DESIGN OF SWAY FRAMES

BY

SUBHI OKDEH

A Thesis Submitted for the Degree of
Doctor of Philosophy

Department of Civil Engineering
University of Aston in Birmingham

May 1980

TO MY WIFE LAYLA, AND

CHILDREN RANA AND ADNAN

SUMMARY

Subhi Okdeh

A Thesis submitted to the University of
Aston in Birmingham

For the Degree of Doctor of Philosophy

1980

The aim of the work presented in this thesis is to produce a direct method to design structures subject to deflection constraints at the working loads. The work carried out can be divided into four main parts. In the first part, a direct design procedure for plane steel frames subjected to sway limitations is proposed. The stiffness equations are modified so that the sway in each storey is equal to some specified values. The modified equations are then solved by iteration to calculate the cross-sectional properties of the columns as well as the other joint displacements. The beam sections are selected initially and then altered in an effort to reduce the total material cost of the frame. A linear extrapolation technique is used to reduce this cost. In this design, stability functions are used so that the effect of axial loads in the members are taken into consideration. The final reduced cost design is checked for strength requirements and the members are altered accordingly.

In the second part, the design method is applied to the design of reinforced concrete frames in which the sway in the columns play an active part in the design criteria. The second moment of area of each column is obtained by solving the modified stiffness equations and then used to calculate the minimum column depth required. Again the frame has to be checked for all the ultimate limit state load cases.

In the third part, the method is generalised to design pin-jointed space frames for deflection limitations. In these the member areas are calculated so that the deflection at a specified joint is equal to its specified value.

In the final part, the Lagrange multiplier technique is employed to obtain an optimum design for plane rigidly jointed steel frames. The iteration technique is used here to solve the modified stiffness equations as well as derivative equations obtained in accordance to the requirements of the optimisation method.

Key words: SWAY FRAMES STEEL CONCRETE SPACE FRAMES OPTIMUM

ACKNOWLEDGEMENTS

The author wishes to express his sincere thanks to Professor K I Majid, BSc, PhD, DSc, CEng, FICE, FIStructE, for his continual help, encouragement and advice throughout this project.

Thanks are also given to my colleague Mr Danny Blackburn, and to Mr R Williams and Mrs C Johns, lecturers in Modern Languages for reading the manuscript.

The author is indebted to the University of Lattakia of the Syrian Arab Republic for providing the scholarship.

LIST OF CONTENTS

SUMMARY
ACKNOWLEDGEMENTS
LIST OF CONTENTS
LIST OF FIGURES
LIST OF TABLES

			Page number
CHAPTER 1		HISTORICAL REVIEW AND SCOPE OF THE PRESENT WORK	1 to 31
	1.1	Introduction	1
	1.2	Design Methods for Strength Requirements	2
	1.2.1	Early Design Methods for Multi-Storey Frames	2
	1.2.2	Rigid-Plastic Design Method	3
	1.2.3	Plastic Design Method for Frame Instability	5
	1.2.3.1	Ultimate-load Design of Multi-Storey Sway frames	6
	1.2.4	Computer Methods of Design	12
	1.2.5	Design of Reinforced Concrete Frames	14
	1.2.6	Design of Pin-Jointed Space Frames	17
	1.3	Design to Deflection Requirements	18
	1.4	Optimum Design	26
	1.5	Scope of Present Work	29
CHAPTER 2		DESIGN OF STEEL FRAMES	32 to 86
	2.1	Introduction	32
	2.2	Contributions of a Member to the Overall Stiffness Matrix	33
	2.3	Member and Joint Specification	37
	2.4	Stiffness Equations for a General Joint in the Frame	39
	2.4.1	Stiffness Equations for a Beam Joint	39
	2.4.1.1	The Vertical Deflection y_j	42

		Page number
2.4.1.2	The Rotation r_j	44
2.4.2	Stiffness Equations for a Column Joint	44
2.4.2.1	The Horizontal Deflection x_j	45
2.4.2.2	The Vertical Deflection y_j	46
2.4.2.3	The Rotation r_j	47
2.4.3	Design Equation for a Column	49
2.4.3.1	Equation for Second Moment of Area of the External Left Column I_1	51
2.4.4	An Example on the Derivation of the Stiffness Equations	52
2.5	Preliminary Beam Sections	54
2.6	Solution of the Modified Stiffness Equations by Iterative Technique	55
2.7	An Example on the Solution of the Modified Stiffness Equations	55
2.8	An Outline of Design Procedure	56
2.9	General Equation for the Sway Deflection Constraints	59
2.10	Universal Beam and Column Sections Used in the Design	62
2.11	Altering the Beam Sections	64
2.11.1	Example on Reducing Beam Sections	65
2.12	Selecting the Internal to External Column Ratio	70
CHAPTER 3	REFINING THE DESIGN PROCESS	87 to 103
3.1	Reduction in the Bending Stiffness of the Columns	87
3.2	Calculation of the Axial Forces	88
3.3	Example on the Axial Load Effect	89
3.4	Economy in Computer Time	92
3.5	Economic Use of Computer Storage	96
3.6	The Design Criteria	97

Page number

3.7	Strength Requirements	100
3.7.1	Forces in a Member	100
3.7.2	Checking the Beam Sections	102
3.7.3	Checking the Column Section for Strength	103
CHAPTER 4	COMPUTER PROGRAM	104 to 118
4.1	Introduction	104
4.2	Input Data	105
4.3	Design for Deflection Requirements	109
4.4	Design for Strength Requirements	111
4.5	Subroutine ITERATE	112
CHAPTER 5	DESIGN EXAMPLES	119 to 151
5.1	A 12 Storey-Single Bay Frame	119
5.2	A 24 Storey Frame with Three Equal Bays	121
5.3	A 35 Storey - Two Unequal Bay Frame	123
5.4	Conclusions	128
CHAPTER 6	DESIGN OF REINFORCED CONCRETE FRAMES	152 to 202
6.1	Introduction	152
6.2	The Principles of Design	153
6.3	Design Assumptions	155
6.4	Reinforcement Requirements	157
6.5	Design Loads	158
6.5.1	Design Loads for Serviceability Limit	159
6.5.2	Design Loads for Ultimate Limit State	159
6.6	Equation for the Minimum Column Depth	160

Page number

6.7	Design of a Beam Section	164
6.7.1	Design of the Beam Section for Ultimate Limit State	165
6.8	Ultimate Limit State for the Columns	167
6.8.1	Construction of Design Chart in CP110	168
6.8.2	Computer Subroutine	171
6.9	Computer Program for the Design of Reinforced Concrete Frames	173
6.9.1	The Program Segments for the Design of Reinforced Concrete Frames	174
6.10	A 15 Storey Reinforced Concrete Frame with Three Unequal Bays	175
6.11	A 24 Storey Reinforced Concrete Frame with Two Unequal Bays	178

CHAPTER 7 DESIGN OF SPACE FRAMES 203 to 251

7.1	Introduction	203
7.2	The Contributions of a Member to the Overall Stiffness Matrix	203
7.3	Modified Stiffness Equations for a General Joint in the Pin Jointed Space Frame	205
7.4	Design Equation for Member Areas	208
7.4.1	Calculation of A_{g1}	209
7.5	Design Procedure	210
7.6	Computer Program	213
7.7	Design Example 1: A Six Member Isostatic Space Frame	214
7.8	Design Example 2: An Eight Member Hyperstatic Space frame	216
7.9	Design Example 3	218
7.10	Design Example 4	219
7.11	Design Example 5	220

		Page number
CHAPTER 8	OPTIMUM DESIGN OF RIGIDLY JOINTED STEEL FRAMES	252 to 289
8.1	Introduction	252
8.2	A Design Example	254
8.2.1	Solution of the Derivative Equations by the Iteration Technique	261
8.3	The Derivatives of the Lagrangian Function for a General Joint in a Frame	262
8.4	The Derivatives of the Lagrangian Function for a Beam Joint	264
8.5	The Derivatives of the Lagrangian Function for a Column Joint	267
8.6	Outline of the Design Procedure	272
8.7	The Computer Program	274
8.8	Design Example 1 - A Four Storey Singly Bay Frame	275
8.9	Design Example 2 - A Four Storey - Two Bay Frame	276
8.10	Design Example 3 - Four Storey Two Unequal-Bay Frames	277
CHAPTER 9	SUGGESTIONS FOR FURTHER WORK	290 to 294
Appendix A	DATA PREPARATION - THE DESIGN OF STEEL FRAMES	295 to 298
Appendix B	DATA FOR THE CONCRETE PROGRAM	299
Appendix C	DATA PREPARATION - THE DESIGN OF PIN-JOINTED SPACE FRAMES	300 to 304
REFERENCES		305 to 311

LIST OF FIGURES

		Page number
CHAPTER 1		
Figure 1.1	The Limited Frame for Beam Design in the Joint Committee's Method	8
Figure 1.2	The Limited Frame for Column Design in the Joint Committe's Method	8
Figure 1.3	The Equivalent Grinter Frame in Wood's Design Method	9
Figure 1.4	Frame Subassemblages in Moy's Method of Design	10
Figure 1.5	Margin of Safety in Codes of Practice over the Years	20
Figure 1.6	A Typical Subassemablage in Anderson and Islam's Design	23
Figure 1.7	The Complete Frame in Anderson and Islam's Method	23
CHAPTER 2		
Figure 2.1	The sign convention and the Positive Direction of the Local and Global Axes	35
Figure 2.2	Numbering of Members and Joints of a Frame	38
Figure 2.3	Specifications for a joint in a Beam	40
Figure 2.4	Configurations of the Member at a Column Joint	40
Figure 2.5	Definition of the Notation Relating the Neighbouring Joints to the Joint Under Consideration	41
Figure 2.6	Intermediate Storey in a Rectangular Frame of Non-Uniform Bay Width	72
Figure 2.7	Single-Storey Single-Bay Frame	72
Figure 2.8	Linear and Non-Linear Deflection Profiles	73
Figure 2.9	Linear Extrapolation to find the Second Moment of Area for the Columns	78
Figure 2.10	The 4-Storey Single Bay Frame	79
Figure 2.11	Deflection Curves for Various Values of k for the Four Storey Single Bay Frame	80

