Title: The influence on unaided vision of age, pupil diameter and spherocylindrical refractive error

Running title: Vision, age, pupil size and refraction

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Background: The aim was to derive equations for the relationship between unaided vision and age, pupil diameter, iris colour and spherocylindrical refractive error.

Methods: Data were collected from 633 healthy right eyes of white subjects aged 20 to 70 years. Subjective spherocylindrical refractive errors ranged from -6.8 to +9.4 D (mean spherical equivalent), -1.5 to +1.9 D (orthogonal component, $\mathrm{J}_{0}$ ) and -0.8 to 1.0 D (oblique component, $\mathrm{J}_{45}$ ). Cylinder axis orientation was orthogonal in $46 \%$ of the eyes and oblique in $18 \%$. Unaided vision (0.3 to +1.3 LogMAR), pupil diameter ( 2.3 to 7.5 mm ) and iris colour ( $67 \%$ light/blue irides) was recorded. The sample included mostly females (60\%) and many contact lens wearers (42\%) and so the influences of these parameters were also investigated.

Results: Decision tree analysis showed that sex, iris colour, contact lens wear and cylinder axis orientation did not influence the relationship between unaided vision and refractive error. New equations for the dependence of the minimum angle of resolution on age and pupil diameter arose from step backwards multiple linear regressions carried out separately on the myopes (2.91.scalar vector +0.51 .pupil diameter -3.14 ) and hyperopes (1.55.scalar vector +0.06 .age 3.45 ).

Conclusion: The new equations may be useful in simulators designed for teaching purposes as they accounted for $81 \%$ (for myopes) and $53 \%$ (for hyperopes) of the variance in measured data. In comparison, previously published equations accounted for not more than 76\% (for myopes) and $24 \%$ (for hyperopes) of the variance depending on whether they included pupil size. The new equations are, as far as is known to the authors, the first to include age. The age-related decline in accommodation is reflected in the equation for hyperopes.

Key words: Vision, ametropia, pupil, age.

Optometry undergraduates have practical classes in which they learn how to relate unaided vision to refractive error. Simulators designed to supplement these classes require equations that relate unaided vision to all sphero-cylindrical refractive errors including myopic and hyperopic astigmats with orthogonal and oblique cylinder axis orientations. It would also be desirable if these formulae accounted for variations caused by pupil size, age-related changes in the ability to overcome uncorrected hyperopia with accommodation and, possibly, iris colour. While equations exist that allow unaided vision to be calculated from any sphero-cylindrical refractive error ${ }^{1}$ or pupil size ${ }^{2}$, the authors know of none that include age or iris colour. The purpose of this study was to derive the desired formulae.

A key feature of this study was to simulate for students the sort of results that would be obtained under the pressures of normal clinical practice rather than those that would be obtained under carefully-controlled laboratory conditions. For this reason it was considered entirely reasonable to utilise data from clinical practice, where short cuts and approximations in clinical methods are used which would not be tolerated in the research laboratory, and older (presbyopic) patients may predominate.

Raasch ${ }^{1}$ showed, by analysing a large sample of military records ${ }^{3}$, that scalar vectors derived from sphero-cylindrical refractive errors were the best means of relating unaided LogMAR vision to uncorrected refractive errors in.

To calculate a scalar vector it is first necessary to convert measured spheres, cylinders and cylinder axes into mean spheres ( M in equation 2 ) together with orthogonal ( $\mathrm{J}_{0}$ in equation 4 ) and oblique ( $\mathrm{J}_{45}$ in equation 5 ) astigmatic components ${ }^{4,5}$.
$M=$ sphere $+($ cylinder $/ 2)($ equation 1$)$
$\mathrm{J}_{0}=-0.5 \mathrm{x}$ (cylinder $\mathrm{x} \cos [2 \mathrm{x}$ cylinder axis]) (equation 2 )
$\mathrm{J}_{45}=-0.5 \mathrm{x}($ cylinder $\mathrm{x} \sin [2 \mathrm{x}$ cylinder axis]) (equation 3 )
The scalar vector, $U$, is then calculated from the mean sphere $(M)$ and both astigmatic components ( $\mathrm{J}_{0}$ and $\mathrm{J}_{45}$ ) using equation 4.
$\mathrm{U}=\mathrm{V}\left(\mathrm{M}^{2}+\mathrm{J}_{0}{ }^{2}+\mathrm{J}_{45}{ }^{2}\right)$ (equation 4)

Raasch derived equation 5 which relates scalar vector, $U$, to unaided LogMAR vision.
$\operatorname{LogMAR}=0.48+(1.07 \times[\log U])+\left(0.46 x[\operatorname{Log~U}]^{2}\right)($ equation 5)
Hyperopes were not included in Raasch's analysis as accommodation was considered too poorly controlled. Pupil size was also not recorded in the military records and so this element was not included in Raasch's calculations either.

