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Toloo M., A. Emrouznejad and P. Moreno (2016) A linear relational DEA model to evaluate two-stage processes with shared inputs, *Computational and Applied Mathematics*, 36 (1): 45–61 ([doi](https://doi.org/10.1007/s40314-014-0211-2)).

The published version is available at:

<https://link.springer.com/article/10.1007/s40314-014-0211-2>.

# A linear relational DEA model to evaluate two-stage processes with shared inputs

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## Abstract

Two-stage Data Envelopment Analysis (DEA) efficiency models identify the efficient frontier of a two-stage production process. In some two-stage processes, the inputs to the first stage are shared by the second stage, known as shared inputs. This paper proposes a new relational linear DEA model for dealing with measuring the efficiency score of two-stage processes with shared inputs under constant returns to scale assumption. Two case studies of banking industry and university operations are taken as two examples to illustrate the potential applications of the proposed approach.

**Keywords:** Data Envelopment Analysis (DEA); Shared inputs; two-stage processes; IT investment; Research income.

## 1. Introduction

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Data Envelopment Analysis (DEA) is a well-known non-parametric mathematical approach for evaluating a set of homogeneous Decision Making Units (DMUs). Charnes et al. (1978) formulated the first DEA model under constant returns-to-scale (CRS) assumption and Banker et al. (1984) extended a DEA model to consider variable returns-to-scale (VRS) assumption. It is assumed that each DMU uses multi-input to produce multi-output and hence there are two types of measures in DEA: inputs and outputs. Nevertheless, in some case, there are some measures that are input for a process and output for another process. This kind of measure is called intermediate measure and such process is named two-stage process. The traditional DEA models treat a two-stage process as a black box and ignore the role of intermediate measures. Some other DEA models have been formulated to look into these black boxes. There are some more complicated network DEA models which are proposed to deal with more than two process with a network structure, i.e. a series structure, a parallel structure, or a mixture of these. Yu and Fan (2009) considered a mixed structure network DEA model to evaluate the performance of multimode bus transit. Tavassoli et al. (2014) formulated a novel slacks-based measure DEA model to assess the efficiency and effectiveness in airline performance. For more details about network DEA models we refer the readers to Kao (2014). The current paper considers a special case of two-stage process where the inputs to the first stage are shared in the second stage, known as *shared inputs*.

Cook et al. (2000) proposed an approach to assess the multi-component efficiency score of a DMU with two stages as a function of the efficiency score of each component. The authors applied their approach to sales and service performance in bank branches. Chen and Zhu (2004) developed an efficiency model for determining the efficient frontier of a two-stage production process and then applied it to evaluate information technology (IT)'s indirect impact on firm performance. Jahanshahloo et al. (2004) formulated a model for deriving an aggregate measure of efficiency with component measurement and then utilized a real data set to validate their method. Amirteimoori and Kordrostami (2005) provided a technique for measuring multi-component efficiency in the presence of imprecise data. Chen et al. (2006) considered a two-stage process with shared inputs in which the second stage uses parts of the first stage inputs in addition to the intermediate measures. Kao and Hwang (2008, 2010) suggested two-stage DEA models where the overall efficiency of a DMU can be decomposed into the product of the efficiencies of the two stages. Chen et al. (2010) proposed another two-stage network model to deal with shared inputs between both stages that cannot be split up in a convenient way. Although their model is based upon additive efficiency decomposition, the weighted average of the efficiency scores of both stages is only evaluated for a certain choice of weights. For review of various models for two-stage systems we refer the readers to Cook et al. (2010).

There are two main approaches in two-stage process DEA models: independent and relational. Independent models evaluate each stage independently and allow an intermediate measure (as an output of the first stage and an input of the second stage) to have different weights for a process, while in a relational model there is an identical weight for an intermediate measure. An interesting result of the relational model is that the overall efficiency can be considered as the product of the two process efficiencies (for more details see Kao 2009).

Chen et al. (2006) considered a sharing of some input resources between two stages and developed an improved two-stage DEA model under CRS assumption. The authors proposed a relational non-linear programming model to evaluate the impact of shared inputs on two stages along with information on how to distribute the shared inputs so that the efficiency is maximized. If there is only one intermediate measure, then their formulated non-linear DEA model converts to a linear program (LP). Nonetheless the proposed non-linear programming is a relational model, Chen et al. (2006) suggested two independent LPs to obtain the efficiency score of each stage and considered an average efficiency for the two-stage process. Clearly, solving two independent LPs leads to different weights for the intermediate measure which violates the given relational assumption. This study overcomes with the problem and introduces a modified two-stage DEA model for dealing with shared inputs. Potential uses are then illustrated with applications to bank industry and university operations. In the first application, we consider deposit as an intermediate measure, meanwhile in the second application intermediate measure is research income (RI).

