

NLSE-based model of a random distributed feedback fiber laser

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ABSTRACT

In this work we propose a NLSE-based model of power and spectral properties of the random distributed feedback (DFB) fiber laser. The model is based on coupled set of non-linear Schrödinger equations for pump and Stokes waves with the distributed feedback due to Rayleigh scattering. The model considers random backscattering via its average strength, i.e. we assume that the feedback is incoherent. In addition, this allows us to speed up simulations sufficiently (up to several orders of magnitude). We found that the model of the incoherent feedback predicts the smooth and narrow (comparing with the gain spectral profile) generation spectrum in the random DFB fiber laser. The model allows one to optimize the random laser generation spectrum width varying the dispersion and nonlinearity values: we found, that the high dispersion and low nonlinearity results in narrower spectrum that could be interpreted as four-wave mixing between different spectral components in the quasi-mode-less spectrum of the random laser under study could play an important role in the spectrum formation. Note that the physical mechanism of the random DFB fiber laser formation and broadening is not identified yet. We investigate temporal and statistical properties of the random DFB fiber laser dynamics. Interestingly, we found that the intensity statistics is not Gaussian. The intensity auto-correlation function also reveals that correlations do exist. The possibility to optimize the system parameters to enhance the observed intrinsic spectral correlations to further potentially achieved pulsed (mode-locked) operation of the mode-less random distributed feedback fiber laser is discussed.

Keywords: Random fibre laser, Rayleigh scattering, NLSE.

1. INTRODUCTION

In past few years, a new type of fibre laser – a random distributed feedback fibre laser – is widely investigated^{1,2}. The laser operates via extremely weak random scattering owing to random Rayleigh backscattering (RS). The gain is provided by a stimulated Raman scattering. Up to date, a number of different random DFB fiber lasers schemes is realized³⁻¹⁷. Namely, random fibre lasers can operate in different spectral bands^{3,4}, be cascaded, i.e. emit higher order Stokes waves³⁻⁵, the laser can be easily tunable^{13,14}, provides multi-wavelength output^{6,10,12}. The noise level of random DFB fiber lasers could be lower than of conventional lasers¹⁸ making them attractive for telecom applications. Random DFB fiber lasers are also applied for sensor applications¹⁹⁻²¹.

A theoretical description of random lasers is challenging in general, see, for example, review²². In the field of random DFB fibre lasers, the latest achievements in the field of description of random DFB fibre laser properties are following. A simple power balance model^{1,17} provides a good description of power performances of the random DFB fibre laser including the generation threshold, the longitudinal generation power distribution¹⁷. The laser could be optimized using the power balance model²³ as well as noise properties could be considered¹⁸. To describe spectral and temporal properties, other models are needed. For example, the random generation could be represented as sets of modes either of passive or active cavity, localized or extended²⁴⁻²⁸. Other approaches based on Maxwell's equations combined with the rate equations of a n-level system do exist^{29,30}. Numerical methods to solve such systems are varying and include Monte-Carlo simulation of a random walk of photons³¹, the finite difference time domain method³²⁻³⁴, the transfer matrix method^{35,36}. Despite a number of methods available to describe spectral and temporal properties of random lasers, none of them are not applied to the description of the random DFB fiber laser. In general, its spectral, temporal and statistical properties were not studied theoretically or numerically until very recently.

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Note that the spectral properties of the random DFB fibre lasers have not been yet studied systematically in experiments. There are only few attempts. As an example of some spectral studies, both power and spectral properties of the forward- and backward-pumped random DFB fibre lasers were measured depending on the ratio between the backward- and the forward-propagating pump wave powers in^{8,9}. It was found that the spectral broadening was more pronounced in the forward-pumped configuration resulting in a 0.9~nm spectral width for 70~mW of the output power. The authors suggested a difference in the longitudinal power distributions as the origin of deviations in the random lasers behavior.

In the recent paper³⁷, we applied a NLSE-based model to describe numerically power, spectral, temporal and statistical properties of the random DFB fibre laser radiation. In the present manuscript, we describe in details the simple NLSE-based model applied for a random DFB fibre laser. To do that we take into account an average energy feedback via random Rayleigh backscattering.

