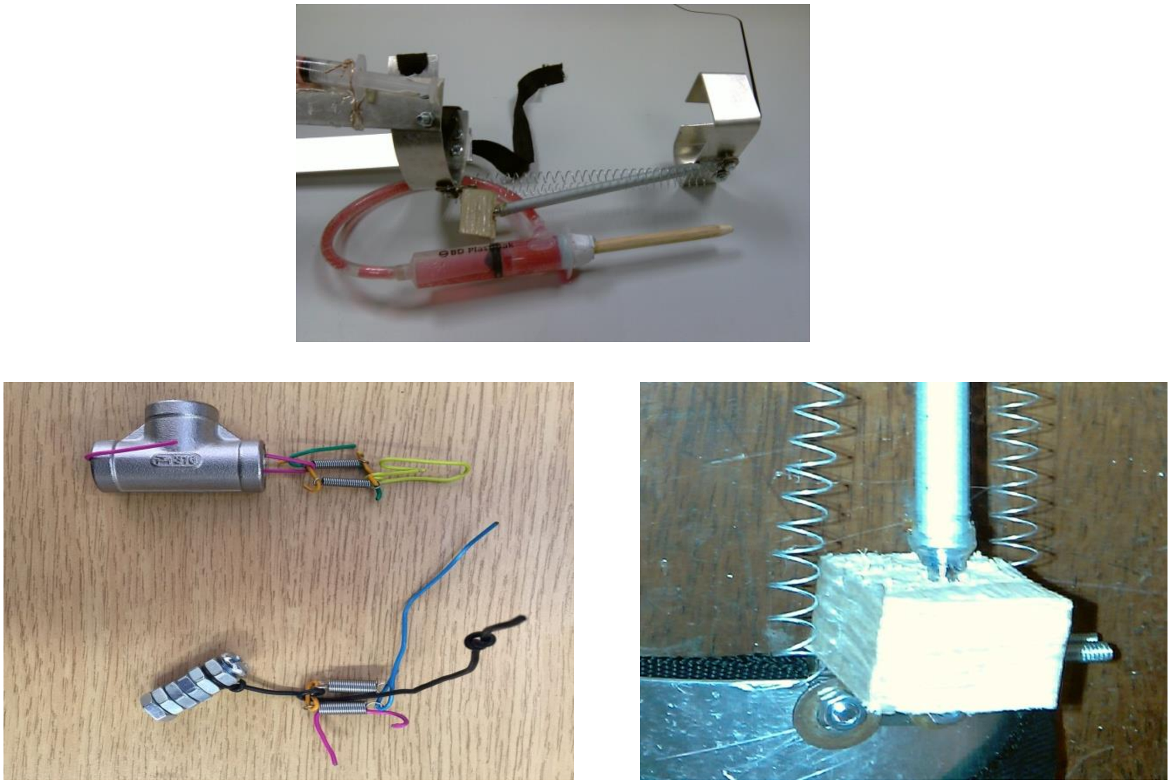
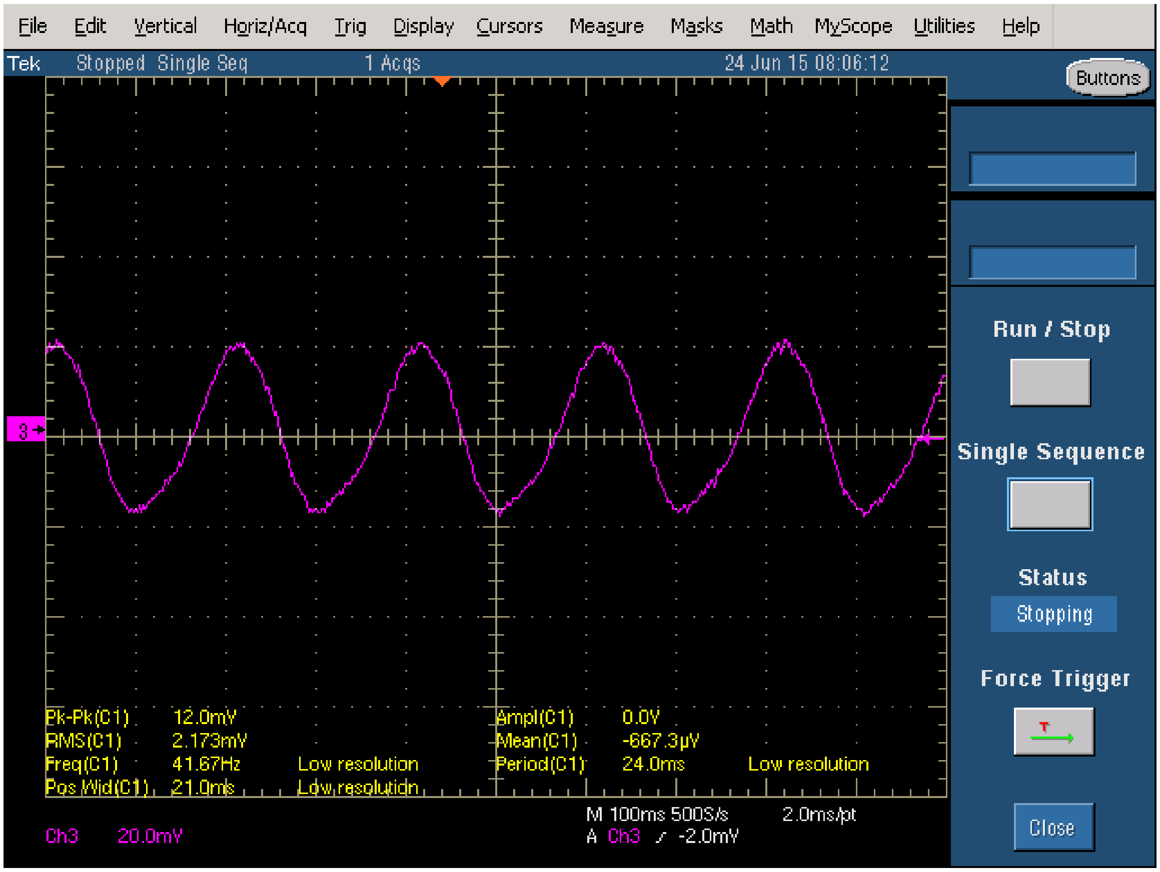


