

Characterization of a multi-resonance ring resonator-based optical device

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ABSTRACT

We describe the linear and nonlinear transfer characteristics of a multi-resonance optical device consisting of two ring resonators coupled one to another and to a waveguide. The propagation effects displayed by the device are compared with those of a sequence of waveguide-coupled fundamental ring resonators.

Keywords: Waveguide-coupled ring resonator, Optical resonance, Pulse propagation effects

1. INTRODUCTION

A waveguide-coupled ring resonator behaves much like a Gires-Tournois interferometer, which is simply a Fabry-Perot interferometer with a 100% reflecting back mirror. Such a device has a unity magnitude response for all frequencies (provided that attenuation due to internal loss is negligible) while imparting a frequency-dependent phase to the transmitted optical field. This frequency-dependent phase response leads to dispersive properties which, for instance, affect the delay and shape of a pulse traversing the resonator.¹ Also, owing to the buildup of circulating intensity, the device may enhance a weak nonlinearity.² However, within a free spectral range, a single resonator can impart only a maximum (on resonance) phase depth of 2π radians. This limited phase depth is only enough to at most:^{3,4} delay a pulse by one pulse width, disperse a pulse at the equivalent of one dispersion length, and/or apply a nonlinear phase shift of 2π . To overcome this limitation, in Refs. 3,4 it has been proposed to use a sequence of waveguide-coupled resonators. Such a system has been shown to behave like an ordinary waveguide with a reduced group velocity, strong dispersion per unit length, and highly enhanced nonlinearity per unit length. However, the configuration considered in Refs. 3,4 requires a long sequence of densely distributed resonators, which may be not straightforward to fabricate. Also, it is critical to achieve the design value for each resonator radius (of the order of a few micrometers), and fabrication-induced variations on the ring sizes must be minimized.

In this paper, we propose an optical structure consisting of two ring resonators coupled one to another and to a waveguide. The basic principle of such a system is that, within the free spectral range of a single resonator, the device exhibits multiple resonances whose number scales as the ratio of the resonator sizes. This implies that, as the ratio of the ring sizes can be made arbitrarily large, the device is able to induce similar propagation effects to those of an arbitrarily long array of fundamental resonators. Furthermore, as we will show below, the density of resonances is conserved when the ratio of the ring sizes is not exactly an integer number, which implies that the tolerances on the ring radii are relaxed.

2. SYSTEM BASICS

Figure 1 shows the configuration of the double-ring resonator device. The couplings are described by transfer matrices in the frequency domain as

$$\begin{pmatrix} \tilde{E}_7(\omega) \\ \tilde{E}_8(\omega) \end{pmatrix} = \begin{pmatrix} r' & it' \\ it' & r' \end{pmatrix} \begin{pmatrix} \tilde{E}_5(\omega) \\ \tilde{E}_6(\omega) \end{pmatrix}, \quad (1)$$

$$\begin{pmatrix} \tilde{E}_3(\omega) \\ \tilde{E}_4(\omega) \end{pmatrix} = \begin{pmatrix} r & it \\ it & r \end{pmatrix} \begin{pmatrix} \tilde{E}_1(\omega) \\ \tilde{E}_2(\omega) \end{pmatrix}, \quad (2)$$

where we assume that the self- and cross-coupling coefficients r' , r and t' , t are independent of the frequency and intensity and that the coupling matrices are unitary such that $r'^2 + t'^2 = r^2 + t^2 = 1$. Propagation along either

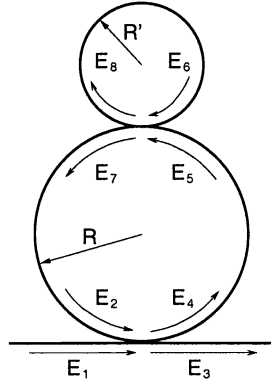


Figure 1. Scheme of the double-ring resonator system.

resonator feedback path occurs via a resonator mode, which may take the form of a mode of a ring waveguide or whispering-gallery mode of a disk or sphere.⁵ Along this feedback path, assuming that the internal attenuation is negligible, the field simply acquires an internal phase shift ϕ' or ϕ such that

$$\tilde{E}_6(\omega) = \exp[i\phi'(\omega)]\tilde{E}_8(\omega), \quad (3)$$

$$\tilde{E}_5(\omega) = \exp[i\alpha\phi(\omega)]\tilde{E}_4(\omega), \quad \tilde{E}_2(\omega) = \exp[i(1-\alpha)\phi(\omega)]\tilde{E}_7(\omega). \quad (4)$$

Furthermore, assuming negligible material dispersion, the internal phase shifts can be written as $\phi'(\omega) = \omega T_{R'}$, $\phi(\omega) = \omega T_R$, where $T_{R'} = 2\pi R'n'/c$, $T_R = 2\pi Rn/c$ are the circumferential transit times. Here, n' and n are the refractive indexes, R' and R are the effective radii of the rings, and c is the speed of light.

