AN INVESTIGATION INTO THE UNDERWRITING OF EXCESS OF LOSS REINSURANCE

Ph.D. Thesis

by

David E. Ayling

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SUMMARY

This thesis sets out to develop the framework for an underwriting system for excess of loss reinsurance. A system is constructed and tested which builds on the strengths but avoids the most important practical limitations of existing judgemental or scientific methods. Key issues in the proposed method are the identification of underwriting decision criteria in the face of uncertainty and the important influence of the market in the decision process.

The research involves the following stages. Firstly, a conceptual framework is provided for tackling the problem effectively in a flexible but comprehensive manner. Secondly, a rigorous empirical examination of underwriting activities over a four year period is undertaken on data collected from an excess of loss underwriting agency operating in the London Market. Thirdly, from relationships established in the empirical investigation, an underwriting system is devised, subjected to sensitivity analysis, and tested in comparison with the performance of a professional excess of loss underwriter.

From the research, the contribution to knowledge is as follows:

1) The first detailed study of excess of loss underwriters' actual decision-making behaviour is provided;

2) Practical application of risk-theoretic techniques is demonstrated as being negligible among excess of loss underwriters;

3) Evidence is provided for the existence of a satisficing model for excess of loss underwriting decisions;

4) An excess of loss underwriting system based on market heuristics is constructed, tested and evaluated;

5) Wider implications of the fact that real units of trade may differ from how they are perceived by market operators are presented and discussed.

The research has important implications for the academic treatment, professional practice, and market supervision of excess of loss reinsurance.

Keywords : Decision-making
Reinsurance
Uncertainty
Underwriting

Submission for the degree of Doctor of Philosophy by David Ernest Ayling.

PREFACE

The subject of this thesis is "excess of loss reinsurance" which is a form of indemnity provided for insurance companies by reinsurance companies. To carry out the investigation the author took on a dual role as a student under the Interdisciplinary Higher Degrees Scheme (I.H.D.) at the University of Aston in Birmingham and as an employee of the Alexander Howden Insurance Group. This combination of experiences results in a thesis which differs from the kind based solely on the evidence of work performed within libraries and laboratories. The interdisciplinary approach to problem-solving combined with practical knowledge of the areas selected for study jointly influence both the character and method of the research.

Two main stages in the research process can be identified. The first stage, from 1976 to 1979, comprised a period of practical experience in the insurance industry during which the author attempted to identify and collect information on excess of loss reinsurance problems. Much of this time was spent in underwriting rooms as an assistant to professional excess of loss underwriters. The second stage, from 1979 to thesis presentation, was spent in contemplation of how the practical problems of excess of loss underwriting could be explained and solved within the framework of the I.H.D. thesis format. The resulting work relies essentially on identification, measurement, analysis of real-life phenomena with a view to exploiting the system to the advantage of an operator in the system. Where possible, conceptual examples from the literature are provided to justify assumptions regarding desired characteristics of reinsurance company behaviour. A prime aim of the research, however, is to demonstrate the inadequacies of such conceptual models for solving the real-life problems of excess of loss reinsurance. Where the conceptual models fail, a satisficing model is set up and demonstrated, via simulation, to be adequate for ensuring reasonable underwriting success in the excess of loss reinsurance market.

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HOW THE THESIS IS ARRANGED

A brief, chapter-by-chapter presentation of the arrangement of information contained in the thesis:

CHAPTER 1 Background to the study

An introduction to the aims and methodology of the thesis.

CHAPTER 2 Introductory Survey

Background information on the role and nature of excess of loss reinsurance.

CHAPTER 3 Reinsurance Company Objectives and the Main Decision Areas

A conceptual framework for the main issues to be tackled in the research.

CHAPTER 4 Theoretical and Practical Aids to Decision-Making for Excess of Loss Reinsurance

A detailed survey of the various approaches available for analysis of key excess of loss underwriting problems.

CHAPTER 5 Excess of Loss Underwriting: The Need for New Knowledge

Identification of remaining problems in excess of loss reinsurance and a proposal for an empirical investigation into real-life underwriting activities.

CHAPTER 6 The Preliminary Analysis

An empirical investigation into the relationships between excess of loss contract details, risk processes and underwriting decision-criteria.

CHAPTER 7 Simulation of Underwriting Strategies

Further research into relationships established in the preliminary analysis with an emphasis on formulation of underwriting strategy using a simulation approach.

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continued

CHAPTER 8 Discussion

Examination of research findings in the light of contemporary literature and market practice and suggestions for future research.

CHAPTER 9 Conclusion

Presentation of the research and the contribution made to knowledge.

CHAPTER 1

BACKGROUND TO THE STUDY

1.1. INTRODUCTION

This investigation attempts to bridge the gap between actuarial and behavioural approaches to the underwriting of excess of loss reinsurance. The method involves an in-depth examination of the criteria for underwriting decisions under conditions of uncertainty and their consequent effects on business performance. An important aspect of the research is to discover whether or not underwriters' pricing and portfolio selection criteria correspond substantially to the theoretical guidelines proposed in the literature. A further aim is to construct and test an underwriting system based on empirical observation of factors which the underwriters themselves take into account in their decision processes. The purpose of this opening chapter is to summarise the broad aims and methodology presented in this study. A general insight into the key problems involved in the underwriting of excess of loss reinsurance, which will be the main concern, is also provided.

1.2 AIMS AND METHODOLOGY

The study sets out to develop a methodology to serve as a practical aid for underwriting excess of loss reinsurance. A prime objective is to investigate whether current practice and performance can be improved using the proposed method. The scheme does not posit a unique and final solution to excess of loss underwriters' problems; many variables which are difficult to quantify and relate are involved in the business which require recourse to judgement and the application of personal skills. The essential ingredients of the process, however, and an approach for dealing with the most important issues, are presented in a systematic and structured manner. It is not suggested that the product of this investigation is an underwriting system that could be taken up at once by the relevant parties; there are bound to be some implementation gaps between the prescriptions of this study and the real world. Nevertheless, a prime aim is to identify and minimise these gaps so that the workable potential of the system proposed can be scrutinised and evaluated. Where possible, recommendations and suggestions for practical implementation of the proposals will be included in the text.

The key problems of excess of loss reinsurance dealt with in the study are those of pricing and portfolio construction which arise as a result of the characteristically high degree of uncertainty prevalent in the business, but which must be reconciled for efficient and prudent business performance. These aspects are the key problems of the business because, through incremental contribution to company reserves and simultaneous acquisition of liability, they are the primary determinants of business performance. Influences on pricing and portfolio construction decisions include both internal (organisational) and external (market) pressures. Desired performance requirements must be achieved via underwriting strategy contrived to deal with the idiosyncracies and uncertainties of the excess of loss market. The underwriter's job could, therefore, be described as the creation of financial order out of potential chaos. To deal with the problems, as in many other areas characterised by uncertainty, great emphasis is usually placed on the capabilities of human judgement and techniques for spreading risk. Such procedures are basic to the insurance industry but, as will become apparent, they are of special importance for excess of loss reinsurance. They are the only methods which have been found workable to date. No satisfactory method exists, however, for linking the judgemental procedures of an individual excess of loss underwriter building his portfolio with the overall company financial plan. Reasons for this lack of communication between underwriter and general management include difficulties in describing an excess of loss underwriter's decision process and the fact that a long time period exists between when an underwriting decision is made and when the consequences of it are known. This renders timely control difficult or impossible. This study sets out to rectify this situation.

It will be demonstrated that an underwriter's judgemental procedures can be described in a manner which can contribute towards the formulation of an underwriting plan which allows greater control of the business. A method by which a plan can be designed to achieve improved business performance will be proposed and tested on data from the aviation excess of loss market. The approach employed in the study is firstly, to describe how current portfolio and pricing decisions are made in practice; secondly, to consider relevant theoretical models which could be applied to the situation, and thirdly, to synthesise a new workable method from the most 'useful' aspects of both theoretical and practical approaches to the problem.

It will be demonstrated that pricing and portfolio problems are by no means independent of each other. Excess of loss pricing decisions, according to the actuarial literature, are concerned with the estimation of low probabilities of the occurrence of high financial loss and the amount which should be charged to ensure that, in the event of such a loss, funds will be available to indemnify the insured, or in a severe case, prevent insolvency. Portfolio decisions are concerned with the excess of loss contract selection process and the consequent risk management of a portfolio consisting of many possibly catastrophic risks. The issues linking the two decision areas are those of profitability and financial stability which form part of overall company objectives. The inter-relationship of pricing and portfolio decisions and their relationship to overall company objectives will be described and incorporated in the proposed system.

A central argument of this study is that the time has come for more organised planning of excess of loss underwriting activities. The growth of formal planning in other areas of the insurance industry and recent methodological developments have helped open the way to this possibility. The main factor preventing organised planning of excess of loss business to date has been the sheer impracticality of the proposals put forward. The schemes proposed in the past have held a strong actuarial or risk-theoretic bias and, although useful for describing the problem, have found little practical application in the industry. The proposed system draws on actuarial and risk-theoretic principles but, in contrast to previous solutions, is strongly biased towards an understanding of observed market behaviour. Indeed, the possibility of assessing risk in a useful fashion on individual excess of loss contracts is dismissed as impractical. The approach taken, for reasons of pure workability, is to assess the value of an excess of loss contract to a portfolio in terms of its relation to expected market performance. The law of large numbers which is employed usefully in all insurance operations plays an important role in the present approach, but where the law is usually employed to ensure, within reasonable bounds, the achievement of a probabilistic expected value, it is employed in the present methodology to achieve a certain standard of portfolio performance which can only be measured in relation to the general level of market performance. A possible criticism of this approach is that a planning system constructed on this basis suffers from a lack of precision in absolute terms. This criticism must stand until the day that catastrophic occurrences become predictable. The present analysis provides only measures of planned performance which seek to improve on general market results. This approach is new in its analytical content only; excess of loss underwriters have informally planned their activities on the basis of past market experience for a long time. Informal planning systems, however, cannot be communicated easily to formal planners concerned with overall company performance. By the proposed method the ability to plan, control, and communicate excess of loss reinsurance underwriting procedures and performance is shown to be possible.

The study presents and examines the following issues:

1) Existing practical and theoretical methods for dealing with excess of loss underwriting problems;

2) The limitations of present practice and theoretical suggestions for planning excess of loss underwriting activities;

3) A statistical analysis of excess of loss underwriting behaviour and performance over four years of an underwriting agency (Trimark Ltd.) operating in the London Aviation Excess of Loss Market;

4) The case for a workable underwriting system based on measures of market performance and analysis of underwriting behaviour;

5) The formal presentation and testing of an excess of loss underwriting system based on the above considerations;

6) A simulated implementation of the proposed system using authentic data from the aviation excess of loss reinsurance market. The results of the simulation are compared with those of a professional underwriter operating in the same market.

7) Wider implications of research findings for excess of loss reinsurance market behaviour and supervision.

The methodology employed for the research can be described, in general terms, as comprising the four stages of empirical observation, modelling, simulation and interpretation. It would be untrue to say, however, that a fixed format was devised at the outset and adhered to rigidly throughout the research. Rather, the results of each stage determined, to a large extent, the nature and direction of the next step in the investigation. At each stage of the process, conceptual frameworks are raised and compared with previous approaches and methods to allow sufficient depth of analysis and interpretation for the further understanding of the nature of excess of loss underwriting.

1.3 BACKGROUND INFORMATION

Due to the specialist nature of excess of loss reinsurance underwriting, two introductory chapters (Chapters 2 and 3) are provided in the thesis as a background for the ensuing analysis. These chapters describe the role and practice of excess of loss reinsurance in general terms and also provide a conceptual framework for the study of reinsurance problems. The more detailed literature survey on decision-making for excess of loss underwriting begins in Chapter 4.

CHAPTER 2

INTRODUCTORY SURVEY

2.1 INTRODUCTION

This chapter describes the nature and role of excess of loss reinsurance in the insurance market. An insight into the complexity of the market and the nature of the risks against which excess of loss reinsurance offers protection is provided as a background to the problems to be considered in later chapters. Attention is also paid to the important excess of loss contract which defines the risk to be transferred, states the conditions of the agreement between ceding company and reinsurer, and explains the procedure for settling losses.

2.2 THE REINSURANCE MARKET

Reinsurance is an international business: large risks are spread all over the world. The City of London has enormous business connections and a large reinsurance market has developed over the years. The industry is complex with interlocking and overlapping markets loosely connected by tradition and practice; consequently it is not easy to describe its constitution. The following, rather simplified, classification of the market should, however, provide an adequate framework for present purposes. In London the reinsurance market consists of Lloyds, professional reinsurance companies, direct writing insurance companies, and reinsurance brokers. Besides the London reinsurance market there are, of course, insurance and reinsurance markets in almost every country. The small size of many of these national markets, their insufficient capacity and other factors, drive insurers to place their business in the international market, particularly in London with companies at and around Lloyds.

The professional reinsurance companies, by definition, limit their activities to reinsurance and do not conduct direct business. The direct writing insurance company, whilst having as its main objective insurance business, nevertheless, may accept reinsurance either out of reciprocity with another insurer (i.e. where risk is exchanged between companies with no premium charge) or for a separate non-reciprocal reinsurance account.

There are no differences in the methods of dealing with reinsurance business between a professional and a direct writing company except, possibly, in the way business is acquired and in the choice of underwriting policy. The reinsurance brokers perform a similiar function to the insurance brokers in the direct market. Indeed, it is quite possible, in some sections of the market, for one broker to fulfill both roles. The difference, however, lies in the fact that insurance brokers act between the insuring public and the direct writing insurance companies, while the reinsurance brokers are the link between ceding companies and reinsurers. The reinsurance broker's functions are, essentially, the acquisition, placement and subsequent administration of the reinsurance contract.

The institution of Lloyds (which dates back to 1688 when Mr. Edward Lloyd opened his coffee house in Tower Street) is not an insurance company, has no shareholders and does not, itself, undertake insurance business. It is an international market, a society of individual underwriters accepting risks for profit. Lloyds does not deal directly with the public but operates through Lloyds Brokers. While it is not easy to estimate the volume of business transacted by Lloyds, there is no doubt that the volume is large, and that the influence of Lloyds extends into other areas of the market.

Besides the above players in the reinsurance market place there are also many miscellaneous underwriters who transact reinsurance. These reinsurers tend to be small and rely on London acceptances to provide a portfolio of foreign business. Some may act on behalf of groups of insurers, both domestic and foreign, and some may have acceptance powers on behalf of overseas companies whom they serve and advise.

2.3 THE ROLE OF REINSURANCE

The purpose of reinsurance is generally taken to be the reduction of risk or, in statistical terms, to minimise the spread of losses around their expected value. Insurers (and reinsurers) cede risk in order to reduce the total risk for which they are liable. As insurance spreads the original risk, reinsurance spreads the risk more widely and, as an insurance company protects its insured policyholders, the function of reinsurance is to protect insurance companies from large losses in a similar manner. An insurance company might seek the services of a reinsurer in any of the following situations:

1) to enable or improve the handling of large risks;

2) to stabilise technical results;

3) to protect against the possibility of a concentration of losses resulting in a total loss which exceeds the insurer's financial resources;

4) to spread risk by department or geographically as a precaution against catastrophe;

5) as an avenue for entry into and out of markets;

6) in a merger situation in order to handle the outstanding policies of the carrier;

7) for technical help or administration facilities such as the use of a computer on special problems.

The overall effect of these measures is the creation of underwriting capacity for individual companies and, therefore, for the industry as a whole. It should be noted, however, that reinsurance is not the sole means available for achieving increased underwriting capacity; enlargement of reserves and more efficient underwriting policies also increase capacity. The decision to buy reinsurance is weighed against its cost and other available alternatives.

2.4 THE VARIOUS FORMS OF REINSURANCE

There are many types of reinsurance available in the market, each devised for a specific purpose. The basic forms currently in use are facultative, quota share, surplus, stop loss and excess of loss reinsurance. These forms may be classified as being either 'proportional' or 'non-proportional'. With proportional reinsurances, the reinsurer participates in losses, premiums and liability with the same proportional share. Quota share and surplus reinsurance are of this type. Facultative reinsurance may be either proportional or non-proportional. The distinguishing characteristic of a non-proportional reinsurance is that the reinsurer's proportion of the original premium is not the same as his share of the losses; the reinsurer's liability attaches only if the loss has exceeded a fixed amount or percentage of the business retained by the insurer for his own account. Excess of loss and its adapted form - stop loss - are examples of non-proportional reinsurances.

Each type of reinsurance will now be considered briefly. Although this study centres on the analysis of excess of loss reinsurance, information on the other forms is included here since an excess of loss contract may be arranged to reinsure companies already dealing in one or all of the various reinsurance forms. The information serves to emphasise the complexity of the risk situation which can exist from drawing up a single excess of loss contract. In a situation where a reinsurer indemnifies another reinsurer the reinsurance is known as 'retrocession'.

2.4.1 Facultative reinsurance

Facultative reinsurance is the oldest form of reinsurance; examples have been recorded as early as the fifteenth century. Basically, it is a contract where each party to the transaction has a free choice in arranging the matter. Although facultative reinsurance has, in recent years, declined with the increased popularity of treaty reinsurances, it is still important as a means of reinsuring individual risks outside the scope of existing treaty forms, or in order to absorb a surplus after treaty facilities have been exhausted. Substantial amounts of, particularly, large fire, aviation and marine risks are reinsured facultatively and also various classes of liability risks. Facultative reinsurance is sometimes referred to as 'specific reinsurance' since it usually refers to a single risk (or a specific group of risks on one policy). The transaction stands entirely by itself and separate from any other reinsurance agreement between two (or more) companies, which may be struck before or after. Facultative reinsurance is hardly ever the sole form of an insurer's outgoing reinsurances but is regularly purchased in addition to existing treaty arrangements.

2.4.2 Quota share reinsurance

Quota share reinsurance is described by Golding (1965, p.148) as 'a form of reinsurance under which the ceding company is bound to cede and the reinsurer to accept a fixed share of every risk which the ceding company may insure in an agreed section of the business'. In practice the expression 'fixed share' is understood as a fixed, proportional share since claims are reimbursed in the same proportion as premium is paid to the reinsured. The view is sometimes held that quota share reinsurance is not reinsurance at all because the loss ratio of the reinsured business is identical to that of the retained account. In addition, the reinsured company cedes off premiums on a large number of small risks where reinsurance would, under some circumstances, seem unnecessary. The quota share reinsurance is always arranged on a treaty basis, and is a form of reinsurance which is 'favourable' to the reinsurer in that it provides for cessions of every risk, great or small, good or bad.

2.4.3 Surplus reinsurance

Surplus reinsurance is the classic and probably the most widely used form of reinsurance. Its outstanding feature is, in terms specified in the reinsurance contract, that the ceding company is entitled to fix its retention in accordance with the 'quality' of the risk. The surplus reinsurance thus enables the insurer to practice anti-selection of risks against the reinsurer within an agreed framework. The technical effects of this process are threefold. Firstly, it has a homogenising effect on the ceding company's net account. Secondly, it prevents the net account from being affected by large losses because the large risks in the portfolio have been reinsured insofar as they exceed the retention. Thirdly, although the reinsurer does not participate in each and every risk, he still bears a share of many small losses, viz., those partial losses which affect the larger risks. Thus, to some extent, surplus reinsurance also provides some protection against worsening of loss experience on the entire portfolio. Surplus reinsurance facilities are not limited in number; in addition to the ordinary surplus treaty (which is called a 'first surplus treaty'), an insurer may arrange for further surplus treaties. A 'second surplus treaty' would receive a share of the surplus only after the first surplus has received the full amount to which it was entitled, and so on.

2.4.4 Excess of loss reinsurance

The excess of loss form of reinsurance contract with which this study is concerned is such that the reinsurer pays for the whole, or some fixed proportion of the amount by which the original total claim, arising out of any one event exceeds a fixed sum called the priority. Usually, an upper limit is placed on the amount by which the claim may exceed the priority. If cover is required beyond the highest upper limit available under a contract, another contract is arranged which commences at the point where the other liability terminated, the priority for the second contract being the sum of the initial priority and the upper limit of the first contract (see Figure 2.1). The process continues until full cover is obtained. Events covered under the contract are largely determined by the reinsured company, but the contract usually contains a list of exclusions such as war risks or nuclear hazards, risks more properly covered elsewhere (for example, an aviation risk on a marine account), and business raising special problems for premium calculations (such as excess of loss reinsurance of reinsurances issued by the ceding company). Some of these exclusions may occasionally be brought into the cover as special acceptances on payment of a suitably assessed additional premium.

The lower layers of an excess of loss protection are often referred to as 'working layers' because they are the ones most often involved in losses; as a general rule there are more small losses than large ones. Working layers end at the ceding company's underwriting limit on individual policies. Further layers are known as 'catastrophe layers' since they will not be touched without the occurrence of an event which causes simultaneous losses on two or more of the reinsured's original policies. Working and catastrophe excess of loss reinsurances have the following similarities and differences:

2.4.4.1 <u>Similarities between working and catastrophe covers</u>

1) For both types of cover the premium paid by the cedent is calculated independently from the premiums charged to the original insureds.

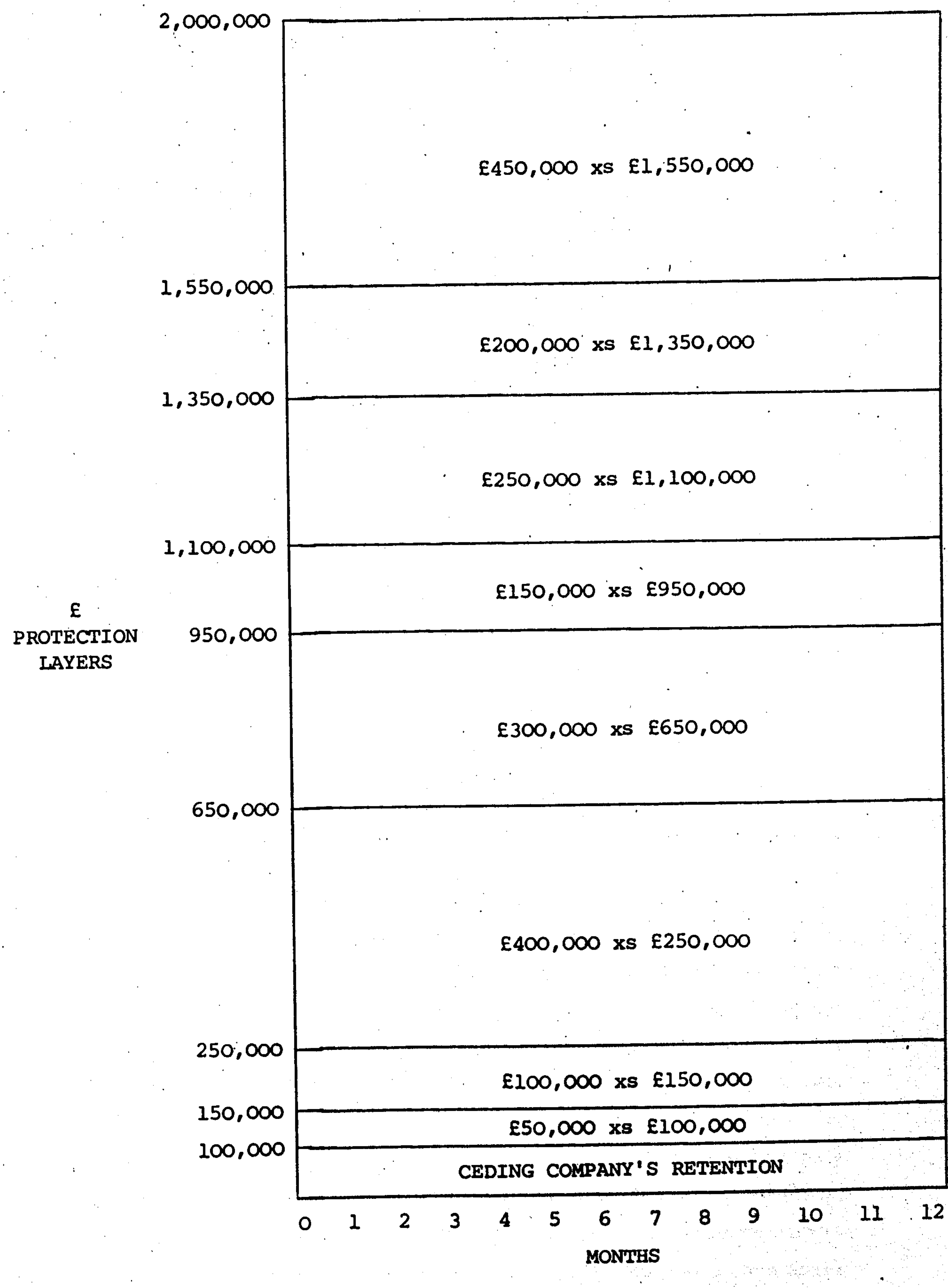
2) No ceding commission is paid to the cedent by the reinsurer.

3) Losses are normally settled within a reasonably short period of time after the cedent notifies the reinsurer of a loss.

4) The cedent's overall protection may be built into layers with different reinsurers in each layer.

5) The reinsurer does not expect that every agreement will pay for itself or that every loss will be recouped. It is possible for one party to make a profit while the other makes a loss.

Figure 2.1 Illustration of an excess of loss protection programme



6) Neither type of cover protects the cedent against a general deterioration in the trend of losses.

7) Both types of cover require that "any one event" be defined; for example, the damage caused by an earthquake for which amount the reinsurer is liable may be limited to, say, 48 hours.

8) Both covers provide an increase in underwriting capacity for the ceding company.

9) Stabilisation of results is provided by both types of cover. Heavy losses are spread over a period of years and between companies.

2.4.4.2 Differences between working and catastrophe layers

1) For working covers, losses are usually frequent enough to be relatively predictable. For catastrophe covers, on the other hand, the frequency of loss occurrence is so small that losses are relatively unpredictable.

2) Working covers protect low layers of the cedents protection programme while catastrophe covers protect the upper layers.

3) The primary purpose of a catastrophe cover is to protect the cedents loss ratio and surplus from all shock losses. Working covers are more useful to guarantee a smoother cash-flow.

2.4.4.3 The excess of loss protection programme

The excess of loss protection programme for a hypothetical ceding company is illustrated in Figure 2.1. The figure shows the various layers purchased as reinsurance for one year's business. In practice, all layers would not start and finish on the same dates, as shown in the figure, but would be staggered throughout the year. As the ceding company grows, further 'top' layers would be bought to protect its increasing catastrophe potential. When each layer goes onto the market for placement it is highly probable that the liability will be shared between many different reinsurers, each accepting a small percentage of the risk. Since each of these reinsurers will have their own reinsurances arranged, the risk is shared out once again into different areas of the market, and so on. This risk-spreading process renders it difficult for an individual researcher to discover how many companies are likely to be involved in the indemnification process resulting from the occurrence of a single large claim.

2.4.5 Stop loss reinsurance

Stop loss reinsurance is an adapted form of excess of loss cover. As the term implies, the ceding company stops losses at a certain loss ratio computed on the business. Thus, the reinsurer is liable only if the loss ratio for an agreed period exceeds a predetermined percentage of the ceding company's total premium income. The reinsurer's liability is limited to a perçentage of the premium income or, alternatively, to a fixed monetary limit. This form of cover is sometimes called 'excess of loss ratio reinsurance'. Stop loss reinsurance differs considerably from other forms of reinsurance such as surplus or excess of loss cover in that it does not give protection for individual claims, or per event, but protects the entire reinsured portfolio. As with excess of loss reinsurance, a distinction can be made between working and catastrophe protection by stop loss treaty.

2.5 THE SPECIAL NEED FOR EXCESS OF LOSS REINSURANCE

Excess of loss reinsurance is one of the market's answers to the need for increased capacity through the shedding of the possibility of low probability but high value shocks to the reinsured company's cash flow. Capacity is a valuable commodity in the insurance market and, although the literature provides no concise definition of the concept, it is generally regarded as having two desirable properties: firstly, it is a necessary input to the production process and secondly, it increases the soundness of an insurance concern. By purchasing excess of loss reinsurance at an economic price, the advantages to the ceding company are twofold: firstly, indemnity is provided for the low probability, high value losses and secondly, reserves which would otherwise be retained to cover the possibility of catastrophe loss are freed for either growth or increased company stability. Without the increased capacity brought about by the reinsurance of catastrophe-type risks, the supply of insurance at direct level would be significantly reduced and, consequently, more costly.

There are indications that the insurance world's susceptibility to catastrophe is increasing. While the frequency of natural disasters (floods, hurricanes, earthquakes, etc.) is not itself increasing, extensive developments in disaster-prone areas (flood-plains, earthquake fault-zones, etc.) have increased the potential losses resulting from natural disasters. Since insurance is bought to cover the new developments, catastrophic losses have increased in both frequency and severity in recent years. With increased technological progress, there are also many new man-made disasters (for example, the thalidomide drug). It is, therefore, essential to the industry that provisions for dealing with catastrophic occurrences are both available and effective. The excess of loss reinsurers may, in this sense, be regarded as the final line of defence against the adversity of nature and technology. The more common types of catastrophe against which excess of loss reinsurance is purchased are shown in Figure 2.2.

Figure 2.2 Disaster and catastrophe situations requiring excess of loss reinsurance protection

NATURAL DISASTERS	MAN-MADE DISASTERS
- Earthquake	- Bombs and bomb threats
- Floods	- Arson
- Hurricanes	- War
- Tornadoes	- Nuclear hazards
- Fires	- Tenants Acts
- Sandstorm	- Explosions
- Landslides	- Chemical accidents
- Extreme temperatures	- Civil disturbances
	- Demonstrations
	- Blackouts
	- Interruption of essential services

2.6 THE EXCESS OF LOSS CONTRACT

The drafting of an excess of loss contract is a complex business usually handled by a specialist since it must be tailored to fit the particular circumstances of the company or portfolio to be reinsured. Although there are no standard contracts, there are certain clauses which occur quite regularly and which, for practical purposes, can be thought of as standard clauses. These clauses are set out in Appendix A, but the most important clause - the reinsuring clause - is included here since it describes the basic features of the protection afforded by the contract. The reinsuring clause typically states the following:

> This reinsurance is to pay the excess of an Ultimate Net Loss to the Reinsured of £XXXXXX each and every loss with a limit of liability to the Reinsurers of £XXXXXX each and every loss.

The term 'Ultimate Net Loss' refers to the sum actually paid out by a ceding company in settlement of claims after making deductions for recoveries and claims upon other reinsurers. The term 'each and every loss' is also important and is generally understood to mean

> each and every loss (and/or occurrences and/or catastrophe) and/or series of losses (and/or occurrences and/or catastrophes) arising out of one event.

Hence the emphasis is on reinsurance against catastrophe-type circumstances. The monetary limits to this protection are known as the upper and lower limit respectively for the amount limiting the reinsurers liability and the point at which liability commences (as illustrated in Figure 2.1).

The means of application of the contract is also specified in the excess of loss agreement. There are two means of contract application - the "losses occurring method" and the "losses attaching method". By the former method the reinsurer becomes liable for losses occurring between two specified dates. By the latter method, the reinsurer becomes liable for losses incurred as a result of policies written by the reinsured between two specific dates. The methods appear to be equally popular in the market.

A further important matter to be defined in the contract is the part(s) of the reinsured's portfolio(s) to be included in the protection. The variety of portfolio classifications for which excess of loss reinsurance is sought includes the following:

1) Net account. Protection is provided for the ceding company's net liability, excluding the underwriting of existing treaty reinsurers, for a specific amount;

2) Gross account. This protection is for both the ceding company and its treaty reinsurers for a specific amount;

3) Combination of gross and net. This allows treaty reinsurers to opt in or out of the excess of loss reinsurance protection;

4) Whole account cover. This cover provides protection for several classes of business. It can be either 'net' or 'gross' and will probably specify limits for each account separately;

5) Umbrella covers. This provides cover against an event causing a heavy loss in several departments simultaneously. It is a catastrophe-type cover and is normally arranged as a 'last-line-of-defence' mechanism.

Payment methods for excess of loss reinsurance (which are specified in each contract) fall into two main classes - the flat premium method, and the 'minimum deposit' method by which an advance, non-returnable payment is paid by the reinsured with an adjustment procedure applied at the end of the indemnity period (calculated on the basis of the ceding company's premium income). The actual money amount handed over under each of the methods is not likely to vary significantly between the two methods per £ of ceding company premium income; the minimum deposit method simply provides a payment mechanism in circumstances where ceding company premium income is difficult to forecast with reasonable accuracy.

The essential ingredients of the excess of loss contract which are necessary for the present study are included above and in Appendix A. Should further information on contractual matters be required, reference should be made to Golding (1965) and I.I.L. Advance Study No. 201 (1975) which contain further insights into the non-exhaustive subject of excess of loss contract drafting.

2.7 A BRIEF NOTE ON THE DIRECTION OF THE RESEARCH

The information presented in this chapter is intended to provide an introduction to the role and nature of excess of loss reinsurance in the U.K. insurance market. The main points which should be emphasised are firstly, that excess of loss is of special importance in the market place as a means of providing underwriting capacity and secondly, that it is a complex but highly adaptable form of reinsurance protection. Efficient use of resources is a key problem facing individual underwriters and the industry as a whole and, since the market is notorious for its high element of uncertainty, many problems must be faced and solved to achieve this end. In the next chapter, attention is paid to the key problems in the process from the point of view of a reinsurer operating in the excess of loss market. These problems are considered further, along with methods for their solution, in later chapters. Attention returns to the wider needs of the industry in the final chapters of this text where the micro-problems facing individual underwriters are considered for their relevance to the macro-problems facing the industry.

CHAPTER 3

REINSURANCE COMPANY OBJECTIVES AND THE MAIN DECISION AREAS

3.1 INTRODUCTION

The main concern of this study is the analysis of excess of loss underwriting problems. It is, therefore, desirable that the excess of loss underwriting function can be described in terms of its contribution to overall reinsurance company objectives and performance. In this way the major decision areas can be considered along with the problems which must be solved to achieve more efficient performance and attainment of company and portfolio objectives. The purpose of this chapter is to provide an outline of the current conceptual framework of the reinsurance organisation and its operations in keeping with the models of insurance and reinsurance affairs put forward in the contemporary literature. This is followed by a description of the key underwriting decision areas in relation to company objectives. Finally, a brief outline of the problems surrounding the key decision areas is provided as an introduction to the more detailed treatment which follows in the next chapter.

3.2 REINSURANCE COMPANY OBJECTIVES

In common with other commercial organisations, a reinsurance company has corporate objectives which lend direction to decision-making processes within the company for both routine, day-to-day operations and planning and control activities. An appropriate starting point for this investigation, therefore, is an analysis of the reinsurance company objectives towards which, for practical purposes, all decision-making within the organisation is geared. A concise statement of reinsurance company objectives, however, is no straightforward matter; interested parties include management, staff, shareholders, policy holders and regulatory authorities - all of whom may have conflicting ideas on how the organisation should operate. Of these groups, probably the shareholders and management contribute most pressure towards shaping company objectives - subject to constraints posed by other interested parties. Karl Borch, a keen modeller of insurance and reinsurance company affairs, supports this general view but, regarding shareholder influence, concludes that company objectives are likely to have as much influence on the shareholding group as the influence of shareholders in shaping company objectives. He writes (1962, p.167):

> It is natural to assume that the objectives of an insurance company in some way can be devised from the personal objectives of those who control the company ... It may be more realistic to assume that the objectives to be pursued by an insurance company are laid down almost accidentally and that they eventually end up with having stockholders who unanimously approve of these objectives.

Borch's view rests on the assumption that, if shareholders do not agree with company objectives, they will transfer their capital elsewhere. It could be reasonably argued, however, that this view is too simplistic in that shareholders' requirements are geared not only to company objectives but also to the performance of share values in a free market. His view, nevertheless, has certain attractions for (re)insurance company modelling purposes because, given that company objectives are as likely to shape the character of shareholders as vice versa, it directs the search for (re)insurance objectives squarely on the organisation itself. The organisation may, thus, be envisaged as a separate economic entity with its own aims and characteristics operating in an economic environment along with a morass of other entities each with their own aims and objectives. To define these objectives one might employ the convenient goal of profit maximisation from economic theory but this approach is inadequate for a description of (re)insurance operations, even at a conceptual level. Pentikainen (1978, p.22) another insurance model-builder, takes the following view:

> The classical goal which often used to be the only one, is the maximisation of profit, or in more general terms, optimisation of utility of profit. Nowadays this goal may be considered far too narrow and it is suggested that other goals are to be taken into account simultaneously. In addition to profit making and its use for dividends and/or bonuses, the consolidation of the existence of the business, i.e. solvency, is a very important goal, and in addition to this often the expansion of the business.

For the explanation and modelling of insurance affairs, this wider appreciation of company objectives is an important step towards realistic analysis. The benefits of this approach are, however, essentially long-term. In the short-term, acceptance of the need for the wider approach renders the convenient and well documented profit maximisation approach, at least temporarily, redundant until the new concepts are mastered. Donaldson (1971, p.25), a non-insurance specialist whose views on financial resiliency and corporate planning might be considered general enough to encompass the needs of insurance operations, describes the modern concept as "a modification of the maximum concept designed to indicate that maximisation must be consistent with other non-profit corporate goals ... The target profit level becomes the 'best' rather than the 'most' taking all considerations into account". For a reinsurance company it would, therefore, seem appropriate for a ranking system of desirability for all the competing objectives such as profit, stability and growth to be devised to provide an overall company objective based on a constrained maximisation. Two conceptual problems are posed by this approach: firstly, a method must be devised by which subjective feelings of parties with interests in reinsurance company affairs can be taken into account; secondly, a unit of measurement, common to all competing objectives, must be found for the mathematical process which leads to a constrained maximisation. The current solution to these problems is to apply the utility function from economic theory to provide a subjective weighting system for competing objectives, and measures of cash flow (adjusted for risk) to provide a common unit for comparison and maximisation purposes. References to these conceptual devices in connection with reinsurance are common in the literature and include the following from Borch (1962, p.162 and 1967, p.584):

> A utility concept applied to insurance companies must refer to what is usually called the risk situation of the company.

and

> It is clear that the real problem of the company is to specify its objectives in terms of a preference ordering over a set of stochastic processes.

From this conceptual starting point, the actuarial profession has built up an enormous volume of knowledge relevant to individual (re)insurance company problems. The overall objective, from an actuarial point of view, has become to maximise profit and minimise risk-taking within the constraints of the business environment. The onus of the analytical work has been to describe the means by which objectives of risk and return can be met in a structured manner through the basic underwriting functions which determine company performance. The methods, originally devised to solve problems in the direct insurance market, however, have not found such successful application in the reinsurance market or, more specifically, in the catastrophe excess of loss market. Consequently, key decision areas such as reinsurance contract pricing and portfolio composition have remained, for practical purposes, in the realms of human judgement and rule of thumb techniques. The proof of success or failure of such techniques manifests itself only in the form of reported underwriting results which, to date, have escaped any form of "pre fruition" test of adequacy. Before embarking on the quest for methods for solving these problems the key reinsurance decision areas will be presented along with methods put forward for their solution. More detailed discussion on methods for solving the problems follows in later chapters.

3.3 KEY REINSURANCE UNDERWRITING DECISION AREAS

The key reinsurance underwriting decision areas are those which determine the degree to which organisational objectives are met. The quality of the relevant decisions is difficult to determine in the case of excess of loss reinsurance due to the strong influence of "luck" (good and bad) on company performance. Nevertheless, the underwriting process centres around three important decision areas over which the underwriter may exercise skill and judgement. They are:

1) Pricing. This involves arriving at a premium for accepting a reinsurance risk which is acceptable to reinsurer and ceding company alike.

2) Portfolio construction. This process employs the law of large numbers and the principles of spread and diversification to construct a portfolio of risk-taking opportunities (reinsurance contracts) in keeping with company risk constraints and profitability targets.

3) Own reinsurance. Reinsurance of the reinsurer's own portfolio involves decisions on how much risk to cede after its initial acceptance. It is an important underwriting decision aimed at optimising the risk-return potential of a completed reinsurance portfolio.

These decision areas may be regarded as being closely linked for purposes of reinsurance company control and their consequences shape the financial results of a reinsurance company. The actuarial view of the interrelated problems and their relationship to management control are described by Ratcliffe (1976) as follows:

> Apart from the more obvious management functions of underwriting, claims and expense control, the successful management of an insurance operation depends on the solution of three interrelated problems.
>
> 1) Establishing premium rates on which business can be obtained and written with a profitable result;
>
> 2) Establishing "correct" "technical reserves" at the close of each accounting period for unexpired risks and for outstanding claims including I.B.N.R.;
>
> 3) Establishing and maintaining the adequacy of the capital base (the free capital resources) and the required level of reinsurance.
>
> The degree of success of the operation can be measured by the overall return on the capital employed in the business, subject to the evaluation of any capital items in the form of margins in the technical reserves.

"I.B.N.R." referred to in the quotation is the insurance term for claims which have been incurred but not reported.

The excess of loss underwriter's decision process is identifiable as the centre of business activity which shapes and determines the outcome of company performance. His pricing decisions determine the likely amount of underwriting profit made by the company and his selection of risks determines the need for and the nature of company reserves and reinsurance requirements.

This study is concerned, chiefly, with two of the three identified underwriting decision areas; namely, pricing and portfolio construction. The intention is not to disregard the own reinsurance decision as unimportant but to concentrate the research effort squarely on the trading situation between ceding company and the excess of loss reinsurer. By centering the analysis on a single stage in the risk-transfer process a more rigorous analysis is possible than if the framework for study was open-ended and intended to take into account retrocession after retrocession. The decision to keep the range of study narrow is further justified because the own reinsurance problems for the reinsurer are substantially the same as those of the ceding company seeking reinsurance protection and may, from the results of the analysis, be generalised at a later stage. The own reinsurance problem will be dealt with further in Chapter 4 where the risk situation of the portfolio to be reinsured is considered. Before embarking on examination of this and methods put forward as prospective solutions for rating and portfolio construction problems, the following aspects surrounding the risk-acceptance decision should be noted.

3.3.1 The rating decision

Rating decisions are made by the "lead underwriter" on individual excess of loss reinsurances. Important attention must, therefore, be paid by him to arriving at a premium which is acceptable to both ceding company and reinsurance fraternity alike. The risk acceptance process for excess of loss reinsurance illustrates the basic economic law that a transaction takes place only if all parties involved in it think they benefit. In this particular situation, the option of other underwriters in the market to follow the "lead" is expressed in terms of a percentage acceptance (or total rejection) of the opportunity to participate in the agreement at the going rate. If the premium set by a lead underwriter is inadequate the risk will not be fully placed on the market and a new lead must be arranged at a higher price. Of prime importance to the rating decision-maker is the possible effect of contract acceptance on corporate cash flows. At time of negotiation, the premium is, potentially, an immediate inflow of funds to the organisation against which uncertain future cash outflows must be gauged. At the same time, for the ceding company, the premium represents an immediate outflow of funds against which the prospect of the same uncertain future cash outflows must be evaluated. For this reason the excess of loss premium is often referred to as the 'central problem'. For example, Vajda (1955, p.63) writes as follows:

> The assessment of premium rates in Non-Proportional Reinsurance can be considered as the "central problem", not only because rating is the basis of the requisite economic stability of reinsurance, but also because the very decision whether the reinsurance form is or is not opportune or applicable depends very largely on the solution which is to be applied for rating.

The problem to which Vajda considers rating should supply the solution is that of pricing under uncertainty. In the rating process for an excess of loss contract, particularly for the catastrophe covers, not one party to the agreement can claim to understand the true nature of the underlying risk situation. Premium rating for excess of loss reinsurance is, therefore, characterised by a high degree of human judgement and a bargaining process. Of course the process has not escaped analysis of a risk-theoretic nature (as will be discussed in Chapter 4), where the intention has been to identify component parts of excess of loss premium and devise methods for their calculation. The risk-theoretic approach, however, has not yet found practical application in the market as a replacement for the enigmatic qualities of underwriting skill and judgement.

The mechanics of applying the excess of loss rate (however determined) are as follows. Once agreed, the rate is applied not to the amount of liability transferred (as in a direct insurance) but to the premium income of the ceding company. For example, if the rate is 1 percent and the ceding company's premium is £1,000,000, the premium handed over is £10,000. Since premium incomes for ceding companies are not known precisely until the end of a contract period it is common for a "minimum deposit" fee to be paid by the ceding office at the beginning of the contract. Should the final premium exceed this amount, the difference is paid over at the end of the contract period. Should the final premium fall short of the minimum, the difference is forfeited by the ceding company. This apparent accuracy in the appliance of excess of loss rates stems from rigorous record keeping rather than precision in measuring reality as reflected in the rate calculation. The basic question "How much shall be charged for contract acceptance?" has no universally agreed solution for excess of loss reinsurance.

3.3.2 Portfolio construction decisions

Portfolio construction decisions are the problems of single underwriters rather than of a group of interested parties. Left to his own devices the excess of loss underwriter's task is to construct a portfolio to meet the objectives and constraints of profitability and financial stability required by the organisation(s) for which he underwrites. Great uncertainty is present in the decision-making process, the essential questions to be answered being:

1) Is it prudent to take on any part of a prospective excess of loss risk contract?

and (assuming a positive answer to the first question)

2) What percentage of the contract should/can be accepted?

These basic decisions which determine incremental additions to the portfolio are made with only limited knowledge of the likely nature of the individual underlying risk situations. The underwriter, therefore, employs a risk selection process aimed at securing a satisfactory overall risk acceptance by employing the law of large numbers to the best of his ability to ensure a balanced portfolio. This risk spreading process is basic to all insurance operations but is carried out under the greatest conditions of uncertainty in the catastrophe excess of loss reinsurance market. The likelihood that the underwriter can achieve desired portfolio characteristic is, therefore, also at its lowest in this particular market. Some underwriting principles which are likely to be considered in the portfolio selection process in the absence of any clear understanding of the likely consequences stemming from any individual contract acceptances are as follows:

1) Write small amounts of a large number of risks to allow the law of large numbers to apply in relation to loss incidence;

2) Try for a certain permanence of risks upon the books. This allows for alterations over time to premiums charged to ceding companies to be negotiated on the basis of new information on loss experience;

3) Include as wide a variety of risks (by class of business and geographically) as possible to lessen the likelihood of a sudden drain on reserves brought about by a single catastrophic event;

4) Charge as much as you can without losing the business.

The above list is not exhaustive and is presented only to provide examples of the paths of action available which underwriters might chose to follow to maintain and improve their position in the market. Of particular note is that none of the principles described above can be easily translated into measurable terms. For example, there is no means of identifying how many risks constitute a balanced portfolio without adequate information on the individual risks which it comprises. The intention to balance premium rates for a ceding company over a period of years also presents problems since past loss experience is no sure guide to future loss experience - especially where catastrophic events are concerned.

3.3.3 Rating and portfolio construction as a joint decision process

There is a trend among risk theoreticians to combine the decision process of rating and portfolio construction into a single decision-model (for example, Ferrari (1967) and Stone (1975)) which is adapted from the Markowitz model for portfolio selection for investments (Markowitz 1952). The combined model has been received with interest by academics but has found little practical application in the excess of loss market. For the combined decision model for rating and portfolio construction to be successfully applied in the excess of loss market the following conditions would be required.

1) The premium attributed to an individual contract must be capable of division into (a) an expected loss element, (b) a claims variation element, (c) an allowance for expenses and (d) an allowance for variations in price (the market differential) around an expected premium.

2) A method for combining the elements into the portfolio in keeping with desired risk and return constraints must be available to the underwriter.

Most theoretical analysis has centered on the second of these problems despite the fact that the inherent uncertainty of excess of loss business renders the first problem virtually unsoluble. Although the theoretical combined decision-model has not found direct application in the market it is, nevertheless, a useful tool at the conceptual level for describing underwriting behaviour. A basic ingredient which is lacking for practical implementation of the model (which will be discussed further in the next chapter) is the unmeasurability of uncertainty - and this problem will remain with excess of loss underwriters for a very long time.

3.4 EXCESS OF LOSS UNDERWRITING PROBLEMS TO BE TACKLED IN THE STUDY

In this chapter the key problems facing the excess of loss underwriter have been described as rating, portfolio construction and own reinsurance. Of these, the first two problems have been selected for detailed analysis in this study. In tackling them, however, a certain amount of attention must, necessarily, be paid to aspects of the business which are also relevant to the third, own reinsurance, problem. For example, description of the ceding company's portfolio is common to both rating and own reinsurance problems. The study proceeds with a detailed presentation of the problems described in this chapter with special emphasis on those encountered through the process of risk acceptance.

CHAPTER 4

THEORETICAL AND PRACTICAL AIDS TO DECISION-MAKING FOR EXCESS OF LOSS REINSURANCE

4.1 INTRODUCTION

Having identified the main excess of loss decision areas, the purpose of this chapter is to consider in detail the approaches and problem-solving techniques which have been applied to the excess of loss underwriting situation. The information provided in this chapter falls into three broad categories. Firstly, the various approaches to excess of loss problem-solving are introduced and discussed in the light of current market practices. Secondly, attention is paid to both theoretical and practical approaches to the excess of loss rating problem, with particular emphasis on the difficulties involved in adapting probabilistic, conceptual models for use by professional underwriters. Finally, the important subject of portfolio management is presented at both a theoretical and practical level and the possibility of an underwriting control system for excess of loss reinsurance is discussed. Throughout all three topics it is possible to identify large differences between the ways working underwriters and academics approach the problems of excess of loss reinsurance; the operations of the market-place fall far short of the conceptual ideals put forward by some writers. General criticisms of the two approaches are that the conceptual models are unrealistic and unworkable but that the practical approach, on the other hand, although workable, does not lend itself easily to development for formal planning and control activities. The information in this chapter serves as a basis for identification of the problematic areas of excess of loss reinsurance which are chosen for further investigation.

4.2 THEORETICAL APPROACHES TO REINSURANCE PROBLEMS

The excess of loss reinsurance process can be loosely described as consisting of the tacit agreement of many decision-makers to pool the results of many individual risky ventures. Through this process, the many organisations represented in the pool attempt to order their affairs according to their individual goals. This arrangement stems from the natural human tendency to share fortunes and spread risk-taking in the face of uncertainty. Each underwriter is both dependent on and in competition with the other underwriters in the marketplace. To survive in this environment, underwriters are provided with little formal training, the job being learned from example, experience, and underwriting tradition. On entry to the underwriting fraternity, an underwriter's decisions are made according to a combination of intuition and experience. Issues such as personal charisma and length of service are held, in market circles, to be as important as an apparent ability to foretell the future. Specialisation is also an important key to underwriting success in the London Market; the underwriter who has spent the greatest length of time on any individual class of business is the one most likely to be first approached by brokers to set a rate and accept part of a contract placement. An important question is, therefore, "How can the theoretical approach to underwriting offer solutions to the problems of practically-oriented people dealing with the idiosyncracies of a lively, changeable and innovative marketplace?" Theoretical approaches put forward include traditional and behavioural economic theories, risk theory and a group of ad hoc concepts which come under the general heading of decision theory. This section describes these general theories and their applicability to reinsurance problems. More specific attention to the application of the theoretical approaches to real-life excess of loss underwriting follows later in the chapter.

4.2.1 Traditional economic theory

The traditional economic theory of the firm is summarised by Weston (1966, p.32) as follows:

> The existing economic theory of the firm focuses on analysing the conditions for maximising net revenue and analysing causes of shifts in equilibrium position. At equilibrium the marginal rates of substitution between products or factors are equal to the ratios of their price.

The ingredients of the theory may, from this quotation, be identified as revenue maximisation, the concepts of price and equilibrium and the existence of products and factors of production. For excess of loss reinsurance, the revenue maximisation intention may be intuitively acceptable, but the remaining ingredients of the theory become difficult to apply due to the inherent problems of risk and uncertainty. For example, enormous problems are involved in any attempt to describe adequately the dimensions of "indemnity against loss" which is the reinsurers marketed product. Revenues and costs of production are equally difficult to identify, as is evident from the following quotation from Pfeffer and Klock (1974, p.248) in which general insurance operations are compared with those of other firms.

> ... few firms are able to determine their marginal revenue or cost curves. To provide a profit, they rely on marking up the average cost per unit by some given percentage. While prices may be allowed to decline below average costs, prices will not be set below variable cost. Insurers tend to follow the same pricing method but, in addition, must contend with the following problems.
>
> (1) Average costs in insurance are predominantly variable in nature and to a large extent beyond the control of the insurer
>
> (2) Accurate projections of average costs, particularly claims items, require sufficient numbers of exposures to allow the 'law of large numbers' to operate . . .
>
> (3) Prices may violate the economic rule and be set below unit variable cost, if losses and claims expenses are far in excess of those projected by the actuaries . . .
>
> (4) For many insurers, within the constraints of capacity, unit cost does not change significantly with sales
>
> (5) The insurer markup is not solely for profit but also includes an input for contingencies that are somewhat unpredictable, such as losses incurred but not yet reported.

The difficulties of applying economic theory to general insurance problems become even more unmanageable when applied to excess of loss operations where uncertainty is even more predominant. Claims projections become more difficult due to lack of risk homogeneity, the mark up for contingencies becomes larger than the profit mark up, and claims incidence becomes much more variable. The acid test of the applicability of economic theory to insurance operations is its ability to encompass market decision-making. Here, traditional economic theory puts forward the concept of a utility function as the medium by which economic man bases his decisions. This omnipotent law is applicable to the excess of loss reinsurance market only with many modifications to allow for the variety and complexity of real-life decision-making. The applicability of traditional economic theory to real reinsurance problems seems, therefore, very limited. To put it another way, an economist and a reinsurance personality are likely to agree on overall business procedures but, on the subject of day-to-day decision procedures, the economist might become bewildered at the degree of guesswork and irregularity involved in most individual market decisions.

4.2.2 The behavioural theory in economics

The behavioural theory in economics frees economic man from the constraints of behaving as a rational pain-and-pleasure calculating machine. Since "pain-and-pleasure" in the form of positive and negative cash-flows are difficult to predict for excess of loss reinsurance, the behavioural approach would seem more applicable than traditional economic theory. Weston (1966, p.33) summarises the behavioural approach as follows:

> Summarised briefly, the behavioural theory of the firm states that the major functions of theory are to formulate an exhaustive set of general concepts and to specify the critical relationships among the system variables. The exhaustive categories in the behavioural theory are (1) organisation goals, (2) organisational expectations and (3) organisational behaviour.

Three important features of the behavioural approach are identification of matters of key importance, goal measurement and analysis of aspiration levels. From such analyses, more practical decision models can be constructed than those based on traditional economic theory. Although the behavioural approach has been applied with some success to the problems of the direct insurance market (for example, Hershbarger (1975) investigates the amount of capital insurers are willing to invest in insurance operations), the author is unaware of any behavioural-oriented case-study on, specifically, the excess of loss market. Most of the pioneer behavioural studies are of a mainly psychological nature and concerned with how individuals make decisions according to the amount and type of information available (for example, Simon (1955), Lindblom (1959), Baumol & Quandt (1964)). Nevertheless, the micro behavioural theory might find useful application in the excess of loss market where information for decision-making is never complete. The following illustration from Tversky and Khaneman (1974, p.50) might be adapted and compared with the excess of loss underwriter's decision-making process:

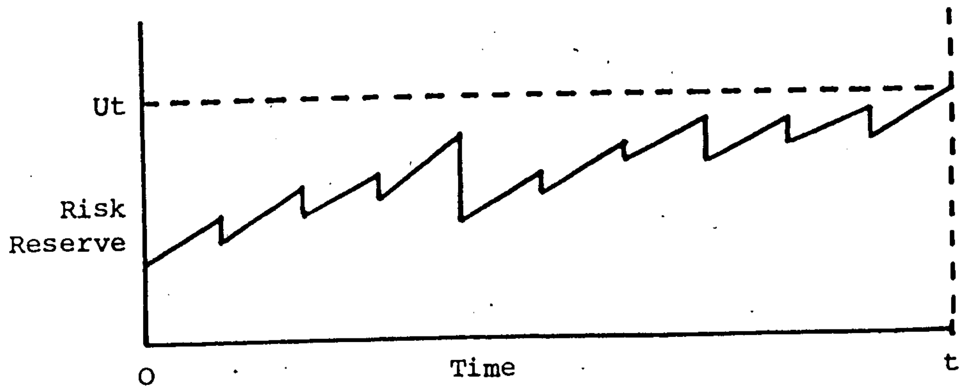
> Suppose one samples a word (of three letters or more) at random from an English text. Is it more likely that the word starts with r or that r is the third letter? People approach this problem by recalling words that begin with r (road) and words that have r in the third place (car) and assess the relative frequency by the ease with which words of the two types come to mind. Because it is much easier to to search for words by their first letter than by their third letter, most people judge words that begin with a given consonant to be more numerous than words in which the consonant appears in the third position.

The lesson to be drawn from the example is that when decisions must be based on experience (as in the case of excess of loss underwriters) the solutions supplied are not necessarily correct. Further, large numbers of decision-makers faced with the same problem might come up with the same wrong answer yet maintain a high degree of agreement among the group. In the absence of any feedback concerning the true solution to the problem, the group might continue under the misapprehension indefinitely. The behavioural approach might therefore be profitably employed in the excess of loss market to establish (a) the (probably familiar) aspects of the business on which underwriters base decisions and (b) the thought process applied to these aspects and (c) whether or not the solutions so arrived at are likely to approximate the real solutions.

4.2.3 Risk Theory

Modern risk theory is often attributed to Filip Lundberg (1932) a Swedish mathematician. Like financial theory, risk theory is concerned with the generation of probability distributions but, unlike financial theory, it encompasses the use of probability distributions in the production function. The risk theory has developed to find many applications in the analysis of insurance and reinsurance problems. The objective has become to provide a mathematical framework capable of handling the random fluctuations in insurance, and to devise various methods for containing their inconvenient effects. Insurance problems are reduced, where possible, to probabilistic models, the primary aim being to control the process by keeping variations in cash flows small in relation to expected results. The theory is widely used by actuaries and continues to provide opportunities for development. The subject matter has, consequently, become complex. The basic features of the theory are, however, illustrated in Figure 4.1 which demonstrates the activity of an insurance business as a stochastic process.

Figure 4.1 The insurance business as a stochastic process



The figure shows the steady inflow of premium income and the sudden outflows of funds for claims settlements. The difference betwen Uo and Ut at the end of a time period denotes the profit (or loss) made. (A need for excess of loss reinsurance might arise if the fluctuations caused by claims were both irregular and large.) Through analysis of the fluctuations in cash flows, useful information may be gleaned on the expected profits to be made and the chances of the business becoming ruined.

There are two divisions within risk theory which examine, essentially, the same problems but from different points of view. One division (Individual Risk Theory) chooses the individual insurance policy as the basic unit of analysis, while the other (Collective Risk Theory) concentrates on the overall effects of the whole range of policies on the insurance portfolio. The fundamentals of each division of risk theory are as follows:

a) <u>Individual Risk Theory</u>

1) Maximum gain accruing from a policy occurs when no claim arises.

2) Maximum loss on a policy is limited by the extent of specified liability.

3) The possible gain or loss from a policy is a random variable.

4) A probability distribution of the random variable exists such that the total area bounded by the function equals 1 (see Figure 4.2 which illustrates the situation where a company has an even chance of gain or loss from a policy).

5) The effect on total cash flow may be determined by combining the expected impacts of the individual probability distributions arising from all the individual policies.

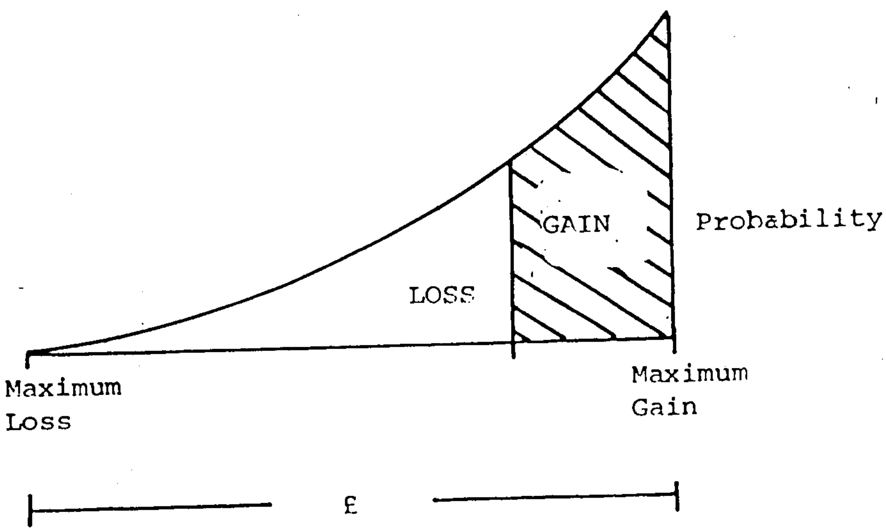
b) <u>Collective Risk Theory</u>

1) The stochastic process for insurance can be determined in terms of random sequences into which a time parameter is introduced.

2) The stochastic process is modelled as comprising (a) a stream of premiums received during a period, (b) the distribution of

individual claims equal to the probability that if a claim should occur it would not exceed a specified amount, and (c) the probability of claims occurrence during the period.

Figure 4.2 The probability function for an individual insurance policy with an even chance of gain or loss



Explanatory note: If the company is to have an even chance of gain or loss under a policy then half the area under the curve should be bounded between zero and the maximum possible gain.

From consideration of the key points of each division within risk theory it may be concluded that the two approaches draw very close to one another on many issues. For example, the ruin problem (the probability that claims will exceed £x in a specified time period) and the measurement of expected profits and losses can be tackled using either approach. The success of applying either approach to insurance and reinsurance problems, however, is dependent on how well the stochastic processes which form the basis of the mathematical models are known in practice. Factors governing these processes for any individual insurance operation are known only partially. The passage of time might allow a company to acquire knowledge of the process through statistical analysis and, eventually, it may be possible to use this knowledge for assessing premium rates, reinsurance requirements and future plans. In order to fully utilise the risk theory, a firm would require an information system for monitoring the stochastic process as it develops and a set of rules for translating the observations into action, with a view to controlling the process so that it develops in a desired fashion. There are, however, some sound theoretical and practical reasons why this ideal state of affairs cannot be reached using risk theory alone. These reasons include the following:

1. Unrealistic assumptions. The risk theory rests on some unrealistic assumptions which limit its ability to reflect reality. These include;

a) the assumption that the value of the business does not change over time,

b) an assumption that laws governing the stochastic processes and probability distributions are completely known, and

c) an inherent assumption that a decision, once made, is irreversible.

2. Divergence from executive attitudes. Risk theory provides a decision technique based, in its simplest form, on calculation of probabilities of various outcomes for various alternative actions, followed by selection of the action with the highest value to the probability of survival. The author's experience is that insurance executives do not make decisions in such a mathematical or structured manner.

3. Difficulties with large claims. Large claims also have the lowest incidence, which makes analysis of the stochastic process difficult and gives rise to the use of approximations, thus detracting from an otherwise scientific approach to insurance problems. Beard (1969, p.41) explains the

difficulty as follows:

> Direct methods of attack on the treatment of F(x) often lead to cumbersome expressions so that it is, in general, not easy to deal with problems concerned with, for example, different methods of reinsurance, net retentions and safety loadings. Furthermore, it is extremely difficult to obtain a broad survey of the problems. Even if the nature of the problems justifies the more detailed computations, simple working approximations are necessary and it follows that one of the problems of applied risk theory is the finding of proper approximations.

The problem of large claims and the use of approximations will be discussed further in Section 4.3.1.

It may be concluded that risk theory, although a powerful analytical tool for insurance affairs, encounters serious limitations to its full application in practice. The problems increase with the size of the fluctuations in the underlying risk process. For excess of loss business these problems reach a peak and present a continuing challenge to risk theoreticians. Some of the solutions put forward will be described later in this chapter. As a generalisation it may be said, however, that for excess of loss business, the risk theory is an appropriate analytical tool but extremely difficult to apply in practice.

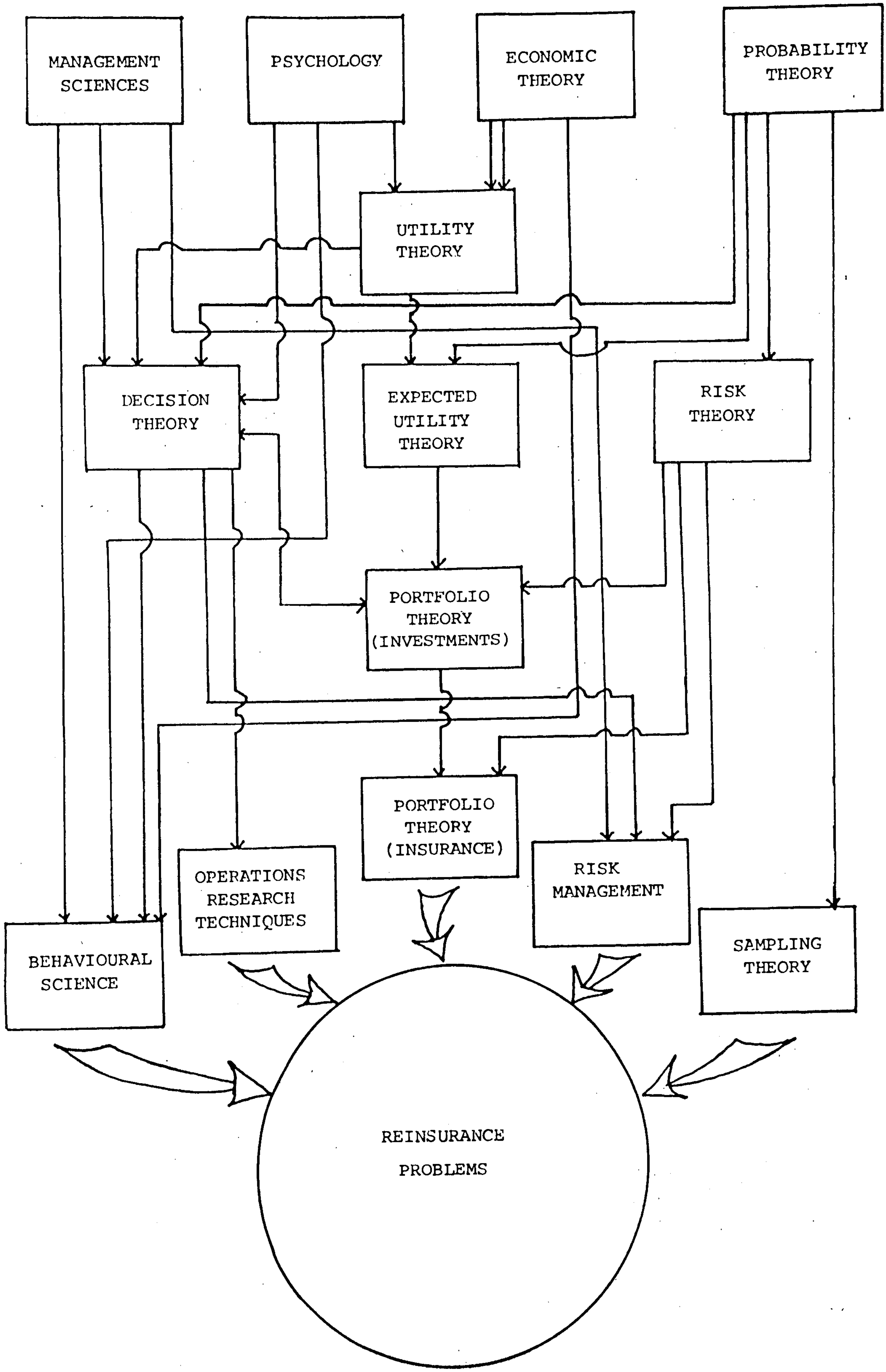
4.2.4 The interdisciplinary approach to reinsurance problem-solving

From the preceding sections it is apparent that, either through lack of development, or inappropriateness of basic assumptions, existing theories of economics and risk have failed to supply useful solutions to many practical reinsurance problems. The remaining knowledge gaps present a challenge to academics and reinsurance professionals alike. A possible reason for these shortcomings is an unwillingness of reinsurance problem-solvers to abandon the straitjackets of the various academic disciplines and to centre the analysis on real-life reinsurance phenomena, thus cutting across both traditional thinking and methodology. Solutions arrived at in this manner might not fit neatly into the older structure of insurance, but could prove more useful for the practically-minded reinsurance executive. Such an approach could, however, be accommodated by the expanding theory of insurance which, from a description of the subject matter of insurance, has grown into a body of a multitude of analytical methods. The emerging focus on insurance goes beyond traditional preoccupation with rating and solvency to market analysis, planning and management techniques, and also embraces theories of efficient markets and equilibrium which provide links with older traditional economic theories. There seems to exist, however, no general theory of reinsurance at present. Theories of insurance deal mainly with risk (which is measurable to some degree) and do not fit neatly with reinsurance which deals mainly in uncertainty and encounters severe difficulties of measurement. The "law of large numbers" becomes difficult to apply where risks are neither homogeneous nor easily measurable and, when dealing with catastrophes, it becomes difficult to find a representative sample. If measurement is impossible, decisions must be made on imperfect information and not necessarily rational. Since neither probability theory nor traditional economic theory can be successfully applied in this situation, the problem-solver must turn to a more behavioural approach aimed at describing and analysing real-life reinsurance activities. This essentially interdisciplinary approach would involve study of the environment, behavioural aspects, quantitative analysis and problem-solving methodology. The main intention of the approach would be to provide workable techniques for decision-making and would probably employ modern decision-science techniques such as those which come under the general heading of operations research. Some of the many disciplines which could be included in this approach to reinsurance problem-solving are illustrated diagrammatically in Figure 4.3. From this figure it can be seen that the range of approaches available is wide and not traceable back to a single origin. The five disciplines or techniques which verge most closely on the reinsurance situation may be regarded as a tool-box for prospective reinsurance problem-solvers. Some examples of their applications to excess of loss reinsurance will be provided in following chapters.