2. NLSE-BASED NUMERICAL MODEL OF THE RANDOM DFB FIBRE LASER

In general, NLSE-based model is the most powerful and widely applied method to describe various fiber optics systems³⁸. In particular, NLSE-based modeling describes well power, spectral, temporal and statistical properties of quasi-CW fiber lasers with conventional cavities made of point-based mirrors including Brillouin lasers³⁹, Ytterbitum-doped fiber lasers^{40,41} and Raman fiber lasers (RFLs)⁴²⁻⁴⁵. We use almost the same model as in⁴²⁻⁴⁵ except the term accounting for the random feedback.

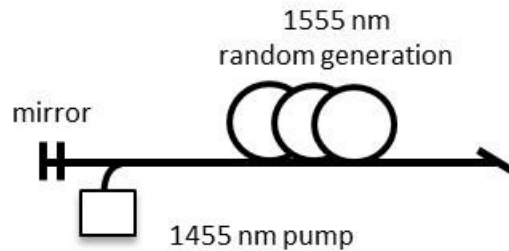


Figure 1. The considered setup.

Here we consider numerically the following random DFB fibre laser scheme, Fig. 1. We consider the laser of length 41 km pumped at 1455 nm by a quasi-CW pump laser (the pump radiation is also governed by NLSEs). In the considered scheme, the highly reflective broadband mirror (centered at 1555 nm) is used at one fibre end. Note that this configuration is equivalent to the configuration with two pump lasers, doubled fibre length and no any mirror in the cavity because of symmetry. Because of the Raman gain, the laser generates near 1555 nm. The equation set is following:

$$\frac{\partial A_p^+}{\partial z^{\pm}} - \frac{1}{v_{gs}} \frac{\partial A_p^+}{\partial t} + \frac{i}{2} \beta_{2p} \frac{\partial^2 A_p^+}{\partial t^2} + \frac{\alpha_p}{2} A_p^+ = i \gamma_p |A_p^+|^2 A_p^+ - \frac{g_p(\omega)}{2} \left(\langle |A_s^{\pm}|^2 \rangle + \langle |A_s^{\mp}|^2 \rangle \right) A_p^+ \quad (1)$$

$$\frac{\partial A_s^{\pm}}{\partial z^{\pm}} + \frac{i}{2} \beta_{2s} \frac{\partial^2 A_s^{\pm}}{\partial t^2} + \frac{\alpha_s}{2} A_s^{\pm} - \frac{\Delta A_s^{Rayleigh}}{\Delta z} = i \gamma_s |A_s^{\pm}|^2 A_s^{\pm} + \frac{g_s(\omega)}{2} \langle |A_p^+|^2 \rangle A_s^{\pm} \quad (2)$$

Here A is complex field envelope, t is a time in a frame of references moving with pump, v_{gs} is a difference between pump and generation Stokes waves inverse group velocities, β_2 , α , γ , g are dispersion, linear attenuation, Kerr and Raman coefficients, ω is the frequency detuning from the center of the gain profile, L is the laser total length, $\Delta A_s^{Rayleigh}$ is given by Eq. (3). Indices “+” and “-” denote generated waves co- and counter-propagating with the pump wave. Here we define the longitudinal coordinate for the generation wave z_{\pm} as $z=0$ at a starting point of the generation wave propagation (either A_s^+ or A_s^-) and $z=L$ at the final point of the propagation, i.e. “+” wave has a coordinate z_+ while propagating, and “-” wave has a coordinate z_- , and both waves propagates in the positive direction of z -axis. At the same time, the longitudinal coordinate value z_+ for the co-propagating Stokes wave A_s^+ corresponds to the value of the longitudinal coordinate $z_-=L-z_+$ for the counter-propagating Stokes wave A_s^- and vice versa. Equations are z -averaged over the dispersion walk-off length of the generation and pump waves, thus the phase in cross-modulation term is zero⁴⁵.

White noise as an initial condition is used to take into account the spontaneous Raman scattering⁴⁶. Raman gain is approximated by the parabola, $g_i(\omega)=g_i - k\omega^2$, where $k = 0.0062 \text{ ps}^2 (\text{W km})^{-1}$, $i = s, p$. The equations are integrated along z using an iterative approach with an integration step Δz , i.e. when integrating equations for A_s^+ , values A_s^- obtained on previous iteration are used, and vice versa.