		Page number
Figure 2.12	First Design Cycle (The Four Storey Single Bay Frame)	81
Figure 2.13	Second Design Cycle	81
Figure 2.14	Modified Second Design Cycle	82
Figure 2.15	Third Design Cycle	82
Figure 2.16	Result of the Reduction of the Third Cycle Beams	83
Figure 2.17	Fourth Design Cycle	83
Figure 2.18	Final Design (The Four Storey Single Bay Frame)	84
Figure 2.19	Optimum Design of the Frame	84
Figure 2.20	Total Mass for Each Design Cycle for the Four-Storey Single Bay Frame	85
Figure 2.21	The Procedure for Economy	86
CHAPTER 3		
Figure 3.1	Five Storey Three Unequal-Bay Frame, Loads and Dimensions of the Frame	90
Figure 3.2	Number of Iterations for Each Design Trial for the Five-Storey Three-Unequal Bay Frame	94
CHAPTER 4		
Figure 4.1) Figure 4.2) Figure 4.3)	Master Program	106 to 108
Figure 4.4) Figure 4.5) Figure 4.6) Figure 4.7)	Subroutine ITERATE	113 to 116
CHAPTER 5		
Figure 5.1	12 Storey-Single Bay Frame - Dimensions and Loads	130
Figure 5.2	The 12 Storey-Single Bay Frame Deflection Profiles	131
Figure 5.3	12 Storey-Single Bay Frame Number of Iterations Per Design Cycle	133

Page number

Figure 5.4	The 12 Storey-Single Bay Frame Material Cost For Each Design	134
Figure 5.5	24 Storey - 3 Equal Bay Frame Dimensions and Loads	135
Figure 5.6	The Deflection Curves for the Design of the 24 Storey Frame Shown in Figure 5.3	136
Figure 5.7	Analysis of Different Designs of the 24 Storey- 3 Equal Bay Frame	141
Figure 5.8	Reduction in the Material Cost of 24 Storey – 3 Equal Bay Frame	142
Figure 5.9	Iteration in Each Design Trial for the 24 Storey - 3 Equal Bay Frame	143
Figure 5.10	35 Storey - 2 Bay Frame	144
Figure 5.11	Deflection Curve for the 35 Storey - 2 Bay Frame	145
Figure 5.12	Column Sway of the 35– 2 Unequal Bay Frame, with Wind from the Left for Various Designs	147
Figure 5.13	Analysis of Different Designs of the 35 Storey - 2 Unequal Bay Frame, Wind from Right	148
Figure 5.14	Cost of the Material for Final Designs Starting with Different Set of Beams Obtained from Anderson and Islam's Design	149

CHAPTER 6

Figure 6.1	Load Combination (1), Different Load Arrangements for Ultimate Limit State	181
Figure 6.2	Load Combination (2) Different Load Arrangements for Ultimate Limit State	182
Figure 6.3	A Typical Column Section	161
Figure 6.4	A Typical Rectangular Section	165
Figure 6.5	A Typical T-Beam Section at Mid-Span	166
Figure 6.6	A Typical Design Curve for Rectangular Column	183
Figure 6.7	Design Stress-Strain Curve for Reinforcement in Tension	183

		Page number
Figure 6.8	A Typical Design Chart for Rectangular Reinforced Concrete Column Section	184
Figure 6.9	Strain Diagrams for Some Significant Points	185
Figure 6.10	The 15 Storey - 3 Unequal Bay Frame. Dimensions and Applied Loads	186
Figure 6.11	Sway Deflection Curve for the 15 Storey Frame Shown in Figure 6.10	187
Figure 6.12	Typical Storey Representation for the 15 Storey Frame	188
Figure 6.13	24 Storey Concrete Frame	193
Figure 6.14	24 Storey Concrete Frame Sway Deflection Profiles	194
Figure 6.15	24 Storey Concrete Frame - Typical Storey Representation	196
CHAPTER 7		
Figure 7.1	A member in a Pin-Ended Space Frame	204
Figure 7.2	Pin-jointed Space Frame, Flow Chart for Subroutine ITERATE	224 and 225
Figure 7.3	6 Member - Isostatic Space Frame	226
Figure 7.4	6 Member Isostatic Space Frame Values of the Member Areas for Each Iteration in the First Design Cycle	227
Figure 7.5	8 Member Hyperstatic Space Frame	229
Figure 7.6	8 Member Hyperstatic Space Frame, Total Volume at Each Design Cycle	232
Figure 7.7	8 Member Hyperstatic Space Frame Number of Iterations Per Design Cycle	233
Figure 7.8	Space Frame (a)-Member and Joint Numbers	234
Figure 7.9	Space Frame (b)-Member and Joint Numbers	234
Figure 7.10	Space Frame (c)-Member and Joint Numbers	235
Figure 7.11	Frames (a) and (b) of Figures 7.8 and 7.9, Total Volume at Each Design Cycle	237

		Page number
Figure 7.12	Frame (c) of Figure 7.10 Total Volume at Each Design Cycle	238
Figure 7.13	The Pyramid Shape Space Frame - Member and Joint Numbers	239
Figure 7.14	The 781 Member Pin-Jointed Space Frame	242
Figure 7.15	Members in Group 3 in the X-Z Plane at y = 0.0 mm	243
Figure 7.16	Members in Group 4 in the X-Z Plane at y = 2000.0 mm	244
Figure 7.17	The 781 Member Space Frame - Various Sections in the Structure	245
Figure 7.18	The 781 Member Space Frame Change in Volume for Each Design Cycle - High Yield Steel Used	246
Figure 7.19	The 781 Member Space Frame - Mild Steel Used. Volume of the Structure at Each Design Cycle	250
Figure 7.20	The 781 Members Space Frame - Mild Steel Used. Number of Iterations per Design Cycle	251

CHAPTER 8

Figure 8.1	Single Storey-Single Bay Frame	255
Figure 8.2	Specifications for a Joint j in a Beam	265
Figure 8.3	Specifications for a General Configuration of a Column Joint j	268
Figure 8.4	Optimum Design Flow Chart	273
Figure 8.5	The 4 Storey Single Bay Frame Optimum Design Using Classical Optimisation	279
Figure 8.6	The 4 Storey Single Bay Frame ΣIL in Each Design Cycle	281
Figure 8.7	A 4 Storey - 2 Equal Bay Frame Dimensions and Loads	282
Figure 8.8	Optimum Design of the Frame Using Classical Optimisation	283

		Page number
Figure 8.9	A 4 Storey - 2 Equal Bay Frame ΣIL per Design Cycle	284
Figure 8.10	A 4 Storey - 2 Equal Bay Frame - The Reduced Cost Design Obtained by the Method of Chapter 2	285
Figure 8.11	A 4 Storey - 2 Unequal Bay Frame Dimensions and Loads	286
Figure 8.12	The 4 Storey - 2 Unequal Bay Frame Optimum Design	287
Figure 8.13	The 4 Storey - 2 Unequal Bay Frame ΣIL per Design Cycle	288
Figure 8.14	The 4 Storey - 2 Unequal Bay Frame - Reduced Cost Obtained Using the Method of Chapter 2	289
CHAPTER 9		
Appendix A		
Figure A1	Example on Data Input for the Steel Design Program	298
Appendix B		
Appendix C		
Figure C1	6 Member - Isostatic Space Frame	301

LIST OF TABLES

		Page number
CHAPTER 1		
CHAPTER 2		
Table 2.1	The convergence of the deflections and second moment of area of the columns to the design values for the single storey frame	71
Table 2.2	Universal Beam Sections for Grade 43 Steel	74
Table 2.3	Universal Column Sections (Grade 43)	75
Table 2.4	Universal Beam Sections for Grade 50 Steel	76
Table 2.5	Universal Column Sections for Grade 50	77
CHAPTER 3		
Table 3.1	The Design of the Five Storey Frame	91
Table 3.2	Vertical Displacements of Joints 1, 2 and 3 of Figure 3.1	95
CHAPTER 4		
CHAPTER 5		
Table 5.1	Designs of the 12 Storey Frame	132
Table 5.2	Design of the 24 Storey - 3 Equal Bay Frame	137
Table 5.3	Anderson and Islam's Design of the 24 Storey - 3 Equal Bay Frame	138
Table 5.4	Design of 24 Storey - 3 Bay Frame Neglecting Axial Stiffness and Axial Load	139
Table 5.5	Cost and Mass of the 24 Storey - 3 Equal Bay Frame Designed by Various Methods	140
Table 5.6	Trial Initial Sections for A 35 Storey 2 Unequal Bay Frame Obtained by Anderson and Islam's Equations Using the Linear Deflection Curve with $\alpha = 1/300$	146

		Page number
Table 5.7	Design of the 35 Storey - 2 Unequal Bay Frame, Combined Loading, Wind from Left	150
Table 5.9	Final Design of the 35 Storey - 2 Unequal Bay Frame for Deflection and Strength Requirements	151
CHAPTER 6		
Table 6.1	Values of γ_f, Serviceability Limit State of Deflection	158
Table 6.2	Values of γ_f, Ultimate Limit State	159
Table 6.3	15 Storey Frame - The Design of the Beam Sections for the Limit State of Deflection	176
Table 6.4	15 Storey Frame - The Steel Area for Beam Sections of the Final Design	189 and 190
Table 6.5	15 Storey Frame - Design of the Column Sections for Deflection Limit State and Ultimate Limit State	191 and 192
Table 6.6	24 Storey Frame - Reinforcement Covers	195
Table 6.7	24 Storey Frame - Reinforcement Ratios Assumed for the Limit State of Deflection	195
Table 6.8	24 Storey Frame - The Design of the Beam Sections for the Limit State for Deflection	197
Table 6.9	24 Storey Frame - The Steel Area for Beam Section of the Final Design	198 and 199
Table 6.10	24 Storey Frame - Design of the Column Sections for Limit State of Deflection and Ultimate Limit State	200 to 202
CHAPTER 7		
Table 7.1	6 Member Isostatic Space Frame Various Design Values	228
Table 7.2	8 Member - Hyperstatic Space Frame, Ratios Between the Member Areas and the Area of Member (1) for Various Design Cycle	230