The buildup factor (i.e., the ratio of circulating intensity to incident intensity) of the upper resonator is derived from Eqs. (1) and (3) as

$$M'(\omega) \equiv \left| \frac{\tilde{E}_6(\omega)}{\tilde{E}_5(\omega)} \right|^2 = \frac{1-r'^2}{1-2r'\cos\phi'(\omega)+r'^2}. \quad (5)$$

This function possesses easily inferable properties. In particular, it is periodic in ϕ' with the period 2π , and in the interval $[(2m-1)\pi, (2m+1)\pi]$, m integer, it peaks at $\phi'_{\text{res}} = 2m\pi$, and is minimum at $\phi' = (2m \pm 1)\pi$. Its maximum and minimum values are given by $(1+r')/(1-r')$ and $(1-r')/(1+r')$, respectively, and the full-width at half-depth (FWHD) of the resonance peak is $O(1-r')$ as $r' \rightarrow 1$. It is easily shown that the amplitude $|\tilde{E}_7(\omega)/\tilde{E}_5(\omega)|$ of the transfer function of the upper resonator is equal to unity for all values of ω , and thus the field traversing the device simply acquires a transmitted phase shift Φ' that exhibits the following internal phase (or frequency) dependence:

$$\Phi'(\omega) \equiv \arg \left(\frac{\tilde{E}_7(\omega)}{\tilde{E}_5(\omega)} \right) = \pi + \phi'(\omega) + 2 \arctan \frac{r' \sin \phi'(\omega)}{1 - r' \cos \phi'(\omega)}. \quad (6)$$

Near resonance ($\phi' \approx \phi'_{\text{res}}$), the transmitted phase shift Φ' becomes sensitively dependent on the internal phase shift ϕ' . A measure of this phase sensitivity is obtained by differentiation of the transmitted phase shift with respect to the internal phase shift^{3,4} to obtain

$$\frac{\partial \Phi'}{\partial \phi'}(\omega) = M'(\omega). \quad (7)$$

In a similar manner, the buildup factor of the lower resonator is derived from Eqs. (2) and (4) as

$$M(\omega) \equiv \left| \frac{\tilde{E}_2(\omega)}{\tilde{E}_1(\omega)} \right|^2 = \frac{1-r^2}{1-2r\cos(\phi(\omega) + \Phi'(\omega)) + r^2}. \quad (8)$$

It is shown that, while the amplitude $|\tilde{E}_3(\omega)/\tilde{E}_1(\omega)|$ of the transfer function of the lower resonator is independent of ω and equal to unity, the phase Φ of the transfer function varies with frequency as

$$\Phi(\omega) \equiv \arg \left(\frac{\tilde{E}_3(\omega)}{\tilde{E}_1(\omega)} \right) = \pi + \phi(\omega) + \Phi'(\omega) + 2 \arctan \frac{r \sin(\phi(\omega) + \Phi'(\omega))}{1 - r \cos(\phi(\omega) + \Phi'(\omega))}. \quad (9)$$

One may see that formulae (8) and (9) are similar to (5) and (6) with ϕ' replaced by the sum $(\phi + \Phi')$. A measure of the sensitivity of the transmitted phase shift to the internal phase shift is obtained from Eq. (9) as

$$\frac{\partial \Phi}{\partial \phi}(\omega) = M(\omega) \left(M'(\omega) \frac{1}{\frac{\partial \phi}{\partial \phi'}} + 1 \right). \quad (10)$$

Note that for $r' = 0$ and $\Phi' = 2m\pi$, Eqs. (8), (9), and (10) reduce to Eqs. (5), (6), and (7) (without superscripts) of a single resonator.