We use following parameters: $\alpha_s = 0.046 \text{ km}^{-1}$, $\alpha_p = 0.055 \text{ km}^{-1}$, $\gamma_s = 1.09 (\text{km} \cdot \text{W})^{-1}$, $\gamma_p = 1.31 (\text{km} \cdot \text{W})^{-1}$, $g_s = 0.36 (\text{km} \cdot \text{W})^{-1}$, $g_p = 0.39 (\text{km} \cdot \text{W})^{-1}$, $\beta_{2s} = 20 \text{ ps}^2/\text{km}$, $\beta_{2p} = 35 \text{ ps}^2/\text{km}$, $1/v_{gs} = -2.3 \text{ ns}/\text{km}$, $L = 370 \text{ m}$, pump power $P = 3 \text{ W}$. Laser mirrors were modeled by supergaussian fiber Bragg gratings (FBGs) of 0.5 nm width.

The Rayleigh backscattering feedback is taken into account via term $\Delta A_s^{\pm \text{Rayleigh}}$ defined as:

$$\Delta A_s^{+\text{Rayleigh}} = \left(\frac{\int_{-\infty}^{+\infty} d\omega |A_s^-(L - z_{\text{prox}}^+)|^2}{\int_{-\infty}^{+\infty} d\omega |A_s^-(L - z^+)|^2} \right)^{1/2} \cdot A_s^-(L - z_{\text{prox}}^+) \cdot e^{i\phi_0 + i\omega\tau_0} \quad (3)$$

Here only the energy income from the Rayleigh backscattering is taken into account via term $\Delta A_s^{\pm \text{Rayleigh}}$ similarly to the balance equation set. Taking into account only an average energy income results in good quantitative description of a random DFB fiber laser power performance within the power balance model^{10,23}. The same approach is used to deal with RS in amplifiers⁴⁸. We follow this approach and add an average term proportional to $\varepsilon = 4.5 \times 10^{-5} \text{ km}^{-1}$ to Eq. (2) (this value may vary depending on fiber NA and fabrication method⁴⁹). At the same time, the generation wave depletion due to the Rayleigh backscattering is considered through the linear losses α . Rayleigh scattering induced energy income to the pump wave is neglected as it is not amplified. A random phase factor $\exp(i\phi_0 + i\omega\tau_0)$ with a random phase ϕ_0 and time τ_0 shifts is used. We do not take into account correlation properties of the random backscattering⁵⁰. As the Rayleigh term includes the optical spectrum of the counter-propagating wave, $A_s^{\mp \text{Rayleigh}}(\omega)$, one needs to save optical spectra at each integration step, which is technically impossible. To deal with that, the optical spectra of the generation waves are saved only at very limited number of points along z coordinate at each iteration (at $N = 50$ z -points in the present case). That means $z_{\text{prox}}(z)$ is a staircase function which approximates z with a set of $N = 50$ steps along z , each of them is a z -coordinate of the closest point where the spectra of counter-propagating wave is saved at the previous iteration.

3. SPECTRAL AND TEMPORAL PROPERTIES OF THE RANDOM DFB FIBRE LASER

We start our consideration from the calculation of the random DFB fibre laser output power as a function of the pump power. The simulation predicts a lasing threshold close to 0.8 W in agreement with experimental observations^{1,2} and analytical calculations^{1,23}. Well above the threshold, the output power grows linearly with pump power that also agrees well with previous experimental observations^{1,2}. The numerically calculated within NLSE-based model power distributions agree qualitatively well with experiment and analytical calculations made within power balance equations¹⁷.

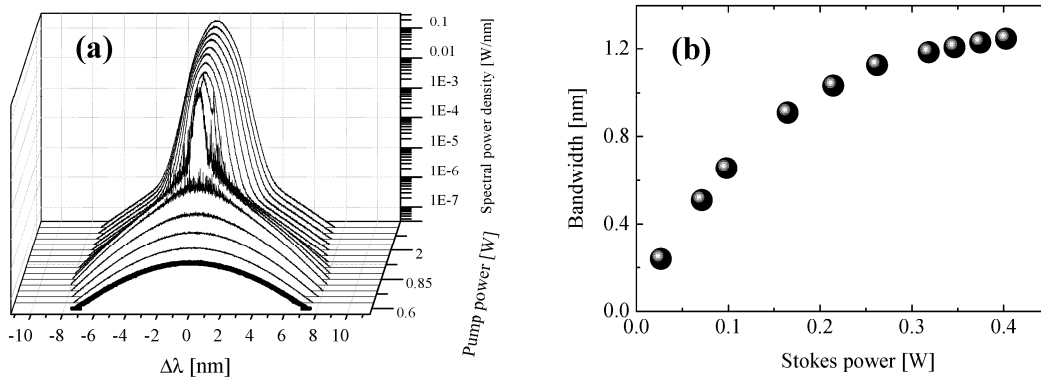


Figure 2. (a) The generation spectrum over the pump power (b) The spectrum rms width depending on the generation power.

