An Investigation into the Steady State Performance of Several Methods of Series Compensating Electricity Transmission Lines

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Doctor of Philosophy

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Summary

There are several methods of providing series compensation for transmission lines using power electronic switches. Four methods of series compensation have been examined in this thesis, the thyristor controlled series capacitor, a voltage sourced inverter series compensator using a capacitor as the series element, a current sourced inverter series compensator and a voltage sourced inverter using an inductor as the series element.

All the compensators examined will provide a continuously variable series voltage which is controlled by the switching of the electronic switches. Two of the circuits will offer both capacitive and inductive compensation, the thyristor controlled series capacitor and the current sourced inverter series compensator. The other two will produce either capacitive or inductive series compensation.

The thyristor controlled series capacitor offers the widest range of series compensation. However, there is a band of unavailable compensation between 0 and 1 pu capacitive compensation. Compared to the other compensators examined the harmonic content of the compensating voltage is quite high. An algebraic analysis showed that there is more than one state the thyristor controlled series capacitor can operate in. This state has the undesirable effect of introducing large losses.

The voltage sourced inverter series compensator using a capacitor as the series element will provide only capacitive compensation. It uses two capacitors which increase the cost of the compensator significantly above the other three. This circuit has the advantage of very low harmonic distortion.

The current sourced inverter series compensator will provide both capacitive and inductive series compensation. The harmonic content of the compensating voltage is second only to the voltage sourced inverter series compensator using a capacitor as the series element.

The voltage sourced inverter series compensator using an inductor as the series element will only provide inductive compensation, and is the least expensive compensator examined. Unfortunately, the harmonics introduced by this circuit are considerable.

Key Words

FACTS, Series Compensation, TCSC,

This thesis is dedicated to my Mother

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Nomenclature

Symbol	Definition	Page
α	Firing angle of the thyristor controlled series capacitor triac (rad)	167
β	Damping Factor = $R/2X$	179
γ	$\arctan\dfrac{k^2+\beta^2-1}{2\beta}$ (rad)	180
γ_n	$\arctan\dfrac{k^2+\beta^2-n}{2\beta n}$ (rad)	186
θ	Phase of the Supply Current (rad)	48
θ_o	Firing angle of the inverter (rad)	177
θ_{ob}	Modified firing angle = θ_o/β (rad)	189
λ	Angle at which the thyristor controlled series capacitor's inductor current returns to zero (rad)	176
a_n	Fourier coefficient of the $\cos n\phi$ term for the inverter currents	185
b_n	Fourier coefficient of the $\sin n\phi$ term for the inverter currents	185
h	$\sqrt{\dfrac{(k^2+\beta^2-1)^2+4\beta^2}{2\beta}}$	180
h_n	$\sqrt{\dfrac{(k^2+\beta^2-n)^2+4\beta^2n^2}{2\beta n}}$	186
I	Magnitude of the Supply Current (A)	48
I_C	Capacitor Current (A)	44

I_{Cn}	Component of the capacitor current of frequency $n\theta$ (A)	187
I_L	Inverter Current for the inverter based compensators Inductor current for the thyristor controlled series capacitor (A)	50
I_{Ln}	Component of the inverter current of frequency $n\theta$ (A)	187
I_S	Supply Current (A)	48
k	Reactance Ratio for the inverter based compensators = 1/XY Resonance Ratio for the thyristor controlled series capacitor = 1/XY	47
n	Harmonic number	185
p_1'	Normalised value of the active power component of the compensating voltage for the current sourced inverter series compensator when $\theta_o = 0$	108
p_1	Normalised value of the active power component of the compensating voltage for the voltage sourced inverter series compensators when $\theta_o = 0$	190
q_1'	Normalised value of the reactive power component of the compensating voltage for the current sourced inverter series compensator when $\theta_o = 0$	107
q_1	Normalised value of the reactive power component of the compensating voltage for the voltage sourced inverter series compensators when $\theta_o = 0$	189
q_2	Dependance of the variaction of V_p and V_q with θ_o	187
R	Resistance (Ω)	176
V_1	Thyristor controlled series capacitor voltage for the first era (V)	168
V_2	Thyristor controlled series capacitor voltage for the second era (V)	169

V_3	Thyristor controlled series capacitor voltage for the third era (V)	173
V_C	Inverter Voltage for the voltage sourced inverter series compensator using an inductor as the series element (V) Series capacitor voltage for all other compensators (V)	178
V_{Cn}	Component of the series capacitor voltage of frequency $n\theta$ (V)	187
V_L	Series Inductor Voltage for the voltage sourced inverter series compensator using an inductor as the series element (V) Inverter voltage for all other inverter compensators (V) Inductor voltage for the thyristor controlled series capacitor (V)	183
V_{Ln}	Component of the series inductor voltage of frequency $n\theta$ (V)	213
V_p	Component of the compesating voltage in phase with the supply current (V)	190
V_q	Component of the compesating voltage in quadrature with the supply current (V)	189
X	Total inductive impedance (Ω)	176
X_{ac}	Impedance of the series inductor for the voltage sourced inverter using an inductor as the series element (Ω)	203
X_{dc}	Impedance of the inductor across the d.c. terminals of the inverter for the voltage sourced inverter using an inductor as the series element (Ω)	192
Y	Total capacitive admittance (S)	167
Y_{ac}	Admittance of the series capacitor for the voltage sourced inverter using a capacitor as the series element (S)	176
Y_{dc}	Admittance of the capacitor across the d.c. terminals of the inverter for the voltage sourced inverter series compensator using a capacitor as the series element (S)	176

Chapter 1
Introduction

1.1 General Background

In the United Kingdom the quality of electricity is determined by the voltage of the supply. This should be sinusoidal at a frequency of 50 Hz, a received voltage that depends upon the rating of the transmission line being used (e.g. 240 volts single phase for domestic use) and no harmonic content. Obviously these are the ideal criteria and in reality are very difficult, if not impossible, to attain, so variations about this ideal are allowed. The variations are detailed in the National Grid Code connection condition cc.6 Technical, Design and Operational Criteria (Grid Code 94). Compensation of some description is required when the quality of electricity generated could become inferior to the standard required.

With the privatisation of the United Kingdom electricity industry into various companies it has become more difficult for the National Grid Company, responsible for electricity transmission to control the supply and demand of electricity. The only equipment the National Grid Company have full control over are the transmission lines, their associated transformers and switch gear. Compensation is increasingly seen as the way to better manage the transport of electricity (Foss 94). Recently there has been interest in using the Statcon to compensate electricity transmission lines, this is a shunt compensation scheme (Trainer et al., 94), (Gyugyi et al., 90), (Schauder et al., 95), (Mori et al., 93). This thesis will look at series methods of introducing compensation to transmission lines.

1) *Shunt compensation* injects a current onto a transmission line. It could be via a direct connection or via a shunt connected transformer. Shunt compensation can be considered as changing the impedance of the load on the transmission line. Examples are switched capacitor/inductor banks, the static var compensator and the Statcon.

2) *Series Compensation* injects a voltage onto a transmission line via a series transformer or via a series connected element i.e. a capacitor. It can be considered as changing the impedance properties of the transmission line. Examples are series capacitors and the Thyristor Controlled Series Capacitor.

It is mainly due to the differences in supply and demand the electricity industry encounters that has resulted in the need for compensation. At present there are essentially five main areas where compensation can be useful. These five types of compensation will be discussed in this chapter and are as follows;

1) Energy Storage

2) Frequency Control

3) Power Transfer

4) Transient Suppression

5) Voltage Level Control

This chapter will then take a brief look at the reasons behind the renewed interest in compensation before focusing on the chosen area of series compensation. It will examine the development of series compensation and outline how it currently operates.

1.2 Types of Compensation

1.2.1 Energy Storage

Once electricity has been generated it cannot be easily stored, so it is essential that the electricity generated closely matches the demand required. This is normally managed well by the National Grid engineers, and any discrepancies in supply and demand are absorbed by the system and even out over time. This is accomplished by electrical plant speeding up to absorb any excess energy and slowing down when there is a deficiency. In any 24 hour period there has always been large fluctuations in the demand for electricity, particularly in the winter. This has led to a genuine interest in methods of storing electrical energy which has been generated during periods of low demand and can be introduced back into the system at periods of high demand. Examples of energy storage methods include pumped water storage, batteries, fly wheels and super-conducting energy storage. There has been, and still is, an interest in using the low loss properties of superconductivity for storage of energy, although this area has yet to become commercially viable. This form of compensation is only viable for fluctuations over a long period of time - many electrical cycles- or even 24 hours for pumped storage.

1.2.2 Frequency Control

In an ideal situation, the electricity generated would exactly match the electricity demand. Electricity generation sets are relatively slow at reacting to changes in demand and can take many hours to reach their full working load from start up. If the supply of electricity is less than the demand then the frequency will drop. This will result in any electrical load which depends upon frequency being effected. For example electrical clocks would become inaccurate and motors would slow down. Similarly, if supply outstrips demand then the electrical frequency will rise with its corresponding effect on any frequency dependent load. Most electrical equipment is built to be supplied by a constant frequency, any large deviation from this frequency can, and has, resulted in catastrophic failure of the system. Therefore there are valid reasons for keeping the frequency constant. There are only two ways of controlling frequency. By varying the generation of electricity or by varying the demand for electricity. In the United Kingdom, under normal operating conditions, the supply and demand of electricity is regulated by monetary methods e.g. high demand equals high cost of electricity. This is an incentive for the supplier to provide electricity and an incentive for the consumer to reduce their consumption of electricity.

1.2.3 Power Transfer

If two parallel transmission lines have differing impedance's, perhaps due to unequal lengths, unequal impedance characteristics, or both, the load will divide unequally between the two.

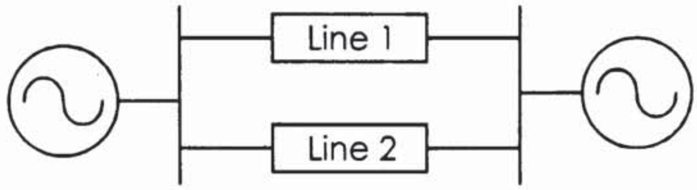

Figure 1.1 Simple Model of Two Parallel Transmission Lines

The situation of parallel lines of unequal impedance has in the past, led to one line running at its thermal limit whilst another was only part loaded (figure 1.1). Series compensation effectively allows the impedance of the two lines to be equalised and so allows both lines to carry their full rated current. Active power dissipated in the system is usually negligible compared to reactive power and so the resistance of transmission lines is generally discounted.

Introducing compensation increases the active power transmitted by the system, with the attendant increased loss dissipation, whilst allowing better use of the capital employed. Obviously the increased losses are tolerable provided there are cost benefits.

The above diagram (figure 1.1) shows two parallel transmission lines. It is possible that one of the lines might take a different route to the other or that the physical construction of one differs from that of the other line. The following example uses different line lengths to illustrate the advantage of capacitive series compensation. Take as an example the resistance per kilometre of both lines is 0.034 Ω, and the reactance per kilometre of both lines is 0.323 Ω (Weedy 1985).

If line 1 has a length of 200 km and line 2 has a length of 100 km, the total resistance of line 1 will be 6.8 Ω, and its reactance will be 64.4 Ω. The second line will have half the resistance and reactance of line 1. The voltage dropped across the two lines is identical then by Ohm's law the current through line1 must be half that of line 2.

If both lines have the same current rating and line 2 is operating on that limit then the first line will not be fully utilised. The fact that line 2 is at its operational limit restricts additional power flow even though line 1 clearly has the capacity. If capacitance is introduced in series with line 1 the total series reactance of that line will be reduced. As a result of this the currents down the two lines will become approximately equal. Line 1 will be able to carry more current and line 2 will still not be exceeding its operational limits. There will be a higher overall power flow.

1.2.4 Transient Suppression

The current and voltage of a circuit are always supposed to be sinusoidal. Disturbances to the flow of electricity result in transient voltages and currents. These disturbances can have serious consequences in terms of a reduction in the quality of electricity supplied and occasionally even cause damage to electrical equipment. A fast acting compensation method (able to respond within one electrical cycle) may be able to reduce the effects of these transients, or even be able to eliminate them. This would ensure a more reliable system providing higher quality electricity.

1.2.5 Voltage Control

Voltage control involves attempting to maintain the voltage received at the load. This is done by generating enough electrical power to match the load and the losses of the transmission line. During times of high load there can be difficulties in keeping the voltage within the specified limits. This is because the electricty generated is essentially of constant power. During times of peak demand the current being drawn increases and consequently the voltage must fall to maintain the balance between the electrical power generated and the electrical power required at the load. This is where voltage control compensation can be very useful. The most common method of controlling the voltage is to use tap-changing transformers, however they are mechanically switched, slow and can be unreliable. This suggests that a fast acting, electronically switched series compensation scheme could be a useful addition to the National Grid.

1.3 Renewed Interest in Compensation

World wide there is considerable interest in the use of electricity compensation. There are good reasons for the interest in this area. In recent years the public have become more conscious of their environment in terms of pollution and aesthetics. Proposals involving a change to the environment are subject to vigorous scrutiny. A consequence of this is that it has become almost impossible to secure planning permission for the construction of new electricity transmission lines, and where it is possible there are usually protests and long delays. This is a large incentive for the transmission companies to improve the utilisation of their existing lines.

In the past it was difficult to control the flow of electricity with any great precision. The improved electronic switching devices now being researched and developed offer the prospect of real time control of reactive power flows and real power transfer. As a result there is a great deal of interest in using power electronics to assist in the control of the transport of electricity. This area of interest has been dubbed FACTS (Flexible A.C. Transmission Systems) (Hingorani, 93). The FACTS initiative is the driving force behind many projects, all of which use power electronics in an attempt to improve the control of the flow of electricity (Gyugyi, 89, 90, 93), (Nelson et al., 95).

The power electronic switches being developed now can withstand high enough power ratings to begin to make them viable for use in high voltage electricity transmission lines. Series compensation in particular could be one area where power electronics makes a useful contribution. This is because traditional series capacitors do not provide sufficiently flexible compensation, they are not fully controllable and they introduce the risk of sub-synchronous resonance and system oscillations. Whereas the power electronic series compensators do offer the prospect of a fully controllable and continuously variable method of introducing a compensating voltage onto a transmission line. They may also be used for the mitigation of sub-synchronous resonance and system oscillations which is an obvious additional advantage.

1.4 Series Compensation

Ideally a series compensator should be a variable reactive element, ranging from capacitive to inductive, and should not introduce any active power losses. In practice, series compensation schemes do have variable reactance but they tend to be either purely capacitive or purely inductive. They also introduce losses, which attempts to minimise.

1.4.1 The Need for Series Compensation

Series compensation has the effect of changing the impedance of a transmission line. In its most probable use, it will effectively add capacitance to the line. This will cause a reduction in the overall reactance of the line and allow a larger load to be supplied. This can be particularly useful when supplying a load through two or more parallel lines.

1.4.2 How Series Compensation Works

An electricity transmission line can be represented by the following simple circuit diagram (figure 1.2) and its behaviour by a phasor diagram (figure 1.3).

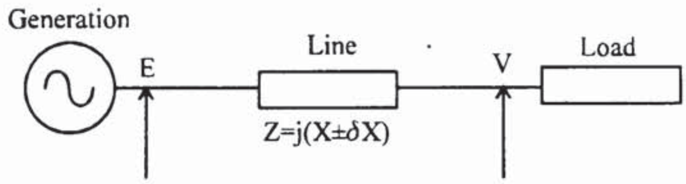

Figure 1.2 Simple Circuit Diagram of an Electricity Transmission Line

The voltage at the load (V) is taken as the reference phasor. The phasor IX represents the voltage dropped across the transmission line, and the generated voltage (E) has to be sufficient to meet the voltage required at the load and the losses of the line. The current through the transmission line (I) is shown lagging the load voltage by a phase angle ϕ, this is typical of most loads.

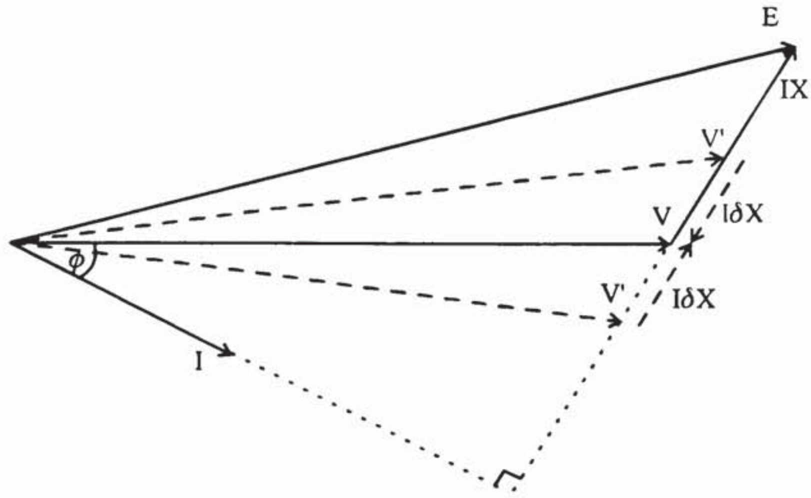

Figure 1.3 Phasor Diagram Showing the effect of Capacitive Compensation

The result of series compensation is represented on the above phasor diagram (figure 1.3) by the phasor $I\delta X$. The magnitude of this phasor represents the degree of compensation and it is usually in the opposite direction to the voltage dropped across the transmission line (i.e. capacitive).

An ideal series compensator would be continuously variable and respond quickly to changes in the system conditions.

1.4.3 Present Methods of Series Compensation

Series compensation was and is used to reduce the reactance of transmission lines. This was facilitated either by inserting one or more fixed capacitors into a transmission line or utilising a quadrature booster. Both these forms of series compensation are examined in a little more detail below.

(a) Fixed Series Capacitor

The most flexible way of providing series compensation using fixed capacitors is to use modules of capacitors in series. This will allow incremental steps of compensation.

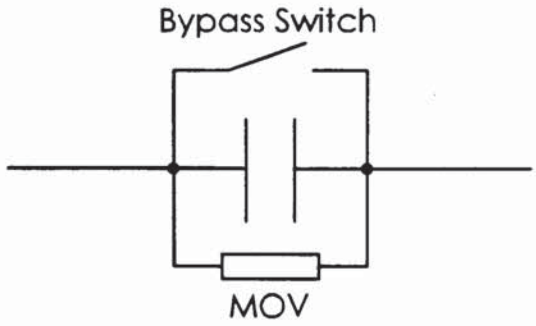

Figure 1.4 Circuit Diagram of Compensation
using Fixed Series Capacitors

The above diagram (figure 1.4) shows a typical representation of a fixed series capacitor. It consists of the fixed size capacitor, a bypass switch to allow the capacitor to be switched in and out of service and a metal oxide varistor (MOV). The varistor protects against over-voltages which can occur across capacitors during short circuit fault conditions and during switching.

The use of fixed capacitors renders the line, in electrical terms, shorter. However, the uptake of this method of series compensation was severely restricted when it was discovered that inserting capacitance can cause a phenomenon which is commonly known as sub-synchronous resonance (SSR).

(b) Quadrature Booster

Another form of series compensation which has been utilised is called the Quadrature Booster or Phase Shifting Transformer (Fyvie, 94), (Nelson, 94).

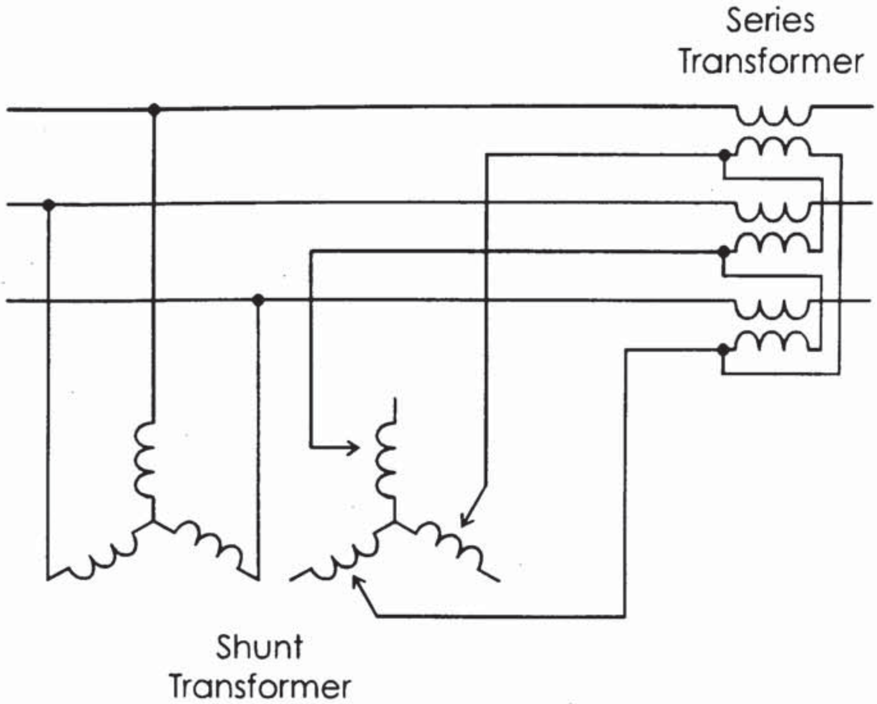

Figure 1.5 Circuit Diagram of a Quadrature Booster

This equipment takes voltage from a shunt transformer between phases. The resultant voltage is inserted onto the transmission line via a series transformer. The circuit diagram of a typical quadrature booster is shown in figure 1.5

A quadrature booster consists of a shunt transformer and a series transformer. The shunt transformer consists of a star connected primary winding. The secondary winding of this shunt transformer is also star connected and has tapping points to allow for incremental variations in the degree of compensation. The secondary winding of the shunt transformer is connected to the primary winding of the series transformer which is delta connected. The resultant voltage introduced back onto the line is in quadrature to the transmission line and it can be either lagging or leading. The quadrature booster method of series compensation has disadvantages:

i) it produces relatively high reactive losses due to the transformers,

ii) it is slow acting.

The quadrature booster is commonly used to compensate for the loss of a transmission line. It allows the load to be carried via alternative routes, there are always additional losses in these circumstances but they are considered acceptable to maintain a secure supply.

1.4.4 Future of Series Compensation

There are several forms of series compensation which need to be examined.

These are :-

(a) Thyristor controlled series capacitor

(b) Voltage sourced inverter series compensator

(c) Current sourced inverter series compensator

(d) Unified power flow controller

(a) Thyristor Controlled Series Capacitor

The thyristor controlled series capacitor is a recently developed method of providing series compensation on transmission lines (Godart et al., 95), (Jalali et al., 94), (Paserba et al., 95). It has several advantages over the capacitor bank methods and can provide real time control over the amount of compensation produced and is likely to require less maintenance due to the use of solid state switches rather than mechanical switches.

The thyristor controlled series capacitor as shown in figure 1.6 consists of a thyristor controlled inductor connected in parallel with a capacitor, this circuit is then connected in series with the transmission line. This configuration allows a continuous and rapidly variable series compensation system.

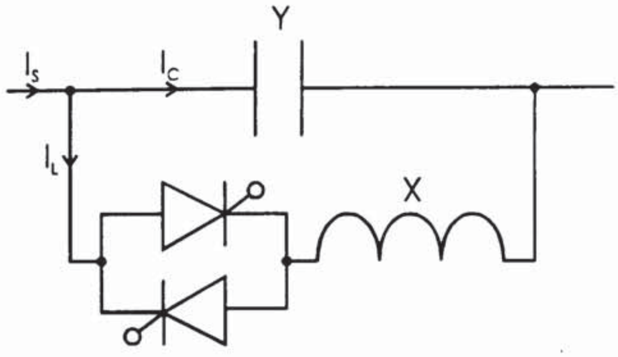

Figure 1.6 Circuit Diagram of a Thyristor Controlled Series Capacitor

The thyristor controlled series capacitor utilises a thyristor controlled reactor (TCR) which is generally switched into the circuit when the transmission line current is non-zero. This has the inevitable consequence of introducing harmonic distortion. The introduction of harmonic distortion to a line needs to be controlled by transmission regulations (Grid Code 94). It may also be that the requirements for capacitance is higher for the TCSC than with the traditional, mechanically switched series capacitor (Larsen et al 1993).

(b) Voltage Sourced Inverter Series Compensator

The voltage sourced inverter series compensator consists of a Statcon serially connected to a transmission line via either a capacitor, an inductor or a transformer. The circuit diagram below (figure 1.7) shows the arrangement of the compensator connected across a series capacitor.

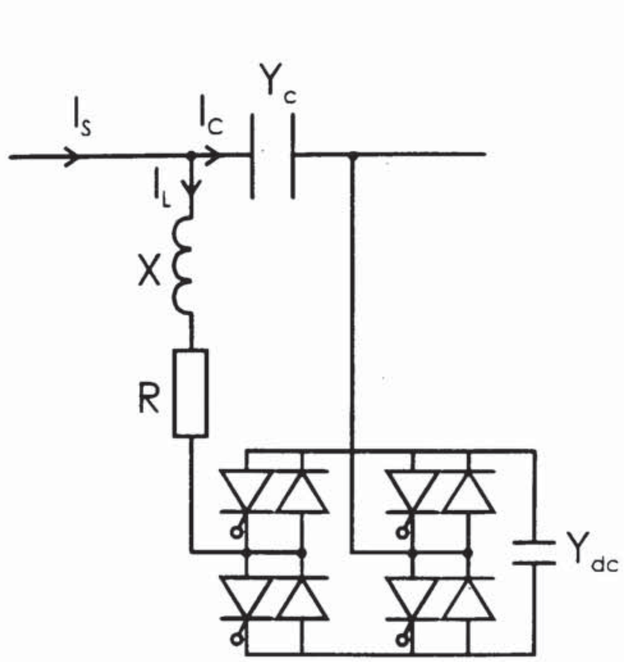

Figure 1.7 Circuit Diagram of a Voltage Sourced Inverter Series Compensator

The voltage sourced inverter series compensator can vary the current from the transmission line to the inverter, by controlling the firing angle of the power electronic switches in the inverter (Gyugui, 89, 93), (Nelson, 93). This has the effect of varying the magnitude of the voltage across the d.c. capacitor and obviously, also the voltage across the a.c. terminals of the inverter. The difference between the voltage across the connections to the transmission line and the voltage across the a.c. terminals of the inverter determine the magnitude and direction of this current.

(c) Current Sourced Inverter Series Compensator

The current sourced inverter series compensator is very similar in its configuration to the Statcon (Boon-Teck, 93). The Statcon is a shunt compensation device which utilises a voltage sourced inverter and a capacitive energy store. Controlling the firing angle of the power electronic switches varies the magnitude of the voltage across the a.c. terminals of the inverter. This controls the magnitude of the current flowing into or out of the compensator. The current sourced inverter compensator is the series equivalent to the shunt connected compensator - the Statcon.

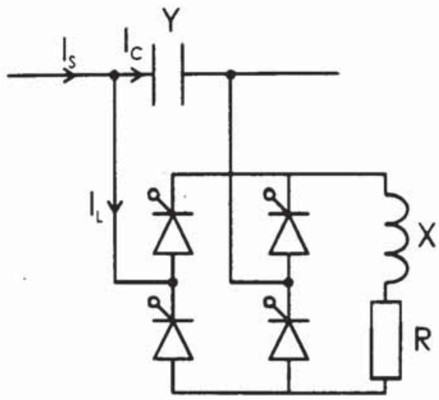

Figure 1.8 Circuit Diagram of a Current Sourced Inverter Series Compensator

The diagram shown above (figure 1.8) shows the configuration of a current sourced inverter series compensator. It consists of a current sourced inverter with an inductive energy store. The resistor shown in series with the inductor represents any active power losses associated with the circuit, such as the switching losses and the losses in the inverter. The inverter is connected across a capacitor.

Control of the compensator is accomplished by controlling the firing of the switches in the inverter. This varies the magnitude of the current flowing through the inverter and this current can be made to flow in both positive and negative directions. This in turn varies the amount of current flowing through the capacitor and, consequently, the magnitude of the voltage across it.

(d) The Unified Power Flow Controller

The ultimate flexible a.c. transmission systems device is at present considered to be the unified power flow controller, this is supposed to be the complete power compensation scheme (Gyugyi 92), (Gyugyi et al. 95), (Mehta et al., 92). However, there are other less ambitious methods of power system compensation such as shunt connected systems (i.e. the Statcon), or series compensation.

The Unified Power Flow Controller consists of two inverters connected together by a d.c. link, one of the inverters is connected to the transmission line via a shunt transformer and the other is connected via a series transformer, this can be seen in the diagram below (figure 1.9).

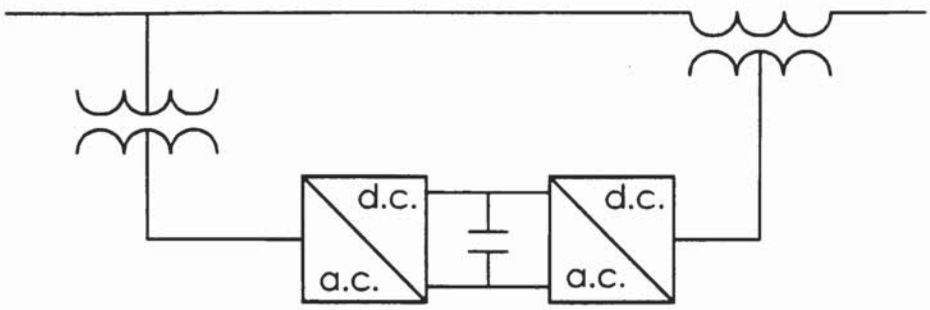

Figure 1.9 Circuit Diagram of the Unified Power Flow Controller

The shunt inverter provides the shunt compensation. This is equivalent to the Statcon. The series connection allows the insertion of a compensatory voltage back onto the transmission line. The connection of the two, via a dc link, allows full control of all forms of electrical compensation, with the added benefit of real power transfer across the link. The perceived desirability of the unified power flow controller is its flexibility when compensating power systems. One of its virtues extolled by Gyugyi was that only one design is necessary for all forms of compensation. In my opinion the main reason for requiring a unified power flow controller is in the situation where the compensation needs of a transmission line are likely to be changing either in the short or the medium term. The requirements of the National Grid may need this flexibility, but it is considered more likely that series or shunt compensation would be the most effective route for providing system support.

This conclusion is based on two factors. Firstly, the unified power flow controller provides series and shunt compensation. The probability of both forms of compensation being required on a single line is low. The second factor is cost. The unified power flow controller requires more components than its alternatives and is therefore more expensive to assemble and maintain.

1.5 Problems Associated with the Proposed Systems

1.5.1 Harmonics

Of the systems examined only the fixed series capacitor and quadrature booster do not produce harmonics. Harmonics are inherent with controlled thyristor firing and numerous attempts have been made to try and eliminate them.

With the thyristor controlled series capacitor the only solution to the problem of harmonics would seem to be filters. However, there are several possibilities that must be scrutinised for the inverter based systems proposed. Three phase systems would produce harmonics of the order $6n\pm1$, and one solution to reducing these is to use filters. Unfortunately, eliminating harmonics of a low order, i.e. 5,7, and 11 is difficult and expensive.

Higher order harmonics are a lot more susceptible to elimination by filters. A 48 pulse Statcon will produce harmonics of order $48n\pm1$. Therefore, another possible solution is to use several phase shifted transformers to produce multiple pulse systems, with 48 pulses being considered the most acceptable configuration. Unfortunately, the multiple transformers configuration is expensive and can be complicated due to the wiring arrangement of its transformers.

Yet another method of harmonic reduction is pulse width modulation (Holtz J., 92), (Turnbull 64), (Patel and Hoft, 73, 74). This method of eliminating harmonics involves precise firing of the power electronic switches several times during one half of an electrical cycle. The frequency of switching is dependent upon how many harmonics are required to be eliminated, and the calculation of the switching times requires equations to be solved by numerical methods. This method of eliminating harmonics is effective but has drawbacks in that it increases the active power losses due to the increased frequency of switching. Another drawback is that it increases the magnitude of the higher order harmonics. There is also the additional complexity of the control system, due to the more elaborate switching pattern. The increase in switching frequency is proportional to the number of harmonics required to be eliminated. This suggests that pulse width modulation is only viable when removing a small number of harmonics.

The most realistic solution for keeping harmonics within statutory limits might well be to use a combination of all the methods described above. For example using 2 phase shifted transformers to produce a 12 pulse system (figure 1.10) would reduce the harmonics to $12n\pm1$. Then using pulse width modulation to remove more harmonics, for example 11^{th}, 13^{th}, 15^{th} and 17^{th}. Filtering could then be used to remove the remaining unwanted harmonics (Kojori et al. 1990)

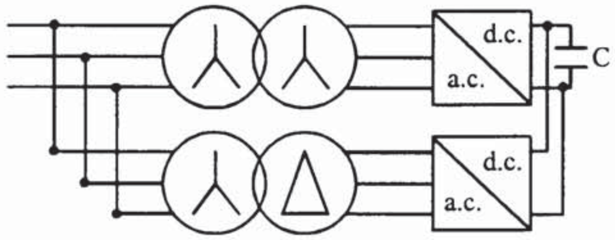

Figure 1.10 Diagram Showing One Configuration of a Twelve Pulse Inverter Circuit

Finally, a new method of eliminating harmonics has been proposed. This is called a multi-level energy store (Menzies and Zhaung, 95), (Rojas et al., 95), (Peng et al., 96). The method involves a number of energy stores being connected together via a number of fully controllable power electronic switches. The correct firing of these switches will produce a stepped waveform which is identical to the multi-pulse arrangement. This has the additional benefit of not requiring extra transformers which must be used in multi pulse arrangements. One example of such a multi-level inverter is shown in figure 1.11 below.

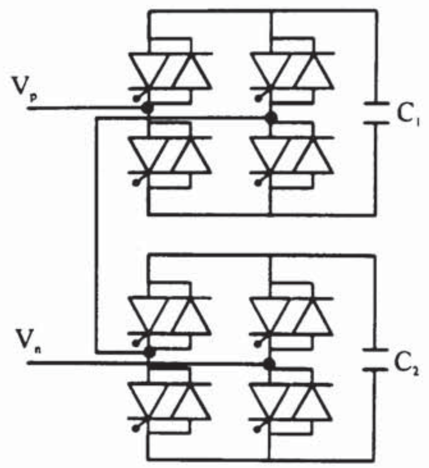

Figure 1.11 Diagram Showing One Configuration of a Two Level Inverter Circuit

1.5.2 Sub-Synchronous Resonance

In any electrical circuit where inductors and capacitors are present there will be a resonant frequency. The introduction of series capacitors onto transmission lines which are naturally inductive will obviously have a resonant condition. When this resonant frequency is in the order of a few Hertz there is the distinct possibility that this can coincide with the frequency of generator shafts. This can cause extensive damage to generation sets. It is because this frequency is below the system frequency, i.e. sub synchronous, that this condition has been called sub synchronous resonance. As a result any plans to install capacitor banks requires extensive analysis and testing before installation can proceed.

1.6 Aims

The aims of this thesis are to examine several methods of introducing a compensating voltage onto a transmission line. These compensation schemes all utilise power electronic switches to provide a quadrature voltage that is continuously variable. The analysis will examine the steady state performance of four different schemes. The first is the thyristor controlled series capacitor. A prototype of this scheme has been built and is operating in

Kayenta, Arizona. The second examines a compensation scheme which utilises a voltage sourced inverter connected across a series capacitor, and the third scheme uses a current sourced inverter again connected across a series capacitor. The final scheme uses a voltage sourced inverter again but uses an inductor as the series element. None of the last three compensation schemes have yet been constructed for use on any transmission system.

In all four cases an analysis of the steady state performance has been conducted. The results of this analysis was compared to simulations produced using the Saber software simulation package. In addition laboratory models of the first two compensation schemes were built in order to make further comparison.

The thesis goes on to look at the advantages of all four schemes, and to point out their respective disadvantages.

Chapter 2
Thyristor Controlled Series Capacitor

2.1 Introduction

The thyristor controlled series capacitor is just one development of the FACTS (Flexible A.C. Transmission Systems) initiative. This area has been of great interest for several years. Interest in the area began when there was the prospect of power electronic switches beginning to achieve the ratings required for high voltage transmission systems.

Power electronic switches can be controlled at speeds which are fast enough to allow real time control over voltages and currents. The advantage of using power electronic switches in series compensation is that it makes the possibility of continuously variable reactance a reality.

The thyristor controlled capacitor has been examined and discussed many times (Karady et al., 93), (Helbing et al., 94), (Nyati et al., 94). A prototype was constructed and is running in Kayenta, Arizona. This would suggest that there is very little left to be discovered about the steady state performance of the thyristor controlled series capacitor. Indeed this method of providing series compensation was only really studied in this project in order to provide background for the introduction to thesis. However, after attempting to simulate the thyristor controlled series capacitor using the software simulator Saber, it was discovered that perhaps there was more to this compensation circuit than the published literature leads one to believe.

All the published literature (Larsen et al., 93), (Renz et al., 93), on the steady state operation of the thyristor controlled series capacitor states that there are only two modes of operation. The capacitive mode and the inductive mode, and the usual mathematical examination of the circuit is based upon the assumption that the current through the thyristor controlled reactor is symmetrical about the zero cross of the capacitor voltage at $\pi/2$ (when the voltage becomes zero whilst going from positive to negative).

The summary of the analysis of the thyristor controlled series capacitor circuit, which is given later in this chapter, does not make this assumption and produces some interesting results as a consequence. The full analysis is given in Appendix A. This chapter presents the results of that analysis and gives an explanation of its meaning, diagrams of simulation plots are also presented as a comparison to the theory and the results of the bench-top model which was built in the laboratory are also shown.

2.2 Circuit Description

The diagram shown below depicts the thyristor controlled series capacitor in circuit form. It consists of a thyristor controlled reactor connected across a series connected capacitor. The system current (I_S) represents the current through the transmission line and is considered to be sinusoidal and constant. The voltage V represents the voltage dropped across the compensator and this is the quantity of most interest from the series compensation point of view. The current through the capacitor is represented by I_C and the inductor current by I_L. The capacitor is represented by its admittance Y and the inductor by its reactance X. Any resistance in the circuit has been neglected in order to simplify the analysis.

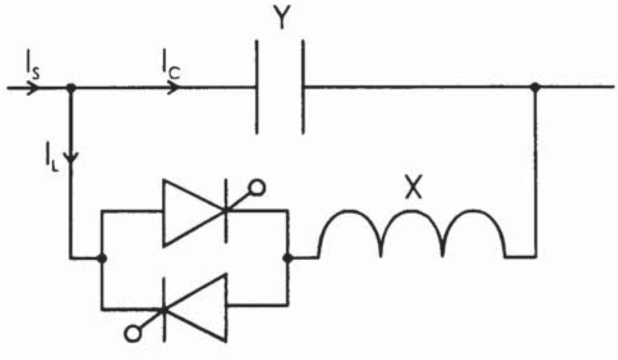

Figure 2.1 Circuit Diagram of the
Thyristor Controlled Series Capacitor

The principle behind the circuit is to insert onto a transmission line a fully controllable and continuously variable voltage that is in quadrature with the current flowing through the line. Transmission lines are almost always inductive and as such any series compensation scheme will almost always be capacitive, in so far as power transfer is concerned. However, if the purpose of the compensator is to mitigate the effects of transients in the circuit then it does not matter whether the compensator is capacitive or inductive, as long as it can provide a fast, continuously variable impedance.

The thyristor controlled series capacitor uses the series connected capacitor to provide the basic capacitive compensation, with the thyristor controlled reactor allowing the continuous variability of the compensating voltage. The inductor is switched into the circuit at a given angle which is defined as α. When the current through the inductor decays back to zero the thyristors cease conduction until the thyristors are next fired allowing conduction once again. The angle at which the inductor current returns to zero and the thyristors block current flow is defined as λ. These two important angles are shown in figure 2.2. The diagram shows both the inductor current flowing between the angles α and λ and the supply current to act as a reference.

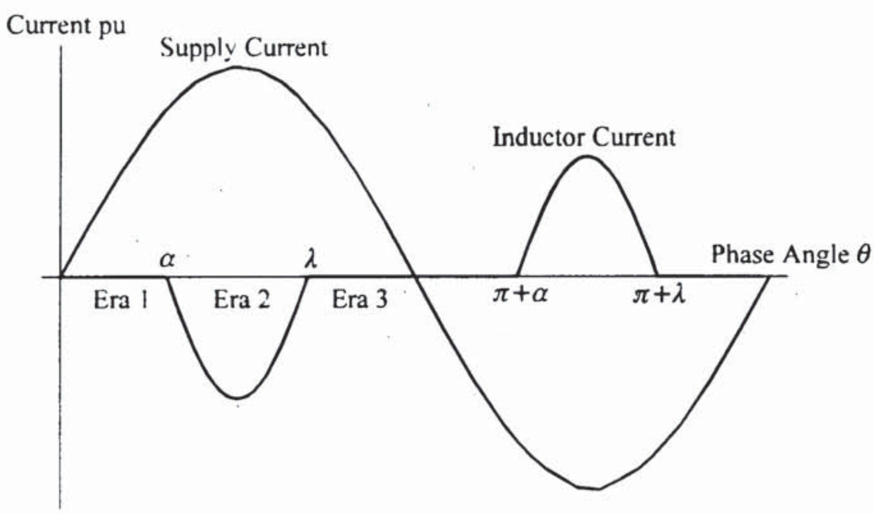

Figure 2.2 Graph Showing an Example of the Inductor Current with Reference to the Supply Current

The thyristors are fired every half cycle. The thyristor controlled series capacitor is operated as a resonant circuit, which depends upon the ratio of the inductor and capacitor to determine that resonant point.

When the thyristors are not allowing conduction the voltage across the compensator is capacitive, i.e. lags the current through the capacitor by $\pi/2$ and the voltage across the compensator is zero at $\pi/2$, as shown by the diagram given below (figure 2.3).

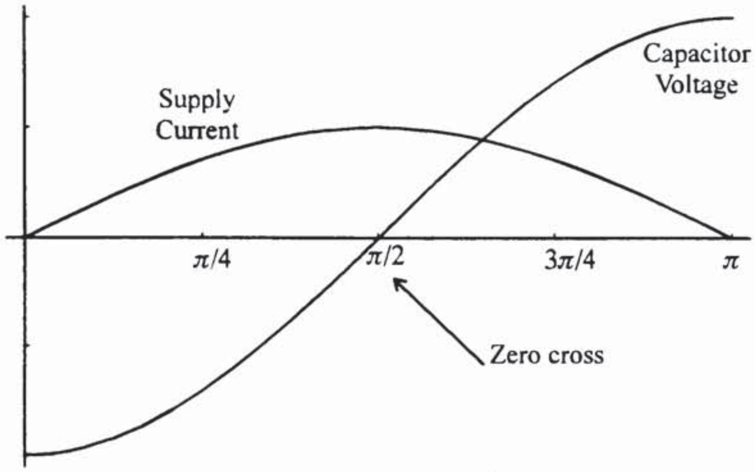

Figure 2.3 Graph Showing the Current through and the Voltage across a Capacitor

If the thyristors are fired just before the zero cross of the compensator voltage then there is a negative voltage across the inductor when the current is allowed to flow through it. This will drive current through the inductor in the negative direction, i.e. from right to left in the circuit diagram shown by figure 2.1. From Kirchoff's current law the current through the capacitor must be the system current I_S minus the inductor current I_L. The summation of these two currents is shown in figure 2.4. The voltage across the compensator V is given by the integral of the current through the capacitor and the effect is to increase the slope of the voltage which is also shown in figure 2.4. This boosts the capacitive compensation provided by the thyristor controlled series capacitor.

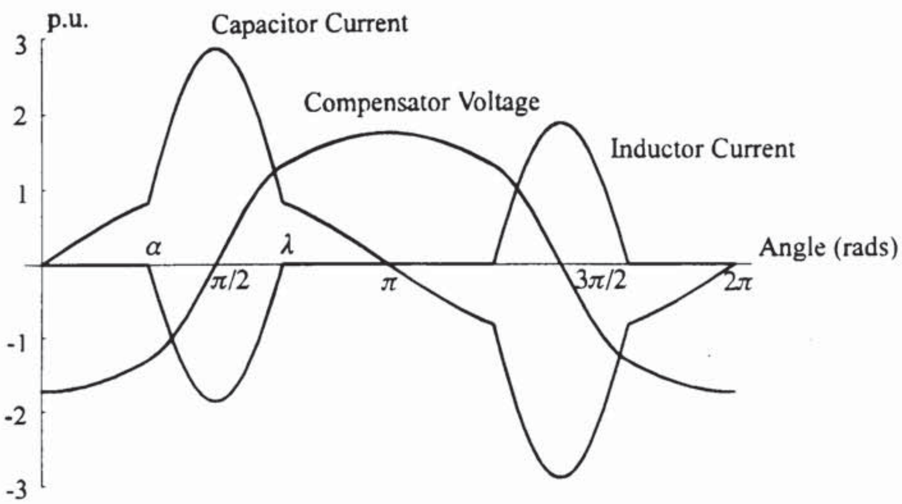

Figure 2.4 Graph of the Inductor and Capacitor Currents and the Resultant Compensator Voltage

If the inductor was allowed to conduct electricity for the whole cycle then the voltage across the compensator would be inductive. This is a valid scenario provided the resonance ratio (k) of the circuit is greater than unity, where:

$$k = \sqrt{\frac{1}{XY}} \tag{2.1}$$

If the thyristors were fired at $\theta = 0$, i.e. full conduction, the circuit becomes a capacitor and an inductor in parallel. The impedance of this condition is given by:

$$Z = \frac{jX}{1 - XY} = \frac{1}{jY}\left(\frac{1}{1 - k^2}\right) \tag{2.2}$$

From equation (2.1) it can be seen that if k is greater than 1 then the impedance of the circuit is inductive. Once an inductive voltage is produced across the thyristor controlled series capacitor it is possible to use the control of the thyristors to create a variable inductor. Increasing the firing angle α from zero up to the resonance angle α_o will increase the inductive voltage across the thyristor controlled series capacitor.