In DEA literature, many scholars considered IT investment as an intermediate measure to the efficiency of firm. In an effort to better model the intermediate measures on firm performance, most popular models are dynamic DEA models (Emrouznejad et al. 2008) or network DEA models (Fare and Grosskopf, 2000). Wang et al. (1997) utilized DEA to study the marginal benefits of IT with respect to a two-stage process in firm-level banking industry. On the other hand, there are many studies have been done to evaluate the efficiency of universities. Obviously, two important aspects in a university are education and research. Some studies considered RI measure as output (see Kwimbere, 1978) and some others regarded it as input (Beasley 1990). As it is explained in Tomkins and Green (1988), there is some confusion over the role of RI. Cook and Zhu (2007) formulated a model for classifying inputs and outputs in DEA and applied their approach to the data of Beasley (1990) for accommodating the appropriate classification of RI. Toloo (2009) illustrated that there is a drawback in the proposed model by Cook and Zhu (2007) and improved it. Toloo (2010) considered alternative optimal solutions and indicated that RI can play an input or an output role and in some other cases both input and output roles. Amirteimoori et al. (2013) proposed a slacks-based measure to deal with flexible measures in DEA. Soares de Mello et al. (2006) utilized a quasi-relational model

with independent multipliers to evaluate engineering post-graduate courses. In this study, we evaluate the impact of IT and RI on the firms and universities, respectively, as two potential applications of the proposed approach.

The rest of this paper organized as follows: Section 2 explains the use of DEA in the measuring efficiency of two-stage process with shared inputs and addresses some issues in the models proposed by Chen and Zhu (2004) and Chen et al. (2006). In Section 3, a revised model to consider shared inputs in two-stage process is introduced. The impact of IT investment on the banking industry and the impact of expenditures on researching and teaching aspects of universities are evaluated in the penultimate section. Section 5 concludes and summarizes the paper.

## 2. Two-stage DEA model with shared inputs

DEA is a nonparametric method of measuring the efficiency of a DMU, such as firms, banks or universities, first introduced by Charnes et al. (1978). DEA has recently gained attention of many authors, hence a numerous applications of DEA is reported in the literature including health sector, educational institutions, banking industries and many more. In many of these applications there is an intermediate measure that can play the role of an output for the first stage and an input for the second stage.

Figure 1 presents a two-stage along with an intermediate measure. The first stage uses  $m$  inputs,  $\mathbf{x} = (x_1, \dots, x_m)$ , to produce an intermediate measure,  $z$ , and the second stage utilizes the output of the first stage to produce  $s$  outputs,  $\mathbf{y} = (y_1, \dots, y_s)$ .

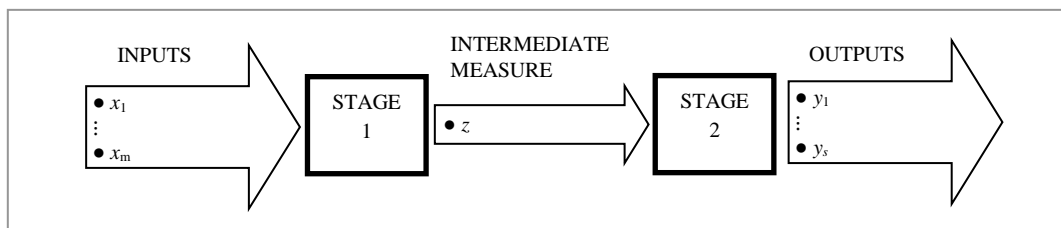


Figure 1. Two-stage process

A DEA Model (1) under CRS assumption can be used for measuring global efficiency of  $DMU_o$  which treats  $z$  as an output:

$$\begin{aligned}
& \max \frac{\sum_{r=1}^s u_r y_{ro} + u z_o}{\sum_{i=1}^m v_i x_{io}} \\
& \text{s.t.} \\
& \frac{\sum_{r=1}^s u_r y_{rj} + u z_j}{\sum_{i=1}^m v_i x_{ij}} \leq 1 \quad j = 1, 2, \dots, n \\
& u_r \geq 0 \quad \forall r, u \geq 0, v_i \geq 0 \quad \forall i
\end{aligned} \tag{1}$$