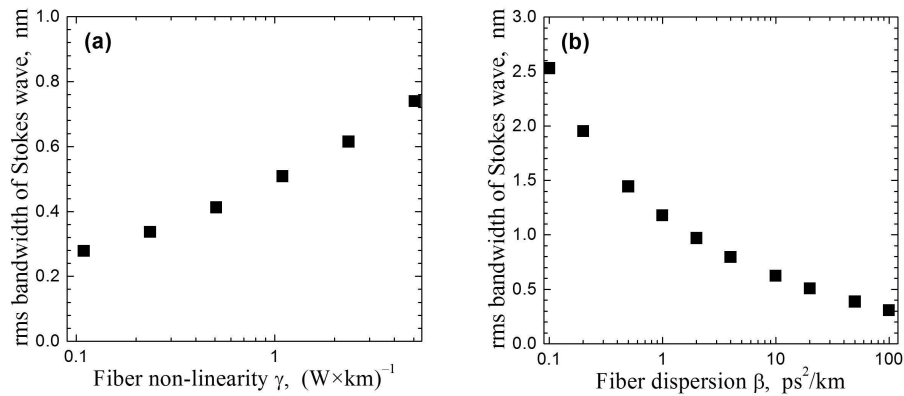


Figure 3. The spectrum broadening at different dispersion (a) and nonlinearity (b) values. Pump power $P=2$ W.

Typical averaged spectra of generated Stokes wave are shown in Fig. 2a. As in experiments, the calculated generation spectrum becomes narrower than the amplified spontaneous emission profile while the pump power increases over the generation threshold indicating that the real lasing is achieved. Note that at later stages of evolution, the spectrum becomes broader with the power, Fig. 2b. Similar nonlinear spectral broadening is observed in some experiments too. We anticipate that such spectral broadening is owing to Kerr nonlinearity and dispersion interplay similar to the processes in the conventional mirror-based fiber lasers⁵²⁻⁵⁴. Thus a simple model taking only an average random Rayleigh backscattering strength provides a good description of both power and spectral properties of random DFB fiber laser.

In the random DFB fibre laser the spectrum broadening law depends strongly on the fibre parameters. Indeed, in our system changing the nonlinearity at fixed pump power and fixed dispersion, we observe that spectrum becomes broader in the systems with larger nonlinearity, Fig. 3a. At the same time, the generation spectrum becomes narrower for systems with higher dispersion, Fig. 3a, if the pump power and nonlinear coefficients are fixed. This could be understood as dispersion prevents to different spectral components to interact nonlinearly in effective way. There are no up to date any experimental data to compare our findings of dispersion and nonlinearity influence on spectrum width. The proper dispersion and/or nonlinear management could be a practical tool to change the spectral properties in real random DFB fiber laser systems.

One more important property which could be calculated within the developed model is the spectrum evolution over the fibre, Fig. 4. The spectrum has a same spectral shape in all points over the fiber which proves the fact the real laser nature of the generation.

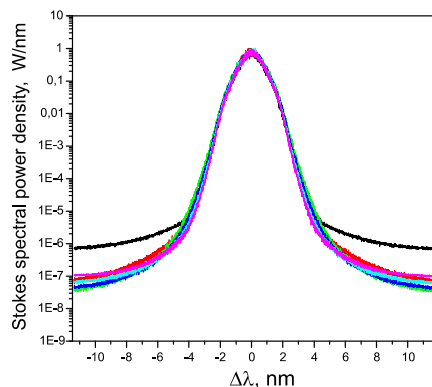


Figure 4. The generation spectrum evolution along the fiber: the spectrum generated at different points along the resonator is shown. The all fiber length is divided by 50 equally spaced points.

Temporal and statistical properties of the random DFB fibre laser radiation could also be calculated within the NLSE-based model. The intensity dynamics reveals highly stochastic nature of the radiation, Fig. 5a. The typical time scale of

fluctuations is ~ 5 ps as it is revealed by the intensity autocorrelation function, Fig. 5b. Note that real-time oscilloscope are limited in bandwidth to 33-60 GHz, so the measured intensity dynamics will be averaged, Fig. 5a.