4.3 THEORETICAL AND PRACTICAL APPROACHES TO THE EXCESS OF LOSS RATING PROBLEM

Attention is now paid to the excess of loss rating problem. The subject is introduced by a brief outline of the ratemaking function in insurance operations, and followed by a description of the information available to the underwriter from which to arrive at an excess of loss rate. Two methods, one employing statistics and probabilities and the other employing human judgement, are then presented to illustrate the excess of loss rating process from both practical and theoretical viewpoints. Finally, the strengths and limitations of these approaches are discussed in terms of their suitability for use in the marketplace.

Figure 4.3 The development of analy… applicable to reinsurance problems



4.3.1 The ratemaking function

The ratemaking function can be described (from Denenberg et al (1974, p.510)) as "the process of predicting claims and expenses involved in assuming risks and then allocating portions of the estimated claims and expenses along with certain margins, to various classes of policyholders". The origins of the essential principles, according to Franckx (1963), date back at least as far as 1762 when the Equitable of London was established, and have taken on an important role in actuarial science ever since. The basic principles of ratemaking are (from Denenberg et al):

1) Reasonableness; that is, they should not be excessive for regulatory or competitive reasons, and

2) Equity; which refers to the fair treatment of individual insureds.

Procedures for implementing these basic principles have, however, been criticised for their limitations, particularly in respect of their reconciliation with the modern theory of the firm. For example, Hickman and Miller (1970, p.586) write:

> Despite large amounts of literature on premium determination which has been developed by the actuarial fraternity and the detailed attention to premium formulae, even in basic texts, the methods for determining insurance premiums seem to puzzle many economists. It has become fashionable in both business and academic circles to ask if recently developed quantitative methods might be profitably used in fixing insurance premiums. These questions may be traced back to the current stress on quantitative methods and to a vague feeling by some that the apparent immunity of premium determination methods to change over time is caused by the reluctance of actuaries to master new ideas.

This criticism of modern actuarial methods is possibly a reflection of the development of the actuarial profession which is deeply rooted in the analysis of life-insurance problems rather than in general insurance. The modern actuarial literature, however, offers many examples of ideas intended to extend the principles of rating into the realms of non-life insurance and reinsurance. Berliner (1976), for example, provides a set of desirable principles for calculation of premium rates whether they be for life or non-life business. These desirable properties (which are not necessarily attainable) are:

a) The problem should be explicitly calculable.

b) The premium calculation principle should lead to a solution T which should be unique.

c) Information is often sparse in practice and, therefore, the less information required for a reasonable premium calculation the better.

d) In practice there is often little time and no auxiliary aids such as computer programs to calculate the premium. It is, therefore, advantageous if the premium calculation principle does not lead to difficult, complicated, time-consuming calculations.

e) The premium should be at least as large as the expected loss cost.

f) The premium should be "finite".

Berliner's ideal properties for premium rates evidence a keen regard for the practical problems of people, other than actuaries, who must arrive at reasonable or satisficing premium rates in order to conduct insurance and reinsurance affairs. The lack of time and information is of particular relevance to the excess of loss underwriters' situation. It must be borne in mind, however, that Berliner's properties are "desirable", not necessarily actual and, indeed, challengeable. For example, the posit that premium should be as large as the expected loss cost need not apply if it can be demonstrated that investment income on premium income offers a higher return on capital than the premium-to-claims differential. Nevertheless, Berliner's set of principles is indicative of a willingness of the actuarial profession to extend their analysis further from their roots in life-insurance to other areas of concern. They also provide a set of criteria against which the quality of complex rating methods such as those for excess of loss reinsurance can be gauged.

4.3.2. Information available for ratemaking

One of the difficulties of applying traditional actuarial methods of rating to excess of loss business which has been mentioned is a lack of information in the right form. This lack of information renders forecasting of claims to arrive at an expected loss figure difficult and susceptible to large errors. There is, however, a certain amount of information available to the excess of loss underwriter which he may consider in his decision-making process. Most of this information is contained or appended to the "slip" which is the document on which an underwriter may subscribe for part or all of a risk. (The underwriter's signature on slip constitutes, in itself, a binding contract of reinsurance.) Other sources of information about particular companies requiring excess of loss protection include general market knowledge and occasional case-studies by conscientious reinsurance brokers. The information on the slip usually includes a history of past losses (if any) but the incidence of claims is usually too low for calculation of a realistic expected loss figure. The amount of information available on any individual risk varies, but the following non-exhaustive list, put forward by Ratcliffe (1974) at the American Institute of Decision Sciences, is offered as information which could be taken into account in the excess of loss rating process (an asterisk indicates that the information is likely to appear on or with the slip):

* 1) The company

* 2) Location of the company

* 3) Class of business reinsured

* 4) Limit desired by the company

* 5) Retention of the company

* 6) Subject premium income

* 7) Rate and premium payment conditions from previous periods (where applicable)

* 8) Commission

* 9) Brokerage

10) Underwriting philosophy and capability of management

11) Financial condition and past performance

12) Qualification of underwriters

13) Overall industry record

14) Accounting procedures

* 15) Limits on original business

* 16) Geographical spread of business to be reinsured

* 17) Type of business to be reinsured

18) Method and statistical input of ceding company's rating procedures

19) Collateral business available from the company

From the above, it is obvious that to take into account all relevant information in an excess of loss rating formula would prove an extremely complex business. It would also be difficult to provide a complete set of objective elements for a formula since many of the listed items are matters which require human judgement. Consequently, the literature available on the excess of loss premium calculation covers a range of approaches from those which are mainly judgemental to those which are mainly objective. The range of approaches is described by Vajda (1955, p.164) in the following passage.

> There are various ways of approaching the determination of the premium rate for a Non-Proportional Reinsurance, but the fundamental distinction is between those which entail reference to the actual structure of the reinsured portfolio as the basis of the rating and those which do not. There are, of course, intermediate methods in which reference to the actual composition of the portfolio is more or less superficial and where other elements, significant to a greater or lesser degree in relation to the portfolio in question, are exactly from past experience.

The bases for the non-proportional premium rates are described by Ammeter (in Vajda 1955) as:

1) Pre-thought rates based on theoretical and statistical considerations;

2) Variable statistical rates based on previous years' experience;

3) Rates verifiable individually (e.g. moderate rate is increased after a large claim);

4) Collective rates (collation of peer group results)

The approaches of both Vajda and Ammeter indicate a preference for statistical analysis, where possible for rating purposes, with a recourse to judgement in the absence of adequate data. For excess of loss working layers, a statistical analysis is often possible, but the following factors surrounding the rating decision would indicate that, for the catastrophe layers, statistical analysis of claims is at least difficult and at most impossible.

1) The time horizon for decision-making is short

2) The pattern of data is random

3) Historical data are not always available

Given this state of affairs, two approaches remain. The first, as described by Vajda, is to base the statistical analysis, not on ceding company's claims experience for the business to be reinsured, but on its entire portfolio, and the second is to employ a high degree of judgement.

4.3.3 The statistical approach to rating excess of loss reinsurance

Actuarial methods have over the years, been developed from their life-insurance origins to aid in many aspects of non-life insurance business. The methods, aimed often at the rating problem, have been most useful in non-life areas such as motor insurance or other types of business where claims experience is plentiful and the subject of insurance fairly homogeneous. For reinsurance, the statistical, actuarial approach has met with less success due to problems of prediction which arise where claims experience is inadequate and risk classes vary by composition and type. Nevertheless, this section presents an outline of the statistical approach to rating excess of loss reinsurance along with its limitations. Some conceptual, probabilistic models of the excess of loss rate are included to illustrate the increasing complexity, over the years, of academic contribution to the rating problem. The complexity of these theoretical models contrasts markedly with the methods of rating employed in the market which are described later in the chapter.

4.3.3.1 <u>The problem of large claims</u>

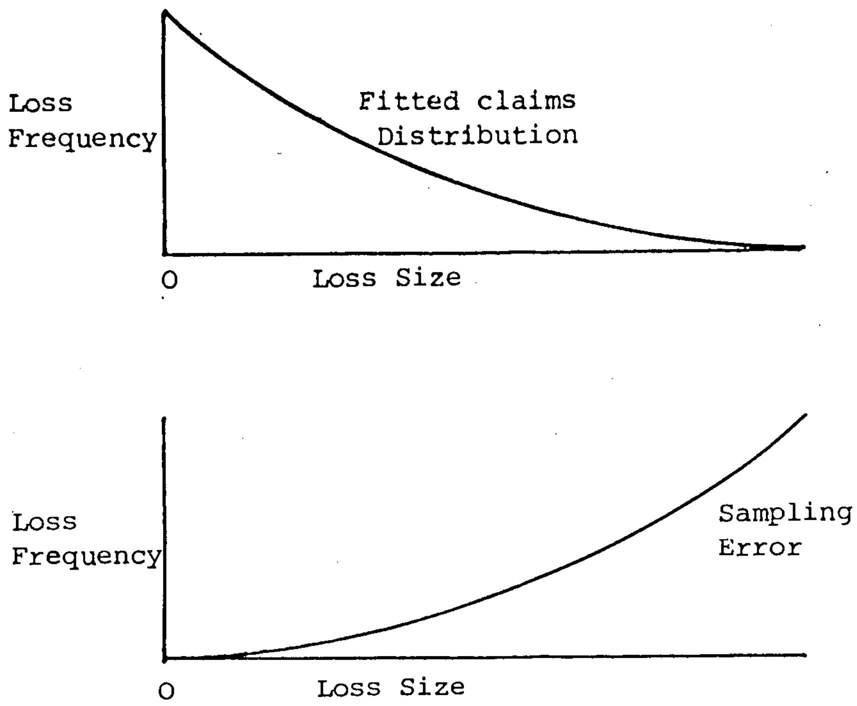
Where possible in insurance, where statistical rating procedures are to be used, the analysis of claims is conducted in two stages; namely, an examination of the number of claims that have occurred, and a study of the distribution of claims amounts. This information can usually be amalgamated to estimate the risk with some precision. In the case of excess of loss reinsurance, however, the number of claims to be expected on a contract per year is small. Consequently, the precision of rating is low. One actuarial solution to this problem would be to generate sufficient data by aggregating past experience from a number of similar contracts for inclusion in the estimate. For excess of loss business this solution cannot be easily applied, however, since both the physical characteristics of the loss and the financial characteristics of the reinsured portfolio vary greatly between companies, thus complicating the identification of peer groups. A second approach to the problem of large claims has been developed which involves detailed analysis and statistical manipulation of the claims distribution. Examples of this type of analytical approach include Eshita (1977), Brown (1977), Pentikainen (1977) and Benktander (1977). Another approach is provided by the work of Hey (1970) and Johnson and Hey (1972) who dismiss the idea of calculating an overall claim amount from an aggregation of experience from similar risk classes on a proportionate basis because it does not adequately reflect the low frequency of large claims. Their suggested alternative approach is to weight the data with factors which are inversely proportionate to the sampling errors of each risk class to produce an approximately unbiased estimator. Taylor (1975) examined the Johnson-Hey method and showed that this estimator was more efficient than the arithmetical mean, in some cases. Straub (1971) suggested yet another approach to the problem which involves treatment of large claims as a separate population, the distribution of which should be studied without any reference to their parent distribution. The methods put forward for consideration of large claims are, however, of mainly academic interest and lack adequate simplification for practical application by excess of loss underwriters.

4.3.3.2 The concept of a risk loading

The concept of a risk loading is a consequence of the difficulties arising from large claims. Since large claims are infrequent, the statistical variation at the tail-end of the claims distribution is relatively large. This is illustrated in Figure 4.4.

From the figure it can be seen that attempts to fit a probability distribution to size and frequency of claims would result in a high sampling error for large claims due to low frequency of data in the large claim range. The statistical variation manifests itself to insurers in the shape of uncertainty about future cash flows and is one of the prime reasons why they might purchase excess of loss reinsurance. By purchasing excess of loss reinsurance, the claims variation is, for the ceding company, restricted to some point below which it is felt the statistical

Figure 4.4 Statistical variation increasing with size of claims



variation is containable. Since the excess of loss reinsurer agrees to remove the uncertainty caused by claims variation from the ceding company's portfolio, a notional charge is included in the premium which is called a "risk loading". Benktander (1971, p.315) explains the concept of a risk loading with regard to reinsurer's profit and fairness as follows:

1) Profit can be considered as the reward which the reinsurer should receive because of his willingness to engage his capital and free reserves in order to take over and carry part of the fluctuation in gross results of the ceding company.

2) Profit, or rather expectation of profit, is thus the price for carrying variance, i.e. a possible fluctuation in the negative direction ..... the price being understood as an addition to the expected average price less costs. Seen from this angle the "price" or expected profit is equal to the loading.

It should be stressed that the risk loading is a notional concept as there is, in practice, no specific calculation included in the rating procedure which takes into account variance around expected values. The complications which would require solving before the measure could be used in practice include the following:

1) Where no past information is available there exists no realistic expected value around which deviations can be measured.

2) Where data is available the distributions are highly skewed, making statistical development difficult.

3) There is a problem as to which parts of the fluctuations should be taken into account, i.e. should all fluctuations be taken into account or only the negative ones?

These issues are explored further by Berliner (1976 and 1977) and Benktander (1971 and 1974). Again, these papers are rather complex to all but the trained actuarial mind and offer little comfort to the practising underwriter.

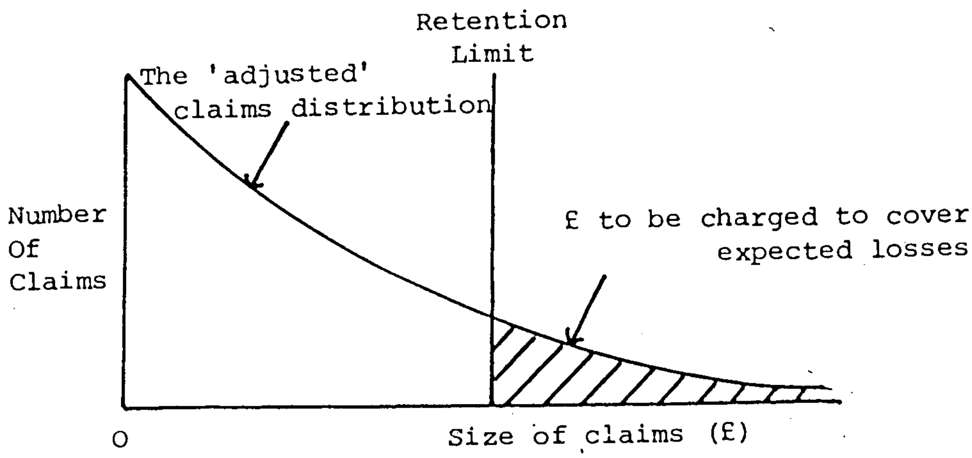
4.3.3.3 Conceptual models of the excess of loss reinsurance premium

Over the years, several formulae and probabilistic models for the excess of loss premium have been put forward. Early attempts were based purely around expected losses but the modern conceptual models also take into account statistical deviations around expected values. Three stages can be identified in the history of modelling of reinsurance premiums. In the beginning the models provided rules of thumb for working underwriters. The second stage produced models which involved advanced statistical and probabilistic techniques and highlighted the inadequacy of the earlier, cruder methods. The final stage, which is also the present stage, is concerned with finding ways in which the complex probabilistic models can be adapted for use in practice. The evidence from the literature is that little success has been achieved in this latter respect. The three models presented to illustrate each stage in the theoretic development are by Tuma (1946), Ammeter (1955) and Harding (1968) respectively. The lack of attempts to improve on the conceptual description of the excess of loss rate since 1968 may be accounted for by the detailed description by Harding (see Section 4.3.3.6) of the large number of problems which would require solutions before the theoretical rating models could be used in practice; the problems of implementation would now appear to offer a greater challenge than those of theoretical development.

4.3.3.4 Tuma's Method (1946)

The first published statistical methodology for rating excess of loss reinsurance was provided by Tuma (1946) who, by his own account, used the formula in his own business operations. His analysis was based on a schematic representation of a ceding company's claims which, using graph paper and a ruler or simple calculus, could be used to establish how many claims of each size would be likely to fall above various retention limits. An illustration of Tuma's graphical method for arriving at the "expected claims" part of the premium is provided in Figure 4.5.

Figure 4.5 Tuma's method for calculating the expected claims element of premium for an excess of loss contract



The main drawback of Tuma's method is that it does not allow for the large element of uncertainty at the tail end of the distribution. The implication is that Tuma either applied his method only to working layers or that he ran the risk of incurring serious cash flow fluctuations without making a charge for it. Besides the expected claims element of the premium, Tuma included an expense loading which, if it was large enough, might have compensated for the lack of a risk loading. The method of apportioning the expense element to the various contracts was firstly, to estimate his total business expenses and secondly, to apportion them to each reinsurance contract in proportion to the total premium income of the ceding company in each case. The eventual premium is arrived at thus (Tuma, 1946, p.24):

> The total rate or gross premium to be charged for an excess of loss treaty is composed of the net premium, which varies considerably within the limits of the cover, and the rate for expenses which is a constant.

It must be assumed that the expenses element includes an allowance for profit.

Before considering more probablistic models of the excess of loss premium, it is important to note that Tuma, despite his relatively crude analysis, was aware of the limitations of his model and did not use it as an exact rating tool. Business acumen was employed to interpret the analytical rate for practical application. For example, he charged a higher rate when faced with a new contract devoid of claims experience. He explains (1946, p.24):

> Reinsurance as a service has to distribute the losses in time, i.e., the surpluses of the active years must be accumulated to be able to pay the deficits of years with larger loss-experience. In other words, the reinsurer has to accumulate reserves from which the deficits of bad years are covered. The financial risk of the reinsurer is therefore larger at the beginning of a reinsurance treaty because at that time reserves are not yet accumulated. For this reason the gross premium of a cover treaty is at the beginning quoted higher than theoretically necessary, and in the following years, if the claim-experience is favourable, the rates are reduced.

From the beginning of analytical descriptions of the excess of loss premium, it must therefore be noted that human judgement plays an important role in the practical implementation of premium rates.

4.3.3.5 Ammeter's model (1955)

Hans Ammeter's excess of loss premium model (in Vajda, 1955) applies actuarial probabilistic concepts to the problem, the emphasis being on the distribution of claims above a retention. Introducing Ammeter's model, Vajda writes (Vajda 1955, p.165):

> In the case of Excess of Loss ..., in the first place the total claims per event (catastrophe) has to be determined from all the policies affected by such an event, in so far as it exceeds the critical value. In the second place, and in a similar way, the total loss to the Reinsurer must be found.

The various types of distribution available for fitting are:

a) $pk = \binom{N}{k} p^k (1-p)^{N-k}$ ... binomial distribution based on the hypothesis: N independent risks of probability p.

b) $pk = e^{-h} \frac{h^k}{k!}$ ... Poisson's distribution, based on the same hypothesis as (a) but for the limiting case $h = Np$ where N is large

c) Various "compound distributions".

d) Finally, without any formal model, pk is assumed to be determined on some statistical basis.

The contribution of probability formulae to the premium estimating procedure is that, providing data is available on small claims, the expectation for large claims can be estimated by examining the tail of the fitted distribution where the large claims might occur. Probabilistic models were, therefore, seen as potentially useful for the marketplace. Ammeter (1955, p.80) writes:

> It might be thought that the theoretical method offers no real advantages since the uncertainty of the empirically determined premium rate is replaced by another uncertainty, namely the uncertainty of the elements required for the theoretical method. In our opinion this judgement cannot be maintained, however, if it is taken into account that these elements may often be fixed by reasonable assumptions.

The basic model for the excess of loss premium provided by Ammeter is shown in Figure 4.6 as a set of formulae based on the Poisson distribution. The formulae were tested for realism (by Ammeter) against a set of actual premium rates for Excess of Loss reinsurance treaties from which it was observed that:

1) the range of fluctuations in the basic probabilities around an estimated mean did not influence the premium;

2) net premium rates for the excess of loss treaties were independent of the size of the ceding companies;

3) the loading element of premium was comparatively heavier for higher limits of self retention.

Figure 4.6 Ammeter's probabilistic model of the excess of loss reinsurance rate

a) Net Premium

$$\pi_M^{EL} = P\int_M^{\infty} (z-M)\ p(z)dz$$

where

P = the expected number of claims

p = the total net risk premium for the year measured by the mean value of the sums at risk

M = the limit of the ceding company's retention

$p(z)$ = the original frequency function

π_M^{EL} = the reinsurance premium to be charged for excess of loss protection above the lower limit (M)

(z, M and P are measured in terms of the mean value of the sums to be paid out)

b) Variance

$$\sigma^2(y) = Pp_2^M + \frac{\pi_M^{EL\ 2}}{h_0}$$

where

$$p_2^M = \int_M^{\infty} (z-M)^2\, p(z)dz$$

h_0 = a parameter measuring the range of basic probabilities. The reciprocal of h_0 is equal to the variance of the unimodal frequency curve.(Pearson type III).

These observations supported the assumptions included in Ammeter's probabilistic model. It was concluded (in Vajda, 1955, p.102):

> ... the theoretical basis proposed makes it possible to choose a loading system adequate from the point of view of the reinsurer's security as well as from the stand point of the ceding company's interest in the reinsurance treaty in question.

The findings of Ammeter provide an example of a comparison between theoretical and actual excess of loss premiums. A comparison between Ammeter's findings and those of the present study is included in a later section of the thesis.

4.3.3.6 Harding's model (1968)

Besides offering a conceptual model of the excess of loss premium, Harding (1968) evaluated the practical difficulties which must be overcome for the theoretical method to be applied in the market. Realising that the greatest difficulties were encountered when losses were difficult to predict, he centred his analysis on Motor Excess of Loss Business. Motor business offered the highest likelihood of reasonably homogeneous risk classes with high claims incidence so, if the theoretical rating method could work anywhere it would work for the reinsured motor portfolio. Like Ammeter, Harding based his model on the Poisson distribution but, unlike Ammeter, employed a truncated log-normal distribution for the amount of claims exceeding the priority (see Figure 4.7). His conclusion was that the

Figure 4.7 Harding's model for calculation of excess of loss premiums for motor insurance

$$PE = (1-p)^{-1} [\pi E + \lambda \sigma E]$$

where p = the rate of commission payable per unit of premium charged

πE = the expected value of the total reinsurance claims

σE = the standard deviation of the total reinsurance claims

λ = the constant proportion of σE required for the loading for expenses, fluctuations, contingencies and profit

PE = the reinsurance premium to be charged

Note:

$$\pi E = G.ME \int_E^{\infty} (Z-E) \frac{d\,F(z)}{1-F(E)}$$

where G = the estimated original gross premium income (O.G.P.I.)

ME = the expected number of reinsurance claims per unit of O.G.P.I.

E = the priority

Z = the total amount of original claims

$F(z)$ = the the log-normal distribution with parameters μ and σ^2

statistical procedures were worthwhile to produce "a premium that the reinsurer can adopt as a reference point from which to negotiate terms that will prove sufficiently profitable in the long run over the whole portfolio". He added, however, that, even with a statistical model, there remained many factors which require subjective assessment. Additional allowances for a realistic premium would include the following:

a) Inflation: An allowance for inflation is required since it affects the size of actual claims, the number of claims which will occur above the priority, and also the gross premium of the ceding company. (The change in gross premium income for the ceding company due to inflation may, alternatively, be allowed for using the minimum deposit system rather than a flat premium.)

b) Ad hoc adjustments: These adjustments would be required in the event of changes in legislative provisions, economic climate or business circumstances. Changes in court attitudes towards the payment of damages is an oft-quoted example of a need for increased premium rates.

c) Risk heterogeneity: For purposes of conceptualising the excess of loss rate, Harding employed the assumption that the distribution of the amount of an original claim is the same for all contracts. In practice, this is not the case and a procedure for ranking risks would be required for a more realistic model. An alternative approach would involve judgemental analysis.

d) Company heterogeneity: This allowance is not to cover differences in types of business written since these would be accounted for in the model in terms of the claims experience produced, but to allow for different companies' gross premium incomes. This allowance would cover differences in the scope of cover granted and loading elements of original premiums. Specimen policies and details of commissions, expenses, profits and statistical details of the reinsured portfolio would be required to arrive at this allowance.

e) The "Accumulation" risk: This allowance would cover the possible inter-dependence of risks on both the ceding company's and the reinsurer's portfolio. This allowance is required to cover the catastrophe potential of the portfolio. For example, a reinsured portfolio with a wide geographically-spread book of business would be less vulnerable to catastrophe hazard than one with all risks centred in a small area.

Harding's work provides, therefore, a more realistic approach than earlier probabilistic models to the use of theoretical rating models in practice, and emphasises the important role of judgement in any workable system.

4.3.3.7 Recent developments for the theoretical rating of excess of loss contracts

Since Harding's paper which highlighted the practical difficulties to be overcome before the conceptual approach to excess of loss rating can become functional, there appears in the literature no suggestion that the theoretical rating models can provide more than a useful reference point for underwriters. The theoretical principles underlying the models have, however, received attention. Three issues which continue to offer a challenge to risk theorists are the fluctuation loadings element of the reinsurance premium, the allowance for accumulation hazard, and new ways of estimating the number of large claims above a retention. The work on these topics includes the following:

a) The risk loading. The study of the risk loading is important for the excess of loss premium since it may account for more of the theoretical rate than the expected claims element. Among the authors who have considered the statistical properties of the risk loading (standard deviation, variance and skewness) are Buhlman (1970), Burrens (1972), Benktander (1971, 1974 and 1977) and Berliner (1976).

b) The accumulation hazard. The accumulation hazard presents a continuing problem for risk theorists and requires the application of estimation procedures to both the underlying risk process and a second, conditional, risk process. The problems are far from soluble with the current state of knowledge. A start has been made, however, by Schmitter (1978) who sets out the problems to be overcome for further development of accumulation hazard estimation procedures.

c) Large claims. Some of the problems of large claims have been mentioned in Section 4.3.3.1. A further technique to be used in the estimation of their probabilities should also be mentioned, namely the "credibility theory". The credibility theory involves, essentially, weighting data according to the degree of confidence in it. The techniques of credibility theory (which is in a state of rapid development) are complex and the existence of sub-divisions such as European Credibility Theory, American Credibility Theory, Bayesian Credibility Theory and the Modern Theory ensure a wide range of approaches to solving the basic problems caused by lack of homogeneous data from a particular source. Using the credibility theory, it is hoped that practical means will be found to extract data from a heterogeneous risk sample in such a way that reasonable forecasts can be made for individual members of that sample. (For an overview of the credibility theory, see Taylor (1974).) Once mastered, such techniques might usefully be employed to combat the difficulties involved in assessing claims potential of excess of loss contracts.

The above advances, however, although applicable to the excess of loss rating problem, have not been combined into a rating formula intended for use in the market.

4.3.4 The judgemental approach to excess of loss rating

Judgemental rates are used for excess of loss contracts where adequate information for a more scientific approach is not available, i.e. most of the time. This attitude to problem solving was neatly expressed by Knight (1921, p.211).

> ... we are likely to do a lot of irrelevant mental rambling and the first thing we know is that we have made up our minds, that our course of action is settled. There seems to be very little meaning in what goes on in our minds, and certainly little kinship with the formal process of logic which the scientist uses in an investigation. There is doubtless some analysis of a crude type involved, but in the main it seems that we "infer" largely from our experience of the past as a whole, somewhat in the same way that we deal with intrinsically simple (unanalysable) problems like estimating distances, weights or other physical magnitudes when measuring instruments are not at hand.

Such an approach has proved the only workable method to date by most practicing excess of loss underwriters for the following reasons:

1) Where claims experience is not available it is the only possible alternative;

2) Timeliness is, up to a point, more important than accuracy (mistakes may be rectified via the law of large numbers);

3) Statistical techniques are not yet available for adequately assessing the accumulation hazard;

4) Rates may be pitched intentionally high or low in order to discourage or encourage various types of business. (In a marketplace the economic laws of supply and demand can affect prices just as much as the "production cost" or the quality of a good.) The need for accurate measurement of risk is consequently diminished.

A further justification for an expedient rating system can be found in the origins of excess of loss reinsurance which was devised to provide a simple solution to the problem of inadequate ceding company capacity. Vajda's comment (1955, p.164) would not seem inappropriate to present-day market operations.

> The main attraction of the form of reinsurance lies in the simplicity of its practical operations, the premiums being fixed simply as a percentage of the original premium income. Where the premium rate is based on an arbitrary estimate all calculations are dispensed with, and also the preparation of all documents and vouchers, on which the computation could otherwise be based, becomes unnecessary ... the merit of the non-proportional form in giving the 'simplest' method lies only in the fact that an accurate calculation is so much more difficult that there is far less inhibition about dispensing with it in favour of an arbitrary estimation.
>
> If it were intended to fix the rate by the application of technical criteria, then - quite apart from the difficulty of finding them - the simplicity of the scheme would naturally have to be relinquished.

In the author's experience, excess of loss underwriters continue to rate contracts in a subjective and expedient manner. From conversation with individual excess of loss underwriters, a common opinion was expressed that the underwriter could not be replaced by an objective, calculating machine.

On the other hand, when asked how the rate was calculated, most underwriters found it difficult to provide an answer which was the least bit scientific. Two common answers included "I just know that that is the rate" and "I am willing to follow Mr. X on that one" (Mr. X being the lead underwriter). Some underwriters cheerfully admitted to not having a systematic approach to the rating problem and did not consider the need for one particularly important. One reason for this apparent lack of concern is the influence of the market which is geared towards "fortune sharing". The spread of risk ensured by the mechanics of the excess of loss contract placement procedure provides a means of sharing not only losses but also over-and-under pricing. The prime aim of the underwriter, in practical terms, might therefore be seen as to ensure participation in as large a number of slips as possible in order to achieve results near the market average. The calculation of an exact rate for individual contracts becomes, in this context, a secondary issue.

4.3.5 Excess of loss rating methods employed in the market

Judgement is employed in the rating of every excess of loss contract - even in the decision whether or not to use a "scientific" rating procedure if one is available. There exist in the marketplace, however, a number of rules-of-thumb or rating methods which underwriters employ in specific cases. Some of these methods were collected in an anonymous and, to a certain extent, tongue-in-cheek article entitled "Excess of Loss Rating Science" (A correspondent, 1977). The methods mentioned include the following:

1) Calculate the burning cost (claims to premiums ratio) and multiply by 10/7. This method corresponds roughly to the "expected value plus a loading" approach to rating but would only work for working layers with some claims experience.

2) As above but increase the loading for I.B.N.R. and inflation.

3) Take the last five years burning cost and divide by two.

4) Guess (the most popular method).

5) Rating on line. This rate is calculated as the average exposure of a number of lines under a contract plus a loading of, say, fifty

percent. This method requires detailed information on the ceding company's lines of business.

6) The market rate. Here a rate is calculated from a sample of similar contracts' claims experience. There are difficulties of establishing a similar group of risks and a computer may be required.

7) Follow the leader.

8) The pay-back system. The underwriter calculates the likely amount of a claim. By dividing this figure by the likely number of years between claims the underwriter arrives at the rate.

9) "A compromise between what the ceding company can afford to pay and what the reinsurer thinks he needs. This happens in most cases but it would be a very dull world if all excess of loss rating were down to a formula which most intelligent 12 year olds could apply."

The anonymous writer concludes:

> ... the dilemma that excess of loss rating suffers from is that actuaries usually require too much information from ceding companies and produce formulas that underwriters cannot understand and are therefore reluctant to apply.

The above comments suggest that scientific rating procedures have not yet replaced the use of judgemental techniques in the excess of loss marketplace.

4.3.6 Limitations of theoretical and judgemental excess of loss rating methods

Both judgemental and scientific methods of rating excess of loss contracts have certain limitations. The judgemental method has always found acceptance in the marketplace, however, and must therefore be acclaimed as the most suitable for practical purposes. Before improvements can be introduced to either judgemental or scientific rating procedures, the following limitations must be overcome.

4.3.6.1 Limitations of theoretical rating methods

1) Lack of data: Even in a theoretical model, only an infinite set of observations could provide an exact solution. For excess of loss business, loss data on individual risks are scarce. For comparison purposes, where experience is to be pooled to provide a better estimate, it is difficult to identify peer groups.

2) Complexity: This limitation applies to the detailed calculation procedures which may not be justified by the degree of accuracy of the result. For example, a detailed calculation method to arrive at an expected loss estimate might not be considered a worthwhile effort since the loading element may account for a much larger proportion of the premium.

3) Information difficulties: Producing underwriting information in a suitable form for statistical analysis involves many difficulties. Firstly, some of the information (for example, ceding company's confidential plans) is not available and, secondly, factors such as I.B.N.R., inflation, and future exchange rates must still be estimated.

4) Failure to account for supply and demand: Scientific rates based on past experience do not allow for price changes brought about by changes in supply and demand. They provide only a theoretical guideline against which an underwriter could guage how much he has over- or under-charged for the business.

5) The need to supplement theoretical rates with judgement: This point is raised by Ammeter (1955, p.80) who writes:

> The empirical method starts from the ceding company's own experience and leads to rates which would have been sufficient in the past. In general this does not lead to the results in which sufficient confidence is felt because the ceding company's experience is often disturbed by fluctuations, so that it is practically impossible to estimate the normal burden on the company of heavy claims. Indeed this uncertainty is the very reason for reinsuring this part of the business.

Even where a statistically-determined expected value can be achieved it is likely that judgement will be required to evaluate whether the trend will continue - especially, as in most cases, where the nature of the ceding company's business is undergoing changes.

4.3.6.2 Limitations of judgemental rating methods

1) No guaranteed opportunity to recover on past mistakes. Most excess of loss contracts are drawn up on a yearly basis with an option to renew at the end of the period. There is no compulsion for either cedent or reinsurer to enter into a long-term agreement. No guarantee exists, therefore, that the opportunity to re-adjust the premium in the light of experience will arise.

2) Lack of control procedures. If scientific rating procedures were employed, the build-up of risk on the reinsurer's portfolio could, theoretically, be monitored to produce a portfolio in keeping with desired risk and return objectives. Since, using judgemental rates, the true nature of the underlying risk is not necessarily considered in a structured manner, this opportunity is foregone.

3) Lack of feedback. Excess of loss reinsurance is notorious for the long time period before eventual notification of claims settle ments, (in some cases, up to twenty years after contract acceptance). The underwriter might (should he live so long) find the premium adjustment required to rectify past inadequacies too great to be included in realistic new rates.

4.4 THE NATURE OF EXCESS OF LOSS PORTFOLIO MANAGEMENT

Portfolio management for excess of loss reinsurance involves all steps taken towards planning, construction and maintenance of a group of reinsurance contracts according to appropriate underwriting criteria. It is a dynamic process since new risks are continually accepted onto the portfolio and old ones expire during the normal course of business. There are five main steps involved in portfolio management (as summarised by Ryan (1978)) which provide a convenient framework for stages involved in the management of an excess of loss portfolio. The main stages are:

1) Definition of portfolio objectives and constraints.

2) Choice of the opportunity set from which the portfolio is chosen.

3) Formulation of decision criteria for building the portfolio.

4) Estimation of the relevant characteristics of the individual opportunities in the set and, on the basis of such estimations, their inclusion in or exclusion from the portfolio.

5) Establishing criteria for monitoring the portfolio through time and for changing its composition whenever and wherever it is deemed necessary.

Each of these stages will be considered further with respect to excess of loss reinsurance.

4.4.1 Definition of portfolio objectives and constraints

Reinsurance company objectives and constraints were described in Chapter 3. These overall requirements are reflected in the objectives and constraints for the various portfolios which comprise the total underwriting business undertaken by the company. They manifest themselves in the shape of real or notional limits on risk-taking, profitability and funds invested. Influences on portfolio objectives and constraints include government regulations, managerial policy and shareholders' requirements. It is convenient to include shareholders' constraints under managerial constraints (that is, to make the assumption that, at any moment in time, shareholders are prepared to accept managerial decisions and preferences or they would move their funds elsewhere). Two main sorts of constraint then remain - statutory constraints which are set outside the organisation, and managerial constraints which are a function of managerial preferences. The exact influence of each of these types of constraint on a single reinsurance portfolio is elusive. The following comments, however, are included to provide an insight into how the limitations on portfolio construction might be determined.

Statutory constraints

Insurance and reinsurance companies are regulated by both company and insurance law - the relevant Acts being the Companies Act 1948 and the Insurance Companies Act 1958. Since these Acts, company law has been updated and insurance and reinsurance companies have been controlled to an increasing degree, firstly, by the Department of Trade (Insurance Division) and, latterly, by the E.E.C. insurance authorities. The solvency regulations which came into operation from August 1978 require the following in respect of overall insurance and/or reinsurance business:

a) A margin of solvency which is the greater of two sums, the first related to gross premium income and the second to relevant claims experience, and

b) A guarantee fund for an individual company. This is equal to one-third of the margin of solvency but is subject to a minimum amount.

The two amounts may be regarded as reference points against which the company's excess of assets over liabilities is gauged. Although the statutory directives do not apply directly to individual portfolios, indirectly effects will be distributed among them. There are likely to be economies of scale operating which would imply the existence of a minimum limit to the amount of funds worth investing in the underwriting process. (Further details of reinsurance regulation and supervision appear in Chapter 5, Section 3.)

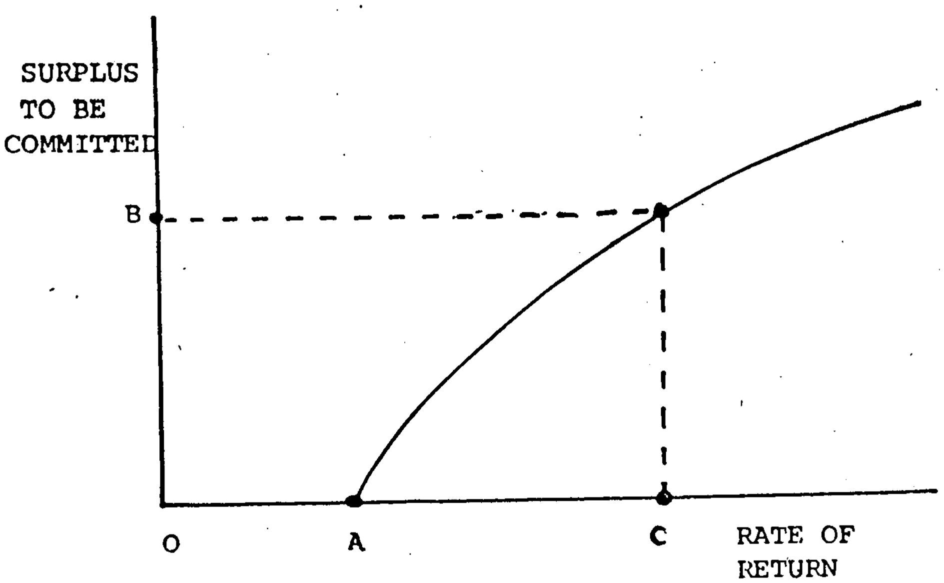
Managerial constraints

Managerial constraints include overall requirements for profitability and risk-taking and the amount of funds to be invested in the business. The relationships between the various managerial constraints may be explained conceptually by the existence of a management utility function which will vary between companies. Hershbarger (1975, p.55) summarises this relationship with regard to insurers' funds:

> The holding of a surplus provides a certain amount of satisfaction or utility to the company. It provides management with a soft cushion for managerial error and a buffer for unexpected catastrophic occurences. When an insurance company releases its surplus through the underwriting function it loses a certain amount of satisfaction; that is, it relinquishes a certain amount of psychological utility. This loss of satisfaction or utility must be offset by something in return. Normally this is accomplished by receiving an adequate rate of return in insurance operations. An insurance company, theoretically, will commit surplus to the underwriting function until the marginal utility of a dollar's worth of surplus equals a dollar's worth of return.

Hershbarger suggested a means for measuring the effects of the managerial utility function (see Figure 4.8) which he calls the "Psychometric Capacity Curve". The curve is intended to illustrate the relationship

Figure 4.8 Hershbarger's Psychometric Capacity Curve



Formula for psychometric capacity curve (PCC)

$$PCC = \frac{e^{c(Xe - Xo)} - e^{-c(X - Xo)}}{e^{c(Xe - Xo)} + e^{-c(X - Xo)}}$$

where c = "a psychological constant"
X = the expected rate of return
Xo = the rate of return at which zero surplus will be committed.
Xe = the expected rate of return

Key to Figure

A = Rate of return at which surplus committed is zero

B = Percentage of surplus which the company will commit at C rate of return between the surplus committed by insurers and the expected rate of return on the business, taking into account managerial preferences. In addition to enabling the supply of insurance to be estimated, Hershbarger suggests that the curve could be used as an insurance planning and control tool.

An alternative approach to managerial constraints is to apply subjective judgements of adequacy to the portfolio using, where possible, the risk theory to provide measures of portfolio stability. This approach is followed by Stone (1975) with regard to the insurance of catastrophe risks to describe "survival" and "stability" constraints. These constraints are described by Stone (p.232) as follows:

> 1) The survival constraint: In order to limit properly the risk of insolvency, an underwriting portfolio will be considered acceptable only if it may be determined that it implies a probability of less than P1 that losses and expenses in a given time period will exceed the total of new premium income and capital funds. The level of P1 will be set, whether formally or implicitly, by management and may be presumed to be extremely low.
>
> 2) The stability constraint: It is generally believed that both management and shareholders harbour a strong preference for some regularity in corporate operating results. In an insurance company this preference may be expressed as a constraint on the loss and expense ratios. Whatever the expected values of these ratios may be, an underwriting portfolio will be considered acceptable only if it may be determined that it implies a probability below P2 that the combined ratio will exceed some target value by X percentage points in any year. The levels of P2 and X will be set, whether formally or implictly, by management.

In order to use the risk theory to provide risk constraints it is of prime importance that the risk is measurable. For excess of loss purposes the required degree of adequacy could not be provided by combining the probability distributions of the individual risks in the portfolio since they are unlikely to be easily measured. Any attempt to employ the risk theoretic approach would therefore require either more suitable estimation procedures than those currently available and/or a collective risk theory approach which centres the analysis on the stochastic process resulting from the whole portfolio. Difficulties using the latter approach would be incurred, however, in forecasting future portfolio results if the population of risks coming onto the portfolio changed to a large degree. Since the universe of excess of loss contracts is constantly changing along with the increasing range of potential catastrophies it is unlikely that the collective risk-theoretical approach could find practical application.

4.4.2 Choice of the opportunity set from which the portfolio is chosen

The underwriter's opportunity set is the population of risks as represented by the excess of loss contracts placed before him in the underwriting room by visiting brokers. The normal format for these contracts is shown in Appendix A. The business opportunities which they represent, however, cannot be described without reference to their riskiness and the possible cash flows which would be generated on acceptance of the contract. Each contract is therefore viewed in terms of its positive and negative features which are:

a) Positive feature: an immediate and certain cash inflow of £x, and

b) Negative feature: the possibility of a future cash outflow of any amount up to £y (where y is much larger than x).

Measurement and description of these features is straightforward for inward cash flows since they are (with some adjustments when premium income exceeds that estimated by the ceding company for the minimum and deposit calculation) certain. For negative cash flows, however, the sort of analysis of expected values and likely variation described earlier in the chapter for calculation of premium rates is required. The pros and cons of scientific or judgemental estimation procedures are common to both rating and portfolio construction. With regard to protential excess of loss claims, judgement is, therefore, the excess of loss underwriter's method for assessing the likely funds outgoings which would result from contract acceptance.

4.4.3 Formulation of decision criteria for building the portfolio

The overriding principles which determine decision rules or criteria for building the portfolio are the principles of spread and diversification. Just as, within a company, business is spread between a number of different lines, the principle is also applied within each portfolio. Diversification among various types of risk, markets and countries reduces the innate possibility of a large loss or chain of losses from a single source. As a decision criterion, the diversification principle for the excess of loss underwriter is applied in a subjective manner taking into account the nature and numbers of contracts available. For a large portfolio, however, or for portfolios constructed from working layer contracts where claims experience is abundant, it may be possible to use mathematical principles to select contracts in such a way that portfolio objectives and contraints are met. In this respect, Ferrari (1967) outlined a Markowitz-type portfolio-theory to "provide an initial report on utilisation selection techniques to suggest the theoretical optimal diversification of lines of insurance written by property and liability companies ... to establish operating criteria for commercial insurance operations". Conceptually, the underwriter's decision criteria can be envisaged in Ferrari's model as a utility trade-off between profitability (or expected returns) and the stability of his portfolio similar to that described by Friedman and Savage (1948). The shape of the desired risk-return relationship will vary between underwriters. In theory, once this relationship for an underwriter has been identified, the decision process could be reduced to a mathematical formula. In practice, however, this is not possible for excess of loss business, partly because the underlying risk processes are not adequately understood and partly because consideration of risk processes is not the sole criterion for underwriting success. An underwriter offering cheaper-than-average market rates in order to encourage new business provides an example of the latter point.

In practice, the underwriter applies a number of constraints to his decisions which, even if they do not correspond exactly with the risk-theoretic approach, ensure a reasonable degree of stability for the portfolio. They are:

a) Limits on exposure for any one risk. In order to guard against the possibility of a number of large losses being concentrated on a few large exposures which might result in insolvency, an upper limit is placed on the amount of liability acceptable on any one contract. This is in keeping with the spread of risk or diversification principle of portfolio composition.

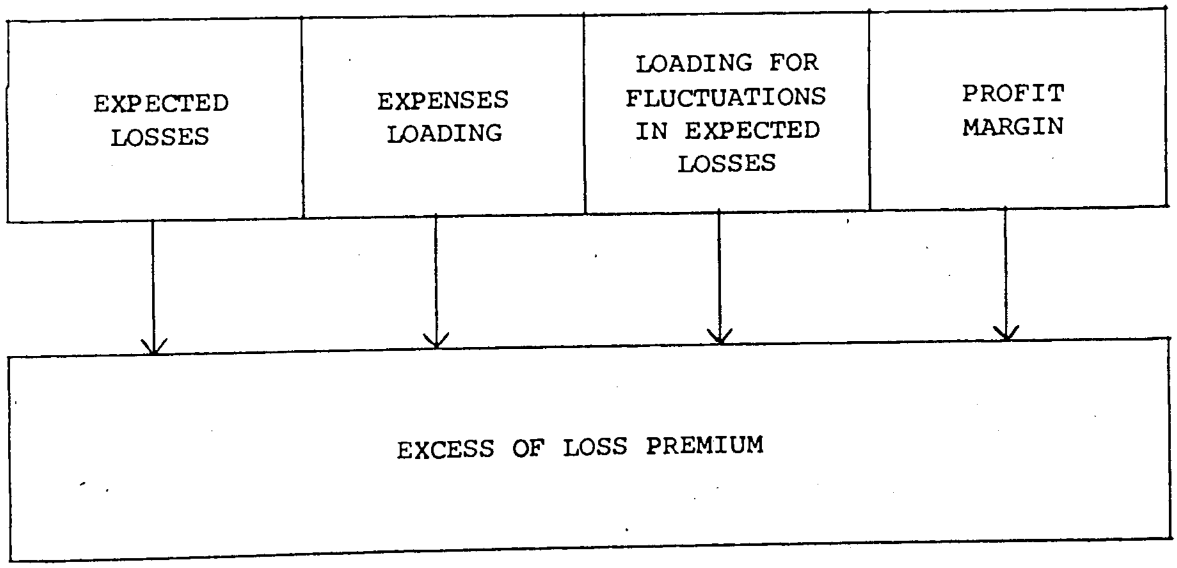
b) Limits on exposure by geographical area and character of risk. These limits on overall liability for specific areas or types of risk are intended to provide a further guard against the catastrophe or conflagration hazard. Normal practice is for each risk to be entered into an "exposure book" which lists, risk by risk, the liability assumed on specific areas or types or risk. A running total allows the underwriter to ascertain his total exposure in any given area at a glance.

c) Other constraints. These include overall requirements for profitability and the desired level of risk-taking. They are essentially determined by underwriting judgement and, except for ad hoc reports to top management of a general nature, are seldom committed to paper.

4.4.4 Estimation of the relevant characteristics of the individual opportunities and the basis of their inclusion in, or exclusion from the portfolio

The estimation of the characteristics of the individual opportunities centres on the elements which comprise the total premium for a contract, as illustrated in Figure 4.9.

Figure 4.9 Elements of the excess of loss reinsurance premium



The difficulties involved in estimating these elements have been described in earlier sections and it is likely that, if they are estimated in practice at all, such estimations are based on judgement. Since these estimations are likely to be inexact it would seem inappropriate to apply a precise mathematical framework to the inclusion or exclusion problem which would require that these elements could be explicitly identified. The conceptual approach is, nevertheless, useful to illustrate the general principles of the risk selection process which can be explained in terms of the "constant utility principle". The constant utility principle requires that the utility, before accepting a contract onto the portfolio, be equal to the expected utility after accepting the premium in return for assuming liability for losses. The expected utility for any income prospect may be calculated, as illustrated by Doherty (1975), using the following equation:

$$\sum_{i=1}^{n} p_i U(A - Li)$$

where p_i = the probability that a loss of size Li will occur

A = the initial wealth

U = the appropriate utility function

n = the number of possible outcomes (including the outcome in which no loss occurs)

Whether or not a contract is accepted onto the portfolio would then be determined by the expected utility of the business opportunity in relation to the utility constant for the portfolio. The underwriter will accept the opportunity only if the expected utility of the contract is greater or equal to the utility constant for his portfolio.

Insurance writers who have employed the principle of constant utility to explain portfolio composition include Ferrari (1974), Benktander (1971), Berliner (1974) and Borch (1978). Detailed attention has been paid by these authors to the nature of the probability distributions employed in the non-life insurance portfolio model but there remain many problems before the concepts could be usefully employed in running the excess of loss business. Examples of where assumptions in the theoretical insurance portfolio models are unrealistic for the underwriting of excess of loss business include:

a) Variability of results. Theoretical insurance portfolio models often assume a symmetrical probability distribution for a line of business. Excess of loss business on the other hand is usually characterised by highly skewed probability distributions;

b) The assumption that historical risk-return trends will continue into the future. Due to the changeable nature of ceding company portfolios over time, extrapolation of historical data would not provide sufficient grounds for assessing future expectations;

c) Complexity of implementation. Even if the problems described in a) and b) could be overcome, implementation of the model would prove a complex business. A computer would be a prime requirement, as would a means of translating management requirements into programmable decisions. Many judgemental variables, such as the utility derived from the contribution to portfolio diversification from various contracts by geographical location and type, would need to be quantified and built into a program which would need continuous updating for new risks accepted and expiring contracts.

To conclude, it is unlikely that the portfolio theory for insurance could be usefully employed in the selection process for excess of loss contracts. The method employed in practice might follow the general principles of the theoretical model but is characterised by judgemental rather than probabilistic estimation procedures.

4.4.5 Criteria for monitoring portfolio performance and for changes in composition

4.4.5.1 Monitoring portfolio performance

In practice, the monitoring of excess of loss business is a routine book-keeping task. The three most important recording systems are:

a) Individual risk cards. One card is prepared for each contract of reinsurance underwritten. Details on these cards include information taken from the slip and the debit and credit cash-flows generated from the acceptance (including estimated future cash flows where a claim has been reported but the £ amount due not yet paid);

b) The exposure book. This book contains a list of £ liability written on contracts falling under various geographical or class of business headings. The book provides, in effect, the total funds which would be required if every contract on the portfolio incurred a 100% claim;

c) The claims book. Cash flows (past and notified) traceable to individual claims are summarised and constantly updated in the claims book. For record-keeping purposes claims events are given a number on notification under which the £ amounts paid or estimated as owed to ceding companies affected by the event are monitored. For catastrophe business, the number of reinsureds affected under a single claims number is likely to be large.

The excess of loss underwriter is, consequently, well-informed of business progress in terms of exposure and notified cash flows. He is able to produce a summary of the latest cash flow situation, an example of which is shown in Figure 4.10, for top management at suitable time intervals which, for assessing actual profitability, would seem adequate. In Figure 4.10 ingoing and outgoing cash flows are summarised under the headings of premium income, protection costs (own reinsurance), settled and outstanding claims, recoveries from our own reinsurers, and gross and net loss ratios. Further analysis of the portfolio, however, requires some form of statistical analysis to make allowance for its risky characteristics. Statistical measures which might be employed include the following, suggested by Tuma (1946), for analysis of insurance portfolios:

a) Mean Loss Ratio

b) Median Loss

c) Standard deviation

d) Intensity of dispersion ((c) divided by (a))

e) Range of dispersion (difference between smallest and largest loss ratios)

f) Extreme positive deviation (largest loss ratio minus (a))

g) Extreme deviation measured ((f) divided by (c))

The analysis would also require adjustments for size of £ liability afforded under each contract to provide a more meaningful analysis of the

Figure 4.10 A typical business progress table for an excess of loss reinsurance portfolio.

	Net Premium before Protection	Protection costs	CLAIMS		RECOVERIES		LOSS RATIOS (Percentages) GROSS		NET	
			Settled	Outstanding	Settled	Outstanding	Settled	Incurred	Settled	Incurred
December 1969	687823	196358	5253	183261	-	-	.76	27.41	1.07	38.36
December 1970	887076	238980	54687	389257	-	-	6.16	50.05	8.44	68.50
December 1971	1067493	217601	120412	413455	-	3925	11.28	50.01	14.17	62.35
December 1972	1142302	217459	238308	379746	-	4146	20.86	54.11	25.77	66.38
December 1973	1140665	217462	302347	382640	-	20401	26.51	60.05	32.75	71.99
December 1974	1214279	230449	410125	332880	-	21013	33.78	61.19	41.69	73.39
December 1975	1315038	253748	535324	351692	-	20190	40.71	67.45	50.44	81.68
December 1976	1519830	296303	739373	289415	-	-	48.46	67.69	60.45	84.08
December 1977	1383781	265435	675044	249951	-	-	48.78	66.85	60.36	82.71
January 1978	1383679	"	675048	"	-	-	48.79	"	60.37	"
February	"	"	"	245444	-	-	"	66.52	60.85	82.32
March	"	"	"	244622	-	-	"	66.47	60.37	82.24
April	1384398	"	686574	231240	-	-	49.59	66.30	61.40	82.08
May	1385246	"	694413	209119	-	-	50.27	65.37	62.19	80.86
June	1386369	"	711861	207691	-	-	51.35	66.33	63.51	82.03
July	"	"	711823	206945	-	-	51.34	66.27	63.50	81.96
August	"	"	712004	201158	-	-	51.36	65.87	63.52	81.46
September	"	"	713885	196197	-	-	51.49	65.64	63.69	81.19
October	"	"	715383	196985	-	4	51.60	65.81	63.82	81.39
November	1315629	"	716579	196700	-	4	51.71	65.91	63.97	81.53
December	1385620	"	716787	211140	-	4	51.73	66.97	63.99	82.84

Source: Alexander Howden Insurance Group risk position in terms of £ expected cash flows to provide a "risk profile" of the portfolio. The output from this sort of statistical analysis might provide useful supplementary information to the sort of information included in reports such as the one illustrated in Figure 4.10. In the author's experience, however, such analyses are not carried out in practice on any regular basis. Some possible reasons for this are:

a) Underwriters are not familiar with the relevant statistical techniques and, even if an actuary was employed for the task, would find the results difficult to interpret for business improvement.

b) Records are not kept in a form from which the required data could be quickly abstracted.

c) Computer requirements. The volume of data and complexity of calculations would necessitate computer facilities if timely information on the current state of the portfolio is to be provided.

d) The analysis cannot improve on actual results. Since actual profit can be ascertained without a detailed statistical analysis the time and effort involved in the calculations is not likely to be considered worthwhile. Top management are often concerned with actual profits rather than what they might have been.

The case for statistical analysis of the reinsurer's portfolio, however, becomes stronger when the prospect of control of the portfolio is considered. In this area there remain problems of estimation where cash flows are likely to be large and infrequent but the theoretical foundations for excess of loss portfolio control have been laid down. The aim is to achieve and maintain portfolio control and constraints through application of the principle of spread. The general principles behind the process are described by Stone (1975, p.236) as follows:

> ... for any given risk, there exists some number n such that n risks of identical size and distribution will constitute an acceptable portfolio under the constraints described by the model and under the assumption of pricing to a target loss and exposure ratio ... A proper spread is said to be achieved when the Exposure Ratio of the portfolio, determined by the number and characteristics of its component risks is below that level defined by the Stability Constraint to be the maximum acceptable ratio.

Stone's analysis extends beyond the portfolio itself to the underwriter's rating strategy showing the two processes of rating and portfolio composition to be linked at least in a structured, theoretical manner. The link between rating and portfolio composition is provided by the degree to which the underwriter has achieved his portfolio objectives and constraints. For example, if the underwriter has already achieved his profitability portfolio constraint, he might underwrite business which would improve his chances of achieving the stability constraint at a price which is lower than the market average. Under these circumstances, the premium rate is shown to be dynamic in response to portfolio composition. Stone hypothesises an Economic Pricing Formula (below) which would take account of the relevant factors.

$$P = \frac{L}{1-r-e-d}$$

where P = the premium charged

L = long-term annual loss expectations

e = the expense ratio

r = the profit ratio

d = a differential based on portfolio composition considerations

Figure 4.11 illustrates how premium, exposure and the number of contracts in the portfolio would interact, according to the formula, to determine portfolio performance. The model provided by Stone has not, however, been extended to provide a basis for practical planning and control of the excess of loss portfolio. Some limitations to Stone's model include the following:

a) The hypothetical assumptions regarding the statistical analysis of the portfolio and included contracts are unrealistic. Estimation of, in particular, expected losses would require judgemental analysis.

b) No precise formula is provided for the calculation of d, the capacity or portfolio composition factor.

c) Risk constraints and performance are measured on an assumption of independent risks. The accumulation hazard is, therefore, not taken into account in the model.

Figure 4.11 Stone's framework for analysis of a high capacity risk portfolio

(i)	EXPOSURE RATIOS UNDER INCREASING SPREAD				
Number of independent risks in portfolio	1	1,000	10,000	100,000	1 mill.
Aggregate expected loss of portfolio	$100,000	$100 mill.	$1,000 mill.	$10,000 mill.	$100,000 mill.
Aggregate standard deviation of portfolio	$1 mill.	$32 mill.	$100 mill.	$316 mill.	$1,000 mill.
Aggregate exposure ratio	1,000%	32%	10%	3.2%	1%

(ii)	IMPACT OF PREMIUM DIVERGENCE (100 INDEPENDENT RISKS)			
Divergence of economic premium	0%	50%	100%	200%
Total premium collected	$16.7 mill.	$25 mill.	$33.3 mill.	$50 mill.
Aggregate expected loss of portfolio	$10 mill.	$10 mill.	$10 mill.	$10 mill.
Aggregate standard deviation of portfolio (approximate)	$10 mill.	$10 mill.	$10 mill.	$10 mill.
Expected loss ratio	60%	40%	30%	20%
Probability that loss will exceed .64 (approximate)	50%	30%	14%	1%

Continued ...

Figure 4.11 (continued)

(iii)	NECESSARY PREMIUM DIVERGENCE UNDER INCREASING SPREADS				
Number of independent risks insured	100	500	1,000	10,000	110,000
Divergence (increase) over economic premium necessary to satisfy stability constraint	200%	100%	60%	15%	0
Expected underwriting profit ratio at spread point; 1 - expected loss ratio - .36 expense ratio	44%	34%	27%	13%	4%

Source: Stone (1975

EXPLANATORY NOTE:
(i) Increasing the number of independent risks on a portfolio increases portfolio stability
(ii) Increasing premiums contributes to portfolio profitability
(iii) Processes (i) and (ii) jointly contribute to portfolio profitability and stability

To conclude, it would appear that portfolio details for excess of loss business are recorded in the market to provide information on which the underwriter can base his essentially judgemental decisions. A theoretical basis of a more refined, probabilistic portfolio control is provided, with particular relevance to catastrophe or high capacity business, by Stone but there remain some serious problems which prevent practical application of the model.

4.4.5.2 Criteria for changes in composition

The criteria for changes in the excess of loss portfolio, in common with other types of portfolio, are either that constraints are being (or are likely to be) exceeded in their relation to desired objectives, or that objectives are not being met. In the conceptual portfolio (Markowitz type) model the need for changes in composition arises when actual or expected future performance deviates from the "efficient frontier" which represents the acceptable combination of risk-taking and expected returns. The factors to be taken into account are the financial situation represented by the portfolio (that is, as a unit of business activity contributing to overall company performance) and its degree of riskiness. Should it be decided that the current or expected financial and risk mix deviates from desirable levels, changes in portfolio (or, rather, in the likely consequences of writing the portfolio) may be brought about by arranging own reinsurance in one or more of the many forms available in the market as described in Chapter 2. The important decision which must be made for each portfolio, in respect to changes in composition or own reinsurance, is the selection of a retention limit for the amount of risk retained in relation to the financial situation represented by the portfolio. This decision is by no means straightforward; due to problems of portfolio measurement the retention limit question cannot be reduced to a matter of applying the correct formula, and human judgement must be employed. Information which might prove useful for analysis of own reinsurance requirements includes the following for the original underwritten portfolio:

1) Premium income by class;

2) Losses by class;

3) Details of present and past reinsurance arrangements including retention levels, profits (from own written business), commissions (inwards and outwards) and investment profits;

4) Estimation of the maximum possible loss which could occur for both individual classes of business on the portfolio and for the portfolio as a whole;

5) Previous claims on reinsurers;

6) The price of reinsurance protection.

In respect of the retention limit question, a number of semi-scientific methods and factors can be identified which, although unlikely to be strictly applied, may enter into the judgemental process. These include the following (from Rubin 1969/70):

1) The maximum retention might be selected as a multiple of the average claim that the company/portfolio can maintain without risking insolvency.

2) Retention limit might be set from analysis of past experience. To determine the proper level for a retention, the (re)insurer studies the spread of losses by size for each class. The point at which loss frequency begins to drop is the one to begin considering as the best level of retention.

3) Set the retention level (for individual risks) in proportion to the profit loading factor in the premium rate. The idea, here, is to keep more risk on profitable business and less on unprofitable business.

4) Set the retention limit in inverse proportion to the premium rate (for individual risks). This method promotes the idea that the premium rate is a rough measure of the risk. A highly rated class of business deserves a small retention and vice versa.

The above methods are geared towards situations where loss experience is plentiful, as potentially for excess of loss working layers, and where the notional elements comprising premium rates are well known. The following methods, also from Rubin, are for catastrophe-type retention limits:

5) Set the retention level with regard to company size and concentration of risk.

6) A formula which would stabilise the underwriting result but not take into account the reinsured's capital and free assets is:

$$M = X\% \times P$$

where	M	=	Maximum limit of retention per event
	P	=	Premium earned net of reinsurance
	X%	=	A low percentage (For reinsurance of direct business Rubin estimates X as 1 for fire business, 3 for casualty business and less than 5 for marine business)

This approach stipulates that the claims ratio will not be affected by more than X% by any single event.

7) A formula which provides for the balancing of risk in proportion to assets to even out fluctuations in business results is given by:

$$M = Y\% \times (C+S)$$

where	C+S	=	Capital plus surplus
	Y%	=	a low percentage representing the maximum number of claims which, if not payable from premium income, would consume all capital and surplus

This approach stipulates that no single event may reduce capital and surplus by more than Y%.

8) A formula that relates the retention level to the cedent's liquid assets is given by:

$$M = Z\% \times A$$

where	M	=	the retention limit per event
	A	=	convertable assets

Z%	=	an estimate of how many times per year large claims will require immediate payment.

This approach aims at ensuring funds for immediate payment of claims.

The above methods collected in Rubin's survey, in the U.S.A., illustrate that the criteria for selection of a retention limit are many and varied. The methods employed in practice are likely to differ between reinsurers according to the underwriter's or portfolio manager's experience and intuition. In fact, the retention limit question could be described as displaying similar problems of applying a balanced mix of analytical and judgemental techniques to those met in the excess of loss rating and portfolio composition functions. As with rating and portfolio composition the retention limit problem can be solved theoretically, but the theoretical model is difficult to implement in practice.

An example of a theoretical solution to the retention problem is given by Pentikainen (1952) in respect of the maximum retention for a ceding company. The solution is given by:

$$M = \frac{(U + S)^2}{4P}$$

where	S	=	the security loading for one year
	P	=	the annual net risk premium as a multiple of the average claim
	U	=	the initial risk reserve
	M	=	the maximum retention limit as a multiple of the average claim

The approach has a collective risk theoretic basis and develops the maximum retention formula in terms of annual net risk premium, the initial risk reserve and a security loading. Pentikainen estimated that if the maximum retention limit is less than fifty times the average claims amount, based on a large sample of claims (more than 500), the error in using the formula will be less than 3% of the value calculated by the formula. The method consists of determining the amount of adverse fluctuations that is 99% certain of not being exceeded. To implement this method for the excess of loss portfolio would prove difficult, however, because it is unlikely that the large sample of claims required would be available from past experience.

4.4.5.3 A comment on the theoretical approach to own reinsurance problems

Although the own-reinsurance problem is not selected for special study in this thesis, the purpose of this short section is to provide an outline of the possible courses of action on this important aspect of reinsurance affairs. The general comments made in this thesis on the wide gap between theoretical and practical approaches to rating and portfolio construction can be extended to the own reinsurance problem. For further information on the theoretical approach to own reinsurance, however, the relevant literature falls under the three general headings of deductible selection for insurance buyers (for example, Murray (1970), Arrow (1974), Smith (1976) and Doherty (1977)), the retention limit problem for reinsurers (for example, Benktander (1975) and Nitzan and Rosen (1977)) and generalised actuarial models (for example, Saito (1976). There are similarities between all three approaches since they all deal with aspects of the general risk-transfer problem of which reinsurance of excess of loss portfolios is but one example.