The resonance angle is the angle at which the voltage across the thyristor controlled series capacitor will become infinite. This is shown is figure 2.5 below.

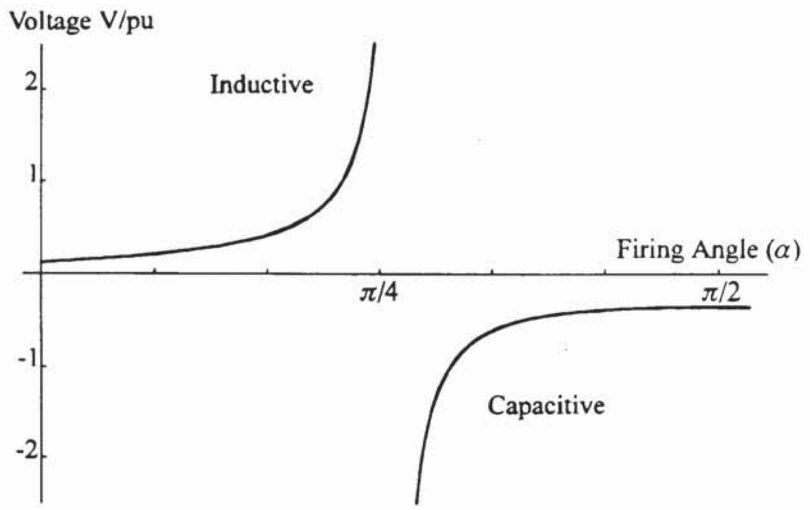

Figure 2.5 Graph Showing the Voltage across the Thyristor Controlled Series Compensator at the Resonance Angle α_o

Trying to change the circuit from capacitive mode to inductive mode by switching the firing angle from after the resonance angle to before the resonance angle and vice versa does not work and this is discussed later in this chapter.

2.3 Circuit Analysis

This section outlines the analysis conducted on the thyristor controlled series capacitor. The complete analysis is given in Appendix A.

For analysis of the thyristor controlled series capacitor the current flowing down the transmission line is taken to be a constant sinusoidal current source of peak magnitude I, giving the expression:

$$I_S = I \sin\theta \tag{2.3}$$

This assumption was made to make the analysis easier. It is not considered unreasonable provided the impedance of the compensator is small compared to the overall impedance of transmission line and its load. In practice the transmission system is based upon a constant voltage source with a load impedance. In addition the transmission line current may not be sinusoidal, however generally the amout of current harmonics should be very small and would not effect the compensator performance. These practicalities might have some impact on the analyses given in this thesis, but it not considered that there would be any significant changes to the results shown.

The cycle shown in figure 2.2 needs to be split into three eras and the voltages and currents calculated in each of them. The circuit diagrams for each era are shown in figures 2.6, 2.7 and 2.8 which can be found later in their respective sections. The first era is defined with the zero cross of the system current being the beginning of the era and the firing of the thyristors being the end of the era. The zero cross of the system current occurs at $\theta = 0$ and the firing of the thyristors is at $\theta = \alpha$. The second era is defined by the duration of the current through the inductor. Current is allowed to flow through the inductor from $\theta = \alpha$ and up to $\theta = \lambda$ where the current through the inductor naturally returns to zero. It should be noted that the angle at which the inductor current returns to zero is a consequence of the circuit conditions and not a control parameter like the firing angle α. The third era is defined by the return of the thyristors to their current blocking state at $\theta = \lambda$ as its start and the zero cross of the system current at $\theta = \pi$ as the end of the third era.

2.3.1 The First Era

In the first era the thyristors are not conducting so the inductor is not connected to the circuit as shown in figure 2.6.

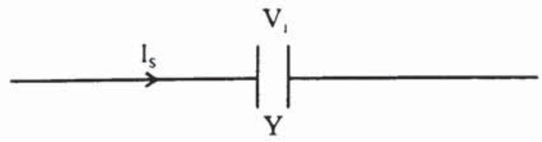

Figure 2.6 Circuit Diagram for the First Era

The circuit can be considered a simple series capacitor and the current through the capacitor must equal the system current and therefore the voltage across the compensator is given by the integral of the system current. The constant of integration will be solved later in the analysis.

$$I_C = I \sin\theta \tag{2.4}$$

$$V_1 = C_1 - \frac{I}{Y}\cos\theta \tag{2.5}$$

2.3.2 The Second Era

In the second era the thyristors are allowed to conduct and the circuit becomes a capacitor and an inductor which are connected in parallel as shown in figure 2.7.

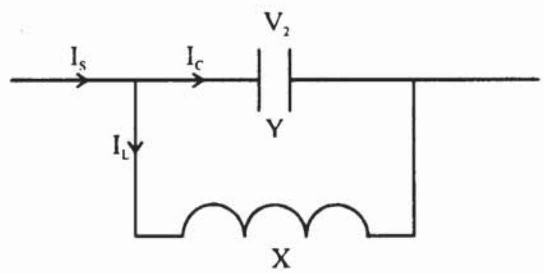

Figure 2.7 Circuit Diagram for the Second Era

Again Kirchoff's current law asserts that the summation of the currents through the capacitor and the inductor must equal the system current. These currents can be represented by the voltage across each component V_2.

$$I_L = \frac{1}{X}\int V_2\, d\theta \tag{2.6}$$

$$I_C = Y\frac{dV_2}{d\theta} \tag{2.7}$$

This leaves a second order differential equation to be solved, where k is the resonance ratio and is defined by equation (2.1).

$$\frac{d^2V_2}{d\theta^2} + k^2V_2 = \frac{I}{Y}\cos\theta \tag{2.8}$$

The general solution to this type of differential equation (2.8) takes the following form.

$$V_2 = \frac{I}{Y}\left(a_1\cos k\theta + a_2\sin k\theta + \frac{\cos\theta}{k^2-1}\right) \tag{2.9}$$

By using the expression for the voltage across the compensator, equation (2.9) to obtain the current through the inductor and by applying the fact that this current is defined to be zero at the beginning and end of the era (α and λ), allows the constants a_1 and a_2 to be found in terms of the control angle α and the, as yet unknown, angle λ.

The voltage across the compensator and the currents through the capacitor and inductor are now given by the following equations.

$$V_2 = \frac{I}{Y}\left\{\frac{k[\sin\alpha\cos k(\theta-\lambda) - \sin\lambda\cos k(\theta-\alpha)]}{(k^2-1)\sin k(\lambda-\alpha)} + \frac{\cos\theta}{k^2-1}\right\} \tag{2.10}$$

$$I_C = I\left\{\frac{k^2[\sin\lambda\sin k(\theta-\alpha) - \sin\alpha\sin k(\theta-\lambda)]}{(k^2-1)\sin k(\lambda-\alpha)} - \frac{\sin\theta}{k^2-1}\right\} \tag{2.11}$$

$$I_L = I\left\{\frac{k^2[\sin\alpha\sin k(\theta-\lambda) - \sin\lambda\sin k(\theta-\alpha)]}{(k^2-1)\sin k(\lambda-\alpha)} + \frac{k^2\sin\theta}{k^2-1}\right\} \tag{2.12}$$

2.3.3 The Third Era

When $\theta = \lambda$ the current through the inductor becomes zero and once again the thyristors block the flow of current. The inductor no longer plays any part in the analysis as shown in figure 2.8.

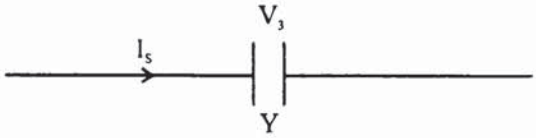

Figure 2.8 Circuit Diagram for the Third Era

The current through the capacitor is once again equal to the system current and the voltage across the compensator is given by the integral of this current.

$$V_3 = C_3 - \frac{I}{Y}\cos\theta \tag{2.13}$$

2.3.4 Eliminating the Unknown Constants

The unknown constants can be found by using the continuity of the capacitor voltage at the boundary of eras 1 and 2 and at the boundary of eras 2 and 3. In addition the half wave anti-symmetry of the voltage at π is also used.

The voltage across a capacitor must be continuous and this allows the constants of integration C_1 and C_3 to be determined. Therefore at the boundary point of the first and second eras $\theta = \alpha$, the expressions for the voltage across the compensator for the first era and the voltage across the compensator for the second era can be equated. The constant C_1 is then expressed in terms of α and λ.

$$C_1 = \frac{I}{Y}\frac{k}{(k^2-1)}\left\{\frac{\sin\alpha\cos k(\lambda-\alpha) + k\cos\alpha\sin k(\lambda-\alpha) - \sin\lambda}{\sin k(\lambda-\alpha)}\right\} \tag{2.14}$$

The same condition also applies to the voltages of the second and third eras which provide an equivalent expression for the constant C_3.

$$C_3 = \frac{I}{Y}\frac{k}{(k^2-1)}\left\{\frac{\sin\alpha - \sin\lambda\cos k(\lambda-\alpha) + k\cos\lambda\sin k(\lambda-\alpha)}{\sin k(\lambda-\alpha)}\right\} \tag{2.15}$$

In addition to this, the voltage across the compensator is half-wave anti-symmetric in the steady state. So at $\theta = \pi$ the voltage of the first era must be the negative of the voltage of the third era. This equation produces the relationship between the constants of integration C_1 and C_3. The relationship being that one is the negative of the other. It is now possible to obtain an equation for the final unknown quantity, namely the angle λ at which conduction of the thyristors ceases.

$$\cos\left(\frac{\lambda+\alpha}{2}\right)\cos k\left(\frac{\lambda-\alpha}{2}\right)\left\{k\cos\left(\frac{\lambda-\alpha}{2}\right)\sin k\left(\frac{\lambda-\alpha}{2}\right) - \sin\left(\frac{\lambda-\alpha}{2}\right)\cos k\left(\frac{\lambda-\alpha}{2}\right)\right\} = 0 \tag{2.16}$$

The solutions to this equation, with one exception, depend upon the resonance ratio k and the higher the resonance ratio the more solutions there are.

(a) The First Solution

The first solution is the exception which does not depend on the resonance ratio and is obtained by equating the first cosine component of the above expression, equation (2.16), to zero. This solution is given by equation (2.17).

$$\lambda = \pi - \alpha \tag{2.17}$$

This solution is the one which is always given in publications on the thyristor controlled series capacitor and it is the only solution which gives the thyristor controlled series capacitor its useful operational characteristics.

(b) The Second Solution

The second solution is derived by equating the second cosine component of the equation to zero. The solution is given in equation (2.18).

$$\lambda = \frac{n\pi}{k} + \alpha \tag{2.18}$$

Where the integer n is an odd number. Clearly this solution *is* dependant upon the resonance ratio. For a resonance ratio less than one there will be no solution obtained because the thyristors would have to allow conduction for greater than π radians and this would lead to the impossible situation of trying to switch the thyristors on before they had switched off. If the resonance ratio was equal to one then conduction would continue for π radians and it would be equivalent to a parallel capacitor and inductor. This therefore is not a sensible solution. For values of the resonance ratio between one and three there will be just the one solution to this equation.

(c) The Transcendental Function

The third component of equation (2.16) can be rearranged for convenience into the following form.

$$k \tan k\frac{\delta}{2} - \tan\frac{\delta}{2} = 0 \tag{2.19}$$

Where

$$\delta = \lambda - \alpha \tag{2.20}$$

This is a transcendental equation and the easiest method of visualising the solutions to this equation is by plotting it for various values of the resonance ratio. Where the transcendental function becomes zero indicates that there is an additional solution the angle at which the inductor current returns to zero.

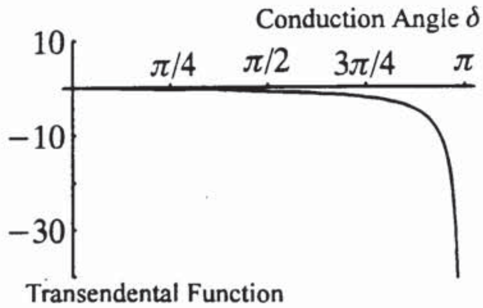

Figure 2.9 Graph of the Transcendental Function for $k = 0.5$

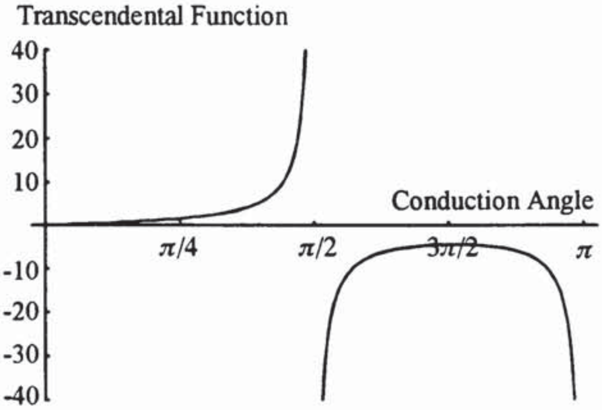

Figure 2.10 Graph of the Transcendental Function for $k = 2.0$

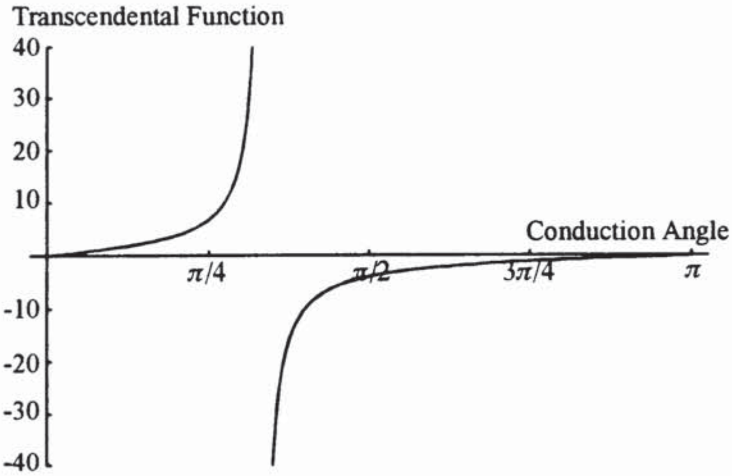

Figure 2.11 Graph of the Transcendental Function for $k = 3.0$

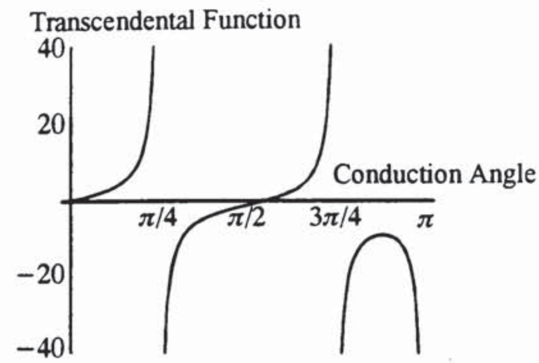

Figure 2.12 Graph of the Transcendental Function for $k = 4.0$

The graphs show that for values of k less than three there are no solutions to the transcendental equation for $\delta < \pi$.

2.3.5 Choosing the Operating State

Keeping the compensator in one stable operating condition is easier if there are fewer conditions to choose from. This means that the value of the resonance ratio needs to be less than or equal to three. This ensures no solutions to the transcendental equation (2.18), and only two stable states are possible.

If the resonance ratio was less than unity the circuit would be always capacitive, regardless of the firing angle of the thyristors. Indeed, once the thyristors are fired and current allowed to flow through the inverter, the current would not return to zero in time for the thyristors to be fired half a cycle later. This would be equivalent to a series capacitor with an inductor connected in parallel.

When one examines the voltage across the compensator and the currents flowing through both the capacitor and the inductor for the operating condition of $\lambda = \pi/k + \alpha$ it becomes obvious that this is not a useful operating state for the compensator. If there were no resistance present (as has been assumed in the analysis) in the circuit then an infinite voltage would be dropped across the capacitor. When there is resistance present the compensator operates with very high losses.

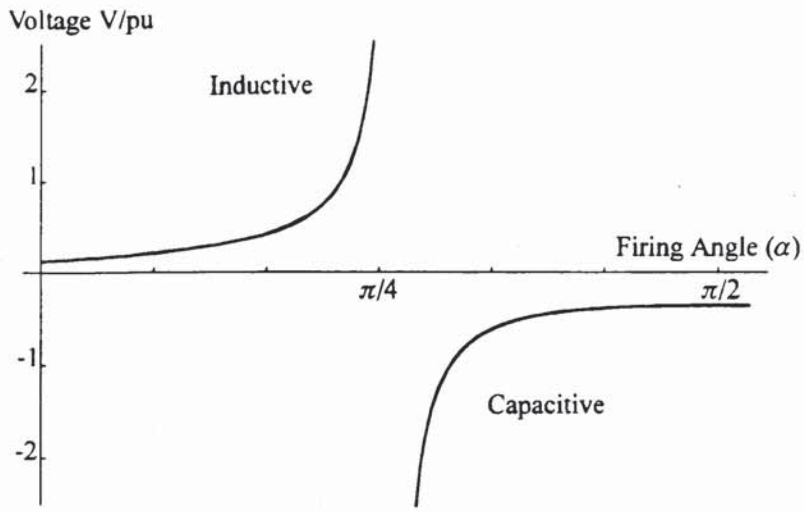

Figure 2.13 Graph Showing the Magnitude of the Compensator Voltage against Thyristor Firing Angle α

This leaves just the solution of $\lambda = \pi - \alpha$. The voltages dropped across the capacitor will be either capacitive or inductive. Figure 2.13 shows how the magnitude of the voltage across the compensator will vary with the firing angle of the thyristors. It shows that as the resonance angle α_o of the compensator is approached from the angle $\pi/2$ the voltage across the thyristor controlled series capacitor is capacitive and increases significantly at the resonance point. If the thyristor firing angle is increased from zero the voltage across the thyristor controlled series capacitor is inductive and again the magnitude increases as the resonance point is reached.

This is the best operating state for the thyristor controlled series capacitor and is the only condition ever mentioned in published literature.

In practice the voltage across the capacitor of the thyristor controlled series capacitor will not remain in a single stable operating state for all firing angles. This is because, with one exception, the firing angle of the thyristors always occurs when the voltage across the capacitor is non-zero. If one assumes the first solution for the thyristor controlled series capacitor (i.e. $\lambda=\pi-\alpha$), the analysis suggests that as the firing angle crosses the resonance angle α_o, the voltage across the capacitor changes phase from inductive to capacitive. The fact that the voltage across a capacitor can not be changed instantaneously ensures that the thyristor controlled series capacitor will not change from capacitive behaviour to inductive behaviour by passing through the resonance angle α_o. In the situation where the firing angle α passes from one side of the resonance angle α_o to the other, the voltage will rise to very large magnitudes (infinity in a circuit with no resistance) and then the phase angle of the voltage changes to become extremely lossy.

It is possible to operate the TCSC in inductive mode by allowing full conduction (180°) and then increase the firing angle α from zero up to the resonance angle α_o. Again increasing the angle past the resonance angle α_o will force the TCSC to become very lossy.

2.4 Simulation of the Thyristor Controlled Series Capacitor

It has already been stated that it was simulations of the thyristor controlled series capacitor on a computer that first suggested that the thyristor controlled series capacitor was not always well behaved. The simulations involved using templates of components to create electrical circuits. There are many ready made templates for components in the software package (Saber) and these describe the components in mathematical formulae. The circuit is then represented by multiple equations, and the simulator solves these equations numerically.

Simulations of the thyristor controlled series capacitor showed that the voltage across the compensator did not flip from capacitive to inductive at the resonance point, as one would be inclined to expect from the published literature.

If one assumes the operating condition where the angle at which the thyristors return to blocking current λ, is defined as π - α. The proportion of the cycle that the inductor has current flowing through it increases as the firing angle of the thyristors is decreased from $\pi/2$. Beyond the resonance point the voltage across the compensator should flip to become inductive but the conduction time of the thyristors should keep increasing. Figure 2.14 shows this as does figure 2.16.

By examining the figure 2.15 it can be seen that at the resonance point the current through the inductor does not change direction and the conduction time of the thyristors remains constant but is phase shifted each time the firing angle α is changed.

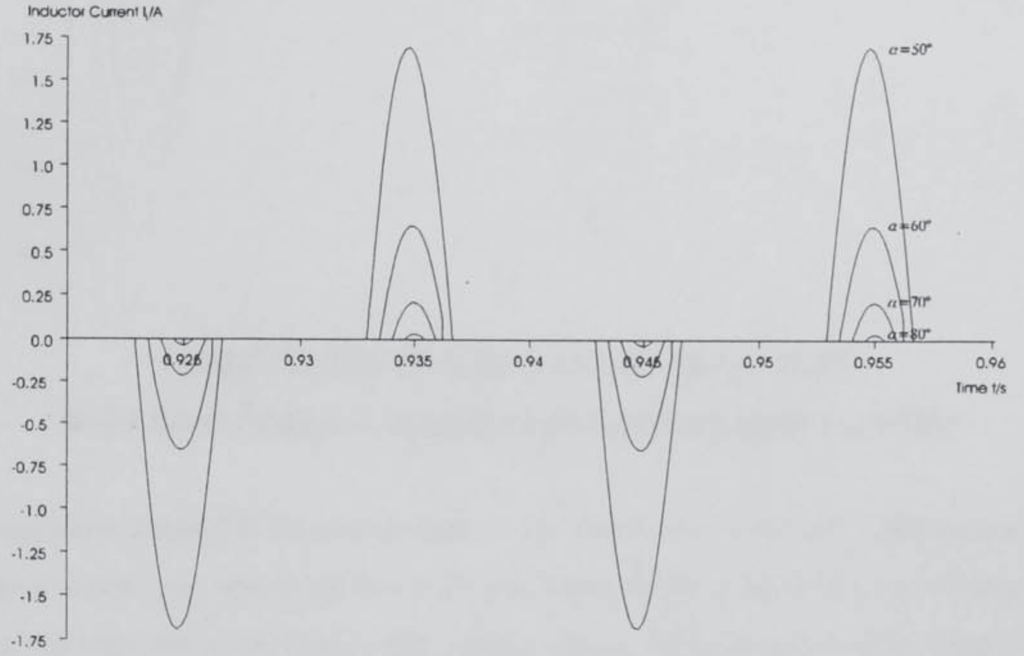

Figure 2.14 Simulation Showing the Inductor Current as the Firing Angle α is varied from $\pi/2$ almost to the Resonance Angle α_o

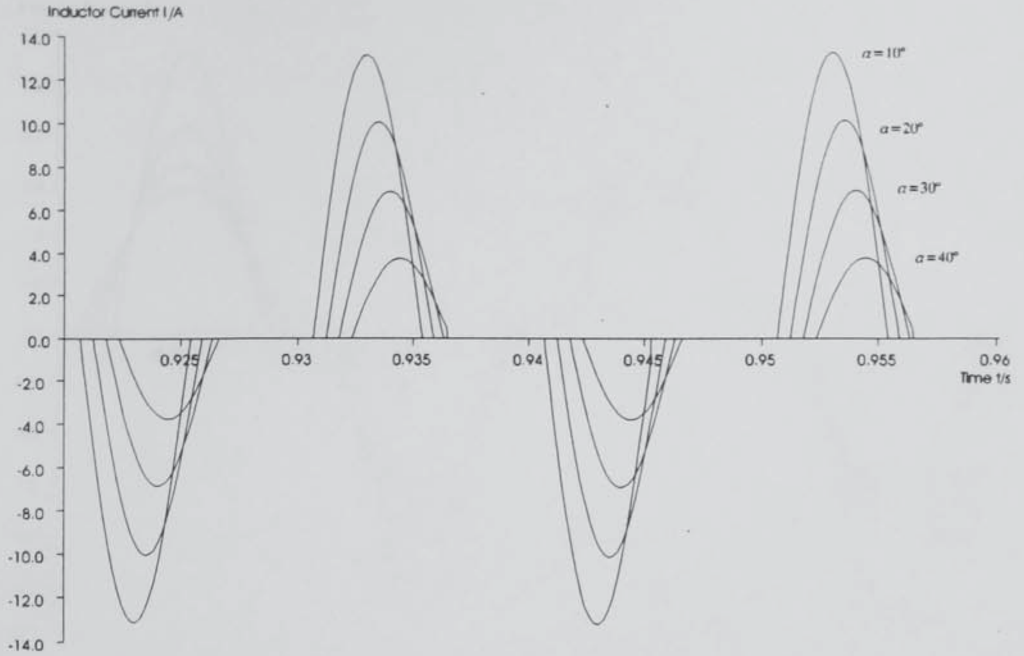

Figure 2.15 Simulation Showing the Inductor Current as the Firing Angle α is varied from the Resonance Angle α_0 to Zero

The resonance angle for the circuit used in the simulations was 45°. Unfortunately the simulator would only approach this angle and was unable to solve the circuit equations for the resonant condition. One of the reasons using a constant sinusoidal current source was to simplify the mathematical analysis and this assumption was also used during the simulations because computer hardware limitations would not produce satisfactory simulations using a constant voltage source with a load impedance.

Varying the firing angle from 0 through to π produces a similar result (figures 2.16 and 2.17) but the inductor current is the negative of that shown in figures 2.14 and 2.15.

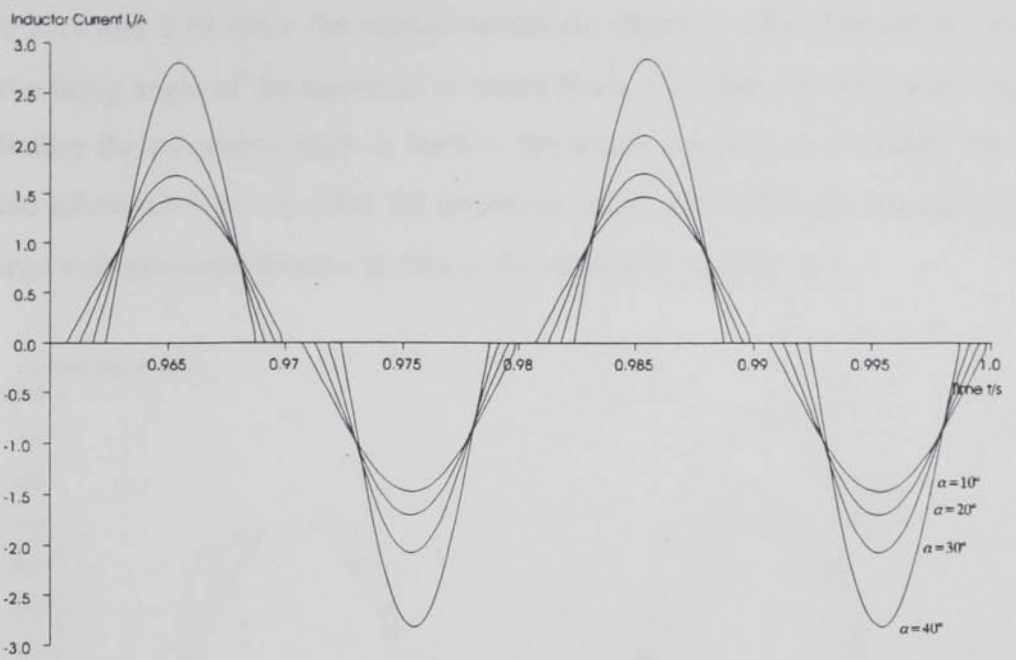

Figure 2.16 Simulation Showing the Inductor Current as the Firing Angle α is varied from 0 to α_o

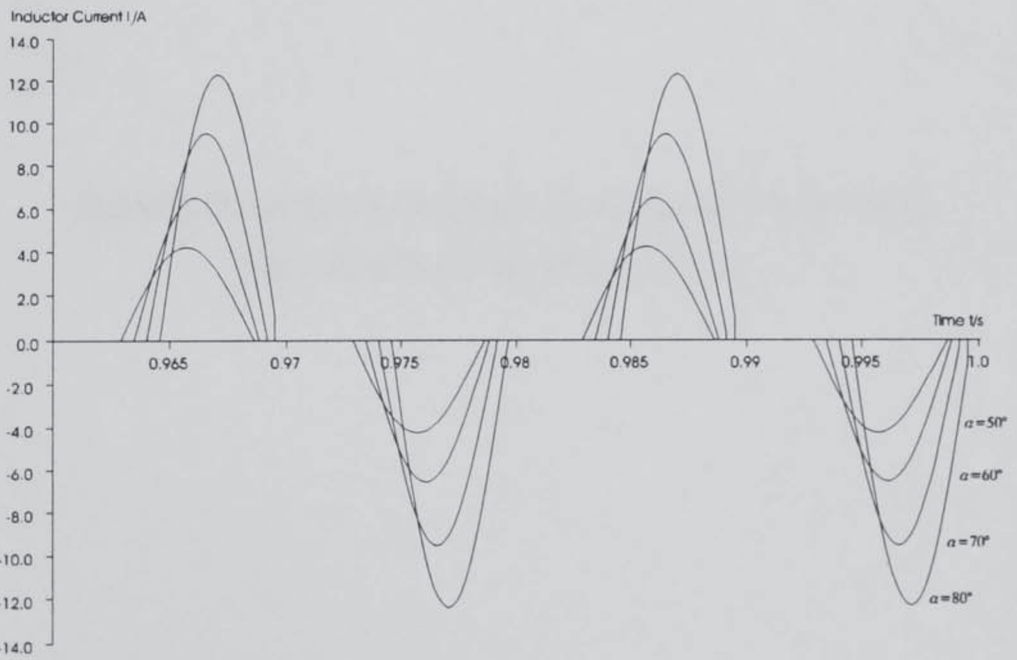

Figure 2.17 Simulation Showing the Inductor Current as the Firing Angle α is varied from α_o to π

Figures 2.18 and 2.19 show the voltage across the thyristor controlled series capacitor when the firing angle of the thyristors is varied from π through the resonance angle, to zero. Before the resonance angle is reached the circuit operates as a variable capacitor using the solution $\lambda = \pi - \alpha$. After the resonance angle is exceeded the voltage becomes very large with increased losses and adopts the solution $\lambda = n\pi/k + \alpha$.

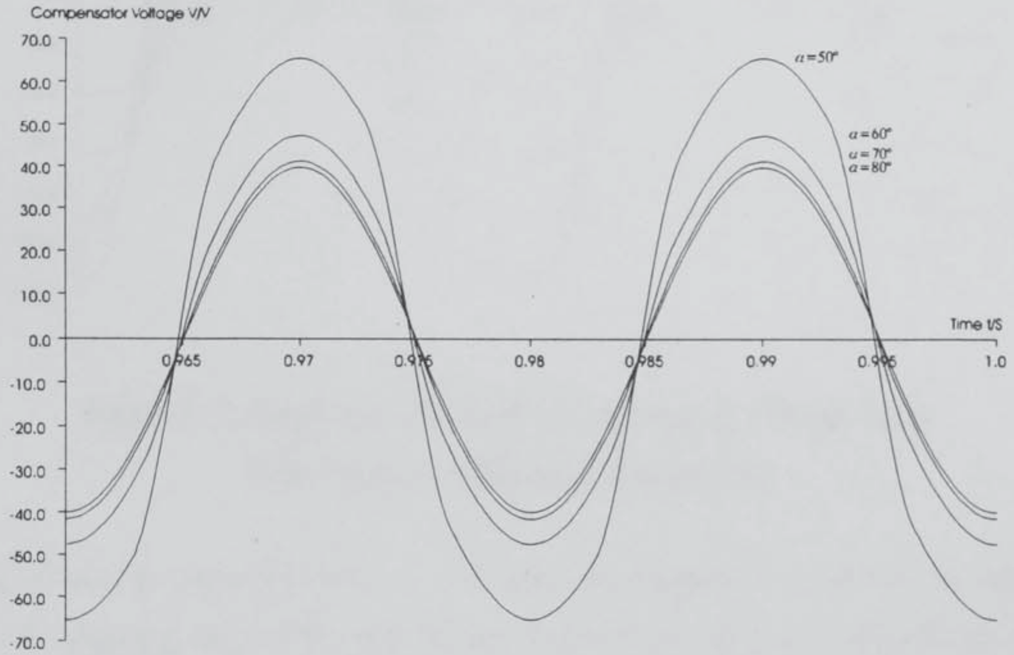

Figure 2.18 Simulation Showing the Compensator Voltage as the Firing Angle α is varied from π to the α_o

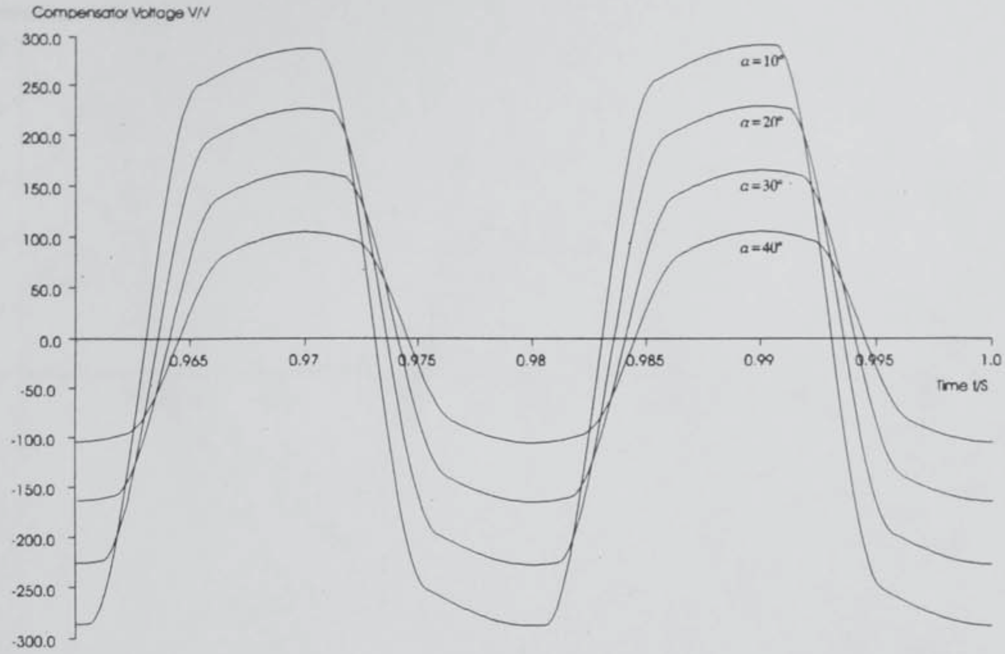

Figure 2.19 Simulation Showing the Compensator Voltage as the Firing Angle α is varied from α_o to Zero

It was possible to make the thyristor controlled series capacitor operate in its inductive mode by allowing full conduction of the thyristors so that the voltage across the compensator was inductive. Then firing the thyristors at an angle before the resonance point. Again if the firing was conducted after the resonance point the thyristor controlled series capacitor changes operation from one state to another. This is shown in figure 2.20.

Figure 2.20 Simulation Showing the Compensator Voltage as the Firing Angle varied from Zero to a_o

Figure 2.21 shows that the thyristor controlled series capacitor voltage changes to the solution $\lambda = n\pi/k + \alpha$ when the firing angle α occurs after the resonance angle when the voltage was initially inductive.

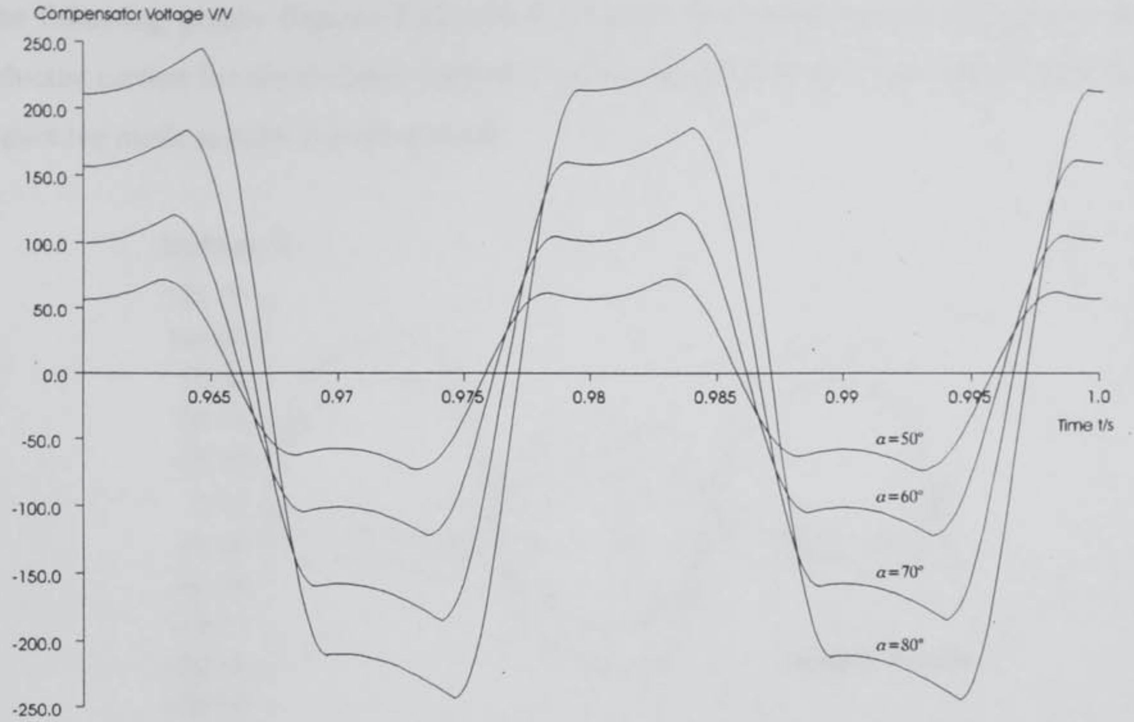

Figure 2.21 Simulation Showing the Compensator Voltage as the Firing Angle varied from a_o to π

As explained earlier the thyristor controlled series capacitor is unable to make the transition from capacitive mode to inductive mode because the switching of the thyristors always occurs whilst there is a voltage across the capacitor.

2.5 A Small Scale Physical Model

In order to develop a greater confidence in the analysis presented earlier it was deemed useful to construct a small scale physical model of the thyristor controlled series capacitor. The model was a simple circuit of a capacitor and an inductor controlled by a pair of back to back thyristors. The supply for the thyristor controlled series capacitor model was provided by the single phase 230V mains. Control of the thyristors was accomplished using an EPROM to provide the switching pattern, with ultimate control being the operator of a dash-pot.

The following graphs (figures 2.22 and 2.23) show the compensator voltage and the inductor current for the thyristor controlled series capacitor model operating in both the capacitive mode and the inductive mode.

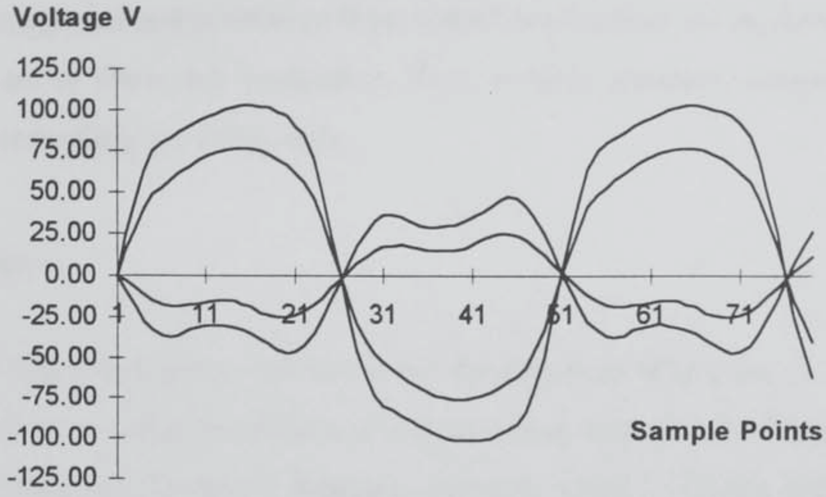

Figure 2.22 Graph of the Compensator Voltage

Operating in Capacitive and Inductive Mode

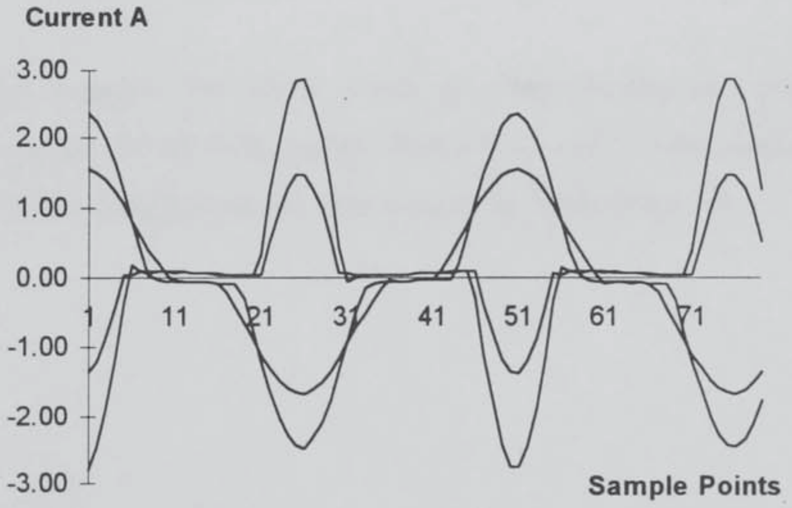

Figure 2.23 Graph of the Inductor Current

Operating in Capacitive and Inductive Mode

These graphs were chosen as an illustration of the thyristor controlled series capacitor operating in its most useful role. The model did not change from capacitive to inductive mode just by varying the firing angle of the thyristors, for reasons outlined in earlier sections. The model of the thyristor controlled series capacitor behaved as the theory given earlier suggested and in order to force inductive behaviour the thyristors needed to be switched on to allow full conduction. Then variable inductive compensation was produced by controlling the firing angle.

2.6 Discussion

The thyristor controlled series capacitor is not the simple compensation circuit that it at first appears to be. It certainly will behave in a capacitive mode and an inductive mode as both the literature and the theory suggests. However trying to change from capacitive mode to inductive mode instantaneously forces the thyristor controlled series capacitor from one stable solution $\lambda = \pi - \alpha$ to another stable solution $\lambda = n\pi/k + \alpha$. A graph of the voltage across the capacitor is shown below (figure 2.24), for the solution of the thyristor switch off angle $\lambda = \pi - \alpha$.

The mathematics suggests that this is a state in which the thyristor controlled series capacitor will operate for all firing angles. However in practice this forces the voltage across a capacitor to change from one value to another instantaneously.

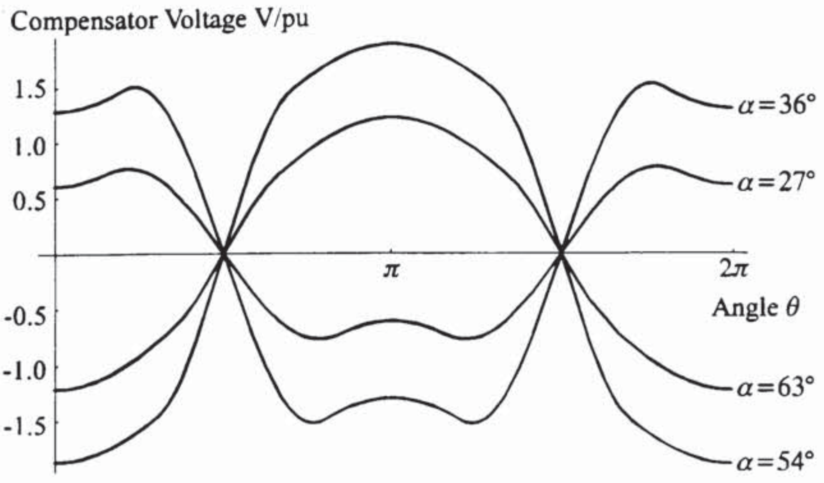

Figure 2.24 Graph Showing the Compensator Voltage for Several Firing Angles using the Condition $\lambda = \pi - \alpha$

The inability of the thyristor controlled series capacitor to switch from capacitive mode to inductive mode by crossing the resonance angle α_o forces the circuit to switch from one operating state where $\lambda = \pi-\alpha$ to another operating state where $\lambda = n\pi/k + \alpha$. This solution introduces large losses.

Voltage harmonics are a considerable problem for the thyristor controlled series capacitor. The third harmonic can be as high as 12% for some thyristor firing angles and the fifth harmonic can be as high as 1.5% (Helbing and Karady 1993). These figures are in line with simulations results which are shown in figure 2.25 below.