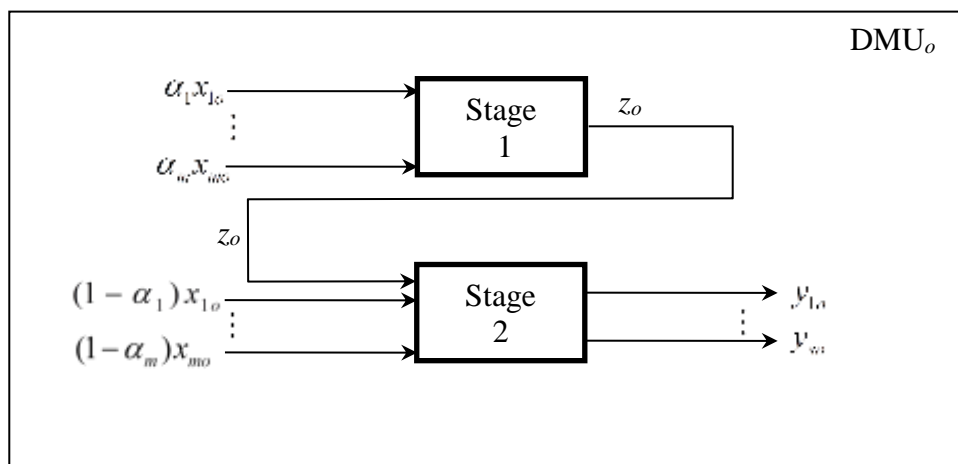
where  $n$  is the number of DMU's and  $o$  refers to the DMU under assessment, so  $o \in \{1, 2, \dots, n\}$ . The set of variables  $\mathbf{v} = (v_1, \dots, v_m)$ ,  $\mathbf{u} = (u_1, \dots, u_s)$  and  $u$  represents the weight of input  $\mathbf{x} = (x_1, \dots, x_m)$ , outputs  $\mathbf{y} = (y_1, \dots, y_s)$  and intermediate measure  $z$ , respectively. The first restriction guarantees that the efficiency for any DMU, evaluated with the set of weights selected by DMU $_o$ , never exceeds unity. The second nonnegativity restriction forces the weights (relative importance) no to be negative.

If the intermediate measure is excluded and the DMU is considered as a black box the above model can be written in a standard DEA model (2) as follows:

$$\begin{aligned}
& \max \frac{\sum_{r=1}^s u_r y_{ro}}{\sum_{i=1}^m v_i x_{io}} \\
& \text{s.t.} \\
& \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1 \quad j = 1, 2, \dots, n \\
& u_r \geq 0 \quad \forall r, v_i \geq 0 \quad \forall i
\end{aligned} \tag{2}$$

Chen and Zhu (2004) have shown neither DEA model (1) nor DEA model (2) can correctly characterize the two-stage process for efficiency evaluation. Chen et al. (2006) showed that in some cases some inputs are directly associated with both stages, that is even in the second stage they should be treated as inputs to the DEA model and thus being defined as shared inputs. The authors proposed each input should be divided into two parts, sharing each input between both stages. As a result, it was assumed that the share of input  $x_i$  for the first and the second stage is  $\alpha_i x_i$  and  $(1 - \alpha_i) x_i$ , respectively, where  $0 \leq \alpha_i \leq 1$  is a variable which controls the parts of the inputs that are consumed by each stage. If all input  $x_i$  is

used by the first stage, then  $\alpha_i = 1$  and similarly  $\alpha_i = 0$  clarifies that the second stage utilized all input  $x_i$ .



**Figure 2.** Shared inputs in a two-stage process

As results a non-linear relational DEA model (3) under CRS assumption was proposed (Chen and Zhu, 2004, Chen et al. 2006) which uses the same set of input/output weights for two stages as generally illustrated in Figure 2.

$$\begin{aligned}
& \max \frac{1}{2} \left[ \frac{uz_o}{\sum_{i=1}^m v_i \alpha_i x_{io}} + \frac{\sum_{r=1}^s u_r y_{ro}}{\sum_{i=1}^m v_i (1 - \alpha_i) x_{io} + uz_o} \right] \\
& \text{s.t.} \\
& \frac{uz_j}{\sum_{i=1}^m v_i \alpha_i x_{ij}} \leq 1 \quad j = 1, 2, \dots, n \\
& \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i (1 - \alpha_i) x_{ij} + uz_j} \leq 1 \quad j = 1, 2, \dots, n \\
& 0 \leq \alpha_i \leq 1, \forall i \\
& u_r \geq 0 \quad \forall r, u \geq 0, v_i \geq 0 \quad \forall i
\end{aligned} \tag{3}$$

Note that the objective function is the average of the efficiencies from both the first stage and the second one, by only taking into account the portion of inputs allocated to each stage but using the same weights in both stages. As well, the restrictions guarantee that efficiencies from both stages are less than unity for any DMU while making sure that the all variables are nonnegative.

## 2.1. Some issues on the two-stage DEA model

Chen et al. (2006) claimed that the model is separable; hence they suggested solving two independent LPs models. However this is incorrect since it is already assumed that the proposed model is relational. In fact, the authors considered the following two separated DEA models:

