Stochastic Anti-Resonance in a Fibre Raman Amplifier

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***Abstract*—Stochastic anti-resonance, that is resonant enhancement of randomness caused by polarization mode beatings, is analyzed both numerically and analytically on an example of fibre Raman amplifier with randomly varying birefringence. As a result of such anti-resonance, the polarization mode dispersion growth causes an escape of the signal state of polarization from a metastable state corresponding to the pulling of the signal to the pump state of polarization.This phenomenon reveals itself in abrupt growth of gain fluctuations as well as in dropping of Hurst parameter and Kramers length characterizing long memory in a system and noise induced escape from the polarization pulling state. The results based on analytical multi-scale averaging technique agree perfectly with the numerical data obtained by direct numerical simulations of underlying stochastic differential equations. This challenging outcome would allow replacing the cumbersome numerical simulations for real-world extra-long high-speed communication systems.**

Keywords*—*stochastic anti-resonance; fibre Raman amplifier; telecommunication systems; multi-scaling; stochastic differential equations

I. Introduction

Most processes in nature are affected by noise leading to a complexity of dynamics. Noise cannot be considered as only disturbing factor because it can modify essentially phase space and evolution of a system. One has to note three main phenomena in this context: i) *stochastic resonance*, when a noise enhances system response to an external periodic perturbation, ii) *stochastic anti-resonance*, when an external periodic perturbation enhances a noise action, and iii) *coherence resonance*, when a noise creates some new coherent-like states in a system [1,2].

In this work we present the results of extensive numerical simulations of a fibre Raman amplifier with inherent stochastic birefringence, which demonstrates the resonant enhancement of noise within a confined range of the polarization mode dispersion parameter. Such an enhancement manifests itself through resonance-like growth of gain fluctuations due to escape of the signal state of polarization from the "polarization trapping" state [3], threshold-like dropping of the Kramers length and the Hurst parameter. The average polarization state remains "localized" but its sensitivity to an input state of polarization vanishes with the growth of polarization mode dispersion parameter. In this sense, a Raman gain in the vicinity of stochastic anti-resonance plays a role of a "depolarizer" reducing the polarization dependent gain.

II. Model

The model for analysis of fibre Raman amplifier with randomly varying birefringence is based on the results of [4,5]. Transition to the reference frame, in which the birefringence vector on the Poincaré sphere is $\boldsymbol{W}_i = (2b_i,0,0)$, results in representation corresponding to random wandering of $\boldsymbol{W}_i$ in horizontal plane and rotation of the unit signal $\mathbf{s} = (s_1, s_2, s_3)$ (i=s) and pump $\mathbf{p} = (p_1, p_2, p_3)$ (i=p) Stokes vectors around the birefringence vector with the frequencies b_i=$2\pi/L_i$ (L_i is a polarization beat-length). Excluding of average scalar gain by the means of normalization of signal power to $G_{ave} = \exp\left(\int_0^L gP_0(z')/2\,dz' - \alpha_s L\right)$ (L is a fibre length, g is a Raman gain coefficient, α_s is a signal loss coefficient, $P_0(z) = P_{in}\exp(-\alpha_p z)$ is a pump power, α_p is a pump loss coefficient, and P_{in} is an input pump power) results in the following set of stochastic differential equations describing an evolution of signal and pump states of polarization [5]:

$$\begin{aligned}
&\frac{ds_0}{dz} = \frac{g}{2}P_0(z)x,\\
&\frac{dx}{dz} = \frac{g}{2}P_0(z)s_0 - 2(b_p - b_s)(p_3 s_2 - p_2 s_3),\\
&\frac{d\mathbf{s}}{dz} = \frac{g}{2}P_0(z)s_0\mathbf{p} + 2b_s\begin{pmatrix}0\\ -s_3\\ s_2\end{pmatrix} + \beta\begin{pmatrix}s_2\\ -s_1\\ 0\end{pmatrix},\\
&\frac{d\mathbf{p}}{dz} = 2b_p\begin{pmatrix}0\\ -p_3\\ p_2\end{pmatrix} + \beta\begin{pmatrix}p_2\\ -p_1\\ 0\end{pmatrix},\\
&\frac{d\theta}{dz} = \beta(z).
\end{aligned}\tag{1}$$