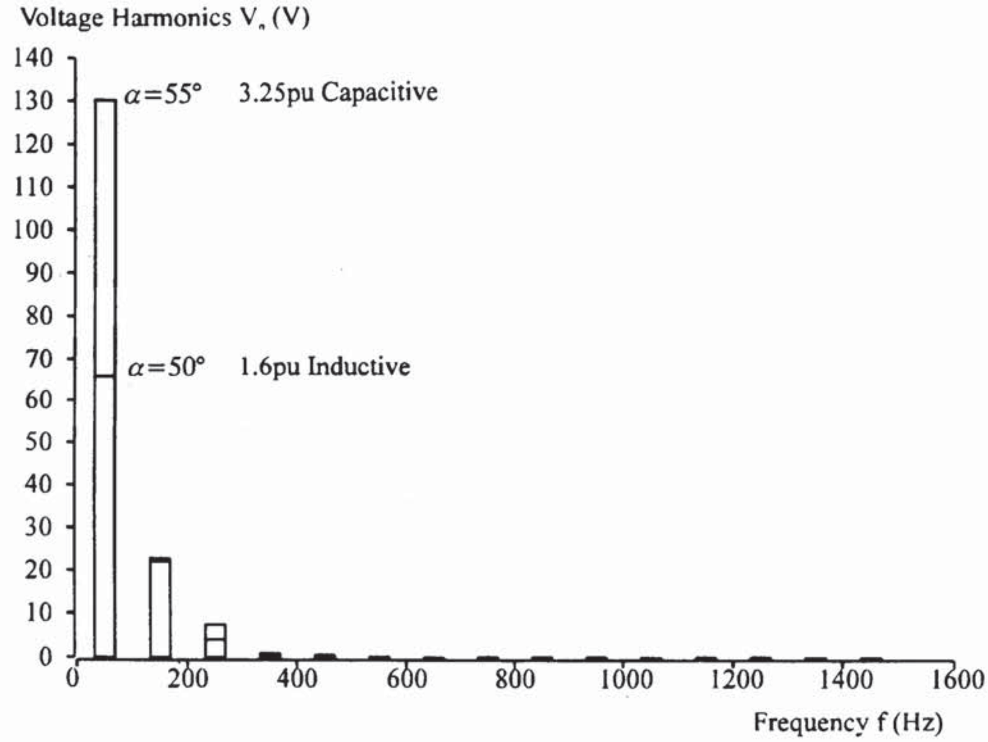

Figure 2.25 Simulation Graph Showing the Magnitude of the Voltage Harmonics

2.7 Conclusion

The thyristor controlled series capacitor will provide both capacitive and inductive compensation. The degree of compensation is continuously variable and controlled by a thyristor switched reactor.

There is more than one steady state solution to the system equations and care must be taken when choosing the compensator parameters (namely k) in order to minimise the number of solutions.

Practical experience has shown that it is impossible to change from capacitive mode to inductive mode quickly. The voltage across the thyristor controlled series capacitor must be forced into the correct phase first. For capacitive operation, this is done by taking the inductor out of the compensation circuit, allowing a capacitive voltage to build up. For inductive operation, the inductor must be allowed to remain continuously in the compensation circuit. Again this enables an inductive voltage to build up across the compensator.

Switching across the resonance angle without creating the correct voltage across the thyristor controlled series capacitor causes the circuit to switch from one solution ($\lambda = \pi - \alpha$) to another ($\lambda = n\pi/k + \alpha$). This second solution introduces very high voltages and increased losses.

Chapter 3

Single Phase Voltage Sourced Inverter Series Compensator Using a Capacitor as the Series Element

3.1 Introduction

The voltage sourced inverter series compensator is essentially a Statcon connected across a series capacitor as shown below (figure 3.1). The basic purpose of the equipment is to provide a variable voltage across the a.c. capacitor, by injecting a current through the series capacitor in addition to the phase current.

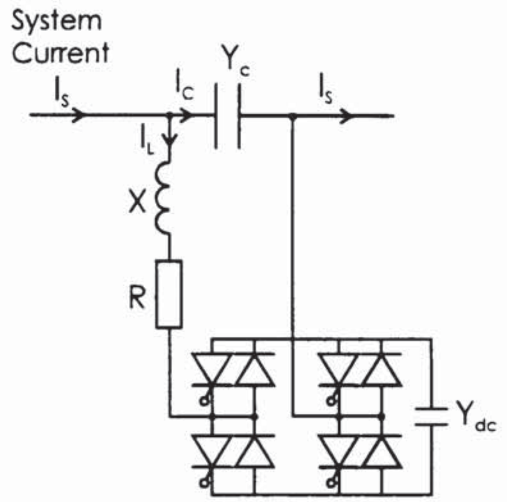

Figure 3.1 Circuit Diagram of the Single Phase Voltage Sourced Inverter Series Compensator

It will be shown that the single phase voltage sourced inverter can be used to vary the voltage across a series capacitor and that this voltage is essentially capacitive. The fact that control is obtained using power electronic switches means that the voltage sourced inverter series compensator is suitable for the control of transient changes in the flow of electricity.

3.2 Circuit Description

The series compensation circuit shown by (figure 3.1) consists of an a.c. capacitor connected in series with the transmission line. There needs to be inductance present in the circuit and this could be in the form of a simple reactor, or it could be due to the leakage reactance of a series transformer connecting the compensator to the transmission line. Without this inductance the circuit would not be a second order differential system and the effect of the firing angle of the inverter would not produce the desired compensating action. The resistance shown in the circuit diagram is a representation of any losses in the circuit, i.e. the switching losses and the losses in the inductor. A voltage sourced inverter connects the d.c. capacitor to the a.c. side of the circuit.

Varying the firing of the electronic switches by a few degrees either side of the reference angle will vary the magnitude of the voltage across the d.c. capacitor. The reference firing angle is defined as zero when the voltage across the a.c. capacitor crosses zero going from positive to negative.

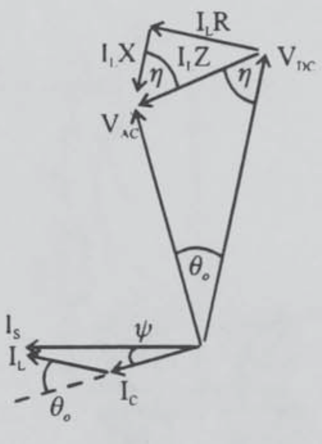

Figure 3.2 Phasor Diagram of the Action of the Firing Angle on the Voltage Sourced Inverter Series Compensator

Provided the phase difference between the angle at which the electronic switches are fired and the reference voltage (V_{ac}) is not too large, then the active power dissipated in the circuit will be significantly lower than the reactive power produced. The variation of the voltage across the d.c. capacitor will be quite large in comparison.

This difference between the phase angle of the a.c. capacitor voltage and the inverter voltage has been called the firing angle and it is denoted by θ_o. Control of this angle is fundamental to the operation of the compensation circuit. When the firing angle is zero, the a.c. capacitor voltage is out of phase with the d.c. capacitor voltage by 180°. The magnitude of the voltage across the a.c. terminals of the inverter is approximately equal to the magnitude of the voltage across the a.c. capacitor. The difference being attributed to the resistance of the compensator and any inductance in the circuit.

When there is a positive firing angle the voltage across the a.c. terminals of the inverter increases to become larger than the voltage across the a.c. capacitor, this forces current to flow in an anti-clockwise direction around the compensator as shown below (figure 3.3). This reduces the current flowing through the a.c. capacitor and causes a drop in the voltage across the a.c. capacitor.

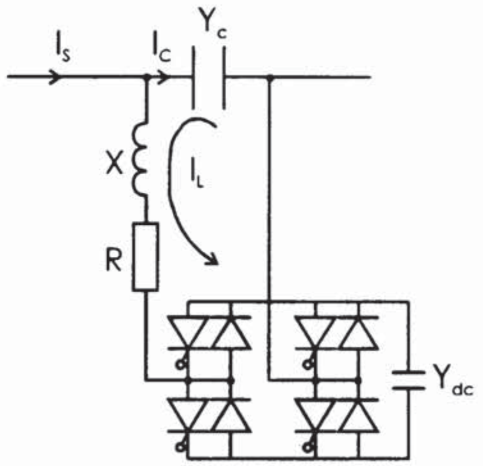

Figure 3.3 Diagram of the Circuit Acting to Decrease the A.C. Capacitor Voltage

When a negative firing angle is used the voltage across the d.c. capacitor is reduced and becomes smaller than the voltage across the a.c. capacitor. This forces current in a clockwise direction as shown below (figure 3.4). Correspondingly, the current through the a.c. capacitor is increased and the voltage across the a.c. capacitor is also increased.

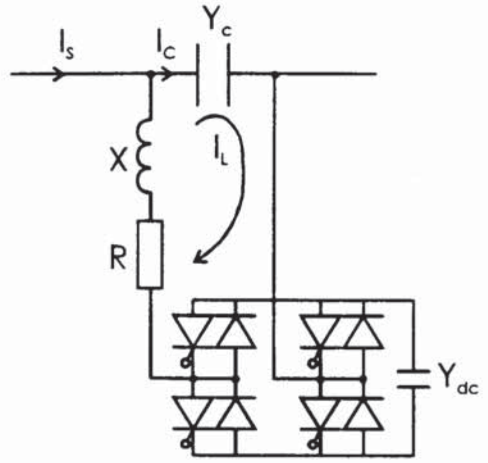

Figure 3.4 Diagram of the Circuit Acting to Increase the A.C. Capacitor Voltage

The firing of the switches in the inverter can be controlled accurately. The high switching frequency provided by these devices means the speed of response of the circuit to variations in system voltage could be quick.

3.3 Circuit Analysis

Analysis of the compensator is made easier because there is half-wave anti-symmetry about $\theta = n\pi$. This allows the circuit to be analysed for one half of an electrical cycle. This symmetry allows a much easier circuit to be drawn for the compensator, as is shown below (figure 3.5).

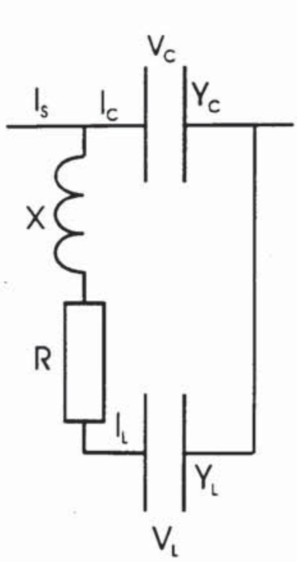

Figure 3.5 Simplified Circuit Diagram of a
Single Phase Voltage Sourced Inverter Series Compensator

The phase current is defined by the following equation.

$$I_S = I \sin\theta \tag{3.1}$$

The circuit diagram depicted above (figure 3.5) shows that for the first half of the cycle the circuit is relatively simple and one for which all the voltages and currents can be found by solving the following second order differential equation. The mathematics involved are documented in Appendix B.

$$\frac{d^2 I_L}{d\phi^2} + 2\beta\frac{dI_L}{d\phi} + (k^2 + \beta^2)I_L = Ir(k^2 + \beta^2)\cos(\phi + \theta_o) \tag{3.2}$$

For this type of second order differential equation the solution is given by the following equation (3.3).

$$I_L = Ir(k^2 + \beta^2)\{a_1 \cos(\phi + \theta_o) + a_2 \sin(\phi + \theta_o) + a_3 e^{-\beta\phi} \cos k\phi + a_4 e^{-\beta\phi} \sin k\phi\} \tag{3.3}$$

The firing angle of the inverter θ_o determines the phase angle of the inverter ϕ. The phase difference between the voltage across the a.c. capacitor V_C and the voltage across the inverter V_L, is the inverter firing angle. As the voltage across the a.c. capacitor lags the supply current by $\pi/2$, the phase angle of the inverter can be related to the phase angle of the supply current θ by the following equation.

$$\phi = \theta - \theta_o - \pi/2 \tag{3.4}$$

The unknown constants a_1 and a_2 can be determined by substituting equation (3.3) back into the differential equation (3.2) and equating coefficients of sine and cosine. The constants a_3 and a_4 are found using the boundary conditions of the inverter current and the voltage across the d.c. capacitor when $\theta = 0$ and when $\theta = \pi$. This results in an expression for the inverter current shown by equation (3.5). All the other unknown voltages and currents in the circuit can then be determined from this equation.

$$\begin{aligned} I_L &= Ir(\beta^2 + k^2)\frac{\sin(\phi + \theta_o + \gamma)}{h} \\ &+ Ir\frac{(\beta^2 + k^2)^2 \cos(\theta_o + \gamma)}{he^{-\beta\pi}(k\sinh\beta\pi - \beta\sin k\pi)}(Ne^{-\beta\phi}\cos k\phi - Me^{-\beta\phi}\sin k\phi) \end{aligned} \tag{3.5}$$

This equation (3.5) for the inverter current is shown graphically in the diagram below (figure 3.6). It shows that the current through the inverter can be made to change direction by varying the firing angle about zero.

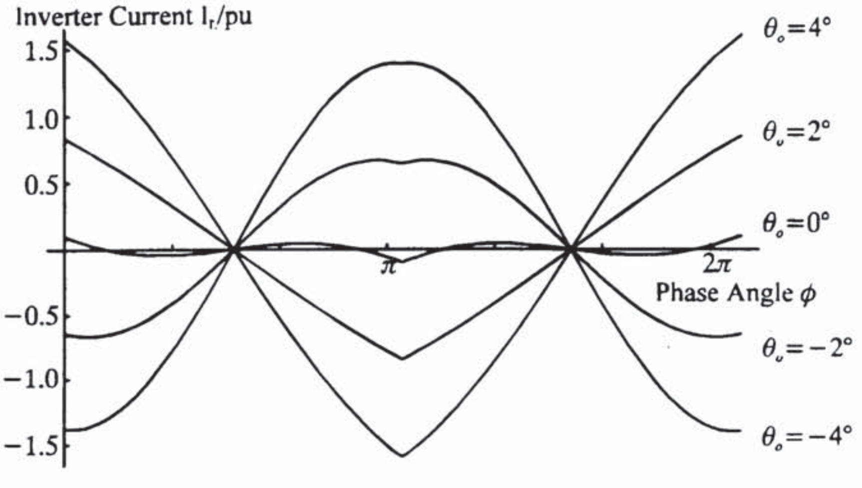

Figure 3.6 A Graph Showing the Response of the Inverter Current to Changes in the Firing Angle of the Electronic Switches

For all the time based graphs shown in this thesis the per unit quantities are based upon the following. The base current is the magnitude of the constant sinusoidal current source *I*, and the base voltage is the magnitude of the voltage dropped across the series element (capacitive or inductive) when no compensation is being used. For example with a capacitive series element for admittance *Y*, the base voltage would be *I*/*Y*.

The difference between the inverter current and the supply current gives the current through the a.c. capacitor. This gives rise to the following equation for the current through the a.c. capacitor.

$$\begin{aligned} I_C &= I\cos(\phi+\theta_o) - Ir(\beta^2+k^2)\frac{\sin(\phi+\theta_o+\gamma)}{h} \\ &\quad -Ir\frac{(\beta^2+k^2)^2\cos(\theta_o+\gamma)}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}(Ne^{-\beta\phi}\cos k\phi - Me^{-\beta\phi}\sin k\phi) \end{aligned} \tag{3.6}$$

This is shown by the following graph (figure 3.7). It can be seen that the current through the a.c. capacitor is the supply current minus the current flowing through the inverter and that it is almost entirely capacitive.

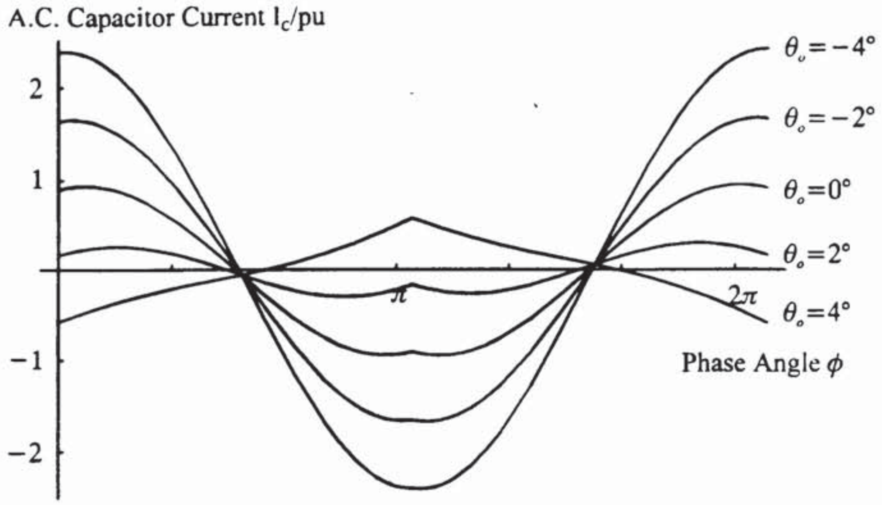

Figure 3.7 A Graph Showing the Current through the A.C. Capacitor for a Range of Firing Angles

The voltage across the d.c. capacitor is obtained by integrating the inverter current and dividing by the impedance of the capacitor. The equation for this voltage is shown by equation (3.7).below. It has been written in terms of the voltage across the a.c. capacitor.

$$V_L = \frac{I}{Y_C}(r-1)(\beta^2+k^2)\frac{\cos(\phi+\theta_o+\gamma)}{h} + \frac{I}{Y_C}(r-1)(\beta^2+k^2)\frac{\cos(\theta_o+\gamma)\{(\beta N - kM)e^{-\beta\phi}\cos k\phi - (\beta M + kN)e^{-\beta\phi}\sin k\phi\}}{he^{-\beta\pi}(k\sinh\beta\pi - \beta\sin k\pi)} \tag{3.7}$$

Plotting this equation (3.7) for several firing angles produces the graph shown below (figure 3.8).

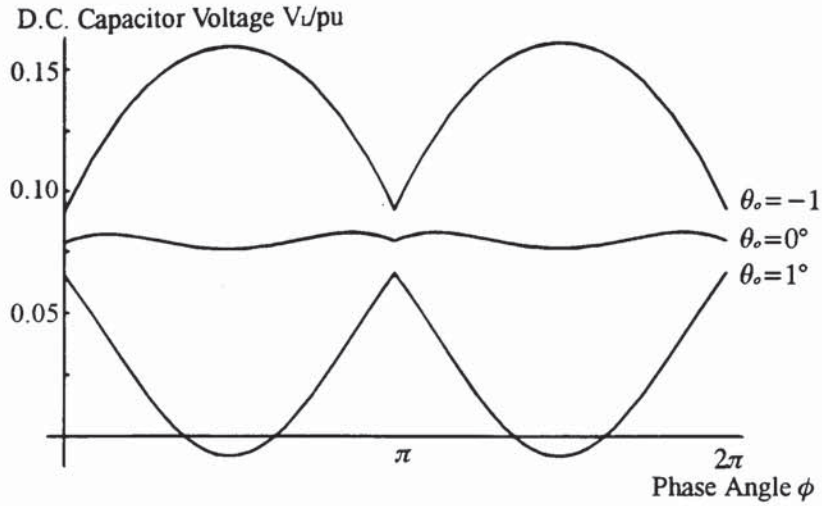

Figure 3.8 Graph Showing the D.C. Capacitor Voltage for Several Values of the Firing Angle

The above diagram (figure 3.8) shows the response of the d.c. capacitor voltage to the firing angle of the inverter. There is a change in the shape of the voltage from domes to trough when the firing angle of the inverter changes from a negative firing angle to a positive firing angle. It is this that causes the current through the inverter to change phase by 180°.

The voltage across the a.c. capacitor is of most interest from a compensation point of view and this is given below.

$$
\begin{aligned}
V_C = {} & \frac{I}{Y_C}\left\{\frac{rk(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)\cos(\theta_o+\gamma)}{h(k\sinh\beta\pi-\beta\sin k\pi)}\right\} \\
& +\frac{I}{Y_C}\left\{\sin(\phi+\theta_o)+r(\beta^2+k^2)\frac{\cos(\phi+\theta_o+\gamma)}{h}\right\} \\
& +\frac{I}{Y_C}\frac{r(\beta^2+k^2)\cos(\theta_o+\gamma)\{(\beta N-kM)e^{-\beta\phi}\cos k\phi-(kN+\beta M)e^{-\beta\phi}\sin k\phi\}}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}
\end{aligned}
\tag{3.8}
$$

This equation takes a lot of examination without providing much insight into what is actually occurring to the voltage across the a.c. capacitor when the firing angle of the inverter is varied. By plotting the equation as is shown below (figure 3.9) it is obvious that varying the firing angle of the inverter produces corresponding changes in the magnitude of the a.c. capacitor voltage. What is perhaps more surprising is the apparent lack of harmonic distortion.

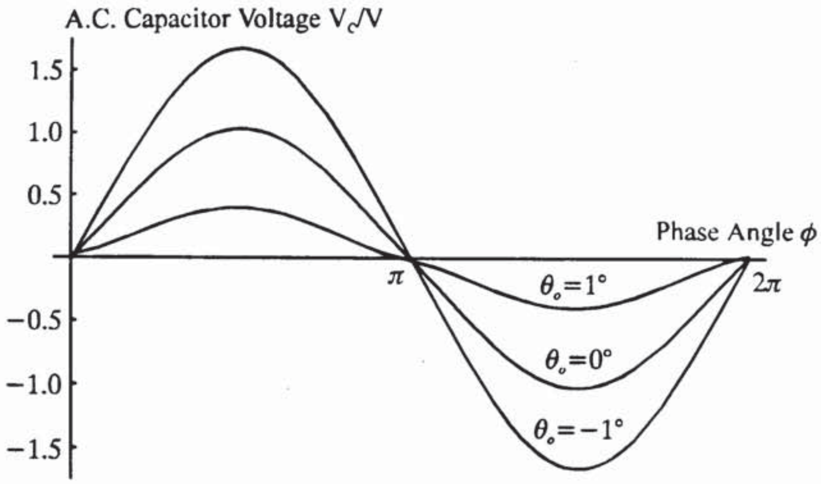

Figure 3.9 Graph Showing the A.C. Capacitor Voltage for Several Values of the Firing Angle

This graph shows that the magnitude of harmonics introduced into the compensating voltage is very small. In fact, visually the voltage across the a.c. capacitor appears to be sinusoidal. Inevitably there are harmonics present and these are examined later in this chapter.

Obviously harmonics are introduced to the system because of the electronic switches and it would be useful to know the magnitude of the fundamental component of this compensation voltage and the harmonics that are introduced. A fourier analysis was then conducted to determine the harmonic content of the a.c. capacitor voltage. The resulting equation is given below.

$$V_{CN} = \frac{I}{Y_C}\left\{\sin(\phi+\theta_0) + r(\beta^2+k^2)\frac{\cos(\phi+\theta_o+\gamma)}{h}\right\} - \frac{I}{Y_C} r(\beta^2+k^2)\frac{hq_2}{\beta}\cos(\theta_o+\gamma)\sum_{n=1}^{\infty}\frac{\cos(n\phi+\gamma_n)}{nh_n} \tag{3.7}$$

There are three further equations which can be developed from this harmonic dependant equation (3.7) for the voltage across the a.c. capacitor. These are the active and reactive components, the fundamental of the a.c. capacitor voltage, and the magnitude of the harmonics (third and higher). The firing angle of the inverter switches is the control variable for the compensator, so these three voltage components are arranged in terms of the firing angle. It has been found that the degree of compensation is linearly dependant upon a modified firing angle θ_{ob}. Where this modified firing angle is within the approximate range of $-1 \leq \theta_{ob} \leq 1$. The modified firing angle is the ratio of the firing angle of the inverter θ_o and the damping factor β.

$$\theta_{ob} = \frac{\theta_o}{\beta} \tag{3.8}$$

The quadrature component of the voltage across the a.c. capacitor is obviously of greatest interest because it is this value that provides the series compensation. The equation below gives a good approximation of the quadrature voltage V_q.

$$V_q = -1 + r(k^2+\beta^2)(q_1 + q_2\theta_{ob}) \tag{3.9}$$

It is readily apparent that the quadrature voltage across the a.c. capacitor is linearly dependant on the firing angle of the inverter switches. There is a basic capacitive voltage dropped across the a.c. capacitor and varying the firing angle of the inverter allows a variation of this basic capacitive voltage by approximately ±20%. The component of quadrature voltage is shown below (figure 3.10).

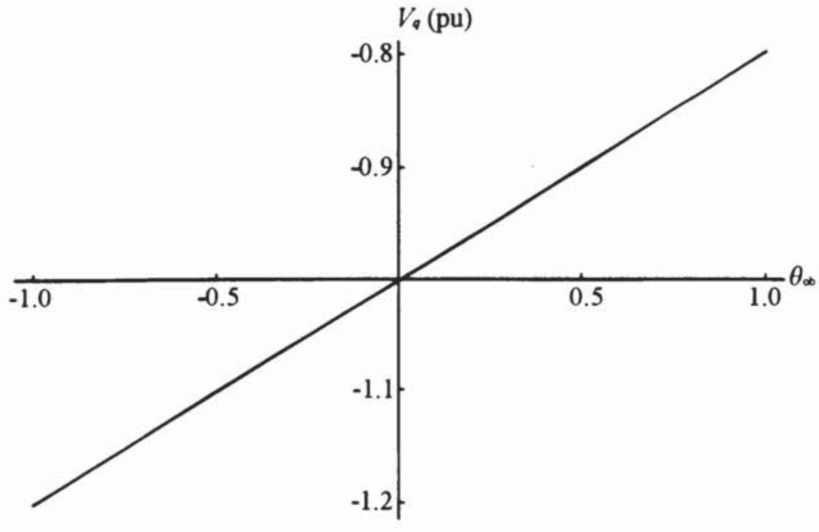

Figure 3.10 Graph Showing the Reactive Component of the A.C. Capacitor Voltage against the Modified Firing Angle

The active power absorbed by the compensation scheme is indicated by equation (3.10).

$$V_p = r\beta(k^2 + \beta^2)(p_1 + q_2\theta_{ob}^2) \tag{3.10}$$

The losses of the system depend upon the square of the modified firing angle of the inverter switches. Provided the modified firing angle is less than unity the reactive power component will be greater than the active power losses. The graph shown below (figure 3.11) depicts this relationship.

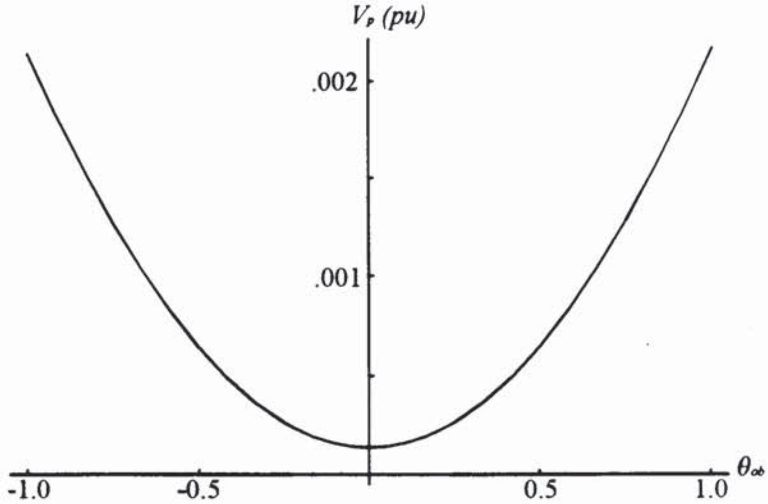

Figure 3.11 Graph Showing the Active Component of the A.C. Capacitor Voltage against the Modified Firing Angle

The magnitude of the harmonics greater than the third are defined by the following.

$$V_n = -r(k^2 + \beta^2)\frac{2q_2}{nh_n}(1 - \theta_o \tan\gamma) \tag{3.11}$$

The harmonic component of the voltage across the a.c. capacitor is shown graphically below (figure 3.12). The magnitude of the harmonics decrease very quickly from the third harmonic and can be considered negligible by the fifteenth harmonic. The magnitude of the harmonics is significantly effected by the reactance ratio k, especially for the lower harmonics.

This suggests that the reactance ratio should be as small as possible in order to keep the harmonics to a minimum.

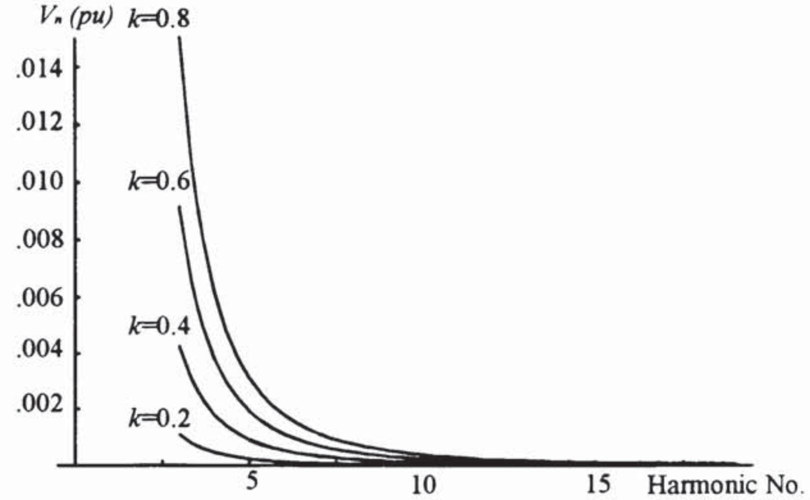

Figure 3.12 Graph Showing the Magnitude of the Harmonics of the A.C. Capacitor Voltage for Several Values of the Reactance Ratio (k)

3.4 Optimal Component Sizes

In order to choose the optimal component sizes for the series compensator several issues must be addressed. The size of the a.c. capacitor is fixed by the degree of compensation required, i.e. the magnitude of the capacitive voltage. Next the size of the d.c. capacitor needs to be decided.

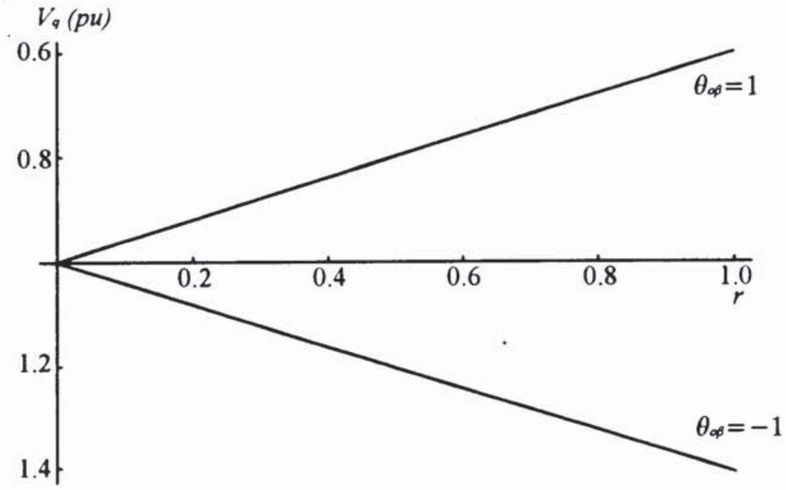

Figure 3.13 Graph Showing the Reactive Power Component of the Fundamental Voltage against the Capacitance Ratio

Figure 3.13 above, shows the response of the quadrature component of the a.c. capacitor voltage (V_q) to the capacitance ratio (r). The capacitance ratio is the ratio of the impedance of the a.c. capacitor to the total capacitive impedance. This capacitance ratio is zero when the impedance of the a.c. capacitor is zero and one when the d.c. capacitor is zero. Obviously, in practice, this ratio can never have the values of 1 or 0, but any value between.

The relationship is a linear one and the degree of compensation is at a maximum when the capacitance ratio approaches 1. This means that the d.c. capacitor is larger than the a.c. capacitor.

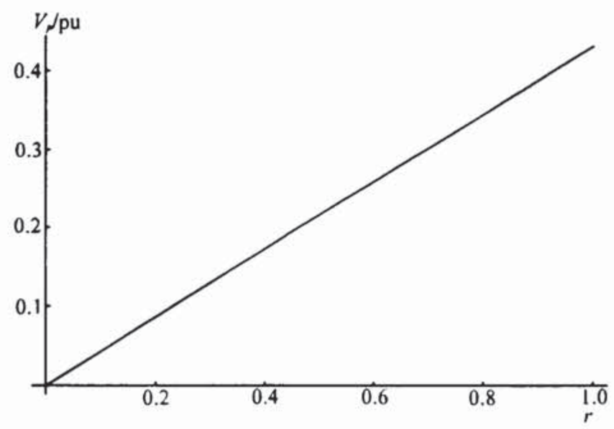

Figure 3.14 Graph Showing the Active Power Component of the Fundamental Voltage against the Capacitance Ratio

The graph above (figure 3.14) shows the response of the active power component of the fundamental component of the a.c. capacitor voltage (V_p) to the capacitance ratio (r). Again this is a linear relationship and the losses are at a maximum when the d.c. capacitor is much larger than the a.c. capacitor.

From the equation describing the quadrature voltage, the response of the voltage to the modified firing angle is dependant on the ratio of the a.c. capacitor to the total capacitance (r), the constant q_2 and $k^2+\beta^2$. The damping factor β, is the 1/2 the ratio between the resistance in the circuit and the inductive reactance of the compensator and the reactance ratio k, is the ratio of capacitive reactance to the inductive reactance. The damping factor is going to be as small as possible with 0.01 being chosen as a reasonable value. The constant q_2 plotted against $1/k$ (Figure 3.15) shows that for values of k up to one the magnitude of q_2 remains fairly constant. This graph applies for a wide range of values of β.

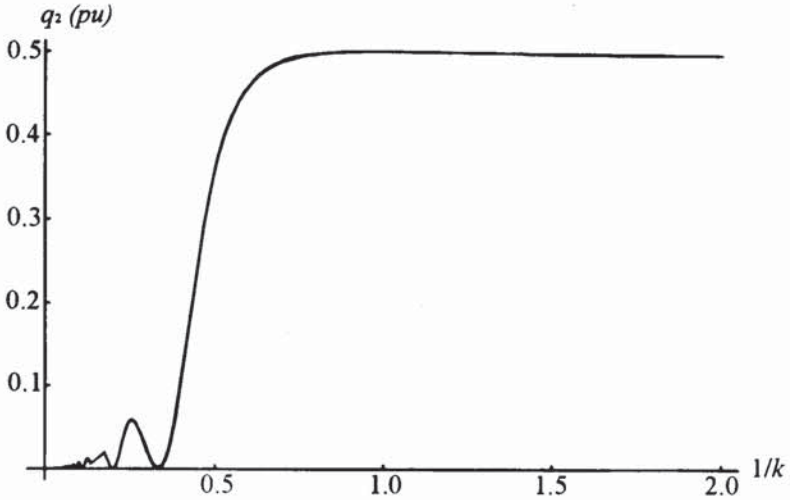

Figure 3.15 Graph of q_2 against $1/k$

This graph suggests that k should be less than unity. Plotting the harmonics of the a.c. capacitor voltage for various values of k less than one shows that the magnitude of the harmonics increases as the reactance ratio increases (figure 3.12). This implies that k should be as small as possible.

Therefore other components of the compensator parameters should be examined. From the equation for the reactive component of the fundamental voltage, equation (3.9), it is obvious that the constant q_2 needs to be as large as possible. This will produce the greatest response to the firing angle. Examining the equation (3.10) for the active power component of the fundamental voltage and plotting its response to the modified firing angle (figure 3.10) shows that the losses in the circuit are very small compared to the reactive component. Also, by plotting the constant p_1 against the reciprocal of the resonance ratio ($1/k$) shows that this constant is very close to zero when k = 1. This is shown in figure 3.16.

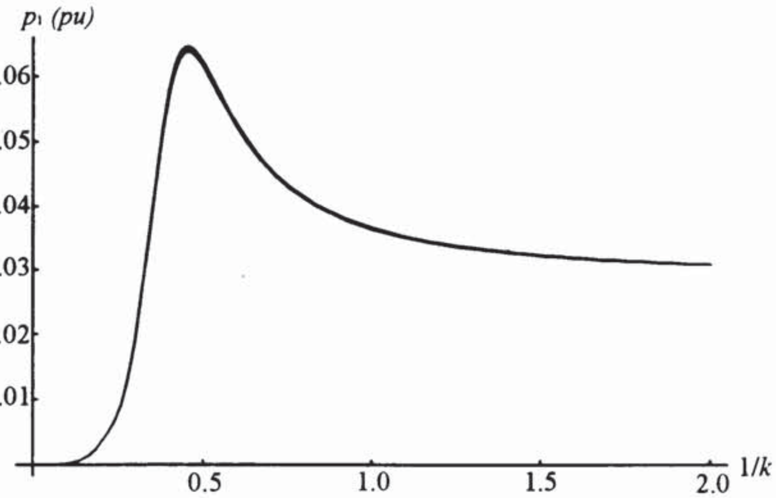

Figure 3.16 Graph of the Constant p_1 against $1/k$

The graph of the constant q_1 is given in figure 3.17 below. The graph also shows that this constant is zero at $k = 1$.

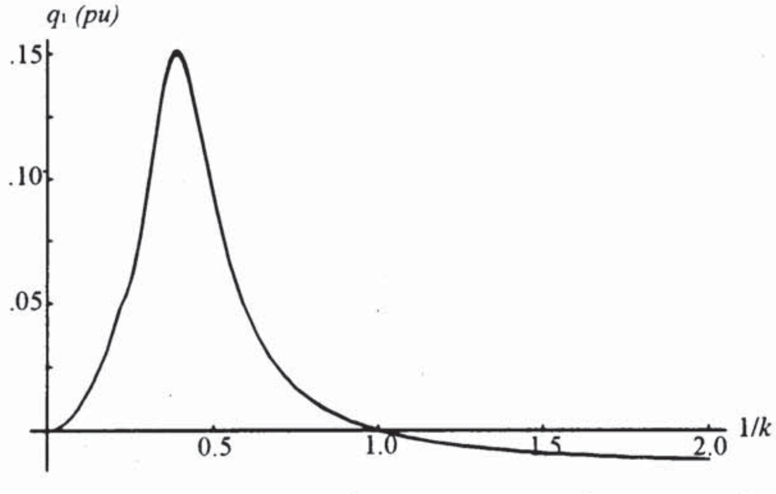

Figure 3.17 Graph of the Constant q_1 against $1/k$

Figures 3.15 to 3.17 were all plotted for several values of β between 0.01 and 0.1. There is no noticeable difference in these graphs, which suggests that changes in β have little effect on the performance of the compensator provided the value chosen remains less than 0.1.

The next graph (figure 3.18) shows the response of the reactive component of the fundamental voltage to the reactance ratio. It is shown for the modified firing angles of ±1.

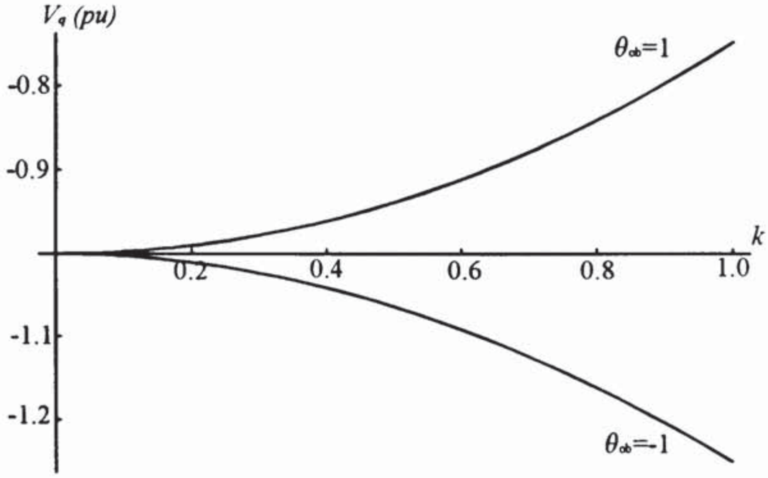

Figure 3.18 Graph Showing the Response of the Quadrature Voltage to Changes in the Reactance Ratio

These graphs show that the reactance ratio should be chosen to be as close to unity as possible, with the consequence that the harmonic content of the voltage is increased.

3.5 Simulation Results

The software simulation package Saber was used throughout the analysis of the compensator. Initially it was used to examine various ideas for the configuration of the compensator and to determine the response of the compensator to variations of the firing angle of the inverter switches. Then, after the algebraic analysis had been completed, the simulation package was used to check the resulting equations which described the compensation system.

The simulator uses templates to build circuits, these are mathematical representations of electrical components, i.e. a capacitor. Many electrical components are provided with the software package, and it is a relatively simple process to create new templates. The single phase voltage sourced inverter series compensator used a combination of pre-built and user built templates.

For the purposes of modelling the compensator, the diagram shown in figure 3.19 was used. The circuit was built using various templates, the system current I_S was provided by an a.c. current source template, a capacitor template was used for both the a.c. and d.c. capacitors etc.

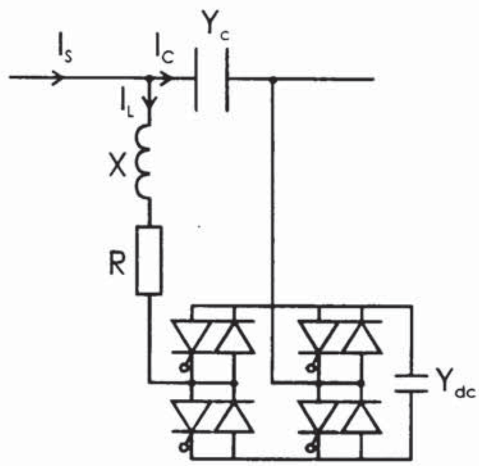

Figure 3.19 Circuit Diagram of the Simulated Series Compensator

The simulated voltage across the d.c. terminals of the inverter is shown in figure 3.20 this compares well with the equivalent graph obtained from the algebraic analysis. The response of the voltage to the firing angle is clearly evident

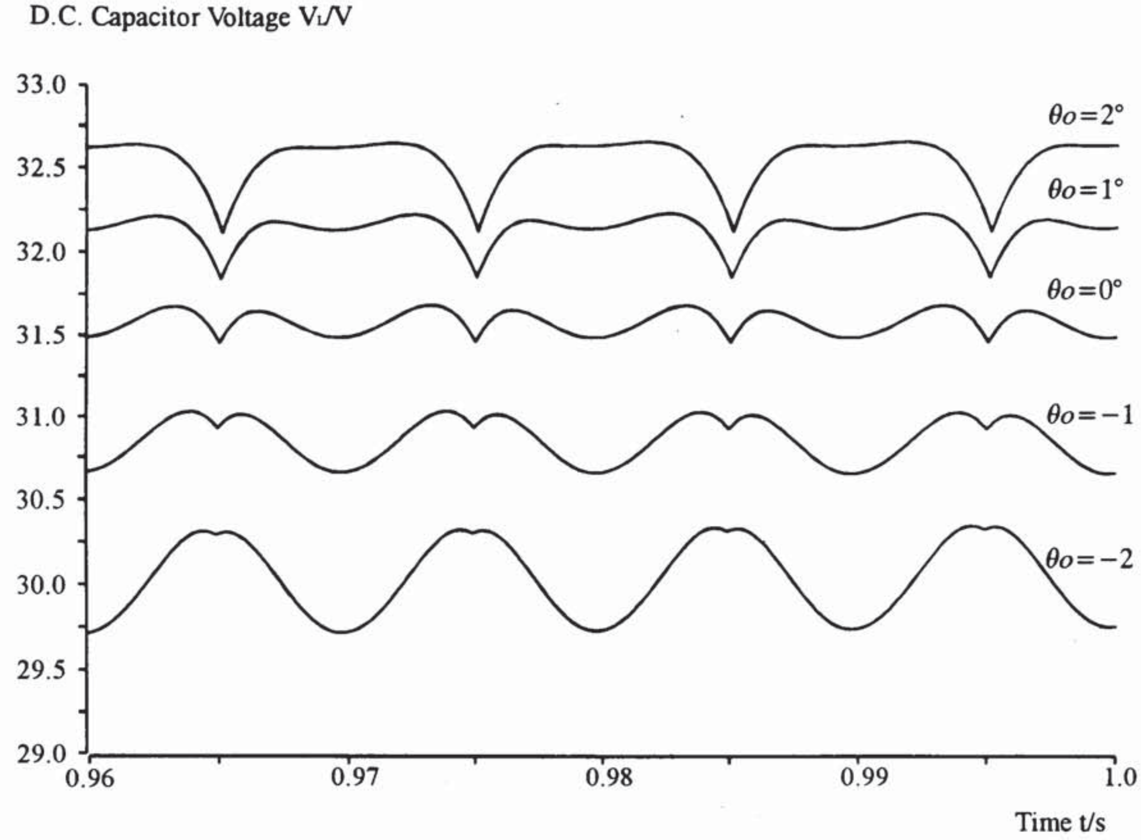

Figure 3.20 Graph Showing the Simulation of the D.C. Capacitor Voltage for Several Values of the Firing Angle

The voltage across the a.c. capacitor is shown by figure 3.21. It also compares favourably with the graphs produced from the previous analysis.

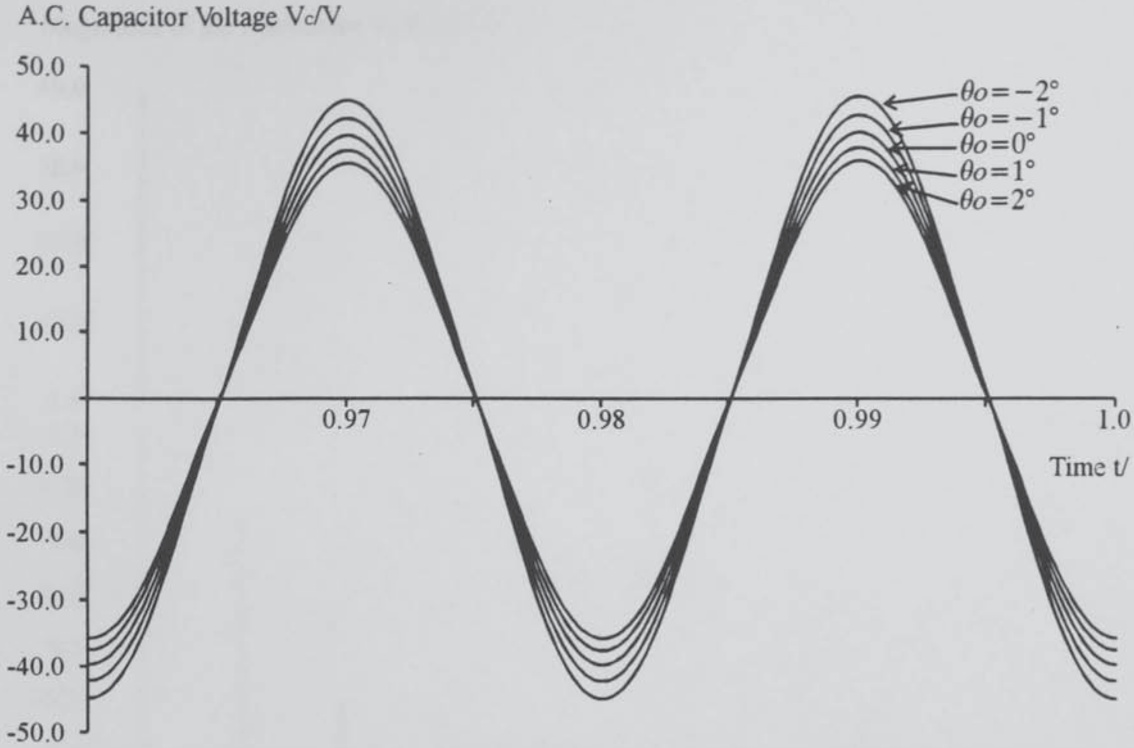

Figure 3.21 Graph Showing the Simulation of the A.C. Capacitor Voltage for Several Values of the Firing Angle

It can be seen from the graph of the ac capacitor voltage that the magnitude of the quadrature voltage can be varied by controlling the firing angle.