4.5 SUMMARY OF CURRENT APPROACHES TO PORTFOLIO CONSTRUCTION AND CONTROL FOR EXCESS OF LOSS REINSURANCE

The elements of excess of loss portfolio management have been described with reference to both the literature and current market practice. As with the rating problem, there exists a problem of reconciling the theoretical and practical approaches to provide a more structured approach to excess of loss underwriting. Both approaches would be enhanced by increased measurability of the risk processes yet, given this increase in measurability, the need for excess of loss reinsurance would decline due to greater ability of ceding companies to understand and control the likely consequences of their own portfolio acceptances; there would be less need for reinsurance. Seen in this light, it is apparent that the true demand for excess of loss reinsurance is rooted firmly in the need to cope with uncertainty which is, by definition, unmeasurable and not susceptible to extrapolation from past experience. The role of probabilistic techniques to provide bases for rates and portfolio composition from analysis of past experience is, consequently, limited in application and value. Judgemental techniques, on the other hand, although demonstrated by market practice to be workable, are not easily susceptible to modelling and development given the current state of knowledge. The two main causes of difficulty in this respect are firstly, due to the inability of judgemental decision-makers to describe the bases on which their decisions are founded and secondly, because there is no satisfactory criteria on which judgements under uncertainty can be evaluated, even in the light of experience. Agreement on a decision is the usual test of a judgemental decision-maker's correctness and this must suffice for a more structured approach of testing the decision against organisational objectives. Without further research, therefore, the excess of loss underwriter must choose between two methods of problem solving: the first is the well-developed yet unusable probabilistic approach, and the second is a workable, judgemental approach which cannot be justified analytically or be developed or communicated with enough precision to form the basis of a structured method of planning and control.

In the next chapter the consequences of these limitations on excess of loss underwriting are discussed further and the areas where new knowledge is required are described, along with suggestions for research aimed at providing the required information.

CHAPTER 5

EXCESS OF LOSS REINSURANCE UNDERWRITING: THE NEED FOR NEW KNOWLEDGE

5.1 INTRODUCTION

This chapter outlines remaining problems for the excess of loss underwriter and the catastrophe reinsurance industry which are not soluble using existing methods and techniques. Special attention is paid to the reasons for failure of these methods and techniques, and also to the possible consequences for individual reinsurers and the industry as a whole. The case for a new direction of research into excess of loss reinsurance will be presented along with proposals to tackle the most important issues. This line of research will be continued and developed throughout the remaining chapters of the thesis.

5.2 REMAINING PROBLEMS FOR THE EXCESS OF LOSS UNDERWRITER

The excess of loss underwriter operates in an industry which is notorious for its uncertainty and the consequent ill-structured nature of its problems. These problems do not lend themselves easily to treatment by developed analytical methods. Where parameters of probability distributions associated with future outcomes are not known, underwriting activities based on expected outcomes are impractical. Where the essential stage of measurement cannot be performed, even the systematic risk management procedures of identification, measurement and control, as developed by Mehr and Hedges (1963), are of only limited assistance. This section underlines those aspects of the excess of loss underwriter's work which suffer due to the lack of meaningful measurement techniques.

5.2.1 Rating problems

The problems facing the excess of loss reinsurer in connection with premium setting were described in Chapter 4. The current situation is that, despite a large body of analytical knowledge, the excess of loss underwriter must employ judgement to cope with day-to-day pricing decisions. Since the use of expected values in underwriting decisions has largely failed, the underwriter is in the difficult position of not knowing, even with the lapse of time, if his pricing decisions are accurate. Feedback information, in terms of whether or not the portfolio showed a profit, forms only a basis for judgements on past overall performance. For the evaluation of past individual pricing decisions, the analysis of feedback information becomes more complex or impossible. These problems are traceable to the characteristics of the excess of loss contract which describes the basic unit of trade. Since every excess of loss contract is different and relevant data on past experience are scarce, no straightforward method is available for allocating costs to individual contracts or for arriving at accurate premiums. The implication is that individual contracts are usually priced either above or below the true (should it ever be determined) price due to the inability of the underwriter to apply a consistent pricing method. In the short-term, therefore, buyers of excess of loss reinsurance may be quoted rates which cannot be justified except on vague judgemental grounds. Only in the very long run can a ceding company form an opinion on whether or not it has been treated fairly at the hands of the reinsurer. The basic ratemaking principles of reasonableness and equity cannot, therefore, be shown to apply for excess of loss premium rates. A further shortcoming of current ratemaking systems is inadequate measurement of the elements which comprise premium (expected losses, loadings, etc.) and this prevents detailed control of the excess of loss portfolio. Individual premium rates, theoretically, provide the source data on which the construction and analysis of the portfolio is based, but where, as for excess of loss business, the source data is heterogeneous and scarce, analysis and control of the portfolio becomes difficult. Methods of estimation are therefore required which employ human judgement to compensate for the lack of objectivity.

5.2.2 Portfolio problems

The consequences of measurement problems and inaccurate methods of analysis of the excess of loss portfolio include difficulties in gauging current and future performance, the danger of exceeding desired or prudent levels of risk taking and the possibility that resources are underutilised. The inability to gauge performance creates difficulties for achieving desired levels of risk-taking. From an unintentionally constructed, "over-risky" portfolio could result an inadequate own reinsurance protection programme and hence the possibility of insolvency. Insolvency could also result from a portfolio which underutilises risk-taking resources and fails to produce the required profit on which to continue trading. The benefits of achieving a balanced portfolio are expressed by Stone (1975 p.239) as follows, with regard to insurance capacity and constraints of return, stability and survival.

> In an uncertain insurance environment, capacity is worth money. To construct a portfolio of a given size and a given expected return within the stability and survival constraints is worth more than to construct an identically sized portfolio with the same return outside of the constraints. A portfolio with substantial excess capacity is even more valuable (it can be used elsewhere). Certainty and stability are positively priced commodities in the framework of modern business theory.

With the existing level of knowledge, underwriters of excess of loss reinsurance are severely limited in their ability to gauge portfolio capacity to any degree of accuracy. Consequently, the dangers of over risk-taking and inability to identify spare capacity on the portfolio persist and are likely to cause inefficiency of business operations. The need for increased efficiency is not only a problem for individual underwriters seeking balanced results but also bears, on a larger scale, on the problems of the industry. As the final line of risk-bearers in the insurance industry, any improvement in efficiency for excess of loss reinsurers should, through competition, benefit direct insurers and, ultimately, the insurance-buying public in the form of lower premium rates. A further consideration is that effective use of resources for protection against large losses is essential for economic and technological progress on both a national and international basis. The problem of inadequate capacity is not unknown in the insurance industry, as is apparent from the following quotation from the Post Magazine (own correspondent (1975, p.132):

> There have been times when there has been talk within the market about insufficient capacity and lack of facilities for handling major risks. Usually, however, capacity has expanded, and no serious problems have been encountered. It looks, however, as though there may very well be serious capacity problems in the future. Now, the position has been reached where the size of ships and the insured values, including cargoes, have increased with exceptional speed.

Such limitations of the market are caused by a lack of funds invested in insurance operations and/or inefficient use of those available. New knowledge aimed at reducing uncertainty and increasing control of the insurance portfolio is essential if limitations of market capacity are to be reduced. For the excess of loss portfolio, where uncertainty is at its greatest anywhere in the market due to the catastrophic nature of the risks covered, there is therefore an important need for improved, workable systems of portfolio construction and control.

5.3 THE PROBLEM OF SUPERVISION

In Britain, regulation of insurance company foundation, control and winding up procedures is by legislation. Regulation of these main stages in the life of a company is facilitated, where possible, by efficient measurement procedures. The regulation of insurance companies is, however, essentially, a process of subjective decision-making on the basis of more or less scientific measuring tools. Financial ratios are examples of such measuring tools which provide useful rules of thumb for subjective decision-making. An important rule of thumb for insurance regulation is that premium income provides an indication of the risk assumed by a company. For example, Revell (1975, p.55) writes:

> In some respects the legislation applies a rule of thumb to prudential problems. This is particularly noticeable in the requirement for a fixed initial amount of capital and in the solvency margin, which relates the free assets to the premium flow; there is a sliding scale, but no consideration other than the level of premium income is taken into account.

From the preceding sections it is apparent that, for excess of loss business, premiums charged are less likely to reflect risk assumed than for any other type of (re)insurance business. The implication here is that, unless measuring tools more accurate than those based on premium income are found, the performance and degree of uncertainty at which individual reinsurers operate cannot be effectively assessed by parties with an interest in the organisation. The vehicle of supervision which dictates the type and extent of (re)insurance information to be made available for external parties with an interest in (re)insurance affairs in the UK is the returns required by the Department of Trade and Industry (D.T.I.) under the Insurance Companies (Accounts and Forms) Regulation 1968 (S.I.N.1408). The returns include, for each financial year of a (re)insurer's performance, the following information:

Schedule 1

a) Balance Sheet

b) Profit and Loss Statement

Schedule 2

a) Fund Revenue and Accounts

b) Premium Analysis

Schedule 3 (Statistical Return)

a) a summary of existing reinsurance arrangements

b) premiums received and claims paid

c) development of claims settlement

d) long-term business (e.g. life assurance)

Schedule 3 of the returns contains the type of information on which a risk analysis of company operating performance might be performed. The amount of information provided, however, differs between companies and, apart from the minimum requirements of details of actual cash flows, provides an inadequate starting point for a risk analysis based on expected values in the case of excess of loss business. This point is raised by Abbott et al (1974, p.227) with specific reference to outwards excess of loss reinsurance.

> On account of the complexity of outwards reinsurance arrangements, companies are allowed to give a very general summary of the situation in Schedule 3, Part I. Some have taken advantage of this, others have given great detail. It is difficult to relate the information, even when given in great detail, to Part III of the Returns. Many treaties relate to excess of loss and unless one knows the excess limit and the distribution of claims amount by size, it is impossible to tell what effect the treaty may have on the position of the company, or on the actual technical reserves necessary in any particular fund.

For companies transacting high volumes of inwards excess of loss reinsurance, the problems become even more acute; even if outwards reinsurance arrangements can be adequately described, assessment of their compatibility with inwards business is complicated by risk measurement difficulties of the underwritten portfolio. As a result reinsurer solvency, in which a ceding company may well have an interest for more than a decade, cannot be easily assessed. The information available from the D.T.I. returns is of mainly descriptive value from which supervisors may base value judgements on reinsurer solvency. There is, therefore, a need for development of analytical measures for excess of loss reinsurance to aid supervisors in their judgemental processes and improve upon the current emphasis on premium income as a rough measure of risk.

5.4 THE NEED FOR A DEPARTURE FROM TRADITIONAL PROBABILISTIC APPROACHES TO EXCESS OF LOSS REINSURANCE PROBLEMS

The key problems for excess of loss underwriters and, indeed, all persons concerned with the business can be traced to the basic risk unit which cannot be adequately described using probability measures. This basic deficiency renders rating, portfolio construction, control and any attempt at analysis of excess of loss business a largely judgemental process. Further, since excess of loss business is, essentially, a long-term business, feedback on the required judgemental decisions is often too late for corrective action to be taken, or out of date for the treatment of new types of risk entering the market. Given the failure of current probabilistic methods to cope with the problems of excess of loss reinsurance, it would seem appropriate to take a wider view of the system in order to harness both theoretic probabilistic and practical judgemental approaches to the problem. There are, of course, those who believe that useful new knowledge can only be achieved through further understanding of the laws of probability. For example, Miller (1980, p.1) writes:

> We must stay with the idea of there being an underlying certainty which is but dimly perceived by the human mind, if we are to do anything constructive about improving the confidence and skill with which people tackle uncertainty; uncertainty must be alleviated by increasingly acute perception of probability.

It could be argued for excess of loss underwriting, however, that such a solution is, with the present state of knowledge, impossible. Since the probability distributions are not known, the alternatives for a practicing underwriter might be described as either waiting until his uncertainty is "alleviated by accute perceptions of probability" or making his (possibly incorrect) decision now and getting on with the business. It would be a reasonable assumption that most excess of loss underwriters take the second of these courses of action and, although probabilistic reasoning may be applied up to a point, they include a large proportion of less than scientific reasoning in their decision processes. These procedures, even if unscientific, must be given credit for enabling the excess of loss form of reinsurance to have survived for more than fifty years. The methods employed in the marketplace have grown up to deal with uncertainty which, by definition, exceeds the boundaries of precise and quantitative knowledge. It would, therefore, seem inappropriate to apply analytical methods based on an assumption of measurability of underlying risk processes. The intention should not be to dismiss probabilistic methods as irrelevant to excess of loss reinsurance, but to devise means of coping with their limitations for practical decision-making by underwriters. There remains a large proportion of the decision process which can be explained, not by resort to the assumption that underlying risk processes are known, but to a more realistic assumption that the individual decision-maker has developed a set of rules for coping with the uncertain, largely unmeasureable environment. These rules may or may not be adequate for the task at hand but must exist, nevertheless, and are, therefore, worthy of study. By identification and analysis of these non-scientific rules for decision-making, important contributions to knowledge of the strengths and weaknesses of excess of loss underwriting systems may be gained. Opportunities for research in this field are wide and involve, essentially, comparing unscientific judgemental rules for decision-making with relevant scientific and objective methods.

An illustration of a non-scientific, yet practical, decision-making procedure is provided by the example of the ancient Egyptians who, when faced with the problem of the Nile which unpredictably flooded, destroying property and drowning livestock, turned to their religion rather than to scientific method. From observation of the heavens they learned, over many years, that when the god/star Sirius appeared on the horizon at dawn the Nile would flood. Their conclusion was that Sirius caused the flood, and, although much time was wasted on useless sacrifice and ritual, they were, nevertheless, able to order their agricultural affairs in a more successful manner. The example of the ancient Egyptians and their observation of Sirius is that of a rule of thumb. Since the Nile's flood itself could not be predicted, attention turned to a more measurable phenomenon: namely the path of Sirius, which could be monitored in a more predictable fashion. Only many thousands of years later was it discovered that large masses of water were affected by movements of heavenly bodies (a process in which Sirius plays a very minor role). The purpose of an analogy between ancient Egyptians and modern excess of loss underwriters is to point out that both groups are faced with problems of uncertainty and that both groups have found rules, other than those which go to the root of the problem, to cope with the situation with reasonable success.

To the practically-minded man, the justification of a rule is of secondary importance to the fact that it works. It is, therefore, suggested that for excess of loss reinsurance, without casting away the developed body of probabilistic methodology, an attempt should be made to identify and evaluate underwriters' decision processes in response to factors not directly related to underlying risk processes. Areas for possible study include market indicators, organisational and commercial arrangements for dealing with problems of uncertainty, business practice and rules of thumb employed by underwriters in the marketplace. Special attention should be paid to those factors which, like the path of Sirius, are measurable and also predictable. Such factors may be perceived as providing a degree of certainty in a highly uncertain environment and worthy of being included in the decision-making process.

5.5 THE NEED FOR A BEHAVIOURAL APPROACH

If probabilistic methods and models have failed to provide practical help to excess of loss underwriters, there remains the problem of analysis of those decision-methods actually employed. By identification, description and tests of these methods it may be possible to evaluate their usefulness and also their limitations. The relevant approach for such studies is the relatively new field of behavioural science which deals with the problems of how individuals search, code and combine information in order to form judgements and decisions. This line of research, which has been applied to many business areas including marketing, finance and accounting, is essentially interdisciplinary and combines theories from, among others, the disciplines of economics and psychology. Since underwriters and reinsurance company managers earn their livings by evaluating information, forming judgements and making decisions, the excess of loss business would seem an appropriate target for study, employing behavioural theories and methods.

The important assumption underlying the behavioural approach is that actual decision-making is not necessarily the same as that implied by traditional economic theories. The emphasis of behavioural research is firstly, on description of the world as it is (rather than as it should be) and secondly, on looking behind the identified behaviour to find reasons for the status quo. By modelling identified decision processes, the aim is to bridge the gap between actual and theoretical behaviour with a view to improving the quality of actual performance, and also identifying opportunities for the development of existing theories. Donaldson (1971, p.12) makes the following comment on the process of behavioural research:

> It involves a serious effort to learn from the businessman himself how he perceives the problem, what he sees as the relevant variables, and how he deals with them analytically in reaching sensible decisions. It means trying to understand his frame of reference instead of imposing one on him. It means taking what is for many academics the giant step of assuming that the average businessman is both intelligent and rational in what he does.

Two of the most important conclusions which have been reached by behavioural scientists from observation of practically-oriented businessmen which are likely to be applicable to the excess of loss underwriter are as follows:

1) <u>The existence of a "bounded" model of reality for decision-makers.</u> The view put forward by Simon (1947) is that management authors should concentrate on what 'is' and not 'what ought to be'. According to his theory of decision-making, 'choice is always exercised with respect to a limited, approximate, simplified "model of the real situation"'. Simon's theory was put forward to describe the situation of a businessman faced with a complex and uncertain business environment, and might therefore be considered as applicable to the problems of the excess of loss underwriter. An important aspect of the simplified model which is employed in decision-making is that it is not necessarily rational. Simon offered the concept of "bounded rationality" to replace that of "subjective rationality" to allow for unique modes and limitations of perceiving, learning, and thinking by decision-makers. If Simon's theory is applied to the excess of loss underwriter, he must be viewed as possessing a thought process involving a "satisficing" model of his environment which enables him

to make decisions on, for example, premium rates and portfolio composition. Decisions reached in this manner are unlikely to be optimal from an economic point of view but, nevertheless, acceptable and within the range of human abilities.

2) The existence of "satisficing" attitudes by decision-makers. March and Simon (1958) put forward the proposition that administrative individuals "satisfice" rather than optimise in their decision processes. The term "satisfice" refers to the way that individuals, when confronted with a decision situation, begin with a search for information and alternatives but are likely to cease the search as soon as an alternative is found that satisfies a subjective minimum standard. In other words, due to the complexity of the problem or time constraints on providing a solution, the search ends before the ideal solution is reached. The evidence with respect to excess of loss rating and portfolio decisions is that, since ideal solutions are not possible using existing scientific methods, a very high proportion of the required decisions are of a satisficing type.

Behavioural studies have been undertaken on insurance matters, although not specifically on excess of loss reinsurance. These studies include those by Hickman and Miller (1970), Gaunt (1972) and Neter & Arthur Williams Jr. (1973) which were geared specifically to the decision process of the underwriter. The results of these studies cannot, however, be directly compared with the excess of loss underwriter's situation since the authors consider underwriting behaviour in response to risky, measurable circumstances rather than to uncertainty. For example, Neter & Arthur Williams Jr. look at underwriters' expected utility functions where the probabilities of outcomes are known. Where cause and effect relationships are highly uncertain, however, as in the excess of loss business, such analysis is impossible unless augmented by a judgemental or satisficing, rather than expected, basis of the utility function. In other words, analysis of a decision-maker in a complex or uncertain situation, such as the excess of loss underwriter, cannot commence until the satisficing model against which decisions are gauged is known.

A behavioural study of excess of loss underwriting would provide information, not only on how decision-makers react to perceived stimuli but also, through development and comparison with theoretical ideals, important conclusions might be drawn about the suitability of individuals' decision-processes for achieving desired results. The primary task, given the present stage of knowledge, is to identify the satisficing model(s) on which excess of loss underwriters base their decisions, and this will be discussed in the next section.

5.6 LIKELY CHARACTERISTICS OF A JUDGEMENTAL DECISION MODEL FOR EXCESS OF LOSS REINSURANCE UNDERWRITING

For the purpose of this section it is assumed that, due to the inadequacy and impracticality of scientific methods for solving excess of loss problems, there exists a manageable, satisficing, non-scientific decision model on which underwriters base their decisions. It is not possible to state at this stage whether or not such a model really exists but, from consideration of the underwriting environment and the types of decisions which are made, it is possible to list some likely and desirable characteristics of such a model should it indeed exist. The characteristics of the underwriters' judgemental decision model might include the following:

1) It will reflect underlyng risk processes, even if actual risk processes are not known with accuracy. If a decision model has become acceptable to underwriters it is likely that one of the main conditions for its acceptance is its predictability. Thus, even though relevant stochastic processes are not known with accuracy, the nature of underlying risk processes will, if only at a subconscious level, have found their way into the decision-making process. For example, after many years of surveying his claims experience, an underwriter is probably able to form a rough idea of what kind of claims experience can be expected from certain types of risk. Decisions based on such analysis, although not necessarily accurate, are, in the long run and via the law of large numbers and the underwriter's learning process, likely to approximate actual risk processes. If such a relationship did not hold to a reasonable degree, the results of underwriting decisions would prove highly erratic and cause discontent among ceding companies over price of cover and, for the reinsurer, a high probability of either inadequate or supernormal profits.

2) The judgemental process will be flexible. Flexibility in the underwriter's judgemental process is required for combining various aspects of his experience to evaluate the consequences of a new action of which he has no experience. This concept may be described mathematically using the credibility theory but, where underwriting judgement is concerned, the mental process is likely to be of a more rough and ready nature. Few, if any, of the underwriter's decisions are based on a single criterion and, consequently, comparisons must be made between alternatives which are not directly comparable. The judgemental process must, therefore, be highly flexible in order to encompass the range of relevant information for decision-making. If this flexibility did not exist, the placement of a new type of risk in the excess of loss market would be impossible.

3) It will hold similarities for different underwriters. The likelihood of similarities in the decision model for different underwriters stems from the fact that the nature of many problems facing them is substantially the same. For example, rating and portfolio problems are common to all underwriters, as are many of the administrative problems involved. A second reason for suspecting similarities in the decision process is the high level of agreement in evidence once a decision affecting many underwriters has been made. This observation applies particularly to the pricing decision which, once made by a single underwriter, is deemed acceptable by the many other signatories on the slip. If this level of agreement did not exist in the market, it would not be possible to place large risks requiring the pooling of underwriting resources of many underwriters.

4) It will encompass the possibility of different strategies between underwriters. The strategic differences are likely to be of two types. Firstly, differences in strategy can be expected between underwriters since their objectives and constraints differ with the types and sizes of account for which they are responsible. The second type of strategic difference is of a competitive nature which might manifest itself in attempts to secure as much profitable business as possible onto portfolios by each underwriter. Since the market underwriting system allows for both competition and collusion, it is likely that these alternative possibilities for strategy are taken into account in the

underwriters' decision model. In an area of high risk or uncertainty the relative performance of competing under writers becomes an important issue. For example, Ryder (1976, p.81) puts forward the view that "a Non-Life Insurance company has to fear only two things. A sudden winding up or a much worse experience than its competitors". Successful competitive strategy under these circumstances becomes even more important than strategy aimed at meeting own desired objectives and constraints as measured in absolute terms. Low underwriting performance shared with the rest of the market is likely to result in a joint effort to raise market rates but low performance of a single firm is less likely to be compensated by an overall rise in future market rates. Both competitive and collusive strategies are therefore likely to play an important role in the judgemental decision model for an excess of loss underwriter.

5) Key parameters of the model will be measurable. The need for measurability is essential in a satisficing decision model, even if the measurment process takes place only at a subconscious level. The subject for measurement need not, however, be the direct cause of the problem at hand. In the Nile flood example, for instance, the ancient Egyptians measured the path of a star rather than any characteristics of the offending waters. For purposes of analysis of the underwriters' decision model the important first step would therefore be to discover the key parameters from which satisficing measurements may be taken.

6) Each possible configuration of key parameters should lead to a deterministic decision. This is a corrollary to (5). The requirement is that, having identified key parameters, each possible combination of them should lead to a different decision. Should an underwriter be faced with the same collection of decision variables on more than one instance, the resultant decisions should be identical. A decision model which cannot be shown to demonstrate these characteristics is imperfect since it goes only part-way to describing the whole process.

7) Once identified, the calculation method and results should be suited for repetition and checking. This requirement is mainly for the testing of empirical studies on the identified decision process. Should a model

of the process satisfy the test of repetition, further development may be possible. If not, further research into the nature of the model would be required.

These proposed characteristics of an excess of loss underwriters' satisficing decision model will be reconsidered in the discussion section of this thesis (Chapter 8) in the light of findings arising from the research.

5.7 PROPOSAL FOR A SEARCH FOR AN UNDERWRITERS' DECISION MODEL BASED ON SLIP DETAILS

The first problem requiring a solution in the search for an excess of loss decision model is identification of measurable variables in the underwriter's field of reference which might, justifiably, be included in a judgemental decision process. In practice, it is probable that such variables do not include formal probability estimations from past experience since they are either difficult and time consuming to provide or liable to include large margins of error. In the previous section it was suggested that the key variables;

a) reflect, roughly, actual risk processes,
b) are flexible,
c) are available to all underwriters, and
d) are finite.

Possible sources of information which would satisfy all these conditions are the reinsurance contract itself, and its summarised form, the slip. These documents are directly concerned with the risk at hand and, although not necessarily containing information from which probabilistic expectations may be formed, provide a great deal of information on which judgemental decisions could be made. Of the slip and the contract, the slip is the document most likely to have the greatest influence on underwriters' judgemental procedures since, unless past contracts are available for perusal, it precedes the contract in time, the contract itself being the evidence that a decision has actually been made. As a summarised version of the excess of loss contract, the information contained in the slip corresponds to that described in Chapter 4, Section 4.3.2, or as a trimmed-down version of the excess of loss contract format provided in Appendix A.

Very few slips contain all the information suggested by these sources but details common to most slips include:

a) The estimated premium income of the ceding company (E),

b) the lower limit or the point at which liability commences for the reinsurer (L),

c) the upper limit which is the upper boundary to the reinsurers liability (U), and

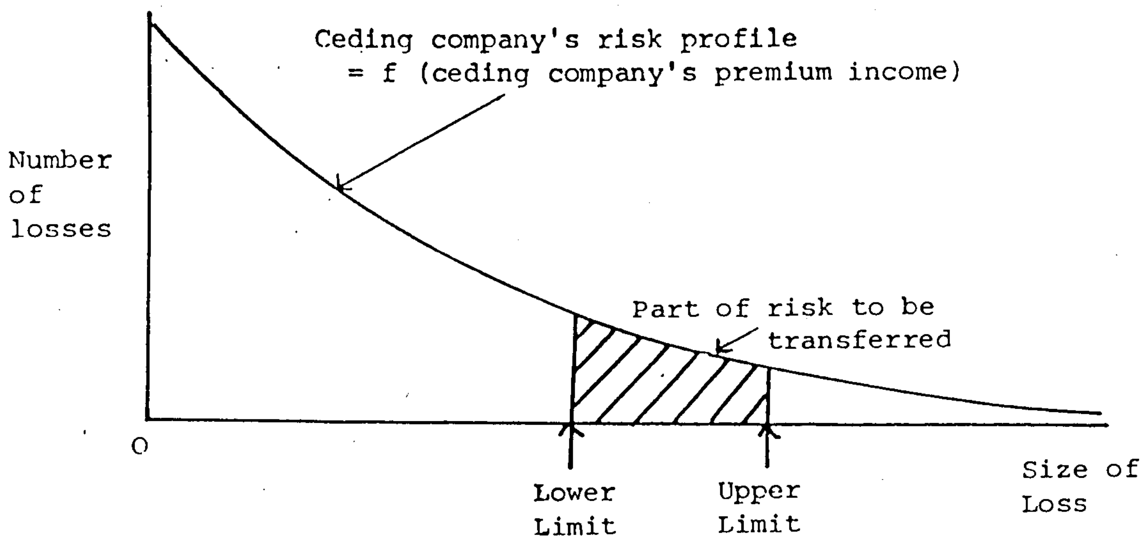
d) (for all but the lead underwriter) the rate which is being charged for the contract, often expressed in terms of a minimum and deposit premium (M).

The other details on the slip are, when available, largely of an unquantifiable nature. Special clauses for exclusion of types of risk or description of the geographical areas covered by the ceding company's business are examples of such unquantifiable details which, even if important in the underwriter's decision procedures, would not lend themselves easily to analytical development. It is therefore proposed that a search for an underwriters' decision model based on variables which are both available to underwriters and also susceptible to mathematical development, namely E, U, L and M, shall be undertaken. The main decision areas to be examined in relation to these variables are the central problems of rating and portfolio construction. Reasons why the underwriter might make these decisions with regard to slip details will now be presented.

5.7.1 Slip details as an influence on rating decisions

The rating decision involves arriving at a finite amount to be charged in return for acceptance of uncertain consequences. Underlying the calculation are considerations of risk processes which are not explicitly known and therefore unsuitable for mathematical development or application of economic principles. The slip details, U, L and E, however, are representative, although not actual measures, of the consequences of risk acceptance. That is, they describe, to a large degree, that part of ceding company's liability which is to be transferred. An illustration of how the slip details might approximate the risk to be transferred is shown in Figure 5.1.

Figure 5.1 A hypothetical approximation of a single excess of loss risk unit from slip details



The figure shows the relatively unknown probability distribution resulting from insurance contracts entered into by a ceding company. The shaded area represents the part of the distribution which is to be transferred by means of an excess of loss contract. The relationship of this distribution to excess of loss slip details and the possible consequences for the premium charged are as follows:

a) Ceding company's premium income (E): the complete distribution resulting from a ceding company's direct business is indirectly related to the ceding company's premium income. This relationship is not perfect or easily measured since premium income is only a rough measure of risk. For example, differences are bound to arise between companies due to the range of different rating and underwriting policies pursued in the market. Nevertheless, estimated premium is the only measurable variable on the excess of loss slip which bears on the gross risk situation faced by a ceding company and, as such, is likely to be taken into account in the underwriter's decision process. For a catastrophe cover, it is likely that an underwriter would charge a higher excess of loss premium for a company with a high premium income than for a company with low premium income (other factors being held constant) since the catastrophe potential, in terms of number and size of individual policies affected by a single event, would be higher in the former case.

b) The lower limit (L): the lower limit to the cover is a rough guide to the number of losses which will affect the reinsured portion of the ceding company's risk distribution. Since, as a general rule, more small losses occur than large ones, it is likely that a cover with a high lower limit would receive fewer losses than, say, a working layer with a low lower limit. Other factors held constant, a layer with a high lower limit might be envisaged as likely to attract fewer claims, and hence a lower premium rate than a layer with a low lower limit.

c) The upper limit (U): the size of the upper limit determines the amount of liability for which the reinsurers are liable. Other factors held constant, one would therefore expect a contract with a higher upper limit to be charged a higher premium than one with a lower upper limit (and vice versa).

The three variables (U, L and E) combine to produce a rough measure of the risk situation represented by the excess of loss contract, which, consciously or unconsciously, might influence underwriters in their rating decisions.

5.7.2 Slip details as an influence on portfolio selection decisions

The number of factors likely to influence the underwriter's portfolio selection procedure is large but, if the scientific approach to the problem is to be followed, the process involves portfolio building with regard to expected values and variances of individual risk units. In its simplest form, the portfolio selection problem is concerned with finding a solution to the question: How much (if any) of a given risk should be taken onto the portfolio at any moment in time? For the scientific approach to be productive for portfolio control, using slip details as a basis for decision-making, it would be a requirement that expected losses and variance could, to a reasonable degree, be predicted from E, U and L. This is necessary if the prime aim of controlled portfolio construction is to meet predetermined requirements of risk-taking and return.

An alternative, less sophisticated criterion for portfolio construction would be to attempt to maintain market position, via consideration of slip details, by determining the relative desirability of contracts in relation to an expected or average market price. This approach would require no direct consideration of expected losses but rely more heavily on writing, as far as possible, large amounts of business priced above the market average and less of business priced below it. The average market price might, judgementally, be gauged in relation to the slip details. Justification for such a system would require, not that slip details could be used to measure the risk situation, but rather that contracts with similar slip details were equally liable to incur losses. If this justification is upheld, an underwriter succeeding in writing mainly business priced above the market average, as estimated (consciously or unconsciously) from slip details, would, at the end of an underwriting period, show a higher profit than an underwriter who managed to write only the apparently low-priced risks. In other words, without attempting to meet constraints of risk and return, successful application of this approach to prices could achieve improved relative market position.

There are therefore, two methods which underwriters might employ to construct portfolios via analysis of slip details. The first method, aimed at achieving specified levels of risk and return, would require that slip details were reasonable estimators of risk processes. The second, less sophisticated method would require that slip details provided a means of establishing an expected price level, against which the relative desirability of individual contracts could be gauged.

Figure 5.2 illustrates how slip details might feature in an underwriter's decision process for establishing the relative desirability of various contracts. The example is presented only as a possible way in which, in the absence of further information, relative preferences might be determined with reference to slip details.

Figure 5.2 Slip details as a means of establishing relative contract preferences

Contract	Premium (M)	Upper Limit (U)	Lower Limit (L)	Estimated premium income for ceding company (E)
A	£1,000	£ 50,000	£50,000	£2mill.
B	£1,000	£ 50,000	£50,000	£5mill.
C	£1,000	£ 50,000	£20,000	£2mill.
D	£1,000	£150,000	£50,000	£2mill.
E	£ 750	£ 50,000	£50,000	£2mill.

The figure comprises slip details from five hypothetical excess of loss contracts. If decisions are made solely on the information provided in the figure, it is possible that a preference ordering might arise along the following lines:

1) Contract A is preferred to contract B because an equivalent premium is received for indemnifying a ceding company with a lower premium income.

2) Contract A is also preferable to contracts C, D and E since contract C offers a lower lower limit, contract D has a higher upper limit, and contract E offers a lower premium for assuming a contract which is otherwise, in each case, the same.

A problem arises, however, when one considers the relative desirability of, say, contracts B and C or C and D where the choice is not so obvious and further information on the relative preference value trade-off between the various slip details would be required. The example is, of course, hypothetical and assumes that personal values are attached to details which need not necessarily reflect the risky consequences of contract acceptance. Experimental work is required in order to assess the possible existence and implications of such an approach to decision-making.

5.8 A PILOT RESEARCH PROGRAMME TO EVALUATE THE EXTENT AND JUSTIFICATION OF THE INFLUENCE OF SLIP DETAILS IN EXCESS OF LOSS UNDERWRITING DECISIONS

A pilot study into the role of slip-details in the underwriting process for excess of loss reinsurance is proposed with a view to narrowing the gap between the actuarial and behavioural approaches to the problem. The investigation involves firstly, an analysis of whether or not slip details influence key excess of loss underwriting decisions and secondly, analysis of whether or not their use in this respect is (or could be) justified.

Data requirements, availability and preparation, and the nature and importance of the various relationships to be investigated in the proposed pilot study will now be presented.

5.8.1 The data

The choice of data is an important decision in an empirical investigation. The main criteria for data selection are that the sample should be large and comprehensive enough to provide meaningful results and that the population for which the sample is taken is relevant to the problem at hand. Data requirements for the current research into the role of slip details in the excess of loss reinsurance underwriter's decision processes will now be discussed under the following headings:

a) Choice of market
b) Choice of a decision-maker
c) Time-span considerations
d) Level of detail
e) Data preparation

a) <u>Choice of market</u>

There is an excess of loss reinsurance market for each major category of direct insurance market (e.g. aviation, marine, fire, life, etc.). Each of these markets is characterised by its own conventions which influence the types of cover provided and, therefore, the types of clauses likely to be included in the contract. Within each market there is, further, a wide variety of possible combinations of clauses to be inserted on a cover. These differences account for the high degree of heterogeneity between covers and render experience rating a complex business. An important consideration when choosing data for an experiment aimed at investigating alternative methods of rating is the degree of heterogeneity permissible within the data sample. Clearly it would be unwise to select data for common statistical treatment from several different markets since claims experience and underwriters' behaviour patterns would probably vary too much to secure reasonable statistical results. Thus it is decided that data for the pilot investigation should be taken from a single market; namely the aviation excess of loss market. The choice of aviation data rather than data from marine, fire or any other market is made largely through practical considerations of data availability. The availability of the data (which is presented in Appendix B) is entirely due to the willingness of Mr. S. Jones, Chief Underwriter and Director of Trimark Ltd., to allow the author to examine and record, for research purposes, the relevant details from his aviation excess of loss account over a period of years.

The excess of loss aviation contracts which form the basis of the data collection cover three main types of loss incurred through aviation activities. These are firstly, damage to or loss of the hull or mechanics of aircraft; secondly, damage to or loss of cargo carried in the aircraft; and thirdly, liability to passengers and third parties. Catastrophic occurrences likely to result in claims on the excess of loss aviation cover include mid-air and airport collisions of aircraft and consequential losses to all parties involved. Such occasions are infrequent but require large sums of money to indemnify insureds in the event of loss and are, therefore, representative of the type of risk requiring excess of loss reinsurance protection. The resulting data reflects the underwriting performance from a heterogeneous group of risks handled in a single market.

b) Choice of decision-maker

From the data collected it is necessary, for a behavioural study of decision-making, that it should be in the right form for analysis of key decisions undertaken by the relevant parties. In the present study, the key decision areas are those of pricing and portfolio selection and the parties whose decisions may be analysed using the data are firstly, Mr. Jones and secondly, all underwriters whose names appear on the slip for each excess of loss contract. The reason for categorising the decision-makers into two distinct sets is to allow the maximum research opportunities from the data available. With particular reference to the rating decision, from the data available, it is not possible to identify those risks which were priced by Mr. Jones, so it would seem appropriate to analyse the pricing decision in overall terms of "an acceptable price to all underwriters willing to take part in the risk". For the portfolio selection decision, on the other hand, the data available refers only to the acceptances by Mr. Jones; the portfolio selection decision must, therefore, be analysed with respect to a single underwriter. The research stance taken could, therefore, be described as an analysis of how a single underwriter makes his decisions in response to decisions already taken by the market.

c) <u>Time-span considerations</u>

Risk acceptance and consequential incurrence of losses is a continuous process which, for purposes of study, might be broken down into any number of time periods. An important consideration is, therefore, the selection of meaningful data periods on which analysis is to be performed. The data collected stems from contracts written between the dates of 1st January 1974 and 31st December 1977 by Mr. Jones for Trimark Ltd., the sample population comprising over three-hundred cases. These dates were selected on the grounds that, pre 1974, the Trimark aviation excess of loss account was relatively small (for example, only 29 cases in 1973); and 1977 was the final underwriting year for data collection if reasonable approximations of eventual claims were to be achieved (adequate information on claims incurred from underwriting activity in any one year becomes available only with the lapse of time). The opportunities for selection of data samples over time are, nevertheless, very wide. The choice of data samples for individual tests will be described in more detail with the presentation of each experiment but, for most tests, the data samples to be employed in the analysis are of one year's duration. By considering the results of identical tests on separate yearly data samples, any relationships established for one data year can be cross-checked with relationships established on other data years.

d) Level of detail

The level of detail provided by the data is shown in Appendix B but the main data requirements for the tests are described here for each contract:

a) the £ value of premium written by the underwriter,
b) the total £ premium for the contract,
c) the £ upper limit of the cover,
d) the £ lower limit of the cover,
e) the £ estimated premium income of the ceding company,
f) the number of claims incurred, and
g) the £ size of each claim incurred.

From Appendix B it can be seen that, in most cases, all these details are available from individual contracts. In some cases, however, notably for estimated premium income for the ceding company but occasionally for premium charged, upper limit and lower limit, certain details were not available for collection. The reasons for these missing data are firstly, because some details were missing from the underwriter's records and secondly, because all the above details are not provided on all contracts. An example of a contact without specified limits would be where a ceding company required cover at all high level layers in the event of existing reinsurance arrangements being suddenly brought to an end by the happening of a catastrophic event. In such a situation the "any one loss" clause is applied and unless secondary arrangements for excess of loss cover are made the ceding company would be its own reinsurer. This apparently limitless type of excess of loss cover which accounts for the lack of detailed data is, in fact, an excess of loss contract against a second or third catastrophic loss.

A decision was made to include cases where certain elements of data were missing on the grounds that, since the underwriters were required to make decisions on limited information, the same situation should be reflected in the data analysis. Some statistical problems arise, however, when data are incomplete, namely those of finding surrogate estimators to fill the data gaps. To deal with these problems, the facility for dealing with missing data on the S.P.S.S. (Statistical Package for the Social Scientist) statistical computer package was employed. Full details of the statistical process employed by the S.P.S.S. statistical package appear in the McGraw

Hill S.P.S.S. Manual but the basis of the missing values procedure is that, where a data gap is encountered in computer calculation procedures, a value is entered which represents the mean average value of all other values in the same data category. This statistical convention is applied consistently on all tests and on all data samples to provide a compatible set of results.

e) Data preparation

The data collected at source from the underwriters' records are not, without further treatment, in a suitable form for statistical analysis. Special treatment is required to adjust the data for the effects of exchange rates on cash flows and also for adjustment of claims experience as recorded on the underwriter's books. The required adjustments to data are described under the following headings:

i) Conversion of currencies.
ii) Adjustments to recorded claims.

i) Conversion of currencies

Payments for premiums and claims, should they occur, are specified in the raw data in terms of either pounds sterling or U.S. or Canadian dollars. In many cases an exchange rate is explicitly stated in the contract but, for others, all payments occurring under the contract are to be paid, should a currency conversion be required, at the going exchange rate on the date of payment. For purposes of statistical analysis it is highly desirable that all cash-flows should be expressed in a single currency. The approach taken to achieve this state of affairs is to convert all monetary values to pounds sterling using, where available, exchange rates specified in the contracts or, in the absence of a specified rate, an estimated rate calculated from monthly average figures. The monthly average rates employed for currency conversion were supplied by Ringstead Insurance Services Ltd., a service company providing data-processing and payments services to Trimark Ltd.. Exchange rates employed in the final adjustment procedure were as follows:

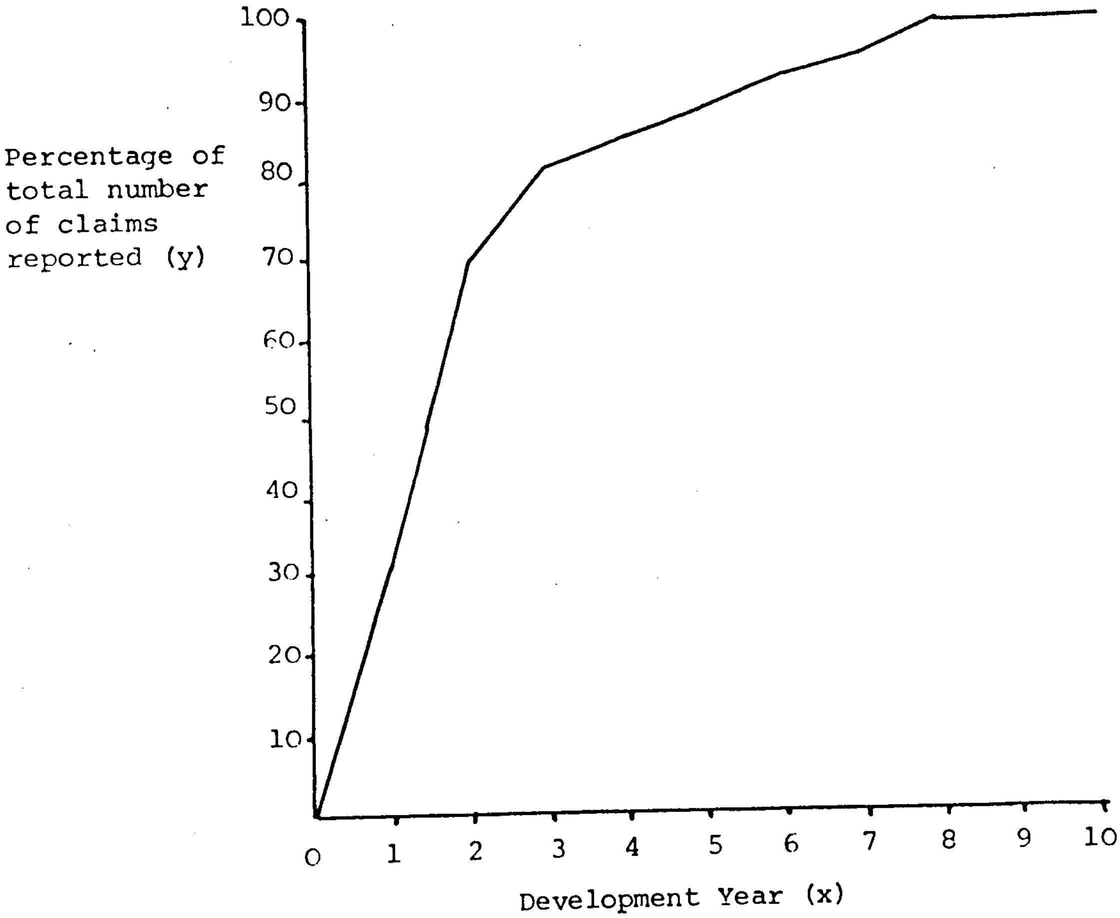
Year	$ equivalent to £
1974	2.338
1975	2.242
1976	1.8178
1977	1.744

ii) <u>Adjustments to recorded claims</u>

Adjustments to recorded claims are required to take into account firstly, the effects of inflation and/or errors in original estimation of claims amounts where claims have been reported but not yet settled and secondly, where claims have occurred but not been reported (I.B.N.R.). Both types of adjustment are required for published insurance and reinsurance accounts and are normally calculated using standard actuarial techniques (for details of these techniques, see Guaschi (1977) or Abbott et al (1974)). These techniques are normally applied for estimation of required adjustments to total claims experience, however, and, for the estimation of I.B.N.R. for present purposes, require adaption for application to individual claims occurrences. Ideally, the data should be adjusted to allow for changes in firstly, the number of claims incurred and, secondly the change in estimated monetary values of the claims. Unfortunately, the first type of adjustment presents severe problems. A possible treatment for the data would be to extrapolate the likely number of claims incurred and reported in each development year, as illustrated in Figure 5.3. Having made such an estimation, however, there is no satisfactory method of allocating the estimates to individual cases. The alternative courses of action are either to make a random allocation which, although random, could not be upheld as truly representative or, alternatively, make no adjustment at all to individual cases but employ the relevant adjustments only to overall claims incurrence, where required, for purposes of inter-period comparisons. The latter approach is to be applied in the present study where circumstances warrant it.

For adjustments to claim size, an adaption of Guaschi's method is employed. The difference in methodology is due only to the fact that Guaschi applies his method to total claims amount for each development year whereas the present study applies the method to average claims size and does not allow for increases in the number of claims reported after the final year of data-collection. The method for arriving at the required adjustments to be applied for each data years is shown in Table 5.1. In the table, values for claims (paid and outstanding) are expressed in index form, the first year of account (T1) being the base year. By monitoring cumulative changes in the index, the percentages to be applied as adjustments to recorded claims size for each year of account are established. Claim size data appearing in Appendix B are presented after application of

Figure 5.3 Method for determining the likely number of claims reported per year



Co-ordinates

x =	1	2	3	4	5	6	7	8	9	10
y =	31.75	70.25	81.89	84.90	88.76	92.91	95.45	100	100	100

Source of data: Trimark Aviation Excess of Loss Reinsurance Account.

Table 5.1 Method for arriving at adjustments to claim size data

PERIOD → / A/C ↓	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10
1970	100	70	89	109	117	234	286	305	272	279
1971	100	48	37	41	38	46	54	47	46	
1972	100	104	118	111	118	107	101	96		
1973	100	854	820	1006	1066	1023	992			
1974	100	93	103	114	118	139				
1975	100	100	125	133	122					
1976	100	251	284	310						
1977	100	81	78							
TOTAL (£) FOR PERIOD T	800	1601	1654	1824	1639	1549	1433	448	318	279
TOTAL (£) FOR T-1	-	-	1576	1514	1517	1410	441	352	272	-
% YRLY INCREASE	100	200	103	116	108	102	102	102	90	103
CUMULATIVE % INCREASE	100	200	206	239	258	263	269	274	246	254
INDEX WITH T10 = 100	39.37	78.74	81.10	94.09	101.57	103.54	105.91	107.87	96.85	100

The data were collected at period T3 of the 1977 account (i.e. 1979). Percentage adjustments for claim size are, therefore, as follows:

A/C 1974 data : 100/103.54 = 96.58%

A/C 1975 data : 100/101.57 = 98.45%

A/C 1976 data : 100/ 94.09 = 106.28%

A/C 1977 data : 100/ 81.10 = 123.30%

Note: The data appearing in the run-off triangle represent the changes in average claim size (paid + outstanding) with base period T1 = 100 as recorded in the underwriter's records for the aviation excess of loss account 1970-1979.

these adjustment factors. It must be stressed that this data has not been adjusted for I.B.N.R. and that care must, therefore, be exercised in interpretation of results to allow for this departure from the normal actuarial treatment of claims.

5.8.2 Relationships to be investigated in the preliminary analysis

The most important relationships to be investigated in the preliminary study are:

a) <u>The relationship of slip details to claims</u>. A test of this relationship will provide evidence on whether or not expected losses can be estimated from slip details and, therefore, whether or not underwriters could be justified in making portfolio and pricing decisions using slip details as risk estimators.

b) <u>The relationship of slip details to price</u>. The purpose of this test is to provide evidence on whether or not slip details are influential in the pricing decision. The interpretation of the result is largely dependent on the quality of the slip details to claims relationship. That is, if slip details can be shown to be predictors of claims and also influence pricing decisions, the use of slip details as estimators in underwriters' decision processes might be justified as being representative of underlying risk processes. If only the slip details to claims relationship holds strong, the implication would be either that underwriters are unaware that a claims approximation method is available using slip details or that they prefer to make pricing decisions without direct regard to claims expectations. If only the slip details to price relationship is upheld, the implication is that pricing decisions are made on the basis of a satisficing model which does not reflect the true risk situation. If neither relationship is strong, the implication would be that slip details are not important in the pricing decision, possibly because they bear no relationship to the underlying risk processes.

c) <u>The relationship of slip details to variations in price</u>. The ability to perform a test on this relationship is dependent on the slip details to price relationship proving strong, thus providing an expected price estimator. Price variations could then be calculated as the difference from slip details. The purpose of a test on the relationship of slip details to price variations is to establish the likely range of variation in price to be expected from any set of slip details.

d) The relationship of slip details to premium loadings. The ability to perform a test on this relationship is also dependent on the ability to establish an expected price from contract details and thereby a measure of price variation. A crude premium loading factor could then be calculated as actual price less claims expectation and price variation. The loading thus calculated might be envisaged as comprising allowances for reinsurers expenses, profit and a charge for carrying variance. The purpose of testing for a relationship between slip details and premium loading is similar to that for the slip details to price relationship, namely to establish whether or not slip details reflect underlying risk processes. If both expected claims and premium loadings hold strong relationships with price, evidence is provided that slip details are useful as risk estimators for ratemaking and portfolio construction purposes.

e) The relationship of slip details to written premium. By analysis of this relationship it is possible to determine whether or not the test underwriter's selection procedure, in terms of premiums written, is influenced by slip details. As with the premium (i.e. the total premium on a contract) to slip details relationship, the implications of this test of portfolio selection procedure can be evaluated only with regard to the ability of slip details to reflect underlying risk processes. For example, if both rating and portfolio selection procedures can be shown, via slip details estimators, to be based on underlying risk processes, the possibility of a structured approach to underwriting planning and control would be established.

f) The relationship of premium to claims, premium loadings and price variations. The purposes of establishing this relationship are firstly, to provide evidence for or against the existence of a risk-based approach to excess of loss rating and secondly, if the former test proves positive, to provide a risk-type measure of premium's component parts which can be compared with those estimated from slip details. If both tests succeed in providing a means of arriving at a premium for an excess of loss contract the degree of similarity between expected premiums arrived at by application of each approach would require further investigation.

g) <u>The relationship of written premium to price, claims, premium loadings and price variations</u>. Tests of this relationship are intended to complement those on the written premium to slip details relationship. The aim is to discover whether slip details are employed in portfolio selection either out of convenience or as a suitable basis for estimating risk processes.

A schematic summary of the relationips to be investigated in the preliminary analysis is provided in Figure 5.4. From the figure it can be seen that the overall purpose of the tests is to evaluate three main general relationships. They are:

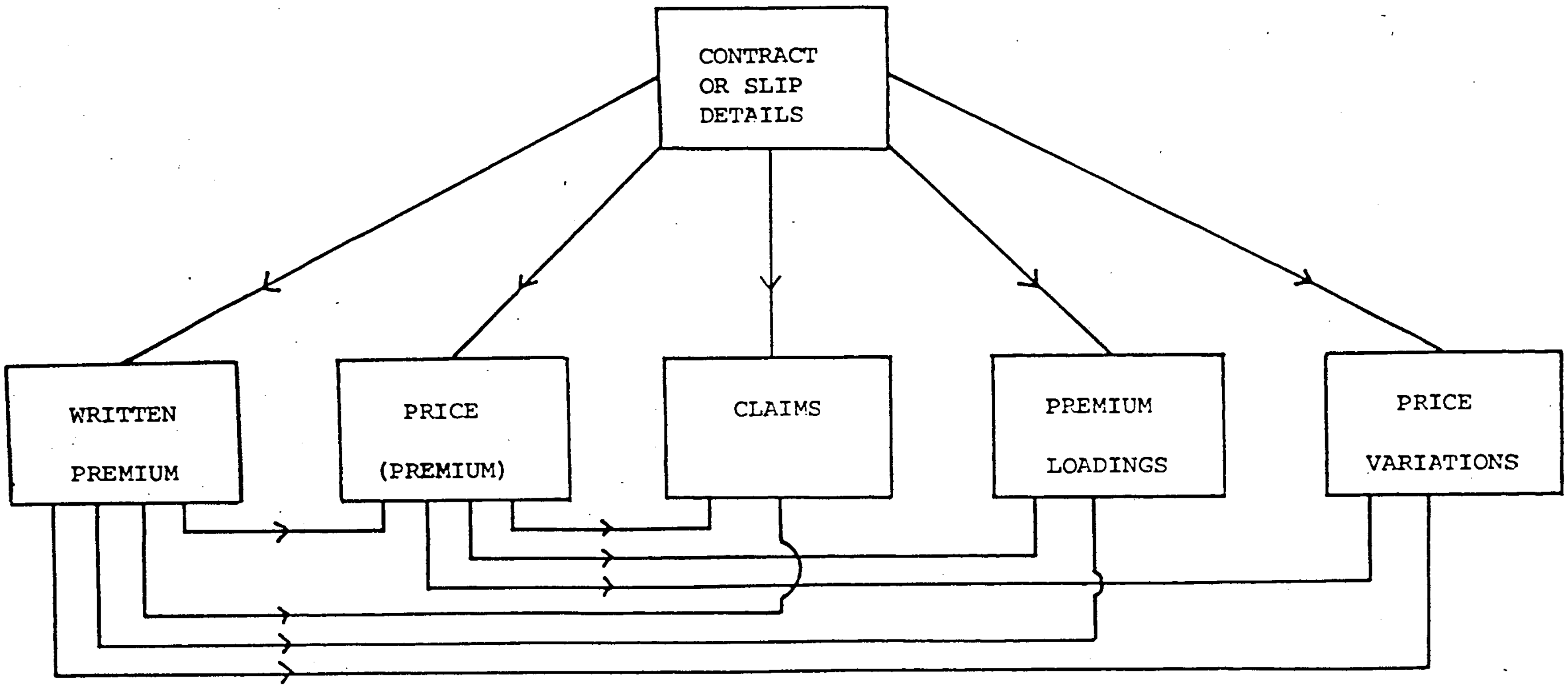
i) The relationship of slip details to key underwriting decisions
ii) The relationship of key underwriting decisions to risk processes
iii) The relationship of risk processes to slip/contract details.

Once the nature of these relationships has been discovered, unless all tests prove negative, it should be possible to cast further light on the nature of the excess of loss underwriters' decision processes. A further possibility is the construction of a decision-model on the basis of the strongest relationships established. The model could then be tested for its strength and weaknesses by simulation.

5.9 SUMMARY OF REMAINING EXCESS OF LOSS UNDERWRITING PROBLEMS AND THE PROPOSED PRIMARY INVESTIGATION

The remaining problems of the excess of loss underwriter have been described in this chapter as stemming mainly from the inadequacy of existing methods to represent, realistically, the risk situation which arises on entry into an excess of loss agreement. Problems caused by this lack of measurability extend beyond the boundaries of the single firm to the insurance industry as a whole and pose special problems for, among others, reinsurance regulators. The key to improved measurability and control lies in the excess of loss underwriting process itself, at the heart of which are

Figure 5.4 Schematic representation of relationships to be examined in the preliminary analysis



the two inter-related problems of rating and portfolio composition. This research proposes a new angle for looking at these key issues, namely an approach which is essentially behavioural but which also employs statistical and risk-theoretic techniques where they can be gainfully employed. A pilot study or preliminary analysis is proposed which examines the role of slip details in underwriting decisions. Slip details are chosen as worthy of special study since they represent a large proportion of the information available to underwriters for making their decisions. The data on which the preliminary analysis will be carried out is provided by an excess of loss reinsurer operating in the Lloyds Market and covers a whole aviation excess of loss account over a period of four years. Some results from the analysis will be representative of market relationships during the period and others will reflect, more specifically, the behaviour of a single underwriter operating in the market. The main relationships to be investigated are shown schematically in Figure 5.4.

At this stage, no attempt at predicting outcomes of the analysis has been made. The results from the analysis will, however, serve as a basis for further analysis to examine the case for prediction of key identified relationships.

CHAPTER 6

THE PRELIMINARY ANALYSIS

6.1 INTRODUCTION

The results of the tests to investigate relationships proposed in Chapter 5 are presented in this chapter, along with the statistical methodology employed and an analysis of the implications of key relationships established. The chapter opens with a general description of the methodology employed for all the empirical tests and a criterion for identifying "satisfactory" and "unsatisfactory" statistical relationships. Information relating to specific tests follows before the results of all the tests are presented. From a synthesis of the results of the preliminary analysis a proposal is made for further research aimed at further examination of the relationships established so far. The further analysis proposed on the basis of relationships established in the preliminary analysis is dealt with in Chapter 7.

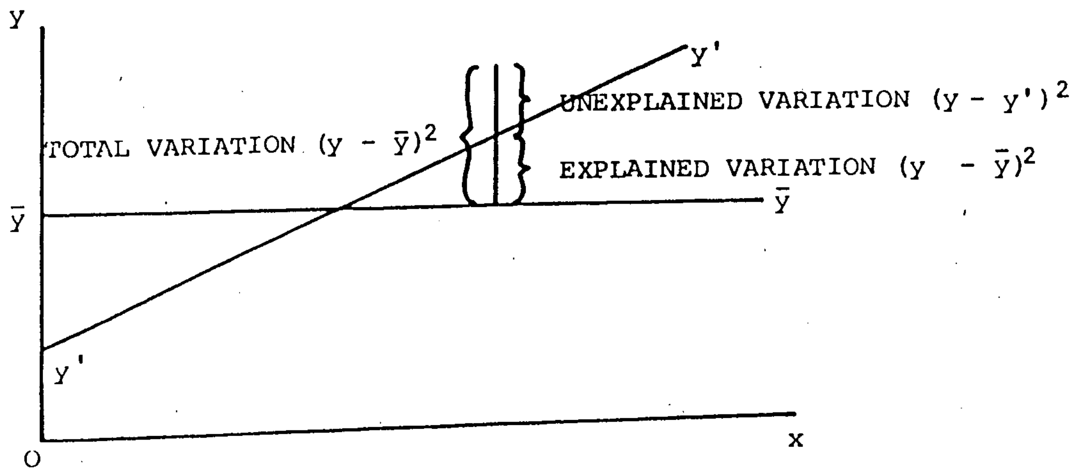
6.2 METHOD FOR ESTABLISHING KEY RELATIONSHIPS IN THE PRELIMINARY ANALYSIS

A multiple regression technique is employed in the preliminary analysis to establish the nature of key underwriting relationships. The approach and the principles involved are basically the same as for two-variable regression and correlation. The introduction of additional variables into the regression model, however, requires the use of a number of additional concepts and techniques. The dependent variable is estimated not from one but, say, in the case of premium (M) being estimated from the upper and lower limits and estimated ceding company premium income (U, L and E), three independent variables - the main justification being the appearance of higher coefficients of determination and correlation. The greater the variation of the dependent variable which the regression equation can explain, the more reliable are the estimates and predictions based upon the model. In theory, it is possible to conceive the inclusion of every variable factor which influences the dependent variable; the explained variation would then be equal to total variation and the perfect estimating equation achieved. In the present analysis, however, only those elements considered of key importance and of quantifiable nature are selected for the estimating equations. It is, therefore, unlikely that all variation will be explained. A problem, therefore, arises of how to distinguish between a "satisfactory" and an "unsatisfactory" relationship in the analysis. The adequacy of a statistical result in this respect, when comparing the degree of association of two or more variables, can only be determined with recourse to judgement. It should also be remembered that, even when the degree of association between variables is high, there is no assurance of any cause-effect relationship. To enable a methodical and structural interpretation of results, however, it is useful if some criterion is applied to identify "good" and "bad" relationships. The statistical measure on which such judgements are made is the coefficient of determination which is calculated using the following equation:

$$\frac{\text{explained variation}}{\text{total variation}} = \text{coefficient of determination}$$

From Figure 6.1 which illustrates a two-variable regression line of best-fit it can be seen that, if explained variation is almost as large as total variation, there is a strong indication that a change in one variable (x) is closely linked to a change in the other (y).

Figure 6.1 Components of total variation in a two-variable least squares regression model



The usual method of calculation for the coefficient of determination is as r^2 where r is the coefficient of correlation for the relationship. The question to be considered is, therefore, what size r^2 should be deemed representative of a reasonable relationship in the present study? As an equitable rule, it would appear fair to apply a criterion of adequacy that the greater part of any relationship tested should be explainable by the degree of association between key variables. This would require an r^2 of at least .5 (or an r of .7071) indicating that at least fifty per cent of statistical variation is caused by the associated variables. An "r^2 of .5 or more" criterion will, therefore, be applied for the two purposes of firstly, dividing, notionally, the results of the preliminary tests into "good" and "bad" categories and, secondly, for identifying relationships from the preliminary analysis which are worthy of further study and development. Of course, detailed results of all the tests are included in an Appendix B for scrutiny and to enable the reader to evaluate the implications of results using any criterion other than the r^2 of .5 rule if so desired.

6.3 TESTS AND REGRESSION PROCEDURES EMPLOYED IN THE PRELIMINARY ANALYSIS

The tests to be included in the preliminary analysis are summarised in Figure 6.2. Three types of test are included, namely "A" tests for estimating key underwriting variables from slip details, "B" tests for estimating individual components of the conceptual premium rate from overall actual premium, and "C" tests which examine an individual underwriter's selection procedure. With one exception, these tests require the regression of one dependent variable on three interdependent variables. The exception (actual written premium as a function of the estimated conceptual components of overall premium) involves the regression of four independent variables on a dependent variable.

A standard and comprehensive procedure is required to ensure an adequate search for any existing relationships (a simple linear regression model might yield misleading results if relationships in the regression model could be more closely defined using a non-linear method). A systematic relationship investigation procedure is, therefore, employed to ensure, within constraints of time available and the computer's capacity to employ extremely large or small numerical values in accurate calculation procedures, that each relationship investigated receives equitable and extensive treatment. The basis of the statistical search is the use of an

Figure 6.2 — Relationships investigated in the preliminary analysis

A TESTS: ESTIMATION OF PREMIUM AND ITS CONCEPTUAL ELEMENTS FROM EXCESS OF LOSS SLIP DETAILS		
A1	Premium	= f (Slip details)
	or (M)	= f (U, L, E)
A2	Claims	= f (Slip details)
	or (C)	= f (U, L, E)
A3	Premium loading	= f (Slip details)
	or (K)	= f (U, L, E)
A4	Price variations	= f (Slip details)
	or (MDIFA)	= f (U, L, E)

B TESTS: RELATIONSHIP OF CONCEPTUAL ELEMENTS OF PREMIUM TO TOTAL PREMIUM		
B1	Premium	= f (Claims, loadings, price variations)
	or (M)	= f (C, K, MDIFA)

C TESTS: SEARCH FOR TEST UNDERWRITER'S PORTFOLIO SELECTION CRITERIA		
C1	Written premium	= f (Slip details)
	or (MW)	= f (U, L, E)
C2	Written premium	= f (Total premium, claims, loadings, price variations)
	or (MW)	= f (M, C, K, MDIFA)

identity (linear) and two non-linear transformations of each variable (see Figure 6.3) which allows the best relationship out of a total of nine variations to be selected for any individual variable's correlation with another. The statistical search for the best multivariate equation in each case is undertaken as follows, in the case of the four-variable model. Firstly, bivariate relationships of the dependent variable with each individual dependent variable are established in turn by applying each of the nine correlation possibilities to each relationship to be investigated. Secondly, the best-fitting transformations of each dependent variable from the bivariate analysis are grouped into three separate multivariate equations. Thirdly, the results of each of the three multivariate equations are compared. From these equations, the one with the highest r^2 is selected as the equation offering the best statistical fit to the relationship. The main steps involved in this process are summarised in Figure 6.4 where, for illustration, a dependent variable (X) is to be predicted from independent variables (A, B and C).

Figure 6.3 Correlation possibilities from transformation of key underwriting variables (four-variable regression model)

		INDEPENDENT VARIABLES		
		Identity	Reciprocal	Logarithmic
DEPENDENT VARIABLES	Identity			
	Reciprocal			
	Logarithmic			

Note: All logarithmic transformations are to base 10.

Figure 6.4 **Steps in the regression procedure used in preliminary analysis (four-variable case)**

Step 1

a) From X, calculate 1/X and Log X

b) From A, calculate 1/A and Log A

c) From B, calculate 1/B and Log B

d) From C, calculate 1/C and Log C

Step 2

a) Regress X on A, 1/A, and Log A respectively

b) Regress 1/X on A, 1/A, and Log A respectively

c) Regress Log X on A, 1/A, and Log A respectively

d) Regress X on B, 1/B, and Log B respectively

e) Regress 1/X on B, 1/B, and Log B respectively

f) Regress Log X on B, 1/B, and Log B respectively

g) Regress X on C, 1/C, Log C, respectively

h) Regress 1/X on C, 1/C, and Log C respectively

i) Regress Log X on C, 1/C, and Log C respectively

Step 3

Select best-fit relationships from the bivariate correlations from Step 2 to construct three multivariate equations as follows:

a) X = f (best fit bivariate relationships of A, B and C transformations with X)

b) 1/X = f (best fit bivariate relationships of A, B, and C transformations with 1/X)

c) Log X = f (best fit bivariate relationships of A, B, and C transformations with Log X)

Step 4

From a), b), and c) of Step 3, select the equation which best fits the data to represent the relationship (i.e. the one with the highest r^2.)

Note: All logarithmic transformations are to base 10.

6.4 METHODOLOGICAL NOTES ON INDIVIDUAL TESTS

The method described in the previous section is applied for all tests but certain tests require further explanation and specifications of conventions used in the analysis. Further notes to accompany individual tests are, therefore, included below.

i) Tests involving a premium variable (i.e. Tests A1 and B1)

Where premium (M) is included as a variable in a regression model, the value entered in each case is the 100 percent premium or the price of placing the whole contract, as specified by the Minimum Deposit appearing on the slip. The Minimum Deposit is calculated in practice by multiplying the ceding company's estimated premium income by the excess of loss premium rate percentage.

ii) Tests involving a written premium variable (i.e. Tests C1 and C2)

Written premium variables (MW) are calculated as the 100 percent premium, as described above, multiplied by the percentage of the contract applied for by the underwriter in each case. This percentage represents that applied for by the underwriter. The actual percentage received (the 'signed' percentage) is, in most cases, slightly less than that applied for.

iii) Tests involving a claims variable (i.e. Tests A2, B1 and C2)

The claims variable employed in these tests represents the average £ amount of total claims (including zero claims) that have resulted from each contract case. For example, if no claims occur then the value of the claims variable (C) is zero. If two claims occur, say, one of £100 and another of £200, then C is £150. This form of claims variable is selected in order to approximate a claims situation where only zero or one claims may occur as this is the standard protection provided by an excess of loss contract. (The 'extra' claims employed in the averaging process arise in the underwriter's records from contracts which, although terminated by the occurrence of a claim, were immediately reinstated). The analysis thus centres on the original excess of loss agreement between ceding company and reinsurer rather than on the more complex situation which arises when reinstatements are taken into account.

iv) Tests involving a premium loading variable (i.e. Tests A3, B1 and C2)

Premium loading variables (K) are calculated as follows for each case:

K = M - C - MDIFA

where	M	=	the total contract premium
	C	=	Total claims
	MDIFA	=	M - MEST where MEST is the expected premium for a contract as calculated from the best-fit equation from test A1.