$$\begin{aligned}
& \max \frac{uz_o}{\sum_{i=1}^m v_i \alpha_i x_{io}} \\
& \text{s.t.} \\
& \frac{uz_j}{\sum_{i=1}^m v_i \alpha_i x_{ij}} \leq 1 \quad j = 1, 2, \dots, n \\
& 0 \leq \alpha_i \leq 1, \forall i \\
& u_r \geq 0 \quad \forall r, u \geq 0, v_i \geq 0 \quad \forall i
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
& \max \frac{\sum_{r=1}^s u_r y_{ro}}{\sum_{i=1}^m v_i (1 - \alpha_i) x_{io} + u z_o} \\
& \text{s.t.} \\
& \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i (1 - \alpha_i) x_{ij} + u z_j} \leq 1 \quad j = 1, 2, \dots, n \\
& 0 \leq \alpha_i \leq 1, \quad \forall i \\
& u_r \geq 0 \quad \forall r, u \geq 0, v_i \geq 0 \quad \forall i
\end{aligned} \tag{5}$$

which average of these two objective functions equals to the objective function in Model (3). It is clear that these two functions are not separable since in relational Model (3) the weight  $u$  appeared in both functions (nominator of the first function and denominator of the second function). Solving separate LPs may give different weights to the  $u$  in Model (4) and the  $u$  in Model (5). Similarly, the weights  $v_i$  and also variables  $\alpha_i$  are not independent in the objective functions of the two models.

To avoid this problem, we propose an alternative transformation of Model (3) to a linear programming as explained in the next section.

### 3. A modified two-stage DEA model

This section suggests an alternative transformation of Model (3) to an LP to avoid the issues that we highlighted in the previous section.

Considering the efficiency corresponding to the first stage in Model (3), we can assume the following substitutions, which will allow defining a new set of weights  $w_i$  for the inputs:

$$\forall i, v_i \alpha_i = w_i$$

Note that from  $0 \leq \alpha_i \leq 1$  we obtain  $0 \leq w_i \leq v_i$ .

Hence, for the denominator from the second stage efficiency, the following equalities can be worked out easily:

$$\forall i, v_i (1 - \alpha_i) = v_i - w_i$$

Moreover, it is obvious that the below definitions stand for the variables controlling the shared inputs. These equalities can be used to transform the



restrictions imposed on the variables  $\alpha_i$  into restrictions on the new set of multipliers  $w_i$ .

$$\forall i, \alpha_i = \frac{w_i}{v_i} \quad (6)$$

By applying the substitutions proposed above, Model (3) can be written as Model (7) below:

$$\begin{aligned} & \max \frac{1}{2} \left[ \frac{uz_o}{\sum_{i=1}^m w_i x_{io}} + \frac{\sum_{r=1}^s u_r y_{ro}}{\sum_{i=1}^m (v_i - w_i) x_{io} + uz_o} \right] \\ & \text{s.t.} \\ & \frac{uz_j}{\sum_{i=1}^m v_i x_{ij}} \leq 1 \quad j = 1, 2, \dots, n \\ & \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m (v_i - w_i) x_{ij} + uz_j} \leq 1 \quad j = 1, 2, \dots, n \\ & 0 \leq w_i \leq v_i, \forall i \\ & u_r \geq 0 \quad \forall r, u \geq 0 \end{aligned} \quad (7)$$

Without any loss of generality, we can set both denominators to be equal to 1, as it was first stated by Charnes et al. (1978):

$$\begin{aligned} \sum_{i=1}^m w_i x_{io} &= 1 \\ \sum_{i=1}^m (v_i - w_i) x_{io} + uz_o &= 1 \end{aligned}$$

From the above pair of equalities, it can be clearly inferred that:  
 $\sum_{i=1}^m (v_i - w_i) x_{io} + uz_o = 2$ .

Thus, the fractional programming, Model (7), by setting the denominators to unity and transforming the fractional restrictions into linear ones, can be written as the following linear programming, Model (8):

$$\begin{aligned}
& \max \frac{1}{2} \left[ uz_o + \sum_{r=1}^s u_r y_{ro} \right] \\
& \text{s.t.} \\
& \sum_{i=1}^m w_i x_{io} = 1 \\
& uz_j - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, 2, \dots, n \\
& \sum_{i=1}^m (v_i - w_i) x_{io} + uz_o = 2 \\
& \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m (v_i - w_i) x_{ij} + uz_j \leq 0 \quad j = 1, 2, \dots, n \\
& 0 \leq w_i \leq v_i, \forall i \\
& u_r \geq 0 \quad \forall r, u \geq 0
\end{aligned} \tag{8}$$

Note that, unlike the linear model proposed by Chen et al. (2006), the above LP Model (8) applies the same set of weights for all functions and restrictions, which is correct since any of the restrictions is independent from the rest of them.

#### 4. Applications

This section utilizes two real data sets to verify the proposed approach. The first application deals with evaluation the impact IT on each stage of the bank's operation with deposit as an intermediate measure. The impact of general and equipment expenditures on each researching and teaching stages of universities along with RI intermediate measure is evaluated in the second application.

##### 4.1 Application in the banking industry

IT investment represents the largest capital expenditure for many firms, hence because of the importance of such investments, there has been a commonly concern for a framework to measure the impact of IT on firm performance. In fact the impact of IT has been perceived in almost every part of a production process, including strategic relevance, process control, research and development, customer service, coordination, costs, etc. In other words, IT is shared amongst all the parts of the production process.