Page number

Table 7.3	8 Member - Hyperstatic Space Frame. Member Areas in the Final Design both Stress and Deflection Requirements are Satisfied	231
Table 7.4	8 Member - Hyperatatic Space Frame Member Forces, and Member Areas when Only Stress Requirements are satisfied	231
Table 7.5	Joint Coordinates and external applied loads for Frames (a), (b) and (c) of Figures (7.8), (7.9) and (7.10)	235
Table 7.6	Final Designs (in which deflections and stress requirements are satisfied) for Frames (a), (b) and (c) shown in Figures (7.8), (7.9) and (7.10)	236
Table 7.7	The Pyramid Shape Space Frame-Joint Coordinates and External Applied Loads	240
Table 7.8	The Pyramid Shape Space Frame-Design Values at the Final Cycle	241
Table 7.9	The 781 Member Space Frame - High Yield Steel - Member Areas in the First Group and Values of α for Each Design Cycle	247
Table 7.10	the 781 Member Space Frame - Various Designs High-Yield Steel Used	248
Table 7.11	Values of α for Mild Steel Space Frames	249
CHAPTER 8		
Table 8.1	The 4 Storey - Single Bay Frame Lagrange Multipliers for Each Joint in the Frame	280
Table 8.2	The 4 Storey - Single Bay Frame Number of Iterations per Design Cycle	280
CHAPTER 9		
APPENDIX A		
Table A1	Data for Column Joints 4, 5 and 6 of the Frame of Figure A1	297
APPENDIX C		
Table C1	Data for Members 2, 3, 4, 5 and 6 of the Frame C1	302
Table C2	Data for Joints 2, 3, 4, 5 and 6 of the Frame of Figure C1	303

CHAPTER ONE

HISTORICAL REVIEW AND SCOPE OF THE PRESENT WORK

1.1 INTRODUCTION

In multistorey buildings the wind load is usually resisted either by a stiff core or by specially designed bracing frames. In such structures the frames may be designed to resist vertical loads only. However, such construction may, in some cases, be found undesirable in order to satisfy other functional requirements of the building. In these cases the transverse frames are designed as plane sway frames and the horizontal forces are taken by rigid frame action.

There is a wide range of possibilities open to the designer by which the member properties of the frame can be obtained. This depends on the design criteria imposed. Fundamentally, the object of the design is to find the most economical structure consistent with the safety and serviceability requirements. Thus, the design of sway frames often includes the following:-

1) Safety - based on the ultimate strength of the structure (strength requirements)
2) Serviceability - the deflection of the structure should not adversely affect the appearance of the structure (deflection requirements)
3) In addition the cost of the structure may be as small as possible.

Methods of designing a sway frame may be classified into three categories. In the first category the strength requirements are considered to be of overriding importance. In the second type both the strength and deflection requirements are considered to be important; and in the third, the optimum design is aimed at and in which it is possible to satisfy all the design criteria simultaneously. Thus, in this chapter the design methods are reviewed according to the above classification.

1.2 DESIGN METHODS FOR STRENGTH REQUIREMENTS

1.2.1 Early Design Methods for Multistorey Frames

The first detailed investigation into the design of multistorey building frames was carried out by the Steel Structure Research Committee (SSRC) from 1929 to 1936. Before this period, the accepted method of design for frames braced against wind was that now known as 'simple design' (Allwood, et al, 1961). In this method the beam-column connections are considered to be pinned. Thus, the beams are assumed to be simply supported. The columns are designed for axial load, as well as for the end moments caused by the beam reactions acting at an assumed eccentricity with respect to the column axis. The design is based on linear elastic behaviour of the structural material. Sections are chosen so that the maximum stresses in a member do not exceed certain permissible values.

An extension of this method to the design of rigidly-jointed sway frames, in which there is no wind-bracing, is explained in the Steel Designers Manual by Allwood, et al (1972). The internal forces in the members due to vertical loading are calculated as above. Those due to wind loading are found by making the frame statically determinate. This is done by assuming points of contraflexure at both the mid-height of the columns, and the mid-span of the beams. At this stage of design connections are taken to be rigid. The total internal-member forces are then obtained by superposition. The Steel Designers Manual gives three simple design methods, all of which use the above assumption. The most commonly used of the three methods is the portal method which assumes that each bay acts as a simple portal and the total horizontal load is divided between the bays in proportion to the spans of the bays.

The final report of the Steel Structures Research Committee and the work by Baker (1954) showed that practical joints made of rivets or high tensile black bolts do not act as being pinned under the effect of vertical loading. The rigidity of the joints produces restraining moments at the ends of a beam reducing the maximum design moments by some 17% to 25%, and causing large end moments in the columns. They have also shown that all the columns in a frame do not necessarily bend in double-curvature under wind loading, particularly in frames with weak beams. This clearly means that the assumption of a hinge at the mid-height of a column is not always true. However, the safety of the above methods probably arises from a judicious choice of numerical values for permissible stresses, eccentricities, etc.

It has long been realised that if the individual members of a frame are designed according to the requirement to satisfy the allowable stresses, then the multistorey frame has a greater load-carrying capacity than permitted by the elastic design method. This has led to the application of rigid-plastic theory in structures, which enables the design of a frame to be based on its overall ultimate strength.

1.2.2 Rigid-Plastic Design Method

This was first developed in Cambridge by Baker, Horne, and Heyman (1956). One assumption made in the analysis of structures by the plastic theory is that the strains and deformations in the structure, at the limit of proportionality, are small compared to those at the onset of strain hardening. The theory therefore neglects the elastic strains and deformations. It assumes that the structure remains undeformed until suddenly a number of fully plastic hinges develop at discrete sections in the structure which converts it into a collapsing mechanism. At the begining of collapse (the instant when the structure is just converted into a mechanism with the formation of the last hinge) the structure is isostatic and, the bending moments throughout the structure can be calculated by using the static equilibrium of forces alone.

The assumption of negligible deflections in the pre-collapse state is generally applicable in frames braced against sidesway and in very few cases of low-rise sway frames where the effect of wind load is very small. In general the horizontal deflections in sway frames are of major importance and can not be neglected. They cause rapid deterioration of the overall stiffness of a frame. In fact, Wood (1958) shows that it is possible for a frame to lose all its stiffness, and therefore collapse, before the occurrance of a full mechanism. This results in the frame collapsing at a load factor less than the plastic failure load. He has shown that in a four-storey one-bay frame, after the formation of only two hinges, the modified critical load of the remaining parts of the frame is less than half of the original load. Furthermore, the hinges formed did not correspond to those of the rigid-plastic mechanism either in their position or in the load levels at which these hinges formed. Thus, the overall effect of instability in multi-storey sway frames is significant. For this reason, in recent years several methods have been proposed to include the instability of the frame into the rigid-plastic design.

1.2.3 Plastic Design Methods for Frames Instability

Heyman (1960), proposed an approach to the ultimate load design of multi-storey sway frames by assuming a system of plastic hinges which involved collapse in both beam and columns. Hinge moments of the beam and columns are calculated with the factored load. Heyman suggests the use of the beam hinge moments calculated to design the beams according to full plastic moment values. For the columns, because of the possibility of instability effects, Heyman proposed that the columns should be designed to remain elastic under the combined action of the hinge moments and axial forces. As a safeguard against instability it was suggested that a frame should be elastic at the working load without allowing excessive sway deflections. A method was given for the estimation of these deflections which made no allowance for the reduced frame stiffness due to instability.

Heyman's design method has the advantage of directness and simplicity. Frame instability is not considered, but there is a degree of conservatism in the design procedure introduced by the elastic design of the columns. No suggestion was made as to what action should be taken if the deflections were not satisfactory.

Holmes and Gandhi (1965) proposed a design method in which an allowance was made for instability. This was restricted to regular rectangular frames in which collapse was assumed to occur by beam, combined and sway mechanisms in the upper, middle and lower regions respectively. An initial design, obtained by considering this pattern of hinges, was modified to allow for:

i) axial load effects, considering the reduction of column stiffnesses due to compressive axial forces by using the 'stability functions' m, n and 0;

ii) the reduction of beam stiffnesses, due to the formation of plastic hinges; and

iii) points of contraflexure not occurring at the mid-height of columns.

The design procedure was repeated until two successive designs were identical.

More recently Holmes and Sinclair-Jones (1970) have used the same principle and developed a more accurate method for modifying the initial design, and also for dealing with boundary regions of the frame, such as the top most storey, the bottom storey and the external columns.

All the above methods reviewed in this section made use of an assumed pattern of hinges, which might not correspond to that which would form in practice. In fact, Majid and Anderson (1968), and Anderson (1969) showed that the assumed pattern of hinges was unrealistic and far from the actual behaviour of the frame at collapse. Furthermore, there can be no certainty of either the safety or the economy of a design obtained by such methods.

The above methods did not design for deflection although some of them pointed to the significance of excessive deflection in tall frames.

1.2.3.1 Ultimate-Load Design of Multi-storey Sway Frames

An empirical expression for the actual failure load of a frame λ_f is given by Merchant (1954) as:

$$1/\lambda_f = 1/\lambda_c + 1/\lambda_p \tag{1.1}$$

where λ_c and λ_p are the elastic critical load and plastic failure load respectively. Approximate methods for calculating λ_c are given by Wood (1974), Horne (1975), Bolton (1976), Moy (1976a), Williams (1977) and Williams (1979). A rapid calculation of λ_p for use in equation (1.1) is given by Neal and Symonds (1952).

The B/20 Draft Specification (1977) permits the use of equation (1.1) within certain limits. These are:

i) If the ratio of $\lambda_c/\lambda_p \geqslant 10$, then λ_f is limited to λ_p

ii) If $\lambda_c/\lambda_p < 4$, then the above equation cannot be used and a full non-linear elastic-plastic analysis is necessary.

The reason is that for this category of frames equation (1.1) will give higher values of λ_f than those given by accurate elastic-plastic analysis.