Smith ${ }^{2}$ derived equations 6 and 7, based on the theoretical relationship between vision (expressed as the minimum angle of resolution, MAR), refractive error (scalar vector, U ) and pupil diameter. Use of equation 6 or 7 depended on whether the level of ametropia was high or low; a cut-off of between 1-2 dioptres was hinted at. Based on data from previous studies, Smith found that the constant, k , was typically 0.83 but ranged from 0.55 to 1.33.

MAR $=k \times U \times$ pupil diameter (equation 6 for high levels of ametropia)
$\operatorname{MAR}=\mathrm{V}\left[1+(\mathrm{kx} \mathrm{U} \mathrm{x} \mathrm{pupil} \mathrm{diameter})^{2}\right]$ (equation 7 for low levels of ametropia)

New equations derived in the present study were compared to those of Raasch ${ }^{1}$ and Smith ${ }^{2}$. It has also been suggested that since uncorrected astigmatism with oblique axis orientation has more of a detrimental effect on vision than that with orthogonal cylinder axis orientation, the oblique component $\left(\mathrm{J}_{45}\right)$ in equation 4 for calculating the scalar vector could be weighted by a factor of two ${ }^{4}$. The need for this adjustment was also investigated in the present study.

## Methods

## Research Ethics

This study received approval from the Research Ethics Committee at Aston University and adhered to the tenets of the Declaration of Helsinki.

## Practice setting

Data were collected in a medium-sized multiple practice (Specsavers, Salisbury, England). All data were collected by the principal author (RMR, an experienced optometrist) and the same examination room and equipment was used throughout.

Sample profile
All subjects were white. The influence of race ${ }^{6}$ could have been studied but other ethnic groups were too scarce to allow meaningful analysis.

Many subjects were also contact lens wearers. Exclusion of these subjects would have substantially reduced the size of the sample. In this case, numbers were sufficient to allow investigation of the effects of contact lens wear.

Other exclusion criteria were:
a. Corrected visual acuity less than 6/7.5, as adopted by Elliott and Cox ${ }^{7}$. Anything less than this could be indicative of pathology or amblyopia.
b. Inability to read all letters on the $1 / 20$ line of the test chart. This was because (1) the patient chair prevented nearer testing distances and (2) non-chart based methods such as counting fingers and hand movements would have to be used at lower levels of vision, making meaningful analysis difficult. This approach was also taken by Patel et al. (2008) ${ }^{\text {8 }}$;
c. Use of medication (e.g. sedatives) which could affect judgement;
d. Use of medication which could affect vision, pupil size, iris colour or any of the factors measured. This included medications such as anti-malarials or tamoxifen (both can affect vision), co-codamol (affecting pupil size), prostaglandin analogues (affecting iris colour) ${ }^{9}$;
e. Previous surgery on or affecting the right eye (from which data were collected);
f. Presence of ocular or systemic conditions likely to affect any of the factors measured, including pregnancy, diabetes, cataract, glaucoma ${ }^{10}$;
g. If RMR suspected the subject of malingering, giving false or misleading results or having poor subjective responses;
h. Unexpectedly good unaided vision at reduced testing distances that bring the chart closer to the far point of myopic subjects ${ }^{11}$.

Data were collected from right eyes only as inclusion of both eyes would have complicated statistical analysis ${ }^{12}$.

## Lighting

Examination room lighting was kept at a constant photopic level during each eye examination. The illuminance incident at the subject's eye was measured using a digital Standard ST-1308 Light Meter (TENMA) by placing the light detector in front of the subject's right eye to ensure that light levels reaching the eyes were consistent throughout the day and throughout the study. This was checked periodically throughout the study. Readings ranged from 87-117 lux (mean, standard deviation: 106, 7 lux). Luminance of the chart used to record unaided vision was also periodically checked using a Spectra Mini-spot Silicon Cell SpotMeter (Photo Research, Division of Kollmorgen Corporation, Burbank, California, USA). Readings ranged from 176 and $208 \mathrm{cdm}^{-2}$ (mean, standard deviation: 195, $9 \mathrm{cdm}^{-2}$ ). This was within the acceptable range of $80-320 \mathrm{cdm}^{-2}{ }^{13}$.