Figure 5. 5 Nonlinear Energy Sinks

5.3 Experiment Simulations and Results

The experiments are based on the numerical methods in Chapter 3. Few results from few experiments have been compared with the numerical system presented results.



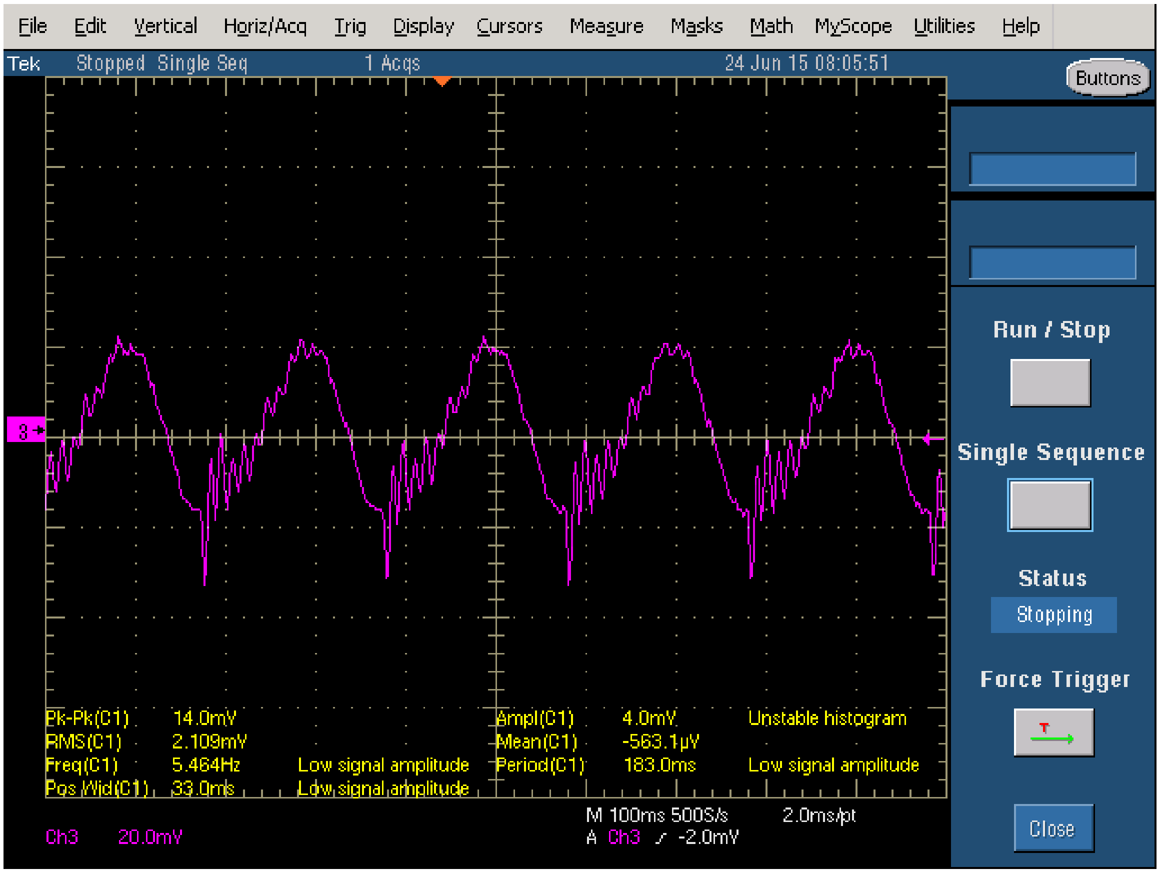
```
Tek    Stopped  Single Seq          1 Acqs                    24 Jun 15 08:06:12

Pk-Pk(C1)     12.0mV                               Ampl(C1)     0.0V
RMS(C1)       2.173mV                              Mean(C1)     -667.3µV
Freq(C1)      41.67Hz     Low resolution           Period(C1)   24.0ms      Low resolution
Pos Wid(C1)   21.0ms      Low resolution
                                                       M 100ms 500S/s       2.0ms/pt
Ch3      20.0mV                                        A  Ch3  ⁄  -2.0mV
```

