ECAI 2012
Luc De Raedt et al. (Eds.)

doi:10.3233/978-1-61499-098-7-474

Reconciling OWL and Non-monotonic Rules for the Semantic Web

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Abstract. We propose a description logic extending $\mathcal{SROIQ}$ (the description logic underlying OWL 2 DL) and at the same time encompassing some of the most prominent monotonic and non-monotonic rule languages, in particular Datalog extended with the answer set semantics. Our proposal could be considered a substantial contribution towards fulfilling the quest for a unifying logic for the Semantic Web. As a case in point, two non-monotonic extensions of description logics considered to be of distinct expressiveness until now are covered in our proposal. In contrast to earlier such proposals, our language has the "look and feel" of a description logic and avoids hybrid or first-order syntaxes.

1 Introduction

The landscape of ontology languages for the Semantic Web is diverse and controversial [11]. In terms of expressive ontology representation formalisms, this is most clearly reflected by the two W3C standards RIF [15] and OWL [10], where the former—the Rule Interchange Format—is based on rules in the wider sense including Datalog and logic programming [12], and the latter—the Web Ontology Language—is based on description logics [1]. In terms of both academic research and industrial development, these two formalisms cater to almost disjoint subcommunities.

While the different paradigms often focus on different perspectives and needs, the field and its applications would as a whole benefit if a certain coherence were retained. This coherence could be achieved through the use of a *unifying logic*, one which reconciles the diverging paradigms of the Semantic Web stack.[4] Indeed, several proposals have been made towards creating such a unified logic, but the quest remains largely unfulfilled.

Two major rifts have been identified which need to be overcome for a reconciliation to occur. This is particularly true in the case of OWL and rule languages. The first rift is due to a fundamental conceptual difference in how the paradigms deal with *unknown* information. While OWL adheres to the so-called *Open World Assumption* (OWA), and thus treats unknown information indeed as unknown, rules and logic programming adhere to the so-called *Closed World Assumption* (CWA) in which unknown information defaults to *false*, i.e., the knowledge available is thought to be a complete encoding of the domain of interest. The second rift is caused by the decision to design OWL as a decidable language: Naive combinations (e.g., in first-order predicate logic) of rule languages (such as Datalog) and OWL are undecidable, and thus violate this design decision.

In principle, the second rift cannot be completely overcome. Nevertheless, several decidable combinations of OWL and rules have been proposed in the literature, usually resulting in a *hybrid* formalism mixing the syntax and sometimes even the semantics of rules and description logics (see the survey sections of [18, 17]). The most recent proposal [20] rests on the introduction of a new syntax construct to description logics, called *nominal schemas*, which results in a description logic which seamlessly—syntactically and semantically—incorporates binary DL-safe Datalog [24] (and as we will see in Section 3.1, even incorporates unrestricted DL-safe Datalog).

While we would argue that the introduction of nominal schemas constitutes a major advance towards a reconciliation of OWL and rules, expanding OWL with nominal schemas by itself does nothing to resolve the first of the rifts mentioned above. And so the approach described in [20] can only be viewed as a partial reconciliation.

The main purpose of the present paper is to build upon the work of [20], addressing that first rift. We show that nominal schemas allow not only for a concise reconciliation of OWL and Datalog, but also that the integration can in fact be lifted to cover established closed world formalisms on both the OWL and the rules side. More precisely, we endow $\mathcal{SROIQ}$, the description logic underlying OWL 2 DL, with both nominal schemas and a generalized semantics based on the logic of minimal knowledge and negation as failure (MKNF) [7, 22]. The latter, and thus also the extension of $\mathcal{SROIQ}$ based on it, is non-monotonic and captures both open and closed world modeling. We show that it in fact encompasses major ontology modeling paradigms, including (trivially) OWL 2 DL and its tractable fragments [10] but also unrestricted DL-safe Datalog, MKNF-extended $\mathcal{ALC}$ [7], hybrid MKNF [23], description logics with defaults [2], and the answer set semantics for Datalog with negation [9]. This means it also covers various ways of expressing the closure of concepts and roles, and also of expressing integrity constraints.

The plan of the paper is as follows. In Section 2 we introduce the syntax and semantics of our new logic, called $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$, and we show that decidable reasoning in a sufficiently large fragment of it can be realized. In Section 3, we show how $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ encompasses many of the well-known languages and proposals related to the integration of OWL, rules, and non-monotonicity. Section 4 concludes with brief remarks on related work and a discussion of future work.

An extended version of the paper (including proofs, examples, and many more details in Section 2.3) is available at http://centria.di.fct.unl.pt/∼mknorr/resources/KHM-ECAI12ex.pdf.

2 MKNF DL $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$

Our work is based on the description logic (*DL*) $\mathcal{SROIQV}(\mathcal{B}^s,\times)$. It extends $\mathcal{SROIQ}$ [11, 13] with concept products [19] and Boolean

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constructors over simple roles [25]. Importantly, it also incorporates nominal schemas [20]. These represent variable nominals that can only bind to known individuals. It has been shown that none of these extensions affect the worst case complexity of reasoning in $\mathcal{SROIQ}$ [20, 19]. We refer to [1, 11] for a detailed account on DLs in general and to [20] for $\mathcal{SROIQV}(\mathcal{B}^s,\times)$ in particular.

Following the work in [7], where the DL $\mathcal{ALC}$ is augmented with two *modal operators* $\mathbf{K}$ and $\mathbf{A}$, we define such an extension for $\mathcal{SROIQV}(\mathcal{B}^s,\times)$. The modal operator $\mathbf{K}$ is interpreted in terms of minimal knowledge, while $\mathbf{A}$ is interpreted as autoepistemic assumption and corresponds to $\neg\mathbf{not}$, i.e., the classical negation of the negation as failure operator $\mathbf{not}$ used in [22] instead of $\mathbf{A}$. The extension to $\mathcal{SROIQV}(\mathcal{B}^s,\times)$ is non-trivial in so far as this DL is significantly more expressive than $\mathcal{ALC}$ and, in particular, the usage of modal operators in role expressions is considerably more advanced.

We introduce the syntax and semantics of our proposed language, called $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ in the nomenclature introduced in [7], and we provide some remarks on a reasoning procedure.