Several methods have been proposed for measuring impact of investments in IT on the productivity and efficiency (Banker et al. 1990). Also many researchers have shown that there is a positive relationship between IT investment and production performance of firms. However it is not easy to measure the effect of IT investment in firm performance. In the literature of production research and production efficiency, two approaches are widely used for assessing IT investment: Stochastic Frontier Analysis (SFA), Data Envelopment Analysis

(DEA). DEA is also used in many recent applications in efficiency of banks or financial institutions (Akther et al. 2013, Barros et al. 2012, Emrouznejad and Anouze, 2009, Emrouznejad and Anouze, 2010, Toloo et al. 2009) as well as efficiency assessment of bank branches (Paradi et al. 2011, Paradi and Zhu, 2013 and Ebrahimnejad et al. 2014) and prediction for banking failure and success (Premachandra et al. 2011). Some other DEA developments can be found in Toloo (2013, 2014).

To demonstrate the use of Model (8) and to compare the results with that of published by Chen et al. (2004, 2006) we use the same dataset as used by Chen and Zhu (2004) page 17, which consists of 27 cases from 22 firms in the banking industry during the years 1987-1989. In the first stage, which represents the collection of funds from its customers, the banks use fixed assets (proxy for capital), number of employees (proxy for labor), and IT investment as inputs to generate deposit as an intermediate measure. In the second stage, banks use the deposit generated in the previous stage as a source of funds to invest in securities and to provide loans. The return from these activities and the risk taken, represented by the bank's profit and fraction of loans recovered, are used as two outputs in the second stage. As a result, the DEA inputs are considered as (i) fixed assets (denoted as  $F$ ), (ii) IT budget (denoted as  $I$ ), and (iii) employees (denoted as  $E$ ). Also, profit (denoted as  $P$ ) and the fraction of loan recovered (denoted as  $R$ ) are outputs in the DEA model. The intermediate measure – deposits - denoted as  $D$  is an output in stage 1, named as Deposit, and input in stage 2, named as Loan. In addition, it is assumed that each input should be divided into two parts, sharing each input between both the deposit and loan stages, as shown in Figure 3.

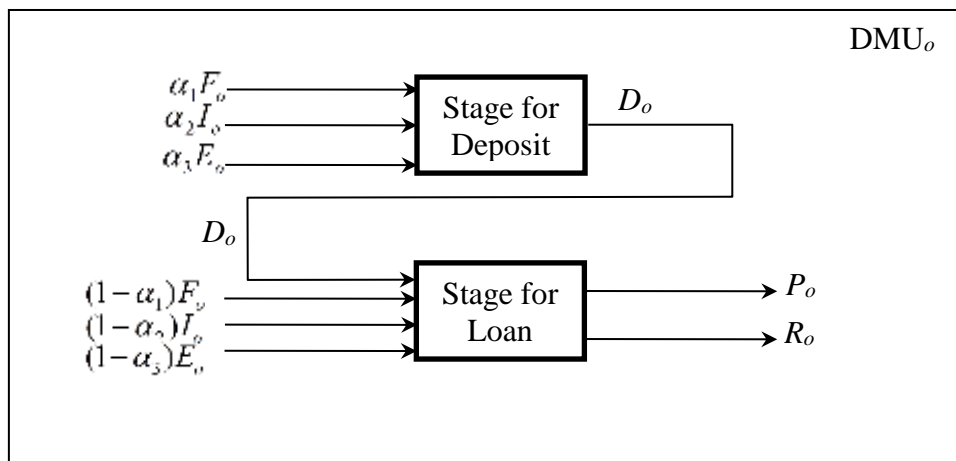


Figure 3. Shared inputs in a banks' operations

Table 1 shows the optimal solutions for Model (8), where each column reports the optimal values for the transformed weights and the original ones

corresponding to the three inputs. Variables controlling the shared inputs can be computed by using equations (6).

**Table 1: Optimal solutions for Model (8)**

Bank No.	$w_1^*$	$v_1^*$	$w_2^*$	$v_2^*$	$w_3^*$	$v_3^*$
1	0.31	0.82	0	0	0.06	0.06
2	0.24	0.56	0	0	0.04	0.04
3	0.18	0.49	0	0	0.03	0.04
4	0.31	1.41	0	0	0.06	0.06
5	0.26	1.32	0	0.09	0.05	0.05
6	0.01	0.03	0.36	0.57	0.01	0.01
7	0.06	0.09	0	0.71	0.02	0.02
8	0.06	0.06	1.8	7.55	0.07	0.07
9	0.01	0.03	0	0	0.01	0.01
10	0.03	0.03	1.1	4.64	0.04	0.04
11	0.03	0.03	1.09	4.64	0.04	0.04
12	0.08	0.08	1.98	8.61	0.06	0.06
13	0	0	0	0	0.08	0.08
14	0.04	0.07	0	0.62	0.02	0.02
15	0.04	0.07	0	0.46	0.02	0.02
16	0.05	0.14	1.46	3.4	0.05	0.05
17	0.54	0.54	0	5.33	0.1	0.1
18	0.07	0.07	0	0	0.04	0.04
19	0.13	0.26	0	0	0.07	0.07
20	0.27	0.96	0	0	0.15	0.15
21	0.27	0.34	0	0	0.15	0.2
22	0.33	0.89	0	0	0.06	0.06
23	0.32	0.88	0	0	0.06	0.06
24	0.03	0.06	0.98	0.98	0.04	0.04
25	0.09	0.09	2.3	7.87	0.07	0.07
26	0.32	0.32	0	0	0.06	0.06
27	0.07	0.07	1.32	6.77	0.07	0.07