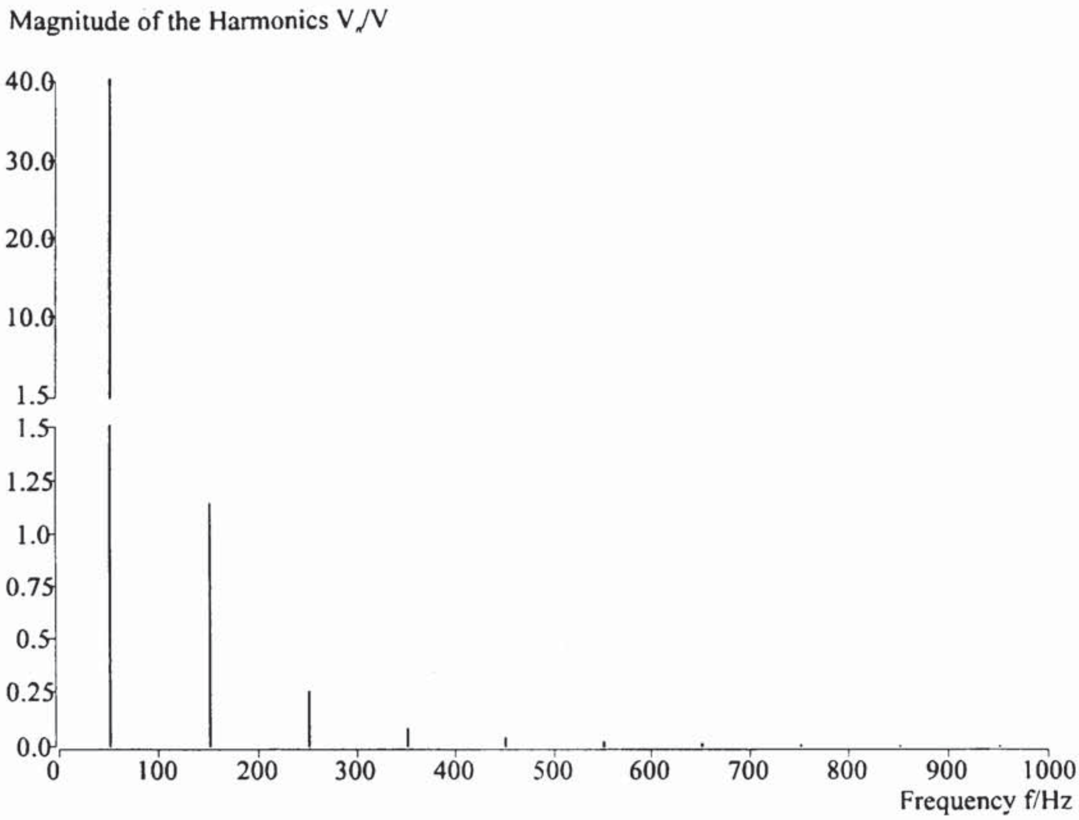

Figure 3.22 Graph Showing the Magnitude of the Harmonics for the A.C. Capacitor Voltage

The harmonic component of the inverter current will circulate around the compensator and there will be no current harmonics introduced to the transmission line. The harmonic component of the voltage across the a.c. capacitor has already been shown in the algebraic analysis to be relatively small and this can be readily seen in figure 3.22.

3.6 Practical Results

A bench-top model of the single phase voltage sourced inverter series compensator was also built as a further check on the analysis described previously in this chapter. Another reason for building the model was as a demonstration model for any interested visitors. The author did not make any contribution to the control system of the models, only in the specification of the size and type of passive components.

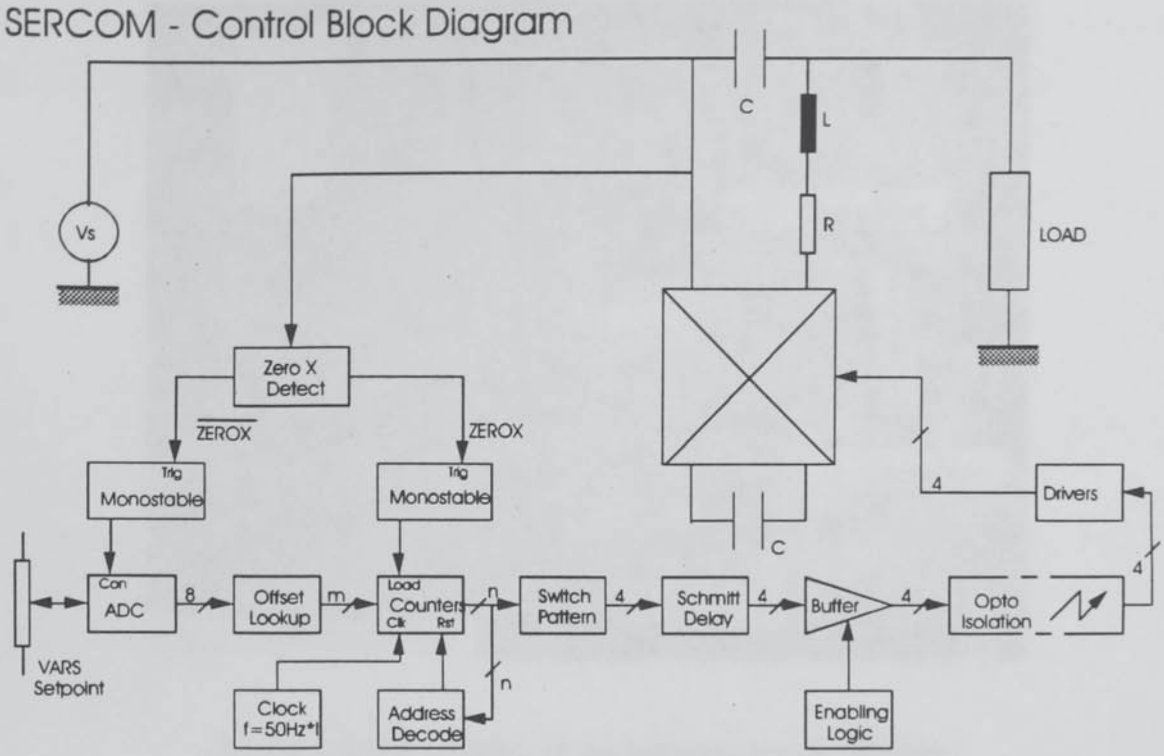

Figure 3.23 Circuit Diagram of the Small Scale Model of a Single Phase Voltage Sourced Inverter Series Compensator

The model was built using real components (as opposed to software) connected as is shown in figure 3.23. The single phase 230V, 50Hz mains was used to provide the power and a variable load was connected at the other end. This load was a combination of resistance and inductance. The a.c. capacitor and the rest of the compensator was connected between the mains supply and the load. A photograph of the single phase voltage sourced inverter series compensator model is shown in figure 3.24.

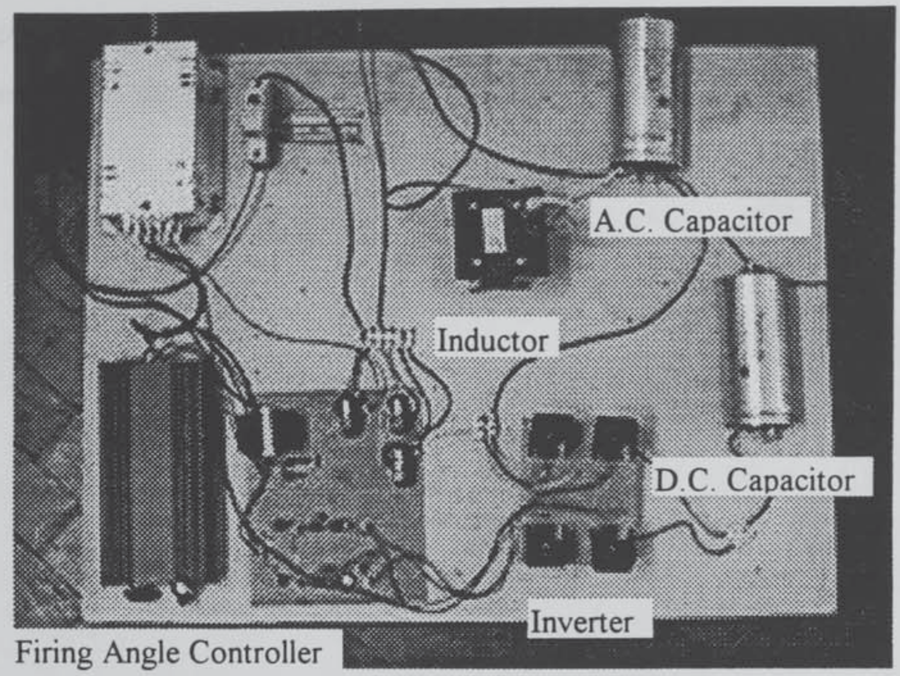

Figure 3.24 Photograph of the Voltage Sourced Inverter
Series Compensator Bench Top Model

Figure 3.25 shows the d.c. capacitor voltage for several firing angles. The data used to create the graph came from the bench top model shown in figure 3.24.

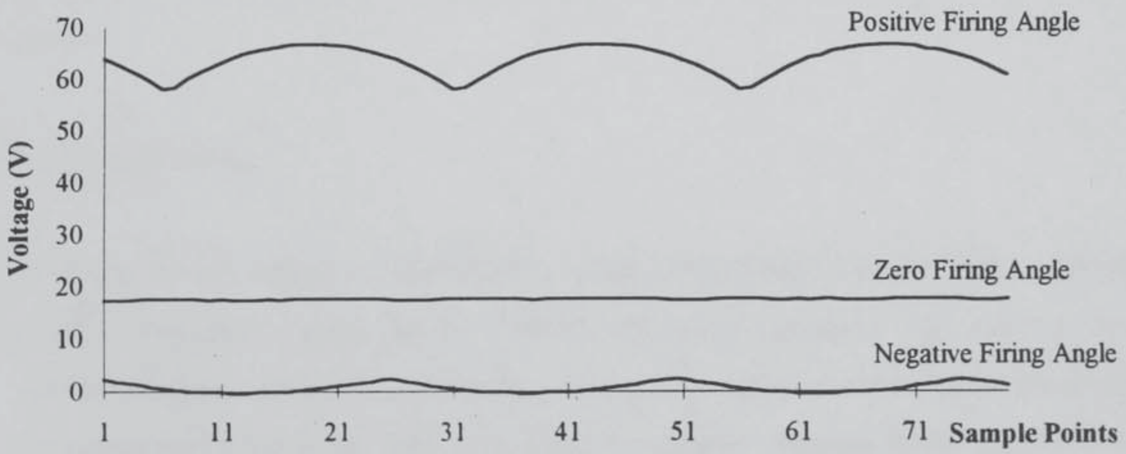

Figure 3.25 Graph Showing the D.C. Capacitor Voltage
for Several Values of the Firing Angle

Figure 3.26 shows the a.c. capacitor voltage for several firing angles. A zero firing angle is equivalent to just the a.c. capacitor in the circuit without any switching occuring.

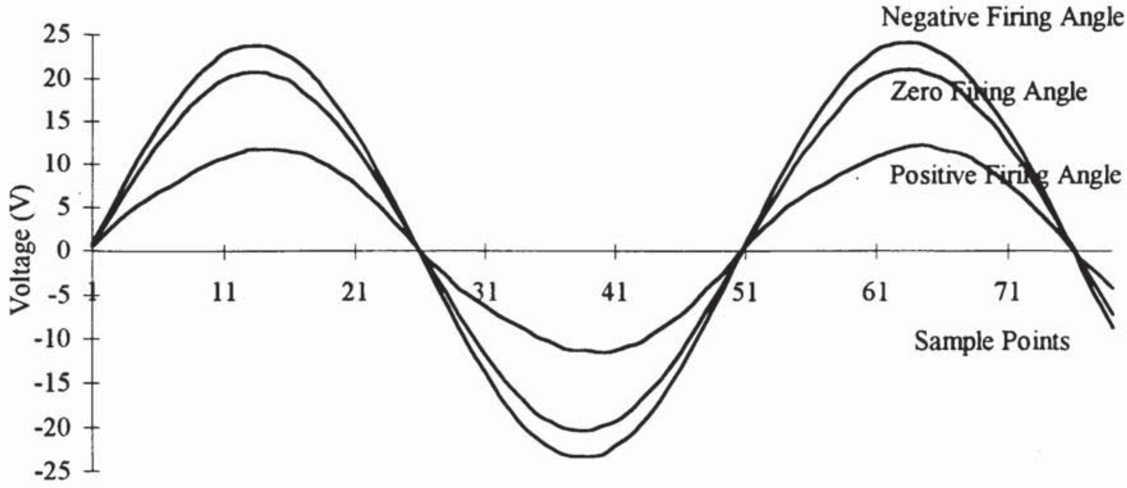

Figure 3.26 Graph Showing the A.C. Capacitor Voltage
for Several Values of the Firing Angle

The results obtained from the model were compared to the waveforms produced by the software simulations, and the mathematical analysis. The model allowed the voltage across the a.c. capacitor to be controlled by the firing of the electronic switches in the inverter.

3.7 Conclusions

The single phase voltage sourced inverter series compensator, can be used to provide a variable capacitive voltage across a series connected capacitor. The control of this variable voltage is obtained via the firing of the power electronic switches in the inverter. The compensator could be used to respond to transient changes in the system voltage due to the fast response of the electronic switches.

The series compensator described in this chapter offers a continuously variable quadrature voltage which can be varied by approximately plus or minus twenty percent on the basic voltage dropped across the series capacitor.

The magnitude of the harmonics introduced to the compensating voltage by the switching in the inverter are relatively small.

Chapter 4

Single Phase Current Sourced Inverter Series Compensation

4.1 Introduction

The basic purpose of series compensation is to improve power flow in electricity transmission lines. Historically whenever series compensation has been deemed necessary, capacitance was introduced to reduce the overall reactance of the transmission line, series compensation has always been synonymous with capacitors.

Until recently, series compensation has been provided by fixed series capacitors which had the disadvantage of being inflexible to changes in the system, through changes in the amount or location of electricity generation/demand or changes in the line network. There was also the potentially more serious disadvantage of provocation of sub-synchronous resonance.

Whilst capacitive compensation is likely to remain the dominant form of series compensation, there are occasions where inductive compensation may be useful. The first of these is in situations where transmission lines are lightly loaded and the shunt capacitance of the lines becomes a dominant effect. Inductive series compensation could also provide a fast acting compensation scheme to reduce the effect of disturbances on the system. In some situations the main requirement is fast response and whether the compensation is inductive or capacitive is usually not critical.

The current sourced inverter series compensator is one possible solution to all the requirements of a series compensator. It will introduce a capacitive voltage in series with a transmission line for the classic compensation requirements, and it can also be used to provide an inductive voltage in series with the transmission line. The use of an inverter made up of power electronic switches introduces a fast response to transient conditions, and the flexibility of a continuously variable series quadrature voltage from capacitive to inductive.

This chapter describes the single phase current sourced inverter series compensation. An overview of the mathematical analysis of the steady state performance of the compensator is given. The algebraic analysis is detailed in Appendix C. Simulations of the circuit were conducted and a selection of the results obtained are shown in order to make a comparison with the results obtained by the mathematical analysis.

4.2 Circuit Description

The current sourced inverter series compensator is shown in circuit form in figure 4.1 and it shows a transmission line with an a.c. capacitor connected serially. A current sourced inverter is connected across the a.c. capacitor. An inductor makes the d.c. energy store and all the resistive elements in the compensation circuit are lumped together as a single resistor in series with the d.c. energy store.

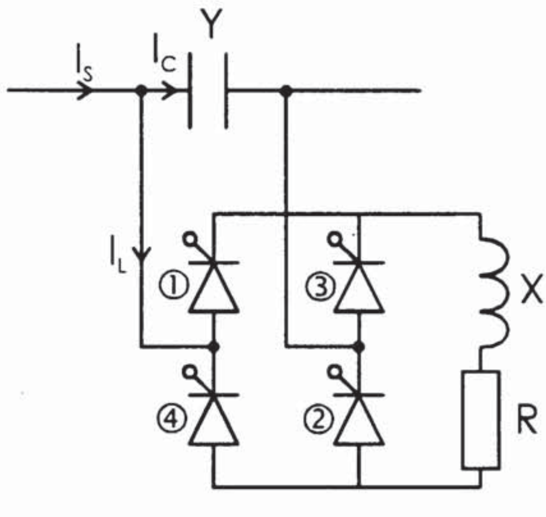

Figure 4.1 Circuit Diagram Showing the
Single Phase Current Sourced Inverter Series Compensator

The compensator works by using the power electronic switches in the inverter to regulate the magnitude of the current through the d.c. energy store. At any one time only two of the four electronic switches are switched on, allowing current to flow. It is always the two switches in a diagonal that are switched at any one time (i.e. switches ① and ②, or ③ and ④).

The timing of the switching is defined by the reference switch, in this case switch ①. The first switch is fired, allowing current to flow, when the voltage across the capacitor crosses through zero, from a positive to a negative voltage, this is described as a zero firing angle. The other two switches are then fired half a cycle later. Firing in this manner allows a d.c. current to build up in the inductive energy store. This is seen at the a.c. terminals of the inverter as a square wave with a domed top.

If the firing angle is advanced slightly the magnitude of the current through the energy store increases and the magnitude of the ripple rises. When the firing angle is delayed the magnitude of the current through the d.c. energy store is reduced and the ripple changes from domes to troughs. The current through the inductor for the three conditions positive, zero and negative firing angles are shown in figure 4.2 below.

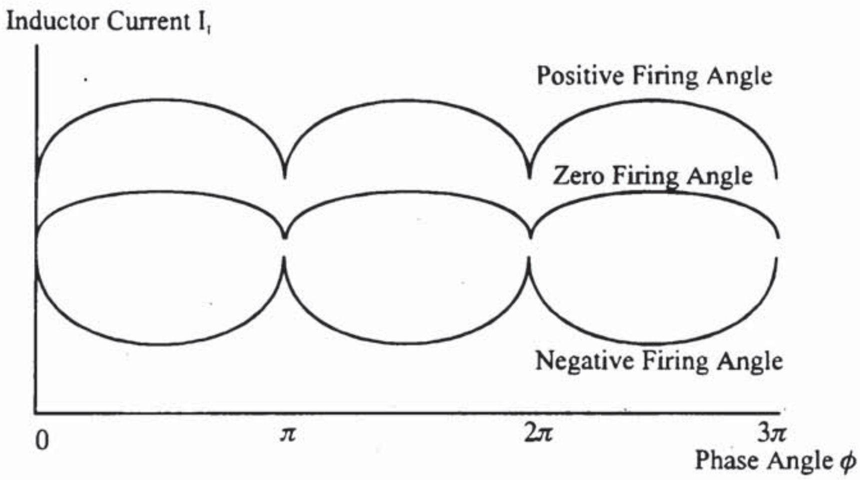

Figure 4.2 Diagram of the Inductor Current
for a Positive, Zero and Negative Firing Angle

Obviously, when all the inverter switches are prevented from allowing current to flow, the circuit acts as a simple fixed series capacitor. In order to avoid introducing any compensation a by-pass switch would be needed. For a zero firing angle the current through the inductor is similar to the current through the capacitor, the exact ratio is dependant upon the size of the reactive components.

As the firing angle of the inverter switches is advanced, the magnitude of the current through the d.c. energy store will increase, and in order to obey Kirchoff's current law the corresponding current through the capacitor must be reduced. This will reduce the compensating voltage injected onto the transmission line. Correspondingly, if the firing angle of the switches in the inverter is delayed then the current through the d.c. energy store is reduced and the current through the capacitor must increase. It is this variation in the magnitude of the current through the inverter which allows the voltage dropped across the capacitor to be controlled.

4.3 Mathematical Analysis

The algebraic analysis for the single phase current sourced inverter series compensator was conducted in a very similar manner to the single phase voltage sourced inverter series compensator. Once again it is possible to simplify the analysis by using the symmetry of the circuit. This allows the analysis to be completed for one half of a cycle with the second half of the cycle being the negative of the first. There is the one obvious exception of the current on the d.c. side of the inverter. A simplified circuit diagram between any two switching instants is shown in figure 4.3 below.

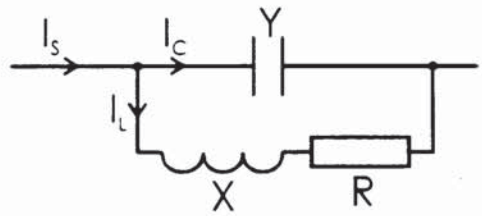

Figure 4.3 Simplified Circuit Diagram of the
Single Phase Current Sourced Inverter Series Compensator

As can be seen for the above diagram the circuit simplifies to an inductor and a resistor connected across a capacitor. It is assumed that the supply current I_S is sinusoidal and not significantly effected by changes made within the compensator. This is a reasonable assumption provided that the impedance of the compensation circuit is small compared to the impedance of the rest of the transmission circuit.

The reference firing angle of the electronic switches is zero when the supply current is zero and the rate of change of supply current with respect to time is negative. The analysis is conducted with the respect to the angle ϕ.

Where:

$$\phi = \theta - \theta_o \tag{4.1}$$

The full analysis is provided in Appendix C and the following paragraphs show the more important equations developed from that analysis.

In figure 4.3, the current through the capacitor I_C can be written in terms of the voltage V across it. This voltage across the capacitor can also be written in terms of the current through the inverter I_L., giving an equation describing the capacitor current in terms of the inverter current. Kirchoff's current law relates the capacitor current and the inverter current to the supply current I_S. The supply current is a sinusoidal source of magnitude I. Rearranging the resultant expression will provide the differential equation shown by equation (4.2) below.

$$\frac{d^2 I_L}{d\phi^2} + 2\beta\frac{dI_L}{d\phi} + (\beta^2 + k^2)I_L = I(\beta^2 + k^2)\sin(\phi + \theta_o) \tag{4.2}$$

The symbol β is the damping factor of the circuit and is defined as:

$$\beta = \frac{R}{2X} \tag{4.3}$$

k is the reactance ratio.

$$k = \sqrt{\frac{1}{XY} - \beta^2} \tag{4.4}$$

The general solution to this type of differential equation is given by equation (4.5) below.

$$I_L = I(k^2 + \beta^2)\{a_1 \cos(\phi + \theta_o) + a_2 \sin(\phi + \theta_o) + a_3 e^{-\beta\phi} \cos k\phi + a_4 e^{-\beta\phi} \sin k\phi\} \tag{4.5}$$

The constants a_1 and a_2 are determined by substituting the equation (4.5) back into the differential equation (4.2), the terms for a_3 and a_4 cancel out leaving terms for a_1 and a_2 which can be evaluated by comparing coefficients of sine and cosine.

In order to obtain the constants a_3 and a_4 the voltage across the compensator first needs to be evaluated and a knowledge of the currents and voltage at the boundary conditions of zero and π are used. The current through the d.c. side of the inverter has the condition of $I_L(0) = I_L(\pi)$, and the voltage has the condition $V(0) = -V(\pi)$. These two conditions allow the constants a_3 and a_4 to be determined in terms of known system parameters.

When the constants a_1, a_2, a_3 and a_4 are substituted back into equation (4.5) the expression for the current through the inverter the following equation (4.6) is obtained.

$$\begin{aligned} I_L &= -I(k^2 + \beta^2)\frac{\cos(\phi + \theta_o + \gamma)}{h} \\ &+ I\frac{(k^2 + \beta^2)\cos(\theta_o + \gamma)}{he^{-\beta\pi}(k \sinh \beta\pi + \beta \sin k\pi)}[(kM + \beta N)e^{-\beta\phi} \cos k\phi + (kN - \beta M)e^{-\beta\phi} \sin k\phi] \end{aligned} \tag{4.6}$$

where

$$M = 1 + e^{-\beta\pi} \cos k\pi \tag{4.7}$$

and

$$N = e^{-\beta\pi}\sin k\pi \tag{4.8}$$

The current through the inverter can be plotted from this equation and this is shown in figure 4.4 below. The d.c. current has been shown for several firing angles. The change from domes to troughs indicates where the compensator changes from inductive to capacitive behaviour. This and all the other time dependant graphs in this chapter used the values of $\beta = 0.01$ and $k = 1.0$.

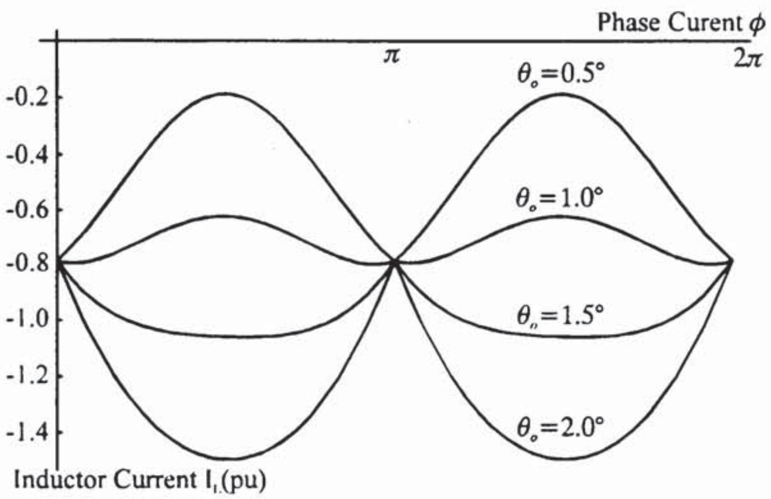

Figure 4.4 Graph Showing the Current through the D.C. Side of the Inverter for Several Values of the Firing Angle

The current through the series capacitor is found by subtracting the inverter current (I_L) from the supply current (I_S). This equation is shown below.

$$I_C = I\left\{\sin(\phi+\theta_o) + (k^2+\beta^2)\frac{\cos(\phi+\theta_o+\gamma)}{h}\right\}$$
$$-I\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{he^{-\beta\pi}(k\sinh\beta\pi+\beta\sin k\pi)}[(kM+\beta N)e^{-\beta\phi}\cos k\phi + (kN-\beta M)e^{-\beta\phi}\sin k\phi] \tag{4.9}$$

The plot of this equation is shown below in figure 4.5. Again the current through the series capacitor is shown for several firing angles.

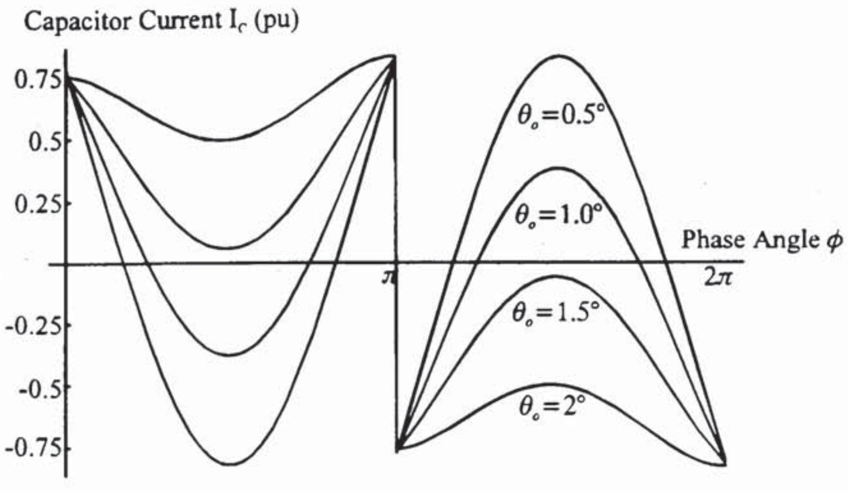

Figure 4.5 Graph Showing the Capacitor Current for Several Values of the Firing Angle

The voltage across the compensator is the sum of the voltage drops across the resistor R and the inductor X, it is given below in equation (4.10).

$$
\begin{aligned}
V = {} & \frac{I}{Y}\frac{\sin(\phi+\theta_o+\gamma)-2\beta\cos(\phi+\theta_o+\gamma)}{h} \\
& +\frac{I}{Y}\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{he^{-\beta\pi}(k\sinh\beta\pi+\beta\sin k\pi)}(Ne^{-\beta\phi}\cos k\phi - Me^{-\beta\phi}\sin k\phi)
\end{aligned}
\tag{4.10}
$$

The expression for the voltage across the compensator is very similar to the equation obtained when evaluating the current through a single phase Statcon. The main difference being the addition of a phase shift which is derived from the losses in the series compensator.

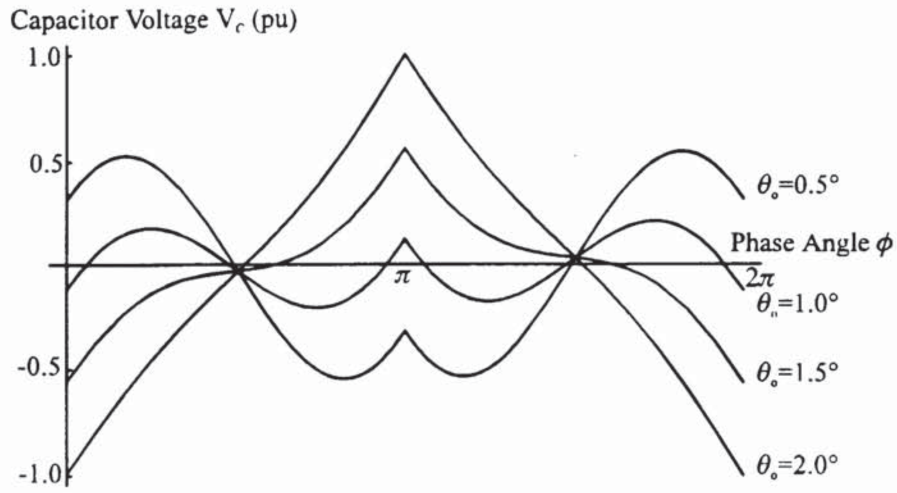

Figure 4.6 Graph Showing the Voltage across the Series Compensator for Several Values of the Firing Angle

The voltage across the compensator is shown by figure 4.6 above. It is controlled by the firing angle of the inverter and it has the capability to act inductively as well as capacitively. Although it is obvious from the graph that the harmonics are quite significant.

In order to examine more closely the relationship between the fundamental voltage across the compensator and the control variable, the firing angle of the inverter, a fourier analysis was conducted on the compensator circuit equations. Equation (4.11) below gives the compensator voltage harmonics.

$$V_n' = \frac{I}{Y}\left\{\frac{\sin(\phi+\theta_o+\gamma)-2\beta\cos(\phi+\theta_o+\gamma)}{h} - \frac{q_2 h}{\beta}\cos(\theta_o+\gamma)\sum_{n=1}^{\infty}\frac{\sin(n\phi+\gamma_n)}{h_n}\right\} \tag{4.11}$$

Where:

$$q_2 = \frac{4\beta k(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)}{\pi h^2(k\sinh\beta\pi+\beta\sin k\pi)} \tag{4.12}$$

The fourier analysis allowed the fundamental component of the compensator voltage to be studied and it is given in equation (4.13) below.

$$V_1 = \frac{I}{Y}\left\{\frac{\sin(\phi+\theta_o+\gamma)-2\beta\cos(\phi+\theta_o+\gamma)}{h} - \frac{q_2}{\beta}\cos(\theta_o+\gamma)\sin(\phi+\gamma)\right\} \quad (4.13)$$

This expression for the fundamental component of the compensator voltage was further split into the component in quadrature with the supply current and called V_q and the component in phase with the supply current denoted by V_p.

$$V_q = q_1' + q_2\theta_{ob} \quad (4.14)$$

This voltage has been expressed in terms of a modified firing angle θ_{ob}.

$$\theta_{ob} = \frac{\theta_o}{\beta} \quad (4.15)$$

The voltage V_q represents the useful contribution of the circuit to the compensation of the transmission line and it is obvious that the voltage is linearly dependant upon the firing angle of the inverter. The constant q_1' is made up of two other constants; q_1 which is identical to the constant q_1 obtained in the single phase voltage sourced inverter series compensator and in the analysis of the Statcon (Hill 95), and q_0 which is an additional constant derived from the algebra in Appendix C.

Where the constants q_0 and q_1 are defined below.

$$q_1 = \frac{\sin\gamma}{h}(1-2q_2) \quad (4.16)$$

and

$$q_0 = -\cos^2\gamma \quad (4.17)$$

The graph of the reactive component of the compensator voltage is shown in figure 4.7 shown below. This graph clearly shows that the reactive component of the fundamental voltage across the capacitor varies linearly with the firing angle of the inverter. It also shows that both capacitive and inductive compensation are possible.

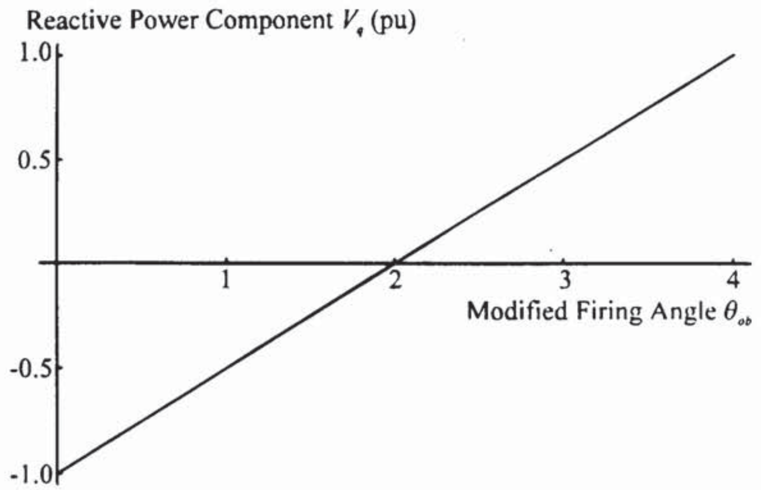

Figure 4.7 Graph Showing the Quadrature Component of the Compensator Voltage against the Modified Firing Angle

The active power component of the capacitor voltage is found by taking the components of $\sin\theta$ from the expression for the fundamental voltage, equation (4.13). This voltage V_p represents the active power dissipated by the compensator and the equation (4.18) is depicted below.

$$V_p = \beta(p_1' + q_2\theta_{ob}^2) \tag{4.18}$$

The dependence between the active power component of the compensator voltage and the firing angle of the inverter is obviously a square relationship so as the firing angle increases the active power losses are going to increase significantly. The constant p_1' is also the sum of the p_1 constant from earlier in Chapter 3 and a new additional constant p_0.

Where

$$p_0 = 2\frac{\sin\gamma}{h} \tag{4.19}$$

and

$$p_1 = \frac{\cos\gamma}{\beta h}(1-2q_2) \tag{4.20}$$

A plot of the active power component of the compensator voltage V_p is shown in figure 4.8. It can be seen from this graph that the losses are very small when compared against the reactive power component. This remains valid provided the firing angle of the inverter is small.

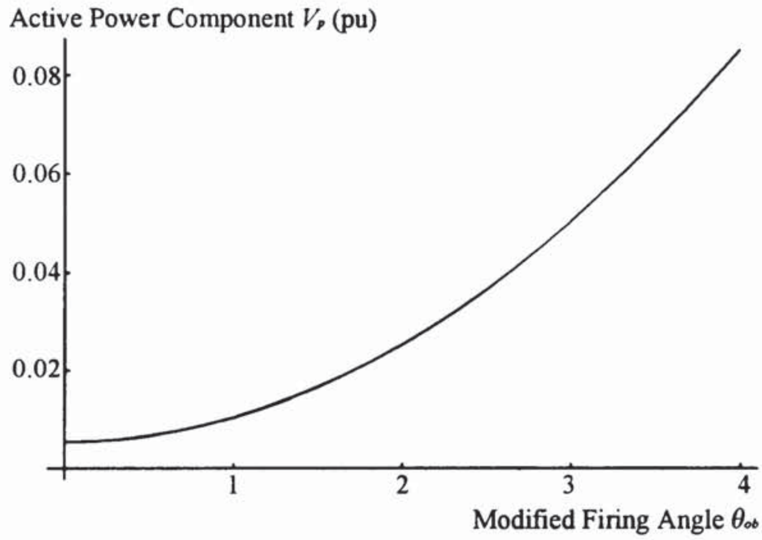

Figure 4.8 Graph Showing the Active Component of the Compensator Voltage against the Modified Firing Angle

The harmonic component of the voltage (all odd harmonics above the fundamental) across the capacitor is given by equation (4.21). Any individual harmonic will respond linearly with the firing angle.

$$V_n = \frac{2q_2}{h_n}(\theta_o \tan\gamma - 1) \tag{4.21}$$

The magnitude of the harmonic voltages is shown below in figure 4.9. It shows that the third harmonic exceeds 10% of the fundamental.

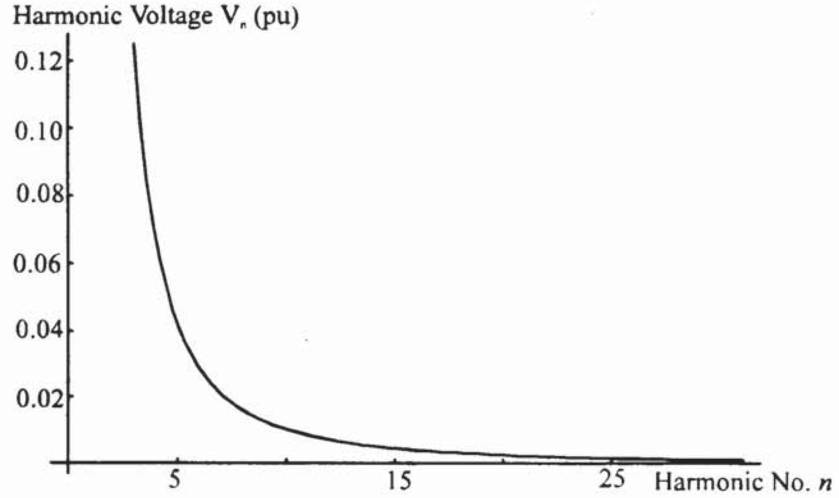

Figure 4.9 Graph Showing the Magnitude of the Harmonic Component of the Compensator Voltage

4.4 Simulation Results

The compensator was created on a software simulator in order to compare the results of the algebraic analysis with a numerical analysis. The simulation package allows the individual components of the single phase current sourced inverter series compensator to be connected together as in a real circuit and then the simulator conducts a numerical analysis of the circuit. All the currents and voltages in the circuit are calculated for discrete time segments, these individual calculations are then plotted to give a graphical description of the response of the circuit.

The current across the d.c. terminals of the inverter is shown in figure 4.10 below and it compares favourably with the graph shown in figure 4.4.

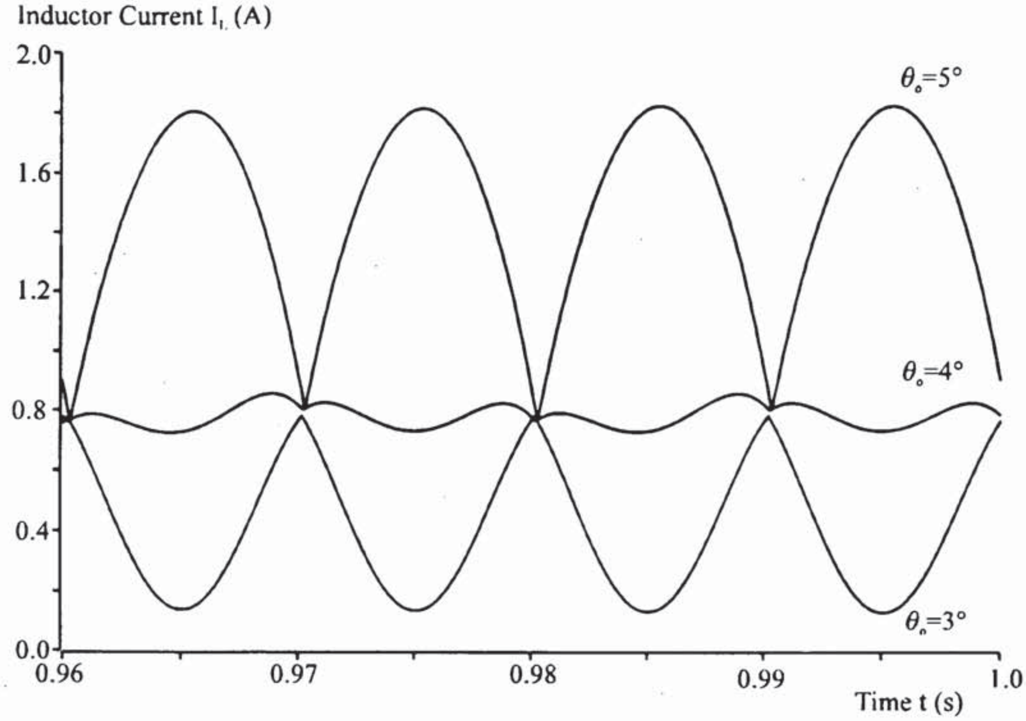

Figure 4.10 Simulation Graph of the Current across the D.C. Terminals of the Inverter

The graph shows the simulation plots of the current through the inductor on the d.c. side of the inverter. The graph also shows how this current varies for several different firing angles.

The next, figure 4.11, shows the voltage dropped across the capacitor, this is the compensating voltage and its response to the firing angle is shown below. In order to make a comparison with the graphs plotted earlier from the algebraic analysis, it should be noted that 1 pu voltage on the earlier graph (figure 4.6) is approximately equivalent to 40 volts on the graph shown below.

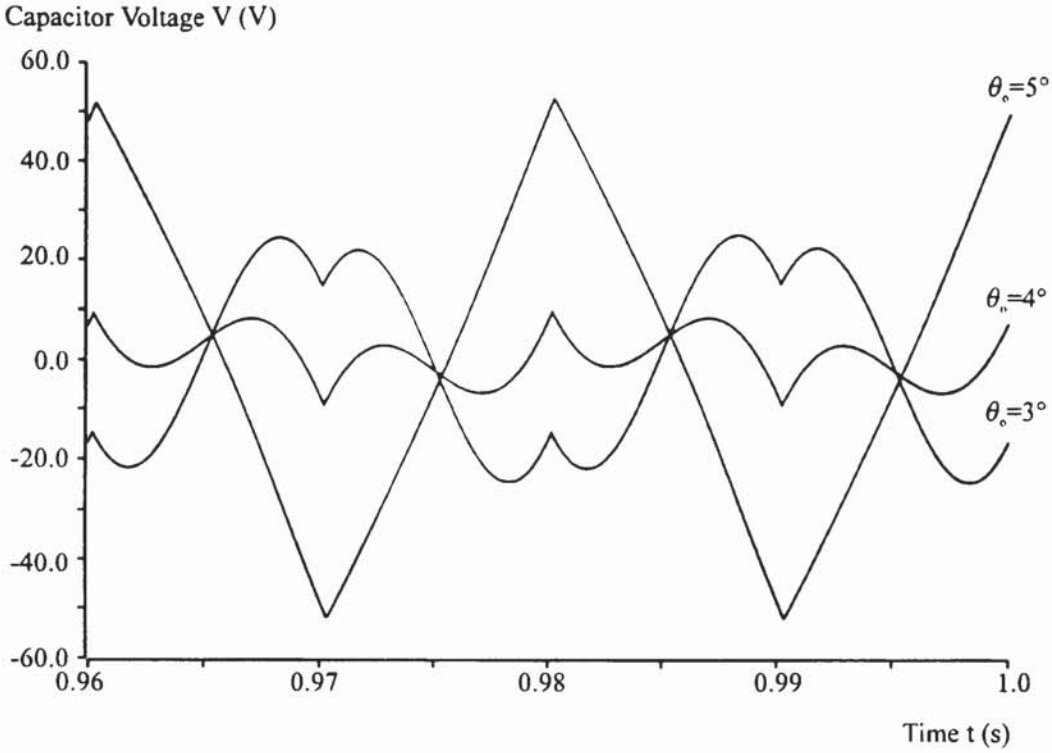

Figure 4.11 Simulation Graph of the Voltage across the Series Capacitor

The magnitude of the harmonics of the simulation circuit were also evaluated by conducting a fourier analysis of the waveshapes. The magnitude of the harmonics of the simulated circuit is shown in figure 4.12 below.