2.1 Syntax

We consider a signature $\Sigma = \langle N_I, N_C, N_R, N_V\rangle$ where N_I, N_C, N_R, and N_V are pairwise disjoint and finite sets of *individual names*, *concept names*, *role names*, and *variables*. Role names are divided into disjoint sets of simple role names N_R^s and non-simple role names N_R^n. In the following, we assume that Σ has been fixed. We define concepts and roles in $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ as follows.

Definition 1 *The set of $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ concepts C and (*simple/non-simple*) $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ roles R (R^s/R^n) are defined by the following grammar.*

$$
\begin{aligned}
\mathsf{R}^s &::= N_R^s \mid (N_R^s)^- \mid U \mid N_C \times N_C \mid \neg\mathsf{R}^s \mid \mathsf{R}^s \sqcap \mathsf{R}^s \mid \mathsf{R}^s \sqcup \mathsf{R}^s \mid \\
&\quad\ \mathbf{K}\mathsf{R}^s \mid \mathbf{A}\mathsf{R}^s \\
\mathsf{R}^n &::= N_R^n \mid (N_R^n)^- \mid U \mid N_C \times N_C \mid \mathbf{K}\mathsf{R}^n \mid \mathbf{A}\mathsf{R}^n \\
\mathsf{R} &::= \mathsf{R}^s \mid \mathsf{R}^n \\
\mathsf{C} &::= \top \mid \bot \mid N_C \mid \{N_I\} \mid \{N_V\} \mid \neg\mathsf{C} \mid \mathsf{C}\sqcap\mathsf{C} \mid \mathsf{C}\sqcup\mathsf{C} \mid \\
&\quad\ \exists\mathsf{R}.\mathsf{C} \mid \forall\mathsf{R}.\mathsf{C} \mid \exists\mathsf{R}^s.\mathsf{Self} \mid \leqslant k\,\mathsf{R}^s.\mathsf{C} \mid \geqslant k\,\mathsf{R}^s.\mathsf{C} \mid \mathbf{K}\mathsf{C} \mid \mathbf{A}\mathsf{C}
\end{aligned}
$$

In the above, U is the *universal role*, $\top$ and $\bot$ are the top and bottom concepts, and k is a non-negative integer. Concepts of the form $\{a\}$ with $a \in N_I$ are called *nominals*, while concepts of the form $\{x\}$ with $x \in N_V$ are called *nominal schemas*. The set of all *concept products* $\mathsf{R}_{C\times D}$, *inverse roles* R^-, and the related function $\mathrm{I}nv : \mathsf{R} \to \mathsf{R}$ are all defined as in [20] except that $C, D \in N_C$.

Definition 2 *Given roles $R, S_i \in \mathsf{R}$, a generalized* role inclusion axiom (RIA) *is a statement of the form $S_1 \circ \cdots \circ S_k \sqsubseteq R$, where $R \in \mathsf{R}^s$ only if $k = 1$ and $S_1 \in \mathsf{R}^s$. A set of RIAs is* regular *if there is a strict partial order $\prec$ on R such that*

- *if $R \notin \{S, Inv(S)\}$, then $S \prec R$ if and only if $Inv(S) \prec R$; and*
- *every RIA has the form $R \circ R \sqsubseteq R$, $Inv(R) \sqsubseteq R$, $R \circ S_1 \circ \cdots \circ S_k \sqsubseteq R$, $S_1 \circ \cdots \circ S_k \circ R \sqsubseteq R$, or $S_1 \circ \cdots \circ S_k \sqsubseteq R$ with $S_i \prec R$ for each i with $1 \le i \le k$.*

An RBox axiom *is an RIA. A* TBox axiom *(or* general concept inclusion *(GCI)) is an expression $C \sqsubseteq D$ where $C, D \in \mathsf{C}$. An* ABox axiom *is of the form $C(a)$ or $R(a,b)$ where $C \in \mathsf{C}$, $R \in \mathsf{R}$, and $a, b \in N_I$. A $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$* axiom *is any ABox, TBox, or RBox axiom, and a $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$* knowledge base *(KB) is a finite, regular set of $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ axioms.*

Additionally $\mathcal{SROIQ}$ admits RBox axioms that directly express the empty role, role disjointness, asymmetry, reflexivity, irreflexibility, symmetry, and transitivity. As shown in [20], all these can be expressed in $\mathcal{SROIQV}(\mathcal{B}^s,\times)$ anyway, so we omit them here. In contrast to [20], we explicitly allow the usage of complex concepts and roles in the ABox to simplify the presentation in the next section. This difference is only syntactic, as such expressions can easily be reduced by introducing new concept and role names.

2.2 Semantics

The semantics of $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ is a generalization of the semantics of $\mathcal{SROIQV}(\mathcal{B}^s,\times)$ [20] and that of $\mathcal{ALCK}_{\mathcal{NF}}$ [7]. We first recall two basic notions from [20].

An *interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of a domain $\Delta^{\mathcal{I}} \neq \emptyset$ and a function $\cdot^{\mathcal{I}}$ that maps elements in N_I, N_C, and N_R to elements, sets, and relations of $\Delta^{\mathcal{I}}$ respectively, i.e., for $a \in N_I$, $a^{\mathcal{I}} = d \in \Delta^{\mathcal{I}}$, for $A \in N_C$, $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, and, for $V \in N_R$, $V^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. A *variable assignment* $\mathcal{Z}$ for an interpretation $\mathcal{I}$ is a function $\mathcal{Z} : N_V \to \Delta^{\mathcal{I}}$ such that, for each $v \in N_V$, $\mathcal{Z}(v) = a^{\mathcal{I}}$ for some $a \in N_I$.

As is common in MKNF-related semantics used to combine DLs with non-monotonic reasoning (see [7, 14, 16, 23]), specific restrictions on interpretations are introduced to ensure that certain unintended logical consequences can be avoided (see, e.g., [23]). We adapt the standard name assumption from [23].

Definition 3 *An interpretation $\mathcal{I}$ (over Σ to which $\approx$ is added) employs the* standard name assumption *if*

(1) N_I^* extends N_I with a countably infinite set of individuals that cannot be used in variable assignments, and $\Delta^{\mathcal{I}} = N_I^*$;
(2) for each i in N_I^*, $i^{\mathcal{I}} = i$; and
(3) equality $\approx$ is interpreted in $\mathcal{I}$ as a congruence relation – that is, $\approx$ is reflexive, symmetric, transitive, and allows for the replacement of equals by equals [8].