## Measurement of unaided vision

Unaided vision was measured using a computerized LCD chart (SC-2000 [UK Type], Nidek, Japan) with a mirror to take the viewing distance to 6.5 metres. The chart displayed 9 lines of 5 high contrast non-serif black letters on a white background (see Table 1). The ten $5 \times 5$ construction 'Sloan' letters ${ }^{14}$ were used in this chart. Switching between two charts of the same design reduced the effects of subjects memorising letters. It can be seen from Table 1 that the size progression was not strictly logarithmic so that this chart was not optimum for LogMAR scoring.

| Testing distance (m) | Snellen denominator (m) | LogMAR (line) | LogMAR (letter) |
| :--- | :--- | :--- | :--- |
| 1 | 20 | 1.301 | - |
| 1 | 15 | 1.176 | 0.025 |
| 1 | 12 | 1.079 | 0.019 |
| 2 | 20 | 1.000 | 0.016 |
| 2 | 15 | 0.875 | 0.025 |
| 3 | 20 | 0.824 | 0.010 |
| 3 | 15 | 0.699 | 0.025 |
| 3 | 12 | 0.602 | 0.019 |
| 3 | 10 | 0.523 | 0.016 |
| 6.5 | 20 | 0.488 | 0.007 |
| 6.5 | 15 | 0.363 | 0.025 |
| 6.5 | 12 | 0.266 | 0.019 |
| 6.5 | 10 | 0.187 | 0.016 |
| 6.5 | 7.5 | 0.062 | 0.025 |
| 6.5 | 6 | -0.035 | 0.019 |
| 6.5 | 5 | -0.114 | 0.016 |
| 6.5 | 4 | -0.211 | 0.019 |
| 6.5 | 3 | -0.336 | 0.025 |

Table 1. LogMAR line and letter values calculated at each testing distance for the chart used to measure unaided vision in this study.

A frosted occluder was used to cover the left eye to minimise consensual pupil dilation. A letter-by-letter recording method ${ }^{15}$ would have been desirable but was too time consuming for use on subjects attending a busy clinic for eye examinations. If subjects could not read the chart at 6.5 m they were taken to $3 \mathrm{~m}, 2 \mathrm{~m}$ and then 1 m . Subjects were asked to guess at lower lines in order to reduce variability ${ }^{16}$. Confusion between similar letters was counted as a correct response ${ }^{17-19}$. This applied to the letter groups $O / D / Q / C, Y / T / V, P / R$ and $H / N / M$. If letters were read incorrectly on more than one line, the lowest line with at least two letters correctly read was recorded as the LogMAR line ${ }^{19,20}$. That is, if a letter was read incorrectly on a line above the recorded LogMAR line it was not counted in this scoring method.

Each subject's unaided vision was initially recorded in terms of the viewing distance, the Snellen denominator value of the lowest line read and the number of letters read incorrectly on the lowest line. This information was used to calculate LogMAR vision. Table 1 shows LogMAR line and letter values calculated at each testing distance. LogMAR line values were calculated using equation 8.

LogMAR (line) $=\log _{10}($ Snellen denominator $/$ testing distance) (equation 8)
Line intervals were calculated by subtracting the LogMAR value of a line from that of the line above. Dividing line intervals by 5 (the number of letters on each line) then gave the LogMAR letter values shown in Table 1. As it was not possible to calculate a LogMAR letter value for the $1 / 20$ line, subjects unable to correctly read all letters on this line were excluded (see exclusion criterion b).

In addition, myopic subjects reading further down than expected on charts read at distances of less than 6.5 mwere also excluded (see exclusion criterion $h$ ). The method of doing this can be understood by studying Table 1. For example, a myope unable to read the 6/20 line would be moved to a distance of 3 m and would not be expected to read lower than the $3 / 10$ line. Ability to read below this line was taken as an indication that unexpectedly good unaided vision had arisen because the 3 m chart was closer to the far point of that myopic subject. As all data had been collected before this issue had been realised, data from any myopic subject who had read further down than expected at closer testing distances was removed retrospectively.

Subjects were asked not to squint or turn their heads in order to read more letters. No attempt was made to control accommodation during the measurement of unaided vision as the ability to accommodate in order to reduce hyperopic blur at different ages was one of the factors explored in this study.

Measurement of pupil diameter
Vertical pupil diameter was measured using a template rule (Matheson Optometrists, UK) held at the temporal edge of the subject's pupil. This blocked less light than if the rule was held above the pupil for horizontal diameter measurements and avoided obstruction by the nose. The rule had semi-circular templates along its edge from 1 mm to 9.5 mm in diameter in 0.5 mm intervals. Others have measured pupil diameter in a similar fashion ${ }^{7}$. Although more precise automated pupil measuring devices exist ${ }^{9}$, none of these were suitable for data collection in practice. Spectacles and contact lenses were removed so that measurement conditions were the same as
those used to record unaided vision. Subjects were asked to fixate a 6.5/20 letter or, if unable, the top of the letter chart. The left eye remained uncovered throughout measurement to avoid unwanted consensual pupil dilatation. Three measurements were taken and averaged.

Measurement of iris colour

Iris colour was included as a parameter in the present study as it was hypothesised that an eye with a lighter-coloured iris could have more light scatter, and therefore worse vision. Iris colour was classified according to an establishedIris Colour Classification System ${ }^{21}$. A paper copy of the grading standards was visually compared to the view of each subject's right iris observed using a white light beam on a slit lamp in the fully illuminated consulting room.