Figure 5. 6 Linear energy output without NES

Different result method has been used in this chapter compare with Chapter 3. It is found the real life factors such as the angel of the spring, the moving direction of the measurement object are uncontrollable. The environment affects the result every different. Therefore, it has been decided to use the displacement reduction to display as the results. But the finding parameters from Chapter 4 have been tested in this section. The results may not display the efficiency of tremor reduction, but the amount of reduction has been calculated.

Firstly, a linear energy wave has been generated. The primary system has been vibrated under the condition of 4.94 Hz. No nonlinear impact or energy pumper has been applied on the system. According to the reading of the results, the maximum amplitude of the linear wave is ±0.19m. The result refers the direct linear wave signal of the primary system.

Secondly, a force impact has been applied on the primary system for causing the nonlinearity signal for later use.



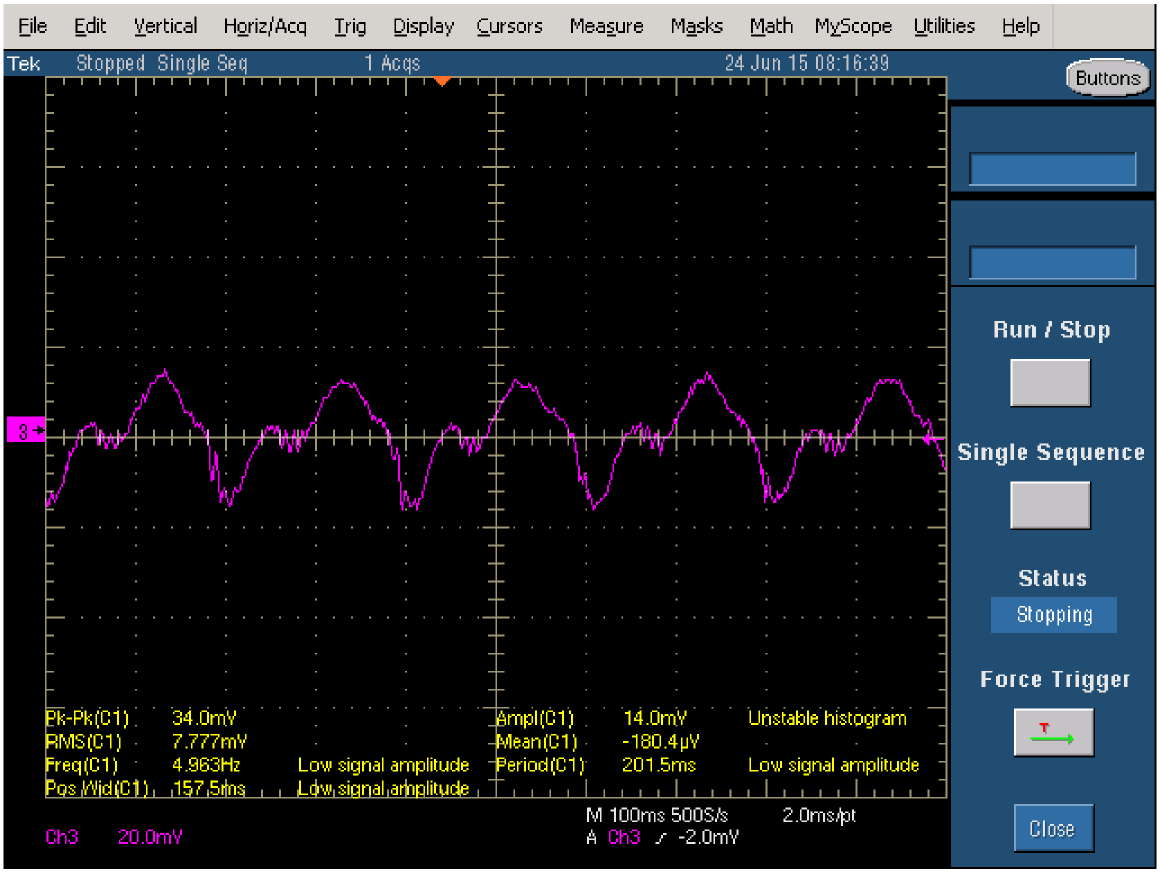
```
Pk-Pk(C1)     14.0mV                          Ampl(C1)     4.0mV      Unstable histogram
RMS(C1)       2.109mV                         Mean(C1)     -563.1µV
Freq(C1)      5.464Hz   Low signal amplitude  Period(C1)   183.0ms    Low signal amplitude
Pos Wid(C1)   33.0ms    Low signal amplitude
                                              M 100ms 500S/s        2.0ms/pt
Ch3      20.0mV                               A  Ch3  ⟋  -2.0mV
```