The first two conditions define $\mathcal{I}$ as a bijective function, while the third ensures that we still can identify elements of the domain.

It was shown in [23, Proposition 3.2] that we cannot distinguish between the consequences of first-order formulas under standard first-order semantics and under the standard name assumption. Therefore, we use the standard name assumption in the rest of the paper without referring to it further.

As an immediate side-effect, we note that the variable assignment is no longer tied to a specific interpretation. Similarly, we simplify notation by using Δ without reference to a concrete interpretation.

The first-order semantics is lifted to satisfaction in MKNF structures that treat the modal operators w.r.t. sets of interpretations.

Definition 4 *An* MKNF structure *is a triple $(\mathcal{I},\mathcal{M},\mathcal{N})$ where $\mathcal{I}$ is an interpretation, $\mathcal{M}$ and $\mathcal{N}$ are sets of interpretations, and $\mathcal{I}$ and all interpretations in $\mathcal{M}$ and $\mathcal{N}$ are defined over Δ. For any such $(\mathcal{I},\mathcal{M},\mathcal{N})$ and assignment $\mathcal{Z}$, the function $\cdot^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$ is defined for arbitrary $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ expressions as shown in Table 1.*

$(\mathcal{I},\mathcal{M},\mathcal{N})$ and $\mathcal{Z}$ satisfy *a $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ axiom α, written $(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z} \models \alpha$, if the corresponding condition in Table 1 holds. $(\mathcal{I},\mathcal{M},\mathcal{N})$* satisfies *$\alpha$, written $(\mathcal{I},\mathcal{M},\mathcal{N}) \models \alpha$, if $(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z} \models \alpha$ for all variable assignments $\mathcal{Z}$. A (non-empty) set of interpretations $\mathcal{M}$* satisfies *α, written $\mathcal{M} \models \alpha$, if $(\mathcal{I},\mathcal{M},\mathcal{M}) \models \alpha$ holds for all $\mathcal{I} \in \mathcal{M}$. $\mathcal{M}$* satisfies *a $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ knowledge base KB, written $\mathcal{M} \models KB$, if $\mathcal{M} \models \alpha$ for all axioms $\alpha \in KB$.*

Note the small deviations of the semantics of $\{t\}$, $\leqslant k\,S.C$, and $\geqslant k\,S.C$ in Table 1 compared to, e.g., that in [20]. These are necessary to ensure that the semantics of these three constructors works as intended under standard name assumption.

So far, we have extended the semantics of [20], considering not only an interpretation but also two sets of interpretations each of which is used to interpret one of the modal operators. We have also provided a monotonic semantics for $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ KBs where the two sets used are identical (hence the interpretation of the two modal operators is exactly the same). Now, we will define a non-monotonic MKNF model in the usual fashion [7, 14, 16, 23]: $\mathcal{M}$ is fixed to interpret $\mathbf{A}$, and a superset $\mathcal{M}'$ is used to interpret $\mathbf{K}$ to test whether the knowledge derived from $\mathcal{M}$ is indeed minimal.

Table 1. Semantics of $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$

Syntax	Semantics
A	$A^{\mathcal{I}} \subseteq \Delta$
V	$V^{\mathcal{I}} \subseteq \Delta \times \Delta$
a	$a^{\mathcal{I}} \in \Delta$
x	$\mathcal{Z}(x) \in \Delta$
$\top$	Δ
$\bot$	$\emptyset$
$\{t\}$	$\{a \mid a \approx t^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}\}$
$\neg C$	$\Delta \setminus C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
$C \sqcap D$	$C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}} \cap D^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
$C \sqcup D$	$C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}} \cup D^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
$\forall R.C$	$\{\delta \in \Delta \mid (\delta,\epsilon) \in R^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$ implies $\epsilon \in C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}\}$
$\exists R.C$	$\{\delta \in \Delta \mid \exists\epsilon$ with $(\delta,\epsilon) \in R^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$ and $\epsilon \in C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}\}$
$\exists S.\mathsf{Self}$	$\{\delta \in \Delta \mid (\delta,\delta) \in S^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}\}$
$\leqslant k\,S.C$	$\{\delta \in \Delta \mid \sharp\{([\delta]_\approx,[\epsilon]_\approx) \in S^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$ and $[\epsilon]_\approx \in C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}\} \leq k\}$
$\geqslant k\,S.C$	$\{\delta \in \Delta \mid \sharp\{([\delta]_\approx,[\epsilon]_\approx) \in S^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$ and $[\epsilon]_\approx \in C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}\} \geq k\}$
$\mathbf{K}C$	$\bigcap_{\mathcal{J}\in\mathcal{M}} C^{(\mathcal{J},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
$\mathbf{A}C$	$\bigcap_{\mathcal{J}\in\mathcal{N}} C^{(\mathcal{J},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
V^-	$\{(\delta,\epsilon) \in \Delta\times\Delta \mid (\epsilon,\delta) \in V^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}\}$
U	$\Delta \times \Delta$
$A \times B$	$\{(\delta,\epsilon) \in \Delta\times\Delta \mid \delta \in A^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$ and $\epsilon \in B^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}\}$
$\neg S$	$(\Delta\times\Delta) \setminus S^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
$S_1 \sqcap S_2$	$S_1^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}} \cap S_2^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
$S_1 \sqcap S_2$	$S_1^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}} \cup S_2^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
$\mathbf{K}R$	$\bigcap_{\mathcal{J}\in\mathcal{M}} R^{(\mathcal{J},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
$\mathbf{A}R$	$\bigcap_{\mathcal{J}\in\mathcal{N}} R^{(\mathcal{J},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
$C(a)$	$a^{\mathcal{I}} \in C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
$R(a,b)$	$(a^{\mathcal{I}},b^{\mathcal{I}}) \in R^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
$C \sqsubseteq D$	$C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}} \subseteq D^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
$R_1 \circ \dots \circ R_n \sqsubseteq R$	$R_1^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}} \circ \dots \circ R_n^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}} \subseteq S^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$

Interpretation $\mathcal{I}$; MKNF structure $(\mathcal{I},\mathcal{M},\mathcal{N})$; variable assignment $\mathcal{Z}$; $A,B \in N_C$; $C,D \in \mathsf{C}$; $V \in N_R$; $S_{(i)} \in \mathsf{R}^s$; $R_{(i)} \in \mathsf{R}$; $a,b \in N_I$; $x \in N_V$, $t \in N_V \cup N_I$; $\circ$ composition of binary relations.