The first condition is stated above in general applicable to braced frames. A method for designing a braced frame for ultimate load is given by the Joint Committee's second report (1971). In this method it was assumed that it is sufficiently accurate for design purposes to consider only that limited part of the frame to which the member is directly connected. The limited frames for a beam and a column design are shown in Figures 1.1 and 1.2, in which the neighbouring members are assumed to be fixed. However, this design method is approximate and it is only applicable to non-sway frames.

For sway frames, when λ_c/λ_p does not exceed the limits imposed by the above conditions, equation (1.1) can be used to calculate λ_f. This involves the calculation of λ_c. Wood (1974) uses a substitute frame based on the work of Grinter (1937). Figure 1.3b shows the substitute frame used by Wood for the original frame shown in Figure 1.3a.

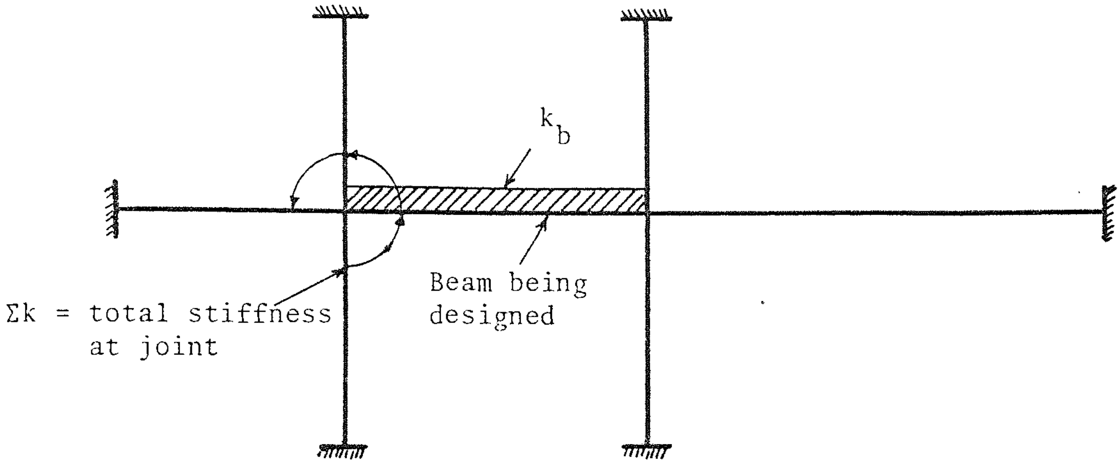


FIGURE 1.1 THE LIMITED FRAME FOR BEAM DESIGN IN THE JOINT COMMITTEE'S METHOD

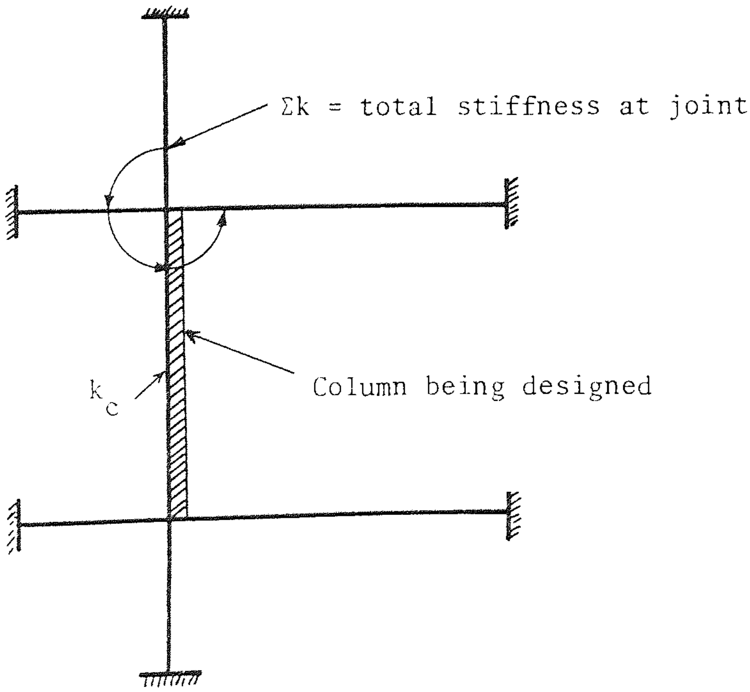
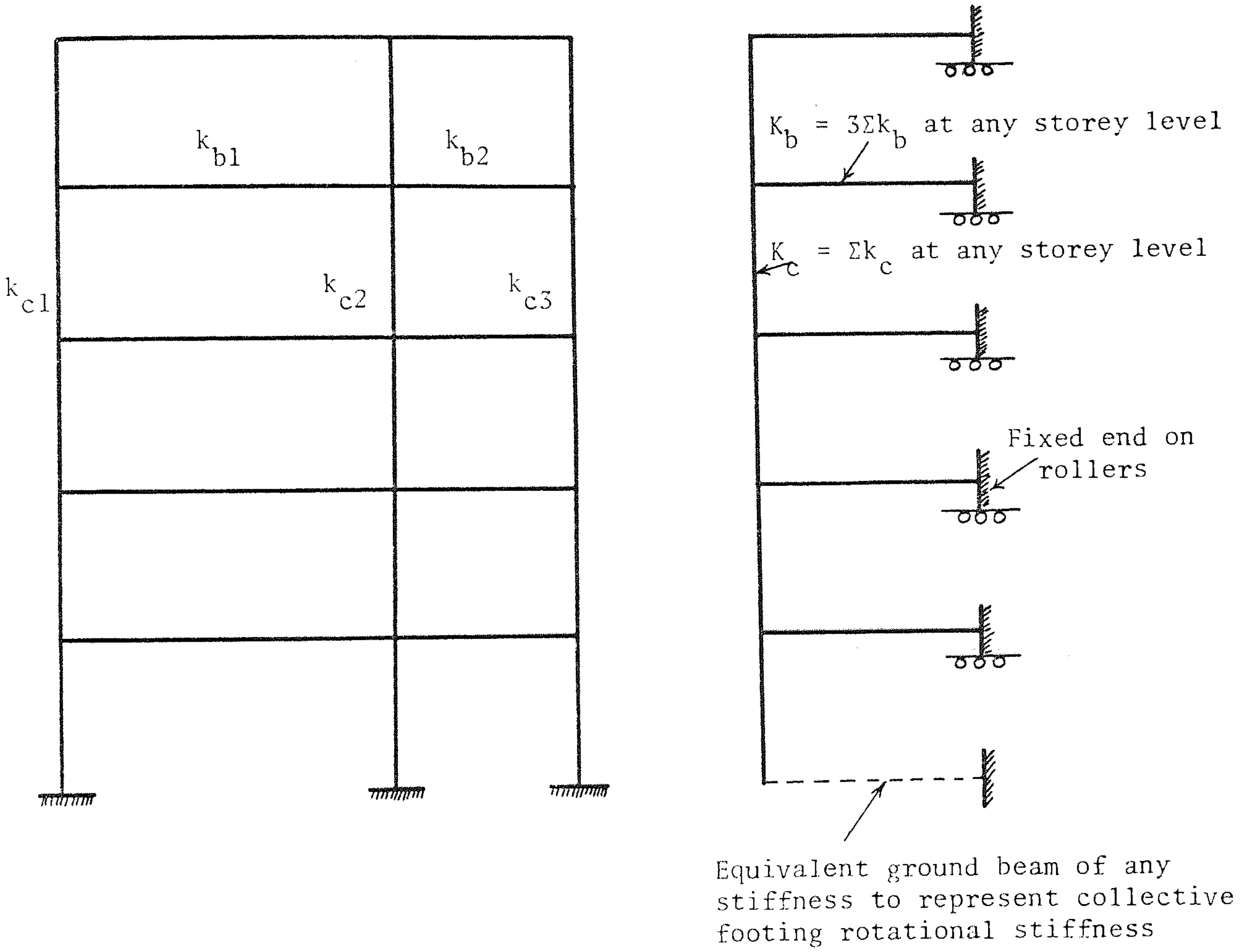


FIGURE 1.2 THE LIMIT FRAME FOR COLUMN DESIGN IN THE JOINT COMMITTEE'S METHOD



(a) The Frame

(b) Equivalent Grinter Frame

FIGURE 1.3 THE EQUIVALENT GRINTER FRAME IN WOOD'S DESIGN METHOD

The stiffness K_b of a beam in the substitute frame at any level is given by $K_b = 3\ \Sigma k_b$, where k_b is I/L for a beam in the real frame, I is the second moment of area, and L is the span of the real beam. The stiffness of the column of a Grinter frame at any storey level is the sum of the stiffness of the columns in the real frame. As the wind and vertical loads on the frame increase, the rotational stiffness of each joint in the substitute frame decreases gradually. Frame instability is said to have taken place when the rotational stiffness of one of the joints of the substitute frame becomes zero or negative. The load factor at this stage is the elastic critical load.