Figure 5. 7 Primary system with nonlinear energy wave

The peak readings of the nonlinear energy wave are +0.22m and -0.31m. According to the readings in figure 5.7, nonlinearity is shown at the peaks of the signal waves.

In Chapter 3, a numerical system of one NES attached the primary system has been presented. A related experiment has been done for comparing the results.

The primary system (the arm model) has been attached by a single NES. The system has been running with nonlinear signal wave under the impacted force.

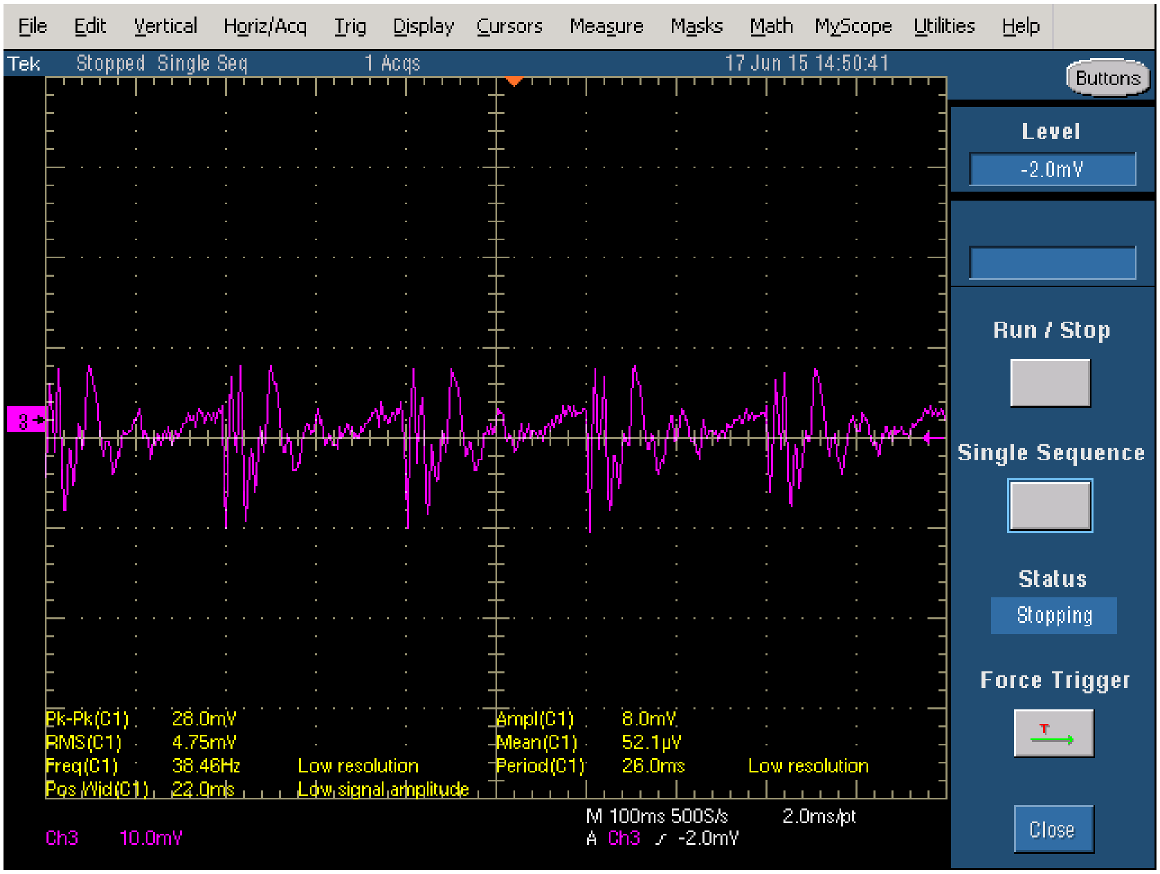


```
Tek   Stopped  Single Seq          1 Acqs                    24 Jun 15 08:16:39

Pk-Pk(C1)    34.0mV                              Ampl(C1)     14.0mV       Unstable histogram
RMS(C1)      7.777mV                             Mean(C1)     -180.4µV
Freq(C1)     4.963Hz     Low signal amplitude    Period(C1)   201.5ms      Low signal amplitude
Pos Wid(C1)  157.5ms     Low signal amplitude
                                                        M 100ms 500S/s       2.0ms/pt
Ch3     20.0mV                                          A  Ch3  ⟋  -2.0mV
```

Figure 5. 8 One NES attached on primary system with impacted nonlinear vibration

From the figure 5.8 the peak readings of the nonlinear energy wave are +0.16m and -0.158m. Not as well as the numerical method, the vibration has been reduced 20%.