3. TRANSFER CHARACTERISTICS

First, we describe the characteristics of the double-resonator structure when the intrinsic material nonlinearity is negligible, such that $n = n' = n_0$. In this case the internal phase shifts ϕ and ϕ' are simply related as $\phi = (R/R')\phi'$. We take the ratio of the resonator sizes to be $R/R' = a + \epsilon$, where a is a positive integer number, to account for deviations from an integer value that may be caused by manufacturing imperfections.

It can be shown that, when $\epsilon = 0$, the buildup factor M is a periodic function of ϕ' with the period 2π . Multiple resonances occur within one period, whose number N_0 increases linearly with increasing values of a . Specifically, it is found that

$$N_0 = \begin{cases} a + 1, & \text{if } a \text{ odd,} \\ a + 2, & \text{if } a \text{ even,} \end{cases} \quad (11)$$

for any interval $[(2m - 1)\pi, (2m + 1)\pi]$, m integer. A minimum of M occurs at the center of the periodicity interval, and either minima (if a odd) or maxima (if a even) occur at the extrema of the interval. The other locations of extremum values of M are of the form $\phi' = 2m\pi \pm f(r')$, i.e., they depend only on r' and are symmetrically distributed with respect to the center of the interval. All the extremum values of M exhibit the same dependence on r as for a single resonator: $M_{\max} = (1 + r)/(1 - r)$, $M_{\min} = (1 - r)/(1 + r)$. The FWHM of the resonance peaks is a function of both r and r' , but exhibits the most sensitive dependence on r , being $O(1 - r)$ as $r \rightarrow 1$ for any value of r' .

Now we take ϵ in the form $\epsilon = p/q$, where p and q are integer numbers without common divisors, and $0 < \epsilon \leq 1/2$. It can be shown that, while most of the features observed for $\epsilon = 0$ still apply in this case, the period of M in ϕ' becomes $2|q|\pi$, and the number of resonance peaks per period varies with a , p , and q . Specifically, denoting by \mathcal{N} the number of peaks per period when p and q are of the same sign, we find that

$$\mathcal{N} = \begin{cases} (a + 1)|q| + |p| + 1, & \text{if } a \text{ odd, and } p, q \text{ odd or } p \text{ odd, } q \text{ even,} \\ (a + 1)|q| + |p|, & \text{if } a \text{ odd, and } p \text{ even, } q \text{ odd,} \\ (a + 1)|q| + |p| + 1, & \text{if } a \text{ even, and } p \text{ odd, } q \text{ even or } p \text{ even, } q \text{ odd,} \\ (a + 1)|q| + |p|, & \text{if } a \text{ even, and } p, q \text{ odd.} \end{cases} \quad (12)$$

When p and q are of opposite sign, the number \mathcal{N}' of peaks per period is found to be

$$\mathcal{N}' = \mathcal{N} - 2|p| \quad (13)$$

for any combination of a , p , and q . The important feature emerging from Eqs. (12) and (13) is that the density of modes N_ϵ in an interval of length 2π is almost conserved with respect to the case $\epsilon = 0$. Indeed, suppose we increase $|q|$ while keeping $|\epsilon|$ to a fixed value, we obtain

$$N_\epsilon = \begin{cases} N_0 \pm |\epsilon|, & \text{if } a \text{ odd,} \\ N_0 - 1 \pm |\epsilon|, & \text{if } a \text{ even.} \end{cases} \quad (14)$$

