

Lyot-filter based multiwavelength random distributed feedback fiber laser

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ABSTRACT

Multiwavelength lasing in the random distributed feedback fiber laser is demonstrated by employing an all fiber Lyot filter. Stable multiwavelength generation is obtained, with each line exhibiting sub-nanometer line-widths. A flat power distribution over multiple lines is also obtained, which indicates the contribution of nonlinear wave mixing towards power redistribution and equalization in the system. The multiwavelength generation is observed simultaneously in first and second Stokes waves.

Keywords: Fiber laser, Multiwavelength, Lyot filter, tilted fiber Bragg grating, Random laser, Random distributed feedback fiber laser

1. INTRODUCTION

Multiwavelength fiber lasers (MWFLs) are of significant interest to the scientific community due to their application diversity. MWFLs find applications in DWDM systems, optical fiber sensors, spectroscopy, etc. They can be broadly classified in terms of the gain medium used, and/or the procedure used to generate the multiple wavelengths. MWFLs have been demonstrated using lasers employing erbium doped fiber,¹ semiconductor optical amplifier,² stimulated Raman scattering (SRS),³⁻⁶ stimulated Brillouin scattering,⁷ or a hybrid of the above.⁸⁻¹¹ The procedure used to generate the multiple wavelengths can be either passive or active. For example, passive elements such as Fiber Bragg gratings,^{3,4,12} Fabry Perot filter,¹³ Sagnac loop mirrors,¹⁴ and Lyot filters^{15,16} have been used. Examples of 'active' nonlinear processes are Stimulated Brillouin scattering, nonlinear polarization rotation,¹⁷ or four wave mixing.¹⁸ For a passive element based MWFL, the individual wavelengths and their inter-spacings are solely determined by the specifications of the incorporated device. Thus tunability (wavelengths and/or spacings) can be achieved in such MWFLs.^{5,10,19}

The choice of gain media is a factor determining the stability of the generated lines. Inhomogeneous gain media are the primary choice for making MWFLs, as they provide more stability, in comparison to homogeneous media. In case the latter are used, special measures need to be taken – for example, cryogenic cooling,²⁰ hybrid gain balance,⁸ or four wave mixing in photonic crystal fibers.²¹ In this regard, gain obtained via the stimulated Raman scattering process is very attractive. While the Raman gain coefficient is small, this is compensated by its THz order bandwidth, and the availability of ultra-low loss, ultra-long gain spans made available in the form of conventional telecommunication fibers. Multiwavelength generation employing the Raman gain mechanism has been demonstrated in many configurations.^{3,4,12,19,22} In all of these, the ease of generation of the lines is attributed to the inhomogeneous nature of the gain mechanism.^{23,24} However, this aspect is still open to debate, as it was shown experimentally that Raman gain can exhibit a homogeneous gain saturation at high

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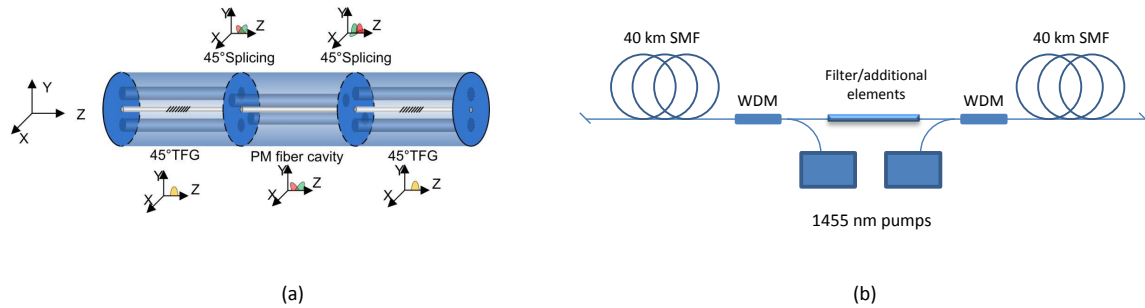


Figure 1. (a) Schematic representation of the all fiber Lyot filter. (b) Experimental configuration of the random DFB fiber laser based on all fiber Lyot filter.

powers due to pump depletion effects.²⁵ Furthermore, due to the high nature of powers employed to achieve Raman gain, cooperation between different linear and nonlinear processes cannot be ruled out.^{26,27} Particularly, four wave mixing has been shown to influence the generation spectrum in Raman fiber lasers,^{28–30} leading to a redistribution of energy from the central modes to side-bands, allowing them to reach lasing threshold levels. Thus, the issue of the exact nature of the gain mechanism in SRS aided systems remains an area of active study.