Williams (1977) modified the substitute frame of figure 1.3b to obtain a close lower bound on λ_c, which is supposed to produce a safe design by equation (1.1). More recently, Williams (1979) presents a unified, exact treatment of the substitute frames which was used by Wood to approximate unbraced multi-storey frames. The method is only exact for the substitute frame and can be used to calculate the exact sway deflection for this frame

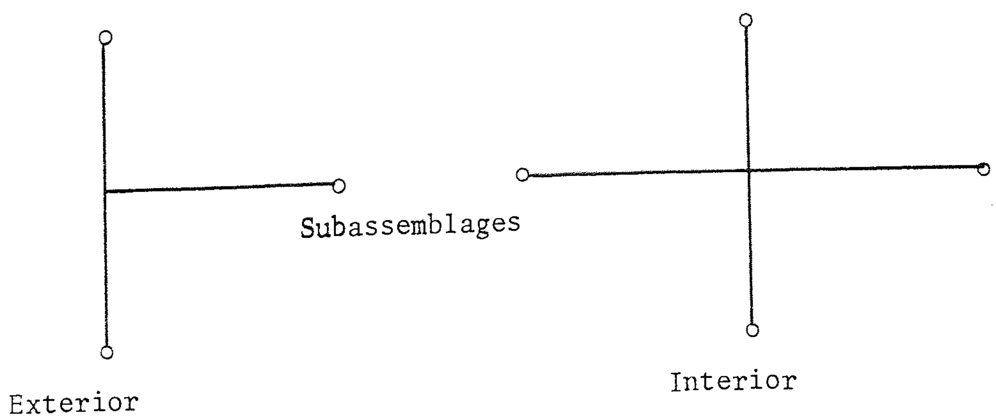


FIGURE 1.4 FRAME SUBASSEMBLAGES IN MOY'S METHOD OF DESIGN

Moy (1976a) proposed a method for calculating λ_c when the frame is subjected to vertical loads only. In this method points of inflection are considered to develop at the mid-lengths of the members. With such an idealization, a storey is assumed to consist of a series of subassemblages composed of two columns and of one or two restraining beams (Figure 1.4) depending upon whether the subassemblage is an exterior or an interior one. The concept upon which the method was based is that the stability of a whole frame is a function of storey stiffnesses. As the gravity loads increase, the storey stiffnesses decrease gradually. Frame instability is said to have taken place when one storey stiffness becomes zero or negative. Moy accounts for the sudden change in the storey stiffness, when a plastic hinge forms in a member. This was reflected by a sharp drop in the storey stiffness. However, his method ignores the wind loads on unbraced frames, which have a major effect on the frame instability.

In all the above methods reviewed in this section the frame is idealized employing mathematical models to depict the characteristics and behaviour of the real structure. These models may not accurately represent the real structure. All the methods calculate λ_c, but the design using equation (1.1) also involves the calculation of rigid-plastic failure load λ_p. Finding the right mechanism for a large multistorey frame is a complex operation. Moreover, the design using this equation may still involve a large amount of iterative calculations to obtain a safe design.

A solution obtained by using the Merchant-Rankine formula (equation 1.1) may not satisfy the specified limitations on the horizontal deflections. Even when the facility for calculating these deflections exists, as in the method by Williams (1979), no recommendations are made to improve the design if the deflections are found to be excessive. Furthermore, the final design obtained by any of these methods may not be the most economical one.

These methods were intended to provide the engineer with a manual design method or an easy method to program on a desk or pocket calculator. However, the lack of accuracy in these methods and the rapid development of digital computers after 1956 made it possible to carry out more accurate analysis of a frame.

All the above methods are in no way design methods. This is because they select the cross section of the members and spend a great deal of time to carry out the "Analysis" of the frame. Another weakness of these methods is that they do not design the frames to satisfy the deflection (or sway) requirements. Once an analysis shows that the deflection in a frame is excessive, the above methods do not suggest methods for reducing it.

1.2.4 Computer Methods of Design

The matrix methods of structural analysis were first used by Livesley (1956), who developed methods to analyse elastic frames with or without considering the instability effects due to axial forces in the members. This method was developed by, amongst others, Livesley (1959), and Jennings and Majid (1965), who prepared programs which can be used to trace the actual load deflection behaviour of a frame up to collapse.

Livesley, Jennings and Majid, used the matrix displacement method of structural analysis in the above programs. In this method, the unknown joint displacements are obtained by solving the matrix equation

$$\underline{L} = \underline{K}\,\underline{X} \qquad (1.2)$$

where $\underline{L}$ is the external load vector, $\underline{K}$ is the overall stiffness matirx, and $\underline{X}$ is the vector of joint displacements. Member forces are then calculated using these joint displacements. Livesley (1956) introduced the effect of axial loads into the above equations by using stability 'ϕ functions' to modify the member stiffnesses. Iterative calculation is necessary as the axial forces in the members, required to calculate the stability functions, are unknown initially.

The elastic-plastic analysis program proposed by Jennings and Majid led Horne and Majid (1966) to suggest a method for the design by computer, of sway frames under proportional loading. A satisfactory structure was obtained by repeated cycles of elastic-plastic analysis and redesign.

The design criteria which were adopted permitted collapse to occur only when a minimum load factor was exceeded. To satisfy these criteria, restrictions were placed upon the formation of plastic hinges in the beam and the columns. These restrictions were:

1) No plastic hinge should develop in a beam below the load factor of unity, and the frame should be entirely elastic under the working load.
2) Under combined vertical and wind loading, no plastic hinge should develop in a column below a specified permissible load factor.

The method required that each proposed design is subjected to an accurate analysis. As a result, points of weakness and overstress are revealed, and can be removed. The final design satisfies the above design criteria, with efficient distribution of material throughout the frame.

However, because of the large storage required for the overall stiffness matrix in all the above methods, Jennings (1966) developed a compact method for the storage and solution of the stiffness equations.

This technique was used by Majid and Anderson (1968) to develop an efficient method for linear and non-linear elastic-plastic design. The criteria used are those adopted by Horne and Majid, stated above.

The elastic-plastic design method is based on very realistic design criteria. However, the provision for the hinges not to form in columns below a permissible load factor, and in beams below the working load does not prevent excessive deflections in the frame. In fact, the frames designed by Majid and Anderson showed that the sway deflection of the frames were excessive, although the frames themselves had a reasonable reserve of strength.

For example, for a fifteen storey frame the sway deflection ratio for the top storey at working load was 1/91. This is more than three times that required by the B/20 Draft. Furthermore, the elastic-plastic method is yet another method of repeated analysis in which the members are modified because an analysis reveals that such modifications are necessary.

1.2.5 Design of Reinforced Concrete Frames

Until recently two methods have been in general use for the design of structural concrete members: the permissible stress method and the load factor method. These have sometimes been used as alternatives and sometimes used in combination.

The permissible stress method was used in England prior to 1957, for all reinforced concrete members. In the subsequent

Code of Practice (CP 114, 1957) it was retained as an alternative to the load factor method for flexure, with and without compression, and remained the basis of design for shear and bond.

In the permissible stress method, the moments and forces acting on a structure were calculated from the actual values of the applied loads, but the limiting permissible stresses in the concrete and the reinforcement were restricted to only a fraction of their true strengths, in order to provide an adequate safety factor. For example, in CP 114 (1957), the permissible tensile stress in steel reinforcement was normally the yield stress divided by the factor 1.8, while the permissible compressive stress for concrete in bending was the cube strength divided by the factor 3. The method assumed a linear stress-strain relationship and a constant modular ratio of steel to concrete.

The permissible stress method models the behaviour of a section under service loads fairly well, but it gives an unsatisfactory indication of condition as failure approaches. This is because the assumption of a linear relationship between stress and strain in the concrete no longer remains true, and thus the distribution of stress in the concrete differs from that under service load.

The load-factor method of design was introduced into CP 114 to overcome this disadvantage. Theoretically, this method involves the analysis of sections at failure, the actual strength of a section being related to the actual load causing failure, with the latter being determined by 'factoring' the design load. The 1957 code for reinforced concrete (1967 reprint) permitted the use of this method with a central load factor of 1.8. However, in order to allow for the fact that there is a greater variation in the strength of concrete than in the strength of steel, it was specified that in calculating the strength, the cube strength of concrete should first be reduced to the ratio of 1.8/3. This brings the load factor method into line with the alternative permissible stress method.

In the above two methods only failure has so far been considered. A member was designed to have an ultimate resistance larger than the applied load. However, it has been noticed that serviceability, no less than safety, is a requirement in structural design. This led to the concept of a limit state design used in the present Code of Practice CP 110:1972.

Many text books have been published to explain the principles of this method (Reynolds and Steedman, 1974), (Allen, 1977), (Bennett, 1973), (MacGinley, 1978) (Astill and Martin, 1975). With this method, the design of each individual member or section of a member must satisfy two separate criteria: the ultimate limit-state, which ensures that the probability of failure is acceptably low, and the limit-state of serviceability, which ensures satisfactory behaviour under service (i.e. working) loads. The principle criteria relating to serviceability are the prevention of excessive deflection, excessive cracking and excessive vibration.

Although CP 110 outlines serviceability limit-state calculations to ensure the avoidance of excessive cracking or deflection, a full analysis of every section was considered to be time consuming. For this reason the new code specified certain limits relating to bar spacing, span over depth ratio, slenderness, etc., and if these criteria are not exceeded, more detailed calculation was considered to be unnecessary. As it stands the code does not impose any limit on sway deflection of frames. The only limit is that the depth of the cross section of a concrete column should not be less than $(H_e/30)$, where H_e is the effective storey height.

1.2.6 Design of Pin-Jointed Space Frames

Pin-jointed space frames are a good solution to the problem of spanning large areas without intermediate supports. Their relative lightness combined with relatively high stiffness makes them attractive as an alternative to beam or grillage systems.

It appears that the design of a safe and serviceable space truss is considered to be a straightforward process, which requires a standard linear analysis followed by member sizing. Thus, research into the design of pin-jointed space frames can be classified into:

i) Research into the carrying capacity of the whole structure.

ii) Research into the behaviour of structural elements.

Little attention seems to have been given to investigation into the behaviour of the complete space frame. However, very recently (Schmidt, et al, 1980) carried out full scale tests on 9.60 m space trusses. The tests showed that premature collapse of the complete truss was obtained, below that predicted by a conventional elastic analysis. This was due to two factors:

a) The significant magnitude (about 9% yield values) of the initial member forces due to the fabrication of the joints and the members of the truss, and not attributable to the weight of the structure itself.

b) The reduced collapse load of a compression member below its average peak load obtained from the element tests.

Therefore, Schmidt's tests showed that a design based on conventional elastic analysis, modified by a load factor, may possess a smaller margin of safety at collapse than its commonly supposed. Schmidt's paper show that the mid-span deflection of the truss was somewhat greater than the limit of span/360 specified by the B/20 draft for maximum allowable deflection at working load.

Schmidt did not suggest the design of the space frame for deflection limitation. Their proposals to modify the conventional elastic analysis to match the test results were to make two modifications to the analysis:

i) A set of initial lack of fit and assembly forces should be added to the externally applied load before the analysis is carried out.

ii) The carrying capacity of compression members should be reduced by some 20% from the theortical value

Although these modifications improved the analysis considerably, it is impossible to estimate the initial forces in the members without actual measurements. This is because, the initial forces depend on the fabrication and erection procedures. Thus, Schmidt's analysis cannot be carried out before actually building the truss.

