

Optimisation of polymer optical fibre based interferometric sensors

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ABSTRACT

A numerical model for studying the performance of polymer optical fibre-based interferometric sensors is presented. The strain sensitivity of Fabry-Perot and two-beam interferometric sensors is investigated by varying the physical and optical properties corresponding to frequently used wavelengths. The developed model was used to identify the regimes in which these devices offer enhanced performance over their silica counterparts when used for stress sensing.

Keywords: POF, interferometers, strain sensitivity, stress sensing

1. INTRODUCTION

Polymer optical fibres (POFs) have received increased interest in recent years due to their different material properties compared to silica based optical fibres (SOFs). Biocompatibility, a higher failure strain and the greater elasticity [1] of POFs are potential advantages over SOFs and these characteristics are useful in the sensor application field. The much lower Young's modulus of POF compared to SOF means that POF sensors have much less effect on any compliant structures that are being monitored and also renders POF based sensors much more sensitive to fibre stress than those fabricated from SOF [2]. The lower Young's modulus also offers sensitivity enhancement when POF is used to sense acoustic waves [3]. However, some considerable drawbacks still exist in POF technology, perhaps the main one being the high fibre loss, which needs to be taken into account when designing interferometric sensors constructed from POF. In the absence of loss the strain sensitivity of a fibre interferometer is proportional to its cavity length, however when interferometers are constructed from POF, increasing the cavity length can result in sensitivity reduction at some point due to the attenuation along the optical path. The loss leads to a reduction in the signal level obtained from two-beam interferometers, while in the case of Fabry-Perot interferometers the finesse can also be significantly reduced. Therefore, evaluating the cavity length that maximises the sensitivity is important in order to optimise the performance of a POF based interferometric fibre sensor.

In this work, a numerical model has been developed in the MATLAB environment where strain sensitivity can be determined for each cavity length of the interferometer. A comparison of strain sensitivities between POF and SOF based interferometric sensors has been made and their optimum cavity length for maximum sensitivity to strain has been found. Moreover, stress sensitivity of POF based sensors is found to be higher than SOF based ones because of their different elastic properties. The results show that POF based interferometric sensors are good candidates for stress sensing.

2. ANALYSIS AND MODELING

2.1 Fabry-Perot interferometer

A pair of partially reflective, parallel and optical flat mirrors is considered in this model, which is the common arrangement of an etalon (Figure 1). Mirror dimensions are not taken into account in this 1-dimensional numerical modelling. The distance L between mirrors creates a cavity where constructive interference occurs if the transmitted beams are in phase. For an input electric field E_0 normal to the first mirror, part of it reflects back and the remaining is transmitted into the cavity. The process continues as depicted in Figure 1. Therefore, the electric fields can be written as:

$$E_1 = t_1^2 E_0 + r_1^2 E_4 \quad E_2 = E_1 e^{j k L} e^{-\frac{\alpha}{2} L} \quad E_3 = r_2^2 E_2 \quad E_4 = E_3 e^{j k L} e^{-\frac{\alpha}{2} L} \quad E_t = t_2^2 E_2 \quad (1)$$

where α is the attenuation across the cavity, r is the reflectance of mirrors and t is the transmittance of mirrors considering that $t=1-r$. k is the wavenumber, related to the wavelength (λ_0) in vacuum and the refractive index (n) of the material by the equation $k = \frac{2\pi n}{\lambda_0}$.

Combining the equations above (1), the transmission electric field E_t can be calculated as:

$$E_t = \frac{t_1^{\frac{1}{2}} t_2^{\frac{1}{2}} E_0 e^{jkl} e^{-\frac{\alpha}{2}L}}{1 - r_1^{\frac{1}{2}} r_2^{\frac{1}{2}} e^{2jkl} e^{-\alpha L}} \quad (2)$$

For an input $E_0=1$ the transmission intensity $I_t = E_t E_t^*$ of Fabry-Perot interferometer equals:

$$I_t = \frac{t_1 t_2 e^{-\alpha L}}{1 - \left(2 \cos(2kl) r_1^{\frac{1}{2}} r_2^{\frac{1}{2}} e^{-\alpha L} \right) + r_1 r_2 e^{-2\alpha L}} \quad (3)$$