Measurement and classification of refractive error
Refractive error was initially estimated by averaging 3 to 5 readings by autorefraction (Tonoref II, Nidek, Japan) followed by subjective refinement to give the final refractive correction. The left eye was occluded with a frosted occlude during subjective refinement. The spherical correction was adjusted to ensure the accommodation was active ${ }^{13}$. A Jackson Cross Cylinder was used to refine the cylinder axis and power. The same was repeated for the left eye. The left eye was then blurred to approximately $6 / 12$ with a +1.00D lens and spherical refinement was performed binocularly, the left fogged eye having relaxed accommodation to give the highest positive/least negative result. This 'push the plus' methodology has been advocated by ${ }^{22,23}$.

Sphero-cylindrical refractive errors were converted to components $M$ (equation 1 ), $\mathrm{J}_{0}$ (equation 2) and $J_{45}$ (equation 3) prior to calculation of scalar vector $U$ (equation 4).. Identification of myopes and hyperopes was complicated by the presence of astigmatism. The steps taken to solve this now follow. The first step was to identify ametropes as all subjects with scalar vectors (U) of greater than 0.50 D . Then, all ametropes were classified as being myopic if the single focal point (in the absence of astigmatism) or the two focal lines (in the presence of astigmatism) were negative in power and hyperopic if they were positive. This approach was considered better than using the mean sphere to classify myopes and hyperopes as it avoided the problem of spherocylindrical powers such as $+1.00 /-2.00 \times 180$ that looks like an astigmatic hyperope in negative cylinder form and an astigmatic myope ( $-1.00 /+2.00 \times 90$ ) in positive cylinder form. The mean sphere would have been misleading as, in this case, it comes to zero; giving the impression that there is no ametropia. The scalar vector, on the other hand, amounts to 1.00D and correctly identifies ametropia but because one focal line is positive and the other is negative then this case is classed as a neither being a myope or a hyperope and is therefore excluded from the analysis. Subjects with cylinder powers of >0.50DC were further classified in terms of cylinder axis orientation. For this purpose, absolute values for the $\mathrm{J}_{0}$ and $\mathrm{J}_{45}$ components were compared to determine which was highest. By this means subjects were classed as having either orthogonal ( $J_{0}$ highest) or oblique ( $\mathrm{J}_{45}$ highest) cylinder axis orientations. Use of 0.50 D as a cut-off between refractive groups was in keeping with previous studies ${ }^{24}$.

Calculation of Blur Sensitivity Ratio (BSR)

A BSR was calculated for each subject in order to simplify statistical analyses, as described later. This ratio represents the first derivative or slope of the curve that relates unaided vision to blur
magnitude. That is, it is the rate of change of MAR as the blur magnitude changes. It was calculated by dividing the minimum angle of resolution (MAR) by the scalar vector (U). For example, a-1.00 D myope who could read $6 / 12(M A R=2)$ would have a BSR of $2 / 1=2$.

Repeatability

All measurements were repeated on two occasions (mean and standard deviation of test-retest interval: 17, 12 days) in 31 individuals in order to generate coefficients of repeatability (COR) ${ }^{13}$ for the parameters measured (Table 3).

Statistical analyses

SPSS 21.0 (IBM SPSS Statistics) was used to perform all statistical analyses.

Decision tree analysis (DTA) using the chi-squared automatic interaction detection (CHAID) method was applied to determine the hierarchical influence on the dependent variable (BSR) of each nominal independent variable i.e. sex, contact lens wear, iris colour, refractive group (myopes, hyperopes) and cylinder axis orientation (orthogonal, oblique). Both DTA and CHAID have been previously used to carry out multivariate analyses in the field of optometry ${ }^{25,26}$. BSR was used here because this single value encapsulated the relationship between unaided vision and ametropia for each subject. The question being asked of DTA was whether this relationship was altered by the independent variables.

Step backward multiple linear regression ${ }^{27}$ was used to generate formulae describing the relationship between the independent (MAR) and independent variables (age, pupil diameter and scalar vector U) for myopic and hyperopic eyes.