In calculating K, the intention is to isolate that part of the total premium which remains when claims and price variations (see (v) below) are removed. The degree of success of identifying K is, therefore, dependent on the quality of MEST (the best-fitting estimator of M) as arrived at in Test A1. Providing K can be reasonably estimated in these circumstances, it may be regarded as comprising allowances in the premium for reinsurers' expenses, profit and claims variance.

v) Tests involving a price variation variable (i.e. Tests A4, B1 and C2)

The price variations variable employed in the tests is a measure of price variation for contracts with similar slip details in terms of U, L and E. The calculation, as explained in (iv) is:

MDIFA = M - MEST

where	M	=	the total contract premium
	MEST	=	the best estimate (calculated separately on each data sample) of M from contract details from test A1.

The accuracy of MDIFA is, therefore, dependent on the success of the A1 test. If the MEST estimates from the former test are characterised by low statistical significance, no attempt will be made to estimate MDIFA.

6.5 DATA SAMPLES EMPLOYED IN THE PRELIMINARY ANALYSIS

Four data samples (for four sets of tests) are employed in the preliminary analysis which spans the underwriting years 1974 to 1976. The temporary exclusion of 1977 data from the analysis is in order to reserve a data test-bed for possible future tests on identified key relationships (as will be explained further in a later section). The four data samples employed in the preliminary analysis are 1974 data, 1975 data, 1976 data, and 1974 to 1976 inclusive data respectively. From a comparison of results between tests on individual data years, information may be gained on the nature of short-term relationships. These may, in turn, be compared with results of the three-year, or long-term data sample. Differences between long and short-term results will provide information on the degree to which relationships established on short-term samples fluctuate from year to year.

6.6 RESULTS OF THE PRELIMINARY ANALYSIS

The results of the preliminary analysis are now presented along with comments as to their meaning in relation to the overall analysis. Only summarised statistical results are presented in the main text; the more detailed results appear in Appendix C along with a glossary of notations employed in the analysis. Occasionally, reference is made to the more detailed results in the appendix, for example when a single independent variable is seen to explain the greater part of variation of the dependent variable. However, the emphasis of results presentation in the main text is to identify multivariate equations which have satisfied the "r^2 of more than .5" criterion. Results of the tests for each data sample are described under the following headings:

i) Estimation of premium and its components from slip details (A tests);

ii) The relationship of premium to premium components estimated from slip details (B tests);

iii) An underwriter's selection procedure (C tests).

6.6.1 Estimation of premium and its components from slip details (A tests)

Summarised results of the A tests are shown in Table 6.3. The table comprises the coefficients of determination for the best-fit relationships arrived at using the methodology described in Section 6.3. Further information, including the transformations employed in the estimate and the beta coefficients, is provided in Appendix C.

Table 6.1 Coefficients of determination for best-fit estimates of premium and conceptual elements of premium from slip details

TEST	DEPENDENT VARIABLE ESTIMATED FROM SLIP DETAILS	Overall r^2 (Independent variables are U,L, E)			
		1974 data	1975 data	1976 data	1974-6 data
A1	Premium (M)	.5244	.6308	.6742	.5606
A2	Claims (C)	.4485	.5654	.1340	.0571
A3	Premiums loading (K)	.0641	.5883	.2223	.0605
A4	Price variation (MDIFA)	.2537	.5814	.1296	.5294

Interpretations of the results of each of the A tests are now presented.

Test A1 Estimation of premium from slip details

From Table 6.1 it is apparent that, employing the r^2 of more than .5 criterion, the premiums for excess of loss contracts on every data sample can be estimated reasonably from slip details. The greater part of the relationship can be seen (from Appendix C) to be explained by the regression of estimated premium income of the ceding company on actual premium charged for the excess of loss contract. The relationship between these two variables (the excess of loss premium and ceding company's premium income) is non-linear except for one (1976) data sample indicating that, in general, the price for cover increases at a slightly decreasing rate in proportion to ceding company's premium income. The effects of upper and lower excess limits are, as indicated by the results, to raise and lower premiums charged respectively but the significance levels of these relationships are very low.

Test A2 Estimation of claims from slip details

The claims estimate does not share the high degree of statistical significance as found with the premium estimate from slip details. The mean average r^2 for single-year data samples is .3826 and, for the three-year data sample, the r^2 is less than .06. A possible explanation for the relatively higher coefficients of determination for the single-year data samples is as follows: given that a large claim has occurred in the market, the distribution of that claim through the market is determined by the contract details which describe the amount of loss to be allocated to each reinsurer. A moderate statistical fit may, therefore, be achieved between claims and slip details in the short run. In the long run, however, there are a number of such short-run relationships to be taken into account. Since the short run patterns are different on each single-year data sample the level of statistical significance of the long run relationship between claims and slip details is, consequently, low. The conclusion from test A2 is that claims cannot be reasonably estimated from slip details.

Test A3 Estimation of premium loadings from slip details

The mean average r^2 for this test on the single-year data samples is .2916 and, for the three year sample, .0605. These results do not satisfy the r^2 of more than .5 criterion of statistical adequacy. It can be seen from Table 6.1 that the best result arises on a data sample with a high r^2 for the claims to slip details, premium to slip details, and price variations to slip details relationships. The implication is that, where all other conceptual elements of the premium can be reasonably estimated, the loading, calculated as the remainder when all other conceptual elements of the premium are removed, may also be reasonably calculated. This state of affairs is demonstrated, however, by the results of only a single data sample and the possibility that this result is mere coincidence cannot be disregarded.

Test A4 Estimation of price variations from slip details

The r^2 for the mean average of the short-term data samples for this test is .3216 and, for the three-year sample, .5294. By application of the r^2

.5 criterion it would appear that price variations can be reasonably estimated from slip details in the long run but not in the short run. The quality of the premium to slip details relationship, which plays an integral part in the calculation of MDIFA, appears to have no consistent influence on the results. The conclusion from test A4 is that price variations may be reasonably estimated from slip details U, L and E in the long run but not in the short run.

6.6.2 The relationship of total premium to estimated premium components (B tests)

Summarised results of the B tests are shown in Table 6.2. The table presents the results of including C, K and MDIFA in a single equation for estimating M on each data sample. Individual coefficients of determination for the relationship of individual variables in the equations and also the overall r^2 results are shown in the table. Further information on the relationships, including betas, is available in Appendix C. The results of the B tests prompt the following comments:

Table 6.2 Coefficients of determination for best-fit estimates of premium from conceptual components of premium

INDEPENDENT VARIABLE	r^2			
	1974	1975	1976	1974-6
Claims (C)	.0258	.0014	.0844	.0214
Premium loading (K)	.0238	.0002	.0557	.0278
Price variation (MDIFA)	.7687	.9588	.3629	.7754
Overall	.7719	.9588	.4583	.7766

Note: The dependent variable in each case is total premium (M).

Test B1 Estimation of premium from claims

From Table 6.2 it can be seen that no reasonable relationship exists between claims and premium charged on any data sample.

Test B2 Estimation of premium from premium loadings

From Table 6.2 it can be seen that no reasonable relationship for the premium to premium loading exists on any data sample.

Test B3 Estimation of premium from price variations

This test examines the relationship between actual premium and variations around an average premium which is estimated from slip details. The results demonstrate high association between variables; the mean average r^2 for the one-year data samples is .6968 and, for the three-year data sample, .7754. From the detailed results in Appendix C, it can be seen that, in most cases (the 1976 data sample is an exception), the relationship is linear, with larger-sized premiums being associated with proportionately larger price variations. The conclusion from Test B3 is that, overall, a reasonable relationship exists between premium size and likely variations in premium around an average estimated from slip details for the data samples under analysis.

6.6.3 An underwriter's selection portfolio (C tests)

The summarised results of the two tests on an underwriter's selection procedure are shown in Tables 6.3 and 6.4. Table 6.3 lists the coefficients of determination achieved when regressing slip details against the underwriter's written line. Table 6.4 presents the results achieved by regressing the total premium, claims, premium loadings and price variations against the underwriter's written line. The results prompt the following comments:

Test C1 Estimation of written premium from slip details

Correlation coefficients on all data samples for both bivariate and overall relationships of slip details to written premium are less than .5. The conclusion is that premiums written by the underwriter cannot be reasonably estimated from slip details.

Table 6.3

Coefficients of determination for best-fit estimates of written premium (MW) from slip details (U, L, and E)

INDEPENDENT VARIABLE	r^2			
	1974	1975	1976	1974-6
U	.2411	.0013	.1768	.0047
L	.2226	.0156	.1550	.0083
E	.3999	.3302	.1293	.2715
Overall	.4411	.3429	.2166	.2789

Note: The dependent variable in each case is written premium (MW).

Table 6.4

Coefficients of determination for best-fit estimates of written premium (MW) from premium (M) and premium components (C, K, and MDIFA)

INDEPENDENT VARIABLE	r^2			
	1974	1975	1976	1974-6
M	.9231	.7296	.5609	.7646
C	.0021	.0060	.0100	.0027
K	.0207	.0041	.0065	.0101
MDIFA	.2840	.3842	.1791	.2988
Overall	.9322	.7390	.5850	.7713

Note: The dependent variable in each case is written premium (MW).

Test C2 Estimation of written premium from total premium and its components

From Table 6.4 a strong relationship is apparent between premium written by the underwriter and the total premium available on the contract. All other bivariate relationships listed fail to reach the r^2 of more than .5 level. From the full equations in Appendix C it is apparent that, for the written premium to total premium relationship, the underwriter writes premium in decreasing proportion to the size of total premium available on the contract.

6.6.4 Summary of results of the preliminary analysis

A summary of the main results from the preliminary analysis is provided in Table 6.5 and Figure 6.5. Table 6.5 presents the evidence provided by A, B and C tests and illustrates the relationships which are shown to exist at a statistical association level requiring an r^2 of at least .5. From the figure it can be seen that the best relationships are as follows:

i) Contract details to total premium

ii) Total premium to written premium

iii) Total premium to price variations as measured from an average price estimated from slip details

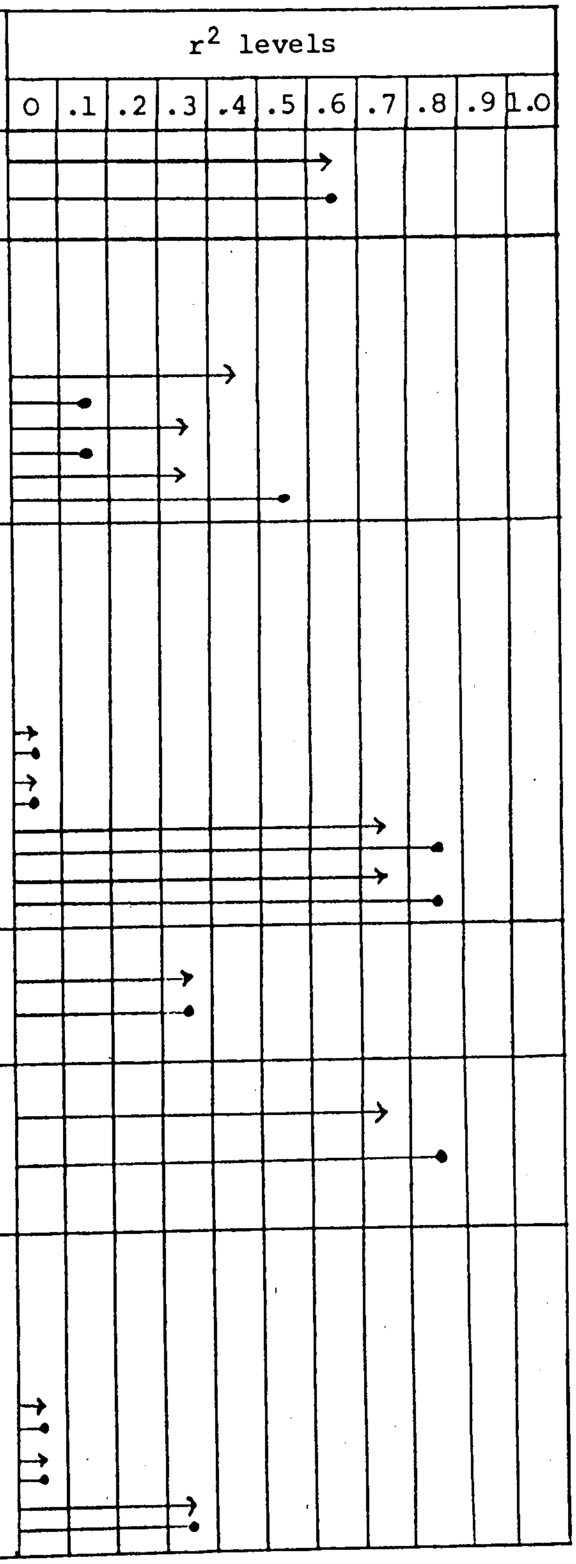
6.7 IMPLICATIONS OF THE RESULTS OF THE PRELIMINARY ANALYSIS

6.7.1 Implications for the use of slip details as approximators for risk processes

From tests A2, A3 and A4 it is apparent that slip details do not provide approximations for underlying risk processes above an r^2 level of .3 of statistical association. In isolation, this result would simply imply that any attempt to estimate risk processes on the basis of excess of loss slip details would be weakly founded. The importance of the result increases, however, when viewed in relation to the result of test A1 which shows that premium can be reasonably estimated from slip details. A situation is therefore shown to exist where premium can be estimated from slip details which do not reflect the underlying risk situation. A conclusion may therefore be drawn that although slip details are not suitable for estimating underlying risk processes they are used, nevertheless, as a guide to establishing premiums.

Table 6.5 Summary of results from the preliminary analysis

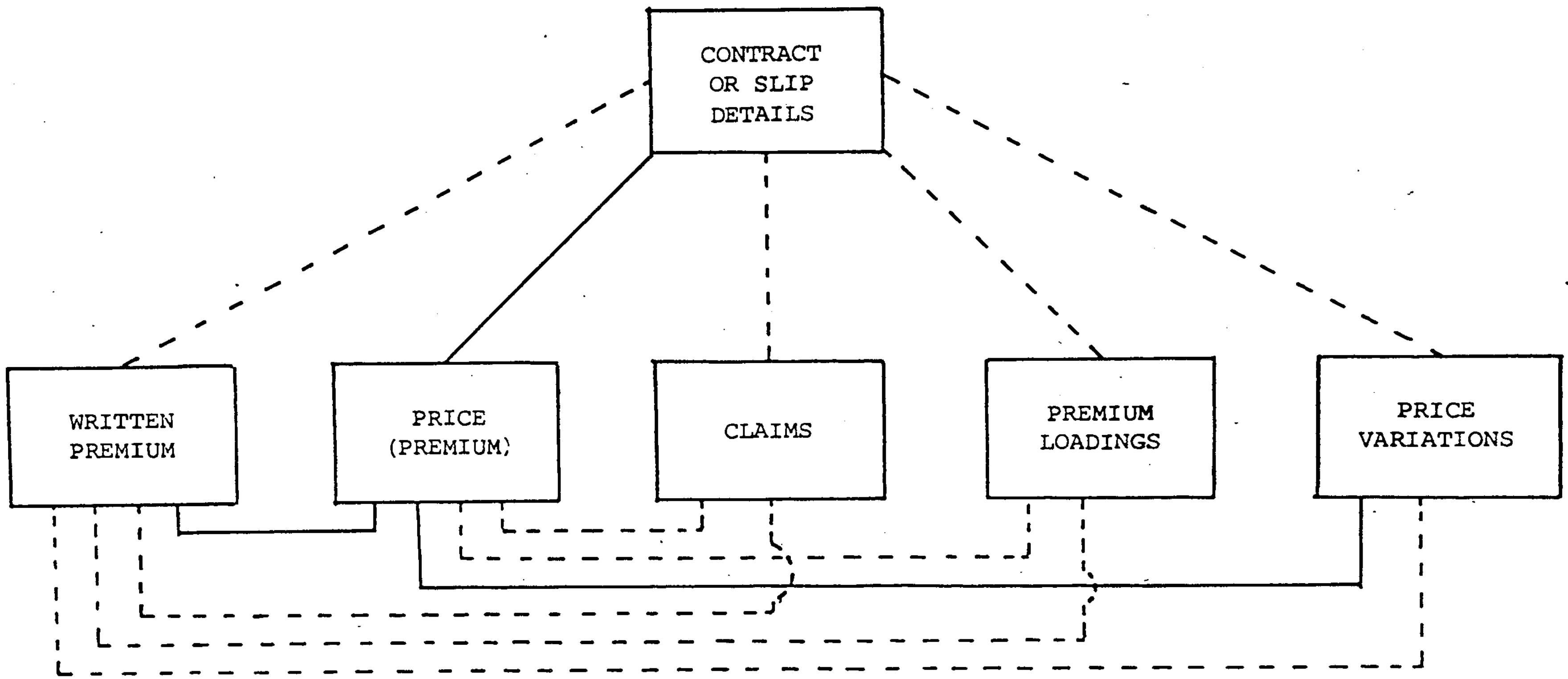
TEST	Evidence sought
A1	Can premium be estimated from slip details?
	Can conceptual elements of premium be estimated from slip details?
A2	(a) Claims
A3	(b) Loadings
A4	(c) Price variations
	Can premium be shown to comprise conceptual allowances for claims, loading and price variations?
B1	(a) Claims
B2	(b) Loadings
B3	(c) Price variations
B4	(d) Overall
C1	Can the test underwriter's written line be estimated from slip details?
C2 (i)	Can the test underwriter's written line be estimated from the premium for a contract?
C2 (ii)	Can the test underwriter's written line be estimated from the conceptual components of premium?
	(a) Claims
	(b) Loadings
	(c) Price variations



Key ———→ denotes mean average r^2 for one-year data samples

———● denotes r^2 for three-year data sample

Figure 6.5 Relationships established in the preliminary analysis



Key

- - - - relationships tested

——— strong relationships ($r^2 \geq .5$) calculated from mean average of single year data samples

Note: Relationships illustrated in the figure are those established for short-term (1 yr.) data samples. In the long run (3 yr.), an additional contract details to price variations relationship holds.

6.7.2 Implications for the application of portfolio theory to excess of loss underwriting problems

The implication of results from the preliminary analysis is that application of a statistical or risk type of portfolio control would not be possible in practice due to the low significances attached to estimates of certain premium components, namely the expected claims and loading elements. From the results of regressing these elements of premium against total premium in a single multivariate equation the evidence suggests that the conceptual view of premium construction (as illustrated in Figure 4.9) would not work in practice. The only element which, conceptually, comprises part of premium and holds a good relationship with total premium is the allowance for price variation. This element is calculated, however, not by actuarial methods but from deviations around an average price calculated on the basis of slip details (which, by tests A2, A3 and A4 are shown not to approximate the risk situation). A conclusion may therefore be drawn that, due to problems of measurability of underlying risk processes, the application of insurance portfolio theory to excess of loss reinsurance is very limited on the evidence of tests undertaken in the preliminary analysis.

6.7.3 Evidence supporting the existence of a satisficing model based on slip details for excess of loss pricing decisions

From test A1, strong support for the existence of an excess of loss pricing procedure based on slip details is provided. The procedure would appear market-wide rather than just attributable to a single underwriter. A possible explanation is that, given the high degree of uncertainty which is characteristic of excess of loss business, a satisficing model is employed which might assume that slip details roughly reflect the underlying risk processes (as illustrated in Figure 5.1). The evidence from tests B1 and B2 is, however, that this assumption is unrealistic and that a premium construction method based on this type of risk approximation method would be ill-founded. An alternative explanation is that an understanding of risk process is not a major concern in the rating of excess of loss contracts but that the satisficing model, based on slip details, is. A justification for such an approach could be that any inadequacies of such a pricing method would be shared equally among participating companies and spread widely via the law of large numbers. If, for example, all reinsurers suffered an abnormal series of losses, premiums could be raised across the board but allocated to specific contracts on the basis of slip details. The law of large numbers and the business spreading effect of excess of loss market mechanics would ensure that the majority of firms managed to recoup past losses. This possibility must remain mere conjecture, however, in the absence of further analysis over a longer time period. The evidence from the present study is simply that excess of loss premiums may be reasonably estimated from slip details. Although a cause-effect relationship cannot be proved using regression techniques, the evidence would certainly support one.

6.7.4 Comments on the professional underwriter's portfolio selection procedure

According to the results of the C tests, the test underwriter selects his portfolio, not with regard to a satisficing model based on slip details or on conceptual components of premiums, but in relation to total premium itself. The decision to disregard the use of contract details in the decision process is justified by the evidence from tests A2, A3 and A4 which demonstrate the inability of this type of estimation procedure to approximate risk processes. The fact that conceptual premium components are not observed to be important in the decision process suggests either that such concepts are not used by the underwriter or that he has attempted but failed to estimate them with any degree of success. These alternative explanations for the results of tests C2a and C2b indicate that a portfolio selection procedure based on expected losses and premium loadings is impractical. The decision rule actually employed, however, although apparently acceptable to the test underwriter, is neither easily justifiable in theoretical terms nor reconcilable with the concept of a risk-approximating satisficing model for market pricing procedure. The identified underwriter's decision rule, in simplified terms, is to underwrite small chunks of contracts which have low premiums and large chunks of contracts which are highly priced, without direct regard to risk processes. The high degree of association between the total premium and slip details is not present between written premium and slip details, indicating that, even if a satisficing pricing model exists in the market, the underwriter chooses to ignore all but the end result of this process (the premium itself) in his portfolio selection procedure. One possible reason for the underwriter's selection procedure being influenced to a large extent by the size of total premium for a contract is that his selection procedure is controlled, to some extent, by market custom. For example, a situation could prevail where brokers require that, if an underwriter is to take any part of a contract at all, the amount must be of at least a predetermined percentage. Brokers might prefer such a placing procedure because it would restrict the number of calls made to underwriting rooms. Although this possibility will not be further expanded in the present study it is, nevertheless, a truism that the underwriter is not sole decision-maker on the matter of how much of a given contract he accepts; the excess of loss broker is in a strong position to influence the size of an acceptance. An alternative explanation of custom dictating underwriting procedures is that the test underwriter, from experience, has found a selection procedure based on premium size workable in the past and, hence, suitable for the present. There may, in fact, be no rational justification for the procedure other than that it is a convenient way to make business acceptance decisions.

6.8 FURTHER RESEARCH REQUIRED FOR ANALYSIS OF EXCESS OF LOSS UNDERWRITING STRATEGY

The preliminary analysis has provided useful information on the nature of excess of loss reinsurance pricing, and an example of the sort of criterion which an underwriter might employ in his portfolio selection process. A notable missing characteristic of the factors identified in the analysis as important for excess of loss decision-making is evidence of well-founded attempts to employ risk measures in the process; more attention is paid in practice to the market and its mechanics than to the pure risks entered into on excess of loss contract acceptance. Evidence from test results would support the view that forces of supply and demand centre not on distinguishable risk units but on excess of loss contracts themselves, with no apparent regard for consequences of contract acceptance. Equal fortune is, to a large degree, ensured primarily by the sharing of contracts, the risky consequences of which cannot be foretold. To cope with this environment, the professional underwriter in this study employs a simple rule of participating as much as possible in small contracts and to a proportionately smaller degree in large ones, the size of total premium income being the main measurable attribute on which he bases his decision. From the evidence of the results of the preliminary analysis it would be unfair to criticise this system on the grounds of being unscientific since, due to measurement problems, the scientific approach aimed at measuring, monitoring and controlling risk processes is demonstrated as being impractical. The underwriter's system achieves, at least, a reasonable spread of the business offered by the market which, in turn, assures a reasonable chance of attaining an average performance for the portfolio compared with the portfolio performances of other underwriters in the market. Nevertheless, a prime aim of this study is to identify means by which underwriting procedures for excess of loss reinsurance might be improved. Improvement, here, means better performance for an individual firm operating in the market and requires identification of strategies which will equip the underwriter to improve his ability to achieve objectives. Further research is, therefore, required in order to determine, test and evaluate various strategies aimed at improving the test underwriter's performance. Required further research will be described under the following headings:

1) Information available for formulation of strategy;
2) A suggested basis for strategy;
3) Further information required for evaluation of the suggested strategy.

6.8.1 Information available for formulation of strategy

The information made available from the pilot analysis for strategy formulation is a knowledge of the quality of key underwriting relationships for the period 1974 to 1976 as illustrated in Figure 6.2. Tested relationships include some relating to risk and others relating to market custom. From a knowledge of these relationships it is now required that information likely to be of use for strategic planning be identified. The first step is to identify those aspects of the results which might aid in the underwriter's attempts to meet his objectives which, for the purposes of this study, are defined as certain degrees of profitability and stability for the underwritten portfolio. The relationships established in the preliminary tests will now be considered in terms of how they might be employed in the process of increasing profitability and stability of the underwriter's portfolio. The most useful relationships are as follows:

a) Premium can be estimated from slip details

This information might be employed by the underwriter firstly, in his pricing decisions and secondly (where price has already been set), in his portfolio selection strategy. Knowledge of price in an area of uncertainty might be gainfully employed in business operations. A problem arises, however, when one considers that, although a strategy aimed at, say, charging a high price or selecting only highly priced contracts might increase the level of earned profit, it is possible that portfolio stability might, consequently, suffer. The tests in the preliminary analysis indicate that neither claims nor premium loadings can be reasonably estimated from either premium or slip details. A strategy arrived at selection of high-priced contracts (relative to the satisficing market pricing model) would not, therefore, be prudent unless further information on the propensity of high-priced contracts to incur losses is available.

b) Likely price variations can be estimated

The fact that likely price variations can be reasonably estimated from the total premium for a contract may be useful to underwriters insofar as it provides a measure of the bargaining range for the contract price. An important consideration is, however, that the reason for the price variations is, from the preliminary analysis, still unknown. Do the variations arise by chance, through imperfections in the bargaining process, or because the higher-priced contracts are genuinely considered more likely to incur losses and/or risk? This question must be answered before the fact that price variations can be estimated can be usefully employed in underwriting strategy.

c) Premiums charged bear little relationship to underlying risk processes

This negative result from the preliminary analysis, although of little value for any attempt at a risk-theoretic approach to underwriting strategy, is important insofar as it directs attention away from such idealistic bases for strategy. In the absence of a workable system requiring use of risk measures, an alternative workable system based on alternative criteria must be found. The test underwriter appears to have already discovered a method which, for him, is workable in that his acceptances are related to the total premium on each contract. Although this system is workable, however, it can be evaluated as successful or unsuccessful only by actual, rather than expected underwriting results. It is, therefore, of little help towards a structured, planning approach to underwriting activities. Even if actual performance is acceptable under the test underwriter's system, the reason for it and, consequently, the basis of future attempts to maintain or improve performance remain unknown.

6.8.2 A suggested basis for strategy

Having considered the new information made available by the preliminary analysis, this section suggests a possible basis of underwriting strategy. The reason that only a 'possible' basis of strategy is suggested is that, from the preliminary analysis alone, not all the required information is available. The most important need for further information centres on the variation of price around an average price estimated from contract details, and the incidence of loss associated with contracts priced at and around the estimated price. From the analysis so far, it is apparent that losses cannot be adequately estimated from premium income on a contract due to a high degree of unexplained variance in the regression equations employed. What is now required is a test to discover whether this identified area of uncertainty is "equally uncertain" over the range of prices attributable to contracts with similar slip details. If it can be demonstrated that contracts priced above the average (as estimated from slip details) are no more likely to incur loss than those priced below it, a source of underwriting strategy would be identified: i.e., by writing more business priced above the market average for contracts with given contract details (and vice versa) an underwriter would improve his relative market position. For complete success, this underwriting strategy would require the following:

1) that contracts priced above an expected market price estimated from slip details are no more likely to incur loss or loss variance than those priced below it;

2) the non-existence of factors preventing the underwriter from achieving desired levels of participation in the various contracts;

3) a method for evaluating consequences to the portfolio from varied degrees of participation in the range of available contracts.

Given all the above idealistic requirements and conditions, the way would be open for an underwriter to plan and achieve super-normal profits compared with those of other underwriters in the market. The strategy followed to achieve this result would be to write the required amounts of premium with due consideration to the extent to which contracts are overpriced (compared with the expected price as estimated from slip details). Since, by 1) above, all similar contracts are deemed to have equally desirable or undesirable consequences, the limiting factor in the selection decision process would not be a measure of variance established from a consideration of risk processes but, rather, a measure of portfolio spread calculated with regard to the size and number of acceptances. The degree to which the underwriter wishes to participate in individual contracts would be determined, not according to risk processes, but with regard to the degree of estimated profitability relative to similar contracts in the market. The intention of the underwriting strategy would be to construct a portfolio, not to desired risk measures of profitability and stability, but, rather, to desired levels of relative profitability and stability as compared with the market average. Even if condition 1) could be shown to apply (which is very unlikely) it is apparent that the strategy which has been described is of a very crude nature insofar as no attempt is made to estimate likely losses incurred on contracts written; the only advantage of the system is that the underwriter employing the strategy is likely to achieve a more satisfactory business performance than his rivals simply because he would receive relatively larger amounts of premium for similar amounts of contract participation.

The purpose of describing the basis of an idealistic underwriting system at this stage, based on implications of the preliminary analysis, is to identify, in advance, constraints on expectations for a more practical system which might be devised. The main limitation is that any underwriting system devised from the information available from this study will, at best, be able to improve relative performance of a portfolio. No new knowledge has been found so far in this investigation which could lead an underwriter to achieve, more successfully, specified absolute levels of risk and/or return on his excess of loss portfolio.

6.8.3 Further information required for evaluation of the suggested strategy

Further information is now required to evaluate the degree to which the conditions and requirements listed in 6.8.2 can be met. Each of these conditions and the methods available for producing the required information on the extent to which they hold are considered below:

a) <u>The degree to which contracts priced above expected price (as estimated from slip details) are more risky than those priced below it.</u>

The first problem is to construct a test from which to discern whether or not contracts priced above an expected price (estimated from slip details) are more likely to incur loss or loss variance than those priced below it. If a regression technique was employed for this purpose, the equation would comprise price variation as the dependent variable and claims, and claims variance as the independent variables. In practice, however, this equation is not easily constructed since some method of attributing loss variance on the whole portfolio to individual contracts would need to be devised. This approach is doomed to failure in the absence of either homogeneous risk groups on which the required estimates could be made or a satisfactory method for estimating the contribution to total portfolio loss and loss variance expectations from the heterogenious characteristics of individual cases. An alternative approach would be to organise the data into several groups according to the degree to which actual price departed from the expected price, and then to regress claims (as a percent of total premium) against premium income for each group. Results of the tests would provide, for each data group, a claims estimator and its associated variance. By a comparison of the results of the separate data groups, a rough measure of the likely loss percentages and loss variances attributable to contracts priced in various ranges of deviation from the expected price (estimated from slip details) could be achieved.

A further possibility would be a sensitivity analysis on a set of simulated portfolios comprising contracts selected in varying amounts according to the degree by which they depart positively and negatively from the expected price estimated from slip details. By monitoring simulated portfolio results (in a series of tests) from participating in contracts priced above and below the expected price as estimated from slip details, the desired information would be gained. A simulation approach will be pursued in this study.

b) <u>The degree to which the underwriter can participate in desirable contracts</u>

The degree to which an underwriter can participate in any single contract lies somewhere between zero and one hundred percent. There are likely to be forces in existence which prevent him from achieving his desired level of participation in a contract, such as the willingness of the broker to allow more than a certain percentage of his client's indemnity to rest with a single underwriter. The brokers' needs to spread indemnity over a wide range of underwriters is two-fold. Firstly, in the event of a large claim, it reduces the possibility that the claim cannot be met because individual reinsurers are unable to release the required monetary sums. Secondly, it allows personal contact and an opportunity to establish goodwill for future placements when they arise. There are, therefore, two constraints on the extent to which an underwriter may participate in desirable business, namely the size of the contract and the degree to which the broker will release business to him. These constraints would require inclusion in a test of the suggested basis for underwriting strategy.

c) A method for evaluating consequences of various strategies

For evaluating the consequences of various strategies, a method is required which does not rely solely on measures of probabilistic expectation for units of comparison. One possibility is the development of an average measure of market performance against which alternative strategies could be evaluated on the basis of simulated results. Another possibility, also requiring a simulation approach, is to compare simulated performances under a series of different strategies. This latter procedure might lead to identification of an optimum strategy which, although not predictable in absolute terms of loss expectations, would at least ensure a higher chance of underwriting success. Before an identified optimum strategy could safely be applied in real underwriting operations it would, however, be necessary to conduct further tests. Examples of further tests which might be considered include repeated simulations on a number of different data samples and attempts to predict relative results. From consideration of a set of results from a series of such tests, the likely consequences of employing the identified optimum strategy in future underwriting operations could be more satisfactorily evaluated.

6.9 PROPOSAL FOR CONSTRUCTION OF AN UNDERWRITING MODEL FOR TESTING PORTFOLIO SELECTION STRATEGIES BASED ON ANALYSIS OF SLIP DETAILS

For further examination of the implications of the preliminary analysis, it would be useful to form a characterisation or model of the relevant section of the real world on which various underwriting strategies can be tested. Since the sample of excess of loss business chosen for the analysis is small relative to the whole market, the evidence generated from this modelling process could not be held out as conclusively representative of all excess of loss underwriting operations. Rather, as Donaldson (1971, p. 73) describes the contribution to knowledge from the modelling process, the intention is to provide 'a serious hypothesis as to intelligent business practice'. The role of the underwriting model is to provide a dynamic framework of analysis or test-bed in which the full implications of various underwriting strategies may be evaluated. The evaluation will thereby be carried out with regard to simulated but realistic results rather than on the basis of hypothesised risk-processes. In this kind of setting, the possible consequences of strategies, in terms of profitability, stability and market penetration, can be expressed in an explicit fashion. The excess of loss simulation model and the results achieved from a series of tests form the subject of Chapter 7 of this thesis.

CHAPTER 7

SIMULATION OF UNDERWRITING STRATEGIES

7.1 INTRODUCTION

The preliminary analysis provides two useful facts about the excess of loss underwriting process; namely that, for the data samples under analysis, the test underwriter's propensity to underwrite can be estimated with reasonable accuracy, as also can price for the contracts in the sample. So far the analysis has provided no firm evidence of a risk-theoretical basis for the estimation of either price (which is shown to be a function of contract details and not of underlying risk processes) or the underwriter's selection procedure (which is shown to be a function of contract price). It would, therefore, seem appropriate to conduct further analysis of the data and the relationships established so far in order to generate information likely to be of use in the theoretical modelling or practical process of excess of loss underwriting. The approach taken to gain this information is to conduct a series of simulation experiments and sensitivity tests geared towards identifying factors important for underwriting success (such as risk-taking and profitability levels) and then to evaluate results for their consistency or variability between data samples. There are two key problems on which the simulation and sensitivity procedures should cast light, and these are:

1) What are the relative potential benefits to the portfolio from business priced at, above and below the expected price calculated by the contract details estimation procedure? The answer to this question, via relative measures, will be used to determine whether variations in price are justified in terms of loss potential or whether they are merely chance variations around an expected price.

2) Can a basis for underwriting strategy be devised from a knowledge of the nature of excess of loss business priced at and around the price expected from analysis of contract details? The answer to this question will determine the degree to which identified relationships are useful for planning underwriting activities in the excess of loss market.

The following procedure is followed with a view to solving the above problems:

a) A model of the excess of loss underwriting process is presented as a vehicle for generating and testing various simulated underwriting strategies.

b) A comprehensive set of simulations are performed on data samples selected from the 1974-76 period.

c) The results of the above simulations are analysed and their implications for excess of loss underwriting planning noted.

d) Conclusions from c), above, are taken into account as the basis of an optimum underwriting strategy to be simulated on 1977 data. The underwriting results from this strategy are compared with those achieved by a practising underwriter selecting from the same set of contract opportunities.

It is hoped that this procedure will provide a useful insight into the workings of the excess of loss reinsurance market of interest to both academic and market practitioners alike, since it combines an analysis of a previously unmeasured market phenomenon with a serious attempt to provide an acceptable basis for underwriting planning.

7.2 OBJECTIVES OF THE SIMULATION PROCEDURES

Before embarking on a description of the underwriting model to be employed in the simulation and sensitivity test procedures, it is useful to state, more formally, the use to which the model will be put. The purpose of the simulations is to test the consequences of various underwriting strategies on short-term (1 year) and long-term (3 year) data samples in as realistic a way as possible, given the quality of the relationships established in the preliminary analysis. The role of the underwriting model in this process is to emulate the activities of a real underwriter in a controllable and structured manner and to enable analysis of causes (underwriting decisions) and effects (consequences to the portfolio). In the modelling process it is, therefore, necessary to introduce measurable objectives against which simulated performances can be compared. In setting these objectives it is necessary to compromise specified (but hypothetical) objectives of the modelled portfolio with the practical objectives of business practice. Having developed a model which represents, within the bounds of available information and expediency, the real world of excess of loss underwriting, the simulation procedure can commence. The objectives of the simulation programme are as follows:

1) To investigate further the relationships established in the preliminary analysis;

2) To model the test underwriter's decision process for portfolio selection;

3) To generate, test and evaluate underwriting strategies based on the professional underwriter's decision process but adjusted to allow for varying degrees of participation in contracts priced above an expected market price estimated from contract details;

4) To scrutinise results of 3), above, to find a basis for optimum underwriting strategy;

5) To conduct sensitivity tests by varying data samples and elements of underwriting strategy between simulations;

6) To identify and evaluate for planning purposes the most useful aspects of the simulated underwriting strategies;

7) To construct and perform a planning test whereby strategy devised from analysis of "past" data is performed on a "future" (1977) data sample.

8) To compare the results of the planning test with the performance of a professional underwriter on the same (1977) data sample.

7.3 THE EXCESS OF LOSS UNDERWRITING MODEL TO BE USED FOR THE SIMULATION PROCEDURES

The excess of loss underwriting model to be employed in the simulation procedures will now be described under the following headings:

1) Underwriting objectives
2) Measures of performance
3) Constraints on underwriting activity
4) Generation of alternative underwriting strategies
5) Evaluation of alternative strategies
6) Selection of an optimum strategy
7) The data testbed

7.3.1 Underwriting objectives

The objectives to be set for the underwriting model require consideration of relevant theoretical, practical and workable issues. The objectives set should be practical insofar as they would be acceptable to practising underwriters, theoretical in that they allow further interpretation and integration with existing underwriting theories, and workable in the context of being capable of providing meaningful results. To simplify the model it is decided to concentrate the analysis on the underwritten portfolio and its cash flows consisting of premiums inwards and claims outwards, without introducing additional complications such as cash flows generated through investment of reserves, or through operations in foreign currencies. The additional considerations are, in fact, likely to effect underwriting activities but, for the present analysis, their effects must be either assumed to be evenly spread over the sample data or as comprising part of the unexplained variation in the statistical estimates. By centering the analysis on the premium and loss characteristics of the portfolio it is possible to devise a straightforward pair of objectives which compromise all the above requirements. The objectives chosen are, therefore, those of profitability and stability for the excess of loss portfolio. These objectives have the advantages of familiarity in the theoretical literature and practicality insofar as practicing underwriters would regard them as desirable characteristics of a portfolio. The method by which they may be assessed and incorporated in an underwriting model based on relationships established in the preliminary analysis will now be explained.

7.3.2 Measures of performance

Measures of performance are required to compare portfolio results from alternative simulated underwriting strategies. The approach taken is similar to that described by Stone (1975) insofar as statistical measures are applied to cash inflows and outflows. A difference is, however, that, where Stone postulates the measures as being constructed from combinations of individual risk units, the measures to be used in the present analysis are calculated on overall portfolio results from simulated underwriting strategies. The formulae used to calculate the measures of performance are as follows:

a) The measure of profitability

$$PROF = \overline{MW}/\overline{CW}$$

where PROF = the measure of profitability

$\overline{MW}$ = the mean average (£) written premium

$\overline{CW}$ = the mean average (£) claims incurred as a result of premiums written

b) The measure of stability

$$STAB = SDCW/\overline{CW}$$

where STAB = the stability measure

SDCW = the standard deviation of (£) claims incurred

$\overline{CW}$ = as above

Each of the portfolio performance measures is calculated on total underwriting results from specified simulated underwriting time-periods.

The profitability performance measure is calculated by dividing mean-average premiums received ($\overline{MW}$) by mean-average claims incurred ($\overline{CW}$), thus allowing profitability to be measured in ratio form which is convenient for comparison of different sized portfolios. A PROF value of more than one indicates that an underwriting profit has been made and a value of less than one indicates an underwriting loss. The stability measure is calculated as the ratio of the standard deviation of incurred claims to the mean average of incurred claims. It is highly likely that, over the range of data samples, the measures will fluctuate in an unpredictable fashion. Reasons for using these ratio-type measures, given their likely variablility, are as follows: firstly, the measures will be employed for comparison of simulated results from a series of underwriting strategies performed on single data samples thus enabling relative movements to be identified. Secondly, comparison of the relative movements, rather than actual results, is convenient for analysis of underwriting performance betwen different time-periods. For the comparisons, a high PROF value is better than a low one and a low STAB value is better than a high one. The objective of the underwriting strategy is, therefore, to maximise PROF and minimise STAB.

7.3.3 Constraints on underwriting activity

In reality there are many constraints on underwriting activity, some of which were described in Section 6.6.3.6. For modelling purposes, however, it is decided that a single constraint should be employed to represent a measure of the degree to which events conspire against the underwriter and prevent him from participating in contracts to his desired level. The constraint is calculated in terms of the percent participation in excess of loss contracts which the underwriter achieves. The calculation for the degree of participation in contracts in the market is as follows:

$$PAR = \bar{X} + 3SDX$$

where PAR = A relative measure of participation in contracts

$\bar{X}$ = the average fraction participation (or the average written line fraction) in contracts calculated on the whole portfolio

3SDX = three times the standard deviation of X

The constraint is that PAR must be less or equal to 1

or $PAR \leq 1$

In statistical terms, the effect of imposing a constraint of $PAR \leq 1$ is that it reduces the probability of simulated underwriting activity unrealistically including the action of writing more than one hundred percent of a contract in the modelling process to less than one percent. PAR, therefore, provides a constraint on both the underwriter's simulated performance and the capacity of the modelling process to produce unrepresentative results. It is important to note, however, that, as a constraint on underwriting, the choice of 1 as a maximum permissable level for PAR is "representative of" rather than "taken from" the real world. As with PROF and STAB, PAR will be employed as a relative measure of portfolio performance for inter-period comparisons. Should the need arise, an alternative measure could also be constructed to describe the underwriter's participation in the market contracts in terms of a percent attainment of PAR.

7.3.4 Generation of alternative underwriting strategies

To provide a realistic basis for modelling procedures, it is decided to base the generated underwriting strategies on those actually adopted by Mr. Jones, the test underwriter. An important requirement is, therefore, that the test underwriter's underwriting behaviour is susceptible to analytical description. From the results of the preliminary analysis (Test C2) it is apparent that reasonable success in this respect can be expected which would increase confidence in the realism of generated results. Two aspects of the test underwriter's selection procedure are, therefore, included in an equation to serve as a model of the underwriter's propensity to accept portions of excess of loss contracts onto his portfolio; namely, the size of a contract's total premium and the degree to which the actual price deviates from expected price. The expected price, in this context, is that estimated from slip details. The equations employed as bases for generation of underwriting strategies are of the following form:

MW = f(M, MDIFA)

where MW = the £ amount of premium the underwriter is prepared to take onto his portfolio for a single excess of loss contract,

M = the total £ premium for the excess of loss contract under consideration,

MDIFA = the difference between M and the expected contract premium estimated from slip details (MEST).

The full equations to be used for the various simulations are shown in Appendix D. The average r^2 for the equations estimated from the single-year data samples is above .75 indicating a high degree of statistical association. This means that over seventy-five percent (on average) of variation on the underwriter's written line decisions can be explained by their relation to total contract premium and price variations (as estimated from slip details) jointly.

Having modelled, as nearly as possible, the underwriter's propensity to underwrite under normal conditions, the next step is to provide a means of generating further, related, contract-acceptance decisions for use in the simulations. The intention behind generation of further indemnity acceptances is to simulate underwriting strategies which take into account, in varying degrees, the difference between actual contract price and the price that would be expected from analysis of slip details. Strategies to be simulated range, therefore, from those where premium written is small in response to apparent underpricing of contracts (according to the price estimate taking slip details into account), to the opposite end of the spectrum with relatively large written premiums being accepted in response to "apparent overpricing" of contracts. The "apparent overpricing" in this context refers to the degree to which the price estimated from the slip details model (as established in test A1 of the preliminary analysis) exceeds the actual total premium for the contract. By including increasingly larger allowances for this apparent overpricing in the simulated underwriting strategies, a set of results is generated from which the extent to which the price variations can be justified in terms of loss experience may be determined. The equations employed to generate this information are of the following form:

$$MWGEN = (a + bM + cMDIFA)((M/MEST)^{Z})$$

where MWGEN = the generated £ written premium taken onto the simulated portfolio for a single contract

a, b and c = constants (determined by the equations shown in Appendix D for each data sample)

M = the total premium for the contract under consideration

MEST = the estimate of price from slip details (from the A1 tests)

MDIFA = the total premium for the contract (M) minus the expected premium (MEST) estimated from slip details

Z = a numerical value (normally between 0 and 3) which can be varied to generate a range of underwriting strategies

The equation is best explained in two stages. The first part of the right hand side of the equation is a model of the professional underwriter's actual underwriting strategy (as shown in Appendix D). This part of the equation will hereafter be referred to as UPU (the underwriter's propensity to underwrite). The second part of the equation serves to adjust the underwriter's participation in contracts priced above that which would be expected (from an estimation based entirely on contract details) by entering various numerical values for Z. Alternative effects of this procedure on the relative value of the generated written premium (MWGEN) compared with actual written premium (MW) are set out in Table 7.1.

Table 7.1 The effect of generated strategies on premiums written on individual contracts

If	M > MEST	and	Z > 1,	then	MWGEN > MW
If	M > MEST	and	Z < 1,	then	MWGEN < MW
If	M < MEST	and	Z > 1,	then	MWGEN < MW
If	M < MEST	and	Z < 1,	then	MWGEN > MW

The overall effect of the equations is that, the larger is Z, the larger will be the simulated participation in contracts where actual price exceeds the price estimated from slip details, and the smaller will be simulated participation in contracts where the opposite actual to estimated price relationship prevails.

7.3.5 Evaluation of alternative strategies

Evaluation of alternative simulated strategies requires analysis of performance measures and constraints on simulated portfolio results. The effect of changes in claims experience and premiums written on portfolio measures of performance are as shown in Table 7.2.

Table 7.2 The effect of claims incurred, premiums written and claims variation on portfolio performance measures

Nature of change (other factors held constant)	Effect on PROF (= $\overline{MW}/\overline{CW}$)	Effect on STAB (= SDCW/$\overline{CW}$)
Claims ↑	decrease	decrease
Claims ↓	increase	increase
Premiums written ↑	increase	no effect
Premiums written ↓	decrease	no effect
Claims variation ↑	no effect	increase
Claims variation ↓	no effect	decrease

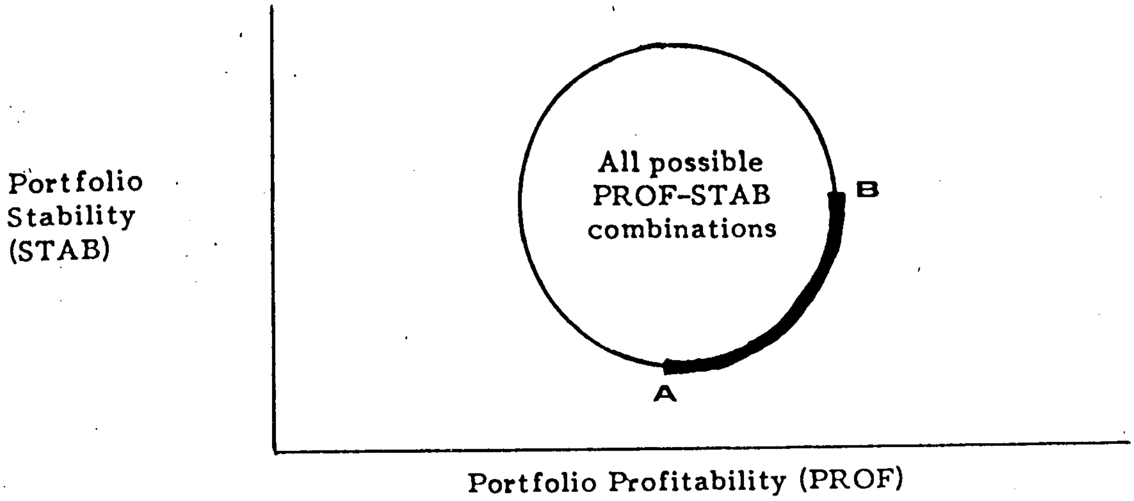
The constraint on underwriting activity (PAR) also requires attention in the performance evaluation process as a limit on the extent to which any particular strategy can be followed. A general rule for evaluating strategy is that strategies leading (or intended to lead) to a high PROF and/or a low STAB are more desirable than ones which would have the oppposite effect.

7.3.6 Selection of an optimum strategy

The optimum strategy is identified with reference to PROF, the profitability measure, STAB, the stability measure, and PAR, the constraint on underwriting activity for any set of generated strategies on simulated portfolio performance. The effects of various Z values on PROF and STAB as constrained by PAR is to generate a set of possibilities resembling the efficient frontier concept from portfolio theory. A difference between the present analysis and the portfolio theoretic model is, however, that expected return and variance of return (from portfolio theory) are replaced by deterministic profitability and stability measures achieved by simulation of various strategies on a set of excess of loss underwriting data.

Figure 7.1 represents (purely for illustration) the possible PROF to STAB relationships generated from the simulations as a circle. The Markowitz criterion for selection of the optimum combination of risk (standard deviation) and expected return is that one should accept only "efficient" portfolios. An efficient portfolio is one with a minimum standard deviation for any expected return and/or the highest possible

Figure 7.1 Hypothetical respresentation of efficient portfolios generated by simulation of underwriting strategies



expected return for a given standard deviation. The equivalent efficient set in the present analysis are those possibilities ranging from a minimum STAB for any value of PROF and/or the highest possible PROF for a given STAB (as illustrated by the arc AB in Figure 7.1). All other combinations are either inefficient or unobtainable. Having identified the efficient frontier, however, it is not possible to judge any single point on it as more or less acceptable than any other point without further information on an underwriter's preferences. If further information is available, the optimum strategy can be identified as the one which leads to a point on the underwriter's highest indifference curve's intersection with the efficient frontier. The only information concerning underwriter's preferences from the data available for the present study is a knowledge of past performance which is inadequate in that it is likely to include large elements of coincidence and luck rather than reflect true underwriting preferences.

In the absence of a suitable guide to underwriting preferences, for the present analysis, an alternative to the algebraic optimizing approach is proposed. Rather than attempt to identify a single strategy as likely to result in optimum performance by employing an unfounded surrogate for underwriting preferences, the intention will be to identify a set of strategies which improve on the actual performance of the test underwriter. A range of strategies will therefore be considered as, potentially, leading to optimum solutions. Although this approach departs from the traditional conceptual model it has the practical advantages of testability and potential use for identification of successful underwriting strategies.

For comparisons of performance between simulations (rather than between simulated results and the test underwriter's actual results), a convenient means of identifying the "best" strategy is by reference to the ratio or percentage of PROF to STAB. Since a large PROF is, generally, desirable and a large STAB undesirable (and vice versa), the most efficient portfolio may be selected as the one with the maxmium PROF and the minimum STAB. This approach assumes a utility trade-off in which STAB and PROF have equivalent values to the underwritten portfolio and is therefore not entirely realistic. Since exact utilities for PROF and STAB are not known, however, and would vary between different companies according to individual preferences, the choice of a single utility value for both PROF and STAB provides at least a consistent basis on which results may be presented and analysed. Should the procedure be required for future analysis, where underwriter's preferences are available, a simple weighting system for PROF and STAB would yield the desired results. For present purposes, however, for comparison of performance between simulated strategies, PROF as a percentage of STAB will be the basic measure for identifying optimum performance and PAR ≤ 1 will be the constraint on underwriting activity.

7.3.7 The data testbed

The data for the simulation tests are the same as for the preliminary analysis except for the addition of a 1977 data sample which serves as a testbed for further testing of relationships and strategies established on 1974 to 1976 data. The exact use of the various data samples will be described, along with description of the simulations performed on them, as the simulations are presented. The bulk of the simulation and analysis will be performed on 1974 to 1976 data. From analysis of these results the possible bases for an underwriting plan are considered and then tested on the 1977 data. Special consideration will be given to the practical potential of the simulation procedure as a basis for planning and, in this respect, on the 1977 data sample, a comparison of results is to be made with those achieved by the test underwriter.

7.4 THE SIMULATION PROCEDURE

7.4.1 The simulation method

The method for simulating and recording results of a test underwriting strategy will now be presented. Computer terminology, where used, refers to the S.P.S.S. package run at U.C.N.W. (University College of North Wales), Bangor, by link to U.M.R.C.C. (University of Manchester Regional Computer Centre), on which the simulations were performed. A detailed understanding of computers or the S.P.S.S. system is not, however, a requisite to understanding the method described below. A schematic overview of the procedure is provided in Figure 7.2. Each step in the process will now be described. (For a detailed description of the S.P.S.S. system see Nie, Hull, Jenkings, Steinberger and Bent (1975).)

(1) Select data sample

The data requirements for performing a simulation on any data sample are the same as for the preliminary analysis (i.e. for each case, total premium income (M), written premium income (MW), upper limit (U), lower limit (L), claims (C) and estimated ceding company premium income (E) are required along with logarithmic and reciprocal transformations of each variable).

(2) Enter the slip details price estimator (MEST)

The slip details price estimator (MEST) is entered into the computer in the following form (e.g. for the 1974 MEST):

```
COMPUTE    MEST = 1/(.68541076E-04
                  -(.20862389E - 10*(U))
                  +(17.208432*(RCPE))
                  +(.26096487E-11*(L)))
```

(See glossary in Appendix C for list of terms)

For each data sample, MEST is entered from the best-fit equation of total premium to contract details as established in Test A1 (listed in Appendix C). The equation is then used in the calculation of MDIFA which represents the difference arising when premium estimated from slip details (MEST) is subtracted from actual total premium (M). MDIFA is a key element in the model of the test underwriter's actual underwriting strategy (UPU).

Figure 7.2 The Simulation Procedure

(1) Enter data into computer, i.e. for each case, (all transformations of total premium, written premium, upper limit, lower limit, E.P.I. and claims).

↓

(2) Enter the required slip details price estimator (MEST) as established on each data sample by Test A1 (as shown in Appendix C).

↓

(3) Enter the UPU for the required data year (the equation which models actual underwriting acceptances for the year, as shown in Appendix D).

↓

(4) Enter the underwriting strategy generating equation (MWGEN) for the lowest Z value (0.005).

↓

(5) Perform the simulation and record the simulated average incurred claims ($\overline{CW}$), the standard deviation of claims (SDCW), average premiums written ($\overline{MW}$), average written line percentage ($\overline{X}$) and the standard deviation of the written line percentage (SDX).

↓

(6) Enter next Z value into the underwriting strategy generator (Z values are 0.05, 0.10, 0.15, 0.20, etc.).

↓

(7) Repeat stages (5) and (6) until the underwriting constraint (PAR) is reached i.e. up to the point where $\overline{X} + 3SDX = 1$.

↓

(8) Construct and tabulate portfolio performance measures from data produced by (each) step (5).

As will be explained further in Section 7.4.2., the choice of MEST for any simulation will not necessarily be the one for the data year on which the simulation is performed but may be selected from a past or future data year as part of the sensitivity analysis.

(3) The basis for strategy generation (UPU)

The basis for strategy generation is provided by the best-fit equations for the underwriter's propensity to underwrite (UPU). The UPU equations established on each data sample are listed in Appendix D. An example of the instruction for entering an UPU into the computer is as follows (for 1974 data):

```
COMPUTE    UPU  = 10**(-.15435228E-01
                  +(.72264889*(LOGM))
                  +(.55268750E-01*(LGMDIFA)))
```

(See glossary in Appendix C for list of terms)

The above equation for the 1974 data would succeed in generating, for each contract case, a written premium which reflects the underwriter's propensity to accept premium income in response to both total premium size and the difference between total premium and the expected premium estimated from slip details.

As with MEST, UPU may be exchanged between data samples for purposes of conducting sensitivity analysis.

(4) The underwriting strategy generator (MWGEN)

Having entered the best-fit estimate of the underwriter's actual premium writing strategy (UPU), it is necessary to enter the underwriting strategy generated into the computer. The strategy generator is entered into the computer in the following form:

```
COMPUTE    MWGEN .05 = (UPU)*(((M)/(MEST))**.05)
```

In the above example, the Z value entered in the equation is .05. For each simulated underwriting acceptance the procedure is to enter UPU, the model of the underwriter's acceptances, which is then multiplied by the actual premium for the contract divided by the slip-details estimate of price taken to the power of .05. The procedure is repeated with a series of Z values being inserted into the MWGEN equation, one after another. For example, MWGEN 1.0 employs a Z value of 1 and represents, roughly, the identity equation of the model of the underwriter's actual underwriting strategy. (In fact, a small difference arises at Z = 1 since MEST is not a perfect estimator of M). Each Z value entered into the MWGEN equation produces a stream of generated written premiums which comprise the total premium accruing to the portfolio for the simulated underwriting strategy.

(5) Simulation and recording of key variables

The simulation procedure involves the following stages in order to produce the data from which performance measures may be constructed and analysed:

i) The MWGEN equation and Z value determine the amount of premium to be accepted for each case.

ii) The simulated written line (from (i) above) determines claims incurred for each case. Claims are allotted pro rata as a percentage of simulated contract acceptance.

iii) Total simulated premiums received is calculated by summation of all individual simulated acceptances.

iv) Total simulated claims incurred is calculated by summing claims incurred on all simulated individual cases.

v) Standard deviation of total simulated claims incurred is calculated on the whole portfolio from details of simulated individual claims incurred.

vi) The average simulated written line percentage and its standard deviation are calculated on all acceptances for the simulated portfolio.

The above procedure involves three main elements; namely, a means of determining the size of the simulated written line, a means of apportioning actual claims according to the simulated participation in each contract, and a means of providing measurements of key portfolio characteristics for tabulation of results.

(6) Tabulation of results

The required column headings for a portfolio simulation results table (illustrated in Figure 7.3) are as follows:

Z value; the value of Z entered for the strategy simulation.

Average written premium ($\overline{MW}$); calculated as mean average of all written lines simulated on the portfolio.

Average claims incurred ($\overline{CW}$); calculated as mean average of all claims incurred on the simulated portfolio.

Average percent contract acceptance ($\bar{X}$); calculated from the mean average of premiums written as a percent of total premium for each contract.

Standard deviation of claims (SDCW); calculated on the whole simulated portfolio.

Standard deviation of per cent contract acceptances (SDX); calculated on the whole simulated portfolio.

Profitability measure (PROF); calculated by dividing $\overline{MW}$ by $\overline{CW}$.

Stability measure (STAB); calculated by dividing SDCW by $\overline{CW}$.

Underwriting constraint (PAR); calculated by adding $\bar{X}$ to three times SDX.

The format for a results table, using the above information, is illustrated in Figure 7.3.

Figure 7.3 Format for a Simulation Results Table

Z Value	$\overline{MW}$	$\overline{CW}$	SDCW	$\bar{X}$	SDX	PROF = $\overline{MW}/\overline{CW}$	STAB = SDCW/$\overline{CW}$	PAR = $\bar{X}$ + 3SDX
.05								
.10								
.15								
.20								
.25								
etc								

From analysis of the simulation results table, the following can be ascertained:

a) the maximum Z value for generating an underwriting strategy with PAR $\leq$ 1;

b) those strategies (if any) which improve upon those employed by the professional underwriter.

Analysis of the simulation results follows in the results section of this chapter.

7.4.2 Description of the simulations

Six types of simulations are to be performed in all. The first five types (on 1974, 1975, 1976 and 1974-6 data samples) are concerned with establishing a pattern of likely performance of underwriting results when Z is increased in the MWGEN equations, and then subjecting the results to a form of sensitivity analysis. The final simulation procedure is performed (on 1977 data) in order to test an underwriting strategy (which has been developed by analysis of the results of the previous simulations) against the actual performance of a practising underwriter with an identical set of excess of loss contract opportunities. A summary of the first five simulation procedures ('A' to 'E') is provided in Table 7.3, along with a brief description of the information sought from each. A more detailed description of each simulation and an analysis of results follows in the remainder of this section. The procedure to be followed for the final ('F') simulation procedure will be described after a synthesis of results from the 'A' to 'E' simulations.

7.5 THE 'A' SIMULATIONS

The 'A' simulations are performed on 1974, 1975, 1976 and 1974-6 data samples separately. For each data sample, the respective UPU and MEST for the period are entered into the simulation procedure using a different Z value to produce each set of results. The purposes of the 'A' Simulations are two-fold. Firstly, information is provided on how excess of loss portfolios are affected by increased participation in business which, by estimation from contract details, is priced higher than its expected price. From consideration of the consequent effects on the simulated portfolio,

Table 7.3 A summary of the 'A' to 'E' Simulations to be performed on data from the 1974 to 1976 underwriting periods

Simulation	Data Sample	UPU	MEST	Information sought
'A' i) ii) iii) iv)	1974 1975 1976 1974-6	1974 1975 1976 1974-6	1974 1975 1976 1974-6	The effect on portfolio performance measures brought about by increases in Z.
'B' i) ii)	1975 1976	1974 1975	1975 1976	Variations in results (cf. 'A' Simulation results) from employing an UPU from the previous period in the strategy generating equations.
'C' i) ii)	1975 1976	1975 1976	1974 1975	Variations in results (cf. 'A' Simulation results) from employing MEST from the previous period in the strategy generating equations.
'D' i) ii)	1975 1976	1974 1975	1974 1975	Variations in results (cf. 'A' Simulation results) from employing both UPU and MEST from the previous period in the strategy generating equations.
'E' i) ii) iii)	1974 1975 1976	1974-6 1974-6 1974-6	1974-6 1974-6 1974-6	Variations in results between short-term periods from using a long-term UPU and MEST combination in the generating equations.

further understanding of the risky nature of such contracts is enhanced. Secondly, information is provided on variation in results between different time periods. Analysis of the results will include identification of optimum underwriting strategy on each sample and their degree of fluctuation between underwriting periods. Since the evidence from the preliminary analysis is that claims patterns vary markedly and unpredictably from year to year, four 'A' Simulations are performed on different data samples in order to produce more representative results than would be produced from a single test.

7.5.1 Results of the 'A' Simulations

The 'A' Simulations are performed according to the procedure described in Section 7.4.1. Full tables of results of 'A' Simulations on 1974, 1975, 1976 and 1974-6 data samples are provided in Appendix E. A summary of the results is shown in Table 7.4 which summarises results of the 'A' Simulation tables shown in Appendix E by grouping results under the appropriate PAR ranges. The figures shown in the table represent the mean-average Z, PROF and STAB values achieved in each respective PAR range for each data sample. The table prompts the following comments:

1) The Z values, which determine the degree of participation or penetration into the market, vary to a large degree between data samples;

2) Without exception, increased values of Z (which generates greater participation in contracts priced above the expected prices estimated from slip details) increases portfolio profitability (PROF) in terms of the ratio of premium income to claims incurred;

3) On 1974, 1976 and 1974-6 data samples, the effects of increases in Z are to increase STAB measures of performance, indicating that the simulated portfolio becomes more unstable. On the 1975 data sample, STAB decreases and then increases with increased Z values.

From the above, with the exception of STAB results on the 1975 simulation, it would appear that increased Z values in the equations employed to simulate underwriting strategy have the effect of increasing profitability and decreasing stability. A possible explanation is that the business priced above the estimated price from contract details contains an extra charge for its variance or capacity content. The method by which this high capacity business can be identified, or how the extra charge is apportioned is not, however, apparent from the results of the 'A' Simulations.

7.5.2 Further analysis of the 'A' Simulation results: the search for an optimum underwriting strategy

Having noted the general effects of increased Z values in the MWGEN equations, namely that both PROF and STAB increase, the purpose of this section is to extract from the results factors which bear on the search for optimum underwriting strategies. The strategy-optimising procedure

Table 7.4

Summary of results for the 'A' Simulations

	Mean-average Z value				Mean-average PROF				Mean-average STAB			
PAR Range	1974	1975	1976	1974-6	1974	1975	1976	1974-6	1974	1975	1976	1974-6
GT. 0, LT. .1												
GT. .1, LT. .2	.150		.325	.125	1.0458		3.1671	1.3797	2.9984		4.8597	4.3829
GT. .2, LT. .3	.525	.175	1.075	.325	1.2169	18.4929	3.2670	1.4159	3.1193	5.5258	4.9562	4.5784
GT. .3, LT. .4	.900	.400	1.825	.625	1.4654	22.9865	3.3824	1.4669	3.2751	4.9820	5.1071	5.0358
GT. .4, LT. .5	1.100	.550	2.350	.800	1.6464	26.4858	3.4759	1.5005	3.3697	4.9166	5.2338	5.4484
GT. .5, LT. .6	1.225	.650	2.725	.925	1.7796	28.9013	3.5498	1.5262	3.4334	4.8924	5.3298	5.7983
GT. .6, LT. .7	1.325	.725	3.025	1.00*	1.8981	30.8849	3.6173	1.3083*	3.4863	4.9118	5.4086	6.0255*
GT. .7, LT. .8	1.425	.800	3.300	1.125	2.0273	33.0299	3.6760	1.5584	3.5417	4.9552	5.4771	6.2657
GT. .8, LT. .9	1.5	.900	3.525	1.150	2.1307	34.4755	3.7329	1.5750	3.5854	4.9860	5.5403	6.5134
GT. .9, LT. 1.00	1.55	.900	3.725	1.225	2.2032	35.9216	3.7805	1.5921	3.6160	5.0131	5.5865	6.7676

*Note: Fluctuations in results occur around Z=1 because MEST is not a perfect estimator of M involves maximising profitability and minimising stability according to some preference function (unknown) within the market participation constraint. Although the underwriter's utility function is unknown, it is, nevertheless, possible to list alternative profitability-to-stability combinations available to the underwriter from the simulation results on which he might exercise his preferences. Since the measures of performance have been chosen in such a way that increases in PROF are desirable and increases in STAB are undesirable, a useful measure of efficiency is the ratio of profitability to stability (expressed as a percentage) which is shown for each data sample by degree of market penetration in Table 7.5. The higher is PROF as a percentage of STAB, the more efficient is the portfolio from which the measures are taken. From Table 7.5 it can be seen that, on the single-year data samples, increased PAR is associated with an increase in PROF as a percentage of STAB indicating that, the more business priced above the expected price estimated from contract details is included on the portfolio, the more efficient the simulated portfolio becomes. The opposite effect, however, occurs on the three-year data sample which employs a long-term UPU and MEST in the MWGEN equations. This latter result requires further explanation since it runs contrary to the results from the three one-year data samples which comprise it. Two reasons for this apparent paradox may be put forward. Firstly, there is no reason to expect a direct mathematical link between the short- and long-term results since the MESTs and UPUs are calculated separately on each data sample and are different; the long-term MEST and UPU are not a mathematical combination of the short-term MESTs and UPUs but the best-fitting estimates calculated on the three-year period. A second possible reason for the difference between long and short-term results is the ever-present possibility of error in simulated results which could occur, in the present example, because UPU and MEST are not perfect estimators of the underwriter's acceptance procedure and contract price respectively. The degree to which the simulation procedure departs from the real-life situation is shown in Table 7.6 which presents PROF as a percentage of STAB for each data sample at various stages of the strategy-modelling process. The first row of Table 7.6 shows the actual PROF as a percent of STAB achieved by the test underwriter. The second row in the table shows the result of modelling the actual performance with an UPU from which a set of underwriting results can be generated. It can be seen that differences between actual and modelled performance are over 30 percent on 1976 and 1974-6 data samples. A further cause of differences between actual and simulated results occurs when the M to MEST ratio is

Table 7.5 PROF as a percentage of STAB for the 'A' Simulations

PAR range	PROF as a % STAB			
	1974	1975	1976	1974-6
GT. 0, LT. .1	-	-	-	-
GT. .1, LT. .2	34.88	-	65.17	31.48
GT. .2, LT. .3	39.01	334.66	65.92	30.93
GT. .3, LT. .4	44.74	461.39	66.23	29.13
GT. .4, LT. .5	48.86	538.70	66.35	27.54
GT. .5, LT. .6	51.83	590.74	66.60	26.32
GT. .6, LT. .7	54.44	628.79	66.88	21.71*
GT. .7, LT. .8	57.24	666.57	67.12	24.87
GT. .8, LT. .9	59.43	691.45	67.38	24.18
GT. .9, LT.1.0	60.93	716.55	67.67	23.53

Note * Fluctuations in results occur around Z=1 because MEST is not a perfect estimator of M.