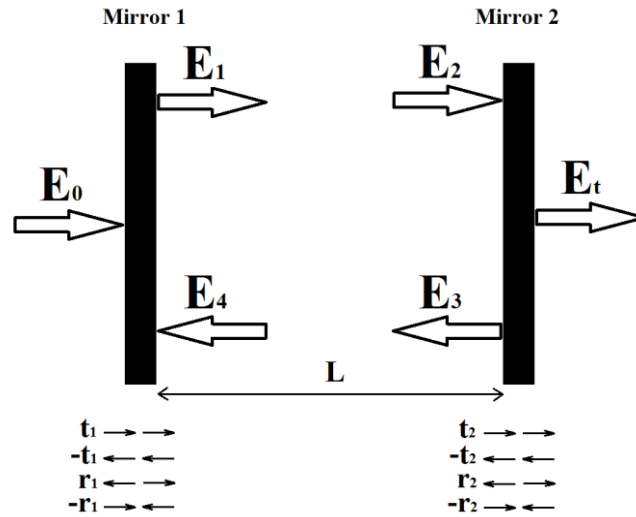


Figure 1: Fabry-Perot interferometer scheme

2.2 Two-beam interferometer

In the case of two-beam interferometer, the same electric field E_0 travels through two different optical paths as depicted in Figure 2. Merging these two beams (neglecting coupler losses) gives the transmission electric field E_t :

$$E_t = E_0 + E_0 e^{-\alpha L} e^{j(2kl)} \quad (4)$$

Considering an input electric field $E_0=1$, the transmission intensity $I_t = E_t E_t^*$ of a two-beam interferometer is:

$$I_t = 1 + e^{-2\alpha L} + 2e^{-\alpha L} \cos(2kl) \quad (5)$$

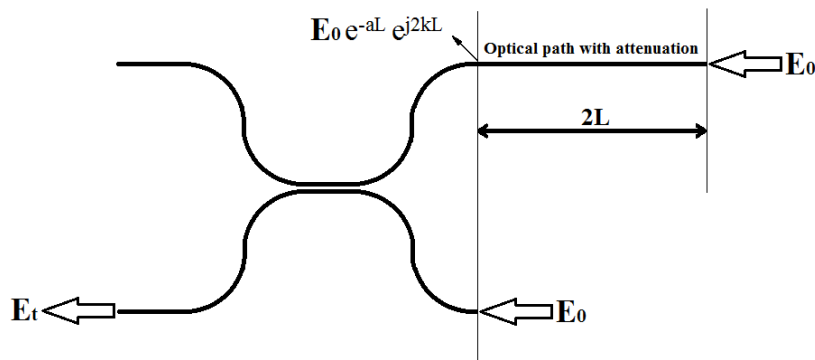


Figure 2: Two-beam interferometer scheme

2.3 Modeling methodology

A numerical model of Fabry-Perot and two-beam interferometer responses has been constructed in the MATLAB environment where the cavity length for maximum strain sensitivity of the sensor can be determined. The model describes the relationship between sensitivity and the key sensor parameters, which are the reflectivity of the mirrors used to form the interferometer, the cavity length and the attenuation. The sensor's strain sensitivity can be determined by differentiating the transmission intensity of interferometer with respects to strain:

$$\frac{dI_t}{d\varepsilon} = \frac{\partial I_t}{\partial n} \frac{\partial n}{\partial \varepsilon} + \frac{\partial I_t}{\partial L} \frac{\partial L}{\partial \varepsilon} \quad (6)$$

For a homogeneous and isotropic elastic material the refractive index change due to the elasto-optic effect [4] is:

$$\frac{\partial n}{\partial \varepsilon} = \frac{n^3(\mu P_{11} + (\mu - 1)P_{12})}{2} \quad (7)$$

where P_{11} and P_{12} are Pockels coefficients, n is refractive index and μ is Poisson's ratio of the material under test. The component $\frac{\partial L}{\partial \varepsilon}$ in equation (6) is actually the cavity length L . The remaining components can be calculated by differentiating the transmission intensity equation with respect to the variables in the expression using MATLAB tools.

3. RESULTS & DISCUSSION

3.1 Optimum cavity length for maximum strain sensitivity

The described model above has been used to determine the optical length providing maximum strain sensitivity of Fabry-Perot or two-beam interferometric fibre based devices. Various parameters have been used during simulations, such as attenuation of fibre, type of material, operating wavelength and reflectivity of mirrors in case of the Fabry-Perot interferometer. The common attenuation α of fibres [5] as well as their refractive indices n for various wavelengths [6-8] are depicted in Table 1. The Pockels coefficients of fused silica are $P_{11}=0.113$ and $P_{12}=0.252$ [9] and the Poisson's ratio is $\mu=0.17$ [10]. For poly(methyl methacrylate) (PMMA) fibre these values are $P_{11}=0.300$ and $P_{12}=0.297$ [11] and $\mu=0.34$ [12]. Since perfluorinated polymer fibre's (PF-POF) Poisson's ratio and Pockels coefficients are still unknown, values of PMMA have been used, assuming the PF-POF to have similar elasto-optic properties.