The design of pin-jointed space-frame elements in tension or in compression is outlined in B/20 draft. To reduce the effect of buckling of the compression elements, smaller compression stresses are used. Design stresses are given in Table (6.2.2) of B/20 draft for different values of the Robertson constant and slenderness ratio.

1.3 DESIGN TO DEFLECTION REQUIREMENTS

The tendency for deflection criteria to dominate the design is necessarily increased by the introduction of more refined design methods, and the introduction of higher-strength steels. High-yield steel provides the frame with more strength than mild steel, and with less expense. Needham (1977) has demonstrated that steel Grade 50 is more economical to use than (mild steel) Grade 43. However, deflection becomes more critical when a higher grade steel is used.

In the design of concrete elements, when the permissible stress design was being used with traditional conservative design methods, problems due to excessive deflection were practically unknown. However, cracking in service has recently become more common. This is due to the use of the limit state design which, because of a more effective use of the concrete compression zone, have resulted in shallow members. More significant, is the introduction of high-strength materials with their correspondingly higher service stresses. For example, in British practice, there has been a progressive increase in permissible stresses with a consequent reduction in overall safety factors. For a concrete having a mean cube strength of 40 N/mm^2 at 28 days, the maximum permissible stresses according to the British codes were as follows (Clarke, 1974):

Date	Maximum permissible stresses
CP114:1957	10 N/mm^2
CP114:1965	12.2 N/mm^2
Draft CP110:1969	13.66 N/mm^2
CP110:1972	18 N/mm^2

(This figure is for ultimate design stress)

This increase in stress together with a reduction in flexural rigidity (EI) due to cracking, with no corresponding increase in modulus of elasticity, results in higher deflections, since deflection in concrete is directly proportional to stress and inversely proportional to flexural rigidity.

Figure 1.5 curve (a) shows how the margin of safety (defined as $\frac{\text{ultimate stress-working stress}}{\text{working stress}}$) for steel reinforcement in tension has reduced successively over the years (Rowe, et al, 1965) and (Clarke, 1974). Curve (b) of Figure 1.5 represents the same information for concrete in bending.

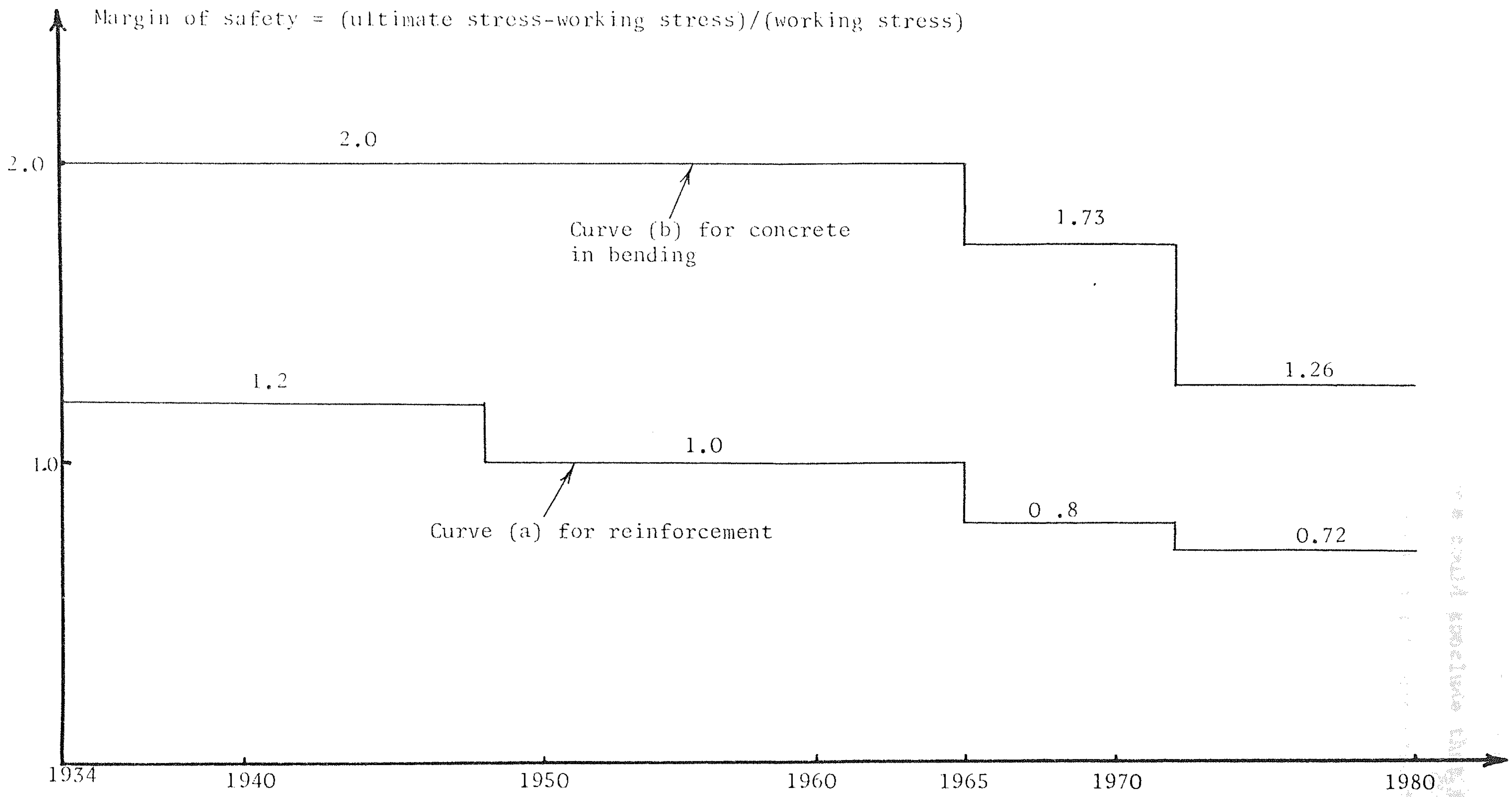


FIGURE 1.5 MARGIN OF SAFETY IN CODES OF PRACTICE OVER THE YEARS

From what has been said above one could conclude that the need for design for deflection requirement has become essential. However, up to now few methods exist for such a design.

Stevens (1960) suggested that the real basis for the design of sway frames should be the prevention of unacceptable deformations under the working loads. A design to satisfy such a criterion was obtained by selecting a curvature pattern that would produce specified deformations, and was compatible with a bending moment distribution in equilibrium with the external loads. Sections were then selected by using moment-curvature charts.

No direct allowance was, however, made for instability. A further defect of the method was that initial assumptions of relative flexural rigidities had to be made for statically indeterminate frames.

A second design method due to Stevens was based upon the collapse state. Maximum overload deformations were specified and used in formulating virtual work equations corresponding to collapse mechanisms in the deformation frame. Member sections were then selected, and the resulting design analysed by approximate methods. If it was found that the specified deformations were exceeded, then a further cycle of design and analysis was necessary.

Although in this method the effect of the specified deflections was included in the virtual work equations, instability effects were ignored, and premature collapse could therefore occur.

Moy (1974 and 1976b) proposed a method for sway frames in which an initial design with adequate strength but inadequate stiffness was corrected to satisfy permissible limits on horizontal deflections. The frame was divided into subassemblages on the basis of the portal method. These subassemblages were shown earlier in this chapter in Figure 1.4, for exterior and interior bays. Expressions for the storey stiffnesses were obtained in terms of the member second moments of area of the storey, and in terms of the permissible sway deflections at working load. Two assumptions were made:

i) The vertical loads were assumed to have a negligible effect on horizontal deflection.

ii) A point of contraflexure was taken to exist under horizontal loading at the mid-height of each column (except in the bottom storey), and at the mid-span of each beam. Thus, the frame was made statically determinate above the bottom storey, and each storey was considered in isolation.

The method is based on hand calculation but it requires a preliminary design and repeated calculation to arrive at a feasible solution.

More recently Anderson and Islam (1979), and Islam (1978) suggested a method for the design of multi-storey frames to sway deflection limitations. The assumptions used are those adopted by Moy, stated above. Expressions relating the sway deflection over a storey height to the second moment of area of the corresponding column and surrounding beam were derived. These expressions were linked to a cost function to obtain an economical solution. A typical internal subassemblage for an intermediate storey in Anderson and Islam's method is shown in Figure 1.6. The complete frame is shown in Figure 1.7. The expression for the second moment of area of an internal column in this subassemblage was given as:

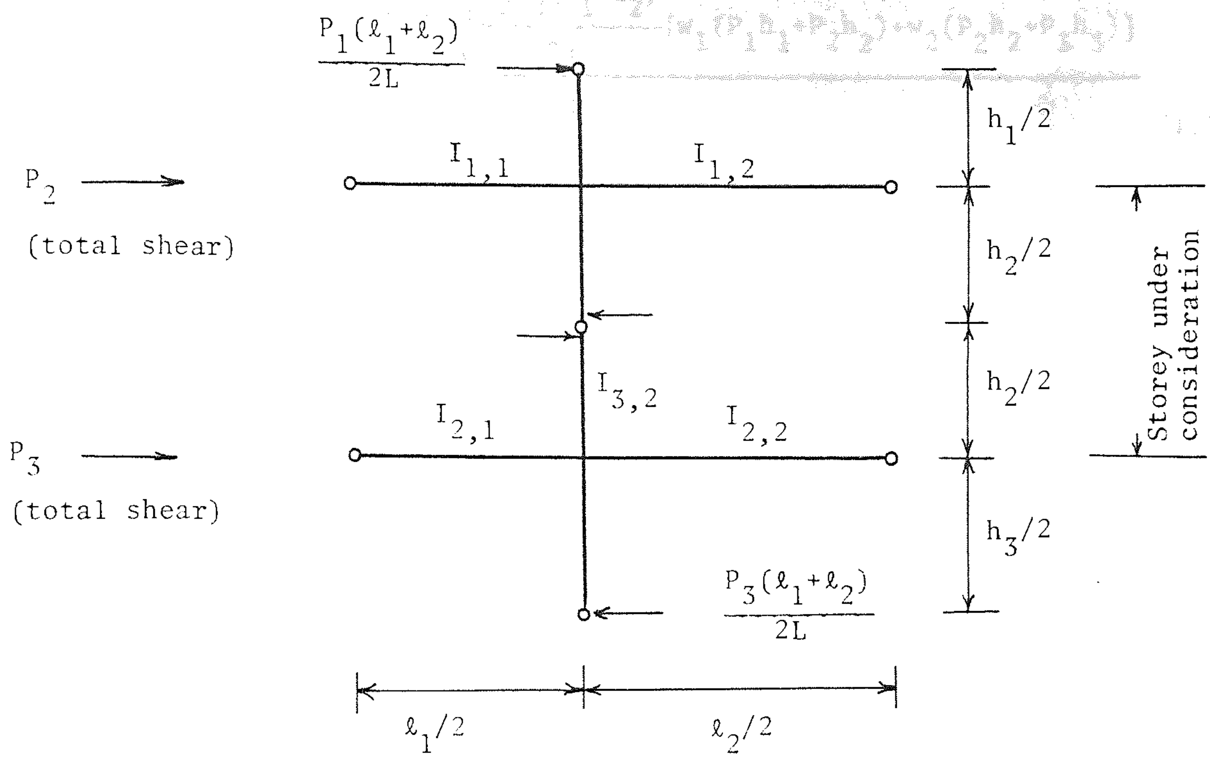


FIGURE 1.6 A TYPICAL SUBASSEMBLAGE IN ANDERSON AND ISLAM'S DESIGN SHOWS THE SECOND COLUMN FROM THE WINDWARD FACE OF A TYPICAL INTERMEDIATE STOREY

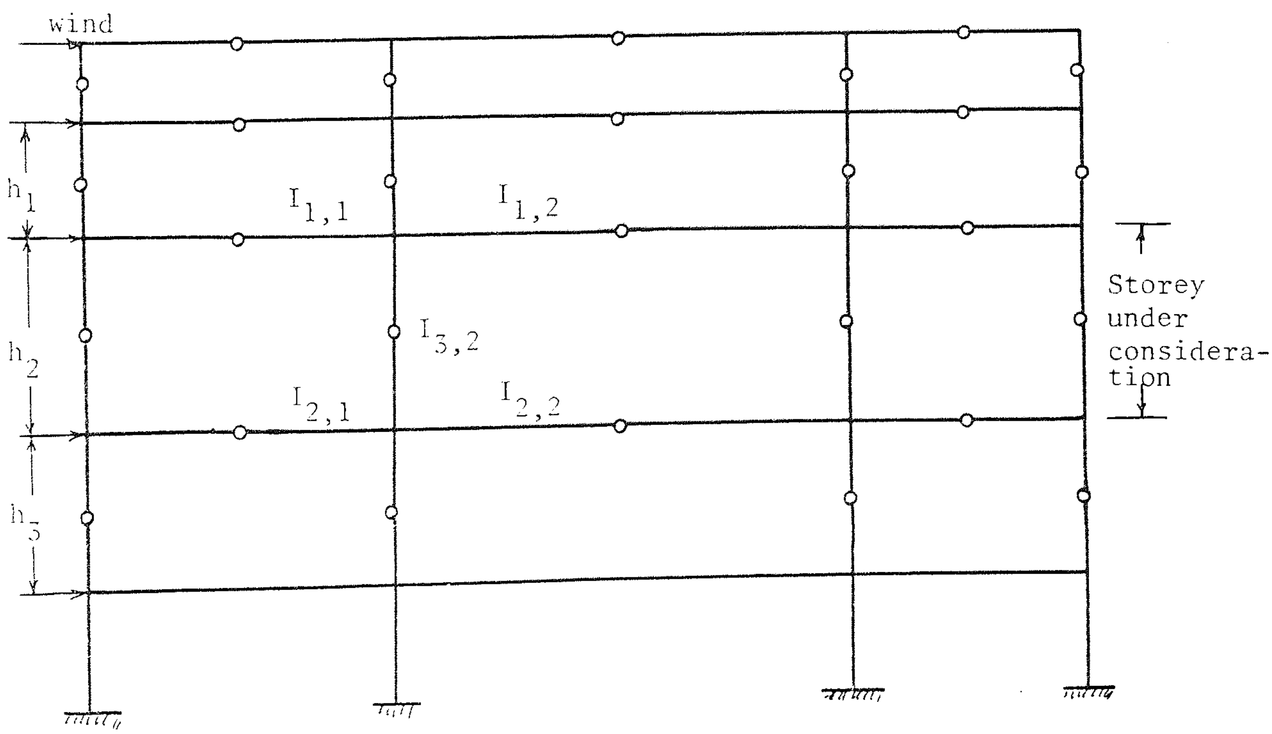


FIGURE 1.7 THE COMPLETE FRAME IN ANDERSON AND ISLAM'S METHOD

$$I_{3,2}=\frac{P_2h_2^3(\ell_1+\ell_2)+h_2\ell_2\sqrt{\frac{P_2h_2(\ell_1+\ell_2)}{w_3}[w_1(P_1h_1+P_2h_2)+w_2(P_2h_2+P_3h_3)]}}{24E\Delta L} \tag{1.3}$$

where

$$w_1 = \frac{q_{1,1}\ell_1^3 + q_{1,2}\ell_2^3 + \ldots\ldots + q_{1,m}\ell_m^3}{2\ell_2^2} \tag{1.4}$$

$$w_2 = \frac{q_{2,1}\ell_1^3 + q_{2,2}\ell_2^3 + \ldots\ldots + q_{2,m}\ell_m^3}{2\ell_2^2}$$

$$w_3=\frac{(q_{3,1}+q_{3,2})\ell_1+(q_{3,2}+q_{3,3})\ell_2 + \ldots\ldots + (q_{3,m}+q_{3,m+1})\ell_m}{\ell_1 + \ell_2}$$

where $q_{1,1}$, $q_{1,2}$ ....., $q_{1,m}$ are the cost factors of the upper beams, $q_{2,1}$, $q_{2,2}$, ..., $q_{2,m}$ are the cost factors of the lower beams, and $q_{3,1}$, $q_{3,2}$, ..., $q_{3,m+1}$ are the cost factors of the columns. In equation (1.3), P_2 is the total horizontal shear in the columns of the storey under consideration, while P_1 and P_3 are those in the storey above and below respectively. The quantities ℓ_1, ℓ_2, ℓ_3, ..., ℓ_m are the bay widths, where m is the number of bays. $L(=\ell_1 + \ell_2 + \ell_3 + \ldots + \ell_m)$ is the total width of the frame. Δ is the horizontal deflection of the storey. h_2 is the height of the storey under consideration, while h_1 and h_3 are those for the storey above and below respectively. E is Young's modulus of elasticity. $I_{3,2}$ refers to the second moment of area of the first left internal column. The expression for the second moment of area of the second left bay beam in the storey under consideration was given by:

$$I_{1,2} = \frac{(P_1h_1 + P_2h_2)\ h_2\ell_2^2\ I_{3,2}}{24E\Delta L\ I_{3,2} - (\ell_1 + \ell_2)\ P_2\ h_2^3} \tag{1.5}$$

when the values of $I_{3,2}$ and $I_{1,2}$ were calculated from equation (1.3) and (1.5) the second moment of area for the remaining columns and beams in the storey under consideration were calculated by proportion using the following two expressions respectively

$$\frac{I_{3,1}}{\ell_1} = \frac{I_{3,2}}{\ell_1 + \ell_2} = \frac{I_{3,3}}{\ell_2 + \ell_3} = \ldots\ldots = \frac{I_{3,m+1}}{\ell_m} \qquad (1.6)$$

$$\frac{I_{1,1}}{\ell_1^2} = \frac{\ell_{1,2}}{\ell_2^2} = \frac{I_{1,3}}{\ell_3^2} = \ldots\ldots = \frac{I_{1,m}}{\ell_m^2} \qquad (1.7)$$

Equation (1.3), (1.5), (1.6) and (1.7) were repeated for each storey.

The values of the cost factors were given in table 2.1 and 2.2 of Islam's thesis for universal column and beam sections. To obtain a quick conservative design, Anderson and Islam have shown that the values of the cost factors can be taken as being equal to one. In fact, their design of the 6 storey, 4-bay fixed base frame shows no change in weight when the real values of the cost factors were used.

Anderson and Islam derived similar expressions to those given by equation (1.3) and (1.5) to deal with the boundary regions of the frame, such as the top-most storey, the bottom storey and the external columns. However, they suggested that a quick design could be obtained if the design of the top storey was considered as the storey below it, and the design of the bottom storey was taken as the one above it.

In equation (1.3) Anderson and Islam took the sway of each storey Δ as being equal to the maximum allowable deflection of $h_i/300$, where h_i is the height of the storey under consideration. However, such a ratio is difficult to be maintained for the ground floor columns, because these are connected to the foundations and therefore deflect less than the others.

The methods of Moy and Anderson and Islam are approximate, as hinges were assumed in the mid-length of each member to avoid considering the frame design in its entirety, and in order to separate it into storey slices. Furthermore, the vertical loads have a considerable effect on the sway especially in non-symmetrical frames. Thus, the effect cannot be ignored.

1.4 OPTIMUM DESIGN

There are two types of optimisation: the classical type (which uses calculus), and the programming type.