## Results

Table 2 shows the frequency of females, contact lens wearers, myopes, hyperopes and astigmats with orthogonal or oblique cylinder axis orientations in each 5 -year cohort from 20 to 70 years of age. Table 3 shows the mean, standard deviation, range and COR for the key variables measured.

| Age <br> cohort <br> (years) | N | Females <br> $(\%)$ | CL <br> wearers <br> $(\%)$ | Myopes <br> $(\%)$ | Hyperopes <br> $(\%)$ | Orthogonal <br> $(\%)$ | Oblique <br> $(\%)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $20-24$ | 54 | 48 | 48 | 87 | 13 | 46 | 15 |
| $25-29$ | 63 | 67 | 68 | 90 | 10 | 44 | 10 |
| $30-34$ | 66 | 67 | 70 | 85 | 15 | 44 | 11 |
| $35-39$ | 64 | 56 | 73 | 75 | 25 | 38 | 20 |
| $40-44$ | 68 | 78 | 51 | 82 | 18 | 46 | 21 |
| $45-49$ | 67 | 63 | 51 | 72 | 28 | 49 | 18 |
| $50-54$ | 67 | 48 | 27 | 54 | 46 | 46 | 22 |
| $55-59$ | 55 | 67 | 24 | 53 | 47 | 55 | 22 |
| $60-64$ | 90 | 53 | 10 | 24 | 76 | 46 | 20 |
| $65-70$ | 69 | 51 | 7 | 22 | 78 | 48 | 22 |
| Overall | 663 | 60 | $\mathbf{6 0}$ | $\mathbf{6 2}$ | $\mathbf{3 8}$ | $\mathbf{4 6}$ | $\mathbf{1 8}$ |

Table 2: Number of subjects ( N ) and percentage (\%) of females, contact lens (CL) wearers, myopes, hyperopes and orthogonal or oblique cylinder axis orientations in each 5-year age cohort.

| Variable | Mean | Standard <br> deviation | Minimum to <br> maximum | COR |
| :--- | :--- | :--- | :--- | :--- |
| unaided vision (LogMAR) | 0.53 | 0.38 | -0.29 to 1.30 | 0.26 |
| Mean spherical equivalent | -0.93 | 2.46 | -6.88 to 9.38 | 0.34 |
| (D) |  |  | -1.50 to 1.87 | 0.16 |
| Orthogonal component (D) | -0.04 | 0.37 | -0.77 to 1.04 | 0.26 |
| Oblique component, (D) | 0.01 | 0.21 | 2.33 to 7.50 | 1.12 |
| Pupil diameter (mm) | 4.22 | 0.84 | 0.20 to 10.09 | 1.46 |
| BSR (minD ${ }^{-1}$ ) | 2.07 | 0.93 |  |  |

Table 3: Mean, standard deviation, range (from measurements on 633 right eyes) and coefficient of repeatability (COR, from repeat measurements made in 31 right eyes) for key variables measured in this study.

Most irides ( $67 \%, 442$ eyes) were classified as grade 1 (light blue or green). The remainder were classified as grade 2 ( $7 \%, 49$ eyes), grade 3 ( $12 \%$, 78 eyes), grade 4 ( $14 \%, 94$ eyes). None were classed as grade 5 (very dark brown). On repeat testing, 28 of the 31 eyes were identically classified, giving an intra-examiner agreement of $90 \%$. The 3 eyes classified differently only altered by 1 grade.

Decision tree analysis (Figure 1) revealed that sex, iris colour and contact lens wear did not influence BSR. Refractive group did have a statistically significant affect such that the BSR was lower in hyperopes than myopes. Cylinder axis orientation only exerted a statistically significant effect in myopes and showed that BSR was lower in astigmats. No statistically significant differences in BSR were found, however, for cylinder axes that were orthogonal or oblique.


Figure 1: Decision Tree Analysis showing the factors affecting the BSR of 663 right eyes. Refractive group ( 414 myopes and 249 hyperopes) had the greatest influence on BSR. Cylinder axis orientation appeared to influence BSR in the myopic group but differences were only found between eyes with ( $n=263$ ) and without ( $n=151$ ) astigmatism; no differences were observed between eyes with orthogonal or oblique astigmatism. Sex and contact lens wear did not influence BSR and, therefore, do not appear in the decision tree.

Step backward multiple linear regression was carried out separately on 414 myopes and 249 hyperopes and. These analyses revealed that pupil sizeand scalar vector U influenced MAR in myopes (equation 9) and only scalar vector $U$ and age influenced MAR in hypermetropes (equation 10).

MAR (myopes) $=(2.91 \times$ scalar vector $)+(0.51 \times$ pupil diameter $)-3.14$ (equation 9$)$

MAR (hyperopes) $=(1.55 x$ scalar vector) $+(0.06 x$ age) -3.45 (equation 10)
ANOVAs carried out on each regression analysis showed that both were statistically significant (for myopes $\mathrm{F}_{2,411}=892.30, \mathrm{P}<0.001$; for hypermetropes $\mathrm{F}_{2,246}=109.77, \mathrm{P}<0.001$ ).