Table (2) shows the correct results as obtained by the proposed Model (8) and the results using Model (3), as published in Chen et al. (2006).

Obviously there are some disagreements between the two models. For instance consider DMU<sub>7</sub>, if we replace  $\alpha_1 = 0.722, \alpha_2 = 0, \alpha_3 = 1$  in Model (3) and solve the resulting fractional linear programming for this DMU, the deposit efficiency and loan efficiency are 0.930 and 1, respectively. These results are achieved by modified DEA Model (8) but we get different results using procedure explained in Chen et al. (2007). According to their model we get the score of 1 for both deposit and loan efficiency which is not correct.

The proposed model in this paper shows that only DMU<sub>18</sub> is full efficient (both deposit and loan efficiency) while in Chen et al. (2007) DMU<sub>7</sub> and DMU<sub>20</sub> are also full efficient. Furthermore, most DMUs get a lower score by Model (8) than by

Model (3), due to a decrease in loan efficiency, i.e. efficiency from the second stage.

For instance, the average efficiency of DMU<sub>24</sub> has dropped from 0.912 to 0.84 because there has been a decrease in loan efficiency while deposit efficiency has remained the same. If we take a close look to another unit, like DMU<sub>14</sub>, it can be seen that average efficiency has decreased as well, due to a drop of the loan efficiency from 0.618 to 0.589. Regarding the variables controlling the shared inputs, DMU<sub>24</sub> underwent a significant rise in  $\alpha_1$  and  $\alpha_3$  while  $\alpha_2$  remained the same, whereas DMU<sub>14</sub> only underwent a little increase in variable  $\alpha_1$ .

The efficiency deterioration in loan efficiency is logical, since Model (8) does not let the value of weights for loan stage to be independent from deposit stage and therefore is far more restrictive than Model (3). It can be inferred that our proposed model provides a greater power of discrimination, which is particularly useful amongst efficient units.

We should clarify that to make our results comparable with Chen et al. (2007) we assumed that  $0 \leq \alpha_i \leq 1$  for  $i = 1, 2, 3$  in Model (3). Equivalent to this, in Model (8) is  $0 \leq w_i \leq v_i$  for  $i = 1, 2, 3$ . However, in practical terms we suggest to assume,  $0 < w_i < v_i$  for  $i = 1, 2, 3$  in Model (8) [equivalently  $0 < \alpha_i < 1$  for  $i = 1, 2, 3$  in Model (3)]. This guarantees that the inputs are assigned to both stages in the production process while the current results show that for some DMUs the IT input is assigned only to one stage of the production process.

**Table 2. Comparison of the results**

Bank No.	Average efficiency		Deposit Efficiency		Loan efficiency		$\alpha_1$		$\alpha_2$		$\alpha_3$	
	Model (3)	Model (8)	Model (3)	Model (8)	Model (3)	Model (8)	Model (3)	Model (8)	Model (3)	Model (8)	Model (3)	Model (8)
1	0.743	0.742	0.639	0.639	0.847	0.846	0.379	0.378	0	0	1	1
2	0.767	0.766	0.651	0.651	0.884	0.881	0.379	0.429	0	0	1	1
3	0.687	0.674	0.518	0.518	0.857	0.831	0.379	0.367	0	0	1	0.750
4	0.799	0.799	0.599	0.599	1	1	0.239	0.220	0	0	1	1
5	0.772	0.77	0.556	0.556	0.988	0.985	0.239	0.197	0	0	1	1
6	0.723	0.714	0.760	0.76	0.686	0.667	0.309	0.333	0.385	0.632	0.999	1
7	1	0.965	1	0.93	1	1	0.722	0.667	0	0	1	1
8	0.714	0.706	0.535	0.535	0.894	0.877	1	1	0.263	0.238	1	1
9	0.630	0.615	0.625	0.625	0.635	0.605	1	0.333	0	0	1	1
10	0.625	0.612	0.496	0.496	0.755	0.727	1	1	0.263	0.237	1	1
11	0.625	0.611	0.495	0.495	0.755	0.727	1	1	0.263	0.235	1	1
12	0.773	0.73	0.669	0.668	0.877	0.792	0.003	1	0.003	0.230	1	1
13	0.931	0.905	0.949	0.949	0.912	0.861	0	0	0	0	0.019	1
14	0.603	0.589	0.588	0.588	0.618	0.589	0.558	0.571	0	0	1	1
15	0.658	0.653	0.658	0.658	0.658	0.648	0.558	0.571	0	0	1	1
16	0.682	0.679	0.665	0.665	0.699	0.694	0.975	0.357	0.264	0.429	1	1
17	0.859	0.787	0.718	0.718	1	0.856	0.150	1	0	0	1	1
18	1	1	1	1	1	1	1	1	0	0	1	1