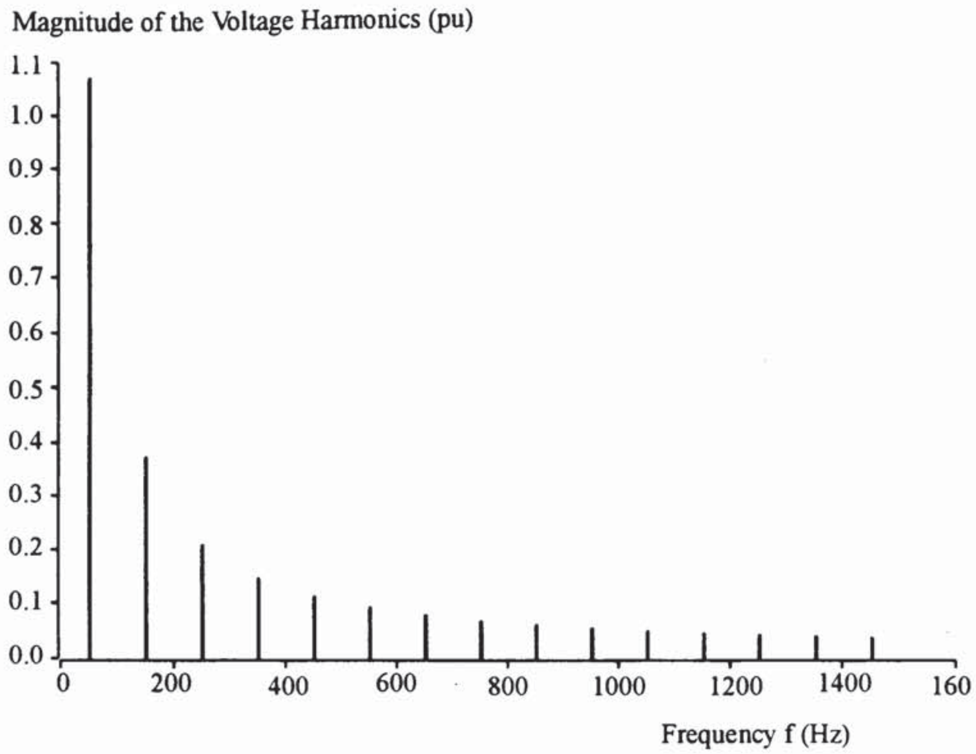

Figure 4.12 Graph Showing the Magnitude of the Harmonics of the Compensator Voltage

The simulation produces harmonic magnitudes that agree with the magnitude of the harmonics calculated by algebraic methods. The above graph shows that the fundamental component was between 35 and 40 volts, and that the third harmonic was approximately 5 volts. This is about 14% of the fundamental and this broadly agrees with the algebraic analysis.

4.5 Discussion

The sizes of the passive components are important to the performance of the compensator. Needless to say the losses of the circuit are important, and should be kept to a minimum. However, some resistance is necessary for the circuit to behave correctly (i.e. provide compensation). The damping factor β is the only component of the compensator effected by the losses in the system. The ratio of the reactance of the inductor to the reactance of the capacitor is the other factor to consider.

There are three equations which need to be considered when choosing the size of the passive components, these are the reactive power component of the compensation voltage V_q, the active power component V_p, and the harmonic component V_n.

Considering first the reactive component of the voltage. This voltage has already been shown to vary linearly with the firing angle. A graph of this voltage against the modified firing angle has been shown in figure 4.7, and the gradient of this voltage is dependant on the constant q_2 given by equation (4.10). The graph shown below shows the relationship between the constant q_2 and the reciprocal of the reactance ratio ($1/k$).

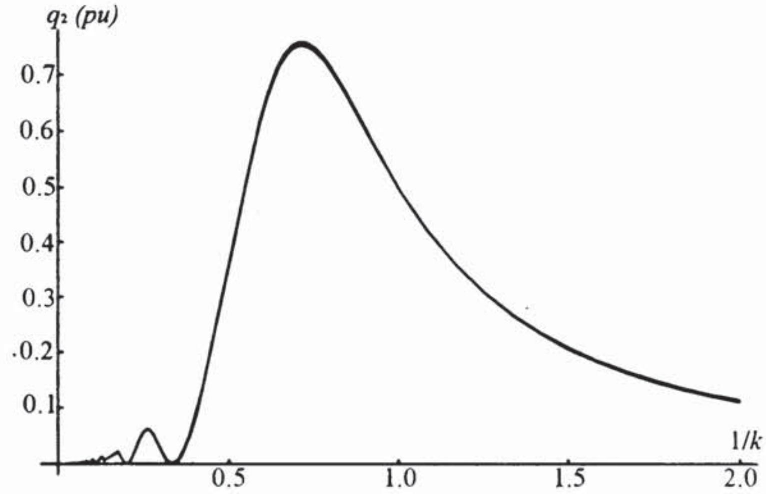

Figure 4.13 Graph Showing q_2 against $1/k$

The graph has been plotted for several values of β ranging from 0.01 to 0.1. It is obvious that the damping factor has very little effect upon the constant q_2. The compensator will have the greatest range of operation if q_2 is as large as possible. The graph shown above shows that the maximum value of q_2 occurs for a value of $1/k$ approximately 0.7, this corresponds k being approximately 1.3. All other values of k give a significantly reduced value for q_2 and as a result a reduced range of compensation.

Next, examining the active power losses in the circuit. This is given by equation (4.16). The losses are largely dependant upon the square of the modified firing angle θ_{ob}. Again the constant q_2 determines the gradient of the losses curve (figure 4.8). The active power dissipated in the circuit needs to be kept to a minimum but the constant q_2 is chosen to provide the maximum compensation. This leaves the constant p_1' which should be minimised and damping factor β obviously needs to be as small as possible.

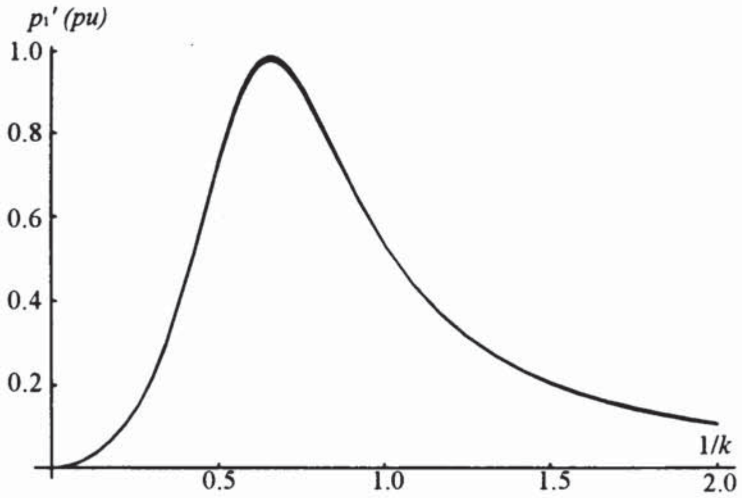

Figure 4.14 Graph Showing p_1' against $1/k$

The above graph shows the constant p_1' against the reciprocal of the reactance ratio ($1/k$) for several values of the damping factor β. It is at a maximum when $1/k$ is approximately 0.7. This is the same as the constant q_2. Ideally $1/k$ would be chosen so that the losses in the compensator are at a minimum. This occurs when k approaches infinity ($1/k$ -> 0), which obviously is impractical, as the compensating voltage would approach zero. The losses are also very small as k becomes less than 0.5 ($1/k > 2$). Unfortunately the q_2 is also significantly reduced for this value of k.

The constant q_1' determines the intercept of the reactive component of the fundamental voltage (figure 4.7). The simulations have shown that a negative firing angle will not provide any compensation. The current through the inverter is zero when the firing angle of the inverter is zero. This means that compensation will only be present when there is a positive firing angle. In turn this implies that there needs to be an intercept on the graph of the reactive power component of compensator voltage.

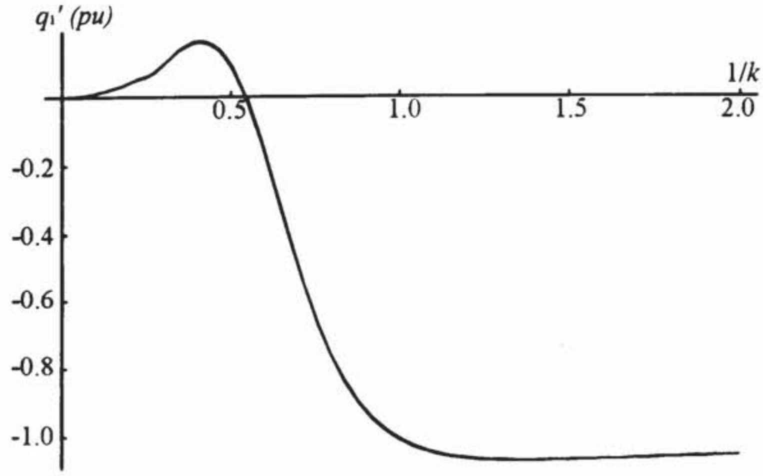

Figure 4.15 Graph Showing q_1' against $1/k$

The above graph shows the constant q_1' against the reciprocal of the reactance ratio ($1/k$) for several values of the damping factor β. Again the damping factor has very little effect on the constant q_1'.

The final choice of the value of the reactance ratio k, comes by examining figure 4.15. In order to obtain a 1 pu capacitive voltage across the compensator the constant q1' needs to have a value -1. This implies that the reactance ratio need to take a value of 1 or lower.

The active power dissipated in the circuit also reduces as the reactance ratio decreases from 1. However, the gradient of the reactive power is also decreased as this ratio reduces from 1. These three graphs (figures 4.13, 4.14, 4.15) tend to suggest that the reactance ratio should have a value of 1 or very close.

The magnitude of the harmonics introduced by the compensating voltage is also important. All the current harmonics will circulate around the compensator and therefore do not need to be considered. The harmonic component of the compensator voltage V_n is shown in figure 4.16 below.

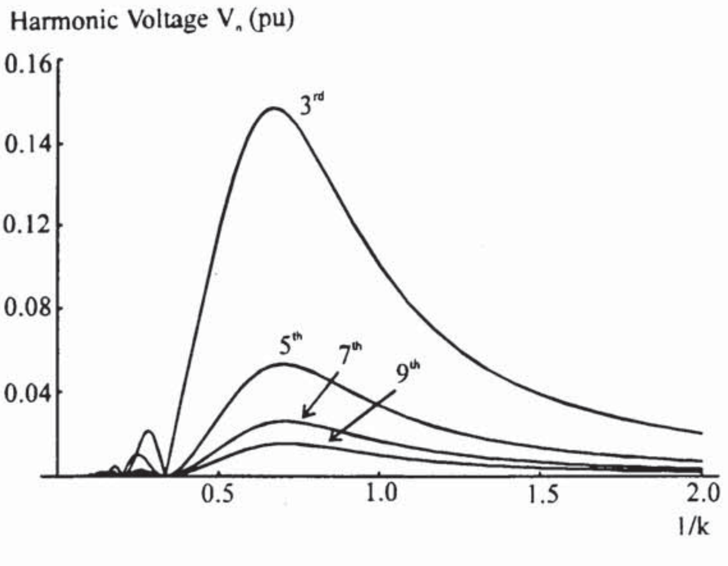

Figure 4.16 Graph Showing the Voltage Harmonics against $1/k$

The above graph shows the magnitude of the compensator voltage harmonics against the reactance ratio. The value of k is significant. The harmonics peak when the reactance ratio (k) is approximately 1.3.

4.6 Conclusions

The single phase current sourced inverter series compensator will offer variable series compensation. The compensator can be used to provide capacitive compensation as well as inductive compensation.

The response of the circuit is linearly dependant upon the firing angle of the inverter and the voltage across the compensator can be varied continuously from inductive to capacitive. This includes zero compensation, however the series compensator would still introduce losses in this condition.

The harmonics are quite large especially when compared to the voltage sourced inverter series compensator using a capacitor as the series element. A three phase circuit would reduce the magnitude of the harmonics but additional measures would still be needed to allow connection to the National Grid.

The reactance ratio k should be chosen to be as close to 1 as possible in order to allow the maximum capacitive compensation with the widest range of available compensation. The damping ratio β has very little effect on the compensation over quite a wide range of values but as small a value of possible should be used in order to keep the losses to a minimum. A value of 0.01 was used throughout the analysis.

Chapter 5

Single Phase Voltage Sourced Inverter Series Compensator Using an Inductor as the Series Component

5.1 Introduction

Series compensation does not need to be typically capacitive, although this is usually desirable. This chapter examines one method of providing series compensation, where the purpose is not to introduce a capacitive voltage to the transmission line. This particular scheme is designed to offer a fast response to transient changes in the transmission line currents. The circuit is basically inductive in nature, with a variable voltage being achieved by altering the firing angle of the inverter.

In some situations it can be useful to have a compensation circuit in which the sole purpose is to maintain a fixed voltage. This can be done with fixed, mechanically switched series components, however this method has the considerable drawbacks of very slow response to changes in the transmission system. This disadvantage can be reduced by introducing electronic switching, but this does not solve the other difficulty of discrete steps in the amount of compensation allowed by fixed inductors and capacitors.

The series compensation scheme examined in the rest of this chapter, uses electronic switches for a fast acting compensation scheme and allows for a continuously variable degree of compensation.

The chapter begins by examining the compensation circuit and explains the way in which the compensation of the transmission line is achieved. Then the algebra, which describes the compensation circuit in mathematical terms, is outlined, in abbreviated form for ease of understanding. The complete mathematical analysis for this compensation circuit is given in Appendix D. Several graphs are shown which provide a clearer picture of the way in which the compensation works, and these can be compared to computer simulation results. The simulations were done using a numerical computer simulation package called Saber. Finally in this chapter a discussion of the compensation circuit is given.

5.2 Circuit Description

The compensation circuit is shown in figure 5.1. The circuit shown is a single phase system.

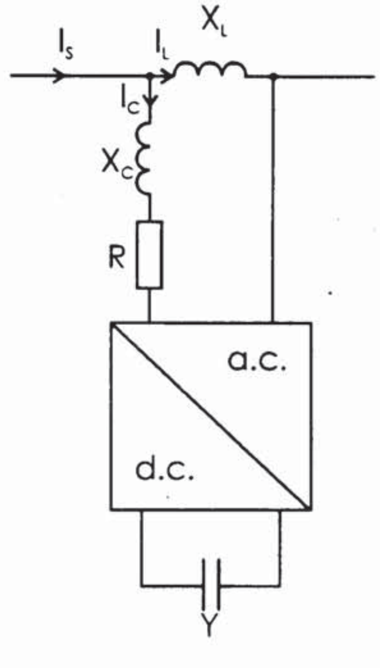

Figure 5.1 Circuit Diagram of the Single Phase Voltage Sourced Inverter Series Compensator Using an Inductor as the Series Element

The circuit consists of an inductor (X_L) connected in series with a transmission line, this provides the basic level of compensation. This could be a purposely created reactor or more probably it would be an existing winding of a transformer. A voltage sourced inverter is connected across this inductor, with another inductor (X_C) separating them. The energy store consists of a capacitor (Y) and the resistor shown in the circuit represents the losses of the compensator as a whole. These losses mainly consist of losses in the inductor and switching losses.

The single phase inverter consists of four power electronic switches with the associated control circuitry. The switches are shown in figure 5.2, they are numbered in order of their firing sequence. The anti-parallel diodes are not depicted and the devices are any suitable, fully controllable, electronic switches.

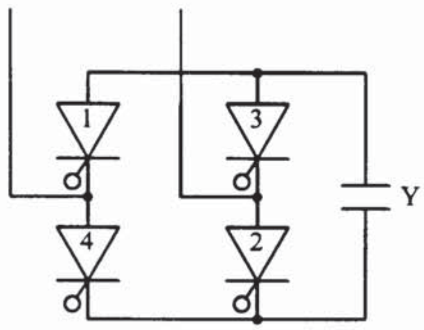

Figure 5.2 Circuit Diagram Showing a Single Phase Voltage Sourced Inverter for the Series Compensator

For this type of compensation circuit the electronic switches fire, or are switched to allow current to flow through them, in diagonal pairs. Switch number 1 is the reference from which all the other switches take their firing signal. The switching pairs are fired 180° apart, i.e. the first two switches are fired, then half a cycle later they are switched off and the other two are switched on.

This firing time for the first switch has been defined as the firing angle (θ_o). The firing angle for this circuit is defined to be zero when the voltage across the inductor (V_L) in series with the transmission line crosses zero, going from positive to negative. As the current through the transmission line before (or after) the compensation circuit provides the reference phase angle (θ), a zero firing angle occurs approximately when the supply current (I_S) is at its peak.

The variation of this firing angle will create a phase difference between the voltage at the a.c. terminals of the inverter and the voltage across the series inductor. A large proportion of the analysis of the compensator was conducted using the phase angle of the inverter voltage (ϕ) and it is therefore necessary to relate the phase angle of the inverter voltage to the phase angle of the reference supply current. This is given in equation (5.1).

$$\phi = \theta - \theta_o + \pi/2 \tag{5.1}$$

It is this phase difference between the inverter voltage and the series inductor voltage that allows the compensator to operate. If there is a small positive firing angle the voltage across the d.c. energy store will fall. The difference between the magnitude of the inverter voltage and the series inductor voltage forces current to flow in an anti-clockwise direction around the compensator. This reduces the current flowing through the series inductor and consequently the voltage dropped across it. This action is shown in figure 5.3.

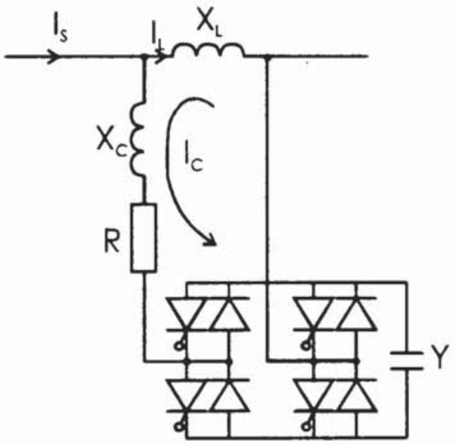

Figure 5.3 Diagram of the Circuit Acting to Reduce the Series Inductor Voltage

When a negative firing angle is used the voltage across the d.c. energy store is increased and becomes larger than the voltage across the series inductor. This forces current in a clockwise direction as shown in figure 5.4. Correspondingly the current through the a.c. capacitor is increased and the voltage across the series inductor is also increased.

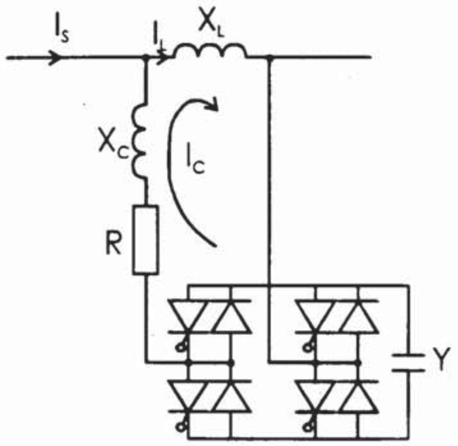

Figure 5.4 Diagram of the Circuit Acting to Increase the Series Inductor Voltage

It will be shown in a later section (5.3) that the response of the voltage across the series inductor is linearly dependant upon the variation of the firing angle of the inverter. The phase difference between the inverter voltage and the voltage across the series inductor requires a degree of resistance in the circuit in order to satisfactorily resolve the voltages in the compensation circuit. These losses are also described mathematically in section 5.3.

5.3 Mathematical Analysis

The analysis of the circuit shown in figure 5.1 can be simplified considerably by using the symmetry of the compensation circuit. The voltage across the d.c. energy store has half-wave symmetry about $n\pi$ and all the other voltages and currents in the compensator have half-wave anti-symmetry. This has the advantage of allowing the compensation circuit to be analysed for the era from 0 to π. Then the equations subsequently derived can be applied to the era from π to 2π by applying the half-wave symmetry.

The circuit diagram is simplified considerably by using this symmetry and the modified circuit is shown in figure 5.5 applies during one half of a cycle.

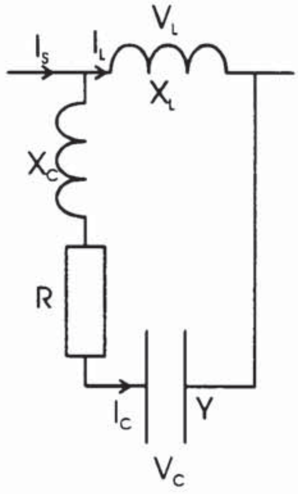

Figure 5.5 Simplified Circuit of the Single Phase Voltage Sourced Inverter Series Compensator

The definitions of the various symbols used are given in Appendix D. As in the previous chapters the supply current (I_S) is taken to be sinusoidal. Then by analysing the voltages across all the components in the compensation circuit and equating the currents at all the nodes it is possible to develop the second order differential equation relating the current through the inverter to the supply current.

$$\frac{d^2 I_C}{d\phi^2} + 2\beta \frac{dI_C}{d\phi} + (k^2 + \beta^2) I_C = Ir\cos(\phi + \theta_0) \tag{5.2}$$

The solution to this differential equation is given by the following general equation (5.3).

$$I_C = Ir\{a_1 \cos(\phi + \theta_o) + a_2 \sin(\phi + \theta_o) + a_3 e^{-\beta\phi} \cos k\phi + a_4 e^{-\beta\phi} \sin k\phi\} \tag{5.3}$$

The constants a_1 and a_2 are found by substituting the general solution to the current through the inverter, equation (5.3), back into the second order differential equation (5.2). The constants a_3 and a_4 cancel out during this substitution and need to be found using boundary conditions. The knowledge of the symmetry mentioned earlier is used to find the constants a_3 and a_4, after first evaluating the voltage across the inverter. The complete equation for the current through the inverter is given in equation (5.4).

$$
\begin{aligned}
I_C &= Ir\frac{\sin(\phi+\theta_o+\gamma)}{h} \\
&+Ir\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}(Ne^{-\beta\phi}\cos k\phi - Me^{-\beta\phi}\sin k\phi)
\end{aligned}
\tag{5.4}
$$

This equation allows the current to be shown pictorially by plotting a graph as shown in figure 5.6, the graph shows the current for several different firing angles.

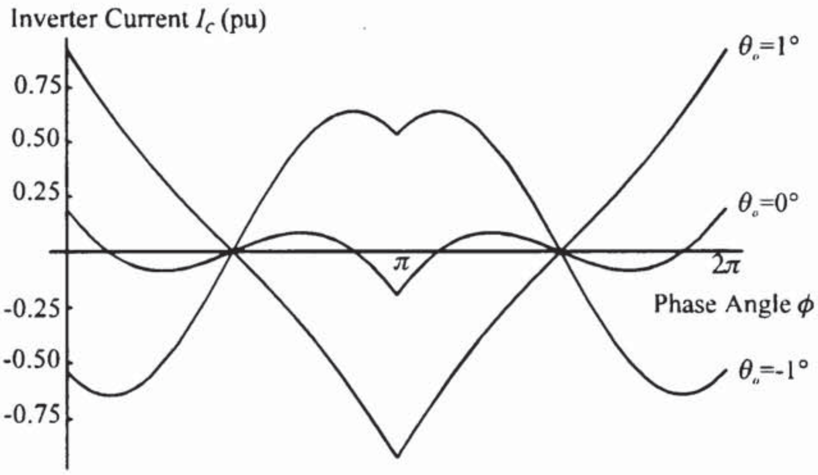

Figure 5.6 Graph Showing the Inverter Current for Several Values of the Firing Angle

The voltage across the d.c. energy store can then be evaluated by integrating the current flowing through the inverter given by equation (5.4). This capacitor voltage is shown in equation (5.5).

$$
\begin{aligned}
V'_C &= -IX_L r(k^2+\beta^2)\frac{\cos(\phi+\theta_o+\gamma)}{h} \\
&+IX_L r(k^2+\beta^2)\frac{\cos(\theta_o+\gamma)e^{-\beta\phi}[(kM-\beta N)\cos k\phi+(kN+\beta M)\sin k\phi]}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}
\end{aligned}
\tag{5.5}
$$

There can be no constant of integration because, if one existed, the voltage across the d.c. energy store would never settle into a steady state. Due to the half wave symmetry of the voltage across the d.c. terminals of the inverter. Simulations and physical modelling has shown this not to be the case.

A graph showing the voltage across the d.c. energy store is shown in figure 5.7 and again the voltages for several different firing angles are shown. It clearly shows the way in which the magnitude of the capacitor voltage changes with respect to the inverter firing angle.

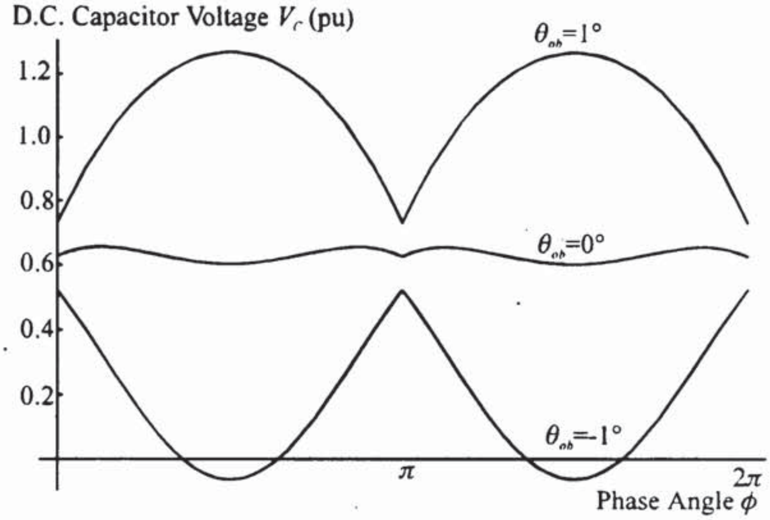

Figure 5.7 Graph Showing the Voltage across the D.C. Capacitor for Several Values of the Firing Angle

The current through the series inductor (I_L) is found by subtracting the inverter current (I_C) from the supply current (I_S). The equation for the series inductor current is shown below.

$$\begin{aligned} I_L = & -I\cos(\phi+\theta_o) - Ir\frac{\sin(\phi+\theta_o+\gamma)}{h} \\ & -Ir\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}(Ne^{-\beta\phi}\cos k\phi - Me^{-\beta\phi}\sin k\phi) \end{aligned} \tag{5.6}$$

A graph of this current is shown below for several firing angles.

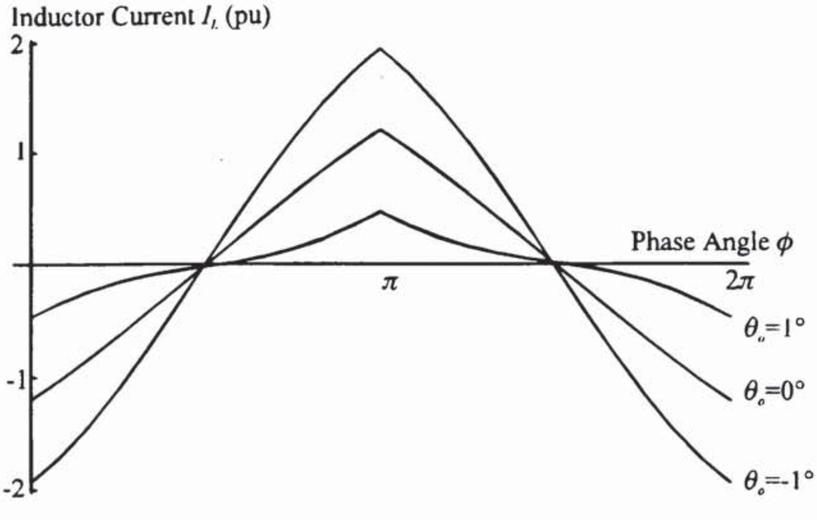

Figure 5.8 Graph Showing the Current through the Series Inductor for Several Values of the Firing Angle

The voltage across the series inductor is the most important as it is the compensating voltage. This voltage is found by first evaluating the current through the series inductor (I_L), equation (5.6), and then by subtracting the inverter current, equation (5.4), from the supply current. The voltage is then found by differentiating the series inductor current and multiplying by the reactance of the inductor. This is given in equation (5.7).

$$V_L = IX_L\left\{\sin(\phi+\theta_o) - r\frac{\cos(\phi+\theta_o+\gamma)}{h}\right\}$$
$$+IX_L r\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{he^{-\beta\pi}(k\sinh\beta\pi - \beta\sin k\pi)}[(\beta N + kM)e^{-\beta\phi}\cos k\phi + (kN-\beta M)e^{-\beta\phi}\sin k\phi] \tag{5.7}$$

Plotting this equation for several values of the inverter firing angle shows that the degree of compensation can be varied by the firing angle as shown by figure 5.9

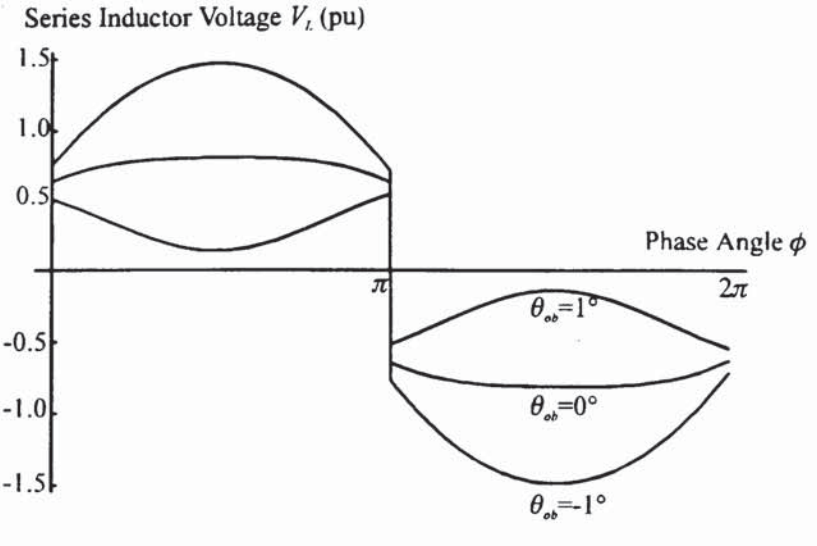

Figure 5.9 Graph Showing the Voltage across the Series Inductor for Several Values of the Firing Angle

In an effort to make the relationship between the voltage across the series inductor and the firing angle easier to see and to examine the harmonic content of the voltage a Fourier analysis was conducted. The algebra was found to be easier to manipulate if the fourier analysis was first conducted on the current through the inverter (I_C). The equation derived for the current through the inverter in terms of its harmonics (I_{Cn}) is given below.

$$I_{Cn} = Ir\frac{\sin(\phi+\theta_o+\gamma)}{h} - Ir\frac{4k(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)\cos(\theta_o+\gamma)}{h\pi(k\sinh\beta\pi-\beta\sin k\pi)}\sum_{n=1}^{\infty}\frac{\sin(n\phi+\gamma_n)}{h_n} \tag{5.8}$$

The voltage across the series inductor can then be found using the same method described earlier, namely subtracting the inverter current from the supply current and differentiating. This equation for the voltage across the series inductor in terms of the harmonics is shown by equation (5.9) below.

$$V_{Ln} = IX_L \sin(\phi + \theta_o) - IX_L r \frac{\cos(\phi + \theta_o + \gamma)}{h}$$
$$+ IX_L r \frac{4k(k^2 + \beta^2)(\cosh \beta\pi + \cos k\pi)\cos(\theta_o + \gamma)}{h\pi(k \sinh \beta\pi - \beta \sin k\pi)} \sum_{n=1}^{\infty} \frac{n \cos(n\phi + \gamma_n)}{h_n} \tag{5.9}$$

The fundamental is of most importance to the compensator and this is obtained from the above equation by setting n=1. However it is easier to see the effect of the firing angle of the inverter on the voltage across the series inductor by splitting the fundamental into the active and reactive power components. As it is the reactive power component that directly contributes to compensation. It shall be given first in equation (5.10).

$$V_q = 1 + r(q_1 + q_2\theta_{ob}) \tag{5.10}$$

Where r is the ratio of series inductance to the total inductance in the circuit. This equation shows that the series compensator will provided a basic level of compensation, $1+rq_1$. Later it will be shown that q_1 is very small and can be neglected, giving a basic compensation of 1 pu. This is the voltage dropped across the series element with no additional compensation being applied. Altering the firing angle of the inverter will give a linear variation in the degree of compensation. The response of the quadrature voltage across the series inductor to variations in the firing angle is shown in figure 5.10

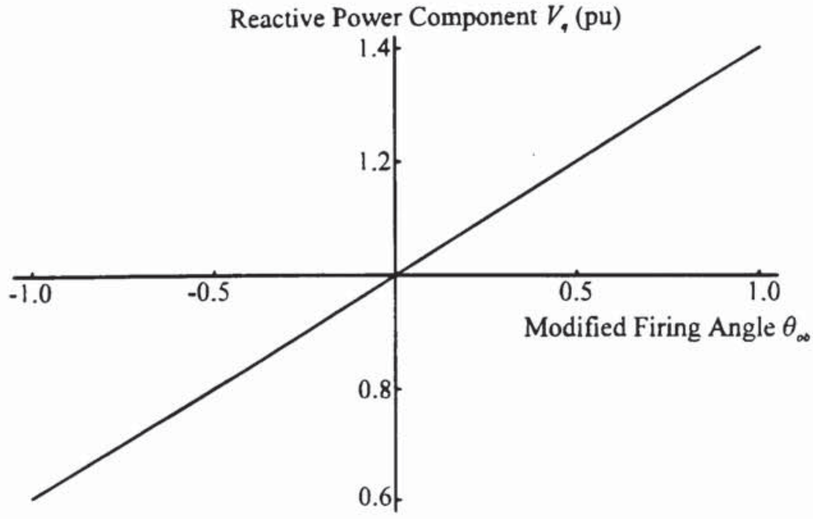

Figure 5.10 The Response of the Reactive Component of the Compensator Voltage to the Modified Firing Angle

The losses of the compensator are represented by the component of the fundamental in phase with the supply current, this equation is given below.

$$V_p = r\beta(p_1 + q_2\theta_{ob}^2) \tag{5.11}$$

Again, later examination of the constants shows that the constant p_1 is very small and can be ignored. This equation shows that the losses are dependant upon the square of the inverter firing angle, and as such if the firing angle becomes large the compensator becomes very lossy. The figure 5.11 below shows this square relationship.

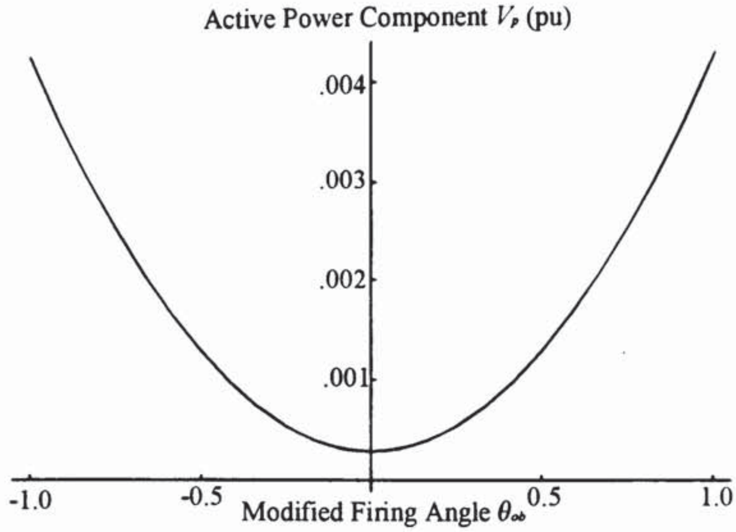

Figure 5.11 The Response of the Active Component of the Compensator Voltage to the Modified Firing Angle

The magnitude of the harmonics is very important as there are tight restrictions on the harmonics allowed to be injected onto electricity transmission lines. The magnitude of the odd harmonics excluding the fundamental is given in equation (5.12) below.

$$V_n = 2q_2 r \frac{n}{h_n}(1-\theta_o \tan\gamma) \tag{5.12}$$

The firing angle of the inverter has a linear relationship with the magnitude of the harmonics with a negative gradient. The magnitude of the harmonics is shown graphically by figure 5.12.

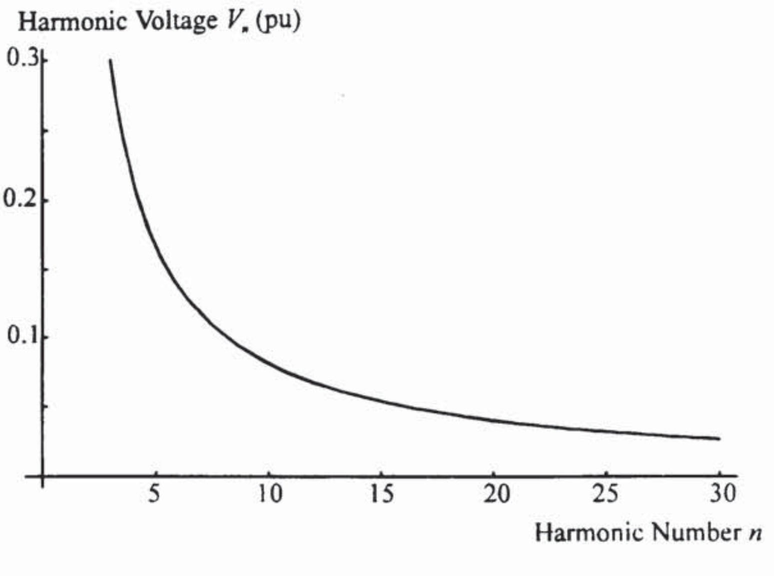

Figure 5.12 Graph Showing the Magnitude of the Harmonic Voltages

5.4 Simulation Results

Simulations were an significant part of the analysis of the series compensators examined in this thesis. Simulations were conducted initially to examine various configurations of series compensator. The algebraic analysis was then conducted, and the simulator was used to check that there was agreement with the algebraic analysis.

The simulation package used was called Saber. The circuit used for the single phase voltage sourced inverter series compensator using an inductor as the series element was very similar to the circuit examined in chapter 3. The major difference being the series inductor replacing the series capacitor in the phase conductor.

The graphs obtained by conducting the simulations were very similar to the graphs obtained by plotting the equations derived from the algebraic analysis. They are shown in this section to act as a comparison to the graphs shown earlier in this chapter.

The first graph shown (figure 5.13) is the current through the inverter for several firing angles. The graph clearly shows that the current changes phase by 180° when the firing angle is changed from a positive angle to a negative one.

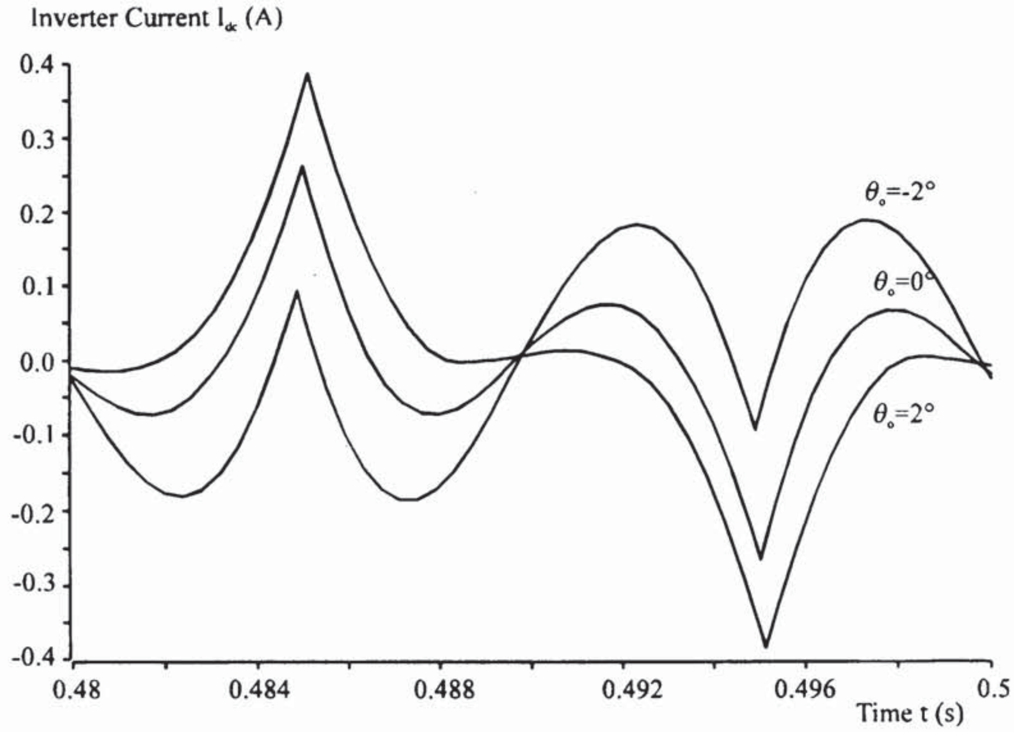

Figure 5.13 Simulation Graph Showing the Inverter Current for Several Values of the Firing Angle

There are some differences between the graphs of the simulated circuit and the plots of the equations developed in the appendices. These are due to slightly different values used for the damping factor and the reactance ratio. The values used for the mathematical equations were chosen to produce the optimum compensation. The values used for the simulations were chosen to provide meaningful results given the limitations of the computer hardware, as mentioned earlier.

Next the simulated voltage across the d.c. terminals of the inverter is given in figure 5.14. The variation of the firing angle shows that the voltage changes from domes to troughs.

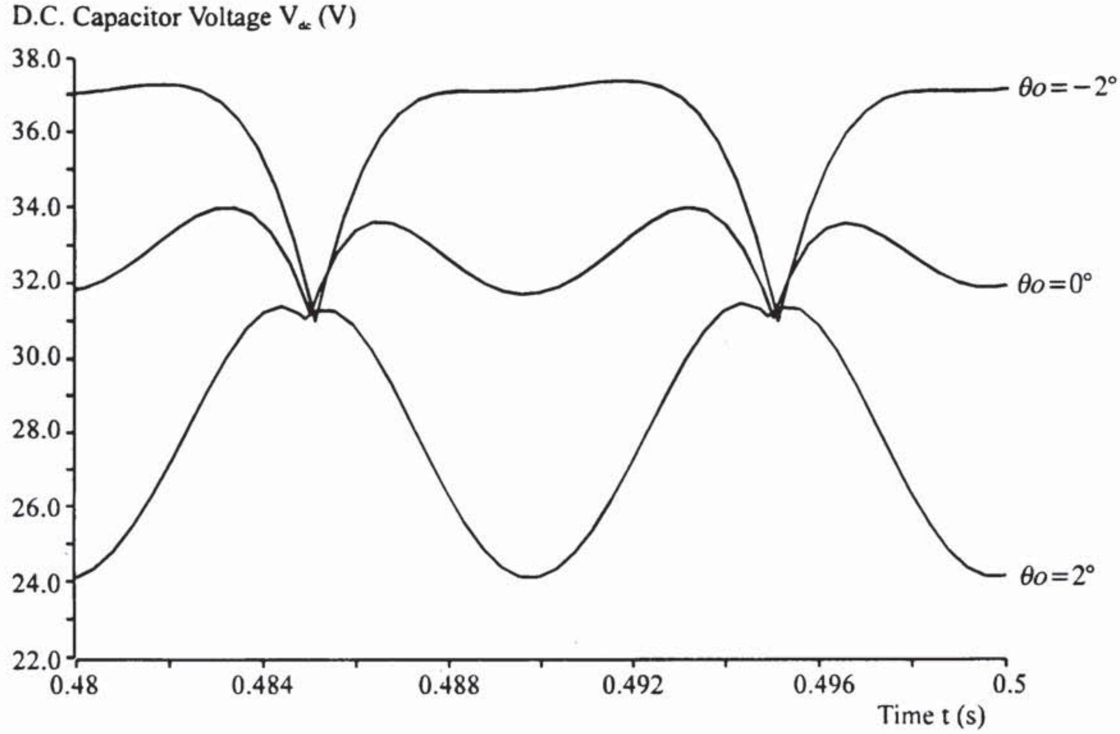

Figure 5.14 Simulation Graph Showing the D.C. Capacitor Voltage for Several Values of the Firing Angle

The current through the series inductor is found by subtracting the inverter current from the supply current. This has a similar effect to the current through the series capacitor shown in chapter 3. The current through the series inductor does not change phase significantly in the manner that the current through the inverter does. Obviously there is a small phase shift due to the firing angle of the inverter. The series inductor current is shown in figure 5.15 below.

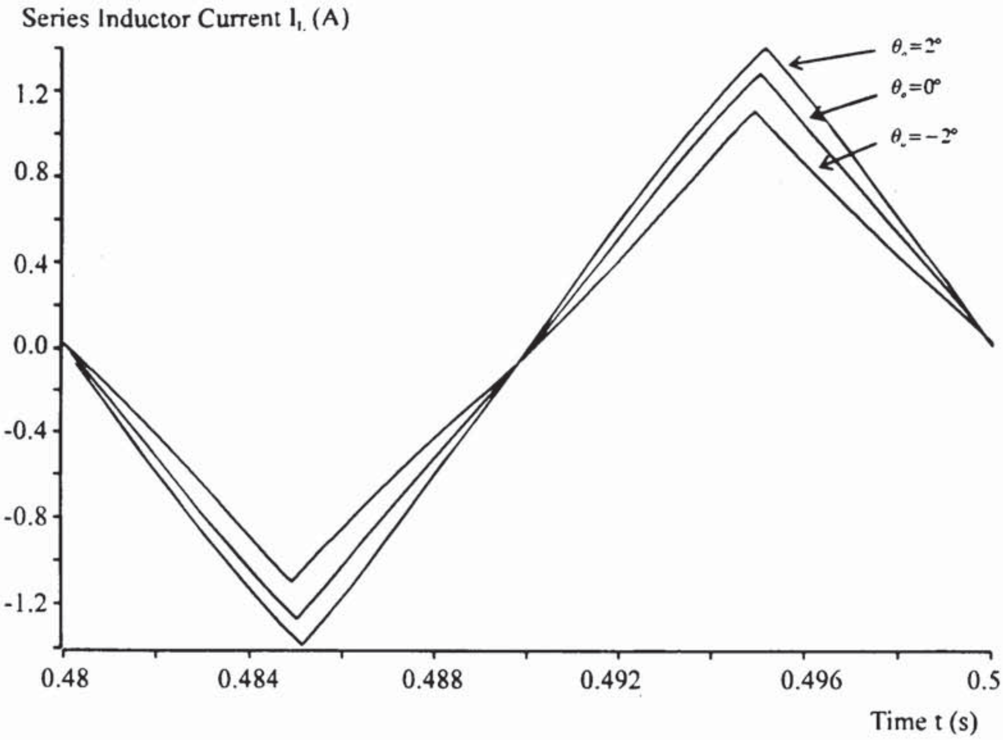

Figure 5.15 Simulation Graph Showing the Series Inductor Current for Several Values of the Firing Angle

The voltage across the series inductor is the most important from a compensation point of view and this is shown in figure 5.16 below. The voltage is obtained by differentiating the current through the series inductor. This differentiation increases the harmonic content of the compensating voltage quite considerably and this is readily apparent in the graph.