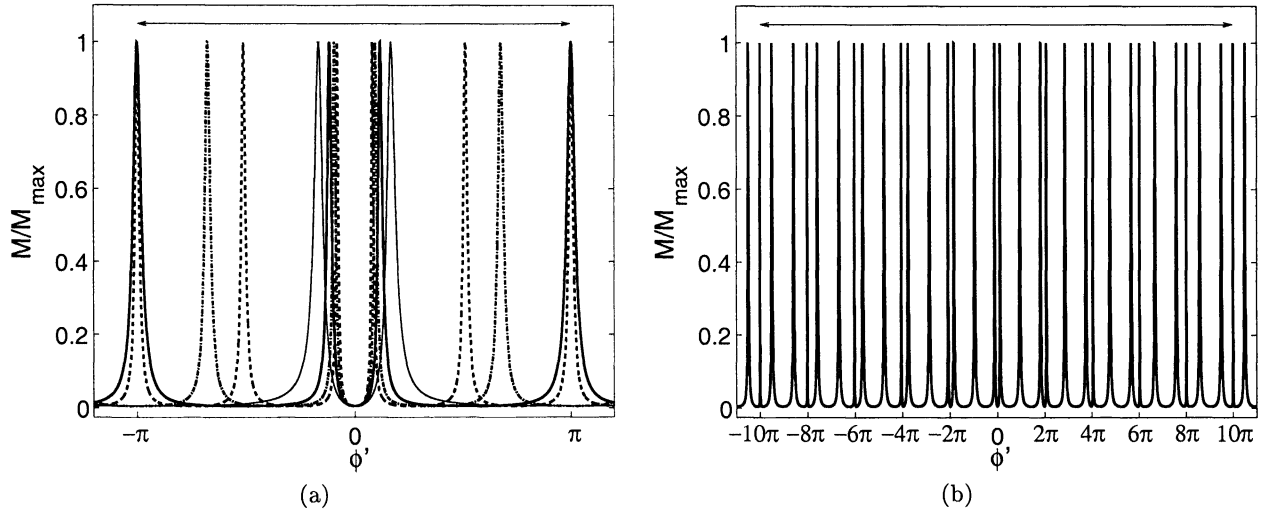


Figure 2. Normalized buildup factor versus the internal phase shift for (a) $R/R' = 1$ (thin solid line), $R/R' = 2$ (solid line), $R/R' = 3$ (dash-dot line), $R/R' = 4$ (dashed line), and (b) $R/R' = 21/10$. Here, $r^2 = r'^2 = 0.75$.

Here, $+|\epsilon|$ is for p and q of the same sign, and $-|\epsilon|$ is for p and q of opposite sign. The conservation of the density of modes is a remarkable fact because it implies that a precise control of the ring resonator sizes is not of crucial importance. Examples of the functional dependence of the buildup factor on the internal phase shift for some values of R/R' are shown in Fig. 2.

The features of $M(\phi'; r, r', R/R')$ and $M'(\phi'; r')$ described above and in Sec. 2 completely determine the functional dependence of the phase sensitivity $\partial\Phi/\partial\phi$ (see its definition in Eq. (10)) on the internal phase shift and the system parameters r , r' , and R/R' . Therefore, we make here only two remarks: $\partial\Phi/\partial\phi$ exhibits the same periodicity in ϕ' as M , and its extremum values occur approximately at the same locations as those of the extremum values of M , this approximation being valid to $O(1-r)^2$ as $r \rightarrow 1$ at least, for any value of r' . Some plots of the transmitted phase shift and phase sensitivity versus the internal phase shift for differing values of r and r' are shown in Fig. 3.

Next, we consider the case when the resonator material possesses a Kerr nonlinearity, i.e., an intensity-dependent refractive index: $n = n_0 + n_2|\tilde{E}_2|^2$, $n' = n_0 + n_2|\tilde{E}_6|^2$. In this case, using Eqs. (2), (4), and (5), we can write the relation between the internal phase shifts ϕ and ϕ' as

$$\phi \approx \frac{R}{R'} \left[1 + \delta \frac{1+r}{1-r} (1 - M'(\phi')) \right] \phi', \quad \delta = \frac{n_2}{n_0} |\tilde{E}_1|^2. \quad (15)$$

Here, parameter δ is a measure of the strength of nonlinearity. We can infer from Eq. (15) that whenever the condition $((1+r)(1+r'))/((1-r)(1-r')) < \delta^{-1}$ is satisfied, the system quantities of interest preserve the functional dependence on the internal phase shift and the system parameters observed in the linear case. Therefore, the above condition gives a limit for the nonlinear behaviour to manifest itself. As an example, if we use the typical values of $n_0 = 1.5$, $n_2 = 3.2 \times 10^{-16} \text{cm}^2 \text{W}^{-1}$, and $A_{\text{eff}} = 50 \mu\text{m}^2$ for a standard single-mode silica fiber, and a typical incident optical power ranging from 5 to 10 W, we find δ to be of the order of 10^{-9} . This implies that all values of r and r' such that $rr' \leq 0.9999^2$ fulfil the above condition. Thus, in what follows we will assume to operate the system within the linear limit.