19	0.770	0.732	0.814	0.814	0.726	0.65	0.005	0.500	0	0	0.018	1
20	0.847	0.847	0.693	0.693	1	1	1	0.281	0	0	0.999	1
21	0.853	0.853	0.707	0.707	1	1	1	0.794	0	0	0.869	0.750
22	0.897	0.854	0.794	0.794	1	0.914	0.098	0.371	0	0	1	1
23	0.890	0.89	0.780	0.78	1	1	0.378	0.364	0	0	0.999	1
24	0.912	0.84	0.930	0.93	0.893	0.75	0.003	0.500	1	1	0.024	1
25	0.693	0.671	0.627	0.627	0.758	0.715	0.997	1	0.126	0.292	1	1
26	0.895	0.758	1	1	0.789	0.515	0.001	1	1	0	0.003	1
27	1	0.971	1	0.942	1	1	1	1	0.244	0.195	0.999	1

## 4.2 Application in assessing university efficiency

Now, we apply the proposed Model (8) to the data sets used in Beasley (1990) for assessing university efficiency (see Table 3). This data set consists of two inputs, namely General Expenditure (GE) and Equipment Expenditure (EE), and three outputs, composed of three types of students (UGS, PGT and PGR), while selecting Research Income (RI) as intermediate measure.

**Table 3 University data adapted from Beasley (1990)**

DMU (University)	Inputs		Intermediate Measure	Outputs		
	General Expenditure	Equipment Expenditure	Research Income	UG Students	PG Teaching	PG Research
1	528	64	254	145	0	26
2	2605	301	1485	381	16	54
3	304	23	45	44	3	3
4	1620	485	940	287	0	48
5	490	90	106	91	8	22
6	2675	767	2967	352	4	166
7	422	0	298	70	12	19
8	986	126	776	203	0	32
9	523	32	39	60	0	17
10	585	87	353	80	17	27
11	931	161	293	191	0	20
12	1060	91	781	139	0	37
13	500	109	215	104	0	19
14	714	77	269	132	0	24
15	923	121	392	135	10	31
16	1267	128	546	169	0	31
17	891	116	925	125	0	24
18	1395	571	764	176	14	27
19	990	83	615	28	36	57
20	3512	267	3182	511	23	153
21	1451	226	791	198	0	53
22	1018	81	741	161	5	29
23	1115	450	347	148	4	32
24	2055	112	2945	207	1	47
25	440	74	453	115	0	9
26	3897	841	2331	353	28	65
27	836	81	695	129	0	37

28	1007	50	98	174	7	23
29	1188	170	879	253	0	38
30	4630	628	4838	544	0	217
31	977	77	490	94	26	26
32	829	61	291	128	17	25
33	898	39	327	190	1	18
34	901	131	956	168	9	50
35	924	119	512	119	37	48
36	1251	62	563	193	13	43
37	1011	235	714	217	0	36
38	732	94	297	151	3	23
39	444	46	277	49	2	19
40	308	28	154	57	0	7
41	483	40	531	117	0	23
42	515	68	305	79	7	23
43	593	82	85	101	1	9
44	570	26	130	71	20	11
45	1317	123	1043	293	1	39
46	2013	149	1523	403	2	51
47	992	89	743	161	1	30
48	1038	82	513	151	13	47
49	206	1	72	16	0	6
50	1193	95	485	240	0	32

Each university can be presented as a two-stage process, where the three types of students are the outputs of the second stage and the intermediate measure – research income – can be used as a source of funds to improve students’ education. This research income can be treated as an outcome from the first stage.

The two inputs are associated with both stages, since expenditure is used for both researching and teaching, and therefore should be treated as shared inputs. The structure of the inputs and outputs from this two-stage illustration can be seen in Figure 4.