Definition 5 *Given a $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ knowledge base KB, a (non-empty) set of interpretations $\mathcal{M}$ is an* MKNF model *of KB if (1) $\mathcal{M} \models KB$, and (2) for each $\mathcal{M}'$ with $\mathcal{M} \subset \mathcal{M}'$, $(\mathcal{I}',\mathcal{M}',\mathcal{M}) \not\models KB$ for some $\mathcal{I}' \in \mathcal{M}'$. KB is* MKNF-satisfiable *if an MKNF model of KB exists. An axiom α is* MKNF-entailed by *KB, written $KB \models_{\mathbf{K}} \alpha$, if all MKNF models $\mathcal{M}$ of KB satisfy α.*

As noted in [7], since $\mathcal{M} \models KB$ is defined w.r.t. $(\mathcal{I},\mathcal{M},\mathcal{M})$, the operators $\mathbf{K}$ and $\mathbf{A}$ are interpreted in the same way, and so we can restrict instance checking $KB \models_{\mathbf{K}} C(a)$ and subsumption $KB \models_{\mathbf{K}} C \sqsubseteq D$ to C and D without occurrences of the operator $\mathbf{A}$.

2.3 Decidability Considerations

In the following, we describe a decidable fragment of $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ which in principle encompasses all the relevant other languages to be discussed in Section 3. Reasoning in this fragment follows in principle the approach from [7] for $\mathcal{ALCK}_{\mathcal{NF}}$ and its refinement from [14]: each model of a knowledge base in $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ is cast into a $\mathcal{SROIQV}(\mathcal{B}^s,\times)$ KB. Consequently, reasoning in $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ is reduced to a number of reasoning tasks in the non-modal $\mathcal{SROIQV}(\mathcal{B}^s,\times)$.

Following [20], we point out that we can simplify reasoning in $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ to reasoning in $\mathcal{SROIQ}(\mathcal{B}^s)\mathcal{K}_{\mathcal{NF}}$ by grounding, i.e., by appropriately substituting nominal schemas by all nominals in all possible ways, and by simulating concept products as shown in [19, 25]. Note that neither grounding nor the material presented in the following are efficient for reasoning. But that does not constitute a problem since we only want to show decidability.

A set of interpretations $\mathcal{M}$ is *first-order representable* (alternatively, *$\mathcal{SROIQ}(\mathcal{B}^s)$ representable*) if there exists a first-order theory ($\mathcal{SROIQ}(\mathcal{B}^s)$ KB) $KB_{\mathcal{M}}$ such that $\mathcal{M} = \{\mathcal{I} \mid \mathcal{I}$ satisfies $KB_{\mathcal{M}}\}$. It is noted in [7] that, for $\mathcal{ALC}$, such a $KB_{\mathcal{M}}$ may be finite or infinite, and it is shown that, in general, models of $\mathcal{ALCK}_{\mathcal{NF}}$ KBs are not even first-order representable. Therefore the notion of subjectively quantified KBs is introduced in [7], and we extend this notion to $\mathcal{SROIQ}(\mathcal{B}^s)\mathcal{K}_{\mathcal{NF}}$ KBs.

Building on the improved formalization in [14], we define that a $\mathcal{SROIQ}(\mathcal{B}^s)\mathcal{K}_{\mathcal{NF}}$ expression S is *subjective* if each $\mathcal{SROIQ}(\mathcal{B}^s)$ subexpression in S lies in the scope of at least one modal operator.

Definition 6 *A $\mathcal{SROIQ}(\mathcal{B}^s)\mathcal{K}_{\mathcal{NF}}$ KB KB is* subjectively quantified *if each expression of the form $\exists R.C$, $\forall R.C$, $\leqslant k\,R.C$, and $\geqslant k\,R.C$ occurring in KB satisfies one of the conditions: R is a $\mathcal{SROIQ}(\mathcal{B}^s)$ role and C is a $\mathcal{SROIQ}(\mathcal{B}^s)$ concept, or R and C are both subjective.*

The overall idea is to avoid expressions that are only partially in scope of a modal operator. Besides not being first-order representable, such expressions yield counterintuitive consequences as shown in [7] (Section 3).

Following [7], we would now proceed to define a set of modal atoms, i.e., subjective expressions, appearing in such a subjectively quantified $\mathcal{SROIQ}(\mathcal{B}^s)\mathcal{K}_{\mathcal{NF}}$ KB. This set can be partitioned into two sets of positive and negative modal atoms, i.e., the atoms that are assumed to hold and the ones that are assumed not to hold. A $\mathcal{SROIQ}(\mathcal{B}^s)$ representation of an MKNF model would be obtained from the modal atoms that are assumed to hold and the part of the considered KB that is free of modal operators. Instead, we restrict $\mathcal{SROIQ}(\mathcal{B}^s)\mathcal{K}_{\mathcal{NF}}$ KBs even further. We use $\mathbf{M}$ to denote either $\mathbf{K}$ or $\mathbf{A}$, and $\mathbf{N}$ to denote either $\mathbf{M}$ or $\neg\mathbf{M}$.

Definition 7 *A $\mathcal{SROIQ}(\mathcal{B}^s)\mathcal{K}_{\mathcal{NF}}$ KB KB is* strictly subjectively quantified *if the following conditions hold:*

1. *each expression of the form $\exists R.C$, $\forall R.C$, occurring in KB, satisfies one of two conditions: R is a $\mathcal{SROIQ}(\mathcal{B}^s)$ role and C is a $\mathcal{SROIQ}(\mathcal{B}^s)$ concept, or R is of the form $\mathbf{M}R'$ with $R' \in N_R$ and C is of the form $\mathbf{N}C'$ with C' a $\mathcal{SROIQ}(\mathcal{B}^s)$ concept;*
2. *modal operators are not allowed inside of statements of the form $\leqslant k\, R.C$, $\geqslant k\, R.C$, and $\exists S.\mathsf{Self}$ nor in RIAs;*
3. *for assertions $R(a,b) \in KB$, we have either $R \in N_R$ or R is of the form $\mathbf{M}R_1$ and $R_1 \in N_R$.*

This restriction is severe, but it approximates the conditions to obtain subjectively quantified $\mathcal{ALCK}_{\mathcal{NF}}$ KBs in [7], which significantly simplifies the following steps outlined before Definition 7.