Table 7.6 PROF as % STAB at various stages in the modelling and simulation procedures

	PROF as % STAB			
	1974	1975	1976	1974-6
Actual Performance	33.03	244.33	43.84	38.89
Model of actual performance (UPU)	33.62	276.78	31.41	27.62
MWGEN at Z = 1	46.67	766.69	21.71	22.91
Max. Z with PAR $\leq$ 1	60.93	716.55	67.67	23.53

included in the strategy generating procedure. The MWGEN equation is

$$MWGEN(Z_i) = (UPU)*(((M)/(MEST))**i)$$

and, since MEST is not a perfect estimator of M, a proportionate change occurs between modelled actual results and the MWGEN results when Z=1 (the identity). These changes can be calculated from Table 7.6. Where PROF as a percentage of STAB for MWGEN at Z=1 is greater than for the UPU model of actual performance, it means that the average M is proportionately greater than the average MEST for that particular data sample (and vice versa). These differences directly affect the simulated written premiums which, in turn, affect the incurrence of simulated losses. From Table 7.6 it is apparent that, for the 1975 data sample in particular, even though the coefficients of determination for MEST and UPU are .63 and .73 respectively, a large change in PROF as a percentage of STAB occurs between modelled performance and simulated performance at Z=1.

The final row of Table 7.6 shows the maximum possible PROF as a percentage of STAB generated by the simulation procedure which still remains within the maximum PAR limit. It can be seen that, despite differences between reality and modelled underwriting strategy, a higher PROF as a percentage of STAB is achieved for each of the single-year simulations than is achieved from either actual or modelled actual performance. The results of the 1974-6 simulation remain contrary to the results of the single-year simulations. A further breakdown of the PROF to STAB relationship at various stages in the modelling process is available in Table 7.7.

7.5.3 Implications for optimum underwriting strategy from 'A' Simulation results

The intention is to decide on an optimum indemnity strategy on the basis of all simulation results. At this stage, it is useful, however, to consider how an optimum strategy might be identified from the results of the 'A' Simulations alone. The assumptions which are necessary to evaluate strategy on the basis of this information are as follows and are necessary because of the limited scenario represented by the 'A' Simulations:

i) Past loss trends will continue in the future
ii) MEST is known for the future
iii) Optimum strategy takes into account profitability and stability but not portfolio size

Table 7.7 Further breakdown of the PROF as a percentage of STAB relationship at various stages in the modelling and simulation procedures

Actual Performance				
	1974	1975	1976	1974-6
PROF	.9793	11.4639	1.3663	1.5992
STAB	2.9649	4.6920	3.2961	3.6475

Model of actual performance (UPU)				
	1974	1975	1976	1974-6
PROF	.9964	15.4576	3.1289	1.3699
STAB	2.9639	5.5847	4.8420	4.3607

MWGEN performance at Z=1				
	1974	1975	1976	1974-6
PROF	1.5501	(39.0342)	3.2581	1.3083
STAB	3.3215	(5.0912)	4.9428	6.0255

MWGEN performance at Max Z with PAR $\leq$ 1				
	1974	1975	1976	1974-6
PROF	2.2032	37.5107	3.8131	1.5979
STAB	3.6160	5.0581	5.6181	6.8534

Note: Bracketed numbers indicate values which are not attainable at Z=1 without exceeding PAR $\leq$ 1 constraint

All these assumptions are unrealistic given the dynamic and unpredictable nature of the excess of loss reinsurance market, but are necessary in order to consider 'A' Simulation results as a basis for planning. By making these assumptions it is possible to determine optimum strategy on the basis of available results. Two scenarios on which optimum underwriting strategy might be based can be considered: firstly, on the basis of evidence from the single-year 'A' Simulations and secondly, on the basis of evidence from the three-year simulation.

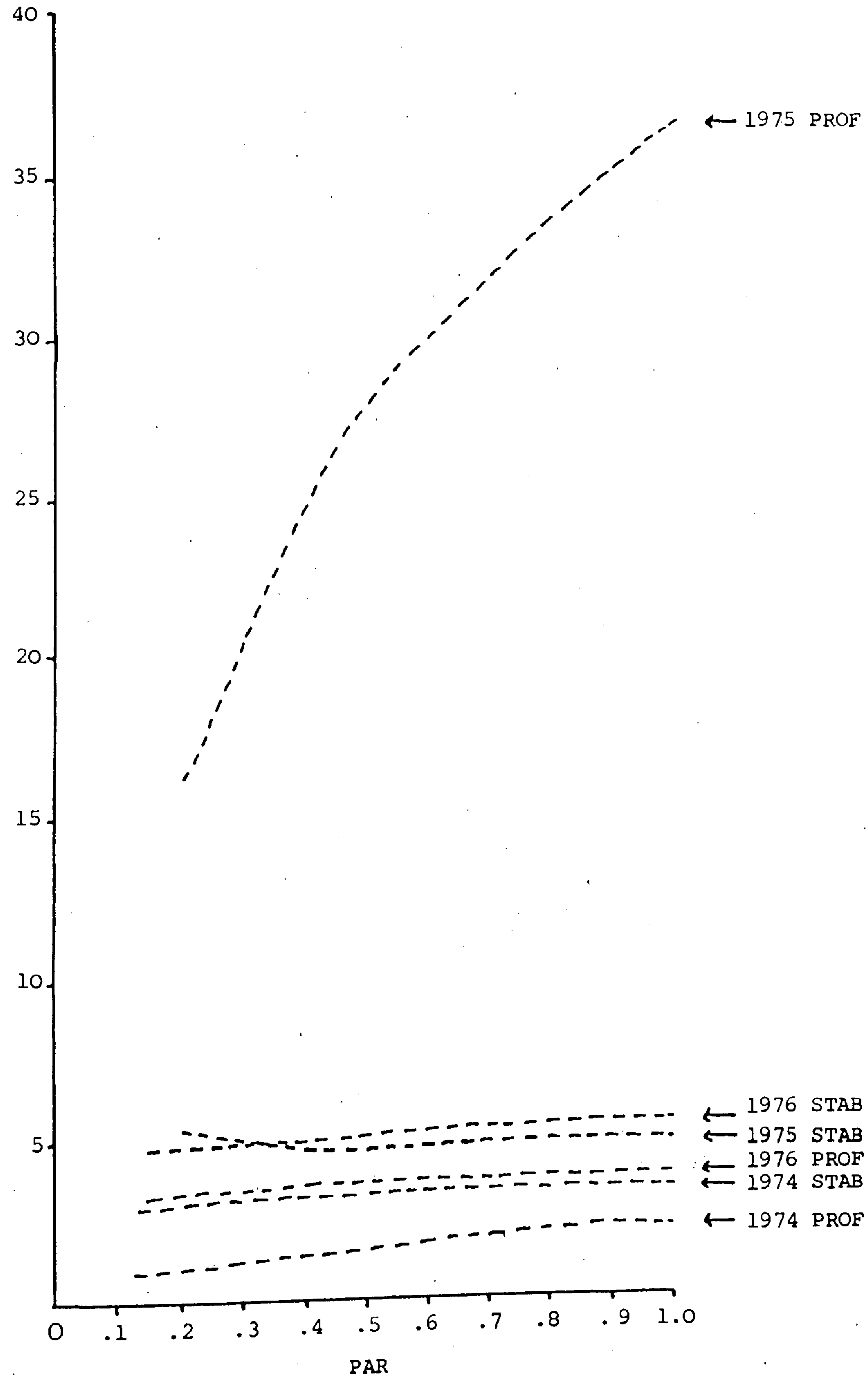
7.5.3.1 Implications for optimum underwriting strategy from the short-term (1 year) 'A' Simulations

A graphical representation of portfolio PROF and STAB measures at various level of market participation for the single-year 'A' Simulations is shown in Figure 7.4. From the graph, which takes co-ordinates from Table 7.4, it can be seen that both PROF and STAB increase with the level of market penetration as the underwriter includes more and more business priced above the expected price (estimated from slip details) onto the portfolio. The selection of an optimum underwriting policy, given the assumptions listed above, is essentially a problem of selecting a PROF to STAB ratio acceptable to the underwriter and then determining the level of market participation required to achieve it. One set of figures which might be employed in such analysis are provided by Table 7.8. The data comprising the table are mean-average performance measures from the 1974, 1975 and 1976 'A' Simulations. From column C of the table, which lists the mean average PROF as a percentage of STAB for each PAR range, a hypothetical underwriter might pick upon a PROF to STAB percentage which satisfies his risk-return preferences and enter the corresponding Z value into the MWGEN equation as the basis for his optimum underwriting strategy. It is likely that, since the PROF to STAB percentage increases with Z, the underwriter would choose an optimum strategy which attempts to participate in the market to the highest possible degree within the constraint provided by PAR.

7.5.3.2 Implications for optimum underwriting strategy from the long-term (3 year) 'A' Simulation

The 1974-6 'A' Simulation provides an alternative planning basis to that provided by arithmetic combination of the results of the three single-year 'A' Simulations. Unlike the results of the single-year simulations, the three-year simulation does not provide a PROF to STAB ratio which

Figure 7.4 Graphical representation of portfolio profitability (PROF) and stability (STAB) for the single-year 'A' Simulations at various levels of market penetration (PAR)



Source of data: Table 7.4

Table 7.8 Data from which optimum strategy may be identified from the results of the single-year 'A' Simulations

PAR range	A	B	C	Z̄
GT. 0, LT. .1	-	-	-	-
GT. .1, LT. .2	-	-	-	-
GT. .2, LT. .3	7.6589	4.5338	169	.5917
GT. .3, LT. .4	9.2781	4.4575	208	1.0417
GT. .4, LT. .5	10.5360	4.5067	233	1.3333
GT. .5, LT. .6	11.4102	4.5519	250	1.5333
GT. .6, LT. .7	12.1334	4.6022	264	1.6917
GT. .7, LT. .8	12.9111	4.6580	277	1.8417
GT. .8, LT. .9	13.4464	4.7039	286	1.9583
GT. .9, LT.1.0	13.9684	4.7385	295	2.0580

Key

Column A = Mean-average PROF for 1974, 1975 and 1976 at various levels of market participation from the 'A' Simulations

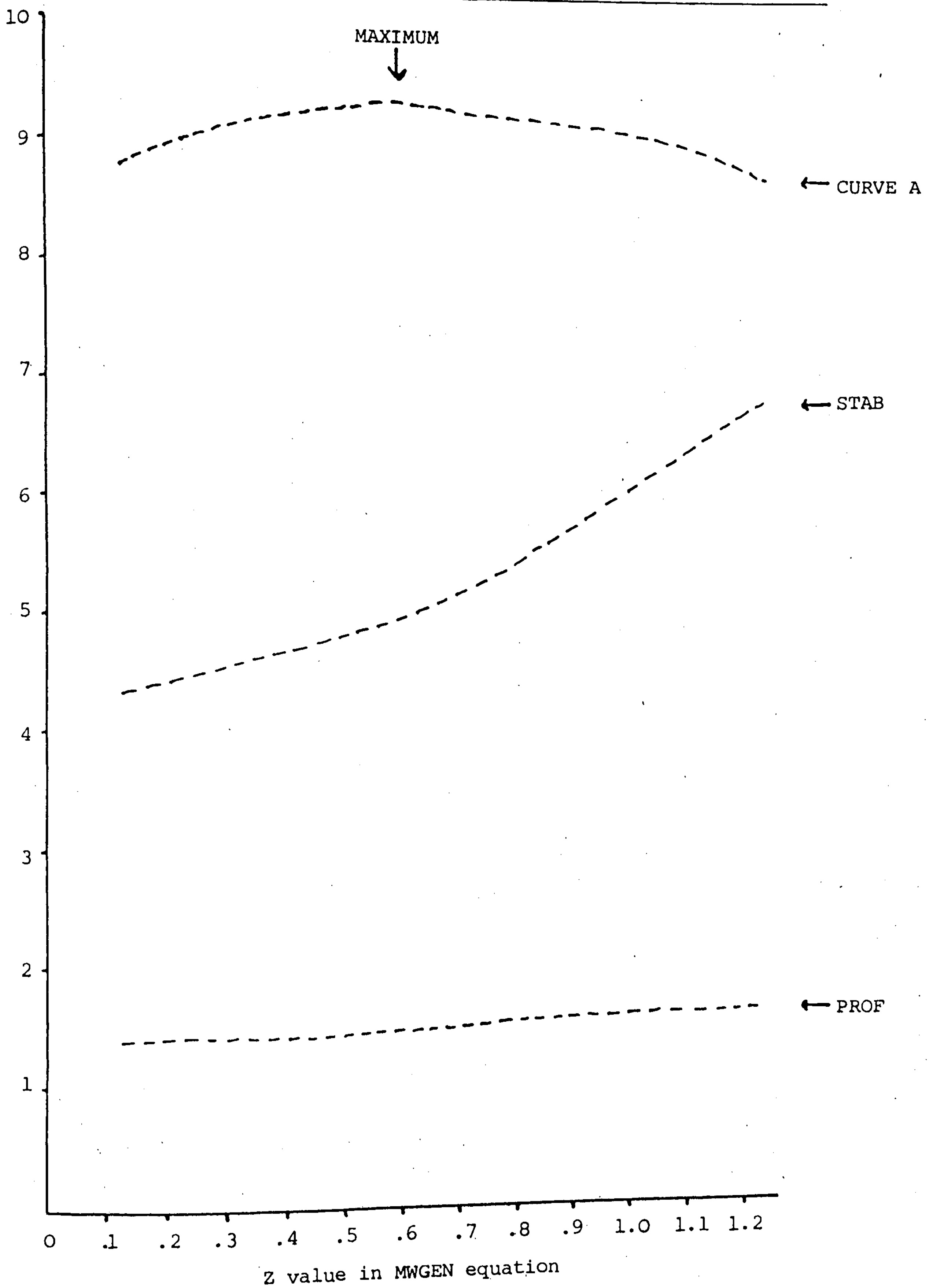
B = The corresponding mean-average STABs

C = A as a percentage of B

$\bar{Z}$ = The mean-average Z value required to generate the respective levels of PAR

constantly increases with the size of Z or PAR. Since PROF still increases with market penetration, however, it is likely that an optimum underwriting strategy would be identified as being at the point where maximum PROF is attainable at a maximum acceptable STAB. Without knowledge of the underwriter's preferences or utility trade-off for PROF and STAB, this point cannot be identified. In Figure 7.5, however, an illustration of this sort of optimum policy selection criterion is presented on the basis that the ratio of £ underwriting profit (premiums less claims) to claims variation (standard deviation of claims incurred on the portfolio) should be at a maximum for the optimum strategy. Using this criterion to represent an underwriter's preferences, a clear optimum policy can be identified at Z = .6. There is no certainty, however, that the most efficient ratio of underwriting profit to standard deviation of claims (curve A on Figure 7.5) as used in this

Figure 7.5 Graphical representation of PROF and STAB and the ratio of underwriting profit to standard deviation of claims for the 'A' Simulation on 1974-6 data



Notes: Formula for Curve A

$$A = \frac{(\overline{MW} - \overline{CW}) \times 100}{SDCW}$$

Source of data: Appendix D, Simulation 'A'(iv) example, is the sole determinant of the underwriter's concept of an optimum underwriting strategy; a wide range of optimizing alternatives might be selected according to different preferences. All that can really be said on the basis of evidence from the three-year simulation is that an underwriter with a distinct preference for profit (no matter what the degree of stability) would select his optimum policy at the highest Z value possible and an underwriter whose main concern was to minimise portfolio instability would choose a low Z value (or not enter the market at all); most underwriters would probably select a point somewhere in between.

7.5.3.3 Short-term vs. long-term evidence for underwriting planning

Two alternative planning scenarios are provided by the short-term and long-term simulation results. Short-term evidence suggests an optimum strategy of maximum market penetration using the highest possible Z value in the MWGEN equation, while the long-term simulation results suggest that an optimum policy might be identified at a Z value before the PAR constraint is reached. Our hypothetical underwriter is, therefore, faced with conflicting evidence as to how his strategy should be chosen. It would be useful if it could be easily determined how much of the conflicting information is caused by short-comings of the simulation procedure compared with reality, and how much is caused by genuine differences in short- and long-term market trends. All we know at this stage, however, is that, if the MWGEN equations (which emulate reality as nearly as possible given the quality of available estimators) are applied to actual data, a conflict of evidence for planning occurs from the two sources of information. Given even the unrealistic assumption that future market losses will continue in the same pattern as in the past, our hypothetical underwriter would select a different planning strategy according to which set of information he took into account.

7.6 THE 'B' SIMULATIONS

The 'B' Simulations are performed on the 1975 and 1976 data samples but employ relationships established on the 1974 data sample in addition to those established on the 1975 and 1976 simulations. The purpose of the 'B' Simulations is to evaluate the sensitivity of portfolio results in response to changes in modelled underwriting strategy (UPU). The particular case to be considered is the use of a 'past' UPU on a 'future' data sample while the price estimator (MEST) remains constant and appropriate for the data sample used in the simulation. For the simulations (two), the only departure in procedure from the 'A' Simulations is that the UPU from the previous year is entered into the MWGEN equation rather than a current one. Thus, the MWGEN equations are constructed in the following way for use in the 'B' Simulations:

1) For the simulation on 1975 data: 1974 UPU and 1975 MEST are used.
2) For the simulation on 1976 data: 1975 UPU and 1976 MEST are used.

The results, when compared with those of the 'A' Simulations on the same data samples, will provide us with the answer to the question 'What happens if a current price estimator (MEST) but a different underlying underwriting strategy (UPU) is used as a basis for portfolio selection?' Analytical development of the results is not an intention of the 'B' Simulations since the equations involved in the computations are complex and, with the highly variable nature of the loss patterns on the data samples, tests of accuracy of any developed model would be possible only to a low degree of significance. From comparison of 'A' and 'B' Simulation results, the only information to be gained is a rough idea of how sensitive the MWGEN equations and consequent portfolio results are to the use of an UPU from a previous period. This information would be important to an underwriter considering whether or not to plan the next period's underwriting strategy on the basis of current practice. The importance of the result lies in its comparison with the results of the 'C' Simulations (see Section 7.7) which experiment with the use of an MEST from a 'past' underwriting strategy on 'future' data. From a comparison of 'B' and 'C' Simulation results it is possible to answer the question 'What is most important in deciding underwriting strategy - to select the correct UPU or the correct MEST?' An insight into the consequences of employing a non-current UPU or MEST is also provided by the comparison of 'B' and 'C' Simulation results.

7.6.1 Results of the 'B' Simulations compared with the results of the 'A' Simulations

The results of the 'B' Simulations are summarised and compared with the results of the 'A' Simulations in Tables 7.9, 7.10 and 7.11. Tables 7.9 and 7.10 compare the results of 'A' and 'B' Simulations on 1975 and 1976 data samples respectively, and Table 7.11 presents the results for both years in terms of percentage changes in results which occur because an UPU from the previous year is entered into the MWGEN equations. The resultant

Table 7.9 Comparison of 'A' and 'B' Simulation results on 1975 data

PAR Range	'A' Sim. Z (a)	'B' Sim. Z (b)	a - b	'A' Sim. PROF (c)	'B' Sim. PROF (d)	c - d	'A' Sim. STAB (e)	'B' Sim. STAB (f)	e - f	'A' Sim. PROF as % STAB (g)	'B' Sim. PROF as % STAB (h)	g - h
GT. 0, LT. .1												
GT. .1, LT. .2												
GT. .2, LT. .3	.175	.125	.050	18.4929	17.1949	1.2960	5.5258	5.4009	.1249	334.66	318.37	16.29
GT. .3, LT. .4	.400	.325	.075	22.9865	20.8691	2.1174	4.9820	5.0852	-.1032	461.39	410.39	50.00
GT. .4, LT. .5	.550	.500	.050	26.4858	24.5948	1.8910	4.9166	4.9350	-.0184	538.70	498.37	40.33
GT. .5, LT. .6	.650	.625	.025	28.9013	27.5514	1.3499	4.8924	4.9033	-.0109	590.74	561.19	29.55
GT. .6, LT. .7	.725	.700	.025	30.8849	29.4783	1.4066	4.9118	4.9130	-.0012	628.79	600.01	28.78
GT. .7, LT. .8	.800	.750	.050	33.0299	30.7881	2.2418	4.9552	4.9257	.0295	666.57	625.05	41.52
GT. .8, LT. .9	.850	.825	.025	34.4755	32.7703	1.0752	4.9860	4.9507	.0353	691.45	661.93	29.52
GT. .9, LT. 1.00	.900	.875	.025	35.9216	34.1725	1.7564	5.0131	4.9775	.0356	716.55	686.54	30.01

Table 7.10 Comparison of 'A' and 'B' Simulation results on 1976 data

PAR Range	'A' Sim. Z (a)	'B' Sim. Z (b)	a - b	'A' Sim. PROF (c)	'B' Sim. PROF (d)	c - d	'A' Sim. STAB (e)	'B' Sim. STAB (f)	e - f	'A' Sim. PROF as % STAB (g)	'B' Sim. PROF as % STAB (h)	g - h
GT. 0, LT. .1												
GT. .1, LT. .2	.325	.625	-.030	3.1671	3.2715	-.1044	4.8597	4.8204	.0393	65.17	67.87	-2.70
GT. .2, LT. .3	1.075	1.650	-.575	3.2670	3.4367	-.1697	4.9562	4.9437	.0125	65.92	69.52	-4.00
GT. .3, LT. .4	1.825	2.372	-.547	3.3824	3.5762	-.1938	5.1071	5.0984	.0087	66.23	70.14	-3.91
GT. .4, LT. .5	2.350	2.875	-.525	3.4759	3.6846	-.2087	5.2338	5.2184	.0154	66.35	70.61	-4.26
GT. .5, LT. .6	2.725	3.225	-.500	3.5498	3.7677	-.2179	5.3298	5.3062	.0236	66.80	71.01	-4.21
GT. .6, LT. .7	3.025	3.525	-.500	3.6173	3.8443	-.2270	5.4086	5.3417	.0669	66.88	71.97	-5.09
GT. .7, LT. .8	3.300	3.800	-.500	3.6760	3.9232	-.2472	5.4771	5.2052	.2719	67.12	75.37	-8.25
GT. .8, LT. .9	3.525	4.025	-.500	3.7329	3.9835	-.2506	5.5403	5.3832	.1571	67.38	74.00	-6.62
GT. .9, LT. 1.00	3.725	4.200	-.475	3.7805	4.0347	-.2542	5.5685	5.5496	.0189	67.67	72.70	-5.03

Table 7.11

Percentage changes in Z, PROF, STAB, and PROF as a percentage of STAB achieved by employing a UPU from the preceding data year in the simulation procedure : 1975 and 1976 data testbed

	% change in Z			% change in PROF			% change in STAB			% change in PROF as % STAB		
PAR range	1975	1976	Mean	1975	1976	Mean	1975	1976	Mean	1975	1976	Mean
GT. 0, LT. .1												
GT. .1, LT. .2		+9.23			+3.30			-0.81			+4.14	
GT. .2, LT. .3	-28.57	+53.49	12.46	-7.01	+5.19	-0.91	+2.26	-0.25	+1.01	-4.87	+6.07	+0.6
GT. .3, LT. .4	-14.25	+29.97	7.86	-9.21	+5.73	-1.74	+2.07	-0.17	+0.95	-10.84	+5.90	-2.47
GT. .4, LT. .5	-9.09	+22.34	6.63	-7.14	+6.00	-0.57	+0.37	-0.29	+0.04	-7.49	+6.42	+0.54
GT. .5, LT. .6	-3.85	+18.35	7.25	-4.67	+6.14	+0.74	+0.22	-0.44	+0.11	-5.00	+6.30	+0.65
GT. .6, LT. .7	-3.45	+16.53	6.54	-4.55	+6.28	+0.87	+0.02	-1.24	-0.61	-4.58	+7.61	+1.52
GT. .7, LT. .8	-6.25	+15.15	4.45	-6.79	+6.72	+0.04	-0.59	-4.96	-2.78	-6.23	+12.29	+3.03
GT. .8, LT. .9	-2.94	+14.18	5.62	-3.12	+6.71	+1.80	-0.70	-2.84	-1.77	-4.27	+9.82	+2.78
GT. .9, LT. 1.00	-2.78	+12.75	4.99	-4.89	+6.72	+0.91	-0.71	-0.34	-0.53	-4.19	+7.43	+1.62

effects on Z, PROF, STAB, and PROF as a percentage of STAB will be described in turn.

i) <u>The effect on Z</u>

The results indicate that the change in UPU affects the Z values required in the MWGEN equations to achieve the equivalent levels of market participation. Variation in recorded Z values is more prevalent at low levels of PAR where differences range from -29 percent to +53 percent compared with the result when a current year UPU is used. At the upper PAR range, the variation is reduced to between -3 and +13 percent. The evidence from the two sets of results is that an underwriting strategy which employs a UPU from a past period would be most likely to produce a Z-to-PAR relationship which emulates the scenario represented by an 'A' Simulation at high levels of market penetration.

ii) <u>The effect on PROF</u>

The percentage variation in PROF for the simulations is less than for Z, PROF variation between 'A' and 'B' Simulations being limited to within 10 percent for each PAR range. There is no immediately obvious pattern in the degree of variation in PROF at high and low PAR levels.

iii) <u>The effect on STAB</u>

The effect on STAB substituting an UPU from the previous year is even less than occurs on PROF. A maximum variation of -4.96 percent is recorded in Table 7.11. Again, no easily identifiable trend in variation in STAB at high and low values of PAR is apparent.

iv) <u>The effect on PROF as a percentage of STAB</u>

Differences between 'A' and 'B' Simulation results for PROF as a percentage of STAB on 1975 and 1976 data samples do not follow the same general pattern. On the 1975 data sample the PROF as a percentage of STAB values are smaller from the 'B' Simulation than from the 'A' Simulation on all PAR ranges (differences are between 4 and 10 percent). A positive change of roughly the same magnitude occurs on the 1976 data sample. It appears that a change in UPU does effect results as represented by the PROF to STAB percentages but that the variation need not be undesirable.

7.7 THE 'C' SIMULATIONS

As with the 'B' Simulations, the 'C' Simulations are performed on the 1975 and 1976 data samples and employ relationships established on 1974, 1975 and 1976 data samples. The purpose of the 'C' Simulations is to examine the sensitivity of portfolio results in response to changes in MEST, the price estimator from slip details. The particular case to be considered is the inclusion of a MEST established on the preceding year's data in a MWGEN equation with a current UPU and a current data sample. The simulations follow the same procedure as the 'B' Simulations except that, where the 'B' Simulations employ a past UPU and a current MEST for the respective data samples, the 'C' Simulations employ a current UPU and a past MEST. MWGEN equations are, therefore, constructed in the following way:

1) For the simulation on 1975 data: 1974 MEST and 1975 UPU are used.
2) For the simulation on 1976 data: 1975 MEST and 1976 UPU are used.

The ensuing results, when compared with those from the 'A' Simulations on the same data samples, provide the answer to the question 'What happens if an underwriter changes his underlying underwriting strategy (UPU) for new market conditions but fails to update his estimate (from slip details) of current market prices (MEST)?' The aim is to produce a set of results from changing MEST in the MWGEN equations which can be compared with the changes perceived in the 'B' Simulations. From the comparison, evidence on which element of the MWGEN equation (UPU or MEST) is most important in attaining accurate predictions for planning purposes will be sought. Synthesis of 'C' Simulation results is conducted in two stages: firstly, a comparison of 'A' and 'C' Simulation results in absolute terms and, secondly, a comparison of 'B' and 'C' Simulation results in terms of percentage changes at the various levels of market penetration.

7.7.1 Results of the 'C' Simulations compared with results of the 'A' Simulations

The results of the 'C' Simulations are summarised and compared with those of the 'A' Simulations in Tables 7.12, 7.13 and 7.14. A comparison of percentage changes in results from the 'A' Simulation for 'B' and 'C' Simulations is provided for key variables in Table 7.15. From these tables it is apparent that the effects of exchanging the MEST in a current MWGEN equation for one from the previous period are much greater than when the

Table 7.12 Comparison of 'A' and 'C' Simulation results on 1975 data

PAR Range	'A' Sim. Z (a)	'C' Sim. Z (b)	a - b	'A' Sim. PROF (c)	'C' Sim. PROF (d)	c - d	'A' Sim. STAB (e)	'C' Sim. STAB (f)	e - f	'A' Sim. PROF as % STAB (g)	'C' Sim. PROF as % STAB (h)	g - h
GT. 0, LT. .1												
GT. .1, LT. .2		.175			31.2790			6.0797			514.48	
GT. .2, LT. .3	.175	.550	-.375	18.4929	34.2772	-15.7843	5.5258	6.1091	-.5833	334.66	561.08	-226.42
GT. .3, LT. .4	.400	.900	-.500	22.9865	38.7127	-15.7262	4.9820	6.1548	-1.1728	461.39	628.98	-167.59
GT. .4, LT. .5	.550	1.125	-.575	26.4858	42.8073	-16.3217	4.9166	6.1820	-1.2654	538.70	692.45	-153.75
GT. .5, LT. .6	.650	1.275	-.625	28.9013	46.4256	-17.5243	4.8924	6.2318	-1.3394	590.74	744.98	-154.24
GT. .6, LT. .7	.725	1.375	-.650	30.8849	49.2017	-18.3168	4.9118	6.2610	-1.3492	628.79	785.84	-157.05
GT. .7, LT. .8	.800	1.475	-.675	33.0299	52.0968	-19.0669	4.9552	6.2641	-1.3089	666.57	831.67	-165.10
GT. .8, LT. .9	.850	1.575	-.725	34.4755	55.5886	-21.1131	4.9860	6.2976	-1.3116	691.45	882.69	-191.24
GT. .9, LT. 1.00	.900	1.650	-.750	35.9216	58.1957	-22.2741	5.0131	6.2899	-1.2768	716.55	925.22	-208.67

Table 7.13 Comparison of 'A' and 'C' Simulation results on 1976 data

PAR Range	'A' Sim. Z (a)	'C' Sim. Z (b)	a - b	'A' Sim. PROF (c)	'C' Sim. PROF (d)	c - d	'A' Sim. STAB (e)	'C' Sim. STAB (f)	e - f	'A' Sim. PROF as % STAB (g)	'C' Sim. PROF as % STAB (h)	g - h
GT. 0, LT. .1												
GT. .1, LT. .2	0.325			3.1671			4.8597			65.17		
GT. .2, LT. .3	1.075	.050	1.025	3.2670	1.5258	1.7412	4.9562	3.8130	1.1432	65.92	40.02	25.92
GT. .3, LT. .4	1.825	.175	1.650	3.3824	1.5101	1.8723	5.1071	4.0020	1.1051	66.23	37.73	28.50
GT. .4, LT. .5	2.350	.350	2.000	3.4759	1.4910	2.3313	5.2338	4.2854	.9484	66.35	34.79	31.56
GT. .5, LT. .6	2.725	.475	2.250	3.5498	1.4802	2.0696	5.3298	4.5004	.8294	66.80	32.89	33.91
GT. .6, LT. .7	3.025	.575	2.450	3.6173	1.4728	2.1445	5.4086	4.6796	.7290	66.88	31.47	35.41
GT. .7, LT. .8	3.300	.650	2.650	3.6760	1.4677	2.2083	5.4771	4.8091	.6680	67.12	30.52	36.60
GT. .8, LT. .9	3.525	.725	2.800	3.7329	1.4634	2.2695	5.5403	4.9429	.5974	67.38	29.61	37.77
GT. .9, LT. 1.00	3.725	.850	2.875	3.7805	1.4578	2.3227	5.5685	5.1081	.4604	67.67	28.53	39.14

Table 7.14

Percentage changes in Z, PROF, STAB, and PROF as a percentage of STAB achieved by employing a MEST from the previous data-year in the simulation procedure : 1975 and 1976 data testbeds

	% change in Z			% change in PROF			% change in STAB			% change in PROF as % STAB		
PAR range	1975	1976	Mean	1975	1976	Mean	1975	1976	Mean	1975	1976	Mean
GT. 0, LT. .1												
GT. .1, LT. .2												
GT. .2, LT. .3	+214.29	-95.35	59.47	+85.35	-53.30	16.03	+10.56	-23.07	-6.26	+67.66	-39.32	14.17
GT. .3, LT. .4	+125.00	-90.41	17.30	+68.41	-55.35	6.53	+34.68	-21.64	6.52	+36.32	-43.03	-3.36
GT. .4, LT. .5	+104.55	-85.11	9.72	+61.62	-67.07	2.73	+25.74	-18.12	3.81	+28.54	-47.57	-9.52
GT. .5, LT. .6	+104.00	-82.57	10.72	+60.63	-58.30	1.17	+27.38	-15.56	5.91	+26.11	-50.76	-12.33
GT. .6, LT. .7	+89.66	-80.99	4.34	+59.31	-59.28	0.02	+27.47	-13.48	7.00	+24.98	-52.95	-13.99
GT. .7, LT. .8	+84.38	-80.30	2.04	+57.73	-60.07	-1.17	+26.41	-12.20	7.11	+24.77	-54.53	-14.88
GT. .8, LT. .9	+85.29	-79.43	2.93	+61.24	-60.80	0.22	+26.31	-10.78	7.77	+27.66	-56.06	-14.20
GT. .9, LT. 1.00	+83.33	-77.18	3.08	+62.01	-61.44	0.29	+25.47	-8.27	8.60	+29.12	-57.84	-14.36

Table 7.15

Percentage deviations in results from the 'A' Simulations for 'B' and 'C' Simulations

	% deviation of Z				% deviation of PROF				% deviation of STAB				% deviation of PROF			
	1975		1976		1975		1976		1975		1976		1975		1976	
PAR range	'B' Sim.	'C' Sim.	'B' Sim.	'C' Sim.	'B' Sim.	'C' Sim.	'B' Sim.	'C' Sim.	'B' Sim.	'C' Sim.	'B' Sim.	'C' Sim.	'B' Sim.	'C' Sim.	'B' Sim.	'C' Sim.
GT. 0, LT. .1																
GT. .1, LT. .2																
GT. .2, LT. .3	-29	+214	+53	-95	-7	+85	+5	-53	+2	+11	0	-23	-5	+68	+6	-39
GT. .3, LT. .4	-14	+125	+30	-90	-9	+68	+6	-55	+2	+35	0	-22	-11	+36	+6	-43
GT. .4, LT. .5	-9	+105	+22	-85	-7	+61	+6	-67	0	+26	0	-18	-7	+29	+6	-48
GT. .5, LT. .6	-4	+104	+18	-83	-5	+61	+6	-58	0	+27	0	-16	-5	+26	+6	-51
GT. .6, LT. .7	-3	+90	+17	-81	-5	+59	+6	-59	0	+27	-1	-13	-5	+25	+8	-53
GT. .7, LT. .8	-6	+84	+15	-80	-7	+57	+7	-60	-1	+26	-5	-12	-6	+25	+12	-54
GT. .8, LT. .9	-3	+85	+14	-79	-3	+61	+7	-61	-1	+26	-3	-11	-4	+28	+10	-56
GT. .9, LT. 1.00	-3	+83	+13	-77	-5	+62	+7	-61	-1	+25	0	-8	-4	+29	+8	-58

UPU is exchanged as in a 'B' Simulation. The effects on Z, PROF, STAB, and PROF as a percentage of STAB will now be considered in turn for the 'C' Simulations.

i) <u>The effect on Z</u>

The results indicate that the change in MEST affects the Z value required to achieve the equivalent level of market participation in the 'A' Simulations to a large extent. Percentage changes compared with the 'A' Simulations range from +214 to -95 at low PAR levels and from +83 to -77 at high PAR levels. These percentage differences are higher than those recorded for the 'B' Simulations.

ii) <u>The effect on PROF</u>

Variation in PROF on comparing 'A' and 'C' Simulation results ranges from +85 to -53 percent at low PAR ranges and from +62 to -61 percent at high PAR ranges. It is interesting to note that, on the 1975 data sample, a higher PROF occurs through entering a non-current MEST in the MWGEN equation than when the correct one (as far as can be achieved by regression techniques employed in the preliminary analysis) is used. The explanation is that there are only a few small claims on the 1975 data sample and, by substituting the MEST from the previous period for the current one, a mathematical change occurs which causes more business to be taken onto the simulated portfolio. Since 1975 was a generally profitable year for the account the result of taking on more business is more profit. The lesson to be learned is that variation in PROF caused by changes in MEST can cut both ways - both an unplanned profit or loss (as in the 1976 example) can be made. Factors to be taken into account include the general quality of business offered during the year, and whether the change in MEST causes more or less business to be taken onto the simulated portfolio. For planning purposes, it is apparent that, for control of underwriting strategy, an accurate MEST is highly desirable since changes (even the substitution of the MEST from the immediately preceding period) would cause large variation from planned performance.

iii) <u>The effect on STAB</u>

Variations in STAB between 'A' and 'C' Simulations are smaller than for PROF. An explanation is that, while a non-current MEST reduces the ability of the MWGEN equations to select 'overpriced' business, thus causing variations in profitability, the loss characteristics of the business written remain relatively constant. The picture to be visualised is one where all contracts lead to fairly unstable portfolio results - but some contracts have higher (relative) premiums than others. By introducing an incorrect MEST, the ability to identify the overpriced contracts is affected more than resultant changes in portfolio loss patterns.

iv) The effect on PROF as a percent of STAB

Variation in this performance indicator is caused by the relative variations of PROF and STAB. From Table 7.14 it is apparent that PROF as a percentage of STAB varies a great deal when the preceding year's MEST is introduced into the MWGEN equations. Percentage variations range from +68 to -39 percent at low PAR ranges to +29 to -58 percent at high PAR ranges. The implication for underwriting planing is that a knowledge of MEST for the future underwriting period is very important for achieving a desired PROF to STAB ratio.

7.7.2 A comparison of results from 'B' and 'C' Simulations and the deviations from 'A' Simulation results

Table 7.15 summarises the percentage deviations from the 'A' Simulation results for the 'B' and 'C' Simulations on 1975 and 1976 data samples. The most noticeable feature of the results is that changes in MEST cause much greater variation in results between 'A' and 'C' Simulations than between 'A' and 'B' Simulations. This effect is noticeable for all measures of performance on all data samples. The price estimator (MEST) must be regarded as an important and illusive planning variable which is subject to relatively severe variation, even in the short run.

7.8 THE 'D' SIMULATIONS

The 'D' Simulations are performed on the same data samples as the 'B' and 'C' Simulations. Relationships from 'past' underwriting periods are superimposed on 'future' data samples as in the 'B' and 'C' Simulations. The case to be investigated by the 'D' Simulations is the effects of using both MEST and UPU from a past period on current period data. Thus, the MWGEN equations are constructed as follows:

1) For the simulation on 1975 data: 1974 UPU and 1974 MEST are used.
2) For the simulation on 1976 data: 1975 UPU and 1975 MEST are used.

Having examined the effect of using either an UPU or a MEST from a previous data sample on a future one, the 'D' Simulations provide information on how a combination of both UPU and MEST from one period affect performance in the next period. Evidence is sought on whether UPU and MEST from a previous period used jointly on a future data sample produce better or worse results than when only one MWGEN component from a previous period (that is, either UPU or MEST) is used

7.8.1 Results of the 'D' Simulations compared with results of the 'A' Simulations

The results from the 'D' Simulations are summarised and compared with results of the 'A' Simulations in Tables 7.16, 7.17 and 7.18. A comparison of percentage deviations at various PAR levels for 'B', 'C' and 'D' Simulations from the relevant 'A' Simulations follows in Table 7.19. A summary of maximum and minimum deviations of 'B', 'C' and 'D' Simulation results from 'A' Simulation results is provided in Table 7.20. The effects on the various measures of performance brought about by employing UPU and MEST from a previous period on a future data sample will now be considered in turn.

i) The effect on Z

A consistent variation in Z from comparing 'A' and 'D' Simulation results is not noticeable in the tables. The simulation on the 1975 data sample produced variations of up to 214 percent at low PAR ranges and up to 83 percent at high PAR ranges. On the other hand, the 'D' Simulation results for the 1976 data sample vary only slightly (a maximum variation of 6 percent) from the results of the 'A' Simulation. The only evidence of a trend in Z results produced by the 'D' Simulation is that simulated Z values lie somewhere between the results from the 'B' and 'C' Simulations for the same PAR ranges.

ii) The effect on PROF

The trends of variations in PROF between 'A' and 'D' Simulations for the various PAR ranges are not consistent between data samples. Differences range from between +78 and -92 percent at low PAR ranges to between +55 and -96 percent at high PAR ranges. The extent of this variation is more than the combined variation observed for 'B' and 'C' Simulations for the same PAR ranges.

Table 7.16 Comparison of 'A' and 'D' Simulation results on 1975 data

PAR Range	'A' Sim. Z (a)	'D' Sim. Z (b)	a - b	'A' Sim. PROF (c)	'D' Sim. PROF (d)	c - d	'A' Sim. STAB (e)	'D' Sim. STAB (f)	e - f	'A' Sim. PROF as % STAB (g)	'D' Sim. PROF as % STAB (h)	g - h
GT. 0, LT. .1												
GT. .1, LT. .2		.125			30.6360			6.0877			503.24	
GT. .2, LT. .3	.175	.475	-.300	18.4929	32.9364	-14.4435	5.5258	6.0939	-0.5681	334.66	540.48	-205.82
GT. .3, LT. .4	.400	.850	-.450	22.9865	37.1289	-14.1424	4.9820	6.1525	-1.1705	461.39	603.48	-142.09
GT. .4, LT. .5	.550	1.075	-.525	26.4858	40.8531	-14.3674	4.9166	6.1877	-1.2711	538.70	660.23	-121.53
GT. .5, LT. .6	.650	1.250	-.600	28.9013	44.8366	-15.9353	4.8924	6.2655	-1.3731	590.74	715.61	-124.87
GT. .6, LT. .7	.725	1.375	-.650	30.8849	47.9391	-17.0542	4.9118	6.2833	-1.3715	628.79	762.96	-134.17
GT. .7, LT. .8	.800	1.475	-.675	33.0299	50.6498	-17.6199	4.9552	6.2844	-1.3292	666.57	805.96	-139.39
GT. .8, LT. .9	.850	1.550	-.700	34.4755	52.9852	-18.5097	4.9860	6.2963	-1.3103	691.45	841.53	-150.08
GT. .9, LT. 1.00	.900	1.625	-.725	35.9216	55.7097	-19.7881	5.0131	6.3216	-1.3085	716.55	881.26	-164.71

Table 7.17 Comparison of 'A' and 'D' Simulations on 1976 data

PAR Range	'A' Sim. Z (a)	'D' Sim. Z (b)	a - b	'A' Sim. PROF (c)	'D' Sim. PROF (d)	c - d	'A' Sim. STAB (e)	'D' Sim. STAB (f)	e - f	'A' Sim. PROF as % STAB (g)	'D' Sim. PROF as % STAB (h)	g - h
GT. 0, LT. .1												
GT. .1, LT. .2												
GT. .2, LT. .3	.175	.175	0.00	18.4929	1.5077	16.9882	5.5258	4.0992	1.4266	334.66	36.78	297.88
GT. .3, LT. .4	.400	.425	-0.025	22.9865	1.4829	21.5036	4.9820	4.5193	0.4627	461.39	32.81	428.58
GT. .4, LT. .5	.550	.575	-0.025	26.4858	1.4721	25.0137	4.9166	4.7842	0.1324	538.70	30.77	507.93
GT. .5, LT. .6	.650	.675	-0.025	28.9013	1.4659	27.4354	4.8924	4.9624	-0.0700	590.74	29.54	561.20
GT. .6, LT. .7	.725	.750	-0.025	30.8849	1.4617	29.4232	4.9118	5.0950	-0.1832	628.79	28.69	600.10
GT. .7, LT. .8	.800	.800	0.00	33.0299	1.4593	31.5706	4.9552	5.1831	-0.2279	666.57	28.15	638.42
GT. .8, LT. .9	.850	.850	0.00	34.4755	1.4570	33.0185	4.9860	5.2707	-0.2847	691.45	27.64	663.81
GT. .9, LT. 1.00	.900	.900	0.00	35.9216	1.4549	34.4667	5.0131	5.3572	-0.3441	716.55	27.16	689.39

Table 7.18

Percentage changes in Z, PROF, STAB, and PROF as a percentage of STAB achieved by employing both UPU and MEST from the previous data-year in the simulation procedure : 1975 and 1976 data testbeds

	% change in Z			% change in PROF			% change in STAB			% change in PROF as % STAB		
PAR range	1975	1976	Mean	1975	1976	Mean	1975	1976	Mean	1975	1976	Mean
GT. 0, LT. .1												
GT. .1, LT. .2												
GT. .2, LT. .3	+171.43	0.00	85.72	+78.10	-91.86	-6.88	+10.61	-25.82	-7.61	+61.50	-89.01	-13.76
GT. .3, LT. .4	+112.50	+6.25	59.34	+61.52	-93.55	-16.02	+23.49	-9.29	7.10	+30.80	-92.89	-31.05
GT. .4, LT. .5	+95.45	+4.55	50.00	+54.25	-94.44	-20.10	+25.85	-2.69	11.21	+22.56	-94.29	-35.87
GT. .5, LT. .6	+92.31	+3.85	48.08	+55.14	-94.93	-19.90	+28.07	+1.43	14.75	+21.14	-95.00	-36.93
GT. .6, LT. .7	+89.66	+3.45	46.55	+55.22	-95.27	-20.03	+27.92	+3.73	15.83	+21.34	-95.44	-37.05
GT. .7, LT. .8	+84.38	0.00	42.19	+53.35	-95.58	-21.12	+26.82	+4.60	15.71	+20.91	-95.78	-37.44
GT. .8, LT. .9	+82.35	0.00	41.18	+53.69	-95.77	-21.04	+26.28	+5.71	16.00	+21.71	-96.00	-37.15
GT. .9, LT. 1.00	+80.56	0.00	40.28	+55.09	-95.95	-20.43	+26.10	+6.86	16.48	+22.99	-96.21	-36.61

Table 7.19 Percentage deviations in results from 'A' Simulations for 'B', 'C' and 'D' Simulations

i)

PAR range	% deviation in Z					
	1975			1976		
	'B'	'C'	'D'	'B'	'C'	'D'
GT. .2, LT. .3	-29	+214	+171	+53	-95	0
GT. .3, LT. .4	-14	+125	+113	+30	-90	+6
GT. .4, LT. .5	-9	+105	+95	+22	-85	+5
GT. .5, LT. .6	-4	+104	+92	+18	-83	+4
GT. .6, LT. .7	-3	+90	+90	+17	-81	+3
GT. .7, LT. .8	-6	+84	+84	+15	-80	0
GT. .8, LT. .9	-3	+85	+82	+14	-79	0
GT. .9, LT.1.0	-3	+83	+81	+13	-77	0

ii)

PAR range	% deviation in PROF					
	1975			1976		
	'B'	'C'	'D'	'B'	'C'	'D'
GT. .2, LT. .3	-7	+85	+78	+5	-53	-92
GT. .3, LT. .4	-9	+68	+62	+6	-55	-94
GT. .4, LT. .5	-7	+61	+54	+6	-67	-94
GT. .5, LT. .6	-5	+61	+54	+6	-58	-95
GT. .6, LT. .7	-5	+59	+55	+6	-59	-95
GT. .7, LT. .8	-7	+57	+53	+7	-60	-96
GT. .8, LT. .9	-3	+61	+54	+7	-61	-96
GT. .9, LT.1.0	-5	+62	+55	+7	-61	-96

Table 7.19 (continued)

iii)

PAR range	% deviation in STAB					
	1975			1976		
	'B'	'C'	'D'	'B'	'C'	'D'
GT. .2, LT. .3	+2	+11	+11	0	-23	-26
GT. .3, LT. .4	+2	+35	+23	0	-22	-9
GT. .4, LT. .5	0	+26	+26	0	-18	-3
GT. .5, LT. .6	0	+27	+28	0	-16	+1
GT. .6, LT. .7	0	+27	+28	-1	-13	+4
GT. .7, LT. .8	-1	+26	+27	-5	-12	+5
GT. .8, LT. .9	-1	+26	+26	-3	-11	+6
GT. .9, LT.1.0	-1	+25	+26	0	-8	+7

iv)

PAR range	% deviation in PROF as a % of STAB					
	1975			1976		
	'B'	'C'	'D'	'B'	'C'	'D'
GT. .2, LT. .3	-5	+68	+62	+6	-39	-89
GT. .3, LT. .4	-11	+36	+31	+6	-43	-93
GT. .4, LT. .5	-7	+29	+23	+6	-48	-94
GT. .5, LT. .6	-5	+26	+21	+6	-51	-95
GT. .6, LT. .7	-5	+25	+21	+8	-53	-95
GT. .7, LT. .8	-6	+25	+21	+12	-54	-96
GT. .8, LT. .9	-4	+28	+22	+10	-56	-96
GT. .9, LT.1.0	-4	+29	+23	+8	-58	-96

Table 7.20 Deviations of 'B', 'C' and 'D' Simulation results from those of the 'A' Simulations (Summary)

Simulation	Z			PROF			STAB			PROF as % STAB		
	+ve	-ve	mean	+ve	-ve	mean	+ve	-ve	mean	+ve	-ve	mean
'B'	+53	-29	41	+7	-9	8	+2	-5	4	+12	-11	12
'C'	+214	-95	155	+85	-67	76	+35	-23	29	+68	-58	63
'D'	+171	-	171	+78	-96	87	+28	-26	27	+62	-96	79

Key

+ve = maximum positive deviation (%) recorded on any PAR range

-ve = maximum negative deviation (%) recorded on any PAR range

mean = mean of absolute values of maximum and minimum percent deviations

iii) The effect on STAB

The values of STAB achieved for the 'D' Simulations vary positively from the equivalent 'A' Simulation results on the 1975 data sample but negatively on the 1976 data sample. The deviations are, however, much smaller than those of PROF (maximum positive and negative deviations are +26 and -26 percent respectively). The implication is that the use of both UPU and MEST from a previous period in the MWGEN equations effects profitability to a larger extent than stability, since the ability to identify business priced higher than that expected from the estimate from slip details is considerably reduced by using non-current estimators.

iv) The effect on PROF as a percentage of STAB

Performance as measured by the PROF-to-STAB percentage improves (compared with 'A' Simulation results) on the 1975 data sample but deteriorates on the 1976 data sample. Variation is high (maximum of +62 percent and a minimum of -96 percent) and is caused mainly by the high variation in the PROF element in the performance measure.

7.8.2 Comment on the results of 'B', 'C' and 'D' Simulations compared with the 'A' Simulation results

Having run and tabulated the results of the 'B', 'C' and 'D' Simulations along with their divergences from 'A' Simulation results, the implications for planning aviation excess of loss underwriting are now considered. The most important contribution from the 'B', 'C' and 'D' Simulations to our knowledge of underwriting planning is to be found in their relative divergence from results of the 'A' Simulation which, by making appropriate assumptions, can be deemed the "ideal" planning information (except for inaccuracies incurred through the modelling process). The information from the 'A' Simulations is "ideal" insofar as it represents a model of underwriting alternatives under perfect information where all market prices and losses are known. Assumptions include: that past loss patterns continue into the future; that MEST is known for the underwriting period, and that the underwriter's underlying strategy (UPU) remains constant. From running the 'B', 'C' and 'D' Simulations it is now possible to examine the situation which arises when these assumptions are relaxed. Since the simulations are, essentially, sensitivity tests involving changes in modelled underwriting strategy on a realistic excess of loss data test-bed, the ensuing results do not lend themselves easily to comprehensive analytical development. This characteristic of the results is apparent from the tabulated consequences of the 'B', 'C' and 'D' Simulations. An important consideration is whether or not the two data test bed periods and the three data samples on which the modelled underwriting strategy elements are formed for 'B', 'C' and 'D' Simulations are adequately representative of excess of loss business to lead to quantifiable conclusions to be taken into account in planning. In this respect it would appear that any quantitative estimate of future underwriting performance would have to be accompanied by a very large allowance for error.

In spite of the simulation procedure's failure to provide predictable results, however, a reasonable understanding of the relative performances to be expected from changes of key underwriting strategy variables in the planning scenario has emerged. From the tabulated results (Tables 7.9 to 7.20) the following general observations can be made:

1) When only UPU is substituted (as in a 'B' Simulation), variation in results (compared with 'A' Simulation results) is small.

2) When only MEST is substituted (as in a 'C' Simulation), variation in results (compared with 'A' Simulation results) is large.

3) When MEST, UPU or both MEST and UPU are substituted, Z is more susceptable to variation than PROF, STAB or the PROF-to-STAB percentage.

In other words, for underwriting planning purposes, the most successful application of the simulation method would be achieved if MEST for the planning period is known but, even then, any attempt to use an exact equation to identify optimum strategy would probably fail due to the likely selection of an inappropriate Z value.

7.9 THE 'E' SIMULATIONS

The 'E' Simulations are designed to test the use of identified long-term (3 year) underwriting relationships as a basis for underwriting strategy on short-term (1 year) data samples. Three 'E' Simulations are performed on 1974, 1975 and 1976 data samples respectively using UPU and MEST established on the 1974-6 data sample for each simulation. The procedure for the 'E' Simulations is, in all other aspects, the same as for the 'A', 'B', 'C' and 'D' Simulations. It will be recalled from Section 7.5.3 that the long-term, 1974-6, 'A' Simulation failed to emulate the short-term, one year 'A' Simulations insofar as the PROF to STAB ratio decreased with increases in Z which ran contrary to the short-term simulation results. The 'E' Simulation provides an opportunity to investigate this result further by examining the effects of using long-term relationships for simulations on the three one-year data samples which comprise the long-term (3 year) sample.

7.9.1 Results of the 'E' Simulations compared with results of the 'A' Simulations

The results of the 'E' simulations are summarised and tabulated in Tables 7.21 and 7.22. It is immediately noticeable from the tables that the use of an identical MEST and UPU for each simulation produces opposing

Table 7.21

Results of employing the 1974-6 UPU and MEST as a basis of underwriting strategy on 1974, 1975 and 1976 data samples

	Mean Z			Mean PROF			Mean STAB			Mean STAB AS % of PROF		
PAR range	1974	1975	1976	1974	1975	1976	1974	1975	1976	1974	1975	1976
GT. 0, LT. .1												
GT. .1, LT. .2	.075	.075	.100	.7025	36.0741	1.4362	3.4229	6.0422	3.8954	20.52	597.04	36.87
GT. .2, LT. .3	.350	.300	.350	.7798	36.7878	1.3331	3.4322	6.0660	4.3492	22.69	606.46	30.65
GT. .3, LT. .4	.675	.575	.625	.9048	38.6202	1.2321	3.4133	6.0647	4.8907	26.51	636.80	25.19
GT. .4, LT. .5	.850	.725	.775	.9964	40.0682	1.1841	3.4105	6.0233	5.1920	29.22	665.22	22.81
GT. .5, LT. .6	.975	.825	.875	1.0620	41.6059	1.1554	3.4126	6.0368	5.3900	31.12	689.20	21.44
GT. .6, LT. .7	1.075	.925	.950	1.1443	43.4049	1.1353	3.4178	6.0431	5.5363	33.48	718.26	20.51
GT. .7, LT. .8	1.150	1.00	1.05	1.2019	44.4416	1.1112	3.4240	6.0260	5.7263	35.10	737.50	19.41
GT. .8, LT. .9	1.225	1.05	1.125	1.2649	45.8476	1.0948	3.4328	6.0305	5.8644	36.85	760.26	18.67
GT. .9, LT. 1.00	1.300	1.10	1.175	1.3323	47.2356	1.0847	3.4442	6.0575	5.9545	38.68	779.79	18.22

Table 7.22 'E' Simulation results as percentages of 'A' Simulation results

	Mean Z			Mean PROF			Mean STAB			Mean STAB AS % of PROF		
PAR range	1974	1975	1976	1974	1975	1976	1974	1975	1976	1974	1975	1976
GT. 0, LT. .1												
GT. .1, LT. .2	50.00		30.77	67.17		45.35	114.16		80.16	58.83		56.58
GT. .2, LT. .3	66.67	171.43	32.56	64.08	198.93	40.81	110.03	109.78	87.75	58.16	181.22	46.50
GT. .3, LT. .4	75.00	143.75	34.25	61.74	168.01	36.43	104.22	121.73	95.76	59.25	138.02	38.03
GT. .4, LT. .5	77.27	131.82	32.98	60.52	151.28	34.07	101.21	122.51	99.20	65.14	123.49	34.38
GT. .5, LT. .6	79.59	126.92	30.28	59.68	143.96	32.55	99.39	123.39	101.13	60.04	116.67	32.19
GT. .6, LT. .7	81.13	127.59	31.40	60.29	140.54	31.39	98.04	123.03	102.36	61.50	114.23	30.67
GT. .7, LT. .8	80.70	125.00	31.82	59.29	134.55	30.23	96.68	121.61	104.55	61.32	110.64	28.92
GT. .8, LT. .9	81.67	123.53	29.79	59.37	132.99	29.33	95.74	120.95	105.85	62.01	109.95	27.71
GT. .9, LT. 1.00	83.87	144.44	31.54	60.47	131.50	28.69	95.25	120.83	106.59	63.48	108.83	26.92

movements in results for measures of performance between different data samples. The effects on each measure of performance will now be considered in turn.

i) <u>The effect on Z</u>

The effect on Z is very similar for each of the 'E' Simulations over the various PAR ranges (see Table 7.21). This evidence is encouraging and indicates that, should an optimum strategy be identified on the basis of 'E' Simulation results, it would be possible to construct an exact MWGEN equation with a specified Z value to be tested on a future data sample. The low degree of variation in Z values at various PAR ranges when MEST and UPU are held constant over the various data samples does not run contrary to results from the previous simulations. When one compares the Z values from the three 'E' Simulations (Table 7.22) with those recorded for the respective 'A' Simulations, however, a much higher degree of variation is apparent. Since the respective data samples are identical for both the 'A' and 'E' Simulations this higher degree of variation must be caused by differences in UPU and MEST for the two sets of simulations.

ii) <u>The effect on PROF</u>

The general level of PROF recorded on each of the data samples varies to a large degree. Causes of this variation are firstly, differences in claims intensity between data samples and secondly, the degree to which the 1974-6 MEST approximates the best-fitting MEST for each particular one-year data sample. From examination of the percentage movements ('E' Simulation results expressed as a percentage of 'A' Simulation results as shown in Table 7.22) it can be seen that these variations may produce either more or less profits than were recorded for 'A' Simulations. There are also differences in the trends of PROF results between data samples in relation to increases in PAR ranges. The results are not encouraging for the prospective use of long-term relationships as a basis for planning.

iii) <u>The effect on STAB</u>

STAB results fluctuate between data samples but, in terms of their percentage divergences from 'A' Simulation results, remain within a 20 percent range of variation. This result would seem to support the view that, using the MWGEN equations as a basis for planning, the most important task is to identify the correct MEST for a data sample since resulting simulated portfolio loss patterns, although irregular between periods, remain fairly stable when UPU is exchanged between MWGEN equations.

iv) The effect on PROF as a percentage of STAB

The variations in PROF as a percentage of STAB follow the same trends as the variations in PROF since STAB variation is relatively small in comparison. On the 1974 and 1975 data samples, results increase with increases in PAR while the 1976 results follow an opposite trend. The percentage deviations from 'A' Simulation results are also contradictory between data samples, with variances of up to 80 percent of 'A' Simulation values. The evidence suggests that a target PROF to STAB percentage in an underwriting plan, using a long-term MWGEN equation, would probably not be met during the course of underwriting operations if past long-term relationships were used as key planning variables.

7.10 THE SIMULATION PROCEDURE AS A BASIS FOR PLANNING

7.10.1 Information available from Simulations 'A' to 'E' for formulation of underwriting strategy

At the beginning of this thesis, a prime aim of the research was stated as being to attempt to formulate and test an underwriting strategy which would improve on the results achieved by Mr. Jones, the practising excess of loss underwriter who kindly offered his data for analysis. The stage of the thesis where this intention can receive serious consideration has now been reached: the remainder of this chapter is concerned with collecting together the relevant facts and implications from the preliminary analysis and simulations to arrive at an optimum strategy to be performed on 'future' (1977) data so that the results can be compared with those actually achieved by Mr. Jones on the same set of excess of loss contract opportunities.

A summary of implications from the simulation results is provided in Table 7.23 and a "track-record" of how simulated results of strategies aimed at employing the highest Z value possible, without exceeding the PAR ≤ 1 constraint, compared with the test underwriter's actual performance is provided in Table 7.24.

Before embarking on the final set of simulations, it is worth considering the planning bases which might be used for the formulation of strategy. The most useful information in this respect is that, according to the evidence available, by employing an MEST calculated on a short-term data sample from which the portfolio is to be constructed in conjunction with the highest Z value that is possible in a MWGEN equation without exceeding the

Table 7.23 Summary of implications for underwriting planning from simulation results

SIMULATION	RESULTS
'A'	For short-term (1 year) simulations, increased values of Z in the MWGEN equations lead to more efficient portfolio performance as measured by the PROF to STAB percentage. For the long-term (3 year) simulations the opposite effect occurs.
'B'	When 'B' Simulation results are compared with 'A' Simulation results, large variations in Z values (up to 53 percent) are recorded but all other measures of performance vary less than 10 percent between the two simulation procedures.
'C'	Large differences in results occur between 'A' and 'C' Simulations. Maximum variation for individual measures of performance between the simulations are 214 percent for Z, 83 percent for PROF, 27 percent for STAB and 67 percent for PROF as a percentage of STAB. Variations in PROF were greater than for STAB for all ranges of PAR on all data samples.
'D'	Large differences in results occur between 'A' and 'D' Simulations. The amounts of variation for individual measures of performance are sometimes larger and sometimes smaller than those which occur between 'A' and 'C' Simulation results but consistently larger than those observed between 'A' and 'B' Simulation results.
'E'	The use of long-term MEST and UPU in MWGEN equations on short-term data samples induces variation in both trends and magnitudes of performance measures between simulations.

PAR $\leq$ 1 constraint, it is possible to achieve the greatest efficiency in terms of the PROF to STAB percentage. Whether the UPU part of the MWGEN equation is chosen from either a past or a current underwriting period appears to have only a small effect on this general result. The procedure is, however, beset by two problems which render the information

Table 7.24 A comparison of simulated portfolio performance with actual portfolio performance for the 1974, 1975 and 1976 data samples

PROF as % of STAB	1974	1975	1976
Actual	33.03	244.33	43.84
Modelled Performance	33.62	276.78	31.41
'A' Simulation at PAR = 1	*60.93	*716.55	*67.67
'B' Simulation at PAR = 1	-	*686.54	*72.20
'C' Simulation at PAR = 1	-	*925.22	28.53
'D' Simulation at PAR = 1	-	*881.26	27.16
'E' Simulation at PAR = 1	*38.68	*779.79	18.22

Note: An asterisk indicates instances simulated portfolio performance is more efficient than actual performance in terms of PROF as a percentage of STAB.

less useful for planning purposes. The first problem is that the exact value of Z to be entered in the MWGEN equation to serve as underwriting strategy cannot be specified since the evidence from the simulation procedures is that the Z value at PAR = 1 varies unpredictably between data samples, even when the MWGEN equation for the data sample on which the simulation takes place is used. This means that an exact underwriting strategy in the form of an equation is likely either to fail to reach the PAR = 1 constraint, or exceed it, rendering the results meaningless. A second problem is that MEST is not likely to be known, in practice, for a future period. It would, therefore, be unrealistic to incorporate it in the underwriting strategy formula. On the other hand, the penalty for using an MEST from a previous period, or on the basis of a long-term trend is highly variable results (as is apparent from 'C' and 'E' Simulations). A possible justification for using the correct MEST (i.e. the one which best-fits the data on which the simulation is performed) for the planned underwriting strategy is that the underwriter might know it in advance -either by instinct, or with the help of a constantly updated calculation procedure on a sample of contracts currently under placement in the market. It should be noted, however, that in order to construct a workable plan, it is necessary to know in advance the price pattern in the market for excess of loss contracts with similar contract details. Whether or not this possibility is realistic will be discussed further in the next chapter. For present purposes, however, the assumption that price is known must be made since the simulations show that, if price is not known, highly variable results occur and ideal planned performance is unlikely to be achieved.

7.10.2 Answers to some questions which an underwriter might ask about the implications of Simulations 'A' to 'E'

At this stage, it would seem appropriate to take a look at the implications of the simulation results for practical portfolio composition and planning purposes. Some key questions (Q) and answers (A) are, therefore, listed below. The questions are intended to be of the type a practising underwriter might ask to clarify the implications of the main findings from the simulation, and their relevance to his underwriting activities. The answers provided are based entirely on the results of the simulation experiments described so far in this chapter.

Q1 Does the simulation procedure offer a means of identifying opportunities for improving short-term (1 year) underwriting performance?

A1 Yes. By selecting a MWGEN equation comprising a current MEST, a past or current UPU and the maximum Z value possible without exceeding the PAR = 1 constraint as a basis for portfolio selection, performance (as measured by PROF as a percentage of STAB) is improved in relation to the model of actual performance.

Q2 What happens if the current MEST is not known and, instead, a MEST from a previous period is used in the equation?

A2 The overall effect on results is the same but a great deal of variation between the results achieved and those recorded when a current MEST is used occurs. These variations can be either positive or negative. The implication is that use of a non-current MEST leads to uncontrollable results.

Q3 Would the use of a MEST calculated on market performance over a longer period (3 years) be any better than a MEST calculated on the immediately preceding period for use in the MWGEN equation?

A3 No. The evidence suggests that a MEST calculated on long-term (3 year) data also provides results which differ (positively and negatively) from those to be expected if a current MEST is employed in the MWGEN equation.