4. PULSE PROPAGATION EFFECTS

We describe here how the proposed multi-resonance system may be used to tailor the linear and nonlinear pulse propagation characteristics of an ordinary dielectric waveguide. Specifically, we consider the configuration in Fig. 1 with a ratio of radii $R/R' = a$ and equal self-coupling coefficients $r = r'$, and we compare such a system

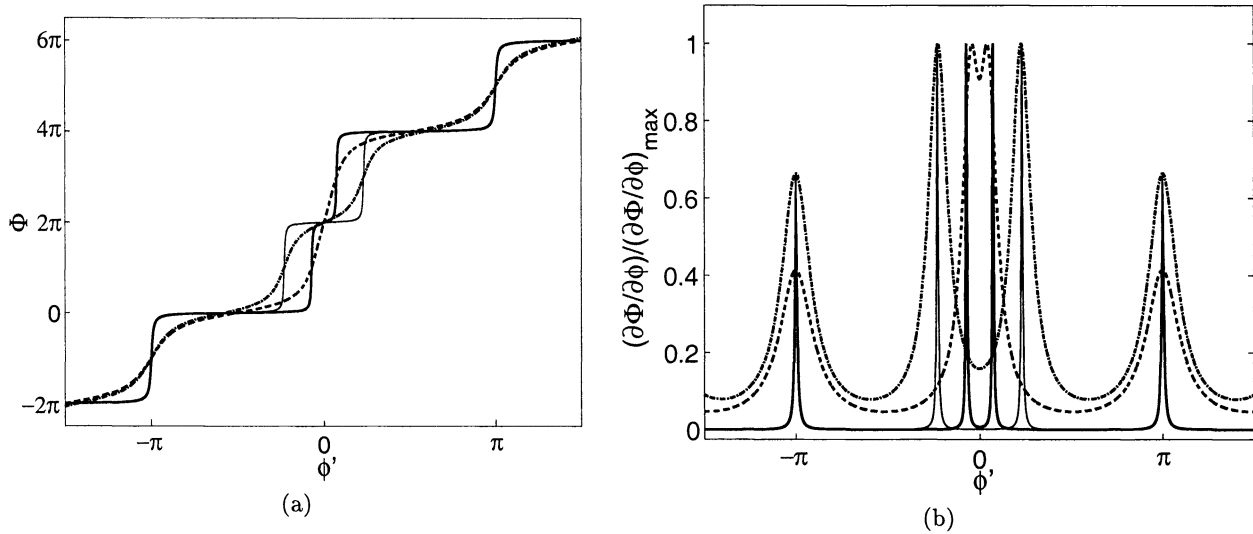


Figure 3. (a) Transmitted phase shift and (b) normalized phase sensitivity versus the internal phase shift for $r^2 = r'^2 = 0.25$ (dah-dot lines), $r^2 = 0.9, r'^2 = 0.25$ (thin solid lines), $r^2 = 0.25, r'^2 = 0.9$ (dashed lines), and $r^2 = r'^2 = 0.9$ (solid lines). Here, $R/R' = 2$.

with a sequence of $(a + 1)$ fundamental resonators with radius R' and self-coupling coefficient r' as regards the effects induced on pulse propagation.

A pulse propagating through either the double resonator device or a single resonator acquires a frequency-dependent phase shift, Φ or Φ' , that contributes to the (accumulated) propagation constant of the coupled waveguide. Near resonance, the contribution by either device to the propagation constant becomes sensitively dependent on the frequency, and this increased phase sensitivity on resonance leads to an increased group delay, or equivalently, an increased pathlength due to recirculation, or still equivalently, a reduced group velocity of propagation. The group delay imparted by the double-resonator system is specifically given by

$$\beta_1^I \equiv aT_{R'} \frac{\partial\Phi}{\partial\phi} = aT_{R'} M \left(\frac{M'}{a} + 1 \right) \xrightarrow{\phi'=\phi'_{\text{res},i}} aT_{R'} \frac{1+r'}{1-r'} \left(\frac{M'|\phi'_{\text{res},i}}{a} + 1 \right), \quad (16)$$

where the last form of this result refers to the situation in which the incident pulse is resonant such that β_1^I almost attains a maximum. Here, $\phi'_{\text{res},i}$, $i = 1, 2, \dots, N_0/2$, are the resonance locations of the device that fall in an interval of length 2π , and they are indexed, e.g., from the center to the extrema of the interval. Similarly, the group delay introduced by the array of resonators is given by^{3,4}

$$\beta_1^{II} \equiv (a+1)T_{R'} \frac{\partial\Phi'}{\partial\phi'} = (a+1)T_{R'} M' \xrightarrow{\phi'=\phi'_{\text{res}}} (a+1)T_{R'} \frac{1+r'}{1-r'}. \quad (17)$$