Table 4 compares the bias, $95 \%$ limits of agreement and coefficients of determination $\left(r^{2}\right)$ for observed versus predicted MAR values arising from equations 9 and 10 to those of Raasch (equation 5) and Smith (for a constant $k$ of $0.55,0.83$ and 1.55) (equations 6 and 7). Although Raasch' s model was only intended for myopes, it is also applied to hyperopes in table 4 to illustrate the need for separate equations for both refractive groups. A cut-off value of 1.5D was adopted when using Smith's equations ( 6 for low ametropia and 7 for high ametropia). New equations 9 and 10 produced better predictions than the others whether this was judged on the bias, limits of agreement or coefficients of determination. For both ametropic groups, Smith's model with a constant, $k$, of somewhere between its lowest ( 0.55 ) and typical ( 0.83 ) values predicted the observations best.

|  |  | $95 \%$ limits of agreement |  |  |  |
| ---: | :--- | :--- | :--- | :--- | :--- |
|  | bias | interval lower | upper | $r^{2}$ |  |
| Myopes ( $\mathrm{n}=414$ ) |  |  |  |  |  |
| Present (equation 9) | -0.01 | 4.09 | -4.09 | 4.08 | 0.81 |
| Raasch (equation 5) | 5.04 | 12.39 | -7.36 | 17.43 | 0.76 |
| Smith (equations 6 and 7) |  |  |  |  |  |
| $k=0.55$ | -0.33 | 4.84 | -5.17 | 4.51 | 0.75 |
| $k=0.83$ | 2.77 | 5.48 | -2.71 | 8.26 | 0.75 |
| $k=1.33$ | 8.34 | 10.57 | -2.24 | 18.91 | 0.75 |
|  |  |  |  |  |  |
| Hyperopes ( $\mathrm{n}=249$ ) |  |  |  |  |  |
| Present (equation 10) | -0.05 | 3.89 | -3.93 | 3.84 | 0.53 |
| Raasch (equation 5) | 4.38 | 14.10 | -9.72 | 18.48 | 0.12 |
| Smith (equations 6 and 7) |  |  |  |  |  |
| $k=0.55$ | 1.26 | 4.73 | -3.47 | 5.98 | 0.24 |
| $k=0.83$ | 3.07 | 6.36 | -3.29 | 9.44 | 0.01 |
| $k=1.33$ | 6.36 | 10.48 | -4.12 | 16.83 | 0.05 |

Table 4. Comparison of the bias, limits of agreement and coefficients of determination ( $r^{2}$ ) for differences between observed MAR values and those predicted using equations from the present study and those of Raasch and Smith.

Unaided vision (MAR) is plotted as a function of scalar vector $U$ in myopes (Figure 2 ) and hyperopes (Figure 3). Measured data are shown as open circles for comparison with modelled data shown as solid or dashed lines. In figure 2 the red and blue circles or lines illustrate the influence of smaller or larger pupil diameters on unaided vision in myopes. In figure 3 the red and blue circles or lines illustrate the influence of younger and older age (reflecting the age-related decline in accommodation) on unaided vision in hyperopes. Figure 2 shows a comparison between the predictions of new equation 9 (solid lines) and the formulae of Raasch (equation 5, black dashed lines) and Smith (equations 6 and 7, red of blue dashed lines). The vertical axis in
figures 2 and 3 is limited to a maximum MAR of 10 as this, conveniently, represents the top line (6/60) on a conventional Snellen chart. The horizontal axis is limited to scalar vector values that reduce vision to 6/60 in the measured data; up to 6.00D for myopes and 7.00D for hyperopes. Both figures allow optometry students to study the effects of blur on unaided vision in myopes with varying pupil size and hyperopes of varying age. While the modelled data (plotted lines) allow students to observe general trends, the measured data (plotted circles) allow students to appreciate that individual variations are commonplace. It can then be understood why their own observations on a fellow student might not always reflect general observations based on the entire class.


Figure 2. Unaided vision (MAR) plotted as a function of defocus (scalar vector, U) in myopes. Red and blue open circles are measurements in subjects with pupil diameters of 3 to 3.9 mm and 5 to 5.9 mm , respectively. Red and blue solid lines represent the application of equation 9 for pupil diameters of 3.5 and 5.5 mm , respectively. Red and blue dashed lines represent the application of Smith's equations (equation 6 for myopia of $>1.5 \mathrm{D}$, equation 7 for myopia of $<1.5 \mathrm{D}$; both with $\mathrm{k}=$ 0.55 as this fitted the myopic data best [see Table 4]) for pupil diameters of 3.5 and 4.5 mm , respectively. The black dashed lines represent the application of Raasch's equation (equation 5) which does not include terms for pupil diameter. The vertical axis shows a maximum MAR of 10 (6 /60 Snellen), representing the top line of conventional Snellen charts. A limitation of equation 9is that the predicted MAR values fall below 0.5 ( $6 / 3$ Snellen) for myopes with small pupils and low levels of defocus.