Q4 From the simulation results, what is the most important requirement for improving portfolio performance using a MWGEN equation as the basis of portfolio selection?

A4 You must know the current MEST. That is, you must know the formula for estimating the market price of excess of loss contracts from slip/contract details for the period during which underwriting is to take place.

The above questions and answers are not intended to be exhaustive but to provide a resumé of the evidence so far which may be useful for evaluating the model for planning purposes. An important point is that certain assumptions would be required to provide a testable scenario for the simulation procedure as a basis for planning. These assumptions are the subject of the next section.

7.10.3 Necessary assumptions for testing the simulation procedure as a basis for underwriting planning

From the discussion on information available for underwriting planning purposes it is apparent that, without making certain assumptions, the simulation procedure, although proving a useful tool for analysis of past trends, has limited value for planning purposes. It is necessary, therefore, to include the following assumptions in the planning scenario before the simulation procedure can be set up and tested in competition with

Mr. Jones with a view to improving on his performance in terms of the PROF to STAB percentage. The required assumptions are:

1) MEST is known for the underwriting period on which strategy is to be carried out (1977);

2) An exact choice of Z value in the MWGEN equation must be replaced with "the highest Z value possible without exceeding the PAR $\leq$ 1 constraint". This requirement assumes a method of determining, in advance, the maximum Z value which can be achieved without exceeding the PAR $\leq$ 1 underwriting constraint.

It is unfortunate for practical planning purposes that these assumptions must be made since the potential use of the simulation procedure as a practical planning aid is consequently diminished. They are necessary for present purposes, however, in order to test the system under ideal conditions. The possibility of relaxing the assumptions will be considered in the next chapter.

7.11 THE 'F' SIMULATIONS

The 'F' Simulations are performed on 1977 data and represent a series of planning scenarios based on information gained in the preliminary analysis and the 'A' to 'E' Simulations. The following simulations are performed:

'F'i) <u>1976 UPU and 1977 MEST on 1977 data</u>

This scenario represents the planning scenario which, with an expectation of slight variation in results from the evidence so far, would secure a maximum PROF to STAB percentage at PAR = 1. The situation represented is one where an underwriter has maintained his contract selection criteria from a previous period but, armed with a 1977 MEST, intends to include as much business priced above MEST as possible on his portfolio. The procedure to be followed corresponds with that of a 'B' Simulation.

'F'ii) <u>1977 UPU and 1977 MEST on 1977 data</u>

This simulation is performed as a control experiment. The intention is to provide information on what would have happened if the underwriter's performance was firstly modelled and then performed with a range of Z values entered in the strategy generating procedure as in an 'A' Simulation. The results from this simulation provide a useful benchmark for comparisons with other 'F' Simulations.

'F'iii) 1976 UPU and 1976 MEST on 1977 data

This scenario represents a more realistic one than 'F'i), above, since it does not require the assumption that MEST is known for the 1977 period. The price estimator from the previous period is used in its place. It is likely, however, that results from this simulation will deviate to a large degree from the results of 'F' Simulations (i) and (ii). This simulation is included to provide further evidence that this is in fact what happens when an incorrect MEST is included in the formula for underwriting strategy.

'F'iv) 1974-6 UPU and 1974-6 MEST on 1977 data

In real life, the basis for planning is often an analysis of long-term trends and it would, therefore, seem appropriate to test their use once again (long-term relationships formed the basis of the 'E' Simulations) for planning purposes. Again, the scenario is more realistic than those requiring a 1977 MEST but large deviations from 'F'i) and 'F'ii) results are to be expected according to the evidence from the 'E' Simulations.

The four 'F' Simulations and the information sought from each are summarised in Table 7.25.

7.11.1 Results of the 'F' Simulations

The results of the 'F' Simulations are summarised in Tables 7.26 (i), (ii), (iii), and (iv). In these tables, details of each performance measure is shown for the planning scenarios represented by each of the 'F' Simulations. As with the previous simulations, the results for each measure of performance will be discussed in turn.

i) The effect on Z

Relatively smaller amounts of variation are apparent between 'F'(i) and 'F'(ii) results compared with deviations of 'F'(iii) and 'F'(iv) results from 'F'(ii). This effect duplicates results from the previous equivalent simulations on 1974, 1975 and 1976 data samples.

ii) The effects on PROF

Again the results emulate, in general, those of the previous equivalent simulations on 1974, 1975 and 1976 data samples. If the 'F'(ii) Simulation

Table 7.25 — A summary of the 'F' Simulations to be performed on data from the 1977 underwriting period

Simulation	Data Sample	UPU	MEST	Information sought
'F'(i)	1977	1976	1977	Whether or not simulated strategy can produce a more efficient portfolio (measured in terms of PROF as a percentage of STAB) than that achieved by a practicing underwriter)
'F'(ii)	1977	1977	1977	Whether or not the implications of the 'A' Simulations are replicated on 1977 data
'F'(iii)	1976	1976	1977	The degree to which a strategy employing key elements estimated from a previous short-term period causes variations from the ideal situation represented by the 'F'(ii) Simulation
'F'(iv)	1974-6	1974-6	1977	The degree to which a strategy employing key elements estimated on long-term trends causes variations from the ideal situation represented by the 'F'(ii) Simulation

results (the simulation when actual UPU and actual MEST are used in the MWGEN equations) are compared with the results of Simulations 'F'(i), 'F'(iii) and 'F'(iv), it is apparent that only small differences occur when a current UPU is exchanged for the UPU from the previous period, but that any attempt to use a non-current MEST leads to highly variable results.

Table 7.26 Results of the 'F' Simulations

(i)

PAR Range	Mean Z value			
	'F'(i)	'F'(ii)	'F'(iii)	'F'(iv)
GT. 0, LT. .1	-	-	-	-
GT. .1, LT. .2	-	.050	.250	.100
GT. .2, LT. .3	.275	.325	.850	.300
GT. .3, LT. .4	.650	.725	1.450	.475
GT. .4, LT. .5	.875	.975	1.850	.575
GT. .5, LT. .6	1.050	1.150	2.150	.650
GT. .6, LT. .7	1.200	1.275	2.375	.700
GT. .7, LT. .8	1.325	1.375	2.550	.750
GT. .8, LT. .9	1.425	1.475	2.700	.775
GT. .9, LT.1.0	1.500	1.550	2.850	.800

(ii)

PAR Range	Mean PROF			
	'F'(i)	'F'(ii)	'F'(iii)	'F'(iv)
GT. 0, LT. .1	-	-	-	-
GT. .1, LT. .2	-	.5769	.8817	.7073
GT. .2, LT. .3	.6021	.6072	.8515	.7869
GT. .3, LT. .4	.6462	.6560	.8213	.8813
GT. .4, LT. .5	.6764	.6908	.8016	.9544
GT. .5, LT. .6	.7018	.7171	.7871	1.0189
GT. .6, LT. .7	.7250	.7369	.7757	1.0683
GT. .7, LT. .8	.7453	.7535	.7688	1.1232
GT. .8, LT. .9	.7622	.7707	.7621	1.1540
GT. .9, LT.1.0	.7752	.7839	.7557	1.1847

Table 7.26 (continued)

(iii)

PAR Range	Mean STAB			
	'F'(i)	'F'(ii)	'F'(iii)	'F'(iv)
GT. 0, LT. .1	-	-	-	-
GT. .1, LT. .2	-	2.8067	4.3608	2.5563
GT. .2, LT. .3	2.8498	2.8276	4.4807	2.5625
GT. .3, LT. .4	2.9329	2.9225	4.7118	2.5725
GT. .4, LT. .5	3.0242	3.0300	4.9159	2.5821
GT. .5, LT. .6	3.1127	3.1233	5.0680	2.5911
GT. .6, LT. .7	3.1988	3.1971	5.2117	2.5984
GT. .7, LT. .8	3.2762	3.2602	5.3132	2.6065
GT. .8, LT. .9	3.3415	3.3258	5.4002	2.6115
GT. .9, LT.1.0	3.3920	3.3766	5.4866	2.6165

(iv)

PAR Range	Mean PROF as % of STAB			
	'F'(i)	'F'(ii)	'F'(iii)	'F'(iv)
GT. 0, LT. .1	-	-	-	-
GT. .1, LT. .2	-	20.55	20.22	27.67
GT. .2, LT. .3	21.13	21.47	19.00	30.71
GT. .3, LT. .4	22.03	22.45	17.43	34.26
GT. .4, LT. .5	22.37	22.80	16.31	36.96
GT. .5, LT. .6	22.55	22.96	15.53	39.32
GT. .6, LT. .7	22.66	23.05	14.88	41.11
GT. .7, LT. .8	22.75	23.11	14.47	43.09
GT. .8, LT. .9	22.81	23.17	14.11	44.19
GT. .9, LT.1.0	22.85	23.22	13.77	45.28

iii) The effect on STAB

For each 'F' Simulation, STAB values increase with the values of Z in the MWGEN equations. The pattern of variances of 'F'(i), 'F'(iii) and 'F'(iv) Simulation results compared with those of 'F'(ii) follows the trend established on 1974, 1975 and 1976 data samples for the previous simulations.

iv) The effect on PROF as a percentage of STAB

The pattern of differences between results of PROF as a percentage of STAB between the four 'F' Simulations follows that of the other measures of performance. That is, more variation is noticeable between 'F'(ii) and 'F'(iii), and 'F'(ii) and 'F'(iv) results than between those of Simulations 'F'(i) and 'F'(ii). As with the previous simulations, replacing a current UPU in the MWGEN equations with a UPU from a previous period produces only minor differences.

7.11.2 'F' Simulation results compared with an underwriter's actual performance

The relevant 'F' simulations to be compared with the underwriter's actual performance are 'F'(i) and 'F'(ii). Simulation 'F'(ii) represents the situation where both MEST and UPU are known for the 1977 data. 'F'(i) represents a slightly more realistic situation where the 1977 MEST is known but the 1976 UPU is still used as a basis of planning. Evidence from the previous simulations suggest that, using either of these combinations, by entering higher and higher Z values in the MWGEN equation (up to the PAR = 1 constraint) the PROF as a percentage of STAB measure of portfolio performance will increase accordingly. The extent of this increase in PROF as a percentage of STAB on previous simulations on 1974, 1975 and 1976 data (see Table 7.24) was to approximately double the test underwriter's actual PROF as a percentage of STAB results. The 'F' simulations on 1977 data are now put forward as a testbed on which to re-test this apparently consistent phenomenon. The relevant results are presented in Table 7.27 along with the equivalent results achieved on 1974, 1975 and 1976 data samples.

From Table 7.27, it can be seen that, for the 1977 data period, the simulation method fails to improve on the test underwriter's actual performance. This result runs contrary to results from the equivalent 1974, 1975 and 1976 data period simulations. Some causes of this change in simulated performance will now be considered.

Table 7.27 Comparison of PROF, STAB, and PROF as a percentage of STAB at various stages in the simulation process for all single-year data samples

PROF as % of STAB	1977	1974	1975	1976
Actual	38.89	33.03	244.33	43.84
Modelled (UPU)	20.86	33.62	276.78	31.41
Current MEST and UPU at Max. Z with PAR $\leq$ 1	23.22	60.93	716.55	67.67
Current MEST, UPU from previous period at Max. Z with PAR $\leq$ 1	22.85	-	686.54	72.02

PROF	1977	1974	1975	1976
Actual	.8355	.9793	11.4639	1.3663
Modelled (UPU)	.5720	.9964	15.4576	3.1289
Current MEST and UPU at Max. Z with PAR $\leq$ 1	.7839	2.2032	35.9216	3.7805
Current MEST, UPU from previous period at Max. Z with PAR $\leq$ 1	.7752	-	34.1725	4.0347

STAB	1977	1974	1975	1976
Actual	2.1483	2.9649	4.9620	3.2961
Modelled (UPU)	2.8090	2.9639	5.5847	4.8420
Current MEST and UPU at Max. Z with PAR $\leq$ 1	3.3766	3.6160	5.0131	5.5685
Current MEST, UPU from previous period at Max. Z with PAR $\leq$ 1	3.3920	-	4.9775	5.5496

1) <u>The level of statistical significance for MEST is low on 1977 data</u>

From Appendix C it can be seen that the coefficient of determination (r^2) for MEST on the 1977 data sample was .48. This value is smaller than those attainable on 1974, 1975 and 1976 data samples where the r^2s average out at .61. The 'C' Simulations demonstrated the likely results of entering a non-current MEST in the MWGEN equations and the present result suggests that a low significance MEST produces similarly unsatisfactory results.

2) <u>A large difference is noticeable between actual and modelled portfolio selection behaviour of the test underwriter</u>

The coefficient of determination for UPU on the 1976 and 1977 data samples are relatively high (r^2s of .58 and .77 respectively) which, <u>prima facie</u>, might lead one to expect a reasonable degree of similarity betwen actual and modelled performance. Differences between actual and modelled performance, however, are not just dependent on the statistical significance of the UPU equation but also on the quality of the estimators employed in its calculation. The particular estimator which causes the variation is again, MEST, as can be explained as follows. The equation for UPU is of the form

$$UPU = a + b(M) + c(MDIFA)$$

or

$$UPU = a + b(M) + c(M\text{-}MEST).$$

By including a MEST characterised by low statistical significance as an element in the calculation of MDIFA, inaccuracies inherent in MEST become included and compounded in UPU. Despite a high degree of statistical significance for UPU, therefore, when UPU is used as a basis for simulating actual performance, a large difference occurs between actual and modelled performance. A question arises from this observation: namely 'Why include MDIFA in the UPU equation at all since the evidence from the betas is that MDIFA plays only a small part in the test underwriter's portfolio selection decisions?' The answer is that the evidence from the preliminary analysis suggests that the underwriter does pay a certain amount of attention to price variations and by including this observation in the UPU model, the realism of the exercise is increased. Besides, the ability and extent to which the underwriter takes the premium estimate from contract details into account is a central theme of the research. The result is, therefore, important as it stands and implies that, if an underwriter bases his portfolio selection decisions partly on total premium and partly on apparent price variations but cannot adequately determine an expected price, his portfolio results would vary a great deal from the situation where he does have an adequate expected price estimator. A further breakdown of the difference between actual and modelled performance is shown in Table 7.28.

Table 7.28 Actual, modelled and simulated performance at Z=1 for the test underwriter for the 1977 underwriting period

Actual Performance

Mean average premium written (MW) = £3515
Mean average claims incurred (CW) = £4207
Standard deviation of claims (SDCW) = £9038
Mean average % contract participation (X) = .1152
Standard deviation of X (SDX) = .1286
PAR = .5010
PROF = .8355
STAB = 2.1483
PROF as a percentage of STAB = 38.89

Modelled performance (UPU)

Mean average premiums written (MW) = £2204
Mean average claims incurred (CW) = £3853
Standard deviation of claims (SDCW) = £10823
Mean average % contract participation (X) = .0419
Standard deviation of X (SDX) = .0506
PAR = .1937
PROF = .5720
STAB = 2.8090
PROF as a percentage of STAB = 20.36

Simulated performance at Z = 1

Mean average premiums written (MW) = £5560
Mean average claims incurred (CW) = £8007
Standard deviation of claims incurred (SDCW) = 24357
Mean average % contract participation (X) = .0864
Standard deviation of X (SDX) = .1278
PAR = .4698
PROF = .6944
STAB = 3.0420
PROF as a percentage of STAB = 22.83

3) <u>A large difference is noticeable between modelled actual performance (UPU) and MWGEN performance at Z = 1</u>

A second source of variation in results occurs because of the difference between modelled behaviour of the test underwriter (UPU) and the MWGEN equation when Z is set at 1 (the identity equation if perfect estimators were available). The cause of this difference can be identified from consideration of the following equations for calculation of UPU and a MWGEN equation.

UPU = a + b(M) + c(MDIFA)

or UPU = a + b(M) + c(M-MEST)

and MWGEN = (UPU)(M/MEST)

From these equations it can be seen that the cause of the difference is, again, the inadequacy of MEST as an estimator of M. From Table 7.27 it can be seen that PROF as a percentage of STAB does not vary to a large degree between modelled performance (UPU) and MWGEN performance at Z = 1 but that PROF, STAB and the premiums and claims factors on which they are calculated do. By multiplying UPU by (M/MEST), a substantial, but proportional, change occurs in simulated portfolio performance. One adverse effect of the substantial change is that simulated premium income from MWGEN at Z = 1 is more than double the actual or modelled actual premium income. Opportunities for increasing the Z value before the PAR = 1 constraint is reached are, consequently, reduced.

From the above considerations, it would appear that the 1977 'F'(i) and 'F'(ii) simulations fail to produce more favourable results than those achieved by the test underwriter because MEST for the period could not be adequately calculated. It is worthwhile to note, however, that in other respects, the results are consistent with those achieved on 1974, 1975 and 1976 data samples. The most important similarity is that, as expected, the PROF as a percentage of STAB measure increased with increases in the value of Z in the MWGEN equations. The fact that it did not increase enough to out-perform the test underwriter was partly due to the inadequacies of estimators used in the modelling process. A further possible cause of the simulation procedure's failure to improve on the test underwriter's portfolio performance is that the nature of the business available in 1977 was substantially different in relation to variations in price, than on 1974, 1975 and 1976 data samples. On 1974, 1975 and 1976 data samples it was apparent that, by simulating increased participation in business priced above the expected market price (as estimated from slip details), increased profits were earned which more than offset increases in portfolio instability. This phenomenon was also recorded on the 1977 data sample (the 'F'(i) and 'F'(ii) simulations) but the extent of the increases in PROF were not as large compared with increases in STAB as those observed on the earlier data period simulations. The question of whether the main cause of this difference in results occurs because of an inadequate MEST or because the nature of the business in 1977 changed is not easily soluble since it would require the availability of a more satisfactory price estimator from contract details which, unfortunately, is not available.

7.12 SUMMARY OF MAIN FINDINGS FROM THE SIMULATION PROCEDURES

A summary of the main findings from the simulation procedures is now presented to conclude this chapter. These findings, along with evidence from the preliminary analysis, form the basis of the discussion in the next chapter.

1. Using the model, it was demonstrated, on all short-term (1 year) data samples, that the quality of business priced above an expected price (estimated from contract details of business of the same period) was such that it decreased stability but increased profitability of the simulated portfolio. Business priced below the expected price had the opposite effect on simulated portfolio results. Using the PROF and STAB measures of portfolio performance it was shown, on all short-term data samples, that the increase in PROF more than offset increases in STAB when, by simulation, strategies were performed aimed at including more and more business priced above the expected price on the portfolio.

2. On a long-term (3 year) data sample the same effects on profitability and stability were recorded as in 1., above. The ratio of PROF to STAB, however, declined (the opposite effect to that recorded on short-term (1 year) data samples) when, by simulation, more and more business priced above the expected price was included in the portfolio.

3. Three out of four times, the increase in PROF as a percentage of STAB brought about by increases in the value of Z in the MWGEN equations up to the underwriting constraint (PAR) produced a more satisfactory result than that achieved by a practising underwriter (on one-year data samples).

4. The failure on one occasion to improve on the professional underwriter's performance using the simulation procedure was traceable to one or both of the following causes. Firstly, the estimator for estimating price from contract details was less statistically significant than on the other three occasions and secondly, the quality of the business may have changed in relation to the contract details thus causing the PAR = 1 constraint to be reached prematurely.

5. The importance of employing a correct or adequate MEST in the simulation procedure was demonstrated via sensitivity analysis which included exchanging MESTs between simulation strategy generating equations and data samples. Uncontrollable results occur when an incorrect MEST is employed in a MWGEN equation.

6. Given the importance of employing a correct MEST in the simulation procedures, it is essential that, if the procedure is to be used for planning purposes, MEST should be known for the period during which planned underwriting activity will take place. Since planning is, essentially, a forward-looking activity, it is unlikely that the future MEST would be known with accuracy. If a surrogate MEST from past (short-term or long-term) experience is employed as a basis of strategy, highly variable portfolio results (compared with the situation where the correct MEST is used) are likely to be recorded. A tendency for these variations to be favourable when market loss experience is low, and unfavourable when market loss experience is high, was noted.

7. The ability of the model and simulation procedure to produce predictable results is highly dependent on the quality of the statistical estimators available for its construction.

8. The model produces the most predictable results on short-term (1 year) data samples and when estimators are characterised by high statistical significance.

9. Use of a correct UPU is not as essential for generating predictable simulated portfolio performance as is the use of a correct MEST.

10. The value of Z at which the PAR = 1 constraint is reached using the MWGEN equations is illusive and highly dependent on the character of contracts in the data sample. The forecasting of an optimum Z value for a future period would therefore be difficult. Perhaps a non-quantitative solution to this problem would be an instruction to an underwriter simply to write as much business priced above the expected price as he can, bearing in mind the need to spread acceptances over a range of contracts.

11. A short, general conclusion from the simulation procedures might be as follows: it is possible to model the market price of excess of loss business which makes it possible to search for and find, using the simulation procedure, improved underwriting strategies. The ability to do this is, however, highly dependent on the quality of estimators used in model building.

CHAPTER 8
DISCUSSION

8.1 INTRODUCTION

This chapter examines the results of both the preliminary analysis and the simulation procedures in the light of the literature and current market practice. Three general topics are chosen for discussion: namely, findings of the preliminary analysis, the simulation procedure and results, and wider issues for which the research findings have implications.

8.2 DISCUSSION ON FINDINGS OF THE PRELIMINARY ANALYSIS

The preliminary analysis involved an empirical investigation into statistical associations between key variables on 242 excess of loss contracts, spanning three years of a professional excess of loss aviation underwriter's portfolio. The findings gave rise to further experiments including the simulation and sensitivity analysis of various underwriting strategies. The purpose of this section is to examine in further detail the implications of the preliminary analysis in its own right and to discuss the importance of results in the light of relevant current knowledge and literature.

8.2.1 Resumé of key findings

A summary of key findings from the preliminary analysis is shown in Figure 6.7 but the main findings are restated here before embarking on the discussion. They are as follows (using the criterion that a coefficient of determination of .5 or more is evidence of a reasonable relationship) for the data analysed:

1. The market price of an excess of loss contract can be estimated from key details (upper limit, lower limit and estimated ceding company's premium income) on the slip or contract;

2. The extent of variations around the estimated price can also be estimated from slip or contract details;

3. An underwriter's portfolio selection procedure is shown to be strongly related to the premium size of individual excess of loss contracts;

4. Premiums for excess of loss contracts are not shown to be representative of underlying risk processes;

5. An underwriter's portfolio selection procedure is shown to be based on neither underlying risk processes nor contract details (other than total premium).

The implications and relevance of these findings will now be discussed under the following headings:

1. Limitations of the preliminary analysis;

2. The results as evidence of a satisficing model for excess of loss underwriting decisions;

3. The results as evidence of an absence of actuarial (risk-based) methods in excess of loss underwriting;

4. A comparison between results of the preliminary analysis and the findings of Ammeter (1955);

5. Implications of the results of the preliminary analysis for Stone's Economic Pricing Formula;

6. The dynamic nature of market pricing criteria.

8.2.2 Limitations of the preliminary analysis

The analytical work in this thesis is based on a sample of 337 aviation excess of loss contract cases (242 of which are used in the preliminary analysis), selected from a vast universe of other aviation excess of loss contracts, the exact number of which is unknown. The results must, therefore, be put into context in relation to the greater aviation excess of loss market and, indeed, the excess of loss market as a whole. Some reasons why the results based on the sample cases may not reflect overall market behaviour are, therefore, included below.

1. Size of sample. The aviation excess of loss contracts selected for analysis in this study form only a fraction of the total number of such contracts placed on the London market during the period 1974 to 1976. As a

sample, therefore, it is unlikely that the data are exactly representative of the market universe of excess of loss contracts placed during this period. A sampling error cannot be explicitly calculated to assess the likelihood of research findings reflecting overall market trends, however, due to the lack of a value (or estimate) for the total number of contracts in existence in the market at the time of data collection. Owing to the large number of observations represented by the data, however, it is unlikely that the sampling error is large enough to render the results obsolete. (See Section 8.3.4 for further consideration of this problem.)

2. Processing errors. Errors can occur at any stages of data processing but may be identified from checking and reverification of results. The methods employed in this study are described in Chapter 5 and the results are available for scrutiny and reverification using the data in Apendix B. The author is unaware of any processing errors as the thesis goes to print.

3. Use of created variables in estimation procedures. Usually, where the original data fails to provide all the required variables in a statistical analysis, substitute or estimated variables can be devised which must be based on certain assumptions. In the present analysis, two such variables are created, namely K and MDIFA, which are described in Chapter 6, Section 4.

For K, an assumption is made that any part of a premium which does not represent expected claims can reasonably be called a premium loading. In the ensuing analysis it was evident that K, thus defined, could not be estimated from contract details, total premium or written premium. A possible shortcoming of this procedure is that K is defined too simply and should take into account factors such as price variations, management expenses and loss variance separately, rather than as combined into a single variable. The main reason for use of a single variable was, however, that, if all elements of a premium loading were to be estimated separately, the amount of unexplained variation encountered in the original estimates would be compounded in the following regression equations, thus increasing the likelihood of misleading results. By combining the loading elements of premium into a single variable, this statistical problem is considerably reduced.

For the calculation of MDIFA, an assumption is made that any variation from MEST, the price estimator from contract details, is a price variation from a heuristic-type pricing model. MDIFA, thus calculated, is shown to relate strongly to total premium but not so strongly to contract details or written premium. It should be noted that the relationship of MDIFA to total premium (M) is, in statistical terms, the relationship of an estimated variable to its variance which, if the original estimator is of reasonable quality, is commonplace in statistical analysis. Any relationships examined which include MDIFA as one of the variables should be evaluated, therefore, in the knowledge that the quality of MDIFA as an estimated variable is entirely dependent on the quality of MEST as an estimator for total premium.

4. Errors in interpretation. These relate to misleading interpretations and conclusions that do not follow from the results. Where interpretations of results are required in the present study, the results on which such interpretations are based are included either in the main text or in an appendix should the reader care to reconsider the evidence according to his own judgemental criteria.

One important area where the author's judgemental criterion is employed is in the decision that relationships characterised by an r^2 of .5 or more are "reasonable" and those which fail in this respect are not. This distinction between reasonable and unreasonable results was useful for interpretation of findings in the main text.

The most important relationship for interpretation in this thesis is the one between contract details and total premium. Since the analysis employs only regression techniques the results cannot be said to imply any cause and effect relationship. Indeed, according to accompanying results, there is little evidence to justify the view on risk-theoretic grounds. All that can be determined from the analysis is that the excess of loss premium can be estimated from slip details for samples of contracts and that the coefficients of determination for the estimates are, on most samples tested, more than .5. Whether this is coincidence or evidence of a heuristic-type rating model is a matter for informed judgement. Since a heuristic method need have no scientific basis the proof of this point, one way or another, would be difficult. The simulation procedures which follow from the preliminary analysis are a test of how an underwriting system might be devised using the fact that price, on most samples, can be estimated from excess of loss contract details, and do not throw further light on whether this relationship is truly heuristic or merely a coincidental market phenomenon.

8.2.3 Evidence of a satisficing model for excess of loss underwriting decisions

The most important result to support the view that a satisficing decision model exists for underwriting excess of loss business is that price (total premium) can be reasonably estimated from slip details. Although, in most cases, the estimated premium income of the ceding company is the most significant slip detail for arriving at the estimate, this is demonstrated as not always being the case. On some tests, the upper or lower limit specified in the contract attracts the highest beta in the estimate (see Appendix C). The satisficing model would appear, therefore, to be complex and involving a continually changing judgemental weighting system for the amount of importance attached by underwriters to individual slip details. To evaluate this phenomenon as evidence supporting the existence of a satisficing model for market decisions, the postulated characteristics of such a system (from Chapter 5) will now be reconsidered.

1. The decision model will reflect underlying risk processes even if actual risk processes are not known with accuracy

This statement would not appear to be true if the risk referred to is of the pure type (rather than business or market risk). If ceding company premium income and other contract details are indeed the main influence on excess of loss premiums, and if it could also be demonstrated that ceding company income and contract details are reasonable measures of risk, then the above statement might be justified. The results indicate that risk cannot be estimated from contract details, however, and it must be assumed that the underlying risk processes are either unmanageable or unimportant for excess of loss pricing decisions. These comments can be applied to the aviation market as a whole over the period studied since the analysis took into account pricing decisions made by a large sample of underwriters.

For the decision process of an individual underwriter, it was demonstrated that the portfolio selection procedure did not reflect underlying risk processes. For him, total premium for the contract was shown to be the most important variable for portfolio selection decisions, even though the evidence from the analysis is that total premium does not reflect the underlying risk processes. The implication is that the underwriter either did not know or did not mind if total premium for an excess of loss contract is an inadequate measure of risk.

The evidence from the study suggests, therefore, that, if the relationships established are characteristic of a satisficing model for excess of loss underwriting decision-making, the satisficing model is, in fact, a poor replica of reality in that it does not adequately reflect the pure risks inherent in underwriting operations.

2. <u>The judgemental process will be flexible</u>

The flexible judgemental process refers to the underwriter's ability to combine influences from familiar experience to arrive at a decision on a new action. The statistical analysis in this study would support the existence of such a judgemental process, particularly in respect of the pricing decision which appears to be based on a flexible weighting of preferences attributed to key contract details. For the portfolio selection procedure of the professional underwriter studied, the flexibility in the judgemental process is achieved via a simplifying rule - to accept business in some (flexible) proportion to the size of the premium for the contract under consideration. A small degree of flexibility is also detected (up to an r^2 of .2) in the professional underwriter's portfolio selection procedures in response to variations in actual price from an expected price estimated from slip details. The evidence from the research is, therefore, that in both rating and portfolio construction decisions, a flexible judgemental decision process can be both identified and measured to a reasonable degree using regression techniques.

3. <u>The decision model will hold similarities for different underwriters</u>

The above statement is supported by the evidence in that, for the price estimate from slip details, a high degree of statistical association is found to exist on most data samples (all except the 1977 data). No similarities in portfolio construction decisions can be identified from the empirical survey because only one underwriter's portfolio construction decision process is investigated. The importance of the excess of loss premium in the portfolio selection process is shown to be consistent, however, for a single underwriter between different time periods.

4. <u>The decision model will encompass the possibility of different strategies between underwriters</u>

This postulated characteristic of the satisficing model is also supported by the results (as evidenced by the price variations around the expected price estimated from slip details). For evidence of variations in strategy between different underwriters in portfolio construction, the results of the preliminary analysis do not provide any information (except, perhaps, that the professional underwriter under analysis was able to try a different strategy on each data sample) because the selection criteria of only one underwriter is examined. Nevertheless, given the nature of the market procedure for excess of loss contract placement, there would appear to be enough freedom for underwriters to select their portfolios in a wide range of combinations within constraints set by the market.

5. <u>Key parameters of the model will be measurable</u>

The key decisions examined by this study are excess of loss contract pricing and the criteria for portfolio selection. According to the results the above statement is correct if one accepts an r^2 of .5 as satisfactory evidence for identification of a key parameter since, at this level of statistical association, price is shown to be a function of contract details and, for one underwriter, portfolio selection is seen to be a function of the total excess of loss premium. Both the individual contract details and the total excess of loss premium are measurable (unlike the risk processes which arise from contract acceptance). The important question which must be asked is, however, "Are they sensible measures?" The example given earlier of the Egyptians monitoring the path of a star rather than waters of the Nile was a sensible yardstick since, although the imagined cause and effect relationship was rather primitive (with apologies to any living disciples of Sirius), it was directly correlated to the problem at hand, namely the Nile Flood. In the excess of loss underwriter's situation, however, whether or not the measure is directly correlated to the problem at hand is not a clear-cut issue. All we know from the analysis is that excess of loss prices are highly associated with contract details (above an r^2 level of .5) and that neither individual contract details nor total premium reflect underlying risk processes. If the problem at hand is the search for a method of reflecting risk processes in excess of loss premiums, the measurable key parameters of the model are, on the evidence of the study, unsatisfactory for decision-making. If, on the other hand, the problem at hand is not to arrive at excess of loss premiums which reflect risk processes but something else - such as to find an equitable way of sharing market experiences - the key parameters of the model could be regarded as being of some value.

6. Each possible configuration of key parameters should lead to a deterministic solution

To satisfy this statement, each combination of key parameters should lead to a different decision and, should an underwriter be faced with the same conditions twice, his resulting decisions should be identical. A decision model which cannot demonstrate these characteristics is imperfect since it goes only part way to describing the whole process. Since, on all estimation procedures in the study, a certain amount of variation remained unexplained, the decision model arrived at in the analysis cannot, therefore, be described as perfect. The only claim made for the present study is that, for certain underwriting decisions, the decision-making criteria employed can be identified statistically with an r^2 of more than .5, indicating that more than half the statistical variation between variables is explained. As a result of the unexplained variation the ability to arrive at a deterministic solution for each combination of key parameters is reduced. The results are, however, indicative of a situation where each possible combination of key parameters "tends towards" (rather than "leads to") a deterministic solution.

7. Once identified, the calculation method and results should be suited for repetition and checking

The method remains constant over several data-samples and results held out as evidence of a satisficing model are consistently accompanied by a high (more than .5) coefficient of determination. The data on which the results are achieved are contained in Appendix B and available for duplication of results using the methodology described in Chapter 6.

8.2.4 Evidence of an absence of risk-based decision criteria used by excess of loss underwriters

In Chapter 4, evidence from the literature and from current market attitudes was presented which indicated that the actuarial or risk-based approach to excess of loss underwriting was, at least, inappropriate and, at most, unworkable for practical purposes. The results of the preliminary analysis provide further empirical evidence that this situation prevails. From the analysis, little direct correlation was observed between risk processes and underwriting decisions.

The main problems involved in applying statistical or risk-based decision criteria to excess of loss reinsurance problems were described, in Chapter 4, as the problem of large claims, inadequate information, hetero geneous risks and lack of data. An additional dimension to the problems might now be put forward; namely, the need to allow for market behaviour in underwriting decisions. From the analysis, it is apparent that a behavioural market phenomenon (that is, the apparent preoccupation with contract details as a basis of price) should be taken into account, to a large extent, along with risk processes. If one postulates two approaches to pricing problems, one being a "cost-plus" approach and the other a "what the market will pay" approach, the evidence from the preliminary analysis is that the second of these two approaches is the one for which a pricing method exists in the excess of loss market. Whether or not this approach is used is not, essentially, a matter of choice since the former method would require the existence of a workable risk-based approach to excess of loss rating problems which does not appear to be available.

8.2.5 A comparison between results of the preliminary analysis and findings of Ammeter (1955)

Ammeter (1955) conducted an empirical study which compared market excess of loss premium rates with those of his conceptual model. As the only survey of this type available in the literature, a comparison of findings with those of the present study would seem appropriate. There are, however, some omissions in the presentation of Ammeter's results which limit the usefulness of the comparison; namely, that Ammeter does not supply information on the size, description or criteria for selection of his data sample. The comparison must therefore be made without knowledge of the class, market or country from which his excess of loss data were selected. Each of Ammeter's findings will, nevertheless, be compared with those of the present study. They are as follows:

1. Fluctuations in basic probabilities do not influence the premium: This specific finding of Ammeter is neither reinforced nor counter-evidenced by results of the present study which is not aimed directly at comparing probabilistic fluctuations with premium rates. A more general observation from the present study, which acts as corollary evidence to Ammeter's findings, is that excess of loss premium rates only reflect the overall, underlying risk-processes to a small degree (as evidenced by the B tests in the present study).

2. Net premium rates for excess of loss treaties are independent of the size of the ceding companies: By "Net premiums", here, Ammeter refers to the expected loss element of gross premium (i.e. gross premium less the allowance for variance). A near-equivalent test from the preliminary analysis is test A3 in which claims (C) is regressed against estimated ceding company's premium income (E) and an average simple r^2 of less than .02 recorded (the .02 value is calculated from the multivariate equations in Appendix C) for the three one-year data samples between 1974 and 1976. The findings of Ammeter and the present study, therefore, are in agreement over the relationship between net premium rates and the size of ceding companies (measured in terms of their premium income).

3. The loading element of premium was comparatively heavier for higher limits of self retention: The results from the preliminary analysis which can be compared with Ammeter's observations in this respect are also those from test A3. From analysis of the multivariate equations in Appendix C, it is apparent that relationships between retention limits (L) and the premium loading estimates (K) on the one-year data samples are characterised by an average coefficient of determination of .22 which is below the r^2 of .5 criterion for adequate evidence of a statistical relationship. It would not therefore be prudent to offer the evidence from the present analysis as either supportive or unsupportive of Ammeter's findings in the absence of a more reasonable statistical relationship.

8.2.6 Implications of results of the preliminary analysis for Stone's Economic Pricing Formula

Stone (1975) hypothesised an "economic pricing formula" which bridged the gap between the rating and portfolio construction decisions for insurance of large risks (not specifically for excess of loss reinsurance) and, therefore, offered a potentially useful tool for analysis of excess of loss reinsurance problems. His basic argument is that, from analysis of mean loss and variance characteristics of many independent risks, an optimum portfolio composition can be envisaged. The degree to which this optimum can be achieved is reflected in the pricing decision in the form of a charge for use of portfolio capacity. As a theoretical model, the attraction is that pricing and portfolio decisions can be seen as being dynamically linked. The present study offers evidence of how the pricing and portfolio construction decisions are made in practice and, although the preliminary analysis does not set out specifically to test Stone's hypothesis, the following comments, nevertheless, can be made on how well Stone's theoretical model corresponds with the practice of excess of loss underwriting:

1. Stone's model requires an estimate of annual loss expectations

From the preliminary analysis, it is apparent that arriving at an expected loss estimate would be problematic. In the A tests, which attempted to estimate claims from both slip details and total premium, the average coefficient of determination achieved was less than .4 when dealing with a large sample (more than seventy cases). For an estimate of expected claims on an individual reinsured portfolio, therefore, unless a reasonable amount of claims experience was already available on each contract, an even lower degree of success would be expected. On the other hand, if loss experience was plentiful on individual risks, the business would not be of the type usually put forward for excess of loss protection. If a judgemental approach is applied as an alternative means of arriving at a figure for expected losses for use in Stone's formulae, the present analysis demonstrates, bearing in mind that the same expected loss figure is to form an important element in premium calculations, that underwriters do not arrive at one with a very high degree of precision. When one considers Stone's economic premium formula

$$P = \frac{L}{1-r-e-d}$$

it can be seen that a lack of precision for the annual loss expectation (L) could lead to severe variability in premium (P) values which would render the formula extremely difficult to use in practice.

2. Stone hypothesises that premiums charged will vary with the composition of the portfolio to which a new contract is to be added

This hypothesis was not formally tested in the preliminary analysis but some of the results have implications for the concepts involved. The main relevant finding is that, although Stone's hypothesis may well hold true to a degree, the preliminary analysis demonstrated that price is also affected by other influences to a large degree, against which changes in response to portfolio composition play only a minor role. The high degree of agreement on price evidenced in the results suggests that premium rates are the products of general market trends rather than phenomena which are influenced by individual underwriters. This result is not surprising when one considers that, for large risks placed by means of an excess of loss agreement, a single underwriter will accept only a small part of the business and must, consequently, charge a premium which is acceptable to all other underwriters likely to participate in the agreement. The larger the contract to be placed, the less is the ability of an individual underwriter to charge premiums in accordance with his own portfolio's needs, and the greater is the need to quote premiums acceptable to the market as a whole. The evidence from the preliminary analysis, therefore, suggests that Stone's hypothesis for the underwriting of large or high capacity risks is unrealistic in that it does not allow adequately for firstly, the difficulties in arriving at a reasonable expected loss estimate and secondly, the important influence of the market in pricing decisions.

8.2.7 The dynamic nature of market pricing criteria

A shortcoming of the risk-theoretical approach to excess of loss reinsurers' problems has been described as the lack of attempts to incorporate the dynamic but uncertain trends of market forces. In the preliminary analysis, the effects of market forces are demonstrated as constituting a very important practical consideration in real-life excess of loss underwriting decision-making - to the extent that risk processes are of only secondary importance. More than 50 percent of variation in prices is shown, in the present study, to be a function of the various combinations of contract details. These are the only easily measurable characteristics which are both available and familiar to all underwriters operating in the market, even though the contract details give no real indication as to the nature of likely future cash outflows resulting from contract acceptance.

Although a strong relationship between the price of an excess of loss contract and its contract details is identified in the preliminary analysis, the dynamic nature of the relationship over time should also be considered since, without a complete understanding of how the various elements in the relationship interact, it cannot truly be claimed that the way in which underwriters (or the market) arrive at pricing decisions has been identified. Recalling the Nile flood example as an allegory, we are at the stage where we know that the Egyptians monitor Sirius to order their affairs, but we do not know the exact point which Sirius must reach in the night sky before the flood arrives. Without such knowledge, only a limited understanding of the process can be claimed.

From the estimates of MEST from contract details in Appendix C, it can be seen that the equations differ between data samples. The reasons for the differences are not identifiable from the preliminary analysis. This knowledge gap was an important impetus for embarking on the secondary analysis using the simulation approach. The discussion on the illusive MEST will, therefore, be rejoined after discussion on implications of the simulation procedure and its results.

8.3 DISCUSSION ON RESULTS OF THE SIMULATION EXPERIMENTS

The simulation experiments involved the modelling and sensitivity analysis of key underwriting decisions on the basis identified in the preliminary analysis, with a view to finding portfolio selection decision criteria which would yield better underwriting results than those achieved by a practising excess of loss underwriter. A methodology was developed using underwriting data from Trimark Ltd. covering the years 1974, 1975 and 1976 and this was then tested for consistency on 1977 data. In this section, further attention is paid to the main problems which arose during the modelling and testing procedures, and also to the implications of the simulation results.

8.3.1 Resumé of key findings

The results of Simulations 'A' to 'F' are tabulated and described in Chapter 7 and the main findings summarised in Section 7.11. An abbreviated list of key findings is presented here for discussion purposes.

1. Using an estimator of price from contract details, it was found that the profitability of a simulated excess of loss portfolio could be increased and stability decreased by taking on more and more business with an actual price above the estimated price.

2. The opposite effect (i.e. stability increased and profitability decreased) occurred when more and more business priced below the estimated price was taken onto the simulated portfolio.

3. The phenomena described in 1. and 2., above, were more apparent on short-term (1 year) simulations than on long-term (3 year) simulations.

4. On three out of four short-term simulations, a more satisfactory ratio of profitability to stability (as measured by PROF and STAB) was recorded than that achieved by a professional underwriter faced with the same business opportunities.

5. Sensitivity analysis revealed that the most important requirement of the simulation process was to have a statistically adequate and relevant price estimator for the time period being simulated. Changes in the underlying underwriting pattern (as represented by UPU) were of less importance.

6. When the simulation procedure was considered as a possible tool for excess of loss portfolio planning the following points emerged:

   i) The underwriter would require a price estimator from contract details for the period in which underwriting activity would take place in order to improve the chances of achieving planned performance;

   ii) Given this estimator, although, normally, "better" underwriting results could be achieved, the extent of improvements cannot be accurately predicted in absolute terms because they are largely dependent on both market loss intensity during the period and the quality of statistical estimators used in the modelling and simulation process;

   iii) A further difficulty, for providing the underwriter with a suitable MWGEN equation as a basis for his planned underwriting strategy, is the prediction of a constant (Z) which exactly maximises portfolio performance within constraints set by the number and characteristics of contracts available in the market.

7. Without further research, and assuming a means of adequately estimating current prices is available, the practical implementation of the simulation procedures suggests that instructions along the following lines should be given to an underwriter with a view to improving portfolio performance: "You must attempt to write as much business as possible which is priced above the price estimated from contract details, but ensure that your selection is spread over enough contracts to achieve the required number of portfolio acceptances. You must rely on your own skill and judgement to determine how much business priced above the estimated price will come your way"

8. The main obstacle preventing the use of the simulation procedure as a portfolio construction tool by excess of loss underwriters is a means of adequately estimating current market prices from a current sample of excess of loss contracts. This problem is not necessarily insurmountable (as will be discussed further in a Section 8.3.4) but presents certain difficulties.

8.3.2 Appraisal of the simulation procedure

A list of objectives for the simulation procedure is described in Section 7.2. The degree of success with which these objectives are achieved is now examined.

Firstly, an objective was to further the investigation from the preliminary analysis, with particular regard to the relationship between the satisficing model of price and the true risky characteristics (in terms of actual losses) of the sample contracts. The simulation procedure produced results under a series of scenarios which enabled further understanding of this relationship.

A second objective was to model the test underwriter's decision process for portfolio selection. Using the UPU equations, a high degree of success was achieved in this respect. The average r^2 for modelling equations, on the four single-year data samples, was more than .75.

Thirdly, the model was used to simulate underwriting strategies which, by various degrees, accepted more of business priced above the satisficing price model and less of business priced below it. The MWGEN equations were the basis of this procedure and were generally successful, despite the ever-present error factor inherent in the estimated variables which formed the basis of the strategy-generating equations.

The fourth objective was to identify an optimum underwriting strategy from improved knowledge of price and loss behaviour and, using the tabulated results of the 'A' Simulations, this was shown to be possible. The optimisation is achieved by monitoring the PROF and STAB measures of performance subject to the PAR constraint, and then selecting the underwriting strategy which maximises the ratio of PROF to STAB as the most efficient. This particular optimising procedure was chosen in the absence of a known underwriters' preference ordering for PROF and STAB and is, therefore, not completely realistic. Nevertheless, the method for producing information onto which an individual underwriter could impose his own utility values was demonstrated as workable by the simulations.

Fifthly, an objective was to conduct sensitivity tests on the model, and this proved to be a useful exercise. Without undertaking these tests the importance for planning purposes of knowledge of the price estimator (MEST) for a particular period would not have been discovered.

The final three objectives were aimed at identifying elements of the simulated strategies which would be useful for planning purposes, for conducting suitable tests, and for comparing the results with those of a professional underwriter. The evidence suggested that, as a basis for planning, a strategy aimed at including as much business as possible priced above the expected price (estimated from contract details) would be a suitable scenario for the final planning test. A necessary assumption for running the test, however, was that the price estimator from contract details was known for the period during which underwriting activity was to be simulated. On running the test, the general result was, as with previous simulations, that a more efficient portfolio was achieved by maximising the amount of business selected which was priced above the expected price (estimated from contract details), subject to the PAR constraint. When the simulated results were compared with those of the test underwriter, however, they were discovered to be less satisfactory: on the 1977 data, the simulation method failed to produce better portfolio results than those achieved by the professional underwriter. This result ran contrary to three previous instances when the technique was applied to earlier single-year data samples where, at the optimum simulated strategy, results proved more efficient than those of the professional underwriter. The overall ability of the simulation procedure to produce more efficient results than those of the test underwriter is summarised in Table 8.1.

Each value in Table 8.1 relates to the ratio of profitability (PROF) to stabililty (STAB) either for a real or a simulated portfolio. The larger the value attached to this ratio, the greater is the efficiency of the portfolio on which it is calculated. It can be seen that, overall, the simulations were able to produce more efficient portfolios than those achieved by the professional underwriter.

Table 8.1 Summary of PROF as a percentage of STAB relationships for actual and simulated (optimum strategy) portfolios on single year data samples

Ratio of PROF to STAB (%)	Data Sample				
	1974	1975	1976	1977	Mean
Professional underwriter	33.03	244.33	43.84	38.89	90.2
Optimum simulated strategy using current UPU and MEST	60.93	716.55	67.67	23.22	217.09
Optimum simulated strategy using current MEST and past UPU	-	686.54	72.02	22.85	260.53

8.3.3 Limitations of the simulation model

All models, by definition, are imperfect replications of reality. Donaldson (1971, p.11) describes the modelling process, thus:

> Such model building inevitably means a retreat from the real world by simplification and abstraction in order to bring to bear the available tools of logic. Once the problem has been classified as to the relevant logical discipline into which it is assumed to fit ... the primary capacities required of the researcher are the technical skills of the disclipline and not an understanding of the real world. While such work is essential in getting the logic of the decision process straight, there is frequently a problem in bringing this logic to bear on real world decision.

In this section, attention is paid to the problems and implications of departures from the real world which entered into the simulation and modelling process described in Chapter 7 of this thesis.

1. The profitability and stability measures

There are many ways in which profitability and stability for a financial process can be measured and it is important to note that PROF and STAB, as employed in the simulation model, are not the sole measures suitable for reinsurance portfolio modelling purposes. They were selected for use in the present study for their simplicity of operation and their resemblance to those suggested by Stone (1975). An important point is that both measures are calculated and tabulated in the results as ratios, with little heed paid to the magnitude of sums involved in the constrained optimisation procedure. The scenario represented is one where portfolio size, in terms of premium income, is completely flexible and unlimited by constraints other than those specified in the model. In real life there are practical considerations, such as the reserve size required by law to support various levels of risk acceptance, and these are not accounted for in the model.

2. Premium volume

As mentioned above, the model does not take into account the £ magnitude of cash flows in the optimising procedure. A further departure from reality, therefore, arises in the model because it is assumed that premium income for the portfolio is determined entirely by market conditions, with no allowance for managerial constraints. For example, no maximum £ written line is specified in the model. The simulation is specifically designed to test the consequences of underwriting strategies embarked upon in response to market conditions, however, and hence the decision to exclude such managerial constraints from the model. From the analysis, only information on portfolio performance in the absence of such managerial constraints is provided.

3. Estimation problems in the modelling process

The following estimation problems were encountered in the modelling procedure:

(a) The underwriter's written line (UPU).

The quality of the estimates of MW from MEST and MDIFA, required for the UPU equations, varied between data samples, although the overall standard was high. Nevertheless, a difference between actual and modelled UPU exists for all simulations.

(b) The estimator of price from contract details (MEST).

The quality of the MEST for each simulation causes fluctuations in the flow of tabulated results from increasing values of Z in the MWGEN equations. The fluctuations occur because MEST is not a perfect estimation of M. That is, the M/MEST part of the MWGEN equation is not equal to 1. The exponential effect of increasing Z smoothes the influence of the difference (between M and MEST) over the simulation results but fluctu ations in results are particularly noticeable around the Z=1 MWGEN equation (for example, see Table 7.4). One cure for variation around the Z=1 results would be to force MEST to take on the same value of M at Z=1. Although fluctuations would thus be smoothed out, however, such an adjustment would imbibe the results with mock precision. Despite the mathematical problems of an imperfect price estimator from contract details, the simulation procedures are nevertheless adequate for demonstrating at least the general trends in results from the portfolio selection procedure based on the decision criteria embodied in the MWGEN equations.

4. The portfolio selection equation cannot distinguish between contracts from various classes of business

By employing an equation to represent portfolio selection criteria, attention is paid only to the quantifiable aspects of contract details. Other description of the risks is ignored completely, with the consequence that no assurance can be given that an adequate spread of business by class or area is achieved. In other words, the contract selection procedure, described in Chapter 7, does not guarantee adequate protection against the excess of loss portfolio conflagration hazard. Although the investigation did not set out to construct a portfolio which reduces the conflagration hazard to acceptable levels, it must be borne in mind that, in practice, such considerations are important to the excess of loss underwriter and would need building into the model, should it ever be intended for use in the market. One way in which the conflagration hazard could be identified and controlled would be to specify additional constraints in the model to reduce the number and £ amounts of contracts selected from any area or class of business to an acceptable figure.

5. The assumption, for planning purposes, that an adequate price estimator for a future period is available is unrealistic

The assumption that an adequate price estimator for use in the portfolio selection equations could be easily predicted from past trends is shown to be unrealistic by the large variations in MEST which occur between data samples. This is an important shortcoming which considerably reduces the possibility of portfolio planning for excess of loss reinsurance using the described simulation procedure. On a more optimistic note, however, it does not rule out the possibility of employing the methodology as an aid to current (rather than planned) underwriting activity. The evidence from the study is that, providing a current price estimator from contract details is available, portfolio performance can be improved. The possibility of providing a workable current price estimator will now be considered.

8.3.4 Towards an on-line estimator of price from contract details

A large body of literature on sampling techniques is available from which a workable method for estimating current expected prices for excess of loss contracts from contract details could be devised. At any one time, a large number of excess of loss contracts are in the process of placement in the market from which a large enough, representative, sample could be taken. Access to the information on contract details would present no particular problem to the professional underwriter who is constantly offered contracts for consideration by visiting brokers. It is unlikely, however, even if a mechanised price-estimating procedure was available to the underwriter, that he could be bothered to run through the procedure of calculating a current expected price every time the prospect of participating in an excess of loss contract arose. A more convenient method would be to employ an on-line expected price estimator which could be constantly updated in the light of new contract information. Such an on-line system would, in its most convenient form, require the frequent input of recent information on latest contract details and prices to produce an up-to-date equation for estimating current expected price.

Two sampling problems arise for devising an on-line price estimator from contract details. The first problem concerns the size of the sample required and the second is the need for a statistical confidence estimate for measuring the likelihood that the sample findings represent the overall, market-wide situation. At the heart of the first problem is the need to select a sample which is large enough to provide satisfactory estimators and also up-to-date (a smaller, more recent sample would be more reflective of current market pricing behaviour). The magnitude of this problem need not be exaggerated, however, since the evidence from the simulation procedures is that a price estimator can be calculated and used successfully from a large sample of contracts covering a one-year time period. An estimator calculated on a large data sample from, say, the immediately preceding six months might, foreseeably, meet with more success than the one-year estimators employed in this study. Should a more rigorous approach be required, a statistical weighting system or credibility approach could be adopted in order to weight the importance of data items (according to their age) in the estimation procedure.

The second sampling problem is to assess the significance of sample results in relation to the universe of other excess of loss contracts in the market - and this requires the calculation of a sampling error or coefficient of variation for the sample. One way of calculating the coefficient of variation for a sample is to use the following equation (from Som, 1973, p.82):

$$e = \sqrt{\frac{1 - P}{Pn}}$$

where P = the universe proportion (the fraction of the sampling universe remaining after the sample has been removed from it)

and n = the sample size

A problem when using the above equation is, however, that the population size, or universe, for any type or class of excess of loss contracts under placement at any one time is probably not known. Estimation procedures would, therefore, be required to arrive at a reasonable figure for universe size. One method which could be applied here has ornithological origins. F.C. Lincoln (1930) used the following equation to estimate the total number of waterfowl on the basis of banding returns:

$$N^* = N_A n/n_A$$

where N^* = estimated number of birds in the universe

N_A = the number of birds captured, banded and then released

n = the number of birds recaptured at a later date

n_A = the number of banded birds in the recaptured groups

The estimator is consistent but biased and requires an accompanying variance estimator for N^* (see Hajeck and Dupac (1967) for method for calculating the variance). The link between waterfowl and excess of loss contracts, here, is that both are amenable to the statistical technique of 'capture and recapture'. Using Lincoln's method, an estimate of the total number of excess of loss contracts for a particular type of business could be achieved by substituting the following elements into the equation:

N^* = estimated number of excess of loss contracts of a particular type in existence during an underwriting period (t).

N_A = the number of excess of loss contracts of a particular type presented to the underwriter during an underwriting period (t).

n = the number of contracts for the particular type of business presented to the underwriter in the following underwriting period (t+1).

n_A = the number of contracts in n which are also renewals of contracts in N_A.

Even if only accepted contracts (rather than all those 'seen') were to form the basis of the sample, all the required information would be available for the above calculations from normal underwriting records.

If the above statistical and sampling problems could be solved, a remaining requirement for an on-line price estimator would be the conversion of the estimation method, described in Chapter 6, into computer software. As can be seen from Figure 6.4, for a four-variable estimate, using regression analysis, twelve regression equations must be produced from which the best one is finally selected as MEST. Without a computer programme to control this procedure, the amount of human effort and time involved is considerable. Given the current state of statistical, computer and technological skills, however, the on-line price estimator is a real possibility which, if the relationships tested in this thesis hold up over time and for other areas of the market, could find useful application by the excess of loss underwriter of the future.

8.4 WIDER ISSUES

Having considered, specifically, the implications of the preliminary analysis and the simulation model, the purpose of this section is to consider further implications of the research. The discussion begins with the possibility of extending the methodology, employed in this study on aviation excess of loss data, to other areas of the excess of loss market. Some of the problems which might then be encountered are also considered. Secondly, some arguments are put forward in support of a pricing system based on heuristics for excess of loss reinsurance. Thirdly, the implications of such a system for market regulators are discussed and the basis for a market supervision system suggested as potentially providing a more accurate means of monitoring firm performance than traditional methods. Finally, the heuristic approach to pricing is presented in a general form and compared with current theories for explaining prices and their movements in other types of market.

8.4.1 The applicability of the methodology to other areas of the excess of loss market

The data used in this study were selected from a single aviation excess of loss reinsurance account and cover the period 1974 to 1977. The question arises: 'Could the methodology be applied successfully to other time periods and/or sections of the excess of loss market?' A brief answer to this question is that, if the necessary information was available, the methodology could be employed - but whether the general trends of results would be repeated cannot be determined, with certainty, in advance. Nevertheless, the following comments on the applicability of the methodology to various other types of excess of loss market are offered here as an indication of some of the likely problems and findings which might arise.

Fire Business

"Fire Business" is a term often used in insurance circles to describe a number of non-life insurance risks - many of which have nothing to do with fire. For example, the "fire department" of an insurance company often combines fire, burglary, pecuniary loss, and accident risks into a single insurance package. The point is that excess of loss protection for a fire portfolio covers a multitude of different kinds of insurance risk, whereas the aviation sample tested comprised only three kinds (hull, cargo and liability). There may, consequently, be several pricing systems to be identified if the contract details method were employed for an analysis of fire business. The success of the methodology would be enhanced by information from underwriters (rather than from legislative documents or textbooks) on what kind of business is perceived in the marketplace as constituting particular excess of loss classes. Care should then be taken to collect data under these more relevant market headings.

Motor Business

According to the literature, if a risk-based excess of loss premium could be employed usefully anywhere in the market, it would be for motor business because the subject of the original insurance is relatively homogeneous. Application of the methodology presented in this thesis to motor excess of loss business would, therefore, be an interesting investigation aimed at finding out if underwriters employ a satisficing rating model even when the risks are relatively measurable.

Marine Business

Marine business, like aviation business, consists mainly of hull, cargo and liability protection. Indeed, there are close historical links between aviation and marine business which, for a small underwriting company, might, even today, be combined on a single excess of loss portfolio. The historical link would lead one to expect similarities in results if the methodology developed in this thesis on aviation business was also applied to marine business. One area where a pricing model would probably be identified is for oil rigs where, in the late 1970s, market personalities met to agree a scale of charges for the various layers of oil-rig liability going onto the market. This oil-rig pricing agreement is a conscious and formal example of the behaviour which, according to the evidence from the present study, also occurs at an informal or unconscious level in the aviation excess of loss market.

The above comments are, of course, based mainly on conjecture and could only be verified by further research in other markets using the methodology applied in this thesis to aviation excess of loss business.

8.4.2 Some arguments in favour of a workable excess of loss pricing system based on heuristics

Operators in the insurance industry are renowned for their willingness to work together on matters of common interest. This willingness manifests itself in market conventions such as the Joint-Hulls agreement (in marine insurance) or the Fire Offices tariff (non-marine business). In the excess of loss market, too, there are examples of underwriters meeting to agree on prices - especially where new, large risks (oil-rigs are a recent example) are likely soon to become familiar business in the market. The purpose of these informal agreements is to achieve a certain unity in prices charged so that, should prices be too low (in the light of experience), all participants in earlier placings would have fared with similar results and would, therefore, be reasonably agreeable to an updated, higher premium for new business. A question arises, however: How do underwriters agree on relative prices for a certain type of risk, for all the different possible combinations of ceding company size and part of the risk to be transferred? The actuarial problems involved in such an exercise are considerable if the intention is to evaluate the various underlying risk processes and allocate to them suitable, fair premium rates. The problem of how to share out the benefits in an equitable fashion remains, even if overall premiums for a certain class of business are deliberately pitched high. One way in which this problem is overcome in practice is through the market placement system which involves many underwriters sharing many risky business opportunities. There remains, however, a need for convenient rules of thumb or working rules to allow different prices to be charged for different contracts - even if the underlying risk processes are unknown. Although it is not proven conclusively in the present study, reasonable evidence is provided that contract details are important elements in this price-allocating system. It would seem appropriate, therefore, to take a closer look at the potential benefits and disadvantages of their role as an informal form of tariffication.

The usual reason for forming a tariff is to maintain a high price for a product or service (whether this be for or against the general economic good). Price maintenance, given the high degree of uncertainty inherent in excess of loss reinsurance, would, therefore, seem useful for excess of loss underwriters. The following points are put forward in support of the idea that contract details are a convenient means for maintaining adequate prices:

1. To keep prices high is a natural desire for underwriters since it leads to higher profits and increased portfolio stability.

2. Since losses are difficult to predict, price must be attached on the basis of more familiar criteria - such as contract details.

3. Given that an underwriter takes part in a large number of market placements, the fact that contract details do not reflect underlying risk processes is, relatively, unimportant - so long as the underwriter, through the law of large numbers, achieves results near the market average.

4. Changes in price trends are easily monitored by underwriters in their daily work by reference to contract details (underwriters see a larger number of contracts than those in which they actually participate). The broker on his rounds is a vehicle for communication in this process.

5. Should prices fall too low, and if similar portfolio results are recorded by many different underwriters, there would be a high degree of agreement in support of raising the general price level, provided that all underwriters employ similar criteria for pricing decisions.

6. Despite a high level of agreement on the general price level, the system would still be flexible enough for different underwriting strategies to be undertaken by different underwriters. For example, an underwriter might specialise in writing either high- or low-layer risks - in which case his results could possibly depart from the average level of market performance. An important point is that only by analysis of contract details would an underwriter know whether the layer was high or low, so the fact that contract details enter underwriters' thought processes can be stated with certainty if such strategies are indeed followed.

The above comments on how an informal form of tariffication could operate in the excess of loss market assumes that contract details are the main guide to the value of a contract. It is worth noting that the comments do not include any suggestion which would lead one to expect market prices to operate in any other way than how they actually do. Two dangers in the system described above are apparent, and these relate firstly, to the market as a whole and secondly, to the individual underwriter.

The danger to the market as a whole is that overall losses might exceed overall premiums plus some safety margin. This could result from any, or all, of the following:

1. A large number of new risks might enter the market at too low a price. In this event, portfolio performance would suffer market-wide due to excessive loss experience in relation to premium income.

2. Communication among underwriters may not be sufficient to ensure that overall market prices are raised when it becomes necessary.

3. Competition from other markets could also prevent overall price increases when they are required. (The effects of over- and under-charging need not necessarily be felt at the same time in all excess of loss markets.)

There are, of course, other possible threats to the excess of loss market (for example, structural difficulties or loss of confidence) but the three examples given above are possibilites which could arise, specifically, if prices are charged, not on a risk basis, but according to a satisficing pricing model based on contract details.

For the individual underwriter, the greatest danger is the possibility of failure to produce portfolio results at or above the market average, thus losing relative market position. Specific causes of this danger, when operating in an environment where price does not reflect loss likelihood, are as follows:

1. Failure to take part in a large enough number of excess of loss contracts to achieve results near the market average. The law of large numbers is at the heart of this process. The mean value which is to be approximated under the law, however, is not, in this context, an absolute loss expectation, but a similarity in overall operating results with other reinsurers operating in the same market. By entering into too few agreements of indemnity, not only is the underwriter's risk of failing to attain performance near the market average increased, but also the weakness of his position for initiating future price increases.

2. General price movements are likely to reflect overall losses in the market for certain types of business - but the way in which the general price level is allocated across various layers in the excess of loss market is likely to be inequitable since the risk processes are not adequately known. As a result, it is likely that, for any type of excess of loss risk, either the upper layers or the lower layers (or both) are likely to be charged below the fair (but unknown) risk premium. A second danger for an individual underwriter, therefore, would be excessive specialisation in either high or low layers for specific types of excess of loss coverage.

Both the above dangers are reduced considerably by standard market practice. Underwriters do attempt to spread their interests over as wide a number of contracts as possible and, although specialisation in particular types of cover is practiced by some underwriters, the consequent expertise is an understanding of what price should be, and this enables the rejection of under-priced contracts. The specialist underwriters for a particular type of cover are also likely to be the lead underwriters for it and, therefore, in a key position to initiate future prices changes.

8.4.3 Suggestions for an excess of loss market supervision system

The results of the preliminary analysis and the implications of the simulations have bearing on existing and potential procedures for monitoring the performance and stability of both individual firms conducting excess of loss reinsurance and the market as a whole. The specific aspect of supervision to which the evidence from the present research relates is the use of premium income as an approximate measure of risk; the results of the preliminary analysis indicate that, for excess of loss business, there is very little correlation between loss patterns and premiums charged. Ratios which include a premium income element in their calculation are, therefore, likely to be misleading for purposes of risk assessment. Further, for monitoring individual firm stability in relation to other firms in the market (rather than in absolute terms), a total premium figure alone is of little value, since it does not take the spread of interests, or the complexity and structure of the portfolio into account. A tentative solution to this problem, based on the implications of findings in this thesis, is presented below.

In the absence of workable risk or premium measures, an understanding of the market itself, and the selection patterns of individual reinsurers is a prospective alternative approach for monitoring firm performance. Rather than attempt to measure risk directly from the experience of individual reinsurers, a system might be envisaged which involves firstly, monitoring market-wide premiums and losses, secondly, assessing the likelihood that individual reinsurers would achieve the expected market level of performance and thirdly, identifying firms likely to achieve results which deviate excessively from the market average. Thus, two performance measures are required - one to assess market performance and the other to assess firm/portfolio performance in relation to general market results.

A rough stability measure might be calculated as follows for a single portfolio excess of loss reinsurer:

$$S = P/N$$

where S = an indication of a portfolio's spread of business

P = total premium written on the portfolio

N = the number of contracts on the portfolio.

The above measure would take into account the spread of business on a portfolio but is still only a rough guide to portfolio stability since no attention is paid to the way the total premium is shared out over various contract acceptances comprising the portfolio. With statistical development, to take into account mean, standard deviation, skewness and other measurable characteristics of spread for premiums over the various acceptances, however, a more sophisticated indicator of portfolio spread could be constructed. Even so, the spread of business on a portfolio is only one factor to be taken into account for monitoring portfolio performance; the quality of the business, in terms of premiums received less actual or likely losses, is also important. Loss incidence for excess of loss business is difficult to predict, however, and premium incomes are likely to be of little value in the estimation process.

A suggested alternative approach to the problem is to calculate the expected loss ratio for a firm via a number of measurements taken on overall loss potential of each class of excess of loss business transacted in the market. The intention would be to build up a composite picture of a firm's loss expectation according to its degree of participation in each business class. The requirements for such a system would be firstly, a market monitoring system to assess likely overall loss ratios for each class of excess of loss business transacted on the market, and secondly, a method for apportioning likely loss ratios in proportion to the amounts of business transacted in each class of business by the individual firm. For a single reinsurer the following measures might be calculated:

$$E^f = \frac{\sum_{i=1}^{n} Ec_i p_i}{P}$$

and

$$MR_f = \sum_{i=1}^{n} k_i \, p_i/P$$

where E_f = the expected loss ratio for the firm

Ec_i = the expected loss ratio for the ith class of excess of loss business written on the portfolio

p_i = the £ amount of premium from the ith class of business on the portfolio

P = total £ premium collected on the portfolio

MR_f = a market risk indicator for the firm

k_i = the probability of achieving Ec_i for each class of business.

MR_f is a measure of market risk in that it measures the likelihood of E_f, calculated for a single firm, following loss expectations for the market as a whole. The calculation of k_i would require analysis of the spread of different classes of excess of loss business over the reinsurer's portfolio. Figures 8.1 and 8.2 provide examples of the calculations of E_f and MR_f for a single, hypothetical excess of loss reinsurer.