Here s_0 is a signal power, x=$\mathbf{s}\cdot\mathbf{p}$ is a projection of signal state of polarization (SOP) to pump SOP, θ is a randomly varying

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angle between $\boldsymbol{W}$ and X-axis of the Poincaré sphere. In the frameworks of fixed modulus model of random birefringence, the noise source $\beta(z)$ can be treated as a Wiener process with

$$\langle \beta(z) \rangle = 0,\ \langle \beta(z), \beta(z') \rangle = \sigma^2 \delta(z - z'), \qquad (2)$$

where $\langle \ldots \rangle$ means averaging over z, and $\sigma^2 = 1/L_c$ (L_c is a birefringence correlation length).

III. Stochastic Anti-Resonance

Stochastic differential equations (1,2) were solved by direct numerical simulations as well as by applying the analytical multi-scale averaging procedure taking into account scales of both regular and random birefringence [5-7]. The dependences of normalized average gain coefficient $\langle G \rangle = 10 \log\left(\langle s_0(L) \rangle / s_0(0) \right)$ and dispersion of its fluctuations $\sigma_G = \sqrt{\langle s_0^2(L) \rangle / \langle s_0(L) \rangle^2 - 1}$ on the polarization mode dispersion parameter D_p [8] are shown in Figure 1. The numerical averaging was performed over ensemble of 100 independent stochastic trajectories.

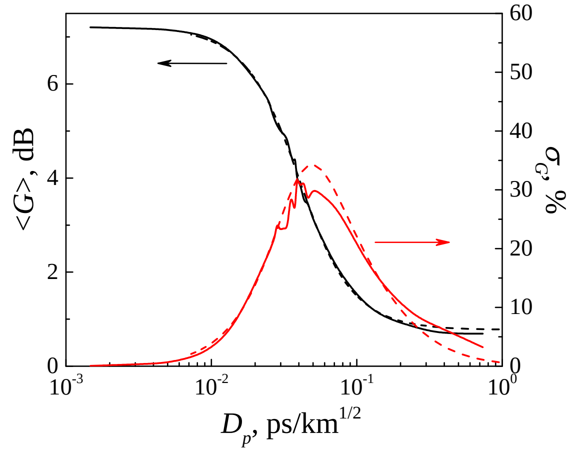

Figure 1. Dependencies of numerical (solid curves) and analytical (dashed curves) averaged gain $\langle G \rangle$ (black) and its dispersion σ_G (red) on the PMD parameter D_p. The initial SOPs are $\mathbf{s}$ = (1,0,0), $\mathbf{p}$ = (1,0,0), s_0=10 mW, P_{in}=1 W.

Figure 1 demonstrates a perfect agreement between the results of analytical multi-scale averaging technique and the direct numerical simulations of Equations (1,2). Thus, cumbersome numerical simulations of long high-speed telecommunication lines can be replaced by comparatively simple analytical methods.

Figure 1 demonstrates an asymptotical behaviour of averaged gain for large and small PMDs. The maximal gain for $D_p \to 0$ ($L_i >> L_c$) and the minimum dispersions correspond to the Manakov's limit [3] when a single averaging scale coincides with L_c:

$$\frac{d\mathbf{S}}{dz} = \frac{g}{2}\left(\mathbf{P}|\mathbf{S}| + |\mathbf{P}|\mathbf{S} \right) - \alpha_s \mathbf{S} + \left(b_s - b_p \right) \exp\left(-2\sigma^2 z \right) \begin{pmatrix} 0 \\ -S_3 \\ S_2 \end{pmatrix},$$

$$\frac{d\mathbf{P}}{dz} = -\frac{g\omega_p}{2\omega_s}\left(\mathbf{P}|\mathbf{S}| + |\mathbf{P}|\mathbf{S} \right) - \alpha_p \mathbf{P}, \qquad (2)$$

where the pump depletion is taken into account, ω_p and ω_s are the pump and signal frequencies, respectively [9]. In this limit, a fibre behaves like an isotropic medium and the gain is defined by polarization pulling caused by attraction of the signal SOP to the pump SOP (i.e., $\langle x \rangle \to 1$) [3]. Such a phenomenon can be described as a trapping of randomly fluctuating particle in a potential well (Figure 2).