The primary system (the arm model) has been attached by a two NES series. The system has been running with nonlinear signal wave under the impacted force.

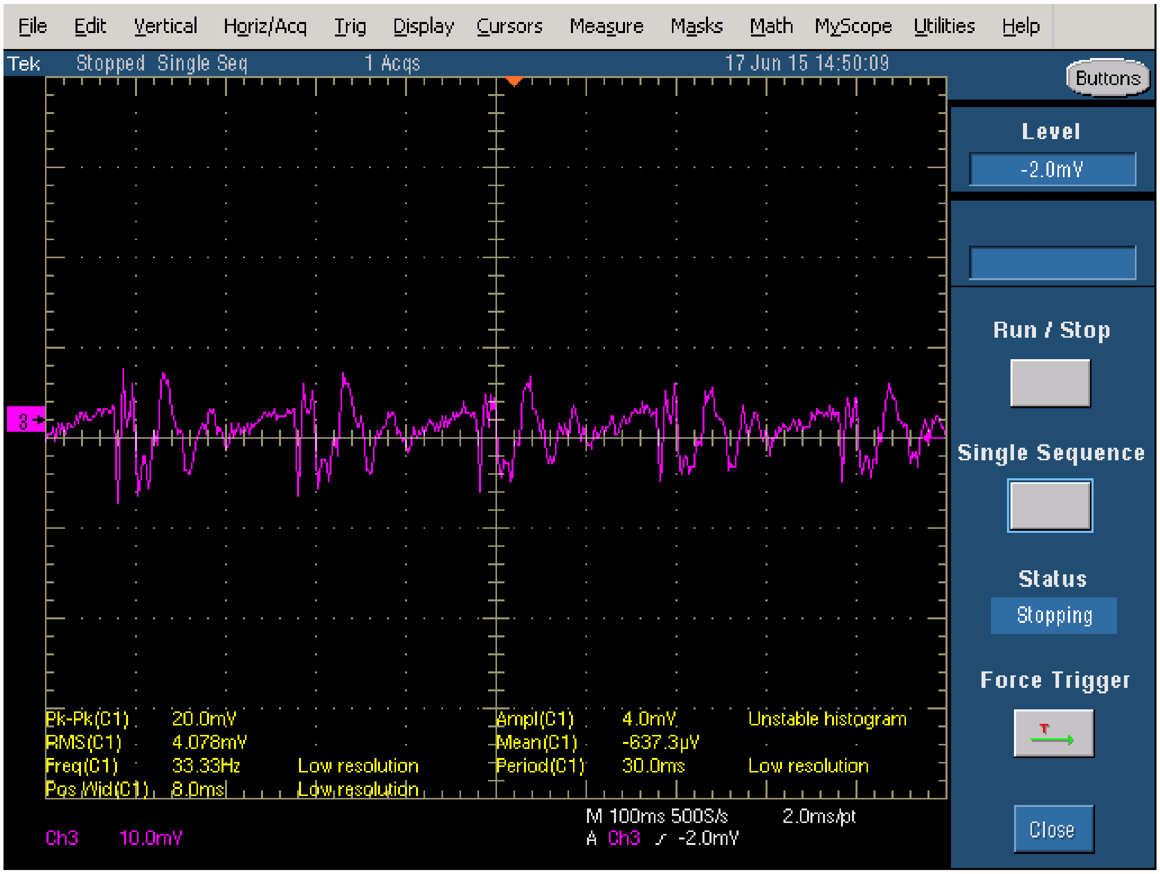


```
Stopped  Single Seq          1 Acqs                17 Jun 15 14:50:41
Level -2.0mV
Status Stopping
Pk-Pk(C1)     28.0mV                            Ampl(C1)     8.0mV
RMS(C1)       4.75mV                            Mean(C1)     52.1µV
Freq(C1)      38.46Hz   Low resolution          Period(C1)   26.0ms    Low resolution
Pos Wid(C1)   22.0ms    Low signal amplitude
                                                M 100ms 500S/s       2.0ms/pt
Ch3      10.0mV                                 A  Ch3  ⌿  -2.0mV
```

Figure 5. 9 Two NESs series connected with the primary system

From the figure 5.9 the peak readings of the nonlinear energy wave are +0.16m and -0.156m. For the peak readings, the maximum amplitude has been reduced 20% of total. Unlike the single NES attached system. Two NESs series connected system also changed the wave form of the signal. The wave has become nonlinearity and the amplitude of the most part of the wave has been significantly reduced.