If, for supervisory purposes, E_f and MR_f were calculated for a number of reinsurers operating in the market, the interpretation of the measures would be as follows:

1. The higher E_f for a particular firm, the higher is the probability of underwriting losses being made. The lower E_f, the higher is the probability of underwriting profits being made;

2. The higher is MR_f the greater is the confidence that can be attached to the E_f estimate;

3. The intention in using the E_f and MR_f measures would be to identify firms where either E_f or MR_f (or both) is too low;

4. Where, for an individual firm, E_f is too high, the risk facing the firm is that of possible business failure due to poor/unlucky selection of business;

5. Where, for an individual firm, MR_f is too low, the risk faced by the firm is that of not spreading underwriting interests widely enough to ensure "equal fortunes" with the rest of the market.

E_f and MR_f could be incorporated into a screening system or early-warning device for identifying excess of loss reinsurers likely to run into difficulties. Before such a system could become a reality, however, the following factors would be required:

1. The availability of data on premiums and losses for the various classes of excess of loss business transacted in the market;

2. A reporting system to supply timely information on market prices and loss levels;

3. The willingness of firms operating in the market to supply details of portfolio composition to supervisory authorities;

4. A method for calculating k, the probability that firm performance will follow general market trends.

The first of the three requirements are concerned with data availability in the right form and access to that data by supervisory authorities. They would require willingness on the parts of excess of loss firms and market supervisors to employ more detailed reporting systems than are currently used. The fourth condition presents a methodological problem the solution to which would require the following factors to be taken into account:

1. As a general rule, one would expect that the larger the number of acceptances from a class of business taken onto a portfolio the higher would be the value of k for that section of the portfolio;

2. The general rule in 1. is complicated by the fact that not all contract acceptances are likely to be for the same amount of premium income. Therefore, the probability that acceptances for a certain class of business on the portfolio would reflect market performance for that class would be characterised by a skewed distribution, in most cases;

3. A further consideration is that differences between actual premiums quoted on the market and theoretical, fair premiums which should be representative of loss expectations are not necessarily distributed evenly throughout the market. There is, therefore, no certainty that a group of contract acceptances taken from a class of business onto a

firm's portfolio would include an even balance of overcharged and undercharged acceptances (in relation to the theoretical, fair premiums).

There would obviously be other methodological and practical problems encountered in setting up a market supervision system for excess of loss reinsurers along the suggested lines and, without further development, the system (as illustrated in Figures 8.1 and 8.2) would provide only a rough guide to reinsurers' performance. The system is included in this thesis, however, as a tentative first step towards a structured market-based system (rather than risk-based) of supervision of excess of loss reinsurers. The evidence from the preliminary analysis is that a supervision system of this type would be more successful than one based on analysis of pure risks for excess of loss reinsurance.

8.4.4 Traditional approaches to pricing compared with research findings

A number of models have been put forward for estimation and prediction of market prices. Many of these studies have centred on stock-market prices and these are, to a certain extent, relevant to the present study. There are certain similarities in that both the stock market and the excess of loss reinsurance market are characterised by the common factors of supply, demand, uncertainty and expectations. To date, the author is unaware of any specific studies on the general level of excess of loss protection prices. The absence of studies in this field is possibly due to the lack of a satisfactory method for combining heterogeneous excess of loss covers to provide a representative index which could be successfully monitored. An analysis of the degree to which excess of loss prices can be demonstrated as conforming with the various theories on price behaviour in stock-markets is, therefore, impossible. It is possible, however, to discuss, more specifically, the results of the present study in relation to approaches which have been tested on the stock-market: namely the technical, fundamentalist and random-walk approaches.

The technical approach involves the use of historical price movements to predict future prices and is highly empirical; the trends identified are not necessarily intelligible so long as they are recurrent and continuous. Fundamentalists, on the other hand, hold that prices are determined and are predictable by expectations about key variables (for example, earnings, dividends, and cost of capital in the stock-market models). In contrast to

Figure 8.1 **Example of the calculation of expected losses (E_f) for an excess of loss portfolio from market indicators**

Class of business written (i)	Expected loss ratio for class (Ec_i)	£ premium written for each class (p_i)	£ claims for each class ($Ec_i\ p_i$)
A	1.2	1,000	1,200
B	.8	20,000	1,600
C	1.5	10,000	15,000
D	.9	10,000	9,000
		41,000	26,800

$$E_f = \frac{\sum_{i=1}^{n} Ec_i p_i}{P} = \frac{26,800}{41,000} = .6537$$

Figure 8.2 **Example of the calculation of market risk (MR_f) for an excess of loss portfolio from market indicators**

Class of business written (i)	Premiums written for class as a proportion of total premiums (p_i/P)	Probability of achieving expected market performance (k_i)	Market risk Indicator ($k_i\ p_i/P$)
A	.0244	.01	.0002
B	.4878	.5	.2439
C	.2439	.3	.0732
D	.2439	.2	.0488
	1.0000		.3661 = MR_f

the technical and fundamentalist approaches, the random-walk approach holds that price changes are brought about by a series of random movements which tend towards an efficient allocation of resources. (For a collection of readings on the various approaches to stock-market price determination, see Elton & Gruber (1972) or Jensen (1972) and, for a more detailed coverage, Lorie and Hamilton (1973) or Dyckman Downes and Magee (1975)). The technical, fundamentalist and random-walk approaches to pricing (from stock-market theory) will now be considered for their applicability to excess of loss reinsurance prices in the light of research findings.

A technical approach would require that excess of loss premiums could be monitored and were reasonably predictable. A difficulty is that excess of loss risks are heterogeneous and, except by the contract details method of estimation described in this thesis, it is not a simple matter to compare prices with either excess of loss contracts or the risks to be transferred. Even the contract details price-estimation process would not lend itself easily to the technical approach to predicting premiums over time, however, due to the relative complexity of the pricing formulae which vary markedly between underwriting periods. It would also require acceptance of the idea that an estimator of the general price level, rather than actual prices, should form the basis of forecasting procedures. The evidence from the study is that price movements in the short-term (one-year), as determined by contract details price estimators, are too variable to be used for planning purposes. This would suggest that use of crude, charting methods for monitoring the changing level of prices would fail to produce the recurrent and continuous patterns required by the empirical, technical approach.

The method of estimating prices from contract details holds some similarities with the fundamentalist approach but also certain differences. The main similarity is the use of key variables as a basis for price prediction, but there the similarity ends. The fundamentalist approach is to predict from measurable, financial characteristics of economic organisations (the actuarial, risk-measurement process would be analogous to the fundamentalist approach if it could be shown to work in practice for excess of loss reinsurance), but the contract details method predicts from heuristics which need have no scientific basis. Expectations enter the fundamentalist models but, in these models, the emphasis is placed on analysis of expectations of actual, measurable financial parameters rather than on subconscious phenomena which appear to be the determinants of price in the present study.

Underlying the random-walk hypothesis is an assumption that markets are efficient in their role as a means of allocating scarce resources; individual price movements need not, however, be observed to follow this overall trend. The evidence from the present study is that, even in the long-term (three years), excess of loss prices do not reflect the underlying risk processes of reinsureds to any high degree - which suggests that the market is not particularly efficient. When considering this evidence it should be remembered, however, that three years for an excess of loss market is not a particularly long time - hardly enough to form a reasonable idea of loss experience - and also that a more identifiable overall relationship might have been observed if the aviation data were combined with data from other types of business being conducted on the market. It is difficult to state how wide a net should be thrown in order to capture what might be, overall, an efficient selection of excess of loss business. An economic interpretation of the random-walk hypothesis involves the assumption (Philippatos, 1973 p.377) that the market "consists of numerous well-informed participants who, by their rational trading moves, adjust prices to reflect all currently available information", and it would seem appropriate to consider the applicability of this assumption to the excess of loss market. The participants are certainly numerous in the excess of loss market, but whether or not the information available is of the type normally included in a rational decision-making process is open to argument. Since the relationship of price to contract details (as demonstrated in this thesis for aviation business) does hold for excess of loss markets, it would certainly be a rational move on a decision-maker's behalf to employ knowledge of the relationship in his decision process. On the other hand, no rational or scientific justification is available to explain the relationships of price to contract details as anything more than an observable market phenomenon.

To conclude, the methodology employed in this thesis goes part way to being classified under each of the recognised approaches to market prices from stock-market theory but does not fit neatly under any one of the general headings of the fundamentalist, technical or random-walk approaches. In the next section attention is paid to whether or not the methodology presented in this thesis constitutes an additional and independent method to the more familiar approaches for explaining price movements.

8.4.5 The heuristic approach to market prices

It has been suggested that the heuristic-based methodology employed in this thesis might rank alongside more familiar approaches to price prediction. The basic factors underlying the new approach, based on findings from the research are, therefore, presented below in generalised form to provide an outline of the shape which might evolve for a heuristic-based theory of market prices.

Key points for the heuristic approach to market prices

1) The true subject of a transaction need have no direct influence over its price (in the present example the true subject of a transaction is the unit of risk to be transferred which is, essentially, an unknown quantity);
2) Price is determined by market heuristics (such as contract details);
3) The heuristics (of which price is a function) need have no scientific or rational basis;
4) Forces of supply and demand act on the market heuristics rather than the real economic good being transacted.

With the exception of point 4) above, all the key points listed have been demonstrated to hold true by the data analysis in this thesis. Point 4) is included by inference from economic theory which holds that price is a function of supply and demand. Each of the points will now be considered in further detail.

1. The subject of a transaction need have no direct influence over its price

This condition runs contrary to the fundamentalist approach and (in the long run) to the random-walk approach to prices. It is unlikely that the condition is upheld in markets which are not characterised by products of a highly uncertain or abstract nature. The condition is most likely to prevail in markets where rigorous or objective description of the economic good being transacted is difficult or impossible.

2. Price is determined by market heuristics

The term "heuristics" refers to convenient attributes connected with the economic good in question which have become an integral part of decision-makers' judgement procedures. Contract details are the basis of the heuristic approach in the present study and, possibly, from future research, the heuristic approach to price might be found to operate in other markets. The difference between the heuristic approach and the fundamentalist approach (which might also predict from variables included in a contract) is that heuristic variables used for predictions need not be meaningful in themselves as quantitative or qualitative measures of performance. They are, rather, phenomena which arise out of decision-makers' practices to "develop strategies that help them to act with limited information and within their computational faculties" (Einhorn 1976).

3. <u>The heuristics need have no scientific or rational basis</u>

This condition implies that a symbol (or symbols) has replaced the subject matter of transactions as the central focus of decision-makers attention and, further, that the symbol is only a satisficing representation of the subject of the transactions. Contract details for aviation excess of loss reinsurance are an extreme example (according to the evidence of the preliminary analysis) of a situation where symbols with little or no relationship to cash flows generated by transactions are, nevertheless, employed market-wide, in a satisficing pricing model. It is possible that similar situations exist in other markets where the heuristics involved in decision-making processes might be more rational than those identified in the present study. For example, ratios are both objective in construction yet subjective in interpretation and often used as a basis for prediction. If prices could be predicted by static ratio analysis, the fundamentalist view would be that price was a function of key relationships encaptured in the ratios. The heuristic approach, on the other hand, would suggest that price was a function of symbols of performance provided by the ratios - but that the ratios need not be adequate representations of reality. Some ratios, for example those applied to judge bank liquidity and stability, are complex and require constant review of the variables to be allowed and disallowed in the computations. Users of such ratios are aware of their representativeness as only a rule-of-thumb guide to decision-making. The question of whether or not such ratios, if they formed part of a pricing equation, were heuristics or objective measures of performance, could be determined only by conducting further tests to assess the validity of the ratios as adequate measures of performance.

4. <u>Forces of supply and demand act on the market heuristics rather than the real economic good being transacted</u>

As mentioned, this thesis offers no empirical test of this condition in the excess of loss market. There are, however, reasonable grounds on which the assumption could be supported: namely that human beings determine market prices and also respond to forces of supply and demand according to their perceptions. If the good being transacted is too complex or unmeasurable for an objective description, a satisficing model of the good might become, for pricing purposes at least, the unit of trade. Sequential price changes, too, might occur according to a perceived satisficing model and since, from economic theory, price changes are a function of supply and demand, one could say that the forces had acted on the satisficing model, rather than the real world, and brought about the price changes. The implication is that at no stage in the process need the true nature of the economic good be completely understood by the participants in the market place. If one accepts this view as plausible, it is possible to further postulate the existence of a market which is perfect (with regard to a satisficing pricing model) yet, at the same time, imperfect (with regard to the true nature of goods being transacted).

The above characteristics of a heuristic pricing system do not combine to form a model which fits neatly into current general economic theories of market behaviour. A concept is introduced where decisions are made according to criteria which have no equivalent theoretical justification in traditional economics. The behavioural evidence from this thesis would, nevertheless, support the existence of such a system in the aviation excess of loss reinsurance market.

8.5 SUMMARY OF DISCUSSION

The discussion in this chapter centred on the findings of the preliminary analysis and simulation procedures undertaken in earlier chapters but extended to further issues in an attempt to present the importance of the findings in context with current knowledge and insurance industry problems.

Specific topics for discussion included the following:

(i) The limitations of techniques applied in the research;

(ii) The existence and nature of a satisficing decision model for excess of loss underwriters;

(iii) A comparison of results with previous investigations and hypotheses relevant to excess of loss reinsurance and the insurance of catastrophic risks;

(iv) The pratical difficulties (and suggestions for their removal) which would be encountered in applying the methodology used in the research as a basis for real-life underwriting plans;

(v) Areas for future research using the methodology presented in this thesis and speculation on consequent research findings;

(vi) A comparison of research findings with general underwriting practices for excess of loss reinsurance with special emphasis on the strengths and weaknesses of the system;

(vii) Suggestions for a market supervision system for monitoring firm performance from market indicators rather than from premium income and loss experience characteristics of individual firms operating in the market;

(viii) The comparison of a heuristic approach to pricing with pricing approaches from stock-market theory.

The conclusions from the discussion, along with the main research findings, are presented in the final chapter of this thesis.

CHAPTER 9

CONCLUSION

9.1 INTRODUCTION

The investigation into the underwriting of excess of loss reinsurance comprised a survey of relevant information and literature on excess of loss underwriters' problems, an empirical study of real-life underwriting behaviour, the construction and testing of an underwriting model based on observable market phenomena, and a discussion on pertinent points arising from the research. In this final chapter, a step-by-step presentation of the research aims, techniques, results, findings and the contribution made to knowledge is provided by way of conclusion.

9.2 PRESENTATION OF THE RESEARCH

1. A description of the London Reinsurance Market, its role and its functions, provided a background to the main research to be undertaken, specifically, into the underwriting of excess of loss reinsurance. The flexibility of the excess of loss reinsurance form was described with reference to basic clauses in the contract of indemnity. In particular, attention was paid to the suitability and use of excess of loss reinsurance for protection of (re)insurance companies against catastrophic events.

2. A conceptual description of reinsurance company objectives in relation to the main underwriting decision areas provided a general framework for the study of excess of loss underwriting problems. The most important areas for decision-making were identified as the rating and portfolio construction functions respectively, since they were key determinants of business success or failure, and closely linked to corporate objectives.

3. Theoretical and practical aids to decision-making for excess of loss reinsurance were described in the light of the literature and current market practice. The main stages of the survey were as follows:

   i) Various general approaches to excess of loss reinsurance problems were described, along with their limitations. Traditional economics, the behavioural theory in economics and the risk-theoretic approach were examined for their applicability to excess of loss underwriting and the view put forward that, although each had implications for excess of loss underwriting, none could provide adequate solutions to the mainly practical problems encountered in running the business from day-to-day.

ii) A need for an interdisciplinary approach aimed at application of various relevant disciplines to the study of excess of loss reinsurance decision-making was noted, before embarking on a more detailed description of the various analytical approaches which have been put forward. These analytical approaches ranged from crude judgemental analysis to intricate risk-theoretic models. Where possible, evidence from the market was included on the extent to which the methods described are actually used in the work of professional underwriters.

iii) Limitations to the various approaches to rating and portfolio construction were described. For the risk-theoretical approach the limitations were described as the problems of providing probability estimates for uncertain events and the consequent complexity of risk-models which rendered them unsuitable for practical decision-making purposes. For the judgemental approach, limitations described included lack of communicability and inadequacy of feedback, which detract from decision-makers' abilities to learn from past experience.

iv) The conceptual link between pricing and portfolio construction decisions was presented to illustrate the opportunities for dynamic portfolio planning, should the practical problems of a joint rating and portfolio-construction system ever be successfully overcome.

4. Needs for new knowledge were considered from the points of view of both professional underwriters and outside parties with interests in the excess of loss underwriting process. For professional underwriters, the remaining problems were described as impediments to improved business planning and portfolio efficiency. For other interested parties and, in particular, market supervisors, the remaining problems described also concerned aspects of efficiency, but in a larger, market-wide context.

Issues such as needs for profitability and portfolio stability for individual underwriters, when extended to the market-wide situation, become the determinants of market profitability and stability.

For both professional underwriters and market supervisors, increased measurability of the excess of loss underwriting process was deemed to be the most important aspect of the business requiring advancement of knowledge.

5. A proposal was made that measurable aspects of actual market behaviour should be investigated. Key aims became to model the excess of loss underwriting process from analysis of empirical data and to develop the model as an aid to underwriting decision-making. The success of this approach would be determined largely by the extent to which behavioural aspects of the excess of loss reinsurance process could be demonstrated as being measurable compared with the measurability of the underlying risk processes. The investigation would involve screening the excess of loss reinsurance process for the existence of a satisficing decision-model by which underwriters ordered their business affairs. It was decided that certain key variables on the excess of loss slip (and/or contract) should provide the basis of the empirical investigation due to their explicit measurability which rendered them firstly, likely to be included in underwriters' judgemental processes and secondly, susceptible to analysis.

6. A preliminary analysis was conducted into the relationship between key contract details and underwriting decisions. Data were collected from an underwriting agency conducting aviation excess of loss reinsurance in the London Aviation Excess of Loss Market. The analysis centered on establishing relationships between contract details, underwriting decisions and risk processes. A regression method for investigating the nature and quality of the relationships was devised and performed on the data. As an aid to description of findings from the preliminary analysis, it was decided that a coefficient of determination (r^2) of .5 or more should be deemed evidence of a reasonable relationship. On application of this subjective criterion for interpretation of results, the following points emerged:

i) Excess of loss premiums could be reasonably estimated from slip or contract details;

ii) The range of likely variations around the expected premium (estimated from contract details) could also be identified;

iii) Claims and (estimated) premium loadings held no reasonable relationship with excess of loss premiums;

iv) For one professional underwriter, portfolio selection decisions were shown to be a function of the total premiums for individual excess of loss contracts.

The above findings were analysed in respect of their implications for the existence of a satisficing model for excess of loss underwriting decisions. Needs for further research, aimed at incorporating relationships identified in the preliminary analysis into an underwriting model, were noted.

7. A theoretical model was presented as a vehicle for testing underwriting strategies based on relationships established in the preliminary analysis. A simulation approach was adopted in order to conduct sensitivity tests and evaluate the model for its potential as an underwriting planning tool. Data samples on which simulations were performed comprised those used in the premiminary analysis, and a further data sample to serve as a testbed for findings established on previous samples. The latter data sample also provided an opportunity to evaluate the usefulness of the model findings as a practical underwriting planning tool, and to set the simulation procedure in competition with a professional underwriter.

For all tests, the criterion for underwriting success was deemed to be the maximisation of the ratio-value of portfolio profitability (PROF) to portfolio stability (STAB) within a constraint (PAR) set by the market (or, more specifically, by the nature and availablity of underwriting opportunities).

8. From the simulations the following points emerged:

i) By simulating underwriting strategies aimed at maximising the amount of business written priced above the expected price (estimated from contract details), portfolio efficiency could be increased in terms of the ratio of PROF to STAB. This result was achieved on all one-year data samples.

ii) On a long-term (3-year) data sample, the procedure was less successful in that the ratio of PROF to STAB did not continuously increase with increases in the amount of business written priced above that estimated from contract details. Nevertheless, certain relationships noted on the short-term samples were duplicated in the long-term simulation: namely that both PROF and STAB increased with increases in simulated acceptances of business priced above the expected premium estimated from contract details.

iii) Sensitivity analysis revealed that the most important requirement in the simulation procedure for achieving the results described in 1) and 2) above, was the availability and statistical adequacy of a price estimator from contract details (MEST) calculated on a data sample on which underwriting strategy is to be simulated. Of lesser importance, in this respect, was the model of the underwriter's underwriting selection procedure (UPU) which was also used in the procedure.

iv) For comparing results of simulated optimum underwriting strategies with the results of a professional underwriter, it was necessary to make two assumptions which detracted from the realism of the simulation procedure as a planning tool. These assumptions were firstly, that an underwriter using the system would be equipped with a MEST for the period during which underwriting activity would take place and, secondly, that a method was available whereby an underwriter could order his contract acceptances in such a way as to take full advantage of underwriting opportunities yet not exceed the PAR constraint.

v) Even with the above assumptions, the professional underwriter achieved a more efficient portfolio, in terms of the ratio of PROF to STAB, than the simulated optimum strategy on the final data sample. This result ran contrary to those of previous simulations on earlier data samples, and explanations for this departure from the general trend of results were sought. Possible explanations included the following: firstly, the r^2 for MEST for the final data sample was low relative to r^2s for MESTs calculated on earlier data samples; secondly, differences between actual performance of the professional underwriter and the model of his performance (which served as a basis for strategy generation) were encountered because of the relative statistical inadequacy of estimated key variables used in the modelling process; thirdly, differences arose when the strategy-generating equations, based on the model of the professional underwriter's actual performance, were constructed, again due to the relative statistical inadequacies of key estimated variables. These three problems were present on the earlier simulations to a lesser degree but proved more critical on the final data sample. It is important to note, however, that, although the final simulation failed to improve on the professional underwriter's performance, the general result that both PROF and STAB increased with increased acceptance of contracts priced above the expected price (estimated from contract details) was duplicated; the rate of increase of PROF compared with that of STAB was the only departure from earlier simulation results.

vi) A retrospective comparison of the professional underwriter's performance, in terms of the ratio of PROF to STAB, with the optimum simulated strategy revealed that, on all but the final data sample, the optimum simulated strategy achieved the highest performance.

9) The preliminary analysis and the simulation procedures were discussed along with implications for their wider excess of loss reinsurance markets. Particular issues raised and examined during the discussion included the following:

i) The evidence from the preliminary analysis was largely supportive of the characteristics postulated as likely of a satisficing model for excess of loss underwriting decisions. It was apparent, however, that the characteristics of the satisficing model evidenced by the results constituted a poor reflection of reality in that they were poorly correlated with underlying risk processes.

ii) The results of the preliminary analysis were compared with findings from an earlier investigation into excess of loss reinsurance conducted by Ammeter (1955) and were deemed, generally, supportive of findings from the earlier study.

iii) The results of the preliminary analysis were considered for their implications for the theoretical model put forward by Stone (1975) which linked pricing and portfolio decisions for the underwriting of large, capacity-consuming insurance contracts. From the comparison, two important points emerged. Firstly, the required estimate of expected claims in Stone's model was deemed impractical since it would be subject to a large margin of error. Due to the key position of the expected claims variable in Stone's formula, the margin for error in estimating expected claims would need to be small in order to prevent large variations in the premium rates to be charged according to the hypothetical model. A second discrepancy between Stone's hypothetical model and the findings of the preliminary analysis concerned Stone's implication that individual underwriters exercised reasonable control over premiums charged for protection against large risks in order to reflect portfolio capacity needs in premiums charged. The evidence from the preliminary analysis in this respect was that individual underwriters are able to exercise only a small degree of influence over reinsurance premiums for large risks. Intuitive reasoning from the knowledge that individual underwriters must, necessarily, quote prices acceptable to the large number of other underwriters who jointly accept large contract placements would support this observation. Market forces were shown in the study to be more important in determining premiums than the risk structure of excess of loss portfolios constructed by individual underwriters operating in the market.

iv) Although the preliminary analysis demonstrated that expected market prices (estimated from contract details) could be reasonably estimated on most data samples, it was also shown that movements in the expected price level between underwriting periods behaved in a dynamic and unpredictable manner. Further, between different time periods, the statistical importance (betas) attached to the various elements from which the expected price level was estimated differed considerably. The price level for excess of loss reinsurance could, therefore, be described as demonstrating a high level of agreement between underwriters but being highly volatile between underwriting periods.

v) Limitations of the simulation procedures were described under two main headings: firstly, the need to achieve a balance between the conflicting aims of accuracy of representation and unnecessary complication in the modelling process; and secondly, problems encountered as a result of using estimated key variables for modelling and strategy-generation purposes. In the modelling and strategy-generating procedures, portfolio selection was simulated using mathematical equations rather than judgemental underwriting skills yet, on balance and within the scenario represented by the model, optimal strategies were successfully identified which, in the majority of cases, produced more efficient portfolios than those achieved by a professional underwriter. It is important to note, however, that the success of the method depended, not on deductive reasoning alone, but on the application of reason to measurable aspects of real-life underwriting behaviour. In other words, the simulation procedure presented in the thesis could augment but never replace the decision processes of excess of loss underwriters.

vi) A specific limitation of the simulation procedure as a practical underwriting tool was selected for further discussion, namely the need for an adequate price estimator (MEST) for the period during which planned underwriting activity is to take place. Planning is essentially a future-oriented business but, from the preliminary analysis, it was demonstrated that the likelihood of arriving at an adequate MEST for a future underwriting period was low. Further, the simulation procedure demonstrated that the use of an inadequate MEST in the strategy generating procedure leads to highly variable results. Attention turned to the possibility of achieving a suitable current (rather than future) MEST. The statistical problems involved in arriving at such an estimate were described as those of finding a representative data sample on which to calculate the estimate and constructing a statistical indicator to assess its universality.

vii) The applicability of the methodology employed in the research on aviation data to other areas of the excess of loss market was considered. An important requirement for success in this respect was deemed to be a knowledge of what underwriters themselves perceive as a market class (for selection of data purposes), since market pricing behaviour need not necessarily fall neatly under the headings described in legislative documents or text-books. A potentially fruitful area of research was suggested as the application of the methodology to motor excess of loss business where risk processes are probably more measurable than in the aviation excess of loss market. From such a study, evidence on the importance of a satisficing pricing model when risk processes are, relatively, measurable would be gained. The relative desirabilities to professional underwriters of either a satisficing or a risk-based pricing system would thereby be investigated.

viii) Some arguments were put forward for why professional underwriters might be content to use a pricing system based on heuristics and a satisficing pricing model. The most important argument was that, as long as the heuristics were similar for all underwriters, an informal price control system could operate to the common good. Certain dangers could arise from such a pricing system, however, in situations where individual underwriters failed to be aware of the latest information on market prices, or where the satisficing pricing model differed greatly from more objective premium measures based on risk processes.

ix) Suggestions were put forward for improving measurement and control over the excess of loss reinsurance market by market supervisors. The basis of the suggestions was that, since market heuristics were demonstrated in the research as being more measurable than underlying risk processes, more attention should be paid to the possibility of incorporating the latter, more measurable phenomena, in market monitoring systems. A basic structure for a market monitoring system, based on analysis of firm performance via market indicators, was outlined and its potential and limitations for market supervision evaluated.

x) The final topic for discussion was a comparison of the heuristic approach to market prices evidenced in the research, with more familiar approaches to the explanation of prices and their movements from stock-market theory. The conclusion was that the approach applied in the research did not fit neatly under any single heading of the more familiar stock-market approaches but evidenced an alternative approach which could be applied, in particular, to markets characterised by cost uncertainty. The bare essentials of a heuristic approach to price determination were described in the light of the research findings, and their economic implications examined. The most important implication for economic theory was that, in markets characterised by cost uncertainty, the forces of supply and demand are likely to act on perceived goods and services rather than real ones -and this could have important consequences for the efficient allocation of resources. In such situations, a strong case for effective market supervision can be made to reduce the dangers of market instability traceable to excessive differences between real and perceived units of trade.

9.3 THE CONTRIBUTION MADE TO KNOWLEDGE

The contribution to knowledge from the research is as follows:

1. The research provides the first, detailed study of underwriters' decision processes for underwriting aviation excess of loss reinsurance.

2. The practical application of risk-theoretical techniques for aviation excess of loss underwriting is shown to be negligible among professional underwriters.

3. The existence and nature of a satisficing model for excess of loss underwriting decisions is investigated.

4. A potential excess of loss underwriting system, based on the use of market heuristics for portfolio selection decisions, is described, tested and evaluated.

5. Impediments to the efficiency and effective supervision of excess of loss reinsurance markets, traceable to differences between real and perceived units of trade, are investigated and suggestions put forward for their removal.

APPENDIX A

THE EXCESS OF LOSS REINSURANCE CONTRACT

Selected clauses from typical excess of loss reinsurance contracts are listed in this appendix with brief explanatory notes. Although the list is not exhaustive it is, nevertheless, comprehensive and provides an insight into the key matters for agreement between a ceding company and its excess of loss reinsurers in the contract-drafting process. Contract clauses listed below are reproduced from selected contracts used in the market.

1) The Reinsuring Clause

e.g. "This reinsurance is only to pay the excess of an ultimate net loss to the reinsured of £xxxx each and every loss with a limit of liability to the reinsurers of £xxxx each and every loss".

Purpose: Definition of monetary limits of indemnity

2) Period Clause

e.g. "This reinsurance covers all losses as herein defined occurring during the period commencing with the 01.01.81 and ending with the 31.12.81 both days inclusive, local standard time at the place where the loss occurs".

Purpose: Definition of time-period during which the agreement is binding.

3) Losses Discovered or Claims Made Clause

e.g. "It is understood and agreed that as regards losses arising under policies and/or contracts covering on a "losses discovered" or "claims made" basis, that is to say, policies and/or contracts in which the date of discovery of the loss or the date when the claim is made determines under which policy or contract the loss is collectable, such losses are covered hereunder irrespective of the date on which the loss occurs provided that the date of the discovery of the loss, in respect of policies and/or contracts on a "losses discovered" basis or the date the claim is made, in respect of policies and/or contracts on a "claims made" basis, falls within the period of this reinsurance.

For the purpose of the foregoing the date of the first discovery of a loss occurrence or the date a claim is first made, shall be the date applicable to the entire loss and the reinsurers shall be liable for their proportion of the entire loss irrespective of the expiry date of this reinsurance provided that such date falls within the period of this reinsurance".

Purpose: Limitation of the recovery procedure to claims made or discovered between two specified dates. An alternative clause may be included which limits claims to those incurred as a result of policies issued by the reinsured between two specific dates -in which case the reinsurance would be said to apply on a "policies attaching" basis.

4) Ultimate Net Loss Clause

e.g. "The term "Ultimate Net Loss" shall mean the sum actually paid by the reinsured in settlement of losses or liability after making deductions for all claims upon other reinsurances and shall include all adjustment expenses arising from the settlement of claims other than the salaries of employees and the office expenses of the reinsured. All salvages, recoveries or payments recovered or received subsequent to a loss settlement under this reinsurance shall be applied as if recovered or received prior to the aforesaid settlement and all necessary adjustments shall be made by the parties hereto.

It is understood and agreed that recoveries under all underlying excess reinsurance treaties and/or contracts (as far as applicable) are for the sole benefit of the reinsured and shall not be taken into account in computing the ultimate net loss or losses in excess of which this reinsurance attaches, nor in any way prejudice the reinsured's right to recovery hereunder".

Purpose: The term "ultimate net loss" restricts claims to the amount of sums actually paid by the reinsured in settlement of losses or liability after making deductions for all recoveries, salvages and claims upon other insurers.

5) Definition of "Each and Every Loss"

e.g. "For the purpose of this reinsurance the term each and every loss shall be understood to mean each and every loss and/or occurrence and/or disaster and/or catastrophe and/or calamity and/or series of loses and/or occurrences and/or disasters and/or catastrophes arising out of one event".

Purpose: This phrase is important because it places no restriction on the number of claims that can be recovered by the cedent under the contract with respect to one event. The only stipulation in the clause is that the claims must arise as a result of the single event in question.

6) Premium Clause

e.g. "The premium for this reinsurance shall be calculated x.xx% of the reinsured's net premium income on the business protected "accounted for" during the period of this reinsurance, subject however to a minimum and deposit premium of £xxxx payable quarterly in advance adjusted as soon as practicable after expiry. The term "net premium income" shall be understood to mean gross premiums less commission, brokerage and profit commission, cancellations and return premiums and less premiums given off by way of reinsurance, recoveries under which incur to the benefit of the reinsurers hereon, and after deduction of premium in respect of business excluded from the protection of this reinsurance".

Purpose: This clause determines the amount and timing of premium payments. The working of a minimum deposit payment system is common in excess of loss contracts. Alternative clauses may specify fixed payments with no opportunity for adjustment.

7) Reinstatement Clause

e.g. "In the event of loss or losses being paid under this reinsurance it is mutually agreed to reinstate this reinsurance up to x full reinstatements of the limits hereof from the time of the occurrence of such loss or losses until expiry of this reinsurance and that an additional premium calculated at pro rata of xx% of the annual premium hereon shall be paid by the reinsured when any loss or losses (or part thereof) requiring such reinstatement are settled".

Purpose: The reinstatement clause provides continuity in the agreement between ceding company and reinsurer. Should a catastrophic claim occur using up all the indemnity afforded by the cover, this clause provides for automatic renewal of cover and an additional premium payment.

8) Exclusions

e.g. "This reinsurance does not cover:

A) Loss or damage caused by war and/or civil war.

B) Financial guarantee or insolvency.

C) Life business other than accidental death and dismemberment".

Purpose: There are likely to be classes of business within a ceding company's portfolio which the reinsurer is unwilling to include in the protection. These are listed in the exclusions clause.

9) Notice of Loss Clause

e.g. "In the event of a claim arising hereunder notice shall be given to the reinsurers through xxxxxxxxx Ltd., as soon as practicable, and all papers in connection therewith shall be at the command of the reinsurers on this reinsurance or parties designated by them for inspection".

Purpose: This clause aims at securing timely notification of claims requiring immediate or future payment by the reinsurers. Details of claims and probable future payments are essential for reinsurers who use the information to plan cash flows and monitor adequacy of reserves.

10) Reinsurance Clause

e.g. "This reinsurance shall be deemed to be subject to the same terms, clauses and conditions as far as they may be applicable hereto and shall pay as may be paid thereon, but subject nevertheless to the terms and conditions of this reinsurance".

Purpose: By this clause, subject to conditions specified in other clauses, the reinsurer agrees to "follow the fortunes" of the ceding company on all the policies in the reinsured portfolio.

11) Other Clauses

Besides the above, a wide range of other clauses are to be found on excess of loss reinsurance contracts. They include the following:

a) Extension of Protection Clause. This provides for an extension of cover should the liability lapse part way through a catastrophic occurence.

b) Currency Conversion Clause. Details of exchange rates are provided for premium and/or claims payments where more than one currency is involved.

c) Inspection of Records Clause. This clause gives the reinsurer permission to examine the ceding company's books should it desire. This may be necessary in the event of a 'false' claim.

d) Ammendments and Alteration Clause. By this clause, the possibility of future endorsement to the contract by agreement of concerned parties is sanctioned.

e) Errors and Omissions Clause. Inadvertent delays, omissions or errors affecting the contract are, by this clause, held not to relieve parties from their responsibilities under the contract.

f) Arbitration Clause. This clause provides a somewhat complicated procedure for settlement of disputes arising from the contract. Arbitrations for U.K. issued contracts are governed by the provisions of the Arbitration Acts 1950 and 1975 and are usually held in London.

APPENDIX B

ABSOLUTE DATA

Contents

1) Absolute Data
2) Currency conversion rates

The data contained in Appendix B represent the details copied from the underwriter's records after adjustment for currency changes and (for reported claims amounts) outstanding payments. A blank space indicates that a particular item of data was not recorded in the underwriter's records.

The data are arranged in columns as follows:

First column (P). The numbers in this column are for identification purposes only. Each number is the underwriter's code for a particular excess of loss contract. The year in which the contractual agreement begins, in each case, can be determined from the first digit of the contract number as follows:

1974 contracts begin with a 1;
1975 contracts begin with a 2;
1976 contracts begin with a 3;
1977 contracts begin with a 4.

Second column (W). This column lists the percentage of each contract that the underwriter is willing to accept i.e. his written premium percentage.

Third column (M). This column lists total (£) premium payable under each contract (specified in the contracts as either a fixed sum or a Minimum Deposit Premium).

Fourth column (U). This column lists the upper limit of liability (£) for which excess of loss reinsurers are liable on each contract.

Fifth column (L). The lower limit (£) or the point at which reinsurers' liability commences is listed in this column for each contract.

Sixth column (E). These data are the estimated premium income (£) for each contract. Where the reinsured did not provide an estimate of premium income for his portfolio, the figure in column E is calculated by dividing the minimum and deposit premium by the adjustment rate specified in the contract.

Seventh column (T). These figures represent the total £ amount of claims incurred on each excess of loss agreement during the year covered by the contract.

Eighth column (N). The number of claims reported under each one-year contract are shown in this column.

Ninth column (C). These figures are calculated as T/N and represent the average, one-off claims amount (£) claimed under each contract.

1) Absolute data

P	W	M	U	L	E	T	N	C
10144	18.750	22901	37500	80000	1145050	13427	1	13427
10284	15.000	30000	200000	75000	1500000	180608	1	180608
10285	15.000	30000	200000	75000	1090909	135336	1	135336
10346	6.000	7500	37500	41500	2000000	0	0	0
10441	15.000	73187	1666666	1666666	2661345	0	0	0
10570	13.000	5000	50000	50000	400000	0	0	0
10648	43.000	16595	26000	37500	1037188	26000	1	26000
10794	20.000	189008	120000	80000	2100089	56999	2	28500
10795	15.000	10000	200000	200000	444444	0	0	0
11417	10.000	2499	95000	100000	20825	0	0	0
12738	14.000	24693	240000	160000	274367	0	0	0
12962	10.000	5692	10000	5000	284600	0	0	0
12963	10.000	6692	25000	7500	669200	7830	1	7830
12996	6.000	10000	137500	37500	285714	46085	1	46085
13119	7.500	4000	33333	9000	320000	6971	2	3489
14133	12.500	15265	48438	39063	381625	48438	1	48438
14135	3.000	67410	40000	30000	1685250	35833	2	17917
14153	4.000	9672	120000	80000	644800	0	0	0
14241	4.000	43940	333333	583333	2510857	0	0	0
14309	10.000	10212	150000	58333	240282	0	0	0
14766	10.000	20080	37500	4000		5965	2	2983
14801	10.000	3395	40000	17500	271600	0	0	0
14836	15.000	8277	100000	133333	384977	0	0	0
14837	3.500	5277	141667	233333	351800	0	0	0
14853	15.000	17573	80000	20000	78102	0	0	0
14888	7.000		40000	6500		2922	1	2922
14902	7.000		37500	16000			1	
15034	17.840	23226	178750	77500	1661968	178750	1	178750
15048	4.500	34328	600000	400000	274624	0	0	0
15065	3.000	3707	85543	384944		0	0	0
15073	7.000		120000	120000		147947	1	147947
15074	7.000		116667	233333		80567	1	80567
15080	10.000	66193	250000	333333	2941911	0	0	0
15129	10.000	3000	100000	100000	200000	0	0	0
15144	6.000	7000	150000	150000	175000	0	0	0
15171	7.500	6700	65000	40000	270800	44965	1	44965
15187	7.500	4770	125000	125000		0	0	0
15218	10.000	27344	300000	400000	1885793	93939	1	93939
15211	7.000							
15212	12.500	54772	40000	40000		0	0	0
15218	7.500	6119	166667	175000	470692	0	0	0
15221	10.000	85901	50000	75000	2863367	19847	4	4962
15240	10.000	7988	50000	45000	285286	0	0	0
15272	10.000	16976	158333	93333	722383	0	0	0
15274	33.000	3750	42500	120000		0	0	0
15283	2.500	62079	400000	450000	2759067	0	0	0
15331	20.000	28318	125000	85000	1132720	0	0	0
15342	7.500	6080	100000	200000	115810	0	0	0
15354	2.500	25000	125000	125000	2083333	15381	1	15381
15357	3.000	21424	333333	350000	952178	27415	1	27415
15358	7.500	27500	400000	600000	2200000	734238	1	734238
15364	5.000	57410	500000	500000	3588125	50000	1	50000
15369	7.500	3395	125000	42333	339500	0	0	0

continued

P	W	M	U	L	E	T	N	C
15376	12.500	8277	100000	100000		0	0	0
15406	7.000							
15411	2.500	31039	250000	600000	2699043	0	0	0
15436	10.000	12699	40000	40000	253980	40000	1	40000
15437	15.000	13500	75000	75000	519231	61103	1	61103
15439	2.000	85543	320787	213858	2737376	320787	1	320787
15440	7.000		535000	535000		94247	1	94247
15443	5.000	27386	240000	96000	3221882	0	0	0
15460	10.000	5255	75000	50000	150143	0	0	0
15474	5.000	33952	250000	250000	4244000	0	0	0
15485	7.500	74157	150000	100000		0	0	0
15492	4.000	27109	250000	50000	1084360	0	0	0
15501	2.500	4277	651574	1086929	305500	0	0	0
15504	3.000	8000	75000	275000	3106796	0	0	0
15508	8.250	5133	74850	21386	62218	2473	1	2473
15515	1.250	19778	75000	70000	1582240	0	0	0
15520	0.175	3000	100000	350000	1714286	0	0	0
15525	7.000	11228				7811	1	7811
15538	10.000	37940	10000	10000	379400	0	0	0
15539	7.000							
15541	2.000	15840	40000	30000	792000	0	0	0
15542	3.330	22831	150000	70000	685616	0	0	0
15567	2.500	27109	250000	50000	1084360	0	0	0
15571	1.080	35821	171000	86400	3316759	171000	1	171000
15572	7.000					267687	1	267687
15576	3.000	46356	200000	140000	1545200	83584	1	83584
15579	0.600	51000	150000	300000	8500000	0	0	0
15583	4.500	41386	125000	65000	919689	0	0	0
15599	2.000	56905	500000	850000	2845250	0	0	0
15603	7.000	11500	150000	220000		0	0	0
15614	3.000	10000	166667	63333	333333	0	0	0
20284	2.500	40000	200000	75000	1600000	0	0	0
20285	2.550	35000	150000	100000	1372549	0	0	0
20346	0.375	7873	37500	41500	2099467	0	0	0
20570	1.250	4000	50000	50000	320000	0	0	0
20648	1.600	17166	40000	37500	1072875	0	0	0
20794	6.000	173809	160000	120000	2896817	0	0	0
20795	2.000	47913	200000	280000	2395650	0	0	0
21109	0.325	6952	120000	120000	21308	0	0	0
21417	12.000	4778	100000	100000	39817	0	0	0
22738	10.000	25150	225000	525000	251500	0	0	0
22962	2.000	5871	10000	7500	293550	0	0	0
23119	1.250	9000	33333	9000	720000	0	0	0
24133	5.850	34532	59375	46875	50291	0	0	0
24135			55000	41000		23500	1	23500
24153	1.500	9798	120000	80000	652600	0	0	0
24241	1.650	41938	333333	583333	2541697	0	0	0
24766	7.000	20211	37500	4000		24580	1	24580
24801	1.250	3498	40000	17500	279840	0	0	0
24836	3.500	8460	241667	133333	241714	0	0	0
24853	18.000	7273	64000	20000	40406	0	0	0
25034	2.250	25420	178750	77500	1129778	0	0	0
25048	12.500	34942	500000	400000	279536	0	0	0
25069	1.400	18487	116667	233500	1320500	0	0	0
25080	2.250	68171	250000	333333	3029822	0	0	0
25171	2.850	7537	80000	40000	264456	0	0	0

continued

P	W	M	U	L	E	T	N	C
25187	7.000	5660	125000	125000		0	0	0
25211	1.450	3631	400000	480000	250414	0	0	0
25212	7.000	9201	60000	60000		0	0	0
25218	1.750	574	208333	212500	32800	0	0	0
25221	1.250	7934	50000	75000	634720	50000	1	50000
25240	2.500	829	50000	45000	33160	0	0	0
25274	7.000	1250	42500	120000		0	0	0
25283	1.150	1645	250000	600000	143043	0	0	0
25331	2.250	14451	125000	125000	642267	0	0	0
25342	5.250	368	100000	200000	7010	0	0	0
25354	2.350	684	150000	100000	29106	0	0	0
25357	2.050	561	333333	350000	27366	0	0	0
25364	5.000	84228	500000	500000	3369120	0	0	0
25369	7.400	3498	125000	38333	349800	0	0	0
25376	12.500	8460	100000	100000	338400	0	0	0
25411	5.000	58165	400000	450000	2585111	21125	1	21125
25436	11.000	13029	40000	40000	400892	0	0	0
25439	6.000	107047	312500	205333	1784117	0	0	0
25440	1.000	55754	520833	520833	2027418	0	0	0
25443	5.000	26341	240000	240000	2634100	0	0	0
25460	10.000	992	75000	50000	28343	0	0	0
25469	27.500	2000	17500	20000		0	0	0
25474	5.500	34978	250000	250000	4372250	0	0	0
25486	5.000	24600	400000	100000		0	0	0
25492	4.000	29956	250000	50000	1331377	0	0	0
25501	2.500	87500	625000	1041667	6250000	0	0	0
25503	15.000	4211	40000	2000	240629	0	0	0
25504	1.000	18000	250000	250000	2769231	0	0	0
25508	33.333	7806	70883	20833	94618	0	0	0
25513	10.000	2342	100000	40000	117100	0	0	0
25520	16.000	2500	100000	450000	1785714	0	0	0
25525	11.000	11708	1250000	2500000	1377412	0	0	0
25538	35.000	21841	10000	10000	218410	0	0	0
25567	4.000	29956	250000	50000	1331378	0	0	0
25571	10.000	98905	560000	240000	3090781	0	0	0
25574	8.160	1088	150000	437500		0	0	0
25576	21.500	47913	200000	140000	1597100	0	0	0
25598	2.960	50762	500000	850000	2538100	0	0	0
25610	5.000	65682	1000000	1000000	2404632	0	0	0
25617	8.000	5000	125000	75000	222222	0	0	0
25621	37.500	41647	116667	33333	1665880	23105	1	23105
25626	10.000	40000	50000			0	0	0
25627	65.000	17841	10417	10417	402051	0	0	0
25630	2.000	15000	175000	275000	1666667	0	0	0
25637	16.500	9375	25000	20000		0	0	0
25649	0.244	52819	1500000	1500000	4225520	0	0	0
25651	22.000	4597	60000	30000	57463	0	0	0
25653	6.000	8690	50000	50000	217250	0	0	0
25655	6.000	31222	354167	62500		0	0	0
30284	22.000	20642	150000	150000	1179543	0	0	0
30285	23.000	26503	120000	120000	1514457	0	0	0
30346	7.000	9973	37500	41500		0	0	0
30580	4.374	7000	100000	100000	560000	0	0	0
30794	27.000	71262	150000	150000	3563100	300000	2	150000
30795	21.000	54009	300000	300000		253916	1	253916
31109	34.000	6801	150000	150000	2267000	0	0	0

continued

P	W	M	U	L	E	T	N	C
32738	21.000	27753	225000	175000	277530	134097	1	134097
32962	10.000	27292	10000	7500	1467312	20000	2	10000
32963	12.000	7891	25000	7500	876778	0	0	0
32996	16.660	7501	60000	45000	468813	0	0	0
33119		5621	40000	9000	449680	3399	1	3399
33924						20828	1	20828
34133	5.640	45695	75250	56250	1087976	11463	1	11463
34135	3.000	48258	56000	42000	1286880	0	0	0
34153	4.000	11152	170000	110000	697000	0	0	0
34241	8.000	53759	625000	750000	2389289	0	0	0
34309	10.000	20501	150000	60000	482376	0	0	0
34766	12.000	26201	37500	4000		35705	1	35705
34801	11.000	13081	40000	17500	1046480	16205	1	16205
34836	16.000	4476	50000	50000	358080	0	0	0
34902	10.000	2976	10000	10000		0	0	0
35034	22.500	81014	315000	37500	2025350	0	0	0
35048	13.000	38429	500000	400000	307432	0	0	0
35065	3.000	20000	125000	333333		0	0	0
35074	13.806	10992	58334	233333	785143	0	0	0
35080	20.000	88764	500000	250000	2367	0	0	0
35171	12.000	8751	80000	40000	307053	0	0	0
35187	8.250	17501	125000	125000		0	0	0
35211	15.000	37957	500000	600000	2617724	100932	1	100932
35212	18.000	56760	75000	75000		0	0	0
35218	2.500	26879	750000	377500	1343950	0	0	0
35220	50.000	1900	60000	40000	12667	0	0	0
35221	11.00	121897	50000	75000	4733864	62190	1	62190
35240	12.000	21161	50000	45000	813885	0	0	0
35283	5.000	32506	312500	750000	2167067	135501	1	135501
35287	7.000	67813	303572	71429		0	0	0
35331	12.000	26879	150000	150000	1264894	150000	1	150000
35336						36243	1	36243
35354	2.500	44706	150000	100000	2079349	0	0	0
35357	4.447	372	333333	350000	719220	0	0	0
35364	4.250	107517	750000	750000	5000791	0	0	0
35369	5.500	8931	200000	49000	637929	0	0	0
35376	12.500	9501	100000	100000	422267	0	0	0
35411	5.000	65012	500000	562500	2889822	21813	1	21813
35437	16.500	26879	75000	75000	1194622	0	0	0
35440	1.000	125000	520833	520833	5000000	0	0	0
35443	5.000	35855	430000	430000	3585500	0	0	0
35469		2500	17500	20000		0	0	0
35474		40806	200000	300000	4411459	215259	2	107630
35492	4.000	28105	250000	400000	1405250	0	0	0
35501	2.500	87500	625000	1000000	6250000	0	0	0
35564	2.500	29254	200000	300000	1772970	0	0	0
35508	34.000	5501	93520	27506	110020	0	0	0
35515	5.000	16378	125000	98000	1310240	0	0	0
35525	11.000	13203	1250000	2500000		0	0	0
35538	100.000	23679	12500	10500	295988	31396	3	10465
35541			62000	15500		42443	4	10611
35567	2.020	28105	250000	50000	1405250	0	0	0
35571	9.227	114717	560000	240000	3584906	0	0	0
35574	5.000	4648	250000	550000		0	0	0
35576	21.500	54009	200000	140000	2160360	0	0	0
35579	4.000	107517	625000	1250000	6476927	625000	1	625000
35599	5.500	54009	625000	1062500	2700450	0	0	0
35603	2.500	23582	400000	150000	786067	0	0	0

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P	W	M	U	L	E	T	N	C
35610	5.000	74009	1000000	1000000	2508780	0	0	0
35617	6.000	7000	200000	100000	28000	0	0	0
35621	28.750	61259	375000	160000	1884892	13557	1	13557
35626	10.000	17879				0	0	0
35627	65.000	22005	15625	15625	517765	0	0	0
35637	18.000	15500	25000	20000		245	1	245
35649	0.272	129622	2750000	2250000	9258714	0	0	0
35651	22.000	5297	60000	30000	66213	0	0	0
35673	17.000	7001	50000	100000	560080	0	0	0
35674	8.500	29030	240000	480000	2150370	0	0	0
35682	25.000	4988	20000	20000		0	0	0
35684	22.000	5801	30000	20000	232040	0	0	0
35685	2.500	21653	240000	240000	1312303	0	0	0
35689	25.000	21503	18750	37500	1264882	13860	2	6930
35697	2.570	16052	583333	2750000		0	0	0
35699	5.000	11952	50000	13333	543273	0	0	0
35708	21.500	1625	27500	27500	162500	0	0	0
35709	11.500	4851	90000	75000	161700	0	0	0
35710	1.890	54009	375000	437500	1661815	0	0	0
40024						666	1	666
40037	4.000	95835	6250000	1250000	5989688	0	0	0
40038	2.000	50138	8250000	3750000	5277684	0	0	0
40055	11.000	21748	60000	45000	724933	0	0	0
40056	11.000	18234	120000	105000	759750	0	0	0
40057	3.000	24028	600000	225000	800933	0	0	0
40058	5.500	4727	375000	825000		0	0	0
40085	10.000	47179	71250	56250	1179475	71250	1	71250
40094	10.000	7017	100000	100000	623733	100000	1	100000
40095	22.100	33679	165000	165000	2172839	113887	1	113887
40096	23.000	26944	132000	132000	1378323	97896	1	97896
40125	10.000	6734	37500	12500	612182	798	1	798
40126	7.500	8307	325000	5000	565102	0	0	0
40129	3.000	36229	330000	330000	2195697	308950	1	308950
40130	3.000	28983	264000	264000	1756545	2969	1	2969
40132	17.000	20734	56250	112500	1884909	0	0	0
40133	8.500	29972	270000	540000	2606261	0	0	0
40143	20.000	7161	30000	20000	477400	30000	1	30000
40161	17.500	27752	75000	75000	1850133	114051	2	57026
40170	7.500	118009	777778	133333	3933633	0	0	0
40171	5.000	36950	333333	133333	3695000	285919	1	285919
40178	17.000	47697	250000	56250	1799887	209602	2	104801
40179	17.000	27017	372500	306250	1801133	224177	1	224177
40180	25.875	65872	211111	97778	2311298	200369	2	100185
40186	3.000	46904	56000	42000	1379529	56000	1	56000
40188	5.000	177615	625000	25000	4440375	625000	1	625000
40189	5.000	111009	875000	875000	5163209	693148	1	693148
40190	2.000	55505	1000000	1750000	5550500	0	0	0
40191	6.000	7521	200000	100000	300840	0	0	0
40192	5.000	16901	125000	98000	1352080	125000	1	124000
40194	6.000	114374	300000	200000	3267829	333458	1	333458
40195	2.000	63073	716743	716743	3409351	573394	1	573394
40196	2.500	47305	860092	1433486	3784400	0	0	0
40199	12.000	15321	40000	40000	510700	40000	1	40000
40202	30.000	16651	20000	20000	456192	0	0	0
40203	18.000	7734	70000	52941	515600	12193	1	12193
40207	10.000	42774	400000	600000	2036857	400000	1	400000
40214	17.500	36170	175000	175000	2127647	140974	1	140974
40227	7.500	100000	375000	375000	4000000	0	0	0

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P	W	M	U	L	E	T	N	C
40228	10.000	35000	375000	750000	4117647	0	0	0
40229	3.000	40000	1125000	2250000	4000000	0	0	0
40230	10.000	7234	225000	90000	192907	0	0	0
40231	30.000	11250	50000	50000		0	0	0
40244	3.000	17202	172018	458717		155339	1	155339
40248	5.000	11454	170000	170000	881077	0	0	0
40254	12.500	5714	100000	100000	207782	4482	1	4482
40263	60.000	21502	15625	15625	537550	14119	1	14119
40264	11.000	13188	716743	3440367	1318800	35541	1	35541
40278	6.000	10601	69444	69444	311794	24571	1	24571
40279	5.000	5300	180555	138888	249418	0	0	0
40289			150000	100000		419524	2	209762
40349	12.500	8021	50000		291673	11899	1	11899
40352	30.000	2874	57175	14285		0	0	0
40370	10.000	6859	150000	150000		0	0	0
40377	19.400	1720	98837	79070		0	0	0
40393	10.000	24335	150000	25000	243350	0	0	0
40394	15.500	8671	165000	495000	21677500	0	0	0
40395	16.500	6937	132000	396000	1734250	0	0	0
40396	2.500	12867	1000000	1000000	171560	0	0	0
40419	27.000	73706	150000	150000	1243533	0	0	0
40420	21.000	77339	300000	300000	3437288	0	0	0
40421	12.000	47638	500000	600000	3285379	0	0	0
40425	5.000	8611	286697	659504	956778	0	0	0
40428	30.000	11424	150000	150000	3808000	0	0	0
40432	5.000	14335	500000	875000	191133	0	0	0
40433	15.000	48928	75000	75000		0	0	0
40443	1.500	16250	750000	123000		0	0	0
40444	2.500	78440	500000	562500	2852364	0	0	0
40445	2.500	37537	312560	750000	2729964	0	0	0
40446	3.000	59989	625000	1062500	2999450	0	0	0
40447	0.770	111009	3250000	2750000	6938063	0	0	0
40448	100.000	25936	12500	12500	345813	0	0	0
40477	20.000	98108	500000	250000	3164774	0	0	0
40478	8.000	61055	625000	750000	3052750	0	0	0
40484	3.000	24514	560000	560000	306425	0	0	0
40490	5.000	28855	1000000	2300000	3847333	0	0	0
40501	2.500	80206	1714286	3142857	6683833	0	0	0
40507	7.500	222018	250000	250000	8073382	231978	2	115989
40508	3.000	138211	1000000	500000	8130059	0	0	0
40525	5.000	10034	243693	522936	250850	0	0	0
40529	18.000	28412	25000	20000		25000	1	25000
40530	12.000	7922	50000	45000	304692	0	0	0
40531	2.500	37266	75000			0	0	0
40542	5.000	16101	329702	573394		0	0	0
40551	2.500	6628	340596	340596	315619	0	0	0
40552	5.000	18335	143349	172018	110054	0	0	0
40553	5.000	10028	143349	315367	111422	0	0	0
40554	1.800	43404	1250000	2750000	7432000	0	0	0
40576	15.000	28335	270000	210000	283350	0	0	0
40577	12.000	39209	600000	480000	348524	0	0	0
40581	12.000	10564	37500	4000		0	0	0
40582	7.000	11211	37500	41500	3449538	0	0	0
40592	10.000	8601	20000		430050	19452	1	19452
40593	15.000	17202	75000		458720	3864	1	3864
40594	10.000	6000	200000	100000	960000	0	0	0

2) Currency Conversion Rates

The currency conversion rates for £ to $ used in preparation of the absolute data are as follows:

YEAR	$ equivalent of £1 sterling*
1970	2.4800
1971	2.4800
1972	2.4800
1973	2.4850
1974	2.3380
1975	2.2420
1976	1.8178
1977	1.7440
1978	1.9310
1979	2.0560

*These rates are the average yearly currency conversion rates used by Ringstead Insurance Services Ltd., the service company supporting Trimark Ltd. for accounting purposes, for each year of account.

APPENDIX C

RESULTS OF THE PRELIMINARY ANALYSIS

Contents

For 2. to 8. above, the best-fit equations are achieved using the method described in Chapter 6 on the following data samples:

1974 data from Appendix B
1975 data from Appendix B
1976 data from Appendix B
1977 data from Appendix B
1974-6 data from Appendix B
1974-7 data from Appendix B

The equations are presented in the following order:

First: equations calculated on 1974, 1975, 1976, and 1974 to 1976 data samples respectively. These estimators are presented and discussed in the preliminary analysis and are used in Simulations 'A' to 'E';

Second: equations calculated on 1977 data for use in the 'F' Simulations;

Third: equations calculated on a 1974 to 1977 data sample. These are not specifically referred to in the text but are included here as additional items of information.

1) Glossary of terms for Appendix C

C	-	Claims incurred (column 9 from Appendix B)
E	-	Estimated premium income for ceding company (column 6 from Appendix B). Note that E followed by - or + and a numerical value means "exponential" rather than "estimated premium income"
K	-	The estimated loading element in total premium for a contract calculated as K = M-C-MDIFA
L	-	The lower limit of liability afforded by a contract (column 5 from Appendix B)
M	-	The total premium payable for an excess of loss contract (column 3 from Appendix B)
MDIFA	-	The estimated price variation element of total premium for a contract. MDIFA is calculated as the difference between MEST and M where MEST is the estimate of M from U, L and E.
MW	-	The premium written by the test underwriter calculated as MW = M/100 x W
U	-	The upper limit of protection afforded by an excess of loss contract (column 4 from Appendix B)

LOGC	-	Log to base 10 of C
LOGE	-	Log to base 10 of E
LOGK	-	Log to base 10 of K
LOGL	-	Log to base 10 of L
LOGM	-	Log to base 10 of M
LGMDIFA	-	Log to base 10 of MDIFA
LGMW	-	Log to base 10 of MW
LOGU	-	Log to base 10 of U
RCPC	-	The reciprocal of C
RCPE	-	The reciprocal of E
RCPK	-	The reciprocal of K
RCPL	-	The reciprocal of L
RCPM	-	The reciprocal of M
RPMDIFA	-	The reciprocal of MDIFA
RCPMW	-	The reciprocal of MW
RCPU	-	The reciprocal of U

*	-	"Multiplied by"
**	-	"to the power of"

2) Estimation of total premium from contract details

Best-fit premium estimator for 1974 data sample					
Dependent variable = RCPM					
Variable	B	F	S	Beta	Simple r
LOGU	.24150302	.46230	.498	.22323	.55674
LOGE	.29810051	36.06514	.000	.53630	.68043
LOGL	.68732063E-01	.04010	.842	.06491	.54248
Constant	.90587843	4.85040	.031		
Overall r = .72416 r^2 = .52441 F = 29.40416 S = 0.000					

Best-fit premium estimator for 1975 data sample					
Dependent variable = RCPM					
Variable	B	F	S	Beta	Simple r
RCPU	.62181018E-01	.00116	.973	.00262	-.03461
RCPE	19.97131900	118.26755	.000	.79530	.79415
L	.14455358E-10	.02234	.882	.01153	-.06099
Constant	.10078119E-01	3.25548	.075		
Overall r = .79422 r^2 = .63079 F = 39.86469 S = 0.000					

Best-fit premium estimator for 1976 data sample					
Dependent variable = M					
Variable	B	F	S	Beta	Simple r
U	.46927341E-02	.36462	.548	.05344	.57418
LOGL	1322.33550000	.39575	.531	.04654	.39420
E	.14344696E-01	80.35805	.000	.76390	.81834
Constant	5176.24611000	.28768	.593		
Overall r = .82108 r^2 = .67417 F = 55.17481 S = 0.000					

continued . . .

Estimation of total premium from contract details (continued)

Best-fit premium estimator for 1974-6 data sample					
Dependent variable = RCPM					
Variable	B	F	S	Beta	Simple r
U	-.14690070E-09	3.7532	.054	-.14930	-.12379
RCPE	17.14352200	293.3329	.000	.73942	.74405
L	.84532237E-10	2.3146	.129	.11711	-.06893
Constant	.81997779E-04	26.3361	.000		
Overall r = .74872 r^2 = .56057 F = 101.20531 S = 0.000					

Best-fit premium estimator for 1977 data sample					
Dependent variable = LOGM					
Variable	B	F	S	Beta	Simple r
LOGU	.54285218	31.32710	.000	.5597	.62717
LOGE	.10972005	15.42967	.000	.3212	.50271
LOGL	-.46338139E-01	.69507	.407	-.08165	.37346
Constant	1.07555830	7.53854	.007		
Overall r = .69427 r^2 = .48201 F = 28.2264 S = 0.000					

Best-fit premium estimator for 1974-7 data sample					
Dependent variable = RCPM					
Variable	B	F	S	Beta	Simple r
U	-.20862389E-10	1.13131	.288	-.0541200	-.10236
RCPE	17.20843200	389.99880	.000	.7330176	.73637
L	.26096487E-11	.01255	.000	.0057100	-.09496
Constant	.68541076E-04	38.40570	.000		
Overall r = .73809 r^2 = .54477 F = 132.83486 S = 0.000					

3) Estimation of claims from contract details

Best-fit premium estimator for 1974 data sample

Dependent variable = RCPC

Variable	B	F	S	Beta	Simple r
RCPU	-2.01163250	13.5938	.000	-.42622	.17657
LOGE	-.60660085E-06	.5125	.821	-.09226	-.14664
RCPL	1.62961420	57.1645	.000	.88089	.59139
Constant	.18745374E-04	1.6255	.206		

Overall r = .66971 r^2 = .44851 F = 21.68741 S = 0.000

Best-fit premium estimator for 1975 data sample

Dependent variable = RCPC

Variable	B	F	S	Beta	Simple r
LOGU	.67948219E-05	6.0579	.016	.216680	-.13828
LCPE	-.16935197E-01	.0755	.784	-.021810	-.08130
RCPL	.18091380	86.6237	.000	.818433	.72522
Constant	-.35775983E-04	6.1387	.016		

Overall r = .75190 r^2 = .56535 F = 30.34998 S = 0.000

Best-fit premium estimator for 1976 data sample

Dependent variable = LOGC

Variable	B	F	S	Beta	Simple r
LOGU	-.49556060	5.5575	.021	-.26795	-.19194
RCPL	11312.99500000	3.9642	.050	.21668	.18586
E	.35662858E-06	6.8795	.010	.30403	.13924
Constant	2.87307370	7.6614	.007		

Overall r = .36611 r^2 = .13404 F = 4.1275 S = 0.000

continued . . .

Estimation of claims from contract details (continued)

Best-fit premium estimator for 1974-6 data sample

Dependent variable = LOGC

Variable	B	F	S	Beta	Simple r
LOGU	-.20084359	2.6877	.102	-.10439	-.13028
RCPE	-19239.05700000	3.6782	.056	-.12074	-.12473
RCPL	6332.08870000	6.3264	.013	.16016	.17781
Constant	2.00942090	10.1861	.002		

Overall r = .23904 r^2 = .05714 F = 4.80771 S = 0.000

Best-fit premium estimator for 1977 data sample

Dependent variable = RCPC

Variable	B	F	S	Beta	Simple r
LOGU	-.15764413E-03	31.0684	.000	-.60787	-.61266
LOGE	.83653961E-05	1.0548	.307	.09159	-.15307
LOGL	-.93090399E-05	.3299	.567	-.06134	-.41603
Constant	.87843075E-03	59.1345	.000		

Overall r = .61961 r^2 = .38392 F = 18.9028 S = 0.000

Best-fit premium estimator for 1974-7 data sample

Dependent variable = C

Variable	B	F	S	Beta	Simple r
RCPU	-.37847204E+09	1.5527	.241	-.16523	-.12527
E	.75473036E+02	8.2550	.004	.35361	.18173
RCPL	-.35644615	.7278	.787	.02554	-.09084
Constant	25163.86400000	10.6656	.001		

Overall r = .20107 r^2 = .04043 F = 4.67680 S = 0.003

4) Estimation of premium loadings from contract details

Best-fit 1974 estimator of K from U, L, E					
Dependent variable = LOGK					
Variable	B	F	S	Beta	Simple r
RCPU	890.23314000	.01090	.917	.01574	.16031
LOGE	.61541820E-01	2.16566	.145	.16281	.13718
RCPL	4503.75600000	1.79258	.184	.20321	.19304
Constant	-.14402774	.39397			
Overall r = .25315 r^2 = .06409 F = 1.82599 S = .149					

Best-fit 1975 estimator of K from U, L, E					
Dependent variable = RCPK					
Variable	B	F	S	Beta	Simple r
LOGU	-.14099108E-04	8.23281	.005	-.25695	.07874
LOGE	-.11058776E-05	1.27555	.263	-.09154	-.08663
RCPL	-.32415842	96.59120	.000	-.83809	-.71661
Constant	.81496683E-04	11.02917	.001		
Overall r = .767 r^2 = .58828 F = 33.33992 S = .000					

Best-fit 1976 estimator of K from U, L, E					
Dependent variable = RCPK					
Variable	B	F	S	Beta	Simple r
RCPU	.62393620	9.7889	.002	.41237	.43292
LOGE	.23018153E-05	3.43312	.068	.18676	.09852
RCPL	.64699494E-01	.36846	.546	.08039	.31612
Constant	-.14531740E-04	3.88616	.052		
Overall r = .47152 r^2 = .22233 F = 7.62397 S = .000					

continued . . .