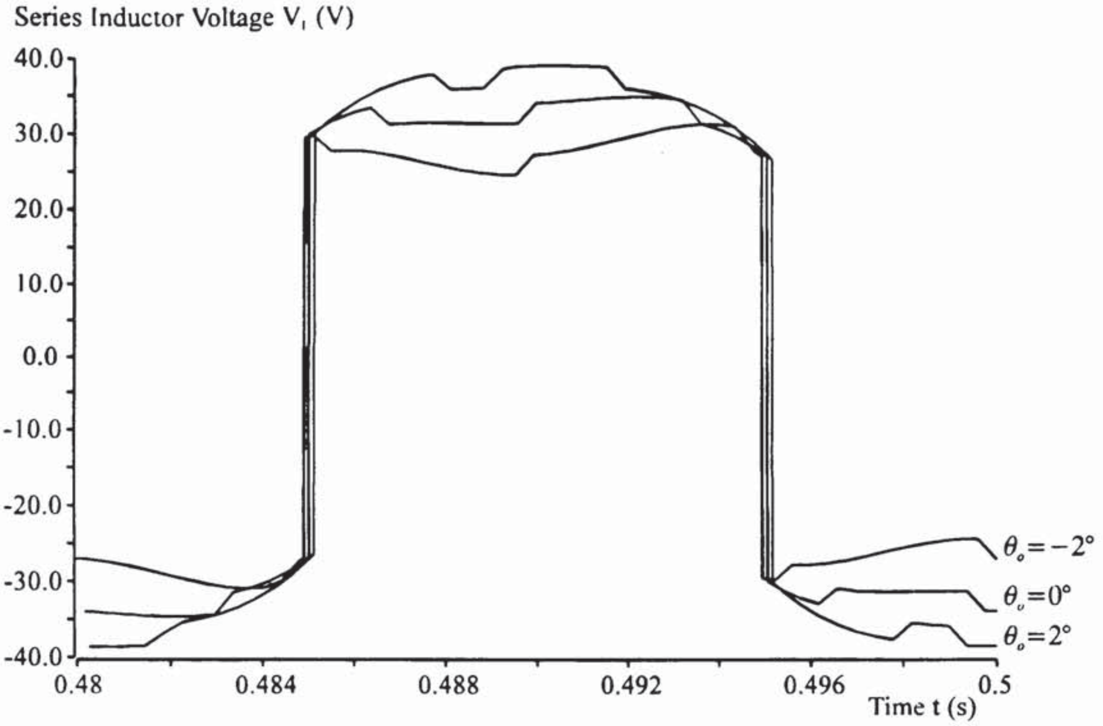

Figure 5.16 Simulation Graph Showing the Voltage across the Series Inductor for Several Values of the Firing Angle

The harmonic content of the voltage across the series inductor is large. This is obvious even by a visual inspection of figure 5.16. Figure 5.17 shows the simulation graph of the magnitude of the harmonics for the series inductor voltage. This graph indicates that the third harmonic is approximately 45% of the fundamental.

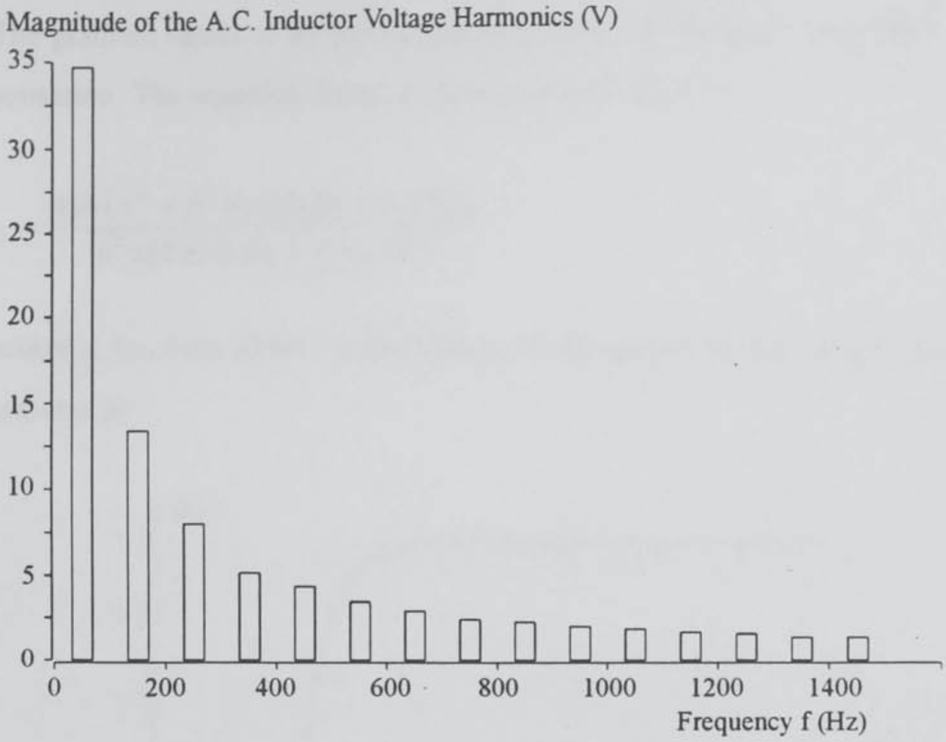

Figure 5.17 Simulation Graph of the Harmonic Content of the Series Inductor Voltage

These simulation graphs agree well with the graphs plotted earlier from the algebraic analysis.

5.5 Circuit Parameters

There are three parameters which affect the degree of compensation obtained by the series compensator. These are the reactance ratio k, the damping factor β, and the ratio of series inductance to the total inductance r. The information that is required is the degree to which these parameters effect the quadrature compensating voltage V_q the losses in the compensator V_p and the harmonics introduced by the compensator V_n.

Examining first the compensating voltage V_q. This voltage is defined by the equation (5.10) given earlier. This voltage has a linear relationship with the control parameter θ_{ob}, where the constant q_2 gives the gradient of the response and q_1 gives the intercept at

θ_{ob}=0. The gradient needs to be the maximum possible as this gives the widest variation of compensation. The equation for q2 is given in equation (5.13).

$$q_2 = \frac{4\beta k(k^2 + \beta^2)(\cosh\beta\pi + \cos k\pi)}{h^2\pi(k\sinh\beta\pi - \beta\sin k\pi)} \tag{5.13}$$

The constant q_2 has been plotted below (figure 5.18) against $1/k$ for several values of the damping factor β.

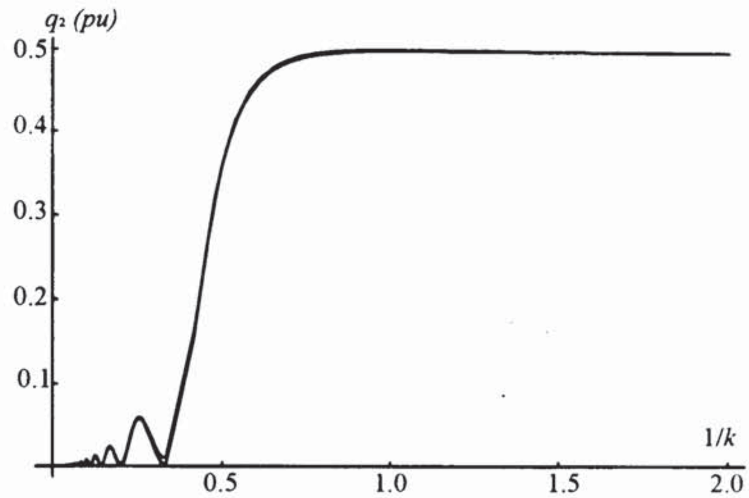

Figure 5.18 Graph Showing q_2 against $1/k$

The graph has been plotted against the reciprocal of the reactance ratio because it is easier to see the response. The constant q_2 was also plotted for several values of the damping factor. For $0.01 \leq \beta \leq 0.1$ and it is obvious that the damping factor has very little effect on the compensator voltage.

The above graph shows that q_2 is at a maximum when k is close to 1 and it remains fairly constant above 0.5. In actual fact q_2 slowly decreases as k reduces from 1. This suggests that in order to maximise the degree of compensation k should be close to 1.

The intercept is given by equation (5.14). The graph of this equation is shown in figure 5.19.

$$q_1 = \frac{\sin\gamma}{h}(1 - 2q_2) \tag{5.14}$$

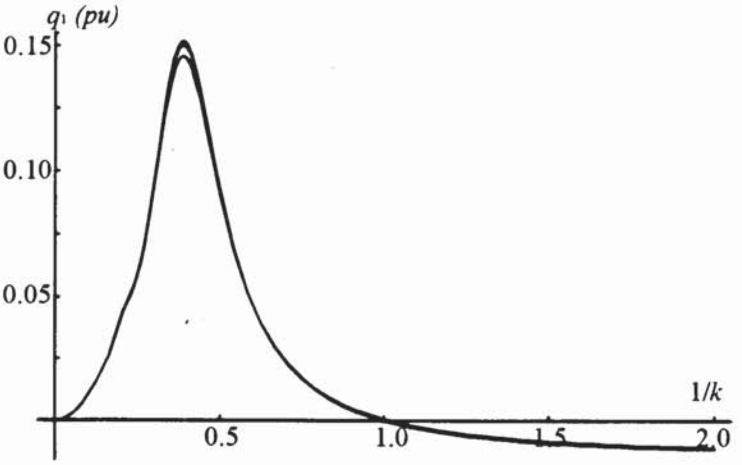

Figure 5.19 Graph Showing q_1 against $1/k$

Again q_1 has been plotted for several values of the damping factor β, and once again the damping factor has very little effect. The graph shows that q_1 is approximately zero when the reactance ratio, k, is close to 1.

The losses of the compensator V_p, is given by equation (5.11). This voltage has a square relationship with the modified firing angle. Again the gradient is dependant on q_2 and the intercept is given by p_1.

The constant q_2 has already been determined from figure 5.18 as 0.5. This indicates the maximum range of series compensation. This leaves the intercept of figure 5.11, the active component of the fundamental compensating voltage. The equation for this constant is given below in equation (5.15).

$$p_1 = \frac{\cos\gamma}{\beta h}(1 - 2q_2) \tag{5.15}$$

This intercept indicates the minimum losses of the compensator and any changes to the firing angle will only increase the losses. The constant p_1 should therefore be chosen to be as small as possible. The graph shown below shows p_1 plotted against the reciprocal of the reactance ratio and it has been plotted for several values of damping ratio.

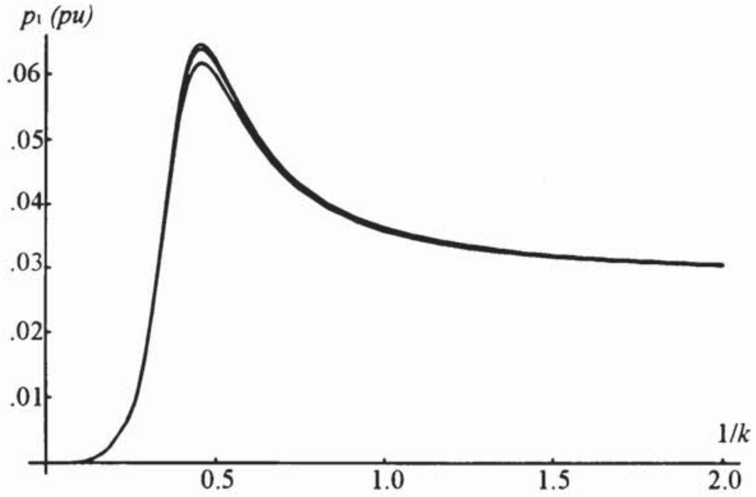

Figure 5.20 Graph Showing p_1 against $1/k$

The above graph shows that p_1 is not sensitive to the damping factor provided it is less than 0.1. and that the losses approach a minimum when the reactance ratio is less than 1. The magnitude of p_1 is very small in the range examined and could be considered to be negligable.

These graphs (figures 5.18, 5.19 and 5.20) suggest that the reactance ratio k should be chosen to be very close to 1.0 in order to maximise the compensation provided and keep the losses to a minimum.

The magnitude of the harmonics are also affected by the reactance ratio. The damping factor β has very little effect whilst it is kept less than 0.1. Figure 5.21 shows how the harmonics are effected by the reactance ratio k.

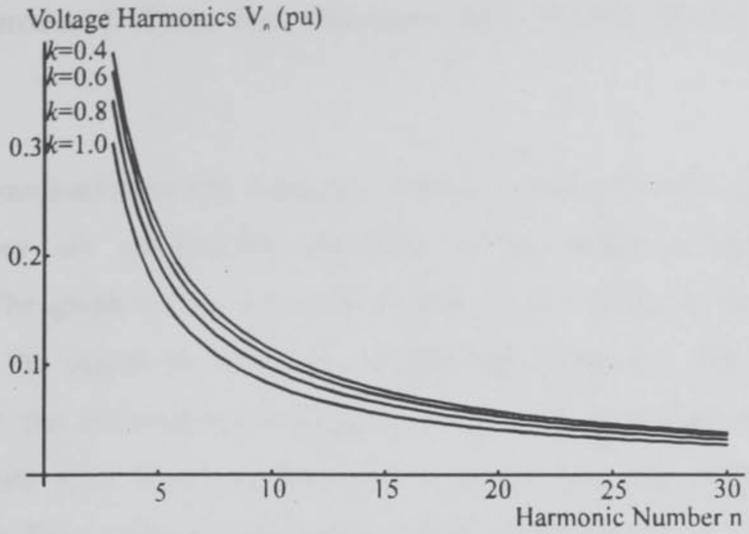

Figure 5.21 Graph Showing the Magnitude of the Voltage Harmonics against the Reciprocal of the Reactance Ratio

The graph shows that the magnitude of the harmonics are not significantly effected by the resonance ratio, apart from the third. This suggests that the total harmonic distortion would be considerably improved by a three phase compensator.

5.6 Conclusions

The single phase voltage sourced inverter series compensator using an inductor as the series element was examined with the belief that there may be a use for a series compensator whose primary role would be to maintain a fixed voltage. This voltage would be inductive. This would avoid the need for an extra, expensive, capacitor as is necessary with the voltage sourced inverter series compensator using a capacitive series element (see Chapter 3).

The analysis showed that the circuit examined would provide a continuously variable inductive voltage of approximately 1±0.2 pu. For the operating range of the compensator the active power dissipated by the circuit would be considerably lower than the compensation produced. These two statements fulfil the reasons for examining the circuit.

However an examination of the harmonic voltage introduced to a transmission line shows that they are considerable obstacles to this being a satisfactory series compensator. The graph of the harmonic voltage (figure 5.12) shows that the third harmonic is in the region of 30% and the fifth only drops to 15%. National Gird regulations state that the maximum magnitude of any single harmonic must be less than 1% and that the total harmonic distortion must be less than 1.5%. Clearly this compensator would need considerable extra circuitry to meet these regulations.

Chapter 6
Discussion of the Results

6.1 Introduction

This chapter compares the series compensators examined in this thesis. The comparison does not examine all aspects but it does highlight the main differences of the compensation systems.

The series compensators were compared according to several different criteria. Firstly, the steady state performance of each compensator was compared, i.e. the amount and range of compensation made available by the circuit. Secondly, the components involved in creating the compensator and their approximate relative costs are explored. Thirdly, the harmonics introduced by the compensators are compared and finally, the benefits of using three phase equipment is examined.

6.2 Compensator Performance

The first area of comparison is the performance of the compensators. The comparison is not as simple as one might assume. Two of the compensators will provide capacitive and inductive compensation and the other two will offer either capacitive or inductive compensation. This alone suggests that differing system requirements will favour one system over another.

Table 6.1, below shows the range of compensation available for each of the compensation schemes examined. The headings are:

TCSC - Thyristor Controlled Series Capacitor;

VSI Capacitive - Voltage Sourced Inverter series compensation using a Capacitor as the series element;

VSI Inductive - Voltage Sourced Inverter series compensation using an Inductor as the series element;

CSI - Current Sourced Inverter.

Range of Output Voltage (pu):			
TCSC	*VSI Capacitive*	*VSI Inductive*	*CSI*
-1 to -3 and 0 to 2	-0.8 to -1.2	0.8 to 1.2	-1 to +1

Table 6.1 Table Showing the Output Range of the Examined Series Compensators

The ranges are shown in per unit form, where the reference is taken to be the voltage dropped across the series element when no switching is occurring.

The thyristor controlled series capacitor has the widest range of available compensation. This is because the circuit operates around a resonance point, but that introduces potential additional hazards. There is a range of capacitive compensation that is unavailable between 0 and 1 pu, which may be a disadvantage for this type of series compensator. The thyristor controlled series capacitor can also operate in more than one stable state, as was described in chapter 2. Efforts must be made to avoid this situation as only one of these stable states is useful for compensation. The others introduce significant losses.

Both of the voltage sourced inverter compensators will offer a basic compensation level of 1 pu, with a variation of approximately $\pm$ 0.2 pu. This variation is controlled by the firing angle of the inverter and is continuously variable. Unfortunately, the compensator with the capacitive series element will only offer capacitive compensation and the series compensator with the inductive series element will only provide inductive compensation. A voltage sourced inverter compensation scheme without any series element would probably allow both capacitive and inductive compensation, with the disadvantage of increasing the harmonic content of the compensating voltage. This makes the voltage sourced inverter series compensators less flexible than either the thyristor controlled series capacitor or the current sourced inverter series compensator. Neither of the two voltage sourced inverter compensators operate around a resonance point. This means that the control system may not need to be quite as sensitive as the control system for the thyristor controlled series capacitor. This factor might reduce costs.

The current sourced inverter series compensator will allow continuously variable compensation between 1 pu capacitive and 1 pu inductive. The compensation circuit is very similar to the thyristor controlled series capacitor. The similarities are so strong that the current sourced inverter series compensator can be made to act in the same manner as the thyristor controlled series capacitor when operating in capacitive mode. This is obtained by altering the firing of the inverter by approximately 180°. Again, the current sourced inverter series compensator does not operate about a resonance point.

6.3 Size of Components

As the thyristor controlled series capacitor has a band of compensation that cannot be accessed it is the minimum compensation requirement that determine the size of the components. This could be a disadvantage for the thyristor controlled series capacitor. If a transmission system has the need for a low minimum degree of series compensation, the size of the capacitor can become large. This is because the thyristor controlled series compensator will only provide a minimum of 1 pu capacitive compensation, so the reactance of the capacitive element needs to be small and therefore the size of the capacitor will increase. Aside from the cost of the power electronic switches the capacitor is probably the most significant cost of the thyristor controlled series capacitor and this could have a considerable effect on the costs of the compensator.

The voltage sourced inverter series compensator with the inductor as the series element will not provide any capacitive compensation, which is one possible disadvantage for this scheme. However, if the requirement for compensation is simply to deal with transients in the electricity supply and to provide stabilising current, the inductive scheme has the advantage of lower costs. The size of the components are dependant upon the degree of compensation required.

The voltage sourced inverter series compensator using a capacitor as the series element will only produce capacitive compensation. The range of compensation is limited when compared to both the thyristor controlled series capacitor and the current sourced inverter series compensator.

The current sourced inverter series compensator is probably the most flexible compensation system. It does not have the restrictions of the thyristor controlled series capacitor in terms of having a limit on the minimum capacitive compensation. In fact as the maximum compensation required is increased the size of the capacitor will decrease, obviously this has to be balanced against the increasing size of the inductor.

Table 6.2 below gives an indication of the costs of the series compensators examined in the thesis. It is assumed that the cost of an inductor is approximately a third of the cost of a capacitor. The cost of the switches has not been considered as all the compensators examined would use similar power electronic switches. The thyristor controlled series capacitor only uses two electronic switches as opposed to the four switches required by the other series compensators. This may be offset slightly by the belief that the other series compensators do not require such a sophisticated control system. Also when a three phase situation is examined the thyristor controlled series capacitor needs exactly the same number of switches as the inverter based series compensators.

	Capacitor	*Inductor*	*Est. Cost*
TCSC	1@1.25pu	1@0.13pu	1.29pu
VSI Cap	2@1.0pu	1@2.0pu	2.67pu
VSI Ind	1@0.5pu	2@1.0pu	1.17pu
CSI	1@0.83pu	1@1.2pu	1.23pu

Table 6.2 Number of Components and Total Costs
for the Single Phase Series Compensators

A cost of 1.00 is the cost of a capacitor providing 1 pu voltage compensation. The comparison of the costs is a little unrealistic as each compensator has very different characteristics. The criteria for comparison were chosen so all the compensators could be compared and was based upon the performance of the two voltage sourced inverter series compensators (1±0.2 pu). The current sourced inverter series compensator and the thyristor controlled series capacitor can easily exceed these criteria. However the table does give a reasonable indication of the relative costs of the compensators.

The voltage sourced inverter series compensator using an inductor as the series element, appears to be the least expensive compensator and this was the main reason for studying this form of series compensation.

The current sourced inverter series compensator is the next cheapest form of series compensation and has the additional advantage of providing both capacitive and inductive compensation.

The thyristor controlled series capacitor is the second most expensive form of series compensation. Although it should be mentioned that the difference in costs between the least expensive three compensators is small, and the calculation of these costs are provided as rough guidelines.

The most expensive compensation method is the voltage sourced inverter series compensator using a capacitor as the series element, this is because it is the only circuit to utilise two capacitors. This compensator is significantly more expensive than the other methods as a result.

None of the costs shown take into account the extra circuitry required to reduce or eliminate voltage harmonics. Elimination of harmonics can be a significant proportion of the total cost of a compensation method. The next section explores the size of these harmonics.

6.4 Harmonics

Table 6.3 shows the magnitude of the 3rd, 5th and 7th order voltage harmonics for the series compensator examined in this thesis. The figures shown were obtained from the simulations conducted on the circuits. The figures for the three phase voltage sourced inverter series compensator are meant as an indication of the benefits of using three phase equipment. They can not be relied upon completely as there have been no subsequent studies conducted, other than an initial exploratory simulation.

Harmonic	*TCSC*	*VSI Cap*	*VSI Ind*	*CSI*	*3ϕ VSI Cap*
3^{rd}	20%	3%	38%	12%	0%
5^{th}	7%	>1%	23%	5%	>1%
7^{th}	2%	>1%	15%	2%	>1%

Table 6.3 Table Showing the Magnitude of Individual Harmonics for the Series Compensators Examined

The above table shows that the voltage sourced inverter series compensator using an inductor as the series element has very large harmonics and it would require a great deal of effort to bring this compensator in line with the National Grid regulations on harmonics which have been detailed in Chapter 1. In fact, with the exception of the three phase voltage sourced inverter using a capacitor as the series element, all of the compensators would require harmonic reduction measures in order to fulfil the National Grid regulations.

Table 6.4 shows the total harmonic distortion introduced by the various compensators to the series compensating voltage.

TCSC		*VSI Cap*		*VSI Ind*		*CSI*		*3ϕ VSI Cap*	
α	*T.H.D.*	θ_o	*T.H.D.*	θ_o	*T.H.D.*	θ_o	*T.H.D.*	θ_o	*T.H.D.*
50°	32.6%	-2°	2.6%	-2°	47.3%	3°	21.1%	-3°	0.5%
60°	12.6%	0°	3.0%	0°	40.4%	4°	83.7%	0°	0.7%
		2°	3.4%	2°	36.6%	5°	12.5%	3°	1.0%

Table 6.4 Table Showing the Total Harmonic Distortion of the Compensating Voltage for the Series Compensators Examined

The total harmonic distortion introduced by the series compensators are, with one exception, in excess of the National Grid regulations. The two tables detailing the magnitudes of individual harmonics and the total harmonic distortion show that the cost of the series compensators are inversely related to the amount of harmonics introduced to the system. It may be that the cost of eliminating harmonics for any of the compensators, on top of the cost of the components, makes one of the compensators more appealing than the others. However, this comparison was not the purpose of this thesis.

The three phase voltage sourced inverter using capacitors as the series elements, met the National Grid regulations for voltage injection and this would have benefits in terms of the cost of equipment. This is examined in more detail in the next section.

6.5 Three Phase Compensators

The electricity transmission system managed by the National Grid plc. is a three phase system. Generally, there are advantages to using three phase equipment over single phase. This chapter will examine some of the possible advantages and disadvantages of using three phase series compensators as opposed to the single phase compensators studied throughout this thesis.

Of all the series compensators examined previously in this thesis only the thyristor controlled series capacitor does not benefit from the translation to three phases. The three phase thyristor controlled series capacitor would consist of three single phase circuits. When compared to the inverter based series compensation schemes this could be a disadvantage. It has already been shown that the harmonic content of the compensating voltage is high. They will not be reduced by a three phase scheme. Table 6.5 shows the number of capacitors, inductors and there probable magnitudes for a three phase series compensation scheme based upon the circuits examined in this thesis. The size of the components is based upon the comparisons made in the single phase comparison (Table 6.2). A value of 3.00pu equates to three fixed series capacitors.

	Capacitor	*Inductor*	*Est. Cost*
TCSC	3@1.25pu	3@0.13pu	3.88 pu
VSI Cap	3@1.0pu + 1@0.67pu	3@2.0pu	5.67 pu
VSI Ind	1@0.33pu	6@1.0pu	2.33 pu
CSI	3@0.83pu	1@0.8pu	2.77 pu

Table 6.5 Number of Components and Total Costs
for the Three Phase Series Compensators

For the inverter based series compensators the advantages of three phases are the same. The current sourced inverter series compensator is shown in three phase configuration below in figure 6.1.

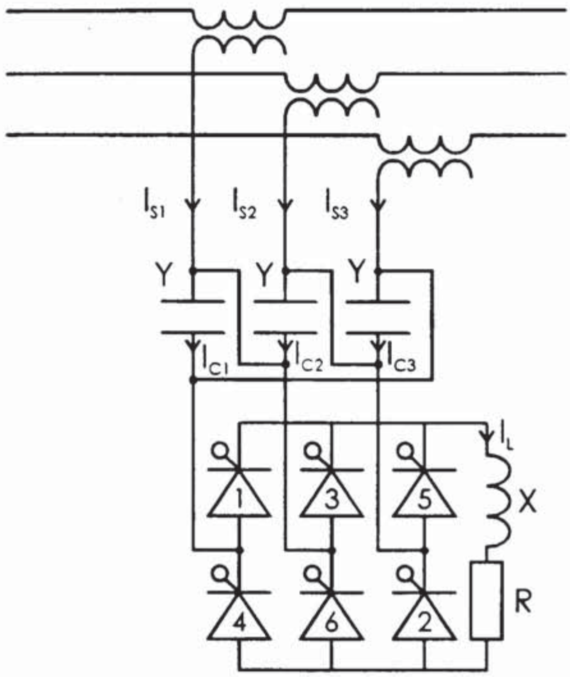

Figure 6.1 Circuit Diagram of the Three Phase Current Sourced Inverter Series Compensator

The number of switches required in the inverter increases from four to six. This is the same as required by three, single phase thyristor controlled series capacitors. The size of the energy store on the d.c. side of the inverter is likely to be smaller than in the single phase equivalent (Hill 94). Unfortunately, a series transformer would be required to connect the inverter based compensator to the transmission line. This would increase costs considerably if the transformer had to be built and installed specifically for the compensator. The increased use of quadrature boosters on the National Grid might allow this type of compensator to be attached to the electricity transmission system at a lower cost.

The harmonic content of three phase inverter based series compensators is considerably improved when compared to their single phase equivalents (Hill 94). The six pulse inverter removes the triplen harmonics and reduces the other harmonics. In the case of the inductive voltage sourced inverter series compensator this is definitely not enough to allow connection to the National Grid without additional harmonic reduction equipment. This is probably also the case for the current sourced inverter series compensator. Although the harmonic content of the 3 phase current sourced inverter series compensator will not require nearly as much equipment to eliminate excessive harmonics.

Initial studies have shown that the three phase voltage sourced inverter series compensator using capacitors as the series elements could be connected to the transmission system without any additional harmonic reduction/elimination measures.

The circuit diagram of one configuration of the three phase voltage sourced inverter series compensator is shown below.

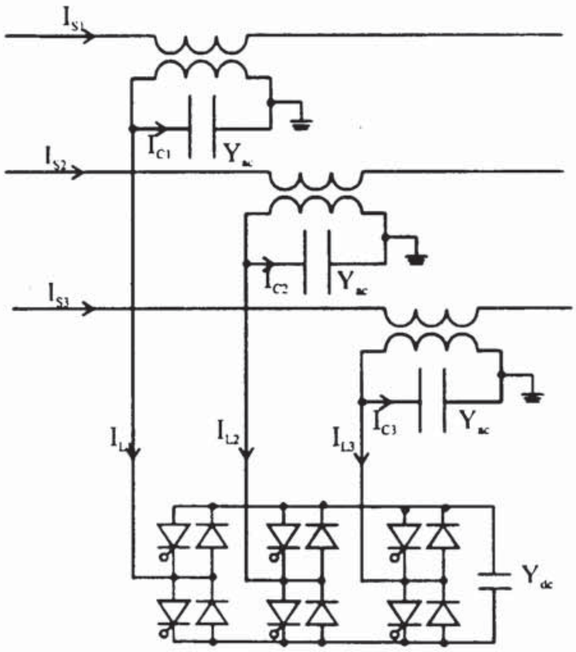

Figure 6.2 Circuit Diagram of a Three Phase Voltage Sourced Inverter Series Compensator using Capacitors as the Series Element

Tables 6.3 and 6.4 indicate that the magnitude of the largest harmonic in the compensating voltage would be less than 1% and the total harmonic distortion would be less than 1.5%. These figures came from a simulation of the circuit which was conducted to make a brief comparison between the single phase and three phase inverter series compensators. Figure 6.3 shows a graph of the magnitude of the voltage harmonics for the three phase voltage sourced inverter series compensator using capacitors as the series elements.

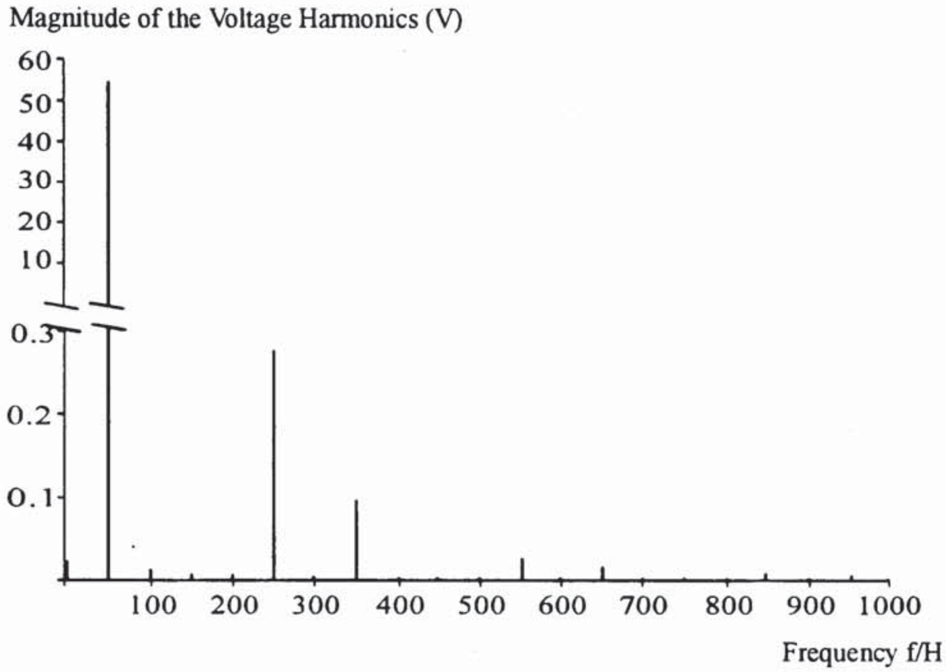

Figure 6.3 Graph showing the Magnitude of the Voltage Harmonics

6.6 Summary of Results

To summarise the points discussed in this chapter.

1) The **thyristor controlled series capacitor** has the widest range and the maximum available compensation and the two voltage sourced inverter series compensators offer the smallest range of compensation.

2) The **voltage sourced inverter series compensator using an inductor as the series element** is the least expensive method of providing series compensation and the voltage sourced inverter using a capacitor as the series element is by far the most expensive.

3) There are harmonics introduced by every series compensator examined but these are kept to a minimum by the voltage sourced inverter series compensator using a capacitor as the series element. The **voltage sourced inverter using an inductor as the series element** introduces the highest level of harmonic distortion.

4) Three phase equipment has advantages for all the inverter based series compensators, in terms of harmonics and relative costs. The price differential between the voltage sourced inverter series compensator using capacitors as the series elements and the thyristor controlled series capacitor is reduced. Also the harmonic content of the **three phase voltage sourced inverter series compensator using capacitors as the series elements** is low enough to allow connection to the transmission system without additional equipment.

Chapter 7
Conclusions

7.1 Introduction

This thesis examined series compensation of electricity transmission lines. More specifically, the steady state performance of four types of series compensator were studied. All four compensators utilise power electronic switches. The thyristor controlled series capacitor, examined in chapter 2, uses back to back thyristors, whilst the other three series compensators examined, all utilise fully-controllable electronic switches in an inverter.

7.2 Analysis

In all cases an algebraic analysis was conducted to provide an expression for the compensation voltage in terms of the firing angle of the electronic switches. A summary of the important formulae are given in the following paragraphs.

7.2.1 Thyristor Controlled Series Capacitor

The compensating voltage is controlled by the firing angle of the thyristors, α. The compensator operates about a resonance angle α_o. The circuit was split into three era and the compensating voltage was analysed for each era.

These three eras are :

$$\text{Era 1} : 0 \leq \theta \leq \alpha \qquad (7.1)$$

$$\text{Era 2} : \alpha < \theta < \lambda \qquad (7.2)$$

$$\text{Era 3} : \lambda \leq \theta \leq \pi \qquad (7.3)$$

The voltages for each era are given by the following equations.

$$V_1 = \frac{I}{Y}\frac{k}{(k^2-1)}\left\{\frac{\sin\alpha\cos k(\lambda-\alpha)+k\cos\alpha\sin k(\lambda-\alpha)-\sin\lambda}{\sin k(\lambda-\alpha)}\right\}-\frac{I}{Y}\cos\theta \quad (7.4)$$

$$V_2 = \frac{I}{Y}\left\{\frac{k[\sin\alpha\cos k(\theta-\lambda)-\sin\lambda\cos k(\theta-\alpha)]}{(k^2-1)\sin k(\lambda-\alpha)}+\frac{\cos\theta}{k^2-1}\right\} \quad (7.5)$$

$$V_3 = \frac{I}{Y}\frac{k}{(k^2-1)}\left\{\frac{\sin\alpha-\sin\lambda\cos k(\lambda-\alpha)+k\cos\lambda\sin k(\lambda-\alpha)}{\sin k(\lambda-\alpha)}\right\}-\frac{I}{Y}\cos\theta \quad (7.6)$$

The thyristor controlled series capacitor will provide both capacitive and inductive compensation. The degree of compensation is continuously variable and controlled by a thyristor switched reactor.

There is more than one steady state solution to the system equations and care must be taken when choosing the compensator parameters (namely k) in order to minimise the number of solutions.

Practical experience has found it impossible to change from capacitive mode to inductive mode quickly. The voltage across the thyristor controlled series capacitor must be forced into the correct phase first. For capacitive operation, this is done by taking the inductor out of the compensation circuit, allowing a capacitive voltage to build up. For inductive operation, the inductor must be allowed to remain continuously in the compensation circuit. Again, this enables an inductive voltage to build up across the compensator.

Switching across the resonance angle without creating the correct voltage across the thyristor controlled series capacitor causes the circuit to switch from one solution ($\lambda = \pi - \alpha$) to another ($\lambda = n\pi/k + \alpha$). This second solution introduces very high voltages and increased losses.

7.2.2 Voltage Sourced Inverter Series Compensator using a Capacitor as the Series Element

The single phase voltage sourced inverter series compensator, can be used to provide a variable capacitive voltage across a series connected capacitor. The control of this variable voltage is obtained via the firing of the power electronic switches in the inverter. The compensator could be used to respond to transient changes in the system voltage due to the fast response of the electronic switches.

The series compensator described in chapter 3 offers a continuously variable quadrature voltage which can be varied by approximately plus or minus twenty percent on the basic voltage dropped across the series capacitor.

$$V_q = -1 + r(k^2 + \beta^2)(q_1 + q_2\theta_{ob}) \tag{7.7}$$

The active power dissipated by the compensator is dependant upon the square of the firing angle of the inverter and is shown below.

$$V_p = r\beta(k^2 + \beta^2)(p_1 + q_2\theta_{ob}^2) \tag{7.8}$$

The magnitude of the harmonics introduced to the compensating voltage by the switching in the inverter are relatively small.

$$V_n = -r(k^2 + \beta^2)\frac{2q_2}{nh_n}(1 - \theta_o \tan\gamma) \tag{7.9}$$

The voltage sourced inverter series compensator using a capacitor as the series element has a relatively small range of available compensation.

7.2.3 Current Sourced Inverter Series Compensator

The current sourced inverter series compensator is one possible solution to all the requirements of a series compensator. It will introduce a capacitive voltage in series with a transmission line for the classic compensation requirements, and it can also be used to provide an inductive voltage in series with the transmission line. The use of an inverter made up of power electronic switches introduces a fast response to transient conditions, and the flexibility of a continuously variable series quadrature voltage from capacitive to inductive.

The compensating component of the series capacitor voltage is given below. It can be continuously and linearly varied from 1pu inductive to 1pu capacitive, by varying the firing angle. Unlike the voltage sourced inverter series compensators a negative firing angle is not used, only a wider range of a positive firing angle.

$$V_q = q_1' + q_2\theta_{ob} \tag{7.10}$$

The active power component of the compensator voltage is a square relationship which is shown by the equation below.

$$V_p = \beta(p_1' + q_2\theta_{ob}^2) \tag{7.11}$$

The harmonic component of the compensating voltage is also linearly dependant upon the firing angle of the inverter.

$$V_n' = \frac{2q_2}{h_n}(\theta_o \tan\gamma - 1) \tag{7.12}$$

The current sourced inverter series compensator is a very flexible method of providing series compensation, with less harmonic content than the thyristor controlled series capacitor.

7.2.4 Voltage Sourced Inverter Series Compensation using an Inductor as the Series Element

The single phase voltage sourced inverter series compensator using an inductor as the series element was examined with the belief that there may be a use for a series compensator whose primary role would be to maintain a fixed voltage. This voltage would be inductive. This would avoid the need for an extra, expensive, capacitor as is necessary with the voltage sourced inverter series compensator using a capacitive series element (see Chapter 3).

The analysis showed that the circuit examined would provide a continuously and linearly variable inductive voltage of approximately 1±0.2 pu. This relationship is given by equation (7.13).

$$V_q = 1 + r(q_1 + q_2\theta_{ob}) \tag{7.13}$$

For the operating range of the compensator, the active power dissipated by the circuit would be considerably lower than the compensation produced. Like the capacitive voltage sourced inverter compensation circuit, the losses are dependant upon the square of the firing angle.

$$V_p = r\beta(p_1 + q_2\theta_o^2) \tag{7.14}$$

However, an examination of the harmonic voltage introduced to a transmission line shows that there are considerable obstacles to this being a satisfactory series compensator.

$$V'_n = 2q_2 r\frac{n}{h_n}(1 - \theta_o \tan\gamma) \tag{7.15}$$

A small range of inductive compensation and the very high harmonic content of the compensating voltage mean that this circuit is unlikely to be used for compensating transmission lines.

7.3 Further Work

7.3.1 Thyristor Controlled Series Capacitor

The thyristor controlled series capacitor produced some unexpected results. It was assumed that there was little to discover about the circuit. However, simulations suggested that the circuit was not all that the published literature would lead one to believe. The literature appears to make an assumption about the symmetry of the compensating voltage and this only partially explains the response of the circuit to changes in the firing angle. By not assuming symmetry an analysis showed that there was more than one stable operating state for the thyristor controlled series capacitor.

These other operating states have not been fully explored by this thesis, neither has the response of the compensator in the region close to the resonance point of the circuit. A full exploration and analysis of these operating states, and the response of the compensator in the region close to the resonance point would be desirable.

7.3.2 Harmonic Reduction/Elimination

Harmonics are present in all the series compensators examined in this thesis. The introduction outlined several methods of reducing the harmonic content of inverter based circuits; pulse width modulation, multiple pulse arrangements and multi-level inverters.

This thesis has not examined the effect of these techniques on the series compensators. Work should be done in this area as harmonic reduction is necessary in order to meet National Grid connection requirements. Perhaps, more importantly, the cost of implementing harmonic reduction on any of the compensators might make one circuit more appealing than any of the others.

7.3.3 Three Phase

All the inverter based series compensator benefit from three phase implementation in terms of reduced harmonics and a reduction in the number and size of the energy storage components. Unfortunately, they also require a suitable series transformer.

Initial studies suggest that the three phase voltage sourced inverter series compensator using capacitors as the series elements could be connected to a transmission line without additional harmonic reduction equipment. This may also be true for the three phase current sourced inverter series compensator.

Further study should be undertaken to analyse the steady state performance of three phase inverter based series compensators.

7.3.4 Transient Performance

All the inverter based circuits should studied for their perfomance in response to disturbances. This could be due fault conditions or due to normal variations of the electricity system. The thyristor controlled series compensator which has been built will already have had a great deal scutiny. The transient performance of the inverter based compensators have not been examined.

7.4 Principle Conclusions

There were several criteria upon which the series compensators were compared. Firstly, the steady state performance of each compensator was compared, i.e. the amount and range of compensation made available by the circuit. Secondly, the components involved in creating the compensator and their approximate relative costs are explored. Thirdly, the harmonics introduced by the compensators are compared and finally, the benefits of using three phase equipment were examined.

The thyristor controlled series capacitor has the widest range and the maximum available compensation and the two voltage sourced inverter series compensators offer the smallest range of compensation.

1) The voltage sourced inverter series compensator using an inductor as the series element is the least expensive method of providing series compensation and the voltage sourced inverter using a capacitor as the series element is by far the most expensive.

2) There are harmonics introduced by every series compensator examined but these are kept to a minimum by the voltage sourced inverter series compensator using a capacitor as the series element. The voltage sourced inverter using an inductor as the series element introduces the highest level of harmonic distortion.

3) Three phase equipment has advantages for all the inverter based series compensators, in terms of harmonics and relative costs. The price differential between the voltage sourced inverter series compensator using capacitors as the series elements and the thyristor controlled series capacitor is reduced. Also the harmonic content of the three phase voltage sourced inverter series compensator using capacitors as the series elements is low enough to allow connection to the transmission system without additional equipment.

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Appendix A
Analysis of the Thyristor Controlled Series Capacitor

A-1 Introduction

The thyristor controlled series capacitor is a circuit that on first inspection appears to be relatively simple. It consists of a series capacitor with a thyristor controlled reactor connected in parallel. The thyristors are fired so that the current is allowed to flow through the inductor. This also enables control of the voltage across the compensator. It is also possible to force the voltage across the compensator to be inductive.

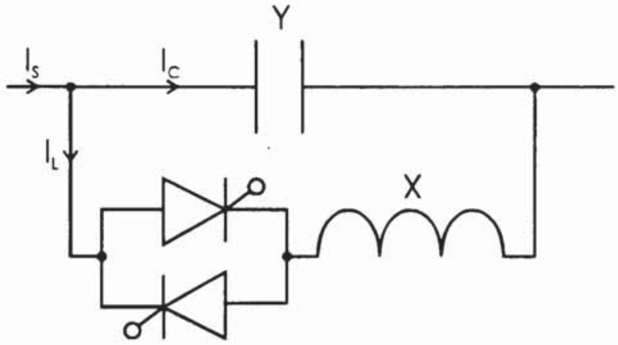

Figure A-1 Circuit Diagram of the Thyristor Controlled Series Capacitor

The thyristors are fired during one half of the system current's (I_S) cycle and the current that is allowed to flow through the inductor (I_L) decays to zero before the end of that half cycle. The thyristor controlled series capacitor, therefore, can be said to have three eras that need to be evaluated in order to derive the mathematical equations that fully describe the operational characteristics of the equipment.

These three eras are :

$$\text{Era 1} : 0 \le \theta \le \alpha \tag{A1}$$

$$\text{Era 2} : \alpha < \theta < \lambda \tag{A2}$$

$$\text{Era 3} : \lambda \le \theta \le \pi \tag{A3}$$

The angle α is when the thyristors are fired and current is allowed through the inductor. The angle λ occurs when the current through the inductor becomes zero and the thyristors block the current once again. These three eras are repeated every half cycle. This is shown graphically by figure A-2.