The primary system (the arm model) has been attached by a two NES series. The system has been running with nonlinear signal wave under the impacted force.

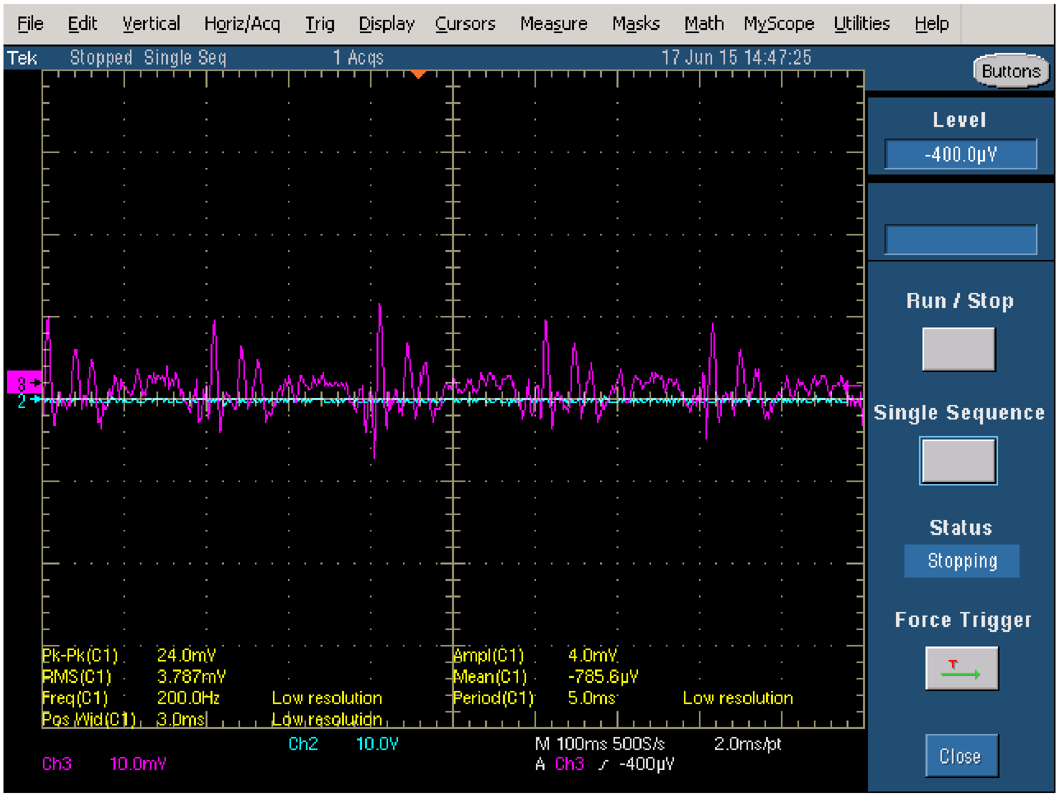


```
Tek   Stopped  Single Seq          1 Acqs                  17 Jun 15 14:50:09

Level -2.0mV

Pk-Pk(C1)      20.0mV                           Ampl(C1)      4.0mV       Unstable histogram
RMS(C1)        4.078mV                          Mean(C1)      -637.3µV
Freq(C1)       33.33Hz      Low resolution      Period(C1)    30.0ms      Low resolution
Pos Wid(C1)    8.0ms        Low resolution

                                                M 100ms 500S/s      2.0ms/pt
Ch3       10.0mV                                A  Ch3  ⎍  -2.0mV
```

Figure 5. 10 Parallel NESs Connection System

From the figure 5.10the peak readings of the nonlinear energy wave are +0.155m and -0.15m. For the peak readings, the maximum amplitude has been reduced 22.5% of total. Similar as the two NESs series connected system also changed the wave form of the signal, the two NESs parallel connected system changed the wave into nonlinearity and the amplitude of the most part of the wave has been significantly reduced.