In this paper, multiwavelength generation is demonstrated in the random distributed feedback fiber laser (RDFB-FL).³⁰ Feedback in the RDFB-FL is obtained via Raman amplification of randomly distributed backward Rayleigh scattering events along the length of the fiber span. While the threshold of this laser is relatively high, the efficiency was noted to be quite comparable to existing CW lasers.^{31,32} A number of different random DFB fiber laser configurations are realized up to date.^{32–38} In particular, random DFB fiber lasers can be multi-wavelength,^{35–37} tunable,^{38,39} can operate in different spectral bands^{40–42} and provide cascaded operation at higher Stokes components.^{40,42} Multiwavelength generation in the RDFB-FL has been demonstrated earlier in the RDFB-FL by employing discrete fiber bragg gratings. The RDFB-FL has been shown to support up to 22 lines spaced apart by 0.8 nm (ITU grid spacing).³⁶ In the present work, multiwavelength lasing in the random distributed feedback fiber laser is demonstrated by employing an all fiber based Lyot filter.⁴³ The Lyot filter operates on the principle of fiber birefringence and polarization interferometry – thus providing a transmission comb of well-defined frequency spacing. Stable comb generation is obtained, with each line exhibiting sub-nanometer linewidth. In what follows, the principle of the Lyot filter is first presented. The experimental configuration and results are then presented, culminating in a discussion of the observations.

2. EXPERIMENTAL METHODS

The all fiber Lyot filter is a type of polarization filter, which comprises of two in-fiber linear polarizers based on 45° tilted fiber grating (TFG) that is inscribed into a polarization maintaining (PM) fiber along its principle axis [46] and a PM fiber as cavity, as shown in Fig. 1(a). The filter has both filtering and polarizing functions, and its bandwidth and free spectral range (FSR) can be adjusted by changing the length of the PM fiber. Detailed description about all fiber Lyot filter has been reported in.⁴³ Here we use a filter having 20 m long PM fiber cavity (birefringence of PM fiber is around 3.47×10^{-4}) which results in a bandwidth of 0.2 nm and a free spectral range of FSR of 0.4 nm.

The all fiber Lyot filter is incorporated in the random DFB fiber laser cavity as shown in Fig. 1(b). We use the following configuration of the random fiber laser - two spans of 40 km standard Corning SMF 28 fiber are pumped from the central point by two Raman fiber lasers at 1450 nm. The Lyot filter is inserted between the two spools of the fiber at the point, ensuring that the feedbacks from both arms are equally effected upon by the filter. At this point, the power of the generated radiation is quite low,⁴⁴ so one can use low-power handling components. Random distributed Rayleigh scattering, amplified by the Raman scattering process, provides the necessary feedback for lasing. Generation is obtained in the region of 1555 nm, with additional generation at 1565 nm at higher powers. Spectral properties were monitored at the laser outputs using an optical spectrum analyzer with a resolution of 0.02 nm.

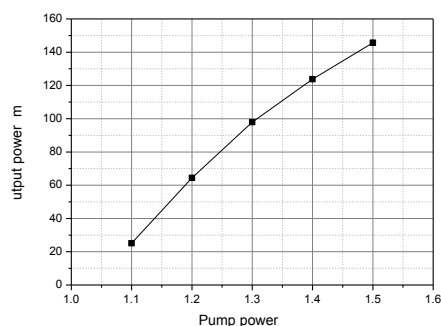


Figure 2. Output power of the multiwavelength random DFB fiber laser based on all fiber Lyot filter