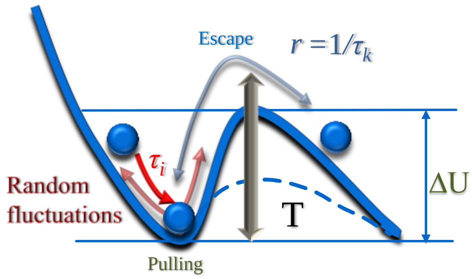

Figure 2. Potential well ΔU created by polarization pulling and modulated by birefringence with the period T defined by beat-length. Fluctuations of SOP caused by random birefringence are characterized by the relaxation length τ_i and by the escape rate r=1/τ_k (τ_k is a Kramers length).

Large PMDs ($L_i << L_c$) result in minimum gain and dispersion (Figure 1) as the pump and signal SOPs are almost decorrelated due to fast polarization beatings (i.e., $\langle x \rangle \to 0$). A "particle" is out potential well that is it escapes from the polarization pulling state (Figure 2). A fibre resembles the polarization maintaining one with rare "kicks" induced by random birefringence. Raman gain is defined by averaged pump SOP and is close to the scalar gain G_{ave} [3].

The most interesting phenomenon appears in the intermedium region of $D_p \approx 10^{-2} \div 10^{-1}$ ps/km$^{1/2}$. The gain fluctuations (Figure 1) and the rate of escape from potential well increase abruptly [4,6]. The last corresponds to the threshold-like drop of the Kramers length [10] in Figure 3. Simultaneously, the Hurst parameter characterizing a long-scale memory in a system [11] decreases from 1 (persistent statistics) for small PMDs and approaches the Brownian limit of H=1/2 (Figure 3). This noise-induced intensification of escape from metastable (polarization pulling) state [12] can be interpreted as a *stochastic anti-resonance*. The noise enhancement distinguishes this phenomenon from the stochastic resonance for which the relative intensity noise is suppressed [1,2]. Nevertheless, there is no complete chaotization in the case of stochastic anti-resonance because the Hurst parameter 0.9>H >1/2 (Figure 3) that is it remains in the region of persistent statistic.

Conclusion

It was shown both numerically and analytically, that a fibre Raman amplifier with randomly varying birefringence demonstrates a resonant-like enhancement of gain fluctuations within diapason of PMDs corresponding to modern telecommunication systems. Such an enhancement can be interpreted as a stochastic anti-resonance and is characterized by abrupt growth of gain dispersion as well as by threshold-like drop of the Kramers length and the Hurst parameter. As was demonstrated analytically, the stochastic anti-resonance is multi-scaling phenomenon and develops when the scales of polarization beat-lengths and correlation length of random birefringence become commensurable. The results obtained provide with a new insight into multi-scaling nature of stochastic phenomena in the periodically driven systems and are usable for simulations of real-world high-speed communication lines.

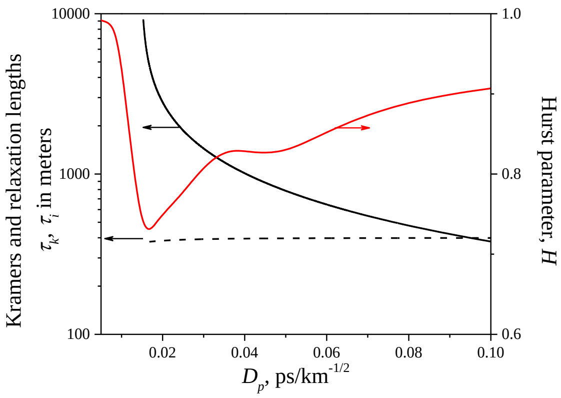

Figure 3. The Kramers (solid black curve) and relaxation (dashed black curve) lengths as well as the Hurst parameter (solid red curve) vs. the PMD parameter D_p.

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