Another experiment has been presented with two NESs parallel connected to the primary system. The components parameters have been chosen with the best result from the latest study of Chapter 3.



```
Tek   Stopped  Single Seq        1 Acqs                   17 Jun 15 14:47:25
Level  -400.0µV
Status  Stopping

Pk-Pk(C1)     24.0mV                          Ampl(C1)      4.0mV
RMS(C1)       3.787mV                         Mean(C1)      -785.6µV
Freq(C1)      200.0Hz    Low resolution       Period(C1)    5.0ms      Low resolution
Pos Wid(C1)   3.0ms      Low resolution
                         Ch2    10.0V              M 100ms 500S/s     2.0ms/pt
Ch3     10.0mV                                     A  Ch3  ⨍  -400µV
```

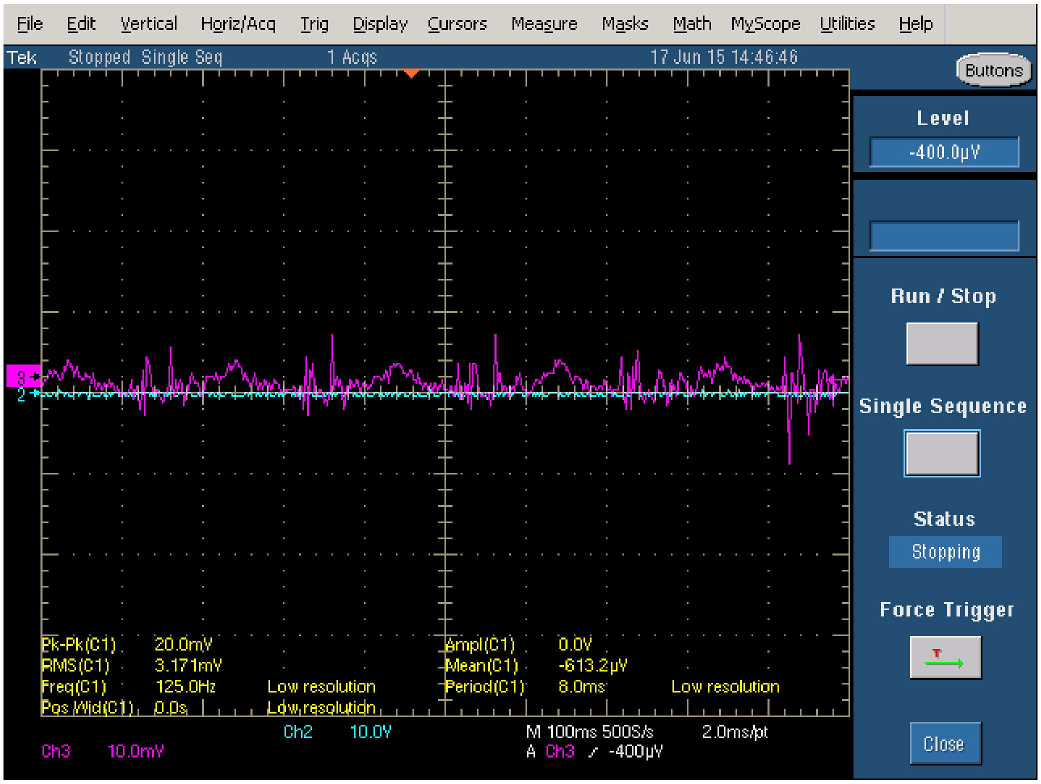
Figure 5. 11 Changing Components with the Best Results for Parallel Structure

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.891; C3=0.012; k1=1; k2=0.90; k3=0.71;

From the figure 5.11 the peak readings of the nonlinear energy wave are +0.22m and -0.14m. For the peak readings, the maximum amplitude has not been reduced, but the lowest peak has been reduced by 54.8% of total. Similar as the two NESs series connected system also changed the wave form of the signal, the system changed the wave into nonlinearity and the amplitude of the most part of the wave has been significantly reduced, and smoother than the random parameter selections.

Last experiment is with two NESs series connected to the primary system. The components parameters have been chosen with the best result from the latest study of Chapter 3.

m1=1; m2=0.1; m3=0.0746; C1=0; C2=0.017; C3=0.012; k1=1; k2=1.0022; k3=0.9936;



```
Pk-Pk(C1)     20.0mV                        Ampl(C1)     0.0V
RMS(C1)       3.171mV                       Mean(C1)     -613.2μV
Freq(C1)      125.0Hz    Low resolution     Period(C1)   8.0ms      Low resolution
Pos Wid(C1)   0.0s       Low resolution
                         Ch2    10.0V                    M 100ms 500S/s    2.0ms/pt
Ch3     10.0mV                                           A  Ch3  ⁄  -400μV
```

Figure 5. 12 Changing Components with the Best result Series Structure

From the figure 5.12 the peak readings of the nonlinear energy wave are +0.14m and -0.2m. For the peak readings, the maximum amplitude has not been reduced, but the lowest peak has been reduced by 70% of total. The signal wave becomes nonlinearity. And by comparing with all the experiments results, this figure shows the smoothest wave form.

By comparing the results from the numerical methods, the results from the experiments did not accurately refer to the numerical. Several environment conditions may affect the reading results. But similar to completion of the numerical method, the experiments presented a vibration reduction of the primary system. Furthermore, they also approved two NESs parallel connected system presents the best results.