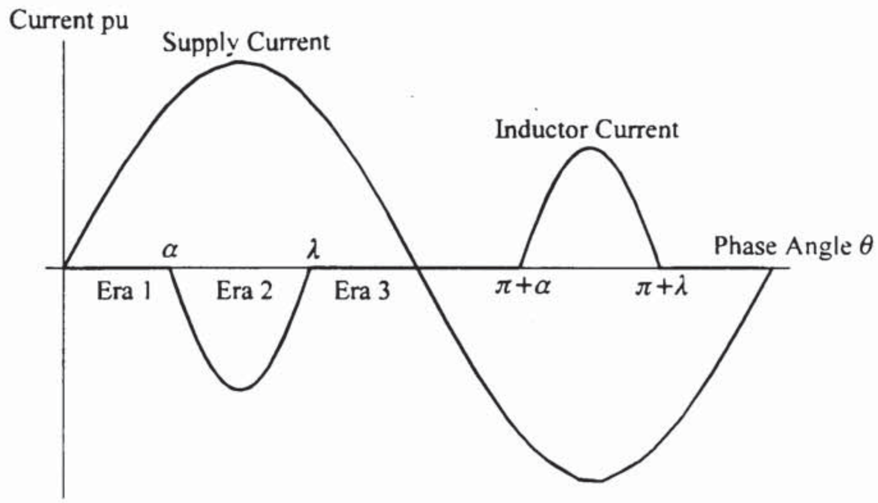

Figure A-2 Graph Showing the Three Era to be Analysed

A-2 The First Era

The circuit for the first era is show below.

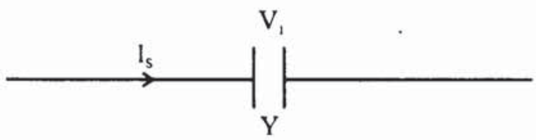

Figure A-3 Circuit Diagram for the First Era

In this condition the thyristors are not conducting and so the current through the inductor is zero. Therefore the current through the capacitor (I_C) must be identical to the system current which is assumed to be sinusoidal.

$$I_C = I \sin\theta \tag{A4}$$

The voltage across the capacitor is obtained by integrating the capacitor current.

$$V_1 = \frac{I}{Y}\int \sin\theta \, d\theta$$

$$= C_1 - \frac{I}{Y}\cos\theta \tag{A5}$$

The constant of integration is denoted by C_1, and will be found later in the analysis.

A-3 The Second Era

The circuit diagram for the second era is shown in figure A-4 below.

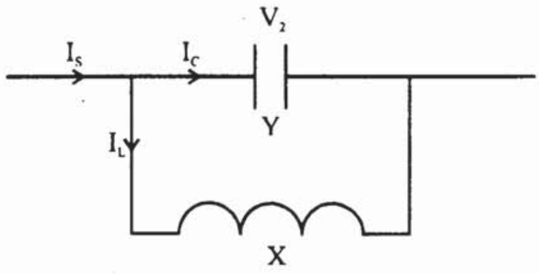

Figure A-4 Circuit Diagram for the Second Era

The thyristors are switched on at $\theta = \alpha$ and they will prevent conduction when the inductor current becomes zero once again, this point has been defined as λ.

A-3.1 Differential Equation

By using Kirchoff's Current Law it can be said that:

$$I_C + I_L = I \sin\theta \tag{A6}$$

The current through the inductor can be written in terms of the voltage dropped across the compensator as follows,

$$I_L = \frac{1}{X}\int V \, d\theta \tag{A7}$$

and the current through the capacitor can also be defined in terms of the voltage across the compensator.

$$I_C = Y\frac{dV}{d\theta} \tag{A8}$$

If equation (A7) and equation (A8) are substituted back into the expression for the currents (equation A6), differentiated once and then the formula is rearranged. This produces a second order differential equation for the voltage across the equipment.

$$\frac{d^2V}{d\theta^2} + \frac{1}{XY}V = \frac{I}{Y}\cos\theta$$

$$\frac{d^2V}{d\theta^2} + k^2V = \frac{I}{Y}\cos\theta \tag{A9}$$

The symbol k is the resonance ratio and is defined by equation (A10) below.

$$k = \frac{1}{\sqrt{XY}} \tag{A10}$$

The general solution for this type of differential equation (A9) is given by:

$$V_2 = \frac{I}{Y}(a_1 \cos k\theta + a_2 \sin k\theta + a_3 \cos\theta + a_4 \sin\theta) \tag{A11}$$

A-3.2 Solution for the Constants a_1, a_2, a_3 and a_4.

Differentiating equation (A11) twice produces the following equation:

$$\frac{d^2V_2}{d\theta^2} = -\frac{I}{Y}(k^2 a_1 \cos k\theta + k^2 a_2 \sin k\theta + a_3 \cos\theta + a_4 \sin\theta) \tag{A12}$$

Making the substitutions of equation (A12) and equation (A11) into the differential equation (A9) produces solutions for the constants a_3 and a_4.

$$(k^2 - 1)(a_3 \cos\theta + a_4 \sin\theta) = \cos\theta$$

By equating coefficients of sine and cosine the constant a_4 is obviously zero and the constant a_3 takes the following form.

$$a_3 = \frac{1}{k^2 - 1} \tag{A13}$$

Therefore the voltage across the compensator can be written as :

$$V_2 = \frac{I}{Y}\left(a_1 \cos k\theta + a_2 \sin k\theta + \frac{\cos\theta}{k^2 - 1}\right) \tag{A14}$$

The current through the capacitor during the second era is derived from equation (A8) and equation (A14).

$$I_C = I\left(-ka_1 \sin k\theta + ka_2 \cos k\theta - \frac{\sin\theta}{k^2 - 1}\right) \tag{A15}$$

The current through the inductor is derived by using equation (A7) and equation (A14) and by adding the constant of integration.

$$I_L = Ik^2\left(\frac{a_1}{k}\sin k\theta - \frac{a_2}{k}\cos k\theta + \frac{\sin\theta}{k^2 - 1}\right) + C_2$$

$$= I\left(ka_1 \sin k\theta - ka_2 \cos k\theta + \frac{k^2 \sin\theta}{k^2 - 1}\right) + C_2 \tag{A16}$$

Adding equation (A15) and equation (A16) together produces the following equation.

$$I_C + I_L = I\sin\theta + C_2$$

Equation (A6) which is derived from Kirchoff's current law indicates that the constant C_2 must be zero. Therefore the current through the inductor is given by:

$$I_L = I\left(ka_1 \sin k\theta - ka_2 \cos k\theta + \frac{k^2 \sin\theta}{k^2 - 1}\right) \tag{A17}$$

The current through the inductor must be zero at $\theta = \alpha$, so using this condition a relationship between a_1 and a_2 is obtained.

$$a_1 \sin k\alpha - a_2 \cos k\alpha + \frac{k \sin\alpha}{k^2 - 1} = 0 \tag{A18}$$

Also the current through the inductor is defined to be zero at $\theta = \lambda$ this produces another relationship between a_1 and a_2.

$$a_1 \sin k\lambda - a_2 \cos k\lambda + \frac{k \sin\lambda}{k^2 - 1} = 0 \tag{A19}$$

Rearranging equation (A18) and equation (A19) into a matrix format gives an expression which is more readily solved.

$$\begin{bmatrix} \sin k\alpha & -\cos k\alpha \\ \sin k\lambda & -\cos k\lambda \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = -\left(\frac{k}{k^2 - 1}\right) \begin{bmatrix} \sin\alpha \\ \sin\lambda \end{bmatrix} \tag{A20}$$

The determinant of this matrix is given below.

$$\det = -\sin k\alpha \cos k\lambda + \cos k\alpha \sin k\lambda \tag{A21}$$

Inverting the two by two matrix gives:

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = -\frac{k}{(k^2-1)\sin k(\lambda-\alpha)} \begin{bmatrix} -\cos k\lambda & \cos k\alpha \\ -\sin k\lambda & \sin k\alpha \end{bmatrix} \begin{bmatrix} \sin\alpha \\ \sin\lambda \end{bmatrix} \tag{A22}$$

Thus the expressions for the constants a_1 and a_2 are given as:

$$a_1 = \frac{k(\sin\alpha\cos k\lambda - \sin\lambda\cos k\alpha)}{(k^2-1)\sin k(\lambda-\alpha)} \tag{A23}$$

$$a_2 = \frac{k(\sin\alpha\sin k\lambda - \sin\lambda\sin k\alpha)}{(k^2-1)\sin k(\lambda-\alpha)} \tag{A24}$$

Substituting the equations for a_1 and a_2 into the equation (A14) for the voltage in the second era gives :

$$\begin{aligned} V_2' &= \frac{I}{Y}\frac{k}{(k^2-1)}\frac{(\sin\alpha\cos k\lambda - \sin\lambda\cos k\alpha)\cos k\theta}{\sin k(\lambda-\alpha)} \\ &\quad + \frac{I}{Y}\frac{k}{(k^2-1)}\frac{(\sin\alpha\sin k\lambda - \sin\lambda\sin k\alpha)\sin k\theta}{\sin k(\lambda-\alpha)} + \frac{I}{Y}\frac{\cos\theta}{k^2-1} \\ &= \frac{I}{Y}\left\{\frac{k[\sin\alpha\cos k(\theta-\lambda) - \sin\lambda\cos k(\theta-\alpha)]}{(k^2-1)\sin k(\lambda-\alpha)} + \frac{\cos\theta}{k^2-1}\right\} \end{aligned} \tag{A25}$$

The current through the capacitor is given by the differential of the above voltage equation (A25).

$$I_C = I\left\{\frac{k^2[\sin\lambda\sin k(\theta-\alpha) - \sin\alpha\sin k(\theta-\lambda)]}{(k^2-1)\sin k(\lambda-\alpha)} - \frac{\sin\theta}{k^2-1}\right\} \tag{A26}$$

The current through the inductor is the integral of the capacitor voltage as shown below.

$$I_L = I\left\{\frac{k^2[\sin\alpha\sin k(\theta-\lambda) - \sin\lambda\sin k(\theta-\alpha)]}{(k^2-1)\sin k(\lambda-\alpha)} + \frac{k^2\sin\theta}{k^2-1}\right\} \tag{A27}$$

A-4 The Third Era

The circuit diagram for the third era is shown in figure A-5 below.

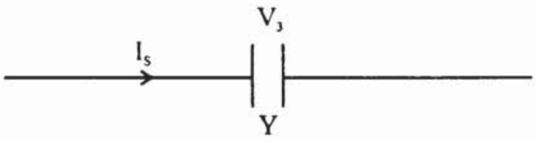

Figure A-5 Circuit Diagram for the Third Era

In this era the thyristors are, once again, not conducting and therefore $I_L = 0$. This gives an expression for the current through the capacitor.

$$I_C = I \sin\theta \tag{A28}$$

The voltage across the capacitor is obtained by integrating the capacitor current.

$$V_3 = \frac{I}{Y}\int \sin\theta \; d\theta \tag{A29}$$

$$V_3 = C_3 - \frac{I}{Y}\cos\theta \tag{A30}$$

A-5 Unknown Constants

Because the voltage across the equipment is half-wave symmetric it is possible to determine the constants C_1 and C_3 from the fact that $V_1|0| = -V_3|\pi|$. Evaluating this condition reveals the following relationship:

$$C_3 = -C_1 \tag{A31}$$

As the voltage across the equipment is continuous, it must follow that the voltage of the first era ,equation (A5), is equal to the voltage of the second era, equation (A25), at $\theta = \alpha$ this provides an expression for the constant C_1.

$$C_1 - \frac{I}{Y}\cos\alpha = \frac{I}{Y}\left\{\frac{k[\sin\alpha\cos k(\lambda-\alpha) - \sin\lambda]}{(k^2-1)\sin k(\lambda-\alpha)} + \frac{\cos\alpha}{k^2-1}\right\}$$

$$C_1 = \frac{I}{Y}\frac{k}{(k^2-1)}\left\{\frac{\sin\alpha\cos k(\lambda-\alpha)+k\cos\alpha\sin k(\lambda-\alpha)-\sin\lambda}{\sin k(\lambda-\alpha)}\right\} \tag{A32}$$

It must also follow that the constant C_3 can be determined by equating the voltage of the second era, equation (A25) and the voltage of the third era, equation (A30), at the condition $\theta = \lambda$.

$$C_3 - \frac{I}{Y}\cos\lambda = \frac{I}{Y}\left\{\frac{k[\sin\alpha-\sin\lambda\cos k(\lambda-\alpha)]}{(k^2-1)\sin k(\lambda-\alpha)}+\frac{\cos\lambda}{k^2-1}\right\}$$

$$C_3 = \frac{I}{Y}\frac{k}{(k^2-1)}\left\{\frac{\sin\alpha-\sin\lambda\cos k(\lambda-\alpha)+k\cos\lambda\sin k(\lambda-\alpha)}{\sin k(\lambda-\alpha)}\right\} \tag{A33}$$

The two expressions for the constants C_1 and C_3 given by equation (A32) and equation (A33) can be equated using the relationship described in equation (A31). This will produce a relationship between α and λ.

$$\sin\alpha\cos k(\lambda-\alpha)+k\cos\alpha\sin k(\lambda-\alpha)-\sin\lambda =$$
$$-\sin\alpha+\sin\lambda\cos k(\lambda-\alpha)-k\cos\lambda\sin k(\lambda-\alpha)$$

$$(\sin\alpha-\sin\lambda)[1+\cos k(\lambda-\alpha)]+k(\cos\alpha+\cos\lambda)\sin k(\lambda-\alpha)=0 \tag{A34}$$

Applying some trigonometric relationships to the above equation (A34) produces an equation for which there are three solutions.

$$-2\cos\left(\frac{\lambda+\alpha}{2}\right)\sin\left(\frac{\lambda-\alpha}{2}\right)\left[1+2\cos^2 k\left(\frac{\lambda-\alpha}{2}\right)-1\right]$$
$$+4k\cos\left(\frac{\lambda+\alpha}{2}\right)\cos\left(\frac{\lambda-\alpha}{2}\right)\sin k\left(\frac{\lambda-\alpha}{2}\right)\cos k\left(\frac{\lambda-\alpha}{2}\right)=0$$

$$\cos\left(\frac{\lambda+\alpha}{2}\right)\cos k\left(\frac{\lambda-\alpha}{2}\right)\times$$
$$\left\{k\cos\left(\frac{\lambda-\alpha}{2}\right)\sin k\left(\frac{\lambda-\alpha}{2}\right)-\sin\left(\frac{\lambda-\alpha}{2}\right)\cos k\left(\frac{\lambda-\alpha}{2}\right)\right\}=0 \tag{A35}$$

The first solution comes from:

$$\cos\left(\frac{\lambda+\alpha}{2}\right)=0$$

This produces the expression for λ.

$$\lambda = n\pi - \alpha \tag{A36}$$

A second solution comes from:

$$\cos k\left(\frac{\lambda - \alpha}{2}\right) = 0$$

This produces the second solution for λ.

$$\lambda = \frac{n\pi}{k} + \alpha \tag{A37}$$

The third solution comes from:

$$k\cos\left(\frac{\lambda - \alpha}{2}\right)\sin k\left(\frac{\lambda - \alpha}{2}\right) - \sin\left(\frac{\lambda - \alpha}{2}\right)\cos k\left(\frac{\lambda - \alpha}{2}\right) = 0$$

Let the angle for which the thyristors are switched on be $\lambda - \alpha = \delta$ and the above equation can now be written as:

$$k\tan k\frac{\delta}{2} = \tan\frac{\delta}{2} \tag{A38}$$

This is a transcendental equation and as such no algebraic solution exists. By plotting this equation it can be shown that for all values of k less than or equal to 3, and for any value of δ less than π other than zero there is no solution. This proves that there are only two solutions for λ within the operational range and these are:

$$\lambda = \pi - \alpha$$

$$\lambda = \frac{n\pi}{k} + \alpha$$

Generally $n = 1$ in order to minimise the number of states that the thyristor controlled series capacitor can operate at. This analysis suggests that there are two stable states provided that the resonance ratio, k, is less than three.

Appendix B

Analysis of a Voltage Sourced Inverter Series Compensator using a Capacitor as the Series Element

B-1 Introduction

The circuit shown below describes the single phase voltage sourced inverter series compensator. The circuit consists of a capacitor which is connected in series with a transmission line. A voltage sourced inverter is connected in parallel with this capacitor, which in turn has a capacitor connected across its d.c. terminals. The resistor is a lumped representation of the losses of the compensator and the inductor represents any inductance in the circuit. Both are necessary components and couple the inverter to the transmission line.

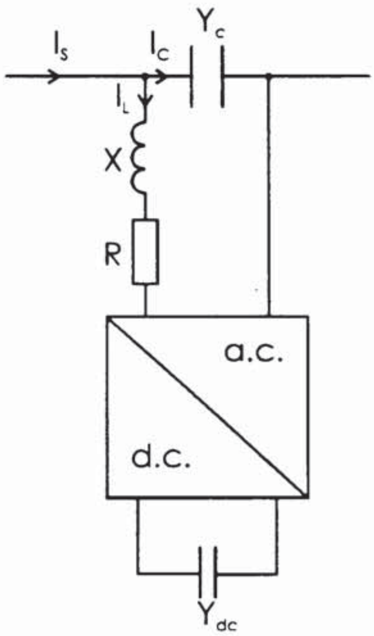

Figure B-1 Circuit Diagram of the Single Phase Voltage Sourced Inverter Series Compensator

The inverter is switched on when the voltage across the a.c. capacitor is approximately zero. The first switching device is fired when the voltage is in the negative half of its cycle. This leads to the relationship between the phase angle θ of the system current I_S and the angle ϕ at which the first device in the inverter is fired.

$$\phi = \theta - \theta_o - \pi/2 \tag{B1}$$

The circuit diagram (figure B-2) below, gives a simple representation of the voltage sourced inverter series compensator when considering the circuit for one half of a cycle.

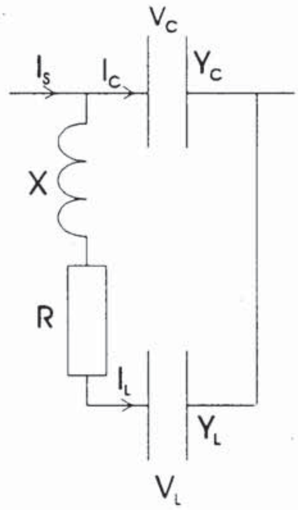

Figure B-2 Simplified Circuit of the Single Phase Voltage Sourced Inverter Series Compensator

B-2 Differential Equation

Using Kirchoff's Current Law, at the node where the system current (I_S) splits into the current through the a.c. capacitor (I_C) and the current through the inverter (I_L), the following equation must apply.

$$I_C + I_L = I_S \tag{B2}$$

The system current I_S is defined as :

$$I_S = I \sin\theta \tag{B3}$$

When the equation (B3) and equation (B1) are substituted into equation (B2) the resulting equation becomes :

$$I_C + I_L = I\sin(\phi + \theta_0 + \pi/2)$$

$$= I\cos(\phi + \theta_0) \qquad \text{(B4)}$$

The current through the a.c. capacitor I_C is defined as,

$$I_C = Y_C \frac{dV_C}{d\phi} \qquad \text{(B5)}$$

and also the voltage across the d.c. capacitor (V_L) is given by:

$$V_L = \frac{1}{Y_L}\int I_L \, d\phi \qquad \text{(B6)}$$

The voltage across the a.c. capacitor must be equivalent to the sum of the voltages across the inductor, the resistor and the d.c. capacitor. This combines to produce the equation below.

$$V_C = X\frac{dI_L}{d\phi} + RI_L + \frac{1}{Y_L}\int I_L \, d\phi \qquad \text{(B7)}$$

By substituting equation (B7) into equation (B5) the current through the a.c. capacitor I_C is found in terms of the current through the inverter I_L.

$$I_C = XY_C\frac{d^2 I_L}{d\phi^2} + RY_C\frac{dI_L}{d\phi} + \frac{Y_C}{Y_L}I_L \qquad \text{(B8)}$$

If this equation (B8) is now inserted into equation (B4) a relationship for the current through the inverter can be obtained.

$$XY_C\frac{d^2 I_L}{d\phi^2} + RY_C\frac{dI_L}{d\phi} + \left(\frac{Y_C}{Y_L} + 1\right)I_L = I\cos(\phi + \theta_o)$$

$$\frac{d^2 I_L}{d\phi^2} + \frac{R}{X}\frac{dI_L}{d\phi} + \frac{1}{X}\left(\frac{1}{Y_L} + \frac{1}{Y_L}\right)I_L = \frac{I}{XY_C}\cos(\phi + \theta_o) \qquad \text{(B9)}$$

For convenience the following definitions will be used :

$$\beta = \frac{R}{2X} \tag{B10}$$

$$k = \sqrt{\frac{1}{X}\left(\frac{1}{Y_L} + \frac{1}{Y_C}\right) - \beta^2} \tag{B11}$$

$$\frac{1}{Y} = \frac{1}{Y_C} + \frac{1}{Y_L} \tag{B12}$$

$$r = \frac{Y}{Y_C} \tag{B13}$$

By making these substitutions - equation (B10) and equation (B11) into equation (B9) - a second order differential equation of the following form is produced.

$$\frac{d^2 I_L}{d\phi^2} + 2\beta \frac{dI_L}{d\phi} + (k^2 + \beta^2) I_L = Ir(k^2 + \beta^2)\cos(\phi + \theta_o) \tag{B14}$$

The general solution to this type of differential equation is:

$$I_L = Ir(k^2 + \beta^2)\{a_1 \cos(\phi + \theta_o) + a_2 \sin(\phi + \theta_o) + a_3 e^{-\beta\phi} \cos k\phi + a_4 e^{-\beta\phi} \sin k\phi\}$$

B-3 Steady State Solution

The steady state solution to the above second order differential equation (B14) takes the form given below in equation (B15). The constants a_3 and a_4 can be ignored for this section because they would cancel each other out later during the analysis.

$$I_{LS} = Ir(k^2 + \beta^2)\{a_1 \cos(\phi + \theta_o) + a_2 \sin(\phi + \theta_o)\} \tag{B15}$$

In order to find the constants a_1 and a_2 the equation (B15) is substituted into the differential equation.

$$-[a_1 \cos(\phi + \theta_o) + a_2 \sin(\phi + \theta_o)] + 2\beta[-a_1 \sin(\phi + \theta_o) + a_2 \cos(\phi + \theta_o)]$$
$$+(\beta^2 + k^2)[a_1 \cos(\phi + \theta_o) + a_2 \sin(\phi + \theta_o)] = \cos(\phi + \theta_o)$$

This equation is solved by equating coefficients of sine and cosine.

Sine :

$$-2\beta a_1 + (\beta^2 + k^2 - 1)a_2 = 0$$

$$a_2 = \frac{2\beta}{\beta^2 + k^2 - 1} a_1 \tag{B16}$$

Cosine :

$$2\beta a_2 + (\beta^2 + k^2 - 1)a_1 = 1 \tag{B17}$$

Making the substitution of equation (B16) into equation (B17) the constant a_1 is determined.

$$a_1 = \frac{\beta^2 + k^2 - 1}{(\beta^2 + k^2 - 1)^2 + (2\beta)^2} \tag{B18}$$

Then inserting the equation (B17) for a_1 into the equation (B16) relating a_2 to a_1 the solution for a_2 is found.

$$a_2 = \frac{2\beta}{(\beta^2 + k^2 - 1)^2 + (2\beta)^2} \tag{B19}$$

This results in the steady state solution now being written as follows :

$$I_{LS} = Ir(k^2 + \beta^2)\left\{\frac{(\beta^2 + k^2 - 1)\cos(\phi + \theta_o) + 2\beta\sin(\phi + \theta_o)}{(\beta^2 + k^2 - 1)^2 + (2\beta)^2}\right\}$$

$$= Ir(k^2 + \beta^2)\left\{\frac{\sin\gamma\cos(\phi + \theta_o) + \cos\gamma\sin(\phi + \theta_o)}{h}\right\}$$

Where :

$$h\sin\gamma = \beta^2 + k^2 - 1 \tag{B20}$$

and

$$h\cos\gamma = 2\beta \tag{B21}$$

Finally the steady state solution can be reduced to the following form.

$$I_{LS} = Ir(k^2+\beta^2)\frac{\sin(\phi+\theta_o+\gamma)}{h} \tag{B22}$$

B-4 Transient Solution

The transient part of the current through the inverter takes the following form,

$$I_{LT} = Ir(k^2+\beta^2)\{a_3 e^{-\beta\phi}\cos k\phi + a_4 e^{-\beta\phi}\sin k\phi\}$$

and thus the complete expression for the current is given by :

$$I_L = Ir(k^2+\beta^2)\left\{\frac{\sin(\phi+\theta_o+\gamma)}{h} + a_3 e^{-\beta\phi}\cos k\phi + a_4 e^{-\beta\phi}\sin k\phi\right\} \tag{B23}$$

In order to calculate the constants a_3 and a_4 it is necessary to use the boundary conditions of both the current through, and the voltage across the d.c. capacitor. The voltage across the d.c. capacitor is given by substituting equation (B23) into equation (B6).

$$\begin{aligned} V_L &= \frac{I}{Y_C}(r-1)(k^2+\beta^2)\frac{\cos(\phi+\theta_o+\gamma)}{h} \\ &+ \frac{I}{Y_C}(r-1)\{(\beta a_3 + k a_4)e^{-\beta\phi}\cos k\phi + (\beta a_4 - k a_3)e^{-\beta\phi}\sin k\phi\} \end{aligned} \tag{B24}$$

The boundary conditions for the current across the d.c. capacitor arise from the knowledge that the current wave-shape is half-wave anti-symmetric :

$$I_L(0) = -I_L(\pi) \tag{B25}$$

Using this relationship in equation (B23) produces an equation relating a_3 to a_4.

$$\frac{\sin(\theta_o+\gamma)}{h}+a_3=-\frac{\sin(\pi+\theta_o+\gamma)}{h}-a_3e^{-\beta\pi}\cos k\pi-a_4e^{-\beta\pi}\sin k\pi$$

$$a_3=-\frac{e^{-\beta\pi}\sin k\pi}{1+e^{-\beta\pi}\cos k\pi}a_4 \tag{B26}$$

The voltage across the d.c. capacitor is half-wave symmetric and produces the following boundary expression :

$$V_L(0)=\mathrm{V}_L(\pi) \tag{B27}$$

Using this relationship on equation (B24) produces another equation relating a_3 to a_4.

$$\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{h}+(\beta a_3+ka_4)=$$
$$\frac{(k^2+\beta^2)\cos(\pi+\theta_o+\gamma)}{h}+(\beta a_3+ka_4)e^{-\beta\pi}\cos k\pi+(\beta a_4-ka_3)e^{-\beta\pi}\sin k\pi$$

$$\begin{aligned}&a_3[\beta(1-e^{-\beta\pi}\cos k\pi)+ke^{-\beta\pi}\sin k\pi]+\\&a_4[k(1-e^{-\beta\pi}\cos k\pi)-\beta e^{-\beta\pi}\sin k\pi]=-2\frac{(\beta^2+k^2)\cos(\theta_o+\gamma)}{h}\end{aligned} \tag{B28}$$

Making the substitution for a_3 from equation (B26) into equation (B28) will give the expression for a_4.

$$\begin{aligned}&a_4[-\beta e^{-\beta\pi}\sin k\pi(1-e^{-\beta\pi}\cos k\pi)-ke^{-2\beta\pi}\sin^2 k\pi]+\\&a_4[k(1-e^{-2\beta\pi}\cos^2 k\pi)-\beta e^{-\beta\pi}\sin k\pi(1+e^{-\beta\pi}\cos k\pi)]=\\&-2\frac{(\beta^2+k^2)\cos(\theta_o+\gamma)}{h}(1+e^{-\beta\pi}\cos k\pi)\end{aligned}$$

$$e^{-\beta\pi}[k(e^{\beta\pi}-e^{-\beta\pi})-2\beta\sin k\pi]a_4=-2\frac{(\beta^2+k^2)\cos(\theta_o+\gamma)}{h}(1+e^{-\beta\pi}\cos k\pi)$$

$$a_4=-\frac{(\beta^2+k^2)\cos(\theta_o+\gamma)}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}(1+e^{-\beta\pi}\cos k\pi) \tag{B29}$$

This equation (B29) is now substituted into equation (B26) to give the equation for a_3.

$$a_3 = \frac{(\beta^2 + k^2)\cos(\theta_o + \gamma)}{he^{-\beta\pi}(k\sinh\beta\pi - \beta\sin k\pi)} e^{-\beta\pi}\sin k\pi \tag{B30}$$

These two equations (B29) and (B30) can be substituted into the equation (B23) for the current through the inverter I_L. In order to make the equation easier to manipulate later the following constants are used. These are defined below.

$$M = 1 + e^{-\beta\pi}\cos k\pi \tag{B31}$$

and

$$N = e^{-\beta\pi}\sin k\pi \tag{B32}$$

The current I_L now becomes:

$$\begin{aligned} I_L &= Ir(\beta^2 + k^2)\frac{\sin(\phi + \theta_o + \gamma)}{h} \\ &+ Ir\frac{(\beta^2 + k^2)^2\cos(\theta_o + \gamma)}{he^{-\beta\pi}(k\sinh\beta\pi - \beta\sin k\pi)}(Ne^{-\beta\phi}\cos k\phi - Me^{-\beta\phi}\sin k\phi) \end{aligned} \tag{B33}$$

The voltage V_L across the d.c. capacitor is now written as.

$$\begin{aligned} V_L &= \frac{I}{Y_C}(r-1)(\beta^2 + k^2)\frac{\cos(\phi + \theta_o + \gamma)}{h} \\ &+ \frac{I}{Y_C}(r-1)(\beta^2 + k^2)\frac{\cos(\theta_o + \gamma)\{(\beta N - kM)e^{-\beta\phi}\cos k\phi - (\beta M + kN)e^{-\beta\phi}\sin k\phi\}}{he^{-\beta\pi}(k\sinh\beta\pi - \beta\sin k\pi)} \end{aligned} \tag{B34}$$

The current through the a.c. capacitor is calculated using equation (B4) and equation (B33).

$$\begin{aligned} I_C &= I\cos(\phi + \theta_o) - Ir(\beta^2 + k^2)\frac{\sin(\phi + \theta_o + \gamma)}{h} \\ &- Ir\frac{(\beta^2 + k^2)^2\cos(\theta_o + \gamma)}{he^{-\beta\pi}(k\sinh\beta\pi - \beta\sin k\pi)}(Ne^{-\beta\phi}\cos k\phi - Me^{-\beta\phi}\sin k\phi) \end{aligned} \tag{B35}$$

As far as the transmission system is concerned the voltage across the a.c. capacitor is of most interest. This voltage can be derived by integrating equation (B35) and multiplying by the impedance of the a.c. capacitor, $1/Y_C$.

$$V_C = \frac{I}{Y_C}\left\{C + \sin(\phi+\theta_o) + r(\beta^2+k^2)\frac{\cos(\phi+\theta_o+\gamma)}{h}\right\}$$
$$+\frac{I}{Y_C}r\frac{(\beta^2+k^2)\cos(\theta_o+\gamma)\{(\beta N-kM)e^{-\beta\phi}\cos k\phi-(kN+\beta M)e^{-\beta\phi}\sin k\phi\}}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}$$

(B36)

The constant C is the constant of integration and can be determined from the boundary conditions of the voltage across the a.c. capacitor, this is given by the following equation.

$$V_C(0) = -V_C(\pi) \tag{B37}$$

Using this condition the constant of integration evaluates to :

$$C+\sin\theta_o + r(\beta^2+k^2)\frac{\cos(\theta_o+\gamma)}{h} + r\frac{(\beta^2+k^2)\cos(\theta_o+\gamma)(\beta N-kM)}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}$$
$$= -C-\sin(\pi+\theta_o) - r(\beta^2+k^2)\frac{\cos(\pi+\theta_o+\gamma)}{h}$$
$$-r\frac{(\beta^2+k^2)\cos(\theta_o+\gamma)\{(\beta N-kM)e^{-\beta\pi}\cos k\pi-(kN+\beta M)e^{-\beta\pi}\sin k\pi\}}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}$$

$$C = -\frac{r(k^2+\beta^2)\cos(\theta_o+\gamma)}{2he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}\{(\beta N-kM)M-(kN+\beta M)N\}$$

$$= \frac{rk(k^2+\beta^2)(M^2+N^2)\cos(\theta_o+\gamma)}{2he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}$$

$$= \frac{rk(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)\cos(\theta_o+\gamma)}{h(k\sinh\beta\pi-\beta\sin k\pi)} \tag{B38}$$

Putting the equation (B38) for the constant of integration into equation (B36) for the voltage across the a.c. capacitor gives.

$$
\begin{aligned}
V_C &= \frac{I}{Y_C}\left\{\frac{rk(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)\cos(\theta_o+\gamma)}{h(k\sinh\beta\pi-\beta\sin k\pi)}\right\} \\
&+\frac{I}{Y_C}\left\{\sin(\phi+\theta_o)+r(\beta^2+k^2)\frac{\cos(\phi+\theta_o+\gamma)}{h}\right\} \\
&+\frac{I}{Y_C}\frac{r(\beta^2+k^2)\cos(\theta_o+\gamma)\{(\beta N-kM)e^{-\beta\phi}\cos k\phi-(kN+\beta M)e^{-\beta\phi}\sin k\phi\}}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}
\end{aligned}
$$

(B39)

B-5 Harmonic Analysis

The elimination or reduction of the harmonic content of the voltage is desirable therefore the investigation of the harmonics of the voltage across the a.c. capacitor is of great interest. For this reason a fourier analysis has been completed. Going back to the current through the inverter I_L the equation for the harmonic content of the current is given by:

$$
\begin{aligned}
I_L &= Ir(\beta^2+k^2)\frac{\sin(\phi+\theta_o+\gamma)}{h} \\
&+Ir\frac{(\beta^2+k^2)^2\cos(\theta_o+\gamma)}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}\sum_{n=1}^{\infty}a_n\cos n\phi+b_n\sin n\phi
\end{aligned}
\tag{B40}
$$

Where

$$
a_n=\frac{2}{\pi}\int_0^{\pi}Ne^{-\beta\phi}\cos k\phi\cos n\phi\,d\phi-\frac{2}{\pi}\int_0^{\pi}Me^{-\beta\phi}\sin k\phi\cos n\phi\,d\phi
$$

and

$$
b_n=\frac{2}{\pi}\int_0^{\pi}Ne^{-\beta\phi}\cos k\phi\sin n\phi\,d\phi-\frac{2}{\pi}\int_0^{\pi}Me^{-\beta\phi}\sin k\phi\sin n\phi\,d\phi
$$

Evaluation of the integral for a_n gives the following equation:

$$a_n = \frac{2}{\pi h_n^2}\{\beta(\beta^2 + k^2 + n^2)MN - k(\beta^2 + k^2 - n^2)N^2\}$$
$$-\frac{2}{\pi h_n^2}\{k(\beta^2 + k^2 - n^2)M^2 + \beta(\beta^2 + k^2 + n^2)MN\}$$

The following constants h_n and γ_n are defined below. They are analogous with the constants h and γ which were defined by equation (B20) and equation (B21)

$$h_n \sin\gamma_n = \beta^2 + k^2 - n^2 \qquad \text{(B41)}$$

and also the definition of γ has its harmonic equivalent.

$$h_n \cos\gamma_n = 2\beta n \qquad \text{(B42)}$$

The constants M and N are replaced by their original definitions, equation (B31) and equation (B32). The constant a_n now becomes:

$$a_n = -\frac{4k}{\pi} e^{-\beta\pi}(\cosh\beta\pi + \cos k\pi)\frac{(\beta^2 + k^2 - n^2)}{h_n^2}$$

$$= -\frac{4k}{\pi} e^{-\beta\pi}(\cosh\beta\pi + \cos k\pi)\frac{\sin\gamma_n}{h_n} \qquad \text{(B43)}$$

The integral for b_n evaluates to give:

$$b_n = \frac{2n}{\pi h_n^2}\{(\beta^2 - k^2 + n^2)MN - 2\beta k N^2\}$$
$$-\frac{2n}{\pi h_n^2}\{2\beta k M^2 + (\beta^2 - k^2 + n^2)MN\}$$

$$= -\frac{4k}{\pi} e^{-\beta\pi}(\cosh\beta\pi + \cos k\pi)\frac{2\beta n}{h_n^2}$$

$$= -\frac{4k}{\pi} e^{-\beta\pi}(\cosh\beta\pi + \cos k\pi)\frac{\cos\gamma_n}{h_n} \qquad \text{(B44)}$$

By substituting equation (B43) and equation (B44) into equation (B40) the current through the inverter gives;

$$I_L = Ir(\beta^2 + k^2)\frac{\sin(\phi + \theta_o + \gamma)}{h}$$
$$-I\frac{4rk(\beta^2 + k^2)^2(\cosh\beta\pi + \cos k\pi)\cos(\theta_o + \gamma)}{h\pi(k\sinh\beta\pi - \beta\sin k\pi)}\sum_{n=1}^{\infty}\frac{\sin n\phi\cos\gamma_n + \cos n\phi\sin\gamma_n}{h_n}$$

$$= Ir(\beta^2 + k^2)\left\{\frac{\sin(\phi + \theta_o + \gamma)}{h} - \frac{hq_2}{\beta}\cos(\theta_o + \gamma)\sum_{n=1}^{\infty}\frac{\sin(n\phi + \gamma_n)}{h_n}\right\} \qquad \text{(B45)}$$

For convenience the constant q_2 has been used and it is defined below.

$$q_2 = \frac{4\beta k(\beta^2 + k^2)(\cosh\beta\pi + \cos k\pi)}{h^2\pi(k\cosh\beta\pi - \beta\sin k\pi)} \qquad \text{(B46)}$$

The current through the a.c. capacitor is derived by substituting equation (B45) into equation (B4).

$$I_{CN} = I\left\{\cos(\phi + \theta_0) - r(\beta^2 + k^2)\frac{\sin(\phi + \theta_o + \gamma)}{h}\right\}$$
$$+Ir(\beta^2 + k^2)\frac{hq_2}{\beta}\cos(\theta_o + \gamma)\sum_{n=1}^{\infty}\frac{\sin(n\phi + \gamma_n)}{h_n} \qquad \text{(B47)}$$

Integrating the a.c. capacitor current and then dividing by Y_C produces the voltage across the a.c. capacitor.

$$V_{CN} = \frac{I}{Y_C}\left\{C_n + \sin(\phi + \theta_0) + r(\beta^2 + k^2)\frac{\cos(\phi + \theta_o + \gamma)}{h}\right\}$$
$$-\frac{I}{Y_C}r(\beta^2 + k^2)\frac{hq_2}{\beta}\cos(\theta_o + \gamma)\sum_{n=1}^{\infty}\frac{\cos(n\phi + \gamma_n)}{nh_n}$$

Again a constant of integration is shown and the previously mentioned boundary condition equation (B37) is used to evaluate it.

$$-C_n - \sin\theta_0 - r(\beta^2+k^2)\frac{\cos(\theta_o+\gamma)}{h} + r(\beta^2+k^2)\frac{hq_2}{\beta}\cos(\theta_o+\gamma)\sum_{n=1}^{\infty}\frac{\cos(\gamma_n)}{nh_n} =$$

$$C_n + \sin(\pi+\theta_0) + r(\beta^2+k^2)\frac{\cos(\pi+\theta_o+\gamma)}{h} - r(\beta^2+k^2)\frac{hq_2}{\beta}\cos(\theta_o+\gamma)\sum_{n=1}^{\infty}\frac{\cos(n\pi+\gamma}{nh_n}$$

$$C_n = \frac{rhq_2}{2\beta}\cos(\theta_o+\gamma)\sum_{n=1}^{\infty}\frac{\cos\gamma_n + \cos(n\pi+\gamma_n)}{nh_n}$$

As all even harmonics are zero, it is only odd values of n that need to be considered. For all odd n $\sin n\pi = 0$ and $\cos n\pi = -1$. Therefore, the constant of integration is zero, and the harmonic expression for the voltage across the a.c. capacitor is given by:

$$\begin{aligned} V_{CN} &= \frac{I}{Y_C}\left\{\sin(\phi+\theta_0) + r(\beta^2+k^2)\frac{\cos(\phi+\theta_o+\gamma)}{h}\right\} \\ &\quad - \frac{I}{Y_C}r(\beta^2+k^2)\frac{hq_2}{\beta}\cos(\theta_o+\gamma)\sum_{n=1}^{\infty}\frac{\cos(n\phi+\gamma_n)}{nh_n} \end{aligned} \tag{B48}$$

B-6 Fundamental Component of the Voltage

The fundamental component of the voltage is probably of greatest interest, and it is written below. The voltage has also been converted to a dimensionless equation by dropping the I/Y_C.

$$V_{C1} = \sin(\phi+\theta_0) + r(\beta^2+k^2)\frac{\cos(\phi+\theta_o+\gamma)}{h} - r(\beta^2+k^2)\frac{q_2}{\beta}\cos(\theta_o+\gamma)\cos(\phi+\gamma) \tag{B49}$$

The exercise of this analysis was to show the relationship between the firing angle θ_o of the inverter, and the active and reactive power of the compensator. In order to accomplish this, equation (B1) needs to be substituted back into the expression for the fundamental voltage which is given in equation (B49).

$$\begin{aligned} V'_{C1} &= -\cos\theta + r(\beta^2+k^2)\frac{\sin\theta\cos\gamma + \cos\theta\sin\gamma}{h} \\ &\quad - r(\beta^2+k^2)\frac{q_2}{\beta}\cos(\theta_o+\gamma)[\sin\theta\cos(\gamma-\theta_o) + \cos\theta\sin(\gamma-\theta_o)] \end{aligned} \tag{B50}$$

To examine the relationship between the reactive power produced by the compensator V'_q and the firing angle θ_o, the component of the a.c. capacitor voltage, equation (B50) that is in quadrature with the system current $I\sin\theta$, must be found.

$$V_q = -1 + r(\beta^2 + k^2)\left\{\frac{\sin\gamma}{h} - \frac{q_2}{\beta}\cos(\theta_o + \gamma)\sin(\gamma - \theta_o)\right\}$$

$$= -1 + r(\beta^2 + k^2)\left\{\frac{\sin\gamma}{h} - \frac{q_2}{2\beta}(\sin 2\gamma - \sin 2\theta_o)\right\}$$

$$= -1 + r(\beta^2 + k^2)\left\{\frac{\sin\gamma}{h}(1 - 2q_2) + q_2\frac{\sin 2\theta_o}{2\beta}\right\}$$

Defining :

$$q_1 = \frac{\sin\gamma}{h}(1 - 2q_2) \qquad \text{(B51)}$$

and making the substitution for small angles of $\sin\theta_o \approx \theta_o$ the quadrature component of the voltage becomes :

$$V'_q = -1 + r(k^2 + \beta^2)(q_1 + q_2\theta_{ob}) \qquad \text{(B52)}$$

where

$$\theta_{ob} = \frac{\theta_o}{\beta} \qquad \text{(B53)}$$

The component of the a.c. capacitor voltage, equation (B50), which is in phase with the system current is examined.

$$V'_p = r(\beta^2 + k^2)\left\{\frac{\cos\gamma}{h} - \frac{q_2}{\beta}\cos(\theta_o + \gamma)\cos(\gamma - \theta_o)\right\}$$

$$= r(\beta^2 + k^2)\left\{\frac{\cos\gamma}{h} - \frac{q_2}{2\beta}(\cos 2\gamma + \cos 2\theta_o)\right\}$$

$$= r(\beta^2 + k^2)\left\{\frac{\cos\gamma}{h} - \frac{q_2}{2\beta}(2\cos^2\gamma - 1 + 1 - 2\sin^2\theta_o)\right\}$$

$$= r(\beta^2 + k^2)\left\{\frac{\cos\gamma}{h}(1 - 2q_2) + \beta q_2 \frac{\cos^2\theta_o}{\beta^2}\right\}$$

Again making the substitution for small angles, θ_{ob} and another new definition,

$$p_1 = \frac{\cos\gamma}{\beta h}(1 - 2q_2) \tag{B54}$$

The active power component of the a.c. capacitor voltage is written as follows.

$$V_p = r\beta(k^2 + \beta^2)(p_1 + q_2\theta_{ob}^2) \tag{B55}$$

B-7 Magnitude Of The Higher Harmonics

The magnitude of the harmonics greater than the fundamental is obtained from the equation for the voltage across the ac capacitor given in equation (B48).

$$V_n = -r(k^2 + \beta^2)\frac{hq_2}{\beta nh_n}\cos(\theta_o + \gamma)$$

$$= -r(k^2 + \beta^2)\frac{hq_2}{\beta nh_n}(\cos\theta_o \cos\gamma - \sin\theta_o \sin\gamma)$$

$$= -r(k^2 + \beta^2)\frac{2q_2}{nh_n}(\cos\theta_o - \tan\gamma \sin\theta_o)$$

Making approximations for small angles gives:

$$V_n = -r(k^2 + \beta^2)\frac{2q_2}{nh_n}\left|1 - \theta_o \tan\gamma\right| \tag{B56}$$

Appendix C
Analysis of a Single Phase Current Sourced Inverter Series Compensator

C-1 Introduction

The current sourced inverter series compensator will provide both capacitive and inductive compensation. The compensation provided is continuously variable.

The circuit consists of a capacitor connected in series with a transmission line. Connected across the series capacitor is a current sourced inverter. This inverter does not need any anti-parallel diodes. The d.c. energy store takes the form of an inductor.

The circuit diagram figure C-1 shows the configuration of a single phase current sourced inverter series compensation scheme. The series capacitor is represented by its admittance Y, the d.c. energy store is represented by the reactance X. All the losses in the compensator are represented by the resistor R.

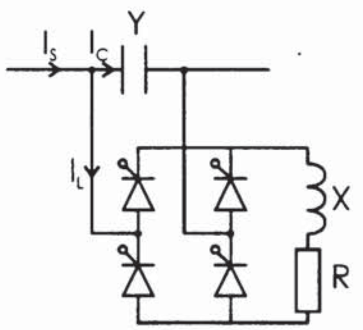

Figure C-1 Circuit Diagram of a Single Phase Current Sourced Inverter Series Compensator

Simulations of the compensator have shown that it will operate both capacitively and inductively. This operational area is defined by the phase angle, ϕ, at which the first switch of inverter is fired. This angle is related to the system angle by the following expression :

$$\phi = \theta - \theta_o \tag{C1}$$

The angle θ_o is the control angle for the compensator. Varying the angle θ_o changes the magnitude of current on the d.c. side of the inverter. The current through the d.c. energy store is half wave symmetric and the current through the series capacitor and the voltage across the series capacitor is half wave anti symmetric. These relationships allow simplifications to be made. It is possible to analyse the circuit for the era $0 \le \phi \le \pi$ and then use the symmetry of the system to produce the equations for the era $\pi \le \phi \le 2\pi$. Examining the compensating circuit for the first half of the cycle allows a simplified circuit diagram shown below in figure C-2 to be used.