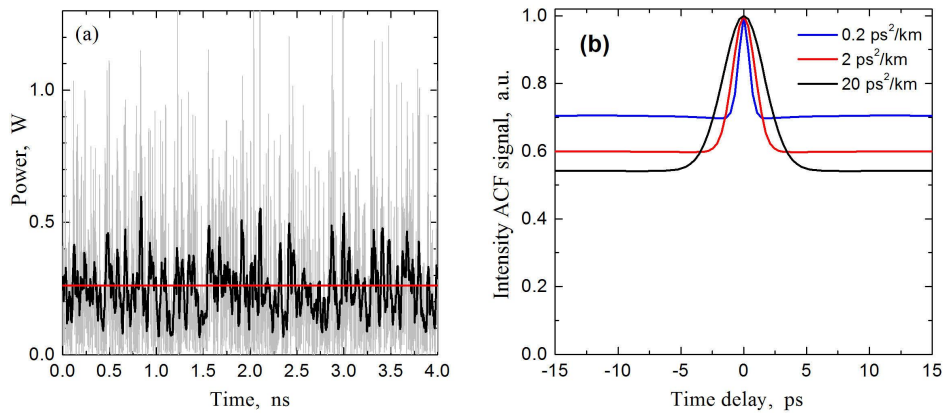


Figure 5. (a) Typical intensity dynamics (grey shows original simulated data, black – smoothed with a bandwidth of 33 GHz, red – average lasing power level), (b) Intensity ACF Pump power is 2 W on all graphs.

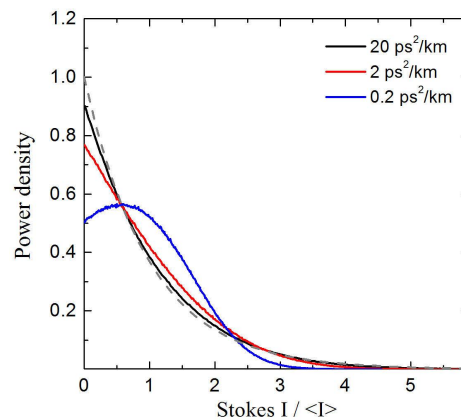


Figure 6. Intensity statistics at different dispersion value. The grey dashed curve corresponds to the exponential statistics of uncorrelated modes.

The more intriguing question is the radiation statistics. We found that the intensity statistics is not completely Gaussian in random DFB fiber laser. The lower the dispersion, the more non-exponential is intensity probability density function (pdf), Fig. 4(c), revealing correlations in radiation. The existence of correlations could be also revealed in the intensity autocorrelation function, which level is higher than 0.5 expected to be in the case of stochastic radiation, Fig. 5b. The intriguing question of non-gaussian intensity statistics in the radiation of the random DFB fiber laser has to be further investigated. Note that the non-gaussian intensity statistics is previously reported in conventional mirror based laser cavities⁴¹⁻⁴⁵. In these systems, numerous longitudinal modes do exist⁵¹. Four wave mixing between different modes results in huge spectral broadening and could be described in terms of wave turbulence⁵²⁻⁵⁴. Four-wave mixing processes lead to a redistribution of energy from the central modes to side-bands, and some partial correlations between different longitudinal modes could arise in these processes. Indications of mode correlations in Raman fiber laser radiation is reported recently in⁵⁵. Moreover, dark and grey solitons can be generated in normal dispersion quasi-CW Raman fiber laser⁵⁶. The origin of the correlations is unknown both in long lasers of conventional cavities and in random fibre lasers.

4. CONCLUSION

Thus in this work we suggested a new full numerical iterative approach that allow one to describe spectral, temporal and coherent properties of DFB fiber lasers based on Rayleigh scattering. The model is based on a set of generalized nonlinear Schrödinger equations for pump and Stokes waves averaged over micron-scale fiber inhomogeneities. Random distributed feedback taken into account has proper power resulted from balance model and spectrum obtained for counter-propagating wave in previous iteration. Calculated generation power and its longitudinal distribution as well as optical spectrum are in good qualitative agreement with previous experimental results. It is shown that increasing the dispersion or decreasing the nonlinear coefficient leads to the narrower generation spectrum providing a possibility to spectral management of random DFB fiber laser generation. Temporal and statistical properties of radiation are also studied. The intensity statistics and intensity auto-correlation function reveal non-gaussian statistics of the random DFB fiber laser radiation what suggests presence of correlations in DFB laser radiation.

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