Here, $\phi'_{\text{res}} = 2m\pi$ is the resonance location of a single resonator in a 2π -interval. Equations (16) and (17) yield a ratio of group delays at resonance of

$$\frac{\beta_1^I|\phi'_{\text{res},i}}{\beta_1^{II}|\phi'_{\text{res}}} = \frac{a}{a+1} \left(\frac{M'|\phi'_{\text{res},i}}{a} + 1 \right). \quad (18)$$

Evaluation of this quantity yields: if $a = 1$ (which corresponds to $i = 1$),

$$\frac{1}{2}(M'|\phi'_{\text{res},1} + 1) = 1, \quad \forall r', \quad (19)$$

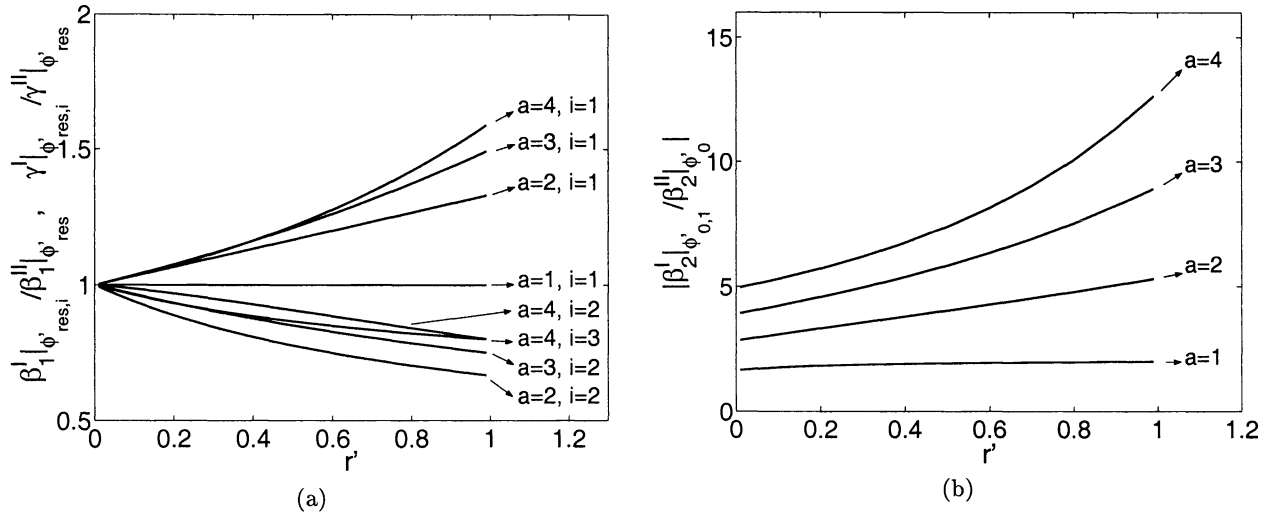


Figure 4. (a) Ratio of the group delays and nonlinearity coefficients at resonance and (b) ratio of the GVD coefficients at the detunings from resonance versus the self-coupling coefficient r' for differing values of a .

and, if $a > 1$,

$$\frac{a}{a+1} \left(\frac{M'|_{\phi'_{res,i}}}{a} + 1 \right) = \begin{cases} 1 + O(r'), & r' \rightarrow 0, \quad i = 1, 2, \dots, N_0/2, \\ \frac{2a}{a+1} + O(1-r'), & r' \rightarrow 1, \quad i = 1, \\ \frac{a}{a+1} + O(1-r'), & r' \rightarrow 1, \quad i = 2, \dots, N_0/2. \end{cases} \quad (20)$$

The interesting feature emerging from Eqs. (19) and (20) is that the group delay of the double-resonator system on the first resonances ($i = 1$) is at least equal to the group delay of the equivalent array of resonators, and can become as large as twice the delay of the array with increasing values of a and r' . Equation (18) is plotted versus r' for the first few values of a in Fig. 4a.