Figure 3. Unaided vision (MAR) plotted as a function of defocus (scalar vector, U ) in hyperopes. Red and blue open circles are measurements in subjects aged 20 to 45 years (pre-presbyopes) and 45 to 70 years (presbyopes), respectively. Red and blue solid lines represent the application of equation 10 for ages of 32.5 and 57.5 years, respectively. Smith's equations (equations 6 and 7) are not applied here as they do not include a term for age. Raasch's equation (equation 5 ) is not applied here as it only strictly relates to myopes. The vertical axis shows a maximum MAR of 10 (6 /60 Snellen), representing the top line of conventional Snellen charts. A limitation of equation 10 is that the predicted MAR values fall below 0.5 ( $6 / 3$ Snellen) forhyperopes with low levels of defocus.

Figures 2 and 3 show that the new equations ( 9 and 10) generated implausible or meaningless levels of unaided vision for low levels of ametropia combined with either small pupil diameters or youth. For example, a scalar vector $U$ of $\pm 0.12$ D gave rise to predicted MAR values of $0.8(\sim 6 / 5)$ using Raasch's equation 5, between $1.0(\sim 6 / 6)$ and $1.6(\sim 6 / 10)$ (using Smith's equation 7 and depending on the $k$ constant and pupil diameter). On the other hand, for the same scalar vector $U$ value, the new equations predicted MAR values of between - 2.1 ( $\sim 6 /-14)$ and $1.0(\sim 6 / 6)$ depending on pupil diameter and age. A simple remedy would be to set a lower limit for the predicted Snellen fraction of, say, 6/5. This, however, would not be realistic as, in real life, the Snellen fraction representing best vision varies from person to person.

In an attempt to arrive at a variable lower limit for use in simulators, further analysis was carried out on the best corrected visual acuity of 150 of the subjects of this study (only these had best corrected visual acuity recorded for the purposes of this study). The age range in this sample was 20 to 69 years (mean, standard deviation: 46,14 years). Pupil diameters ranged from 2.5 to 6.7 mm (mean, standard deviation: 4.1, 0.8 mm ). The best corrected Snellen fraction ranged from 6/7.6 to 6/3 (Snellen denominator mean, standard deviation: 5.6, 0.7). An ANOVA carried out on
the linear multiple regression describing the dependence of best corrected MAR values on age and pupil diameter showed that no statistically significant relationship existed ( $F_{2,147}=0.45, P=$ $0.64)$. As neither age or pupil diameter influenced MAR then it seemed reasonable to quote a cutoff value with upper and lower limits based on 2 standard deviations either side of the mean. These were, for MAR, 1.1 and 0.7 which correspond to Snellen fractions of $6 / 6.6$ to 6/4.0. This lead to equation 11 that could be used to calculate a variable minimum Snellen denominator:

Minimum Snellen denominator at $6 \mathrm{~m}=(\mathrm{S} \times 2.6)+4$ (equation 11)
In equation 11, the value $S$ is a value between 0 and 1 that is randomly generated. The value of 2.6 represents the difference in the Snellen denominator (for a 6 m testing distance) between the upper (6.6) and lower (4.0) limits ( $6.6-4.0=2.6$ ) found in the analysis just described. The value 4 represents the lowest possible Snellen denominator (taken from the lower limits found in the analysis just described).

## Discussion

Table 2 shows that the sample of eyes studied were slightly biased towards females (60\% of the sample) and included many contact lens wearers ( $42 \%$ of the sample). Decision tree analysis (Figure 1) showed that neither of these factors influenced the relationship between vision and ametropia (expressed in the form of the BSR).

Table 2 also shows that cylinder axis orientations were more commonly orthogonal (46\%) than oblique (18\%). This is consistent with previous studies ${ }^{3,28}$. Decision tree analysis (Figure 1) further revealed that orthogonal and oblique cylinder axis orientation had no influence on the BSR. This finding did not support the commonly held view that unaided vision is better for orthogonal cylinder axis orientations due to the Roman alphabet being dominated by vertical and horizontal lines ${ }^{4}$. This indicated that weighting the $\mathrm{J}_{45}$ value in equation 3 by a factor of two ${ }^{4}$ was not necessary. The findings of the present study also disagreed with the majority of previous reports on vision in artificially induced oblique and orthogonal astigmatism ${ }^{18,29-33}$. Previous studies ${ }^{18,34}$ have indicated that susceptibility to artificially induced astigmatism at various axes may be dependent on an individual's natural astigmatism and that the visual system may be more tolerant of real compared to simulated refractive defocus, especially in astigmatism ${ }^{35}$. These observations suggest that the findings of the present study on natural astigmatism may be closer to the truth.

The predominance of light or blue eyes found in the present study (67\%) had been reported in other studies on white populations ${ }^{36}$. Intra-examiner agreement (90\%) was also of the order reported in previous studies ${ }^{21}$. That iris colour did not have a significant effect on BSR was in keeping with a previous study in which no differences in visual acuity were found despite the observation that light blue eyes had poorer contrast sensitivity and greater intraocular stray light ${ }^{36}$.