Based on partitions (P,N) of the modal atoms $MA_\Delta(KB)$ appearing in a strictly subjectively quantified $\mathcal{SROIQ}(\mathcal{B}^s)\mathcal{K}_{\mathcal{NF}}$ KB KB, the notion 'induces', and $Ob_K(P,N)$, a $\mathcal{SROIQ}(\mathcal{B}^s)$ representation of the models of KB, it can be shown that the models of KB are $\mathcal{SROIQ}(\mathcal{B}^s)$-representable (for details, also w.r.t. the just mentioned notions, we refer to the extended version and also [7]).

Corollary 1 *Let KB be a strictly subjectively quantified $\mathcal{SROIQ}(\mathcal{B}^s)\mathcal{K}_{\mathcal{NF}}$ KB, $\mathcal{M}$ an MKNF model of KB, and (P,N) be the partition of $MA_\Delta(KB)$ induced by $(\mathcal{M},\mathcal{M})$. Then $\mathcal{M} = \{\mathcal{I} \mid \mathcal{I} \models Ob_K(P,N)\}$.*

This does not yield a decidable procedure yet, since subjectively quantified $\mathcal{ALCK}_{\mathcal{NF}}$ KBs may have an infinite representation or an infinite number of models [7] and the same holds for strictly subjectively quantified $\mathcal{SROIQ}(\mathcal{B}^s)\mathcal{K}_{\mathcal{NF}}$ KBs. To counter that, a further restriction is introduced in [7].

Definition 8 *Let KB be a $\mathcal{SROIQ}(\mathcal{B}^s)\mathcal{K}_{\mathcal{NF}}$ KB that is strictly subjectively quantified, $\mathcal{A}$ its ABox, and Γ the set of axioms in the TBox of KB that contain at least one modal operator. A concept C in KB is* simple *if, for all expressions of the form $\exists \mathbf{A}R.\mathbf{N}D$ or $\forall \mathbf{A}R.\mathbf{N}D$ in C, D has no occurrence of role expressions of the form $\mathbf{K}R$. KB is* simple *if the following conditions are satisfied:*

1. *only axioms of the form $\mathbf{K}C \sqsubseteq D$ occur in Γ, where C is a $\mathcal{SROIQ}(\mathcal{B}^s)$ concept and no $\mathbf{K}$ operator occurs in $\exists$ and $\forall$ restrictions in D;*
2. *for each $\mathbf{K}C \sqsubseteq D \in \Gamma$, $(KB \setminus (\Gamma \cup \mathcal{A})) \not\models \top \sqsubseteq C$;*
3. *all concept expressions in $\mathcal{A}$ are simple.*

We can show that a decidable reasoning procedure exists.

Theorem 1 *Let KB be a simple $\mathcal{SROIQ}(\mathcal{B}^s)\mathcal{K}_{\mathcal{NF}}$ KB. Then the MKNF models $\mathcal{M}$ of KB can be characterized by a finite subset of $MA_\Delta(KB)$.*

3 Coverage of Other Languages

We now turn to the key results of our proposal, namely that $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ encompasses some of the most prominent languages related to OWL, rules, non-monotonic reasoning, and their integrations. Some results follow trivially from the definitions in Section 2 and from previous work. Some work is needed to show coverage of n-ary Datalog (Section 3.1) and Hybrid MKNF (Section 3.2). $\mathcal{SROIQV}(\mathcal{B}^s,\times)\mathcal{K}_{\mathcal{NF}}$ then encompasses the following.

- $\mathcal{SROIQ}$ (a.k.a. OWL 2 DL).
- The tractable profiles OWL 2 EL, OWL 2 RL, OWL 2 QL.
- RIF-Core [3], i.e., n-ary Datalog, interpreted as DL-safe Rules [24]. Coverage of binary Datalog is shown in [20], while the general case is shown in Section 3.1.
- DL-safe SWRL [24], $\mathcal{AL}$-log [6], and CARIN [21]. Coverage follows from Section 3.1 (alternatively from Section 3.2 in conjunction with [23]).
- $\mathcal{ALCK}_{\mathcal{NF}}$. This follows from our definitions and includes notions of concept and role closure present in this formalism.
- Closed Reiter defaults, a form of non-monotonic reasoning, are covered through the coverage of $\mathcal{ALCK}_{\mathcal{NF}}$. This includes coverage of DLs extended with default rules as presented in [2].
- Hybrid MKNF (see Section 3.2 below).
- Answer Set Programming [9], i.e., disjunctive Datalog with classical negation and non-monotonic negation under the answer set semantics. This follows from the coverage of Hybrid MKNF [23].

3.1 n-ary Datalog

Below, we generalize a result found in [20] on embedding DL-safe rules into $\mathcal{SROIQV}(\mathcal{B}^s,\times)$. Specifically, we extend the result to apply to rules in which predicates of arbitrary arity appear. Some notation and definitions are also adopted from [20].

In the following, RB is a set of Datalog rules defined over a signature $\Sigma = \langle N_I, N_P, N_V\rangle$, where N_I, N_P, and N_V, are sets of constants, n-ary predicates, and variables, respectively. Each rule has the form $A_1,\ldots,A_n \rightarrow H$, where H and each A_i is of the form $P(t_1,\ldots,t_n)$, with $P \in N_P$ and each $t_i \in N_I \cup N_V$. $N_{P,i}$ ($N_{P,>i}$) is the set of predicates of N_P with arity i (greater than i). Also, $\top,\bot \in N_{P,1}$, and $\approx \in N_{P,2}$.