Table 1: Common refractive indices and attenuation of each material for frequently used wavelengths

	PMMA		PF-POF (CYTOP)		Fused Silica	
	n	α (dB/km)	n	α (dB/km)	n	α (dB/km)
$\lambda=0.650 \mu\text{m}$	1.4883	200	1.3410	60	1.4565	3.5
$\lambda=0.850 \mu\text{m}$	1.4840	2000	1.3390	40	1.4525	1.0
$\lambda=1.050 \mu\text{m}$	1.4817	~8000	1.3370	25	1.4498	0.5
$\lambda=1.350 \mu\text{m}$	1.4750	~100000	1.3360	20	1.4464	0.3
$\lambda=1.550 \mu\text{m}$	1.4710	~100000	1.3355	20	1.4440	0.2

Using equation (3) for a PMMA POF based Fabry-Perot sensor at $\lambda=0.850 \mu\text{m}$ and considering 80% reflectivity mirrors, the optimum cavity length for maximum strain sensitivity is 23.74 cm (Figure 3). Unity input power is assumed in all figures. Increasing the reflectivity of the mirrors to 90%, the same maximum sensitivity levels can be achieved with a shorter cavity length of 11.35 cm (Figure 4). However, a PMMA POF based sensor at $\lambda=1.550 \mu\text{m}$, where the typical attenuation is 1dB/cm, has an optimum cavity length of 0.23 cm with 100 times less sensitivity (Figure 5). Beyond that point, any attempt for longer Fabry-Perot cavity fabrication will result not only in a bigger size of the device but also to a significant reduction of its sensitivity. On the other hand, using PF-POF based interferometric sensors at $\lambda=1.550 \mu\text{m}$ with 90% reflectivity mirrors, longer Fabry-Perot cavities can be created (Figure 6) because of the fibre's lower attenuation.