Estimation of premium loadings from contract details (continued)

Best-fit 1974-6 estimator of K from U, L, E					
Dependent variable = K					
Variable	B	F	S	Beta	Simple r
RCPU	.13722559E+09	.32294	.570	.03789	.09977
E	-.10565798E-01	10.42780	.001	-.22661	-.24240
L	-.25238080E-02	.04157	.839	-.01454	-.12269
Constant	-3582.49260000	.24407	.622		
Overall r = .24595 r^2 = .06049 F = 5.10818 S = .002					

Best-fit 1977 estimator of K from U, L, E					
Dependent variable = K					
Variable	B	F	S	Beta	Simple r
LOGU	-9701.5000	.22584	.636	-.06447	-.14331
LOGE	-9025.6807	2.35675	.128	-.17031	-.20210
RCPL	-2195.2300	.03521	.852	-.02493	-.11960
Constant	67239.7410	.66503	.417		
Overall r = .21596 r^2 = .04664 F = 1.48393 S = .224					

Best-fit 1974-77 estimator of K from U, L, E					
Dependent variable = K					
Variable	B	F	S	Beta	Simple r
LOGU	-5041.15800000	.32946	.566	-.05494	-.13144
E	-.85823802E-02	11.36019	.001	-.19332	-.21678
LOGL	-1046.80290000	.02071	.886	-.01390	-.12887
Constant	19372.66700000	.57124	.450		
Overall r = .22566 r^2 = .05092 F = 5.95548 S = .001					

5) Estimation of price variations from contract details

1974 Best-fit MDIFA estimator from U, L, E

Dependent variable = LGMDIFA

Variable	B	F	S	Beta	Simple r
U	.15628994E-05	2.53947	.115	.17116	.24484
LOGE	.38885234	19.32352	.000	.48620	.47895
L	-.15199278	.69954	.405	-.09977	.21126
Constant	.22197587	.91461	.763		

Overall r = .50368 r^2 = .25369 F = 9.06485 S = .000

1975 Best-fit MDIFA estimator from U, L, E

Dependent variable = RPMDIFA

Variable	B	F	S	Beta	Simple r
RCPU	28.87628600	2.54342	.115	.14083	.07480
RCPE	163.93645000	95.42500	.000	.75616	.75243
LOGL	.34705469E-03	.64471	.425	.07087	.02651
Constant	-.29680433E-02	1.67305	.200		

Overall r = .76248 r^2 = .58137 F = 32.40429 S = .000

1976 Best-fit MDIFA estimator from U, L, E

Dependent variable = LGMDIFA

Variable	B	F	S	Beta	Simple r
LOGU	.71398723	1.17397	.282	.43170	.20400
LOGE	.21667397	6.83439	.011	.30302	.34036
LOGL	-.59062366	.90019	.346	-.37213	.16549
Constant	-.65284414	.55498	.458		

Overall r = .36003 r^2 = .12962 F = 3.97124 S = .011

continued . . .

Estimation of price variations from contract details (continued)

1974-6 Best-fit MDIFA estimator of K from U, L, E

Dependent variable = MDIFA

Variable	B	F	S	Beta	Simple r
U	.40485592E-01	20.471640	.000	.38829	.53824
E	.11564553E-01	113.009080	.000	.56297	.69367
L	-.14283636E-01	5.482620	.020	-.18674	.37535
Constant	-2353.91190000	1.741233	.188		

Overall r = .72761 r^2 = .52942 F = 89.25242 S = .000

1977 Best-fit MDIFA estimator from U, L, E

Dependent variable = MDIFA

Variable	B	F	S	Beta	Simple r
U	-.11435156E-01	8.84747	.004	-.35519	-.43217
RCPE	-.50421899E+10	7.98628	.006	-.25888	-.17215
L	-.80984699E-02	2.20056	.141	-.17796	-.36431
Constant	21942.31400000	22.40737	.000		

Overall r = .51274 r^2 = .2629 F = 10.8196 S = .000

1974-77 Best-fit MDIFA estimator from U, L, E

Dependent variable = LGMDIFA

Variable	B	F	S	Beta	Simple r
LOGU	.37614484E-01	.05167	.820	.01735	.36890
LOGE	.51373036	150.70189	.000	.57143	.63042
LOGL	.23030840	2.96076	.086	.12947	.37149
Constant	-1.30810730	7.87479	.005		

Overall r = .6440 r^2 = .41474 F = 78.66008 S = .000

6) Estimation of total premium from claims, (estimated) premium loadings and (estimated) price variations

1974 Best-fit estimator of M from C, K, MDIFA					
Dependent variable = M					
Variable	B	F	S	Beta	Simple r
RCPK	-5853205	.43212	.513	-.03555	-.15416
LOGC	-528.6915700	.58590	.446	-.04219	.16064
MDIFA	1.1031967	252.28047	.000	.88186	.87675
Constant	18084.4740000	92.09251	.000		
Overall r = .87856 r^2 = .77187 F = 90.22413 S = .000					

1975 Best-fit estimator of M from C, K, MDIFA					
Dependent variable = M					
Variable	B	F	S	Beta	Simple r
K	.15399645	.34389	.559	.02735	.01346
MDIFA	1.02034940	1637.57550	.000	.98053	.97916
C	.13960149	.58140	.446	.03581	-.03761
Constant	6802.99020000	59.51594	.000		
Overall r = .97935 r^2 = .95912 F = 547.40636 S = .000					

1976 Best-fit estimator of M from C, K, MDIFA					
Dependent variable = M					
Variable	B	F	S	Beta	Simple r
K	.42258813E-01	.15275	.697	.07970	-.23598
MDIFA	10897.01800000	54.43579	.000	.60874	.60239
C	.15479399	3.48309	.066	.38021	.29043
Constant	18213.10300000	32.41673	.000		
Overall r = .67699 r^2 = .45831 F = 22.56213 S = .000					

continued . . .

Estimation of total premium from claims, (estimated) premium loadings and (estimated) price variations (continued)

1974-76 Best-fit estimator of M from C, K, MDIFA					
Dependent variable = M					
Variable	B	F	S	Beta	Simple r
K	.10586984E-01	.07702	.782	.02394	-.16671
MDIFA	.88051488	797.08177	.000	.87704	.88054
C	.23096775E-01	.45930	.508	.05692	.14635
Constant	12256.96400000	126.2188	.000		
Overall r = .88126 r^2 = .77661 F = 275.80819 S = .000					

1977 Best-fit estimator of M from C, K, MDIFA					
Dependent variable = M					
Variable	B	F	S	Beta	Simple r
K	-.21855053	3.6740	.058	-.65942	-.41176
MDIFA	.71493494	70.42895	.000	.64538	.70636
C	-.14876384	2.09198	.152	-.49717	.38522
Constant	31314.77400000	103.4447	.000		
Overall r = .73204 r^2 = .53588 F = 35.02278 S = .000					

1974-77 Best-fit estimator of M from C, K, MDIFA					
Dependent variable = M					
Variable	B	F	S	Beta	Simple r
C	.11033467E-01	.28427	.594	.03146	.26957
MDIFA	1.04133110	29877.47610	.000	.95125	.95316
K	.82509969E-02	.13757	.711	.02209	-.29488
Constant	656.62515000	291.54369	.000		
Overall r = .95352 r^2 = .90866 F = 1104.2636 S = .000					

7) Estimation of the underwriter's written line from contract details

1974 Best-fit MW estimator from U, L, E					
Dependent variable = LGMW					
Variable	B	F	S	Beta	Simple r
LOGU	.29189874	.91915	.341	.34096	.49401
LOGE	.22551171	28.04777	.000	.51269	.63183
LOGL	-.91005770E-01	.09552	.758	-.10862	.47182
Constant	.77814549	4.86363	.030		
Overall r = .66417 r^2 = .44112 F = 21.04801 S = .000					

1975 Best-fit MW estimator from U, L, E					
Dependent variable = RCPMW					
Variable	B	F	S	Beta	Simple r
U	.18730435E-08	.12859	.721	.042612	.01159
RCPE	350.63885000	34.34013	.000	.577770	.57464
LOGL	.11375068E-02	.49611	.484	.08298	.12475
Constant	-.44190846E-02	.35366	.554		
Overall r = .58554 r^2 = .34285 F = 12.17372 S = .000					

1976 Best-fit MW estimator from U, L, E					
Dependent variable = LGMW					
Variable	B	F	S	Beta	Simple r
LOGU	.36964452	1.33956	.251	.43748	.42045
LOGE	.79113736E-01	3.87889	.052	.21657	.35962
LOGL	-.93076878E-01	.95173	.759	-.11479	.39373
Constant	1.3305785	9.81424	.002		
Overall r = .46543 r^2 = .21663 F = 7.37417 S = .000					

continued . . .

Estimation of the underwriter's written line from contract details (continued)

1974-76 Best-fit MW estimator from U, L, E					
Dependent variable = RCPMW					
Variable	B	F	S	Beta	Simple r
LOGU	-.54985545E-03	.35484	.552	-.08506	.06820
RCPE	277.71788000	88.73834	.000	.51872	.52102
LOGL	.93647784E-03	1.22569	.269	.15811	.09111
Constant	-.84758960E-03	.22220	.638		
Overall r = .52808 r^2 = .27887 F = 30.67915 S = .000					

1977 Best-fit MW estimator from U, L, E					
Dependent variable = LGMW					
Variable	B	F	S	Beta	Simple r
LOGU	.17966487	2.81149	.097	.20659	.30639
LOGE	.11487627	13.85792	.000	.37506	.43194
LOGL	-.31566243E-01	.26427	.608	-.06203	.18724
Constant	1.80531610	17.40119	.000		
Overall r = .46226 r^2 = .21368 F = 8.24317 S = .000					

1974-77 Best-fit MW estimator from U, L, E					
Dependent variable = RCPMW					
Variable	B	F	S	Beta	Simple r
LOGU	-.17229242E-04	.00127	.972	-.00297	.05153
RCPE	279.65496000	126.32696	.000	.52321	.52351
LOGL	.33544205E-03	.71556	.398	.07040	.07002
Constant	0.70910746E-03	.26079	.610		
Overall r = .52790 r^2 = .27868 F = 42.88438 S = .000					

8) Estimation of the underwriter's written line from total premium, claims, (estimated) premium loadings and (estimated) price variations

Best 1974 estimator of MW from M, C, K, MDIFA

Dependent variable = LGMW

Variable	B	F	S	Beta	Simple r
LOGM	.72243142	741.57861	.000	.91293	.96079
RCPE	477.05917000	1.34339	.250	.03422	-.04562
K	-.15301113E-06	.17062	.681	-.01225	-.14376
LGMDIFA	.56619782E-01	9.53996	.003	.10295	.53287
Constant	-.33435753E-01	.12156	.728		

Overall r = .96550 r^2 = .93219 F = 271.51216 S = .000

Best 1975 estimator of MW from M, C, K, MDIFA

Dependent variable = LGMW

Variable	B	F	S	Beta	Simple r
LOGM	.83752067	92.67726	.000	.8293	.85415
K	.58697545E-05	.23337	.631	.04033	-.06383
LGMDIFA	.25646226E-01	.62869	.431	.06668	.61985
LOGC	.68810941E-01	1.53729	.219	.10521	-.07735
Constant	-.44772915	2.15186	.147		

Overall r = .85963 r^2 = .73897 F = 48.83386 S = .000

Best 1976 estimator of MW from M, C, K, MDIFA

Dependent variable = LGMW

Variable	B	F	S	Beta	Simple r
LOGM	.68354258	74.89670	.000	.68500	.74895
K	.48294110E-06	.03155	.859	.03192	-.08045
LGMDIFA	.84957612E-01	4.47170	.038	.16630	.42315
C	.48280251E-06	.05353	.818	.04155	.09985
Constant	.13944191	.18896	.665		

Overall r = .76484 r^2 = .58498 F = 27.83851 S = .000

continued . . .

Estimation of the underwriter's written line from total premium, claims, (estimated) premium loadings and (estimated) price variations (continued)

Best 1974-76 estimator of MW from M, C, K, MDIFA					
Dependent variable = LGMW					
Variable	B	F	S	Beta	Simple r
LOGM	.75155683	474.66998	.000	.85081	.87440
LOGC	.30233103E-01	3.02393	.083	.6279	-.05163
K	.63845920E-06	1.68792	.195	.04641	-.10049
LGMDIFA	.29441495E-01	2.78371	.097	.06455	.54663
Constant	-.13242409	1.11436	.292		
Overall r = .87825 r^2 = .77133 F = 199.85702 S = .000					

Best 1977 estimator of MW from M, C, K, MDIFA					
Dependent variable = LGMW					
Variable	B	F	S	Beta	Simple r
LOGM	.63878832	120.93351	.000	.71240	.84187
K	.50645993E-06	2.60209	.110	.08763	-.20538
RCPC	-71.39210100	.13569	.713	-.02129	-.40348
LGMDIFA	.99958604E-01	26.06233	.000	.30470	.60348
Constant	.31171181	1.66036	.201		
Overall r = .8799 r^2 = .77422 F = 77.15344 S = .000					

Best 1974-7 estimator of MW from M, C, K, MDIFA					
Dependent variable = LGMW					
Variable	B	F	S	Beta	Simple r
LOGM	.75437917	614.41368	.000	.85335	.87108
C	.10990950E-05	1.34067	.248	.11789	.05991
LGMDIFA	.23580117E-01	2.87078	.091	.05608	.53231
K	.12818755E-05	1.57248	.211	.12906	-.14143
Constant	-.10545599	.87063	.351		
Overall r = .87293 r^2 = .76200 F = 265.7388 S = .000					

APPENDIX D

RESULTS OF THE SIMULATION PROCEDURES

Contents

1) Glossary of terms for Appendix D

The following terms appear in Appendix D only. For other terms (which appear in both Appendix C and Appendix D) see the glossary of terms for Appendix C.

$\overline{CW}$	-	The total £ amount of claims divided by the number of contracts written under a simulated underwriting strategy
$\overline{MW}$	-	The mean average £ written line for a simulated underwriting strategy
PAR	-	An indication of market penetration for a simulated underwriting strategy (PAR = $\overline{X}$ + 3SDX)
PROF	-	A measure of simulated portfolio profitability calculated as $\overline{MW}$ divided by $\overline{CW}$
SDCW	-	The standard deviation of £ claims amounts incurred on a simulated portfolio
SDX	-	The standard deviation of % participation in excess of loss contracts on a simulated portfolio
STAB	-	A measure of stability for a simulated portfolio calculated as SDCW divided by $\overline{CW}$
UPU equation	-	An UPU equation is the best-fit equation which estimates MW from M and MDIFA on a data sample. It is a model of the underwriter's propensity to underwrite.
$\overline{X}$	-	The mean average % participation in excess of loss contracts for a simulated portfolio
Z	-	A numerical value employed in the MWGEN equations for generating acceptances for a simulated portfolio

2) UPU equations used in the simulation procedures

1974					
Dependent variable = LGMW					
Variable	B	F	S	Beta	Simple r
LOGM	.72264889	758.9206	.000	.913206	.96079
LGMDIFA	.55268750E-01	9.1903	.003	.100490	.53287
Constant	-.15435228E-01	.0269	.870		
Overall r = .96486 r^2 = .93095 F = 545.99907 S = .000					

1975					
Dependent variable = LGMW					
Variable	B	F	S	Beta	Simple r
LOGM	.81377856	92.3096	.000	.8058	.85415
LGMDIFA	.27310306E-01	.7167	.400	.0710	.61985
Constant	-.33853536	1.3417	.251		
Overall r = .85573 r^2 = .73228 F = 97.10201 S = .000					

1976					
Dependent variable = LGMW					
Variable	B	F	S	Beta	Simple r
LOGM	.68530308	79.1035	.000	.6868	.74895
LGMDIFA	.84889803E-01	4.6309	.034	.1662	.42315
Constant	.13836841	.1930	.662		
Overall r = .76464 r^2 = .58467 F = 57.0122 S = .000					

continued . . .

UPU equations used in the simulation procedures (continued)

1977					
Dependent variable = LGMW					
Variable	B	F	S	Beta	Simple r
LOGM	.64188677	159.3807	.000	.7159	.84187
LGMDIFA	.89506114E-01	23.1522	.000	.2728	.60348
Constant	.29219126	1.9856	.162		
Overall r = .87596 r^2 = .76731 F = 151.68532 S = .000					

1974-76					
Dependent variable = LGMW					
Variable	B	F	S	Beta	Simple r
LOGM	.73604360	483.7051	.000	.8332	.8744
LGMDIFA	.32975585E-01	3.416	.058	.0730	.54663
Constant	-.55415414E-01	.2206	.639		
Overall r = .87642 r^2 = .76811 F = 395.84025 S = .000					

1974-77					
Dependent variable = LGMW					
Variable	B	F	S	Beta	Simple r
LOGM	.74232543	666.8688	.000	.8397	.87108
LGMDIFA	.23211979E-01	2.8825	.090	.0552	.53231
Constant	-.52258377E-01	.2525	.616		
Overall r = .87227 r^2 = .76085 F = 531.29809 S = .000					

3) **Results of the 'A' Simulations**

'A' Simulation on 1974 data (Simulation 'A'(i))

Z value	MW	CW	SDCW	X	SDX	PROF	STAB	PAR
.05	1434	1418	4218	.0370	.0457	1.0113	2.9746	.1741
.10	1492	1451	4333	.0380	.0468	1.0283	2.9862	.1784
.15	1553	1486	4455	.0390	.0481	1.0451	2.9980	.1833
.20	1619	1523	4584	.0401	.0496	1.0630	3.0098	.1889
.25	1688	1561	4720	.0412	.0511	1.0814	3.0234	.1945
.30	1765	1600	4864	.0424	.0528	1.1031	3.0400	.2008
.35	1846	1642	5016	.0436	.0547	1.1242	3.0548	.2077
.40	1933	1685	5176	.0450	.0568	1.1472	3.0718	.2154
.45	2027	1731	5346	.0464	.0591	1.1710	3.0884	.2252
.50	2129	1778	5524	.0479	.0617	1.1974	3.1069	.2330
.55	2238	1828	5715	.0494	.0645	1.2243	3.1264	.2430
.60	2357	1881	5916	.0511	.0677	1.2531	3.1451	.2542
.65	2485	1936	6128	.0529	.0712	1.2836	3.1653	.2665
.70	2623	1994	6353	.0548	.0752	1.3154	3.1861	.2804
.75	2773	2055	6592	.0568	.0795	1.3494	3.2078	.2953
.80	2936	2119	6844	.0589	.0844	1.3856	3.2298	.3121
.85	3113	2187	7112	.0612	.0899	1.4234	3.2519	.3309
.90	3305	2259	7396	.0637	.0959	1.4630	3.2740	.3514
.95	3514	2335	7698	.0663	.1026	1.5056	3.2982	.3741
1.00	3742	2414	8018	.0691	.1101	1.5501	3.3215	.3994
1.05	3990	2499	8360	.0721	.1185	1.5966	3.3453	.4276
1.10	4260	2589	8723	.0753	.1277	1.6454	3.3693	.4584
1.15	4555	2684	9111	.0787	.1380	1.6971	3.3946	.4927
1.20	4876	2784	9524	.0824	.1494	1.7514	3.4210	.5306
1.25	5228	2892	9965	.0864	.1621	1.8077	3.4457	.5727
1.30	5612	3006	10438	.0906	.1761	1.8669	3.4724	.6189
1.35	6033	3127	10945	.0953	.1917	1.9293	3.5002	.6704
1.40	6493	3257	11488	.1002	.2090	1.9936	3.5272	.7272
1.45	6997	3395	12073	.1056	.2280	2.0610	3.5561	.7896
1.50	7549	3543	12703	.1114	.2492	2.1307	3.5854	.8590
1.55	8154	3701	13383	.1176	.2726	2.2032	3.6160	.9354
1.60	8819	3870	14118	.1244	.2985	2.2788	3.6481	1.0199

'A' Simulation on 1975 data (Simulation 'A'(ii))

Z value	MW	CW	SDCW	X̄	SDX	PROF	STAB	PAR
.05	1996	123	672	.0645	.0493	16.2276	5.4634	.2124
.10	2186	128	687	.0686	.0520	17.0781	5.3672	.2246
.15	2398	133	705	.0730	.0552	18.0301	5.3008	.2386
.20	2634	139	726	.0779	.0589	18.9496	5.2230	.2546
.25	2896	146	749	.0832	.0632	19.8356	5.1301	.2728
.30	3188	153	775	.0890	.0682	20.8366	5.0654	.2936
.35	3514	160	806	.0953	.0740	21.9625	5.0375	.3173
.40	3877	169	840	.1022	.0807	22.9408	4.9704	.3443
.45	4282	178	879	.1098	.0855	24.0562	4.9382	.3753
.50	4735	187	923	.1181	.0975	25.3209	4.9358	.4106
.55	5242	198	973	.1273	.1078	26.4747	4.9141	.4507
.60	5809	210	1029	.1374	.1198	27.6619	4.9000	.4968
.65	6445	223	1091	.1484	.1335	28.9013	4.8924	.5489
.70	7157	237	1162	.1606	.1493	30.1983	4.9030	.6085
.75	7956	252	1240	.1741	.1674	31.5714	4.9206	.6763
.80	8852	268	1328	.1890	.1883	33.0299	4.9552	.7539
.85	9860	286	1426	.2054	.2121	34.4755	4.9860	.8417
.90	10992	306	1534	.2236	.2395	35.9216	5.0131	.9421
.95	12266	327	1654	.2438	.2710	37.5107	5.0581	1.0568

'A' Simulation on 1976 data (Simulation 'A'(iii))

Z value	MW	CW	SDCW	$\bar{X}$	SDX	PROF	STAB	PAR
.05	1533	489	2369	.0253	.0445	3.1350	4.8446	.1588
.10	1562	498	2410	.0257	.0454	3.1365	4.8394	.1619
.15	1592	506	2453	.0262	.0462	3.1462	4.8478	.1648
.20	1622	515	2496	.0267	.0471	3.1495	4.8466	.1680
.25	1654	524	2541	.0272	.0480	3.1565	4.8492	.1712
.30	1685	533	2587	.0277	.0490	3.1614	4.8537	.1747
.35	1718	542	2634	.0282	.0499	3.1697	4.8598	.1779
.40	1751	551	2682	.0287	.0509	3.1779	4.8675	.1814
.45	1786	561	2732	.0293	.0520	3.1836	4.8699	.1853
.50	1821	571	2782	.0298	.0530	3.1891	4.8722	.1888
.55	1857	581	2835	.0304	.0541	3.1962	4.8795	.1927
.60	1893	591	2888	.0309	.0553	3.2030	4.8866	.1968
.65	1931	602	2943	.0315	.0564	3.2076	4.8887	.2007
.70	1969	613	3000	.0321	.0576	3.2121	4.8940	.2049
.75	2009	624	3058	.0328	.0589	3.2196	4.9006	.2095
.80	2049	635	3117	.0334	.0602	3.2268	4.9087	.2140
.85	2091	646	3178	.0341	.0615	3.2368	4.9195	.2183
.90	2133	658	3241	.0347	.0629	3.2416	4.9255	.2234
.95	2177	670	3305	.0354	.0643	3.2493	4.9328	.2283
1.00	2222	682	3371	.0361	.0657	3.2581	4.9428	.2332
1.05	2267	695	3439	.0368	.0672	3.2618	4.9482	.2384
1.10	2314	708	3509	.0376	.0688	3.2684	4.9562	.2440
1.15	2362	721	3580	.0383	.0704	3.2760	4.9653	.2495
1.20	2411	734	3653	.0391	.0720	3.2847	4.9768	.2551
1.25	2462	748	3728	.0399	.0737	3.2914	4.9840	.2610
1.30	2514	762	3805	.0407	.0754	3.2992	4.9934	.2669
1.35	2567	776	3884	.0416	.0773	3.3080	5.0052	.2735
1.40	2621	791	3965	.0424	.0791	3.3135	5.0126	.2797
1.45	2677	806	4048	.0433	.0810	3.3213	5.0223	.2863
1.50	2734	821	4134	.0442	.0830	3.3301	5.0353	.2932
1.55	2792	837	4221	.0451	.0851	3.3357	5.0430	.3004
1.60	2852	853	4311	.0461	.0872	3.3435	5.0539	.3077
1.65	2914	869	4403	.0470	.0893	3.3533	5.0667	.3149
1.70	2977	886	4497	.0480	.0916	3.3605	5.0756	.3228
1.75	3042	903	4594	.0490	.0939	3.3688	5.0875	.3307
1.80	3108	920	4694	.0501	.0963	3.3783	5.1022	.3390
1.85	3176	938	4795	.0512	.0978	3.3859	5.1119	.3446
1.90	3246	956	4900	.0523	.1013	3.3954	5.1255	.3562
1.95	3318	975	5007	.0534	.1039	3.4031	5.1354	.3651
2.00	3391	994	5117	.0546	.1066	3.4115	5.1479	.3744

continued . . .

'A' Simulation on 1976 data (Simulation 'A'(iii)continued)

Z value	$\bar{M}W$	$\bar{C}W$	SDCW	$\bar{X}$	SDX	PROF	STAB	PAR
2.05	3467	1013	5229	.0557	.1094	3.4225	5.1619	.3839
2.10	3544	1033	5345	.0570	.1123	3.4308	5.1742	.3939
2.15	3624	1054	5463	.0582	.1152	3.4383	5.1831	.4038
2.20	3705	1075	5584	.0593	.1183	3.4465	5.1944	.4142
2.25	3789	1096	5708	.0608	.1215	3.4571	5.2080	.4253
2.30	3874	1118	5836	.0622	.1247	3.4651	5.2200	.4363
2.35	3962	1140	5967	.0636	.1281	3.4754	5.2342	.4479
2.40	4053	1163	6101	.0650	.1316	3.4850	5.2459	.4598
2.45	4145	1186	6238	.0664	.1351	3.4949	5.2597	.4717
2.50	4241	1210	6379	.0679	.1388	3.5050	5.2719	.4843
2.55	4338	1234	6524	.0695	.1426	3.5154	5.2869	.4973
2.60	4438	1259	6672	.0711	.1466	3.5250	5.2994	.5109
2.65	4541	1285	6823	.0727	.1506	3.5339	5.3097	.5245
2.70	4647	1311	6979	.0743	.1548	3.5446	5.3234	.5387
2.75	4755	1338	7138	.0761	.1591	3.5538	5.3348	.5534
2.80	4867	1365	7302	.0778	.1636	3.5656	5.3495	.5686
2.85	4981	1393	7469	.0796	.1682	3.5757	5.3618	.5842
2.90	5098	1421	7641	.0815	.1729	3.5876	5.3772	.6002
2.95	5218	1451	7816	.0834	.1778	3.5961	5.3866	.6168
3.00	5342	1480	7997	.0853	.1829	3.6095	5.4034	.6340
3.05	5469	1511	8181	.0873	.1881	3.6373	5.4143	.6516
3.10	5599	1542	8371	.0894	.1935	3.6310	5.4287	.6699
3.15	5733	1574	8565	.0915	.1990	3.6423	5.4416	.6885
3.20	5871	1607	8764	.0937	.2048	3.6534	5.4536	.7081
3.25	6012	1641	8967	.0959	.2107	3.6636	5.4644	.7280
3.30	6157	1675	9176	.0982	.2168	3.6758	5.4728	.7486
3.35	6306	1710	9390	.1005	.2231	3.6877	5.4912	.7698
3.40	6459	1746	9609	.1030	.2296	3.6993	5.5034	.7918
3.45	6616	1782	9834	.1054	.2363	3.7127	5.5185	.8143
3.50	6777	1820	10064	.1080	.2433	3.7236	5.5297	.8379
3.55	6943	1853	10300	.1106	.2504	3.7469	5.5586	.8618
3.60	7114	1898	10542	.1133	.2578	3.7482	5.5543	.8867
3.65	7289	1938	10790	.1161	.2654	3.7611	5.5676	.9213
3.70	7469	1979	11043	.1189	.2733	3.7741	5.5801	.9388
3.75	7653	2021	11303	.1218	.2814	3.7867	5.5928	.9660
3.80	7843	2064	11570	.1248	.2898	3.7999	5.6056	.9942
3.85	8038	2108	11843	.1279	.2985	3.8131	5.6181	1.0234

'A' Simulation on 1974-6 data (Simulation 'A'(iv))

Z value	MW	CW	SDCW	$\bar{X}$	SDX	PROF	STAB	PAR
.05	1755	1275	5577	.0518	.0442	1.3765	4.3741	.1844
.10	1871	1353	5942	.0542	.0460	1.3829	4.3917	.1922
.15	1997	1437	6347	.0568	.0481	1.3897	4.4168	.2011
.20	2134	1528	6795	.0595	.0504	1.3966	4.4470	.2107
.25	2283	1627	7294	.0624	.0529	1.4032	4.4831	.2211
.30	2446	1733	7851	.0656	.0558	1.4114	4.5303	.2330
.35	2623	1849	8473	.0689	.0590	1.4197	4.5825	.2459
.40	2817	1974	9170	.0725	.0626	1.4271	4.6564	.2603
.45	3029	2110	9951	.0764	.0667	1.4355	4.7161	.2765
.50	3261	2258	10827	.0805	.0713	1.4442	4.7950	.2944
.55	3515	2419	11813	.0850	.0766	1.4531	4.8834	.3148
.60	3794	2595	12921	.0890	.0825	1.4620	4.9792	.3365
.65	4101	2787	14167	.0950	.0892	1.4715	5.0832	.3626
.70	4437	2996	15571	.1006	.0968	1.4810	5.1973	.3910
.75	4808	3226	17152	.1066	.1055	1.4904	5.3168	.4231
.80	5216	3476	18932	.1331	.1153	1.5006	5.4465	.4590
.85	5666	3751	20938	.1202	.1265	1.5105	5.5820	.4997
.90	6163	4052	23197	.1279	.1392	1.5210	5.7248	.5455
.95	6713	4384	25742	.1362	.1538	1.5313	5.8718	.5976
1.00	6212	4748	28609	.1362	.1776	1.3083	6.0255	.6690
1.05	7994	5148	31837	.1550	.1891	1.5529	6.1843	.7223
1.10	8740	5589	35474	.1657	.2106	1.5638	6.3471	.7975
1.15	9568	6075	39569	.1774	.2350	1.5750	6.5134	.8824
1.20	10488	6612	44180	.1902	.2629	1.5862	6.6818	.9789
1.25	11511	7204	49372	.2041	.2947	1.5979	6.8534	1.0882

4) Results of the 'B' Simulations

1975 MEST, 1974 UPU on 1975 data (Simulation 'B'(i))

Z value	MW	$\bar{C}W$	SDCW	$\bar{X}$	SDX	PROF	STAB	PAR
.05	2114	133	736	.0718	.0567	15.8947	5.5338	.2419
.10	2313	138	752	.0762	.0595	16.7609	5.4493	.2547
.15	2534	144	770	.0810	.0627	17.5972	5.3472	.2691
.20	2779	150	791	.0863	.0665	18.5267	5.2733	.2858
.25	3052	157	815	.0920	.0709	19.4395	5.1911	.3047
.30	3355	165	842	.0982	.0760	20.3333	5.1030	.3262
.35	3693	173	874	.1050	.0819	21.3468	5.0520	.3507
.40	4069	182	909	.1125	.0888	22.3571	4.9945	.3789
.45	4489	191	949	.1207	.0968	23.5026	4.9686	.4111
.50	4958	202	995	.1296	.1060	24.5446	4.9257	.4476
.55	5482	213	1046	.1394	.1167	25.7371	4.9108	.4895
.60	6068	225	1105	.1502	.1290	26.9689	4.9111	.5372
.65	6724	239	1170	.1621	.1432	28.1339	4.8954	.5917
.70	7458	253	1243	.1751	.1595	29.4783	4.9130	.6536
.75	8282	269	1325	.1895	.1783	30.7881	4.9257	.7244
.80	9205	287	1417	.2054	.1998	32.0732	4.9373	.8048
.85	10241	306	1519	.2229	.2246	33.4673	4.9641	.8967
.90	11405	327	1632	.2423	.2530	34.8777	4.9908	1.0013

1976 MEST, 1975 UPU on 1976 data (Simulation 'B'(ii))

Z value	$\bar{M}W$	$\bar{C}W$	SDCW	$\bar{X}$	SDX	PROF	STAB	PAR
.05	1219	382	1836	.0198	.0347	3.1911	4.8063	.1239
.10	1242	389	1866	.0202	.0353	3.1928	4.7969	.1261
.15	1265	395	1896	.0205	.0359	3.2025	4.8000	.1282
.20	1289	402	1928	.0209	.0366	3.2065	4.7960	.1307
.25	1313	408	1960	.0213	.0373	3.2181	4.8039	.1332
.30	1338	415	1993	.0217	.0380	3.2241	4.8024	.1357
.35	1364	422	2027	.0220	.0387	3.2322	4.8033	.1381
.40	1390	429	2062	.0225	.0395	3.2401	4.8065	.1410
.45	1417	437	2098	.0229	.0403	3.2426	4.8009	.1438
.50	1444	444	2134	.0233	.0411	3.2523	4.8063	.1466
.55	1472	452	2172	.0237	.0419	3.2566	4.8053	.1494
.60	1501	460	2211	.0242	.0427	3.2630	4.8065	.1523
.65	1531	468	2251	.0246	.0436	3.2714	4.8098	.1554
.70	1561	476	2292	.0251	.0445	3.2794	4.8151	.1586
.75	1592	484	2333	.0256	.0455	3.2893	4.8202	.1621
.80	1623	493	2376	.0261	.0464	3.2921	4.8195	.1653
.85	1656	501	2421	.0266	.0474	3.3054	4.8323	.1688
.90	1689	510	2466	.0271	.0484	3.3118	4.8353	.1723
.95	1723	519	2512	.0276	.0495	3.3314	4.8410	.1761
1.00	1758	528	2560	.0281	.0506	3.3295	4.8485	.1799
1.05	1794	538	2609	.0287	.0517	3.3346	4.8494	.1838
1.10	1830	548	2659	.0293	.0529	3.3394	4.8522	.1880
1.15	1868	557	2711	.0298	.0541	3.3537	4.8671	.1921
1.20	1906	568	2764	.0304	.0553	3.3556	4.8662	.1963
1.25	1946	578	2818	.0311	.0566	3.3668	4.8754	.2009
1.30	1986	588	2874	.0317	.0579	3.3776	4.8878	.2054
1.35	2027	599	2931	.0323	.0592	3.3840	4.8932	.2099
1.40	2070	610	2990	.0330	.0606	3.3944	4.9016	.2148
1.45	2113	621	3050	.0337	.0621	3.4026	4.9114	.2200
1.50	2158	633	3112	.0343	.0636	3.4092	4.9163	.2251
1.55	2204	645	3176	.0350	.0651	3.4171	4.9240	.2303
1.60	2251	657	3241	.0358	.0667	3.4262	4.8919	.2359
1.65	2299	669	3308	.0365	.0683	3.4365	4.9447	.2414
1.70	2348	682	3376	.0373	.0700	3.4428	4.9501	.2473
1.75	2398	694	3446	.0381	.0718	3.4553	4.9654	.2535
1.80	2450	708	3519	.0389	.0736	3.4605	4.9703	.2597
1.85	2503	721	3592	.0397	.0754	3.4716	4.9820	.2659
1.90	2558	735	3668	.0405	.0773	3.4803	4.9905	.2724
1.95	2614	749	3746	.0414	.0793	3.4900	5.0013	.2793
2.00	2671	763	3826	.0423	.0813	3.5007	5.0144	.2862

continued . .

Results of the 'B' Simulations (continued)

1976 MEST, 1975 UPU on 1976 data (Simulation 'B'(ii) continued)

Z value	MW	CW	SDCW	X̄	SDX	PROF	STAB	PAR
2.05	2730	778	3908	.0432	.0834	3.5090	5.0231	.2934
2.10	2790	793	3992	.0441	.0856	3.5183	5.0340	.3009
2.15	2852	808	4078	.0451	.0878	3.5297	5.0470	.3085
2.20	2916	824	4166	.0461	.0901	3.5388	5.0558	.3164
2.25	2981	840	4256	.0471	.0925	3.5488	5.0667	.3246
2.30	3048	856	4349	.0481	.0950	3.5607	5.0806	.3331
2.35	3116	873	4444	.0492	.0975	3.5693	5.0905	.3417
2.40	3186	890	4542	.0503	.1001	3.5798	5.1034	.3506
2.45	3259	907	4642	.0514	.1028	3.5932	5.1180	.3598
2.50	3333	925	4744	.0525	.1056	3.6032	5.1286	.3693
2.55	3409	944	4850	.0537	.1085	3.6112	5.1377	.3792
2.60	3487	962	4957	.0549	.1114	3.6247	5.1528	.3891
2.65	3567	981	5068	.0562	.1145	3.6361	5.1662	.3997
2.70	3649	1001	5181	.0574	.1176	3.6454	5.1758	.4102
2.75	3733	1021	5297	.0587	.1209	3.6562	5.1881	.4214
2.80	3820	1042	5416	.0601	.1243	3.6660	5.1977	.4330
2.85	3909	1063	5538	.0615	.1277	3.6773	5.2098	.4446
2.90	4000	1084	5664	.0629	.1313	3.6900	5.2251	.4568
2.95	4094	1106	5792	.0643	.1350	3.7016	5.2369	.4693
3.00	4190	1128	5923	.0658	.1388	3.7145	5.2509	.4822
3.05	4288	1151	6058	.0674	.1427	3.7255	5.2632	.4955
3.10	4390	1175	6196	.0689	.1468	3.7362	5.2732	.5093
3.15	4494	1199	6338	.0705	.1510	3.7481	5.2861	.5235
3.20	4601	1223	6483	.0722	.1553	3.7621	5.3009	.5381
3.25	4711	1248	6631	.0739	.1598	3.7748	5.3133	.5533
3.30	4823	1274	6784	.0757	.1644	3.7857	5.3250	.5689
3.35	4939	1300	6940	.0775	.1691	3.7992	5.3385	.5846
3.40	5058	1327	7100	.0793	.1740	3.8116	5.3504	.6013
3.45	5180	1354	7264	.0812	.1791	3.8257	5.3648	.6185
3.50	5305	1382	7432	.0832	.1843	3.8386	5.3777	.6361
3.55	5434	1411	7604	.0852	.1897	3.8512	5.3891	.6543
3.60	5566	1441	7780	.0872	.1953	3.8626	5.3990	.6731
3.65	5702	1471	7604	.0893	.2010	3.8763	5.1693	.6923
3.70	5842	1501	7780	.0915	.2069	3.8921	5.1832	.7122
3.75	5985	1533	7961	.0937	.2131	3.9041	5.1931	.7330
3.80	6133	1565	8147	.0960	.2194	3.9188	5.2058	.7542
3.85	6284	1598	8336	.0984	.2259	3.9547	5.2165	.7761
3.90	6440	1632	8531	.1008	.2327	3.9461	5.2273	.7985
3.95	6599	1666	8730	.1033	.2398	3.9610	5.2401	.8227
4.00	6764	1701	8935	.1058	.2468	3.9765	5.2528	.8462

continued . . .

Results of the 'B' Simulations (continued)

1976 MEST, 1975 UPU on 1976 data (Simulation 'B'(ii) continued)

Z value	$\bar{M}W$	$\bar{C}W$	SDCW	$\bar{X}$	SDX	PROF	STAB	PAR
4.05	6932	1737	9578	.1085	.2542	3.9908	5.5141	.8711
4.10	7106	1774	9803	.1112	.2619	4.0056	5.5259	.8969
4.15	7284	1812	10034	.1139	.2698	4.0199	5.5375	.9233
4.20	7467	1851	10271	.1168	.2779	4.0340	5.5489	.9505
4.25	7655	1890	10513	.1197	.2863	4.0503	5.5624	.9786
4.30	7849	1931	10761	.1227	.2950	4.0647	5.5728	1.0077

5) Results of the 'C' Simulations

1974 MEST, 1975 UPU on 1975 data (Simulation 'C'(i))

Z value	MW	CW	SDCW	X	SDX	PROF	STAB	PAR
.05	1521	50	302	.0355	.0437	30.4200	6.0400	.1666
.10	1582	51	311	.0365	.0450	31.0196	6.0980	.1715
.15	1646	53	322	.0376	.0464	31.0570	6.0755	.1759
.20	1714	55	332	.0388	.0478	31.1636	6.0364	.1822
.25	1786	56	343	.0400	.0495	31.8929	6.1250	.1885
.30	1863	58	354	.0412	.0512	32.1207	6.1034	1948
.35	1944	60	365	.0426	.0531	32.4000	6.0833	.2019
.40	2031	62	377	.0440	.0552	32.7581	6.0806	.2096
.45	2123	64	390	.0454	.0574	33.1719	6.0938	.2176
.50	2222	66	403	.0470	.0601	33.6667	6.1061	.2273
.55	2326	68	416	.0486	.0628	34.2059	6.1176	.2370
.60	2438	70	430	.0504	.0659	34.8286	6.1429	.2481
.65	2557	73	444	.0522	.0692	35.0274	6.0822	.2598
.70	2685	75	459	.0541	.0729	35.8000	6.1200	.2728
.75	2821	77	474	.0562	.0769	36.6364	6.1558	.2869
.80	2967	80	490	.0584	.0814	37.0875	6.1250	.3026
.85	3123	82	507	.0607	.0863	38.0854	6.1829	.3196
.90	3290	85	524	.0631	.0916	38.7059	6.1647	.3379
.95	3470	88	541	.0657	.0975	39.4318	6.1477	.3582
1.00	3663	91	560	.0685	.1040	40.2527	6.1538	.3805
1.05	3870	94	579	.0714	.1112	41.1702	6.1596	.4050
1.10	4092	97	598	.0745	.1190	42.1856	6.1649	.4315
1.15	4332	100	619	.0778	.1275	43.3200	6.1900	.4603
1.20	4589	103	640	.0814	.1369	44.5534	6.2136	.4921
1.25	4866	106	662	.0852	.1473	45.9057	6.2453	.5271
1.30	5164	110	684	.0892	.1586	46.9455	6.2182	.5650
1.35	5486	113	708	.0935	.1709	48.5487	6.2655	.6062
1.40	5833	117	732	.0981	.1845	49.8547	6.2564	.6516
1.45	6207	121	757	.1030	.1994	51.2975	6.2562	.7012
1.50	6612	125	784	.1083	.2156	52.8960	6.2720	.7551
1.55	7048	129	811	.1140	.2334	54.6357	6.2868	.8142
1.60	7520	133	839	.1200	.2529	56.5414	6.3083	.8787
1.65	8031	138	868	.1265	.2742	58.1957	6.2899	.9491
1.70	8583	142	898	.1335	.2975	60.4437	6.3239	1.0260

Results of the 'C' Simulations (continued)

1975 MEST, 1976 UPU on 1976 data (Simulation 'C'(ii))

Z value	MW	CW	SDCW	X̄	SDX	PROF	STAB	PAR
.05	3395	2225	8484	.0936	.0659	1.5258	3.8130	.2913
.10	3724	2451	9526	.0997	.0694	1.5194	3.8866	.3079
.15	4089	2703	10709	.1064	.0735	1.5128	3.9619	.3269
.20	4495	2983	12050	.1136	.0783	1.5069	4.0396	.3485
.25	4946	3295	13575	.1215	.0838	1.5011	4.1199	.3725
.30	5449	3643	15305	.1302	.0901	1.4957	4.2012	.4005
.35	6009	4030	17270	.1396	.0974	1.4911	4.2854	.4316
.40	6633	4463	19502	.1500	.1058	1.4862	4.3697	.4674
.45	7329	4945	22038	.1613	.1156	1.4821	4.4566	.5081
.50	8107	5484	24920	.1737	.1268	1.4783	4.5441	.5541
.55	8975	6087	28194	.1873	.1397	1.4745	4.6318	.6064
.60	9946	6761	31915	.2023	.1545	1.4711	4.7274	.6658
.65	11031	7516	36145	.2187	.1716	1.4677	4.8091	.7335
.70	12246	8360	40952	.2368	.1911	1.4648	4.8986	.8101
.75	13607	9307	46416	.2567	.2135	1.4620	4.9872	.8972
.80	14319	9838	49522	.2668	.2263	1.4607	5.0136	.9457
.85	15131	10368	52628	.2768	.2391	1.4555	5.0337	.9941
.90	18757	12894	67717	.3296	.3019	1.4571	5.2581	1.2353

6) Results of the 'D' Simulations

1974 MEST, 1974 UPU on 1975 data (Simulation 'D'(i))

Z value	$\bar{MW}$	$\bar{CW}$	SDCW	$\bar{X}$	SDX	PROF	STAB	PAR
.05	1566	52	315	.0375	.0467	30.1154	6.0577	.1776
.10	1628	53	325	.0386	.0480	30.7170	6.1321	.1826
.15	1693	55	335	.0398	.0498	30.7818	6.0909	.1892
.20	1763	57	346	.0410	.0510	30.9298	6.0702	.1940
.25	1836	59	358	.0422	.0527	31.1186	6.0678	.2003
.30	1914	61	369	.0436	.0546	31.3770	6.0492	.2074
.35	1997	63	381	.0450	.0566	31.6984	6.0476	.2148
.40	2085	65	394	.0464	.0588	32.0769	6.0615	.2228
.45	2179	67	407	.0480	.0612	32.5224	6.0746	.2316
.50	2279	69	421	.0496	.0639	33.0290	6.1015	.2413
.55	2385	71	435	.0513	.0668	33.5915	6.1268	.2517
.60	2499	73	449	.0532	.0700	34.2329	6.1507	.2632
.65	2620	76	464	.0551	.0735	34.4737	6.1053	.2756
.70	2749	78	480	.0571	.0773	35.2436	6.1538	.2890
.75	2887	81	496	.0593	.0816	35.6420	6.1235	.3041
.80	3035	83	512	.0615	.0862	36.5663	6.1687	.3201
.85	3193	86	530	.0640	.0913	37.1279	6.1628	.3379
.90	3363	89	548	.0665	.0969	37.7865	6.1573	.3572
.95	3544	92	566	.0692	.1031	38.5217	6.1522	.3785
1.00	3739	95	585	.0721	.1099	39.3579	6.1579	.4018
1.05	3948	98	605	.0752	.1173	40.2857	6.1735	.4271
1.10	4173	101	626	.0785	.1255	41.3168	6.1980	.4550
1.15	4415	104	647	.0820	.1344	42.4519	6.2212	.4852
1.20	4674	107	670	.0857	.1443	43.6822	6.2617	.5186
1.25	4954	110	693	.0896	.1550	45.0364	6.3000	.5546
1.30	5255	115	717	.0939	.1668	45.7913	6.2348	.5943
1.35	5579	118	741	.0984	.1797	47.2797	6.2797	.6375
1.40	5929	122	767	.1032	.1938	48.5984	6.2869	.6846
1.45	6306	126	794	.1083	.2093	50.0476	6.3016	.7362
1.50	6714	131	821	.1139	.2262	51.2519	6.2672	.7926
1.55	7153	135	850	.1198	.2447	52.9852	6.2967	.8539
1.60	7628	139	879	.1261	.2649	54.8777	6.3237	.9208
1.65	8142	144	910	.1329	.2870	56.5417	6.3194	.9939
1.70	8697	149	942	.1401	.3112	58.3691	6.3221	1.0737

Results of the 'D' Simulations (continued)

1975 MEST, 1975 UPU on 1976 data (Simulation 'D'(ii))

Z value	MW	CW	SDCW	X	SDX	PROF	STAB	PAR
.05	2482	1630	6360	.0661	.0454	1.5227	3.9018	.2023
.10	2726	1798	7152	.0705	.0481	1.5161	3.9778	.2148
.15	2997	1985	8050	.0753	.0511	1.5098	4.0554	.2286
.20	3298	2192	9070	.0805	.0547	1.5046	4.1378	.2446
.25	3634	2424	10229	.0862	.0589	1.4992	4.2199	.2629
.30	4008	2683	11544	.0925	.0637	1.4939	4.3026	.2836
.35	4425	2971	13039	.0993	.0693	1.4894	4.3888	.3072
.40	4890	3293	14737	.1068	.0758	1.4841	4.4753	.3342
.45	5410	3653	16666	.1150	.0833	1.4810	4.5623	.3649
.50	5990	4055	18859	.1241	.0919	1.4772	4.6508	.3998
.55	6639	4505	21351	.1340	.1018	1.4737	4.7394	.4394
.60	7364	5008	24183	.1449	.1132	1.4704	4.8289	.4845
.65	8176	5572	27403	.1568	.1263	1.4673	4.9180	.5357
.70	9086	6204	31062	.1701	.1413	1.4645	5.0068	.5940
.75	10105	6913	35222	.1846	.1585	1.4617	5.0950	.6601
.80	11248	7708	39951	.2007	.1782	1.4593	5.1831	.7353
.85	12530	8600	45328	.2185	.2007	1.4570	5.2707	.8206
.90	13970	9602	51440	.2381	.2264	1.4549	5.3572	.9173
.95	15586	10728	58390	.2598	.2557	1.4528	5.4428	1.0269

7) Results of the 'E' Simulations

1974-6 MEST, 1974-6 UPU on 1974 data (Simulation 'E'(i))

Z value	$\overline{MW}$	$\overline{CW}$	SDCW	$\bar{X}$	SDX	PROF	STAB	PAR
.05	1559	2237	7722	.0524	.0449	.6969	3.3973	.1871
.10	1656	2339	8066	.0546	.0466	.7080	3.4485	.1944
.15	1762	2446	8429	.0569	.0484	.7204	3.4460	.2012
.20	1876	2560	8811	.0594	.0504	.7328	3.4418	.2106
.25	2000	2680	9214	.0620	.0526	.7463	3.4381	.2198
.30	2135	2806	9640	.0648	.0551	.7609	3.4355	.2301
.35	2283	2940	10090	.0678	.0579	.7765	3.4320	.2415
.40	2444	3082	10566	.0710	.0609	.7930	3.4283	.2537
.45	2619	3232	11071	.0744	.0643	.8103	3.4254	.2673
.50	2811	3391	11605	.0780	.0682	.8290	3.4223	.2826
.55	3022	3560	12172	.0189	.0725	.8489	3.4191	.2994
.60	3252	3739	12775	.0860	.0773	.8698	3.4167	.3179
.65	3506	3929	13416	.0904	.0827	.8923	3.4146	.3385
.70	3784	4131	14098	.0952	.0888	.9160	3.4106	.3616
.75	4090	4346	14826	.1003	.0957	.9411	3.4114	.3874
.80	4427	4575	15603	.1058	.1034	.9677	3.4105	.4160
.85	4799	4819	16434	.1117	.1122	.9958	3.4103	.4483
.90	5209	5079	17323	.1180	.1221	1.0256	3.4107	.4843
.95	5663	5357	18276	.1248	.1332	1.0571	3.4116	.5244
1.00	6032	5654	19300	.1304	.1475	1.0669	3.4135	.5729
1.05	6722	5972	20400	.1402	.1602	1.1256	3.4159	.6208
1.10	7340	6312	21585	.1488	.1765	1.1629	3.4197	.6783
1.15	8025	6677	22862	.1581	.1949	1.2019	3.4240	.7428
1.20	8788	7069	24243	.1682	.2158	1.2432	3.4295	.8156
1.25	9636	7490	25736	.1792	.2395	1.2865	3.4360	.8977
1.30	10581	7942	27354	.1911	.2665	1.3323	3.4442	.9906
1.35	11635	8430	29111	.2041	.2971	1.3802	3.4533	1.0954

Results of the 'E' Simulations (continued)

1974-6 MEST, 1974-6 UPU on 1975 data (Simulation 'E'(ii))

Z value	$\bar{M}W$	$\bar{C}W$	SDCW	$\bar{X}$	SDX	PROF	STAB	PAR
.05	1660	46	278	.0510	.0454	36.0870	6.0435	.1872
.10	1767	49	296	.0534	.0474	36.0612	6.0408	.1956
.15	1884	52	315	.0560	.0498	36.2308	6.0577	.2054
.20	2012	55	336	.0588	.0524	36.5818	6.1091	.2160
.25	2151	59	357	.0618	.0553	36.4576	6.0508	.2277
.30	2303	63	381	.0650	.0587	36.5556	6.0476	.2411
.35	2469	67	406	.0685	.0624	36.8507	6.0597	.2557
.40	2651	71	432	.0723	.0667	37.3380	6.0845	.2724
.45	2850	76	460	.0763	.0716	37.5000	6.0526	.2911
.50	3068	81	491	.0807	.0771	37.8765	6.0617	.3120
.55	3307	86	523	.0854	.0833	38.4535	6.0814	.3353
.60	3569	92	557	.0905	.0904	38.7935	6.0543	.3617
.65	3857	98	594	.0960	.0984	39.3571	6.0612	.3912
.70	4174	105	633	.1020	.1076	39.7524	6.0286	.4248
.75	4523	112	674	.1085	.1180	40.3839	6.0179	.4643
.80	4907	119	719	.1155	.1298	41.2353	6.0420	.5049
.85	5331	127	766	.1232	.1433	41.9764	6.0315	.5531
.90	5799	135	816	.1316	.1586	42.9556	6.0444	.6074
.95	6315	144	870	.1407	.1760	43.8542	6.0417	.6687
1.00	6844	154	928	.1498	.1964	44.4416	6.0260	.7390
1.05	7519	164	989	.1615	.2184	45.8476	6.0305	.8167
1.10	8219	174	1054	.1734	.2440	47.2356	6.0575	.9054
1.15	8995	186	1124	.1864	.2732	48.3602	6.0430	1.0060

Results of the 'E' Simulations (continued)

1974-6 MEST, 1974-6 UPU on 1976 data (Simulation 'E'(iii))

Z value	M̃W	C̃W	SDCW	$\bar{X}$	SDX	PROF	STAB	PAR
.05	2035	1396	5322	.0519	.0429	1.4577	3.8123	.1806
.10	2178	1516	5905	.0545	.0448	1.4367	3.8951	.1889
.15	2332	1649	6561	.0573	.0468	1.4142	3.9788	.1977
.20	2500	1794	7298	.0602	.0491	1.3935	4.0680	.2075
.25	2683	1955	8128	.0634	.0516	1.3724	4.1575	.2182
.30	2882	2132	9061	.0667	.0545	1.3518	4.2500	.2302
.35	3100	2327	10112	.0704	.0577	1.3322	4.3455	.2435
.40	3337	2542	11293	.0743	.0613	1.3127	4.4426	.2582
.45	3597	2780	12623	.0784	.0654	1.2939	4.5406	.2746
.50	3881	3043	14120	.0829	.0700	1.2754	4.6402	.2929
.55	4193	3334	15803	.0878	.0752	1.2576	4.7400	.3134
.60	4535	3657	17698	.0930	.0811	1.2401	4.8395	.3363
.65	4910	4013	19830	.0986	.0878	1.2235	4.9414	.3620
.70	5323	4409	22229	.1047	.0954	1.2073	5.0417	.3909
.75	5777	4848	24928	.1113	.1041	1.1916	5.1419	.4236
.80	6277	5335	27966	.1184	.1140	1.1766	5.2420	.4604
.85	6829	5876	31383	.1261	.1254	1.1622	5.3409	.5023
.90	7439	6477	35229	.1345	.1383	1.1485	5.4391	.5494
.95	8112	7145	39557	.1436	.1531	1.1353	5.5363	.6029
1.00	5836	7888	44426	.1301	.1887	.7399	5.6321	.6962
1.05	9684	8715	49905	.1642	.1894	1.1112	5.7263	.7324
1.10	10600	9636	56070	.1760	.2116	1.1000	5.8188	.8108
1.15	11616	10661	63007	.1888	.2369	1.0896	5.9100	.8995
1.20	12746	11804	70812	.2028	.2660	1.0798	5.9990	1.0008

8) Results of the 'F' Simulations

1977 MEST, 1976 UPU on 1977 data (Simulation 'F'(i))

Z value	MW	$\bar{C}W$	SDCW	$\bar{X}$	SDX	PROF	STAB	PAR
.05	2489	4314	12208	.0461	.0553	.5770	2.8299	.2120
.10	2592	4453	12601	.0475	.0572	.5821	2.8298	.2191
.15	2701	4598	13021	.0490	.0592	.5874	2.8319	.2266
.20	2816	4751	13470	.0507	.0614	.5927	2.8352	.2349
.25	2938	4912	13950	.0523	.0637	.5981	2.8400	.2434
.30	3068	5082	14463	.0541	.0663	.6037	2.8459	.2530
.35	3206	5261	15013	.0560	.0690	.6146	2.8536	.2360
.40	3353	5450	15602	.0580	.0720	.6152	2.8628	.2740
.45	3509	5649	16233	.0601	.0753	.6212	2.8736	.2860
.50	3675	5839	16908	.0623	.0788	.6294	2.8957	.2987
.55	3852	6081	17633	.0646	.0827	.6334	2.8997	.3127
.60	4040	6316	18410	.0670	.0869	.6396	2.9148	.3277
.65	4241	6564	19242	.0696	.0914	.6461	2.9314	.3438
.70	4455	6827	20136	.0723	.0964	.6526	2.9495	.3615
.75	4683	7105	21094	.0752	.1018	.6591	2.9690	.3806
.80	4927	7399	22122	.0783	.1076	.6659	2.9899	.4011
.85	5188	7711	23225	.0815	.1140	.6728	3.0119	.4235
.90	5467	8042	24409	.0850	.1210	.6798	3.0352	.4480
.95	5766	8392	25679	.0886	.1285	.6871	3.0599	.4741
1.00	6086	8754	27043	.0925	.1367	.6944	3.0857	.5026
1.05	6428	9160	28507	.0966	.1456	.7017	3.1121	.5334
1.10	6795	9579	30080	.1009	.1553	.7094	3.1402	.5668
1.15	7189	10025	31768	.1055	.1659	.7171	3.1689	.6032
1.20	7612	10499	33582	.1105	.1773	.7250	3.1986	.6424
1.25	8065	11004	35531	.1157	.1898	.7329	3.2289	.6851
1.30	8553	11541	37625	.1213	.2034	.7411	3.2601	.7315
1.35	9077	12112	39876	.1272	.2182	.7494	3.2923	.7818
1.40	9641	12721	42296	.1335	.2343	.7579	3.3249	.8364
1.45	10248	13370	44898	.1402	.2519	.7665	3.3581	.8959
1.50	10901	14062	47698	.1474	.2709	.7752	3.3920	.9601
1.55	11605	14801	50710	.1550	.2917	.7841	3.4261	1.0301

Results of the 'F' Simulations (continued)

1977 MEST, 1977 UPU on 1977 data (Simulation 'F'(ii))

Z value	MW	CW	SDCW	X	SDX	PROF	STAB	PAR
.05	2292	3973	11151	.0432	.0522	.5769	2.8067	.1998
.10	2386	4099	11501	.0445	.0539	.5821	2.8058	.2062
.15	2485	4231	11875	.0460	.0558	.5873	2.8067	.2134
.20	2590	4370	12274	.0475	.0578	.5927	2.8087	.2209
.25	2702	4517	12702	.0490	.0600	.5982	2.8120	.2290
.30	2820	4671	13159	.0507	.0624	.6037	2.8172	.2379
.35	2946	4834	13648	.0524	.0650	.6094	2.8233	.2474
.40	3079	5005	14173	.0543	.0678	.6152	2.8318	.2577
.45	3221	5186	14734	.0562	.0708	.6211	2.8411	.2686
.50	3372	5377	15335	.0583	.0741	.6271	2.8519	.2806
.55	3533	5578	15980	.0604	.0777	.6334	2.8648	.2935
.60	3704	5719	16671	.0627	.0816	.6396	2.8788	.3075
.65	3886	6016	17413	.0651	.0859	.6459	2.8944	.3228
.70	4081	6254	18208	.0677	.0905	.6525	2.9114	.3392
.75	4288	6506	19061	.0704	.0955	.6591	2.9298	.3569
.80	4510	6772	19976	.0732	.1009	.6660	2.9498	.3759
.85	4747	7055	20958	.0762	.1068	.6729	2.9707	.3966
.90	5000	7354	22011	.0794	.1133	.6799	2.9931	.4193
.95	5270	7671	23143	.0828	.1202	.6870	3.0169	.4434
1.00	5560	8007	24357	.0864	.1278	.6944	3.0420	.4698
1.05	5870	8365	25661	.0902	.1361	.7017	3.0677	.4985
1.10	6202	8744	27061	.0942	.1451	.7093	3.0948	.5295
1.15	6559	9147	28565	.0985	.1548	.7171	3.1229	.5629
1.20	6941	9574	30180	.1031	.1655	.7250	3.1523	.5996
1.25	7351	10031	31915	.1079	.1770	.7328	3.1816	.6389
1.30	7792	10515	33780	.1131	.1896	.7410	3.2126	.6819
1.35	8266	11031	35785	.1186	.2032	.7493	3.2440	.7282
1.40	8775	11580	37940	.1244	.2181	.7577	3.2763	.7787
1.45	9323	12166	40257	.1307	.2342	.7663	3.3090	.8333
1.50	9912	12790	42750	.1373	.2518	.7750	3.3425	.8927
1.55	10547	13455	45432	.1444	.2709	.7839	3.3766	.9571
1.60	11232	14165	48319	.1519	.2917	.7929	3.4112	1.0270

Results of the 'F' Simulations (continued)

1976 MEST, 1976 UPU on 1977 data (Simulation 'F'(iii))

Z value	$\bar{M}W$	$\bar{C}W$	SDCW	$\bar{X}$	SDX	PROF	STAB	PAR
.05	1778	1994	8679	.0266	.0458	.8917	4.3526	.1640
.10	1818	2045	8898	.0272	.0468	.8890	4.3511	.1676
.15	1859	2097	9125	.0278	.0479	.8865	4.3515	.1715
.20	1901	2150	9363	.0284	.0490	.8842	4.3549	.1718
.25	1945	2206	9611	.0290	.0501	.8817	4.3568	.1793
.30	1990	2263	9869	.0296	.0513	.8794	4.3610	.1835
.35	2036	2322	10139	.0303	.0526	.8768	4.3665	.1881
.40	2083	2383	10420	.0310	.0539	.8741	4.3726	.1927
.45	2132	2446	10714	.0317	.0552	.8716	4.3802	.1973
.50	2183	2511	11020	.0324	.0566	.8694	4.3887	.2022
.55	2235	2579	11339	.0331	.0581	.8666	4.3967	.2074
.60	2289	2648	11672	.0339	.0596	.8644	4.4079	.2127
.65	2344	2720	12020	.0347	.0612	.8618	4.4191	.2183
.70	2401	2795	12382	.0355	.0628	.8590	4.4301	.2239
.75	2460	2872	12761	.0363	.0645	.8542	4.4432	.2298
.80	2521	2951	13155	.0372	.0663	.8543	4.4578	.2361
.85	2583	3033	13567	.0380	.0682	.8516	4.4731	.2426
.90	2648	3118	13996	.0389	.0702	.8493	4.4888	.2495
.95	2714	3206	14444	.0399	.0722	.8465	4.5053	.2565
1.00	2783	3297	14912	.0409	.0743	.8441	4.5229	.2638
1.05	2854	3392	15399	.0419	.0765	.8414	4.5398	.2714
1.10	2927	3489	15907	.0429	.0788	.8389	4.5592	.2793
1.15	3003	3590	16437	.0440	.0812	.8365	4.5786	.2876
1.20	3080	3694	16990	.0451	.0838	.8338	4.5994	.2965
1.25	3161	3802	17566	.0462	.0864	.8314	4.6202	.3054
1.30	3244	3914	18167	.0474	.0891	.8288	4.6415	.3147
1.35	3330	4029	18794	.0486	.0920	.8265	4.6647	.3246
1.40	3418	4149	19448	.0498	.0950	.8238	4.6874	.3348
1.45	3510	4273	20129	.0511	.0981	.8214	4.7107	.3454
1.50	3604	4401	20839	.0524	.1014	.8189	4.7351	.3566
1.55	3701	4534	21579	.0538	.1048	.8163	4.7539	.3682
1.60	3802	4672	22351	.0552	.1083	.8138	4.7840	.3801
1.65	3906	4814	23155	.0567	.1121	.8112	4.8089	.3930
1.70	4014	4962	23993	.0582	.1160	.8089	4.8353	.4062
1.75	4125	5115	24867	.0598	.1200	.8065	4.8616	.4198
1.80	4240	5273	25777	.0614	.1243	.8041	4.8885	.4343
1.85	4358	5437	26726	.0630	.1287	.8015	4.9156	.4491
1.90	4481	5607	27715	.0648	.1334	.7992	4.9429	.4650
1.95	4608	5784	28745	.0666	.1382	.7967	4.9697	.4812
2.00	4739	5966	29818	.0684	.1433	.7943	4.9980	.4983

continued . . .

Results of the 'F' Simulations (continued)

1976 MEST, 1976 UPU on 1977 data (Simulation 'F'(iii) continued)

Z value	MW	CW	SDCW	X	SDX	PROF	STAB	PAR
2.05	4874	6156	30936	.0703	.1486	.7917	5.0253	.5161
2.10	5015	6352	32101	.0723	.1542	.7895	5.0537	.5349
2.15	5160	6555	33315	.0743	.1600	.7872	5.0824	.5543
2.20	5310	6766	34579	.0765	.1661	.7848	5.1107	.5748
2.25	5465	6984	35897	.0787	.1725	.7825	5.1399	.5692
2.30	5625	7211	37269	.0809	.1791	.7801	5.1684	.6182
2.35	5791	7446	38698	.0833	.1861	.7777	5.1972	.6416
2.40	5963	7690	40187	.0857	.1934	.7719	5.2259	.6659
2.45	6141	7942	41737	.0882	.2010	.7732	5.2552	.6912
2.50	6325	8204	43352	.0908	.2090	.7710	5.2843	.7178
2.55	6516	8476	45034	.0935	.2174	.7688	5.3131	.7457
2.60	6713	8758	46786	.0963	.2261	.7665	5.3421	.7746
2.65	6917	9050	48611	.0992	.2352	.7643	5.3714	.8048
2.70	7128	9354	50511	.1022	.2448	.7620	5.3999	.8366
2.75	7347	9668	52490	.1053	.2548	.7599	5.4293	.8697
2.80	7574	9995	54551	.1085	.2653	.7578	5.4578	.9044
2.85	7809	10334	56698	.1119	.2763	.7557	5.4865	.9408
2.90	8052	10685	58933	.1153	.2878	.7536	5.5155	.9787
2.95	8304	11050	61260	.1189	.2998	.7516	5.5439	1.0183

Results of the 'F' Simulations (continued)

1974-6 MEST, 1974-6 UPU on 1977 data (Simulation 'F'(iv))

Z value	$\bar{MW}$	$\bar{CW}$	SDCW	$\bar{X}$	SDX	PROF	STAB	PAR
.05	2381	3444	8801	.0535	.0404	.6913	2.5555	.1747
.10	2570	3636	9295	.0568	.0427	.7068	2.5564	.1849
.15	2780	3841	9821	.0603	.0453	.7238	2.5569	.1962
.20	3013	4059	10384	.0641	.0485	.7423	2.5583	.2096
.25	3272	4291	10986	.0684	.0523	.7625	2.5602	.2253
.30	3562	4540	11631	.0730	.0570	.7846	2.5619	.2440
.35	3888	4804	12321	.0781	.0628	.8093	2.5647	.2665
.40	4253	5088	13062	.0837	.0701	.8359	2.5672	.2940
.45	4666	5391	13859	.0900	.0791	.8644	2.5708	.3273
.50	5134	5716	14716	.0969	.0904	.8982	2.5745	.3681
.55	5665	6063	15639	.1047	.1046	.9344	2.5794	.4185
.60	6271	6436	16635	.1134	.1223	.9744	2.5847	.4803
.65	6965	6836	17713	.1231	.1444	1.0189	2.5911	.5563
.70	7762	7266	18880	.1342	.1718	1.0683	2.5984	.6496
.75	8681	7729	20146	.1468	.2058	1.1232	2.6065	.7642
.80	9745	8226	21523	.1611	.2479	1.1847	2.6165	.9048
.85	10980	8762	23023	.1775	.2999	1.2531	2.6276	1.0772

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