Higher-order derivatives of the phase response of either resonator-based structure no longer preserve the pulse shape but rather lead to dispersive effects. In particular, the second derivative of the phase gives rise to the lowest-order group velocity dispersion (GVD). It can be shown that the accumulated GVD induced by the double-resonator system is given by

$$\beta_2^I \equiv a^2 T_{R'}^2 \frac{\partial^2 \Phi}{\partial \phi'^2} = a T_{R'}^2 \left[\frac{\partial M}{\partial \phi'} \left(\frac{M'}{a} + 1 \right) + \frac{M}{a} \frac{\partial M'}{\partial \phi'} \right] \xrightarrow{\phi' = \phi'_{0,1}, r' \rightarrow 1} \pm 4a^2 T_{R'}^2 \frac{3\sqrt{3}}{4(1-r')^2} + \frac{1}{O(1-r')^{3/2}}. \quad (21)$$

The GVD coefficient is very small on resonance ($(M/a)\partial M'/\partial \phi'|_{\phi'_{res,i}} = 1/O(1-r')^{3/2}$ as $r' \rightarrow 1$); however, a small detuning $\phi'_{0,1}$ on the lower or higher side of the resonances $i = 1$ can lead to a large normal (positive) or anomalous (negative) value of the dispersion. Specifically, these detunings are locations of extremum values of $\partial M/\partial \phi'$, and are such that $\phi'_{res,1} - \phi'_{0,1} = \pm(1-r')/(2a\sqrt{3}) + O(1-r')^2$ as $r' \rightarrow 1$. Similarly, the accumulated GVD introduced by the array of resonators is found to be^{3,4}

$$\beta_2^{II} \equiv (a+1) T_{R'}^2 \frac{\partial^2 \Phi'}{\partial \phi'^2} = (a+1) T_{R'}^2 \frac{\partial M'}{\partial \phi'} \xrightarrow{\phi' = \phi'_0, r' \rightarrow 1} \mp (a+1) T_{R'}^2 \frac{3\sqrt{3}}{4(1-r')^2} + \frac{1}{O(1-r')}. \quad (22)$$

In such a case the GVD coefficient is zero on resonance, and the dispersion maxima occur at detunings ϕ'_0 such that $\phi'_0 - \phi'_{res} = \pm(1-r')/\sqrt{3} + O(1-r')^2$ as $r' \rightarrow 1$. Comparing (21) to (22) one may see that the magnitude of GVD induced by the double-resonator system is enhanced with respect to that of the equivalent array of

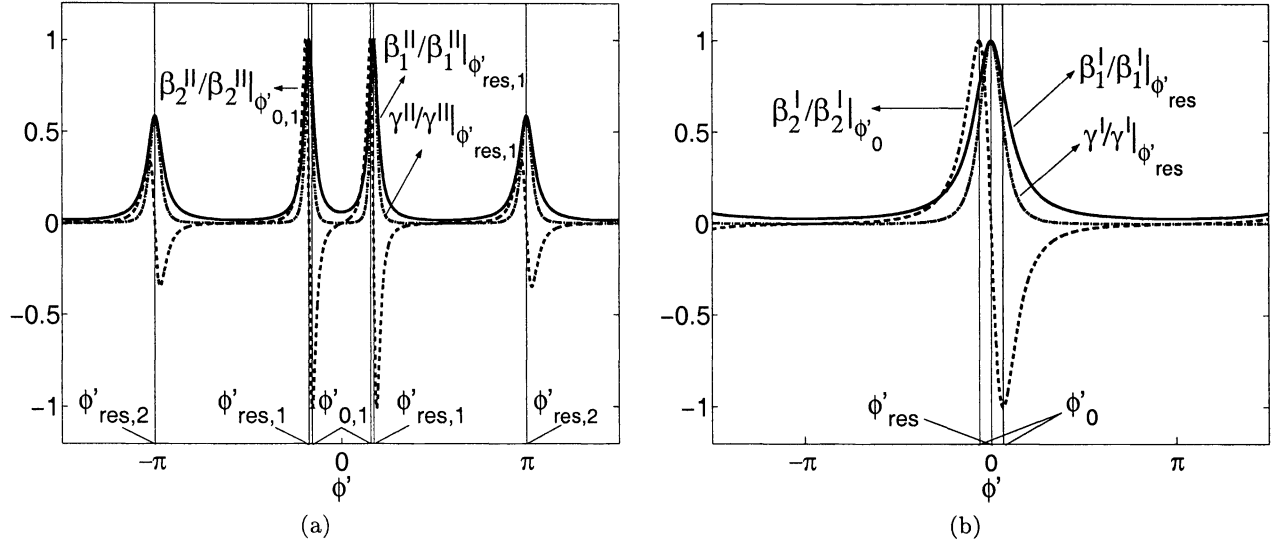


Figure 5. Functional dependence of the group velocity reduction (solid line), GVD (dashed line), and nonlinearity coefficient (dash-dot-dot line) on the internal phase shift for (a) the double-resonator configuration (here, $a = 2$), and (b) an array of fundamental resonators.