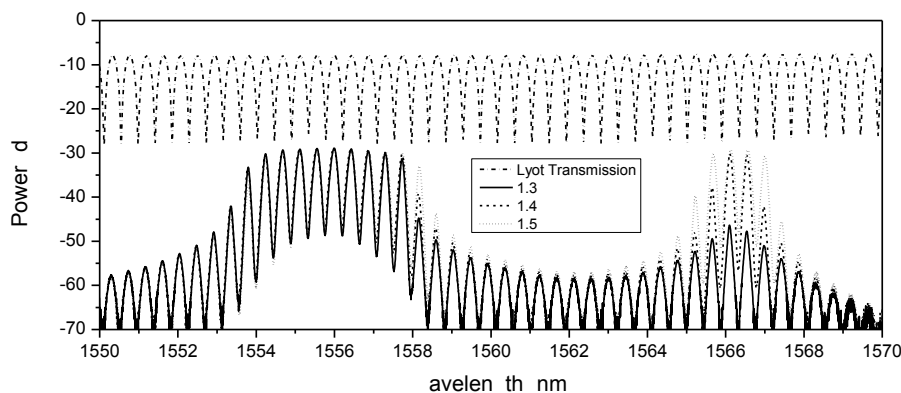


Figure 3. Spectral characteristics of the multiwavelength generation. Note the excellent transmission characteristics of the Lyot filter. The spectra show the growth of the secondary peak near 1566 nm with increasing pump power.

3. RESULTS AND DISCUSSIONS

Figure 2 shows the typical power transfer characteristics of the RDFB-FL with the Lyot filter incorporated. While the threshold is seen to be at 1.1 W, the generation is unstable due to the influence of SBS.³⁰ Stable multiwavelength generation is obtained after 1.3 W of pump power, as shown in Fig. 3. The observed peaks correspond to the transmission profile of the Lyot filter (also shown in Figure 3) – the high loss of transmission being attributed to polarization dependent losses of the tilted fiber Bragg grating. As can be seen in the figure, the pump power level plays an important role in determining the number of lines in the multiwavelength generation. Typical spectral widths of the lines lie between 0.08 to 0.14 nm, the narrower lines being observed at the edges of the generation. However, the generated lines are clearly narrower than the intrinsic width of 0.2 nm of the filter.

It is also interesting to note that the multiwavelength generation is characterized by an extremely flat power distribution near the Raman gain peak. The power variation between the generated lines is less than 0.5 dB. Also, the number of lines generated increases with the pump power. Note that this profile is significantly different from that observed in the conventional RDFB-FL without any elements. To understand whether the filter plays any role in the generation process, the filter was removed from within the configuration and placed at the output. Indeed, as shown in Fig. 4 the filter clearly plays a role in the generation dynamics. These measurements were performed at a pump power level of 3.5 W. When placed at the output, the filter passively filters the output radiation, closely following the intrinsic radiation profile. However, when it is placed inside the cavity, power redistribution is observed, and the generation profile is much flatter. Furthermore, secondary Stokes is generated at these power levels. The linewidth of the secondary Stokes is about 0.08 nm at 0.5 dB level. Note that at the power level of 3.5 W, the additional flattening for the first Stokes wave is due to the cascaded pumping for the second Stokes wave. The flatness could be potentially increased further by opting for a multiple pump