Repeatability (COR) of unaided vision ( $\pm 0.26$ LogMAR) was slightly worse than that $( \pm 0.21$ LogMAR) reported by Elliott and Sheridan ${ }^{14}$ from the dominant eyes of 20 subjects with refractive errors up to $\pm 6.00 \mathrm{D}$. That and other studies found that the variability of unaided vision was higher than for aided visual acuity ${ }^{14,37}$ and increased with optical defocus ${ }^{38}$. So the reduced repeatability
found in the present study might have been the result of taking readings from right eyes, rather than dominant eyes, and from subjects with a broader range of refractive errors.

Repeatability of the mean spherical equivalent ( $\mathrm{M}= \pm 0.34 \mathrm{D}$ ), $\mathrm{J}_{0}( \pm 0.16 \mathrm{D})$ and $\mathrm{J}_{45}( \pm 0.26 \mathrm{D})$ components was similar to previous reports ${ }^{39,40}$.

Repeatability of pupil diameter was $\pm 1.1 \mathrm{~mm}$. Twa et al. ${ }^{9}$ reported $95 \%$ limits of agreement between two examiners (rather than COR of intra-examiner repeatability measured in the present study) semi-circular templates of $\pm 0.72 \mathrm{~mm}$ and $\pm 1.41 \mathrm{~mm}$ in bright ( 1000 lux) and dim (5 lux) light, respectively. The COR in the present study lies between these two and might, in part, be due to the light conditions of the present study (106 lux) falling between those adopted by Twa et al. ${ }^{9}$.

The repeatability of BSR ( $\pm 1.5 \mathrm{minD}^{-1}$ ) has not been reported before now. The typical BSR from 633 eyes was $2.1 \mathrm{minD}^{-1}$. This equates to a 0.32 log unit loss on a LogMAR chart for 1.00 D of blur and is close to the 0.4 log unit loss reported by Elliott and $\mathrm{Cox}^{7}$ in their study on the +1.00 D blur test.

Curiously, Figure 1 showed that BSR was lower in myopic individuals with astigmatism compared to those without. A similar observation was made by Atchison and Mathur ${ }^{18}$ who commented that this was at odds with what was expected from geometric calculations or earlier observations by Atchison et al. ${ }^{33}$ in which spherical and astigmatic focus effected vision to the same degree.

The BSR was mainly influenced by refractive group (Figure 1). Here hyperopes had much smaller values than myopes. This was expected given that pre-presbyopic hyperopes can accommodate to reduce blur and similar observations had previously been made in artificially induced hyperopia ${ }^{41}$. Multiple linear regressions reinforced this notion in that increasing age raised the MAR in hyperopes (equation 10) and had no effect in myopes (equation 9).

Increased pupil diameter only raised the MAR in myopes (equation 9) and this partly agrees with the findings of previous studies ${ }^{41-43}$.

This study has several important limitations. First of all, limiting unaided vision to $1 / 20$ or better has restricted the range of refractive errors that could be included and has possibly truncated the data from those with high refractive errors to those who just managed to see letters on the $1 / 20$ line. Secondly, the scoring system which (a) did not account for letters incorrectly read on lines higher than the lowest one read and (b) allowed confusions between several groups of letters to be overlooked may well have artificially improved unaided vision scores. The implications of this scoring system are that equations 9 and 10 may have underestimated the impact of defocus on unaided vision. Some evidence for this comes from Table 4 in which it can be seen that Smith's equations ( 6 and 7 ) incorporating the lowest $k$ constant ( 0.55 ) show better agreement with the observed unaided vision scores than those incorporating the typical theoretical k constant (0.83). It is possible, however, that the scoring system adopted more closely matches measurements of vision made in the busy practice environment and would, therefore, still provide useful information for optometry students and teachers.

In summary, the observed MAR values (table 4) were most closely modelled using the new equations compared to those of Raasch ${ }^{1}$ and Smith ${ }^{2}$. All equations offered better predictions of unaided vision in myopes compared to hyperopes. This suggests that unmeasured variations in
higher order aberrations and tolerance to blur may predominate in hyperopic eyes. The new equations and those of Smith ${ }^{2}$ lend themselves best for simulating variations in unaided vision for teaching purposes but the new equations have the advantage of being able to simulate variations in unaided vision arising from age (i.e. the age-related reduction of accommodation) in hyperopes. The new equations are complicated, however, by the requirement of an additional equation (equation 11) that provides a variable but feasible minimum Snellen denominator. It could be argued that, although there are limitations to the clinical data collected, the use of such data is one of the strengths of this study as it provides optometry students clinical findings that arise from the "real world" rather than the "laboratory".

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