Some uncontrollable problems have been discovered during the experiments. Once the vibration started, the masses of the NESs resonance not only in the vertical direction but roughly random. This problem may affect the energy absorbing.

5.4 Conclusion

This chapter is referred the theoretic method has been developed in Chapter 3. The aim of this chapter is designing an application for tremor reduction using the targeted energy transfer. The human tremor has been introduced in this chapter. The unwanted energy is aimed to transfer passively by using the nonlinear energy pumping. The experiment has been set up referred to Chapter 3. First testing experiment is with the theoretic method with one NES attached to primary system. The results have been taken for further use. Two other experiments have been done with two different structures of two NESs attached to primary system. Different parameters components have been tested in the experiments. The results are compared and discussed. Due to affects of the testing environment, the efficiency of the tremor reducer model is uncontrollable. Another method has been used to test the amount of tremor reduction. The presented results of model testing, the series connection system reduced larger amount of tremor (70% of original tremor reduction) than parallel connection system (54% of original tremor reduction). These outcomes showed same results were presented from Chapter 4. Refer to the result of chapter 4, some more tests have been taken on the designed model, and the results showed chapter 4 best parameter components are able to consider using in future design.

Chapter 6

Conclusions and Future Work

6.1 Conclusions and Innovations of the Research

Targeted energy transfer has been investigated in theoretically and experimentally in this thesis. The theoretical simulation results and experimental results are presented, compared and discussed.

In first chapter of the thesis, targeted energy transfer and nonlinear energy pumping has been introduced briefly. In the introduction, the system of energy transfer between a linear oscillator and an energy absorber has been presented. Nonlinear energy sink has been briefly introduced in this section. This application has been used in both theoretical methods and experimental tests. TMD has been introduced and compared with NES. The result of comparing offered a background of NES using lately. Tremor reduction has been presented for later application designing and developing. As an application of nonlinear vibration reduction, tremor reduction has been selected as an object to solve with the research. The inconveniences of the patients have been introduced in this section. The direction of designing the application refers to these reasons.

Literature has been reviewed in the second chapter. Published studies of targeted energy transfer have been reviewed. These studies offered core theory of building theoretic model in Chapter 3. The studies introduced field of targeted energy transfer progressing, nonlinear energy attachments being used. This offers a strong background for presenting the numerical method lately. The development of numerical methods of dynamics of NES system has been studied. The equations in Chapter 3 are referred to section. The tremor assessment history has been studied in the last part of this chapter. Different kinds of applications have been studied. These studies offered idea of later design of tremor reduction application. WOTAS gives a remarkable background in theory of the application in Chapter 5.

Based on the reviews from the literature, a study of nonlinear linear energy pumping dynamics has been investigated. Chapter 3 of this thesis has presented a early study of the dynamics. Followed the literature in Chapter 2, a two LOs each attached a NES system has been investigated. This offered an early idea to design the numerical method in Chapter 4.

In Chapter 4 a numerical method has been developed to study nonlinear energy transfer. This method is based on Vakakis' theory which has been reviewed in Chapter 2. In this chapter, a single NES attached to primary system has been presented. By solving the equations by MATLAB, results have been gathered. Lately a development of this system has been tested. Two NESs attached to primary system has been investigated. In this development, two different structure systems have been created, two NESs series connected to primary system and two NESs parallel connected to primary system. Two systems have been tested by MATLAB, the results of different has been presented in Chapter 3. In the tests of system, it is found the different parameters of the components affect the result significantly. Later in section, in order to figure out the best results of the system, genetic algorithm has been introduced and used for picking the parameters of components.

After the parameters have been chosen, another set of tests has been done. Furthermore, some more tests have been done with the non-chosen parameters. In the final section of this chapter, the results have been compared and discussed. This offered theoretical method to design the application in Chapter 5.

By using the knowledge from the reviewed literature in Chapter 2 and theoretical method in Chapter 3, Chapter presented the designed application of tremor reduction. An experiment of application model has been designed. This chapter presents a single NES attached to primary system experiment for the start. The results have been taken and compared with the theoretical results from Chapter 3. Similar to Chapter 3, a set of experiment has been done with two different structure systems: Two NESs series connected to primary system and two NESs parallel connected to primary system. By using the chosen parameters components from Chapter 3, the systems of two different structures have been tested. The results have been compared and discussed in the final section of the chapter.

6.2 Suggested Further Work

For the further work, the model of the tremor reduction assessment would be built more accurate. The result of testing the application model did not show as well as the theoretic results. The environment of the experiment in Chapter 5 is very limited. The vibration direction of the mass on the NESs are required a further control. Both the numerical testing and experiments in the research have been implemented in this thesis. In order to create an application of device using the theory requires following work. The work up to now is based on 2-DOF mode, but human body tremor is more complicated than this mode. A further development is required for final application designed.

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