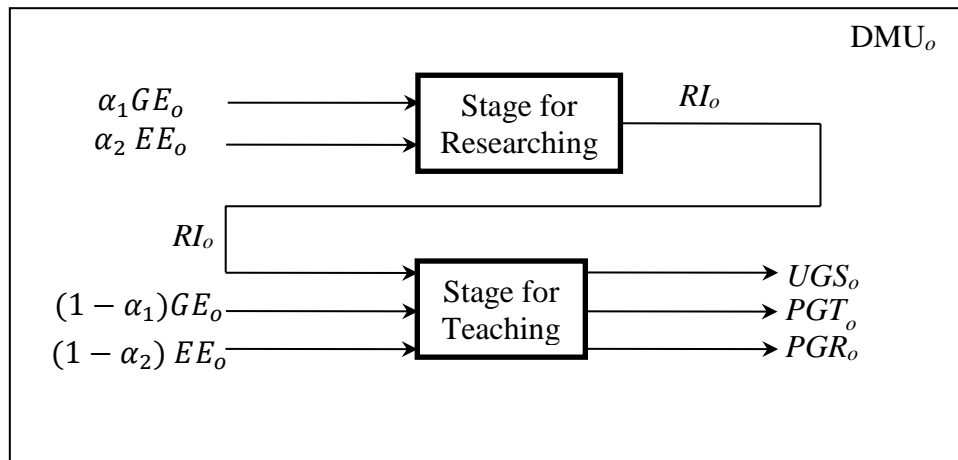


Figure 4. Shared inputs of university illustration

Table (4) shows the results as obtained by the proposed Model (8). First column refers to the average efficiency of the model while the other two columns show the efficiency of each single stage.

Table (4). The results of Model (8)

DMU (University)	Average Efficiency	Stage 1 Efficiency	Stage 2 Efficiency
1	0.668	0.336	1
2	0.475	0.398	0.552
3	0.456	0.103	0.809
4	0.503	0.405	0.6
5	0.575	0.151	1
6	0.664	0.477	0.852
7	1	1	1
8	0.569	0.549	0.59
9	0.479	0.052	0.905
10	0.621	0.421	0.82
11	0.527	0.22	0.834
12	0.536	0.514	0.558
13	0.547	0.3	0.794
14	0.511	0.263	0.759
15	0.5	0.296	0.704
16	0.422	0.301	0.542
17	0.513	0.724	0.303
18	0.448	0.382	0.514
19	0.717	0.433	1
20	0.602	0.539	0.664
21	0.514	0.38	0.647
22	0.525	0.508	0.543
23	0.416	0.217	0.616
24	0.522	1	0.043
25	0.636	0.65	0.622
26	0.385	0.417	0.352
27	0.598	0.54	0.656
28	0.536	0.071	1
29	0.576	0.516	0.635
30	0.589	0.729	0.448
31	0.52	0.35	0.689
32	0.569	0.245	0.893
33	0.642	0.283	1
34	0.638	0.581	0.694
35	0.693	0.387	1
36	0.576	0.329	0.823
37	0.576	0.493	0.659
38	0.558	0.283	0.833
39	0.591	0.435	0.746
40	0.524	0.349	0.698
41	0.639	0.697	0.582
42	0.596	0.413	0.779
43	0.444	0.1	0.789



44	0.587	0.173	1
45	0.598	0.553	0.643
46	0.568	0.528	0.609
47	0.524	0.523	0.526
48	0.64	0.345	0.936
49	0.689	0.453	0.925
50	0.56	0.284	0.837

Model (8) shows that only DMU<sub>7</sub> is full efficient (both stages), while there are two efficient DMUs in the first stage and eight efficient DMUs in the second stage. DMU<sub>24</sub> achieves a great amount of research income, exceeding by far the amount of expenditure, so it makes sense that it is considered to be efficient in the first stage. However, the number of graduates from DMU<sub>24</sub> does not match the expectations so its efficiency in the second stage is very low.

All eight efficient DMUs in the second stage share the same feature: the number of students is high compared to the amount of expenditure and research income. In any case, Model (8) provides a great power of discrimination when assessing efficiency.

## 5. Conclusions and further research

Production processes have several stages with an intermediate measure. In some cases, there are some shared inputs that are shared by stages. Chen et al. (2004, 2006) introduced a relational fractional programming model for evaluating IT impacts on firm performance when shared inputs are presented. This paper first showed their transformation procedure to a linear programming is incorrect. Secondly, the paper proposed a different transformation to improve their relational model for measuring the impact of shared inputs on production processes. Thirdly, we illustrated that the linear program can be utilized in measuring the impact of IT investment on the banking industry and the impact of expenditures on researching and teaching aspects of universities.

One main limitation of the proposed model is that similar to Chen and Zhu (2004) and Chen et al. (2006) we considered the case that there is only one intermediate measure. However, the transformation of the fractional program to a linear program if there is more than one intermediate measure can be done in a similar way. Extending an approach to evaluate for two-stage processes with shared inputs under VRS assumption, with negative data, or with imprecise data can be considered as interesting further research topics. Furthermore, another interesting research topic is developing an independent DEA model to deal with two-stage processes with shared inputs under different scale assumptions.

## Acknowledgements

The research was supported by the Czech Science Foundation (GACR project 14-31593S) and through European Social Fund within the project CZ.1.07/2.3.00/20.0296.

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