Definition 9 *Interpretations $\mathcal{I}$ and variable assignments $\mathcal{Z}$ are defined as in Section 2.2 under the standard name assumption. The function $\cdot^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$ from Table 1 is generalized to n-ary predicates by assigning a relation $P^{\mathcal{I}} \subseteq \Delta^n$ to each $P \in N_P$ with $n > 2$.*

Since Datalog rules are free of modal operators, we need only refer to $\mathcal{I},\mathcal{Z}$. An atom $P(t_1,\ldots,t_n)$ is satisfied *by $\mathcal{I}$ and $\mathcal{Z}$, written $\mathcal{I},\mathcal{Z} \models P(t_1,\ldots,t_n)$ if $(t_1^{\mathcal{I},\mathcal{Z}},\ldots,t_n^{\mathcal{I},\mathcal{Z}}) \in P^{\mathcal{I}}$. A set of atoms $\mathcal{B}$ is* satisfied *by $\mathcal{I}$ and $\mathcal{Z}$ ($\mathcal{I},\mathcal{Z} \models \mathcal{B}$) if $\mathcal{I},\mathcal{Z} \models A_i$ for each $A_i \in \mathcal{B}$. A rule $\mathcal{B} \rightarrow H$ is* satisfied *by $\mathcal{I}$ and $\mathcal{Z}$ ($\mathcal{I},\mathcal{Z} \models \mathcal{B} \rightarrow H$) if $\mathcal{I},\mathcal{Z} \models H$ or $\mathcal{I},\mathcal{Z} \not\models \mathcal{B}$. $\mathcal{I}$* satisfies *a rule $\mathcal{B} \rightarrow H$ if, for all assignments $\mathcal{Z}$, $\mathcal{I},\mathcal{Z} \models \mathcal{B} \rightarrow H$. $\mathcal{I}$ satisfies RB if it satisfies all $r \in RB$.*

DL-safe rules with n-nary predicates can be embedded into an equisatisfiable $\mathcal{SROIQV}(\mathcal{B}^s,\times)$ KB $\mathsf{dl}(RB)$ over the signature $\Sigma = \langle N_I, N_C, N_R, N_V\rangle$. Here, $N_C = N_{P,1}$ and $N_R = N_{P,2} \cup \{U\} \cup S$, where S is a special set of roles: If $P \in N_{P,>2}$ has arity k, then $P_1,\ldots,P_k \in S$ are unique binary predicates associated with P; S is the set of all such predicates.

The rules defining $\mathsf{dl}(RB)$ are shown below. C and R are unary and binary predicates in RB, while P has higher arity.

1. $\mathsf{dl}(C(t)) := \exists U.(\{t\} \sqcap C)$;
2. $\mathsf{dl}(R(t,u)) := \exists U.(\{t\} \sqcap \exists R.\{u\})$;
3. $\mathsf{dl}(P(t_1,\ldots,t_k)) := \exists U.(\exists P_1.\{t_1\} \sqcap \ldots \sqcap \exists P_k.\{t_k\})$;
4. $\mathsf{dl}(A_1,\ldots,A_n \rightarrow H) := \mathsf{dl}(A_1) \sqcap \ldots \sqcap \mathsf{dl}(A_n) \sqsubseteq \mathsf{dl}(H)$;
5. $\mathsf{dl}(RB) := \{\mathsf{dl}(r) \mid r \in RB\}$.

Definition 10 *Given a set of interpretations $\mathcal{M}$ for RB, consider one $\mathcal{I} \in \mathcal{M}$. We define a family fam$(\mathcal{I})$ of interpretations $\mathcal{J}$.*

(a) *To each $(d_1,\ldots,d_k) \in P^{\mathcal{I}}$, assign a unique element e in Δ (i.e., we define a total, injective function from the set of tuples to Δ).*
(b) *For each $C \in N_{P,1}$, $C^{\mathcal{J}} := C^{\mathcal{I}}$.*
(c) *For each $R \in N_{P,2}$, $R^{\mathcal{J}} := R^{\mathcal{I}}$.*
(d) *For each $P \in N_{P,>2}$, if $(d_1,\ldots,d_k) \in P^{\mathcal{I}}$, then $(e,d_i) \in P_i^{\mathcal{J}}$, where e is the element assigned to $(d_1,\ldots,d_k)$ in point (a).*

The standard name assumption applies, so the domain is always the same, and elements in N_I are always mapped to the same $d \in \Delta$.

Any interpretation $\mathcal{J}$ for $\mathsf{dl}(RB)$ can be reduced to an interpretation $\mathcal{I}$ for RB: if $(e, d_1) \in P_1^{\mathcal{J}}, \ldots, (e, d_k) \in P_k^{\mathcal{J}}$, then $(d_1, \ldots, d_k) \in P^{\mathcal{I}}$. And so, for any interpretation $\mathcal{J}$ for $\mathsf{dl}(RB)$, there is an $\mathcal{I}$ for RB such that $\mathcal{J} \in \mathrm{fam}(\mathcal{I})$.

Lemma 1 *Let A be an atom in RB, $\mathcal{I}$ an interpretation of RB, $\mathcal{J} \in fam(\mathcal{I})$, and $\mathcal{Z}$ a variable assignment.*

1. *$\mathcal{I}, \mathcal{Z} \models A$ if and only if $\mathsf{dl}(A)^{\mathcal{J},\mathcal{Z}} = \Delta$. and*
2. *$\mathcal{I}, \mathcal{Z} \not\models A$ if and only if $\mathsf{dl}(A)^{\mathcal{J},\mathcal{Z}} = \emptyset$.*

Theorem 2 *Let RB be a Datalog program. $\mathcal{M}$ is the set of all interpretations $\mathcal{I}$ that satisfy RB if and only if $\mathcal{M}_1 = \{\mathcal{J} \mid \mathcal{J} \in fam(\mathcal{I})$ with $\mathcal{I} \in \mathcal{M}\}$ is the set of all interpretations that satisfy $\mathsf{dl}(RB)$.*

It also follows immediately from Theorem 2 that the tractable language $\mathcal{SROELV}_n$ introduced in [20] encompasses not only $\mathcal{SROEL}$ a.k.a. OWL 2 EL, but also n-ary Datalog.

3.2 Hybrid MKNF Knowledge Bases

Hybrid MKNF knowledge bases [23], which are based on MKNF logics [22], i.e., first-order logic with equality plus two modal operators $\mathbf{K}$ and $\mathbf{not}$, are defined as the combination of a decidable DL knowledge base and a set of rules.