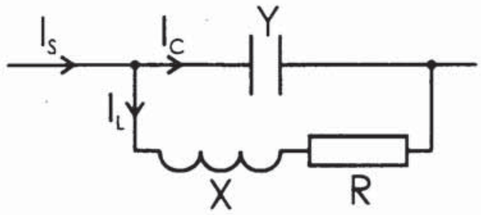

Figure C-2 Single Phase Current Sourced Inverter Series Compensator Simplified for Mathematical Analysis

C-2 Differential Equation

The switching of the devices in the inverter are defined by a reference device. It will be called the first switch (See chapter 4). The first switch is fired when the system current I_S is approximately zero and going negative. Using Kirchoff's current law and making the substitution of equation (C1), the currents flowing around the circuit can be related by the following equation.

$$I_C + I_L = I\sin\theta$$

$$= I\sin(\phi + \theta_o) \tag{C2}$$

The voltage across the inverter can be described as the sum of the voltage drops across the inductor and the resistor.

$$V = X\frac{dI_L}{d\phi} + RI_L \tag{C3}$$

This voltage is the same as the voltage across the series capacitor and so the current through the capacitor is described by :

$$I_C = Y\frac{dV}{d\phi} \tag{C4}$$

Making the substitution of equation (C3) into equation (C4) will produce an equation for the current through the capacitor, in terms of the current through the inverter.

$$I_C = XY\frac{d^2 I_L}{d\phi^2} + RY\frac{dI_L}{d\phi} \qquad \text{(C5)}$$

Then substituting equation (C5) into equation (C2) gives a second order differential equation for the inverter current.

$$XY\frac{d^2 I_L}{d\phi^2} + RY\frac{dI_L}{d\phi} + I_L = I\sin(\phi + \theta_o)$$

$$\frac{d^2 I_L}{d\phi^2} + \frac{R}{X}\frac{dI_L}{d\phi} + \frac{1}{XY}I_L = \frac{I}{XY}\sin(\phi + \theta_o)$$

The damping factor of the inverter current (I_L) is defined as:

$$\beta = \frac{R}{2X} \qquad \text{(C6)}$$

and the reactance ratio of the circuit is given by.

$$k = \sqrt{\frac{1}{XY} - \beta^2} \qquad \text{(C7)}$$

Making the substitution of the equations for β and k into the second order differential equation gives the equation shown below.

$$\frac{d^2 I_L}{d\phi^2} + 2\beta\frac{dI_L}{d\phi} + (\beta^2 + k^2)I_L = I(\beta^2 + k^2)\sin(\phi + \theta_o) \qquad \text{(C8)}$$

The solution to this type of differential equation is shown below.

$$I_L = I(k^2 + \beta^2)\{a_1\cos(\phi + \theta_o) + a_2\sin(\phi + \theta_o) + a_3 e^{-\beta\phi}\cos k\phi + a_4 e^{-\beta\phi}\sin k\phi\}$$
(C9)

C-3 The Constants a_1 *and* a_2

By substituting the above expression for the inverter current equation (C9) back into the differential equation (C8), and by equating coefficients of sine and cosine, will produce equations of a_1 and a_2. The constants a_3 and a_4 will cancel each other out. So in order to shorten the analysis the components of a_3 and a_4 will be temporarily ignored.

$$-a_1\cos(\phi+\theta_o)-a_2\sin(\phi+\theta_o)+2\beta[-a_1\sin(\phi+\theta_o)+a_2\cos(\phi+\theta_o)]$$
$$+(\beta^2+k^2)[a_1\cos(\phi+\theta_o)+a_2\sin(\phi+\theta_o)]=\sin(\phi+\theta_o) \tag{C10}$$

Equating coefficients of cosine.

$$2\beta a_2+(\beta^2+k^2-1)a_1=0$$

$$a_2=-\frac{\beta^2+k^2-1}{2\beta}a_1 \tag{C11}$$

Equating coefficients of sine.

$$-2\beta a_1+(\beta^2+k^2-1)a_2=1 \tag{C12}$$

Substituting equation (C11) into equation (C12) will give an equation for a_1.

$$a_1=-\frac{2\beta}{(\beta^2+k^2-1)^2+4\beta^2}=-\frac{\cos\gamma}{h} \tag{C13}$$

Next making the substitution of equation (C13) into equation (C11) will give an expression for a_2.

$$a_2=\frac{\beta^2+k^2-1}{(\beta^2+k^2-1)^2+4\beta^2}=\frac{\sin\gamma}{h} \tag{C14}$$

Where h is defined as:

$$h=\sqrt{(\beta^2+k^2-1)^2+4\beta^2} \tag{C15}$$

and γ is defined as:

$$\gamma=\arctan\frac{\beta^2+k^2-1}{2\beta} \tag{C16}$$

The current through the inverter now takes on the form shown below.

$$I_L = I(k^2+\beta^2)\left\{-\frac{\cos\gamma}{h}\cos(\phi+\theta_o)+\frac{\sin\gamma}{h}\sin(\phi+\theta_o)\right\}$$
$$+I(k^2+\beta^2)\{a_3e^{-\beta\phi}\cos k\phi + a_4e^{-\beta\phi}\sin k\phi\}$$

$$= I(k^2+\beta^2)\left\{-\frac{\cos(\phi+\theta_o+\gamma)}{h}+a_3e^{-\beta\phi}\cos k\phi + a_4e^{-\beta\phi}\sin k\phi\right\} \qquad \text{(C17)}$$

By inserting equation (C17) back into equation (C3) the voltage across the compensator can be evaluated.

$$V = X\left\{\frac{dI_L}{d\phi}+2\beta I_L\right\}$$

$$= \frac{I}{Y}\left\{\frac{\sin(\phi+\theta_o+\gamma)}{h}+(ka_4-\beta a_3)e^{-\beta\phi}\cos k\phi-(ka_3+\beta a_4)e^{-\beta\phi}\sin k\phi\right\}$$
$$+\frac{I}{Y}\left\{-\frac{2\beta\cos(\phi+\theta_o+\gamma)}{h}+2\beta a_3e^{-\beta\phi}\cos k\phi+2\beta a_4e^{-\beta\phi}\sin k\phi\right\}$$

$$= \frac{I}{Y}\frac{\sin(\phi+\theta_o+\gamma)-2\beta\cos(\phi+\theta_o+\gamma)}{h}$$
$$+\frac{I}{Y}\{(\beta a_3+ka_4)e^{-\beta\phi}\cos k\phi+(\beta a_4-ka_3)e^{-\beta\phi}\sin k\phi\} \qquad \text{(C18)}$$

C-4 The Constants a_3 *and* a_4

The constants a_3 and a_4 are determined from the boundary conditions of the voltage across, and the current through the inverter. Taking first the boundary condition for the voltage. The voltage across the d.c. terminals of the inverter is half wave anti symmetric. This gives rise to the following relationship.

$$V(0) = -V(\pi) \qquad \text{(C19)}$$

Using this condition, equation (C14) on the expression for the inverter voltage, equation (C13), a short equation relating the constant a_3 to a_4 is found.

$$-\frac{2\beta\cos(\theta_o+\gamma)-\sin(\theta_o+\gamma)}{h}+\beta a_3+ka_4 =$$
$$-\frac{2\beta\cos(\theta_o+\gamma)-\sin(\theta_o+\gamma)}{h}-\{(\beta a_3+ka_4)e^{-\beta\pi}\cos k\pi+(\beta a_4-ka_3)e^{-\beta\pi}\sin k\pi\}$$

$$(\beta a_3 + ka_4)(1 + e^{-\beta\pi}\cos k\pi) = (ka_3 - \beta a_4)e^{-\beta\pi}\sin k\pi$$

$$a_4 = -\frac{\beta(1 + e^{-\beta\pi}\cos k\pi) - ke^{-\beta\pi}\sin k\pi}{k(1 + e^{-\beta\pi}\cos k\pi) + \beta e^{-\beta\pi}\sin k\pi} a_3 \tag{C20}$$

The current through the inverter is half wave symmetric, this gives rise to the boundary condition for the current through the inverter.

$$I_L(0) = I_L(\pi) \tag{C21}$$

Using this condition on the equation for the inverter current, equation (C17) another equation relating a_3 to a_4 is obtained.

$$-\frac{\cos(\theta_o + \gamma)}{h} + a_3 = \frac{\cos(\theta_o + \gamma)}{h} + a_3 e^{-\beta\pi}\cos k\pi + a_4 e^{-\beta\pi}\sin k\pi$$

$$a_3(1 - e^{-\beta\pi}\cos k\pi) - a_4 e^{-\beta\pi}\sin k\pi = 2\frac{\cos(\theta_o + \gamma)}{h} \tag{C22}$$

Substituting equation (C20) into equation (C22) will provide a solution for a_3

$$a_3\left\{(1 - e^{-\beta\pi}\cos k\pi) + e^{-\beta\pi}\sin k\pi \frac{\beta(1 + e^{-\beta\pi}\cos k\pi) - ke^{-\beta\pi}\sin k\pi}{k(1 + e^{-\beta\pi}\cos k\pi) + \beta e^{-\beta\pi}\sin k\pi}\right\} = 2\frac{\cos(\theta_o + \gamma)}{h}$$

$$\begin{aligned} &a_3\{k(1 - e^{-2\beta\pi}\cos^2 k\pi) + \beta e^{-\beta\pi}\sin k\pi(1 - e^{-\beta\pi}\cos k\pi)\} \\ &\quad + a_3\{\beta e^{-\beta\pi}\sin k\pi(1 + e^{-\beta\pi}\cos k\pi) - ke^{-2\beta\pi}\sin^2 k\pi\} \\ &\quad = 2\frac{\cos(\theta_o + \gamma)}{h}[k(1 + e^{-\beta\pi}\cos k\pi) + \beta e^{-\beta\pi}\sin k\pi] \end{aligned}$$

$$\begin{aligned} &a_3 e^{-\beta\pi}\{k(e^{-\beta\pi} - e^{-\beta\pi}) + 2\beta\sin k\pi\} = \\ &2\frac{\cos(\theta_o + \gamma)}{h}[k(1 + e^{-\beta\pi}\cos k\pi) + \beta e^{-\beta\pi}\sin k\pi] \end{aligned}$$

$$a_3 = \frac{\cos(\theta_o + \gamma)}{he^{-\beta\pi}(k\sinh\beta\pi + \beta\sin k\pi)}(kM + \beta N) \tag{C23}$$

where

$$M = 1 + e^{-\beta\pi}\cos k\pi \tag{C24}$$

and

$$N = e^{-\beta\pi}\sin k\pi \tag{C25}$$

Inserting this expression for a_3 into equation (C20) also offers a solution for a_4

$$a_4 = -\frac{\cos(\theta_o + \gamma)}{he^{-\beta\pi}(k\sinh\beta\pi + \beta\sin k\pi)}(\beta M - kN) \tag{C26}$$

The current through the inverter is now found by inserting the equations (C23) and (C26) into equation (C17).

$$I_L = -I(k^2 + \beta^2)\frac{\cos(\phi + \theta_o + \gamma)}{h}$$
$$+I\frac{(k^2 + \beta^2)\cos(\theta_o + \gamma)}{he^{-\beta\pi}(k\sinh\beta\pi + \beta\sin k\pi)}[(kM + \beta N)e^{-\beta\phi}\cos k\phi + (kN - \beta M)e^{-\beta\phi}\sin k\phi]$$
(C27)

The current through the series capacitor is found by subtracting the inverter current, equation (C27), from the supply current, equation (C2).

$$I_C = I\left\{\sin(\phi + \theta_o) + (k^2 + \beta^2)\frac{\cos(\phi + \theta_o + \gamma)}{h}\right\}$$
$$-I\frac{(k^2 + \beta^2)\cos(\theta_o + \gamma)}{he^{-\beta\pi}(k\sinh\beta\pi + \beta\sin k\pi)}[(kM + \beta N)e^{-\beta\phi}\cos k\phi + (kN - \beta M)e^{-\beta\phi}\sin k\phi]$$
(C28)

The voltage across the d.c. terminals of the inverter is found by substituting equations (C23) and (C26) into the expression given by equation (C18).

$$V' = \frac{I}{Y}\frac{\sin(\phi + \theta_o + \gamma) - 2\beta\cos(\phi + \theta_o + \gamma)}{h}$$
$$+\frac{I}{Y}\frac{(k^2 + \beta^2)\cos(\theta_o + \gamma)}{he^{-\beta\pi}(k\sinh\beta\pi + \beta\sin k\pi)}(Ne^{-\beta\phi}\cos k\phi - Me^{-\beta\phi}\sin k\phi) \tag{C29}$$

C-5 Harmonic Analysis

In order to find the harmonic content of the compensator voltage a fourier analysis must be conducted. The voltage across the compensator for the n^{th} harmonic is given by the following equation.

$$\begin{aligned} V_n &= \frac{I}{Y}\frac{\sin(\phi+\theta_o+\gamma)-2\beta\cos(\phi+\theta_o+\gamma)}{h} \\ &+\frac{I}{Y}\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{he^{-\beta\pi}(k\sinh\beta\pi+\beta\sin k\pi)}\sum_{n=1}^{\infty}a_n\cos n\phi+b_n\sin n\phi \end{aligned} \tag{C30}$$

The constant a_n is defined as:

$$a_n = \frac{2}{\pi}\int (Ne^{-\beta\phi}\cos k\phi - Me^{-\beta\phi}\sin k\phi)\cos n\phi \, d\phi$$

This equation evaluates to,

$$a_n = -\frac{2k(M^2+N^2)}{\pi h_n^2}(\beta^2+k^2-n^2)$$

$$= -\frac{4ke^{-\beta\pi}(\cosh\beta\pi+\cos k\pi)}{\pi}\frac{\sin\gamma_n}{h_n} \tag{C31}$$

where:

$$h_n\sin\gamma_n = \beta^2+k^2-n^2 \tag{C32}$$

The constant b_n is defined as:

$$b_n = \frac{2}{\pi}\int (Ne^{-\beta\phi}\cos k\phi - Me^{-\beta\phi}\sin k\phi)\sin n\phi \, d\phi$$

This equation evaluates to,

$$b_n = -\frac{4ke^{-\beta\pi}(\cosh\beta\pi+\cos k\pi)}{\pi}\frac{2\beta n}{h_n^2}$$

$$= -\frac{4ke^{-\beta\pi}(\cosh\beta\pi+\cos k\pi)}{\pi}\frac{\cos\gamma_n}{h_n} \tag{C33}$$

where:

$$h_n \cos\gamma_n = 2\beta n \tag{C34}$$

The definitions of γ_n and h_n are analogous with the definitions of γ and h defined earlier. Next, by making the substitution of a_n and b_n into equation (C30);

$$V_n = \frac{I}{Y}\frac{\sin(\phi+\theta_o+\gamma)-2\beta\cos(\phi+\theta_o+\gamma)}{h}$$
$$-\frac{I}{Y}\frac{4k(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)\cos(\theta_o+\gamma)}{\pi h(k\sinh\beta\pi+\beta\sin k\pi)}\sum_{n=1}^{\infty}\frac{\sin\gamma_n\cos n\phi+\cos\gamma_n\sin n\phi}{h_n}$$
(C35)

The following substitution is made to reduce the size of the above equation.

$$q_2 = \frac{4\beta k(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)}{\pi h^2(k\sinh\beta\pi+\beta\sin k\pi)} \tag{C36}$$

The harmonic voltage across the compensator can now be written as:

$$V_n = \frac{I}{Y}\left\{\frac{\sin(\phi+\theta_o+\gamma)-2\beta\cos(\phi+\theta_o+\gamma)}{h}-\frac{q_2 h}{\beta}\cos(\theta_o+\gamma)\sum_{n=1}^{\infty}\frac{\sin(n\phi+\gamma_n)}{h_n}\right\}$$
(C37)

C-6 Fundamental Component

As the firing angle θ_o is the control parameter for the compensator it would be useful to develop equations which illustrate the relationship of the active and reactive power of the compensator to the firing angle. In order to demonstrate this, the fundamental component of the compensator voltage must first be found.

$$V_1 = \frac{I}{Y}\left\{\frac{\sin(\phi+\theta_o+\gamma)-2\beta\cos(\phi+\theta_o+\gamma)}{h}-\frac{q_2}{\beta}\cos(\theta_o+\gamma)\sin(\phi+\gamma)\right\}$$

Next the equation needs to be written in terms of the system phase angle θ. This is done by substituting equation (C1) into the above equation.

$$V_1 = \frac{I}{Y}\left\{\frac{\sin(\theta+\gamma)-2\beta\cos(\theta+\gamma)}{h}-\frac{q_2}{\beta}\cos(\theta_o+\gamma)\sin(\theta+\gamma-\theta_o)\right\} \tag{C38}$$

First, the component of voltage in quadrature with the system current will be examined. The reactive power component of the fundamental voltage is in the direction $\cos\theta$. The voltage has been converted to per unit by dropping the I/Y.

$$V_q = \frac{\sin\gamma - 2\beta\cos\gamma}{h} - \frac{q_2}{\beta}\cos(\theta_o + \gamma)\sin(\gamma - \theta_o)$$

$$= -2\beta\frac{\cos\gamma}{h} + \frac{\sin\gamma}{h} - \frac{q_2}{2\beta}(\sin 2\gamma - \sin 2\theta_o)$$

$$= -\cos^2\gamma + \frac{\sin\gamma}{h}(1 - 2q_2) + q_2\frac{\sin 2\theta_o}{2\beta}$$

$$= q_o + q_1 + q_2\theta_{ob} \tag{C39}$$

Where the constants q_0 and q_1 are defined below.

$$q_1 = \frac{\sin\gamma}{h}(1 - 2q_2) \tag{C40}$$

and

$$q_0 = -\cos^2\gamma \tag{C41}$$

θ_{ob} is the modified firing angle and is defined as shown below.

$$\theta_{ob} = \frac{\theta_o}{\beta} \tag{C42}$$

Next, taking the component of voltage in phase with the supply current.

$$V'_p = \frac{\cos\gamma + 2\beta\sin\gamma}{h} - \frac{q_2}{\beta}\cos(\theta_o + \gamma)\cos(\gamma - \theta_o)$$

$$= 2\beta\frac{\sin\gamma}{h} + \frac{\cos\gamma}{h} - \frac{q_2}{2\beta}(\cos 2\gamma + \cos 2\theta_o)$$

$$= 2\beta\frac{\sin\gamma}{h} + \frac{\cos\gamma}{h} - \frac{q_2}{\beta}\cos^2\gamma + \frac{q_2}{\beta}\sin^2\theta_o$$

$$= 2\beta\frac{\sin\gamma}{h} + \frac{\cos\gamma}{h}(1-2q_2) + \beta q_2\frac{\sin^2\theta_o}{\beta^2}$$

$$= \beta(p_o + p_1 + q_2\theta_{ob}^2) \tag{C43}$$

Where

$$p_0 = 2\frac{\sin\gamma}{h} \tag{C44}$$

and

$$p_1 = \frac{\cos\gamma}{\beta h}(1-2q_2) \tag{C45}$$

The magnitude of the odd harmonics greater than the fundamental is obtained by taking the components of equation (C37) that include and higher harmonics. The development of this equation is detailed below.

$$V_n = -\frac{q_2 h}{\beta h_n}\cos(\theta_o + \gamma)$$

$$= -\frac{q_2 h}{\beta h_n}(\cos\theta_o\cos\gamma - \sin\theta_o\sin\gamma)$$

$$= -\frac{2q_2}{h_n}(\cos\theta_o - \sin\theta_o\tan\gamma)$$

Making the substitution for small angles $\sin\theta_o \approx \theta_o$.

$$V_n = \frac{2q_2}{h_n}(\theta_o\tan\gamma - 1) \tag{C46}$$

Appendix D
Analysis of a Voltage Sourced Inverter Series Compensator using an Inductor as the Series Element

D-1 Introduction

The circuit shown below describes the single phase voltage sourced inverter series compensator. The circuit consists of an inductor which is connected in series with a transmission line. Connected in parallel with this inductor is a voltage sourced inverter, which in turn has a capacitor connected across its d.c. terminals. The resistor and additional inductor are necessary components and couple the inverter to the transmission line.

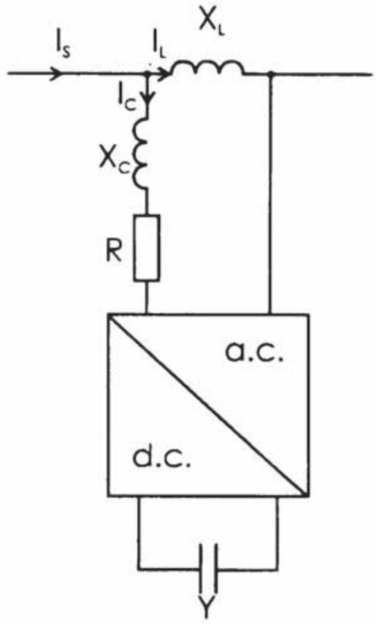

Figure D-1 Circuit Diagram of the Single Phase Voltage Sourced Inverter Series Compensator using an Inductor as the Series Element

The inverter is switched on when the voltage across the a.c. capacitor is approximately zero. The first switching device is fired when the voltage is in the negative half of its cycle. This leads to the relationship between the phase angle (θ) of the system current (I_S) and the angle (ϕ) at which the first device in the inverter is fired.

$$\phi = \theta - \theta_o + \pi/2 \qquad \text{(D1)}$$

The circuit diagram (fig. D-2) below, gives a simple representation of the voltage sourced inverter series compensator when considering the circuit for one half of a cycle.

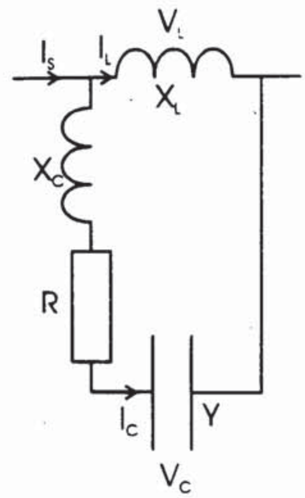

Figure D-2 Simplified Circuit of the Single Phase Voltage Sourced Inverter Series Compensator

D-2 Differential Equation

Using Kirchoff's Current Law, at the node where the system current (I_S) splits into the current through the a.c. inductor (I_L) and the current through the inverter (I_C), the following equation must apply.

$$I_C + I_L = I_S \tag{D2}$$

The system current I_S is defined as:

$$I_S = I \sin\theta \tag{D3}$$

When the equation (D3) and equation (D1) are substituted into equation (D2) the resulting equation becomes:

$$\begin{aligned} I_C + I_L &= I \sin(\phi + \theta_0 - \pi/2) \\ &= -I \cos(\phi + \theta_0) \end{aligned} \tag{D4}$$

The voltage across the a.c. inductor is given in the first instance by writing it in terms of the current through it (I_L).

$$V_L = X_L \frac{dI_L}{d\phi} \tag{D5}$$

The voltage across the d.c. capacitor (V_C) can be written in terms of the current through the inverter (I_C).

$$V_C = \frac{1}{Y}\int I_C \, d\phi \tag{D6}$$

The voltage across the a.c. inductor (V_L) can also be written in terms of the current flowing through the inverter (I_C).

$$V_L = X_C \frac{dI_C}{d\phi} + RI_C + \frac{1}{Y}\int I_C \, d\phi \tag{D7}$$

Making the substitution of equation (D5) into equation (D7), to obtain the current through the a.c. inductor (I_L) in terms of the current through the inverter (I_C).

$$X_L \frac{dI_L}{d\phi} = X_C \frac{dI_C}{d\phi} + RI_C + \frac{1}{Y}\int I_C \, d\phi \tag{D8}$$

Differentiating equation (D4), multiplying by X_L and then making the substitution of equation (D8) will give an expression with the inverter current (I_C) as the only unknown.

$$(X_C + X_L)\frac{dI_C}{d\phi} + RI_C + \frac{1}{Y}\int I_C \, d\phi = IX_L \sin(\phi + \theta_0)$$

This can be rearranged into a second order differential equation as shown in equation (D9) below.

$$\frac{d^2 I_C}{d\phi^2} + \frac{R}{(X_C + X_L)}\frac{dI_C}{d\phi} + \frac{1}{(X_C + X_L)Y} I_C = \frac{IX_L \cos(\phi + \theta_0)}{(X_C + X_L)} \tag{D9}$$

The differential equation can be simplified by using an expression for the sum of the inductance in the circuit as given below in equation (D10).

$$X_C + X_L = X \tag{D10}$$

The ratio of the series inductance to the total inductance in the circuit (r) is given by the equation below.

$$r = \frac{X_L}{X} \tag{D11}$$

The damping factor of the inverter current is shown in equation (D12) below.

$$\beta = \frac{R}{2X} \tag{D12}$$

Finally the ratio of the capacitive and inductive reactance's in the circuit is represented by equation (D13).

$$\frac{1}{XY} = k^2 + \beta^2 \tag{D13}$$

The current through the inverter can now be expressed in terms of the supply current and dimensionless constants by the following second order differential equation.

$$\frac{d^2 I_C}{d\phi^2} + 2\beta \frac{dI_C}{d\phi} + (k^2 + \beta^2) I_C = Ir\cos(\phi + \theta_0) \tag{D14}$$

D-3 Solution for the Inverter Current

The general solution for this type of second order differential equation is given below.

$$I_C = Ir\{a_1 \cos(\phi + \theta_o) + a_2 \sin(\phi + \theta_o) + a_3 e^{-\beta\phi} \cos k\phi + a_4 e^{-\beta\phi} \sin k\phi\} \tag{D15}$$

The constants a_1 and a_2 are found by substituting the general solution for the inverter current (D15) back into the second order differential equation (D14). The constants a_3 and a_4 are cancelled during the substitution and for convenience they are omitted from the following analysis.

$$\begin{aligned}&(k^2 + \beta^2 - 1)a_1 \cos(\phi + \theta_o) + (k^2 + \beta^2 - 1)a_2 \sin(\phi + \theta_o) \\ &-2\beta a_1 \sin(\phi + \theta_o) + 2\beta a_2 \cos(\phi + \theta_o) = \cos(\phi + \theta_o)\end{aligned}$$

The equations for a_1 and a_2 are found by equating coefficients of sine and cosine. Firstly taking the coefficients of sine, to find the constant a_2 in terms of a_1.

$$(k^2 + \beta^2 - 1)a_2 - 2\beta a_1 = 0$$

$$a_2 = \frac{2\beta}{k^2 + \beta^2 - 1} a_1 \tag{D16}$$

Next equating coefficients of cosine.

$$(k^2 + \beta^2 - 1)a_1 + 2\beta a_2 = 1 \tag{D17}$$

The constant a_1 can be then found by substituting equation (D16) into equation (D17).

$$a_1 = \frac{k^2 + \beta^2 - 1}{(k^2 + \beta^2 - 1)^2 + 4\beta^2} \tag{D18}$$

Then inserting equation (D18) back into equation (D16) will provide the constant a_2.

$$a_2 = \frac{2\beta}{(k^2 + \beta^2 - 1)^2 + 4\beta^2} \tag{D19}$$

These equations can be then simplified using the following definitions.

$$h = \sqrt{(k^2 + \beta^2 - 1)^2 + 4\beta^2} \tag{D20}$$

and

$$\gamma = \arctan \frac{k^2 + \beta^2 - 1}{2\beta} \tag{D21}$$

The constants a_1 and a_2 now are written as follows.

$$a_1 = \frac{\sin\gamma}{h} \tag{D22}$$

and

$$a_2 = \frac{\cos\gamma}{h} \tag{D23}$$

The current through the inverter (I_C) given in equation (D15) becomes:

$$I_C = Ir\left\{\frac{\sin(\phi + \theta_o + \gamma)}{h} + a_3 e^{-\beta\phi}\cos k\phi + a_4 e^{-\beta\phi}\sin k\phi\right\} \tag{D24}$$

In order to determine the constants a_3 and a_4 the voltage across the a.c. terminals of inverter needs to be found. This is found using equations (D6) and (D24).

$$V_C = -\frac{Ir}{Y}\frac{\cos(\phi+\theta_o+\gamma)}{h} - \frac{Ir}{Y}\frac{a_3}{k^2+\beta^2}(\beta e^{-\beta\phi}\cos k\phi - ke^{-\beta\phi}\sin k\phi)$$
$$-\frac{Ir}{Y}\frac{a_4}{k^2+\beta^2}(ke^{-\beta\phi}\cos k\phi + \beta e^{-\beta\phi}\sin k\phi) \quad \text{(D25)}$$

The boundary conditions for the current through the inverter and the voltage across the a.c. terminals of the inverter allow the constants a_3 and a_4 to be found. Taking first the current through the inverter, the boundary conditions for which are defined by the following relationship.

$$I_C(\pi) = -I_C(0) \quad \text{(D26)}$$

Applying this relationship to equation (D24) for the current through the inverter will produce a relationship between the constants a_3 and a_4.

$$-\frac{\sin(\theta_o+\gamma)}{h} + a_3 e^{-\beta\pi}\cos k\pi + a_4 e^{-\beta\pi}\sin k\pi = -\frac{\sin(\theta_o+\gamma)}{h} - a_3$$

$$a_3 = -\frac{e^{-\beta\pi}\sin k\pi}{1+e^{-\beta\pi}\cos k\pi}a_4 \quad \text{(D27)}$$

Taking next the boundary relationship for the voltage across the a.c. terminals of the inverter.

$$V_C(\pi) = V_C(0) \quad \text{(D28)}$$

Then applying this relationship to the voltage given in equation (D25).

$$-\frac{\cos(\theta_o+\gamma)}{h} + \frac{a_3}{k^2+\beta^2}(\beta e^{-\beta\pi}\cos k\pi - ke^{-\beta\pi}\sin k\pi)$$
$$+\frac{a_4}{k^2+\beta^2}(ke^{-\beta\pi}\cos k\pi + \beta e^{-\beta\pi}\sin k\pi)$$
$$= \frac{\cos(\theta_o+\gamma)}{h} + \frac{a_3}{k^2+\beta^2}\beta + \frac{a_4}{k^2+\beta^2}k$$

$$a_3[\beta(1-e^{-\beta\pi}\cos k\pi) + ke^{-\beta\pi}\sin k\pi] +$$
$$a_4[k(1-e^{-\beta\pi}\cos k\pi) - \beta e^{-\beta\pi}\sin k\pi] = -2\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{h} \quad \text{(D29)}$$

The constant a_4 can be found by substituting equation (D27) into equation (D29).

$$-\frac{e^{-\beta\pi}\sin k\pi}{1+e^{-\beta\pi}\cos k\pi}[\beta(1-e^{-\beta\pi}\cos k\pi)+ke^{-\beta\pi}\sin k\pi]a_4 +$$
$$[k(1-e^{-\beta\pi}\cos k\pi)-\beta e^{-\beta\pi}\sin k\pi]a_4 = -2\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{h}$$

$$a_4[\beta e^{-\beta\pi}\sin k\pi(1-e^{-\beta\pi}\cos k\pi)+e^{-2\beta\pi}\sin^2 k\pi$$
$$-k(1-e^{-2\beta\pi}\cos^2 k\pi)+\beta e^{-\beta\pi}\sin k\pi(1+e^{-\beta\pi}\cos k\pi)] =$$
$$2\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{h}(1+e^{-\beta\pi}\cos k\pi)$$

$$a_4[2\beta e^{-\beta\pi}\sin k\pi - k(1-e^{-2\beta\pi})] = 2\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{h}(1+e^{-\beta\pi}\cos k\pi)$$

$$a_4 = -\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}(1+e^{-\beta\pi}\cos k\pi) \tag{D30}$$

Then substituting equation (D30) into equation (D27) will produce the expression for a_3.

$$a_3 = \frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}e^{-\beta\pi}\sin k\pi \tag{D31}$$

Once again in order to make the equations more concise the following definitions will be used.

$$M = 1+e^{-\beta\pi}\cos k\pi \tag{D32}$$

and

$$N = e^{-\beta\pi}\sin k\pi \tag{D33}$$

The current through the inverter can now be written as follows.

$$\begin{aligned} I_C &= Ir\frac{\sin(\phi+\theta_o+\gamma)}{h} \\ &+Ir\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}(Ne^{-\beta\phi}\cos k\phi - Me^{-\beta\phi}\sin k\phi) \end{aligned} \tag{D34}$$

The voltage across the d.c. energy store can be found by integrating the current through the inverter and dividing by the impedance of the capacitor.

$$V_C = -\frac{I}{Y} r \frac{\cos(\phi + \theta_o + \gamma)}{h}$$
$$+\frac{I}{Y} r \frac{\cos(\theta_o + \gamma) e^{-\beta\phi} [(kM - \beta N)\cos k\phi + (kN + \beta M)\sin k\phi]}{he^{-\beta\pi}(k \sinh \beta\pi - \beta \sin k\pi)}$$

$$= -IX_L r(k^2 + \beta^2) \frac{\cos(\phi + \theta_o + \gamma)}{h}$$
$$+IX_L r(k^2 + \beta^2) \frac{\cos(\theta_o + \gamma) e^{-\beta\phi} [(kM - \beta N)\cos k\phi + (kN + \beta M)\sin k\phi]}{he^{-\beta\pi}(k \sinh \beta\pi - \beta \sin k\pi)}$$

(D35)

The can be no constant of integration. If a constant of integration existed then, due to the half wave symmetry of the voltage across the d.c. energy store would never settle into a steady state. Simulations and physical modelling has shown this not to be the case.

D-4 Voltage Across the A.C. Inductor

The current through the a.c. inductor is found using equation (D4) and equation (D34), by subtracting the current through the inverter from the supply current.

$$I_L = -I\cos(\phi + \theta_o) - Ir \frac{\sin(\phi + \theta_o + \gamma)}{h}$$
$$-Ir \frac{(k^2 + \beta^2)\cos(\theta_o + \gamma)}{he^{-\beta\pi}(k \sinh \beta\pi - \beta \sin k\pi)} (Ne^{-\beta\phi} \cos k\phi - Me^{-\beta\phi} \sin k\phi) \quad \text{(D36)}$$

The voltage across the inductor in series with the transmission line is found by differentiating the current through the inductor and multiplying by the impedance of the inductor.

$$V_L = IX_L \left\{ \sin(\phi + \theta_o) - r \frac{\cos(\phi + \theta_o + \gamma)}{h} \right\}$$
$$+IX_L r \frac{(k^2 + \beta^2)\cos(\theta_o + \gamma)}{he^{-\beta\pi}(k \sinh \beta\pi - \beta \sin k\pi)} [(\beta N + kM)e^{-\beta\phi} \cos k\phi + (kN - \beta M)e^{-\beta\phi} \sin k\phi]$$

(D37)

D-5 Harmonic Analysis

The above equation (D37) for the voltage across the a.c. inductor does not provide very much assistance in visualising the wave-shape and the equation does not indicate how the voltage varies with changes in the firing angle of the inverter. The most suitable method of mathematically describing the voltage across the a.c. inductor is to conduct a harmonic analysis, then splitting the equation into the fundamental component and the other higher harmonics.

The simplest method of deriving the harmonic voltage across the a.c. inductor is to conduct the fourier analysis on the current through the inverter and then use the equations given earlier to obtain the voltage.

$$\begin{aligned} I_{Cn} &= Ir\frac{\sin(\phi+\theta_o+\gamma)}{h} \\ &+Ir\frac{(k^2+\beta^2)\cos(\theta_o+\gamma)}{he^{-\beta\pi}(k\sinh\beta\pi-\beta\sin k\pi)}\sum_{n=1}^{\infty}a_n\cos n\phi+b_n\sin n\phi \end{aligned} \tag{D38}$$

The constant a_n is found from the following integral.

$$a_n=\frac{2}{\pi}\int_0^{\pi}(Ne^{-\beta\phi}\cos k\phi-Me^{-\beta\phi}\sin k\phi)\cos n\phi\, d\phi \tag{D39}$$

This evaluates in the following manner.

$$\begin{aligned} a_n &= \frac{2}{\pi h_n^2}\{\beta(\beta^2+k^2+n^2)MN-k(\beta^2+k^2-n^2)N^2\} \\ &-\frac{2}{\pi h_n^2}\{k(\beta^2+k^2-n^2)M^2+\beta(\beta^2+k^2+n^2)MN\} \end{aligned} \tag{D40}$$

Where the constant h_n is defined as follows.

$$h_n=\sqrt{(k^2+\beta^2-n)^2+(2\beta n)^2} \tag{D41}$$

This constant is analogous with the definition of h given by equation (D20).

$$a_n=\frac{2k}{\pi h_n^2}(M^2+N^2)(\beta^2+k^2-n^2)$$

$$=-\frac{4ke^{-\beta\pi}}{\pi h_n^2}(\cosh\beta\pi+\cos k\pi)(\beta^2+k^2-n^2)$$

$$=-\frac{4ke^{-\beta\pi}(\cosh\beta\pi+\cos k\pi)}{\pi}\frac{\sin\gamma_n}{h_n} \tag{D42}$$

Again the definition of γ_n is given below and is analogous with the definition of γ given in equation (D21).

$$\gamma_n = \arctan\frac{k^2+\beta^2-n^2}{2\beta n} \tag{D43}$$

The constant b_n is found from the following integral.

$$b_n = \frac{2}{\pi}\int_0^{\pi}(Ne^{-\beta\phi}\cos k\phi - Me^{-\beta\phi}\sin k\phi)\sin n\phi\, d\phi \tag{D44}$$

This integral evaluates in the following manner.

$$\begin{aligned} b_n &= \frac{2n}{\pi h_n^2}\{(\beta^2-k^2+n^2)MN - 2\beta kN^2\} \\ &\quad -\frac{2n}{\pi h_n^2}\{2\beta kM^2 + (\beta^2-k^2+n^2)MN\} \\ &= -\frac{4ke^{-\beta\pi}(\cosh\beta\pi+\cos k\pi)}{\pi}\frac{2\beta n}{h_n^2} \\ &= -\frac{4ke^{-\beta\pi}(\cosh\beta\pi+\cos k\pi)}{\pi}\frac{\cos\gamma_n}{h_n} \end{aligned} \tag{D45}$$

Inserting these two equations (D44) and (D45) back into the expression for the current through the inverter given by equation (D38).

$$\begin{aligned} I_{Cn} &= Ir\frac{\sin(\phi+\theta_o+\gamma)}{h} \\ &\quad -Ir\frac{4k(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)\cos(\theta_o+\gamma)}{h\pi(k\sinh\beta\pi-\beta\sin k\pi)}\sum_{n=1}^{\infty}\frac{\cos n\phi\sin\gamma_n+\sin n\phi\cos\gamma_n}{h_n} \\ &= Ir\frac{\sin(\phi+\theta_o+\gamma)}{h} \\ &\quad -Ir\frac{4k(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)\cos(\theta_o+\gamma)}{h\pi(k\sinh\beta\pi-\beta\sin k\pi)}\sum_{n=1}^{\infty}\frac{\sin(n\phi+\gamma_n)}{h_n} \end{aligned} \tag{D46}$$

The current through the a.c. inductor is then found by subtracting this current from the supply current in accordance with equation (D4).

$$I_{Ln} = -I\cos(\phi+\theta_o) - Ir\frac{\sin(\phi+\theta_o+\gamma)}{h}$$
$$+Ir\frac{4k(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)\cos(\theta_o+\gamma)}{h\pi(k\sinh\beta\pi-\beta\sin k\pi)}\sum_{n=1}^{\infty}\frac{\sin(n\phi+\gamma_n)}{h_n} \tag{D47}$$

The voltage across the inductor in series with the transmission line is then found, as was done before, by differentiating the current through the inductor.

$$V_{Ln} = IX_L\sin(\phi+\theta_o) - IX_Lr\frac{\cos(\phi+\theta_o+\gamma)}{h}$$
$$+IX_Lr\frac{4k(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)\cos(\theta_o+\gamma)}{h\pi(k\sinh\beta\pi-\beta\sin k\pi)}\sum_{n=1}^{\infty}\frac{n\cos(n\phi+\gamma_n)}{h_n} \tag{D48}$$

D-6 The Fundamental Component of the Compensator Voltage

The fundamental component is considered to be most useful and the magnitude of harmonics is usually compared to the magnitude of the fundamental. As a result the fundamental needs to be evaluated. This is done by setting n equal to unity for the above equation.

$$V_{L1} = IX_L\sin(\phi+\theta_o) - IX_Lr\frac{\cos(\phi+\theta_o+\gamma)}{h}$$
$$+IX_Lr\frac{4k(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)}{h^2\pi(k\sinh\beta\pi-\beta\sin k\pi)}\cos(\theta_o+\gamma)\cos(\phi+\gamma) \tag{D49}$$

It is helpful to first convert the equation back in to terms with the supply frequency θ, and then into its sine and cosine components.

$$V_{L1} = IX_L\cos\theta + IX_Lr\frac{\sin(\theta+\gamma)}{h}$$
$$-IX_Lr\frac{4k(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)}{h^2\pi(k\sinh\beta\pi-\beta\sin k\pi)}\cos(\theta_o+\gamma)\sin(\theta+\gamma-\theta_o)$$

$$V_{L1} = IX_L\cos\theta + IX_Lr\frac{\cos\gamma}{h}\sin\theta + IX_Lr\frac{\sin\gamma}{h}\cos\theta$$
$$-IX_Lr\frac{4k(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)}{h^2\pi(k\sinh\beta\pi-\beta\sin k\pi)}\cos(\theta_o+\gamma)\cos(\gamma-\theta_o)\sin\theta \tag{D50}$$
$$-IX_Lr\frac{4k(k^2+\beta^2)(\cosh\beta\pi+\cos k\pi)}{h^2\pi(k\sinh\beta\pi-\beta\sin k\pi)}\cos(\theta_o+\gamma)\sin(\gamma-\theta_o)\cos\theta$$

D-7 The Quadrature Component of the Compensator Voltage

This equation can now be separated into the component in phase with the supply current and the component in quadrature with the supply current. Taking first the component in quadrature with the supply current (i.e. $\cos\theta$), and also using the following definition to simplify further the expression.

$$q_2 = \frac{4\beta k(k^2 + \beta^2)(\cosh\beta\pi + \cos k\pi)}{h^2\pi(k\sinh\beta\pi - \beta\sin k\pi)} \tag{D51}$$

The reactive power component of the voltage across the series inductor is written as follows. The voltage has been expressed in per unit by dropping the IX_L.

$$V_q = 1 + r\frac{\sin\gamma}{h} - r\frac{q_2}{\beta}\cos(\theta_o + \gamma)\sin(\gamma - \theta_o)$$

$$= 1 + r\frac{\sin\gamma}{h} - r\frac{q_2}{2\beta}(\sin 2\gamma - \sin 2\theta_o)$$

$$= 1 + r\left\{\frac{\sin\gamma}{h}(1 - 2q_2) + q_2\frac{\sin 2\theta_o}{2\beta}\right\}$$

$$= 1 + r(q_1 + q_2\theta_{ob}) \tag{D52}$$

Where:

$$q_1 = \frac{\sin\gamma}{h}(1 - 2q_2) \tag{D53}$$

and

$$\theta_{ob} = \frac{\theta_{ob}}{\beta} \tag{D54}$$

The approximation for small angles of $\sin\theta \approx \theta$ was used to further simplify the equation.

This equation for the quadrature component of the voltage across the series compensator has now become a simple linear equation. The voltage responds linearly to changes in the firing angle provided the variation of this angle is kept small.

D-8 The Active Power Component of the Compensator Voltage

In order to obtain an understanding of the losses involved with the circuit, the component in phase with the supply current is next analysed.

$$V_p = r\frac{\cos\gamma}{h} - r\frac{q_2}{\beta}\cos(\theta_o + \gamma)\cos(\gamma - \theta_o)$$

$$= r\left\{\frac{\cos\gamma}{h} - \frac{q_2}{2\beta}(\cos 2\gamma + \cos 2\theta_o)\right\}$$

$$= r\left\{\frac{\cos\gamma}{h} - \frac{q_2}{\beta}(\cos^2\gamma - \sin^2\theta_o)\right\}$$

$$= r\left\{\frac{\cos\gamma}{h}(1 - 2q_2) + \beta q_2 \frac{\sin^2\theta_o}{\beta^2}\right\}$$

$$= r\beta(p_1 + q_2\theta_o^2) \qquad \text{(D55)}$$

Where:

$$p_1 = \frac{\cos\gamma}{\beta h}(1 - 2q_2) \qquad \text{(D56)}$$

This equation shows that the losses of the compensator increase with the square of the firing angle. However provided the circuit components are chosen correctly and the firing angle is kept small then the losses are significantly smaller than the quadrature output.

D-9 The Harmonic Component of the Compensator Voltage

The magnitude of the higher harmonics is obtained from equation (D48) and ignoring the fundamental components of the equation. This information is useful as regulations limit the magnitude of any harmonics allowed onto a transmission system.

$$V_n = r\frac{nq_2h}{\beta h_n}\cos(\theta_o + \gamma)$$

$$= r\frac{nq_2h}{\beta h_n}(\cos\theta_o \cos\gamma - \sin\theta_o \sin\gamma)$$

$$= 2q_2r\frac{n}{h_n}(\cos\theta_o - \tan\gamma \sin\theta_o)$$

Making the approximation for small angles the harmonic content of the voltage across the series inductor (V_n) becomes:

$$V_n = 2q_2r\frac{n}{h_n}(1 - \theta_o \tan\gamma) \qquad \text{(D56)}$$