resonators by

$$\left| \frac{\beta_2^{\text{I}}|_{\phi'_{0,1}}}{\beta_2^{\text{II}}|_{\phi'_0}} \right| = \frac{4a^2}{a+1} + \frac{1}{O(1-r')^{3/2}}, \quad r' \rightarrow 1. \quad (23)$$

This factor approaches $4a$ for large a . A plot of the ratio of GVD values versus r' for the first few values of a is shown in Fig. 4b. By taking higher-order derivatives of the phase of the transfer function, higher-order contributions to the dispersion displayed by both devices may also be acquired.

In addition to inducing a strong group delay and dispersion, a resonator-based system may enhance a weak nonlinearity. Indeed, if the resonator material possesses a Kerr nonlinearity, then the internal phase shift will be intensity dependent. Near resonance, the transmitted phase shift is sensitively dependent on the internal phase shift, which is in turn dependent on an enhanced circulating intensity. The combined action of these effects gives rise to a dually enhanced effective nonlinear propagation constant,² calculated from the derivative of the transmitted phase shift with respect to the input intensity. The accumulated strength of enhanced nonlinearity for the double-resonator structure is specifically given by

$$\gamma^{\text{I}} \equiv \frac{\partial \Phi}{\partial |\tilde{E}_1|^2} = \frac{\partial \Phi}{\partial \phi} \frac{\partial \phi}{\partial |\tilde{E}_2|^2} \frac{\partial |\tilde{E}_2|^2}{\partial |\tilde{E}_1|^2} = a \frac{4\pi^2 R' n_2}{\lambda} M^2 \left(\frac{M'}{a} + 1 \right) \xrightarrow{\phi' = \phi'_{\text{res},i}} a \frac{4\pi^2 R' n_2}{\lambda} \left(\frac{1+r'}{1-r'} \right)^2 \left(\frac{M'}{a} + 1 \right). \quad (24)$$

Here, $2\pi n_2/\lambda$ represents the strength of the intrinsic material nonlinearity. Similarly, the accumulated nonlinearity for the sequence of resonators is found to be²⁻⁴

$$\gamma^{\text{II}} = (a+1) \frac{4\pi^2 R' n_2}{\lambda} M'^2 \xrightarrow{\phi' = \phi'_{\text{res}}} (a+1) \frac{4\pi^2 R' n_2}{\lambda} \left(\frac{1+r'}{1-r'} \right)^2. \quad (25)$$

One may see from Eqs. (24) and (25) that the ratio of the enhanced nonlinearities of the two systems at resonance is equal to that of the respective time delays:

$$\frac{\gamma^{\text{I}}|_{\phi'_{\text{res},i}}}{\gamma^{\text{II}}|_{\phi'_{\text{res}}}} = \frac{a}{a+1} \left(\frac{M'}{a} + 1 \right). \quad (26)$$

In particular, operation of the double-resonator system on the first resonances ($i = 1$) may give a nonlinearity strength twice as large as that of the equivalent array of fundamental resonators. Figure 5 summarizes the main propagation characteristics of the two systems in a graphical manner as functions of the internal phase shift.

5. CONCLUSION

We have described the linear and nonlinear transfer characteristics and some of the propagation characteristics of a ring resonator-based optical device that exhibits multiple resonances within the free spectral range of a single resonator. We have shown that the multi-resonant behaviour of the device leads to pulse propagation effects which are similar to those induced by a sequence of fundamental resonators. The properties of the device were shown to be preserved for arbitrary ratios of the ring radii. Therefore, the proposed device may provide one with an easy-to-implement technique for achieving significant modification of the propagation properties of a light pulse through an optical waveguide, as an alternative to large and densely packed arrays of resonators.

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