Given a DL knowledge base $\mathcal{O}$, a (function-free) first-order atom $P(t_1, \ldots, t_n)$ is a DL-atom if P is $\approx$ or is in $\mathcal{O}$; otherwise it is a non-DL-atom. An *MKNF rule* r has the below form, where H_k, A_i, and B_j are (possibly classically negated) first-order atoms:

$$\mathbf{K}H_1 \vee \mathbf{K}H_l \leftarrow \mathbf{K}A_1, \ldots, \mathbf{K}A_n, \mathbf{not}B_1, \ldots, \mathbf{not}B_m \qquad (1)$$

A program $\mathcal{P}$ is a finite set of MKNF rules, and a *hybrid MKNF knowledge base* $\mathcal{K}$ is a pair $(\mathcal{O}, \mathcal{P})$. The *ground instantiation* of $\mathcal{K}$ is the KB $\mathcal{K}_G = (\mathcal{O}, \mathcal{P}_G)$ where $\mathcal{P}_G$ is obtained from $\mathcal{P}$ by replacing each rule r of $\mathcal{P}$ with a set of rules substituting each variable in r with constants from $\mathcal{K}$ in all possible ways.

In [23], MKNF KBs are considered in which H_k, A_i, and B_j may be arbitrary first-order formulas. Here, the restriction to (classically negated) first-order atoms suffices and simplifies the presentation.

Hybrid MKNF KBs are embedded into MKNF logics. We briefly recall the syntax and semantics of (function-free) MKNF logics.

Let $\Sigma = (N_I, N_P, N_V)$ be a signature and N_P contain the equality predicate $\approx$. The syntax of MKNF formulas over Σ is defined by the below grammar, where $t_i \in N_I \cup N_V$ and $P \in N_P$.

$$\varphi \leftarrow P(t_1, \ldots, t_n) \mid \neg\varphi \mid \varphi \wedge \varphi \mid \exists x.\varphi \mid \mathbf{K}\varphi \mid \mathbf{not}\varphi \qquad (2)$$

Moreover, $\varphi_1 \vee \varphi_2$, $\varphi_1 \supset \varphi_2$, $\varphi_1 \equiv \varphi_2$, $\forall x : \varphi$, $\top$, $\bot$, $t_1 \approx t_2$, and $t_1 \not\approx t_2$ are admitted standard syntactic shortcuts.

First-order atoms of the form $t_1 \approx t_2$ (resp. $t_1 \not\approx t_2$) are called *equalities* (resp. *inequalities*), and $\varphi[t_1/x_1, \ldots, t_n/x_n]$ denotes the formula obtained by substituting the free variables x_i in φ, i.e., those that are not in the scope of any quantifier, by the terms t_i. φ is closed if it contains no free variables. Given a (first-order) formula φ, $\mathbf{K}\varphi$ is called a $\mathbf{K}$-*atom* and $\mathbf{not}\varphi$ a $\mathbf{not}$-*atom*; $\mathbf{K}$-atoms and $\mathbf{not}$-atoms are *modal atoms*. As in n-nary Datalog, N_P contains N_C and N_R. The generalization of interpretations to n-ary $P \in N_P$ also applies.

Let φ be a closed MKNF formula. Given an MKNF structure $(\mathcal{I}, \mathcal{M}, \mathcal{N})$, satisfaction of φ is defined as in [23, Table II]. We say that a set of interpretations $\mathcal{M}$ *satisfies* φ, written $\mathcal{M} \models \varphi$, if $(\mathcal{I}, \mathcal{M}, \mathcal{M}) \models \varphi$ for each $\mathcal{I} \in \mathcal{M}$.

A set of interpretations $\mathcal{M}$ is an *MKNF model* of φ if (1) $\mathcal{M}$ satisfies φ, and (2) for each set of interpretations $\mathcal{M}'$ with $\mathcal{M}' \supset \mathcal{M}$ we have $(\mathcal{I}', \mathcal{M}', \mathcal{M}) \not\models \varphi$ for some $\mathcal{I}' \in \mathcal{M}'$.

An MKNF formula φ is *MKNF-satisfiable* if an MKNF model of φ exists. Furthermore, φ *MKNF-entails* ψ, written $\varphi \models_{\mathbf{K}} \psi$, if $\mathcal{M} \models \psi$ for each MKNF model $\mathcal{M}$ of φ.

The definition above is similar to Definition 5, only that in the earlier definition, sets of (pairs of) individuals are considered, while here the satisfaction relation from [23, Table II] is used. We recall the embedding of hybrid MKNF KBs into MKNF logics.

Let $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ be a hybrid MKNF knowledge base and $\pi(\mathcal{O})$ the transformation of $\mathcal{O}$ into a formula of first-order logic with equality. We extend π to MKNF rules r of the form (1), $\mathcal{P}$, and $\mathcal{K}$ as follows, where $\vec{x}$ is the vector of the free variables of r.

$$\pi(r) = \forall\vec{x} : (\mathbf{K}A_1 \wedge \ldots \wedge \mathbf{K}A_n \wedge \mathbf{not}B_1 \wedge \ldots \wedge \mathbf{not}B_m \supset \mathbf{K}H_1 \vee \ldots \vee \mathbf{K}H_l)$$

$$\pi(\mathcal{P}) = \bigwedge_{r \in \mathcal{P}} \pi(r) \qquad \pi(\mathcal{K}) = \mathbf{K}\pi(\mathcal{O}) \wedge \pi(\mathcal{P})$$

We abuse notation and use $\mathcal{K}$ instead of $\pi(\mathcal{K})$. The following syntactic restriction, similar in spirit to the restriction applied to nominal schemas, ensures decidability. An MKNF rule r is *DL-safe* if every variable in r occurs in at least one non-DL-atom $\mathbf{K}A_i$ in the body of r. $\mathcal{K}$ is *DL-safe* if all the rules in $\mathcal{K}$ are DL-safe.

As argued in [23], reasoning in hybrid MKNF can thus be restricted to ground $\mathcal{K}_G$. We thus use the variable assignment $\mathcal{Z}$ for that purpose and extend the satisfiability relation in [23, Table II] from $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ to $(\mathcal{I}, \mathcal{M}, \mathcal{N}), \mathcal{Z}$. We link satisfiability in the usual way by defining $(\mathcal{I}, \mathcal{M}, \mathcal{N}) \models \varphi$ if $(\mathcal{I}, \mathcal{M}, \mathcal{N}), \mathcal{Z} \models \varphi$ for all $\mathcal{Z}$.

Now, we show that $\mathcal{K}$ can be embedded into an equisatisfiable $\mathcal{SROIQV}(\mathcal{B}^s, \times)\mathcal{K}_{\mathcal{NF}}$ knowledge base $\mathsf{dl}(\mathcal{K})$ over $\Sigma = \langle N_I, N_C, N_R, N_V \rangle$. Again, $N_C = N_{P,1}$ and $N_R = N_{P,2} \cup \{U\} \cup S$, where S is a set of roles defined as in Section 3.1 and $\approx \in N_{P,2}$.