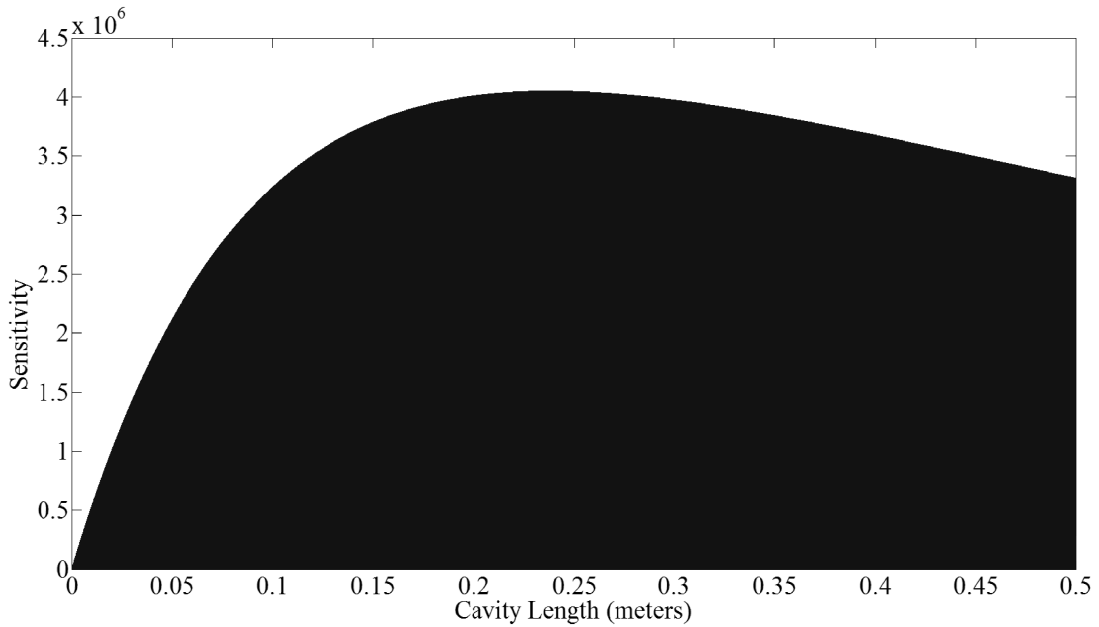


Figure 3: PMMA-POF Fabry-Perot at 0.850 μm with 80% reflectivity mirrors

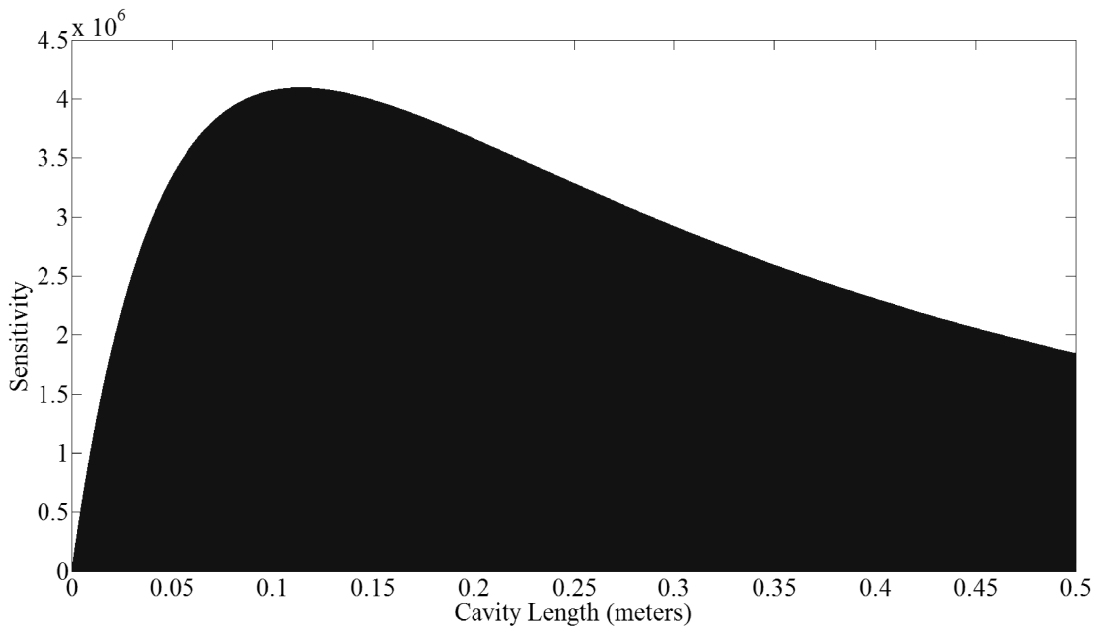


Figure 4: PMMA-POF Fabry-Perot at 0.850 μm with 90% reflectivity mirrors

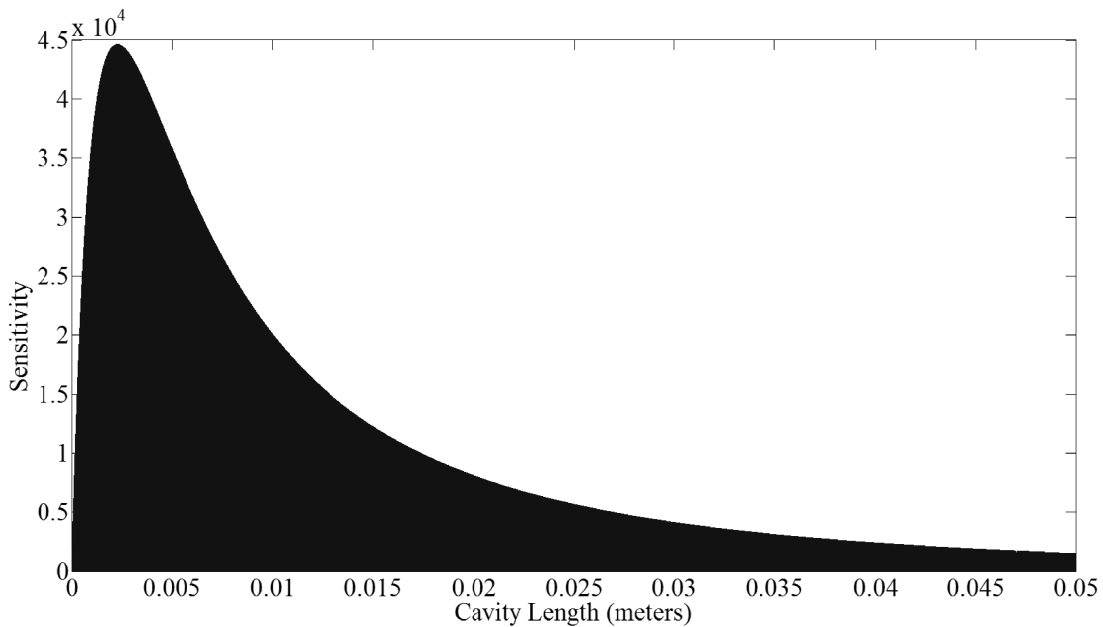


Figure 5: PMMA-POF Fabry-Perot at 1.550 μm with 90% reflectivity mirrors

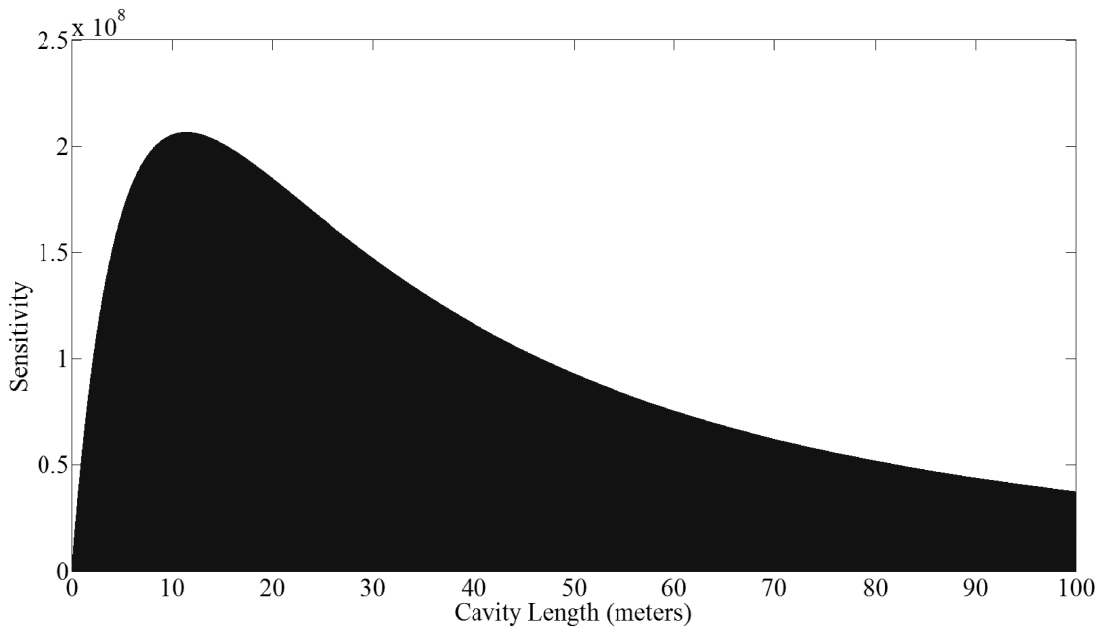


Figure 6: PF-POF Fabry-Perot at 1.550 μm with 90% reflectivity mirrors

The optimum cavity lengths and the achievable maximum sensitivities of a POF based Fabry-Perot interferometers with 90% reflectivity mirrors are depicted in Table 2. The respective optical path lengths for two-beam interferometers can be seen in Table 3.

Table 2: Optimum optical cavity length of a Fabry-Perot interferometer with 90% reflectivity mirrors

	PMMA		PF-POF (CYTOP)		Fused Silica	
	Maximum sensitivity	Optimum Length (meters)	Maximum sensitivity	Optimum Length (meters)	Maximum sensitivity	Optimum Length (meters)
$\lambda=0.650 \mu\text{m}$	5.3687 e+07	1.1391	1.6477 e+08	3.8010	2.6277 e+09	64.778
$\lambda=0.850 \mu\text{m}$	4.0963 e+06	0.1143	1.8877 e+08	5.6672	7.0244 e+09	227.95
$\lambda=1.050 \mu\text{m}$	8.2803 e+05	0.0285	2.4420 e+08	9.1054	1.1363 e+10	457.36
$\lambda=1.350 \mu\text{m}$	5.1342 e+04	0.0022	2.3728 e+08	11.375	1.4714 e+10	758.74
$\lambda=1.550 \mu\text{m}$	4.4624 e+04	0.0022	2.0659 e+08	11.390	1.9208 e+10	1132.5