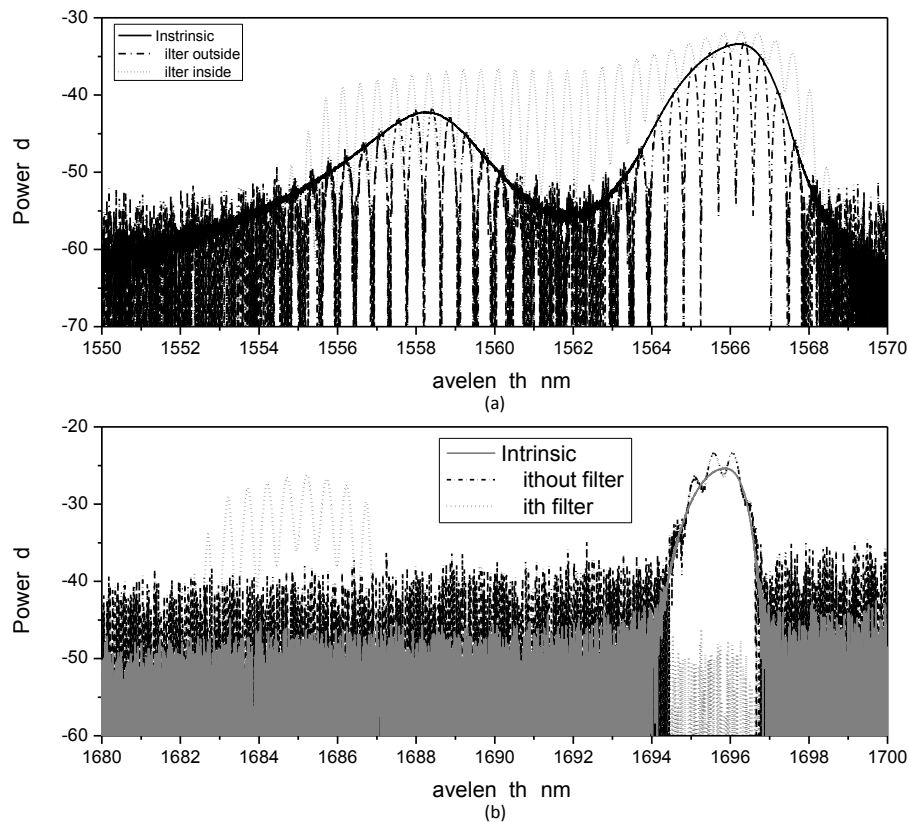


Figure 4. Figures illustrating the role of the filter in the generation process. The figures compare the generation with the filter inside and outside the cavity for the first Stokes (upper row) and the second Stokes cascade (lower row). Pump power was 3.5 W for both sets of figures.

configuration, or by nonlinear broadening of pump linewidth.³⁶

The demonstrated flatness and narrow linewidth suggests that nonlinear processes play an important role in the formation of generation in multiwavelength random DFB fiber laser and different lines could interact nonlinearly with each other. In particular, each generation line could have its own longitudinal distribution of the power along the power thus decreasing the competition of the different lines for the same pump.³⁶ This effect, however, does not account directly for the observed line narrowing when compared with a configuration, where the Lyot filter passively filters the generated radiation as in Fig. 4. Note that existence of possible mode correlations in the random DFB fiber laser has been reported recently in⁴⁵ that resulted in non-Gaussian intensity statistics similarly to the non-Gaussian statistics of conventional Raman fiber lasers^{46–50} as well as in Ytterbium-doped fiber lasers.⁵¹ Note that indications of mode correlations in Raman fiber laser radiation is reported also in.⁵² Moreover, dark and grey solitons can be generated in normal dispersion quasi-CW Raman fiber laser.⁵³ The origin of these correlations is unknown. The presented multi-wavelength random DFB fiber laser could be used as a test bed for experimental investigation of possible nonlinear interaction and correlations as spectral width of individual line (0.08-0.14 nm being equal to 10-18 GHz) is within the electrical bandwidth of real time oscilloscopes. Moreover, simultaneous measurements of the intensity dynamics in different lines could be of interest as it could reveal directly correlations between different lines.

The spectral characteristics can also be altered by changing the physical properties of the Lyot filter being used. For example, the free spectral range can be controlled by simply changing the length of the filter. The lineshapes can be controlled by applying such filters in cascades. The PM fiber in the Lyot filter can in essence be replaced by any suitable birefringent device, allowing room for possibility of wavelength and space tuning.

These properties can also be employed to obtain sub-picometer radiation in the RDFB laser, as demonstrated in an earlier work.⁵⁴

4. CONCLUSION

Here we have demonstrated a multiwavelength RDFB-FL based on Lyot all-fiber filter. The all-fiber architecture of the filter allows for its use in high power configurations – a feat that is quite difficult to achieve with traditional Fabry Perot based devices. The laser generates multiple lines both in first and second Stokes waves. The separation between lines is defined by the transmission profile of Lyot filter. However, the individual linewidth of the generated lines is found to be less than the spectral width of Lyot filter transmission profile. Nonlinear interaction between different lines could play an important role in formation of multiwavelength random DFB fiber laser generation properties. The Lyot filter based RDFB-FL thus offers a unique playground for the study of spectral dynamics in such highly nonlinear random lasing systems.

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