The extension of $\mathsf{dl}(RB)$ to $\mathsf{dl}(\mathcal{K})$ is given below.

1. $\mathsf{dl}(C(t)) := \exists U.(\{t\} \sqcap C)$;
2. $\mathsf{dl}(R(t, u)) := \exists U.(\{t\} \sqcap \exists R.\{u\})$;
3. $\mathsf{dl}(P(t_1, \ldots, t_k)) := \exists U.(\exists P_1.\{t_1\} \sqcap \ldots \sqcap \exists P_k.\{t_k\})$;
4. $\mathsf{dl}(\neg A) := \neg\mathsf{dl}(A)$;
5. $\mathsf{dl}(\mathbf{K}H_1 \vee \mathbf{K}H_l \leftarrow \mathbf{K}A_1, \ldots, \mathbf{K}A_n, \mathbf{not}B_1, \ldots, \mathbf{not}B_m) := \mathbf{K}\mathsf{dl}(A_1) \sqcap \ldots \sqcap \mathbf{K}\mathsf{dl}(A_n) \sqcap \neg\mathbf{A}\mathsf{dl}(B_1) \sqcap \ldots \sqcap \neg\mathbf{A}\mathsf{dl}(B_m) \sqsubseteq \mathbf{K}\mathsf{dl}(H_1) \sqcup \ldots \sqcup \mathbf{K}\mathsf{dl}(H_l)$;
6. $\mathsf{dl}(\mathcal{K}) := \mathcal{O} \cup \{\mathsf{dl}(r) | r \in \mathcal{P}\}$.

The definition of a family $\mathrm{fam}(\mathcal{I})$ (Definition 10) is straightforwardly lifted from RB to $\mathcal{K}$. We lift Lemma 1 to sets of interpretations and the expressions appearing in $\mathcal{K}$.

Lemma 2 *Let F in $\mathcal{K}$ be of the form A, $\neg A$, $\mathbf{K}F_1$, or $\mathbf{not}F_1$ where A is an atom, and F_1 of the form A or $\neg A$, $\mathcal{M}$ a set of interpretations of $\mathcal{K}$, $\mathcal{M}_1 = \{\mathcal{J} \mid \mathcal{J} \in fam(\mathcal{I})$ with $\mathcal{I} \in \mathcal{M}\}$, $\mathcal{I} \in \mathcal{M}$, $\mathcal{J} \in fam(\mathcal{I})$, and $\mathcal{Z}$ a variable assignment. The following hold.*

1. *$(\mathcal{I}, \mathcal{M}, \mathcal{M}), \mathcal{Z} \models F$ iff $\mathsf{dl}(F)^{(\mathcal{J}, \mathcal{M}_1, \mathcal{M}_1), \mathcal{Z}} = \Delta$.*
2. *$(\mathcal{I}, \mathcal{M}, \mathcal{M}), \mathcal{Z} \not\models F$ iff $\mathsf{dl}(F)^{(\mathcal{J}, \mathcal{M}_1, \mathcal{M}_1), \mathcal{Z}} = \emptyset$.*

Hybrid MKNF KBs can be embedded into $\mathcal{SROIQV}(\mathcal{B}^s, \times)\mathcal{K}_{\mathcal{NF}}$.

Theorem 3 *Let $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ be a hybrid MKNF KB. $\mathcal{M}$ is an MKNF model of $\mathcal{K}$ if and only $\mathcal{M}_1 = \{\mathcal{J} \mid \mathcal{J} \in fam(\mathcal{I})$ with $\mathcal{I} \in \mathcal{M}\}$ is a hybrid MKNF model of $\mathsf{dl}(\mathcal{K})$.*

Note that in few cases, the embedding does not yield a simple KB (cf. Definition 8). Consider, e.g., $\bot \sqsubseteq \exists U.(\{a\} \sqcap C)$ and $\mathbf{K}D(a) \leftarrow \mathbf{K}C(a)$. This problem remains open for future work.

4 Related Work and Conclusions

We have proposed $\mathcal{SROIQV}(\mathcal{B}^s, \times)\mathcal{K}_{\mathcal{NF}}$ as an advance towards a unifying logic for the Semantic Web. It covers a wide variety of languages around OWL, rules, and non-montonicity, which have been discussed in the context of Semantic Web ontology languages. While the coverage of our language depends significantly on the integrative strength of hybrid MKNF, our proposal significantly advances on the latter not only in terms of coverage, but also in that it provides a unified syntax, in the tradition of description logics. This syntax rests crucially on the use of nominal schemas and indeed, as discussed in [20], extending OWL 2 DL with nominal schemas is conceptually and syntactically relatively straightforward.

Most closely related in spirit to our endeavor are probably [4, 5, 23]. However, [4, 5] are less encompassing with respect to the languages which can be embedded in it. We have already discussed in detail hybrid MKNF [23], which is covered by our approach.

So, how might the logic proposed here fare as a unifying logic for the Semantic Web stack? The very concept of such a unifying logic currently acts mainly as a driver for research into ontology language development, and as such provides guidance which ensures that languages do not diverge too widely. In this sense, our proposal is valid. However, at the same time there seems to be little discussion in the community on *requirements* for such a unifying logic. In particular, should the unifying logic merely (or primarily) provide a conceptual underpinning, or should it allow practical ontology modeling?

To serve as a conceptual underpinning, it would be desirable to further extend our proposed language to cover further expressive features discussed in the context of Semantic Web ontology languages. In order to develop it into a practical language, strong reasoning algorithms must be developed and implemented, e.g., by incorporating further ideas from [20] and [14], together with modeling guidance and practical use cases.

We believe that our proposal has potential in either direction.

ACKNOWLEDGEMENTS

We acknowledge support by the National Science Foundation under award 1017225 “III: Small: TROn – Tractable Reasoning with Ontologies,” and by the Fundação para a Ciência e a Tecnologia under project “ERRO – Efficient Reasoning with Rules and Ontologies” (PTDC/EIA-CCO/121823/2010). Matthias Knorr acknowledges hospitality and support by the Ohio Center of Excellence in Knowledge-enabled Computing (Kno.e.sis) for a stay at Wright State University in September 2011.

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