Table 3: Optimum optical path length of a two-beam interferometer

	PMMA		PF-POF (CYTOP)		Fused Silica	
	Maximum sensitivity	Optimum Length (meters)	Maximum sensitivity	Optimum Length (meters)	Maximum sensitivity	Optimum Length (meters)
$\lambda=0.650 \mu\text{m}$	4.1183 e+08	21.70	1.2639 e+09	72.38	2.0150 e+10	1243.9
$\lambda=0.850 \mu\text{m}$	3.1423 e+07	2.171	1.4480 e+09	108.56	5.3884 e+10	4343.1
$\lambda=1.050 \mu\text{m}$	6.3518 e+06	0.543	1.8733 e+09	173.75	8.7167 e+10	8663.4
$\lambda=1.350 \mu\text{m}$	3.9384 e+05	0.043	1.8201 e+09	217.11	1.1287 e+11	14457.9
$\lambda=1.550 \mu\text{m}$	3.4231 e+05	0.043	1.5848 e+09	217.25	1.4735 e+11	21718.5

3.2 Strain and stress sensitivity

Neglecting attenuation for the moment, it has been found [13] that PMMA based interferometers have ~14% more strain sensitivity than fused silica ones because of their different strain coefficients. Taking losses into account, the strain sensitivity ratio of polymer and silica based sensors begins to drop as the optical cavity length increases. For example, a PMMA POF based two-beam interferometer operating at $\lambda=0.850\mu\text{m}$ has been found to have higher strain sensitivity compared to its silica counterpart if the optical length is less than 26cm, despite the higher losses.

However, the stress sensitivity of POF based sensors can be even much higher than SOF based devices. The Young modulus of silica and PMMA is 73 GPa and 3.3 GPa respectively, which means approximately 25 times more elasticity for PMMA [14]. Young modulus is related with strain and stress by this equation:

$$E = \frac{\sigma}{\epsilon} \tag{8}$$

where σ and ϵ are stress and strain respectively. Therefore, the stress sensitivity can be calculated by:

$$\frac{\partial I_t}{\partial \sigma} = \frac{\partial I_t}{\partial \epsilon} \frac{1}{E} \tag{9}$$

For instance, the stress sensitivity of a two-beam interferometer operating at $\lambda=1.550 \mu\text{m}$ can be seen in Table 4. It shows those regimes where POF based two-beam interferometric sensor offer enhanced performance compared to SOF. Stress sensitivity of a of Fabry-Perot interferometer with 90% reflectivity mirrors at $\lambda=1.550 \mu\text{m}$ can be seen in Table 5. Some regimes can be seen where POF based Fabry-Perot interferometric sensors offer enhanced performance compared to SOF.

Table 4: Stress sensitivity of two-beam interferometer at $\lambda=1.550 \mu\text{m}$

Optical Length (meters)	PMMA		PF-POF (CYTOP)		Fused Silica	
	Stress sensitivity per MPa	Sensitivity ratio with silica	Stress sensitivity per MPa	Sensitivity ratio with silica	Stress sensitivity per MPa	Sensitivity ratio with silica
0.001	6.35	25.4	6.01	24.0	0.25	1
0.01	51.6	20.5	60.1	23.8	2.52	1
0.1	64.0	2.5	600.9	23.7	25.3	1
1	0.0006	~0	5969.7	23.6	252.7	1
10	~0	~0	56667	22.4	2527	1

Table 5: Stress sensitivity of Fabry-Perot interferometer with 90% reflectivity mirrors at $\lambda=1.550 \mu\text{m}$

Optical Length (meters)	PMMA		PF-POF (CYTOP)		Fused Silica	
	Stress sensitivity per MPa	Sensitivity ratio with silica	Stress sensitivity per MPa	Sensitivity ratio with silica	Stress sensitivity per MPa	Sensitivity ratio with silica
0.001	11.04	14.15	18.52	23.74	0.78	1
0.01	6.08	0.781	184.9	23.76	7.78	1
0.1	0.06	0.001	1828	23.49	77.8	1
1	~0	~0	16285	20.95	777.3	1
10	~0	~0	62242	8.10	7681	1

4. CONCLUSION

The described numerical model for Fabry-Perot and two-beam interferometers has been used to determine their optimum cavity length for maximum strain sensitivity. Results show those regimes where interferometric sensors constructed in polymer optical fibres can offer enhanced stress sensitivity over their silica counterparts.

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