The first type is efficient only when it deals with equality constraints. One method of this type is the Lagrange multiplier technique (Hadley, 1970 edn). For instance, it is required to optimise the function

$$Z = f(x_1, x_2, \dots x_n) = f(x) \tag{1.8}$$

subjected to m equality constraints:

$$g_i(x_1, x_2, \dots, x_n) = b_i, \quad i=1, \dots, m \tag{1.9}$$

The procedure is first to convert each of the equations (1.9) to the form:

$$G_i = \lambda_i [b_i - g_i(x)], \quad i=1, \dots, m \tag{1.10}$$

where the new variable λ_i is known as the Lagrange multipler. The constraints G_i are than added to the objective function Z to form

$$F(x, \lambda) = f(x) + \sum_{i=1}^{m} \lambda_i [b_i - g_i(x)] \tag{1.11}$$

Here $F(x,\lambda)$ is called Lagrangian function. The difference between $F(x,\lambda)$ and $f(x)$ is that $F(x,\lambda)$ has more variables than $f(x)$ and the turning points of $F(x,\lambda)$ also include those of $f(x)$. The turning point for the unconstrained function $F(x,\lambda)$ is found by setting to zero the partial derivatives of $F(x,\lambda)$ with respect to each of the m+n variables x_j, j=1, ..., n and λ_i, i=1, ..., m. Thus

$$\frac{\partial F}{\partial x_j} = \frac{\partial f}{\partial x_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial x_j} = 0, \quad j = 1, \dots, n \tag{1.12}$$

$$\frac{\partial F}{\partial \lambda_i} = b_i - g_i(x) = 0, \quad i=1, \dots, m \tag{1.13}$$

An advantage of Lagrange's method, from a practical point of view, is that the optimum solution can be obtained with relatively simple numerical techniques. However, the method is only useful when the constraints are equalities.

The programming methods, are often more efficient than the classical type. This is because they can deal with inequalities and with relatively large probelms. For these reasons, these methods are often used to design rigidly jointed framed structures. The principles of some of these methods are given in some of the recently published text books (Majid, 1972), (Majid 1974), and (Gallagher and Zienkiewicz, 1973).

Both the rigid-plastic and the elastic design method can be used to formulate the optimisation problems. The former method, however restricts the design constraints to cover strength requirements only and cannot deal with deflection constraints.

In the elastic design method, the design criteria used are that the stresses in the members and the deflections at the joint of a structure should not exceed certain permissible values. It then becomes necessary to express the deflections and the stresses in the structure in terms of the design variables. This can be carried out by employing either of two matrix methods.

In the case of the matrix displacement method the design problem becomes one of finding the sectional properties of the member so that three constraints are satisfied. These are the stiffness equalities and the deflection and the stress inequalities.

An alternative formulation uses the matrix force method to find the same member properties while satisfying the compatibility constraints which are equalities, as well as the deflection and the stress inequalities.

Whichever method is employed the optimum design of a frame is a non-linear problem even if the instability effects due to the presence of high axial forces in the members are ignored. In this case, the linear programming techniques cannot be directly used to solve such problems. Several non-linear programming methods are available to overcome this difficulty but these may lead to complications and increase the computer time when designing large frames. Linear programming methods may, however, be used in the solution of non-linear problems by using an iterative procedure. Each iteration consists of a small linearised step of the problem. Different methods of solving a non-linear problem by such linearisation and iteration, e.g. the cutting plane method, the piecewise linearisation method etc, have been discussed in detail by Majid (1974).

Saka (1975) presented a general computer program for the automatic optimum design of realistic, rigidly jointed multi-storey frames. A non-linear programming algorithm was proposed for the minimum weight design of structures in which the joint displacements and stress constraints were included as design variables. The design problem was formulated using the matrix displacements method. The problem was then linearised using the cutting plane method, and the optimum design was obtained using the simplex method.

Although, this method achieved a design in which all the design criteria were satisfied, it was not applicable to large frames, as it was essentially a procedure requiring extensive storage space and consumed a considerable amount of computer time. These difficulties can be overcome, however, by making use of the backing store of a computer (Al-Pasha 1981-82).

1.5 SCOPE OF PRESENT WORK

The design of multistorey sway frames with realistic wind loading can be governed by the limitations specified by the codes on the permissible horizontal sway of each storey. These are now specified in terms of the storey-heights at the specified unfactored loads (B/20 Draft Specification). It has also been recommended that at a load factor of unity the frame should remain elastic in consideration of both strength and serviceability of structure, (Majid and Anderson (1968),(Moy, 1976) and(Majid, Stojanovski and Saka, 1980).

In general a rational approach to designing sway frame for deflection limitations is not adopted in existing methods, practiced in industry. Furthermore, economy in design is sacrificed in order to obtain a fast manual procedure which is either restricted in its application or inaccurate in its approach.

Many of the recent proposals for improved methods of design (Moy) and (Anderson and Islam 1979) have been based upon assumed rather than actual behaviour. For example, the assumptions of hinges at the mid-point of each member, and the neglect of the vertical loads on the frame etc are proposed to simplify the design methods. Accurate design methods for deflection limitation has not been generally available.

The work presented in this thesis began with an attempt to find an accurate direct design method to limiting horizontal deflections at the unfactored loads. Chapter 2 presents a method for the design of plane rigidly-jointed multistorey steel sway frames to withstand specified values of sway deflections. Using these values, the overall stiffness equations $\underline{L} = \underline{K}\ \underline{X}$ are modified and then solved by iteration not to calculate deflections but to calculate the unknown cross-sectional properties of the columns. Thus the method proposed is that of design as opposed to that of an analysis. Simple linear extrapolation is then proposed to progressively reduce the cost of the material of the frame.

Chapter 3 gives suitable criteria to cover both the strength and the deflection requirements in sway frames. Using stability functions, the axial loads effects are catered for and a refined, more accurate design is obtained. The procedure is then computerized, and an explanation of the computer programming is given in detail in Chapter 4.

Chapter 5 contains the examples solved by this program. The effect of the axial force and a comparison of the results with other methods are pointed out in this chapter.

In Chapter 6 the design procedure is applied to the design of rigidly-jointed multistorey reinforced concrete sway frames. The depth of each column and beam in the frame is calculated first, to withstand specified sway in the columns. The reinforcement of the columns and of the beams are then calculated so that the strength requirements are also satisfied.

The method of Chapter 2 can be used to design pin-jointed space frames, for deflection limitation, by formulating the problem in terms of the unknown cross-sectional areas. This is done in Chapter 7 where a computer program for this purpose is also presented.

In Chapter 2 an approximate extrapolation technique is used to change the beam sections in an attempt to reduce the material cost of the frame. In Chapter 8 the results of Chapter 2 are then used in conjunction with the method of Lagrange multipliers in order to obtain an optimum solution for the design problem. This new extended method is also iterative. The computer program for the optimisation method is also given in this chapter.

CHAPTER TWO

DESIGN OF STEEL FRAMES

2.1 INTRODUCTION

The design of a rigidly jointed sway frames requires that the overall stiffness equations $\underline{L} = \underline{K}\underline{X}$ must be satisfied while, in addition, the stresses and the sway in each member are within upper bounds specified by some limit state design code. In the case of sway frames, it has been found (Anderson, 1969 and Majid and Anderson, 1968) that the deflection or sway requirements are more severe than the stress requirements. For this reason it is possible to satisfy the sway requirements first. If the sway in each column is equated to its upper bound, then the solution of $\underline{L} = \underline{K}\underline{X}$ becomes easier. In this chapter it is assumed that the sway in each member is known and the equations $\underline{L} = \underline{K}\underline{X}$ are solved to calculate the cross sectional properties of the columns. An iterative method is suggested to solve these equations. This chapter only deals with the design of two dimensional rectangular steel sway frames.

For a general joint in a sway frame, expressions are obtained to calculate the deflections in terms of those at the neighbouring joints as well as the second moment of area of the connecting members. The value of the horizontal sway at each storey-level is first specified, and the sway equations are used to evaluate the unknown sectional properties of the columns. The stiffness equations for each joint are solved by an iterative technique. This produces values for the other deflections in the joint as well as the second moment of area of the columns. The beam sections are selected by a preliminary method such as that suggested by (Islam, 1978, and Anderson and Islam 1979). These are then altered in an effort to reduce the total material cost of the frame.

Simple linear extrapolation is used to find the cost of each storey. The beam which produces the least cost is used in the next round of an iteration process, which is repeated until no further reduction in material cost of the frame can be gained. As the iteration process continues, the beam sections, the column sections as well as the ratio between the external and the internal columns all change.

2.2 CONTRIBUTIONS OF A MEMBER TO THE OVERALL STIFFNESS MATRIX

The contributions of a member to the overall stiffness matrix [K] of a frame is:-

$$K = \begin{array}{cc} & \begin{array}{cc} \qquad\text{First end} & \qquad\qquad\text{Second end} \end{array} \\ \begin{array}{c} \\ \text{First end} \\ \\ \\ \\ \text{Second end} \\ \\ \end{array} & \left[\begin{array}{ccc:ccc} A & B & -C & -A & -B & -C \\ B & F & -T & -B & -F & -T \\ -C & -T & e & C & T & f \\ \hdashline -A & -B & C & A & B & C \\ -B & -F & T & B & F & T \\ -C & -T & f & C & T & e \end{array}\right] \end{array} \qquad (2.1)$$

i.e.

$$K = \left[\begin{array}{c:c} K_{11} & K_{12} \\ \hdashline K_{21} & K_{22} \end{array}\right] \qquad (2.2)$$

where

$$A = a\ell_p^2 + bm_p^2$$

$$B = (a-b)\ \ell_p\ m_p$$

$$C = -d\ m_p$$

$$F = am_p^2 + b\ell_p^2$$

$$T = d\ell_p \qquad (2.3)$$

$$e = 4EI/L$$

$$f = 2EI/L$$

$$a = EA/L$$

$$b = 12EI/L^3$$

$$d = -6EI/L^2$$

In equation (2.3), E is the Young's modulus of elasticity, I is the second moment of area for the member, L is the length of that member, ℓ_p and m_p are the direction cosines for the longitudinal P axis of the member, and A is the member area.

For rectangular skeletal frames it is convenient to simplify the $\underline{K}$ matrix and construct separate matrices for beams and columns. It is assumed that the direction of the P axis for a beam is from left to right, while that for a column is vertically downwards. The sign convention adopted for forces and displacements is in accordance with the right hand screw rule, and is shown for a horizontal and a vertical member in Figure 2.1.

For a beam, the stiffness equations are simplified to become:

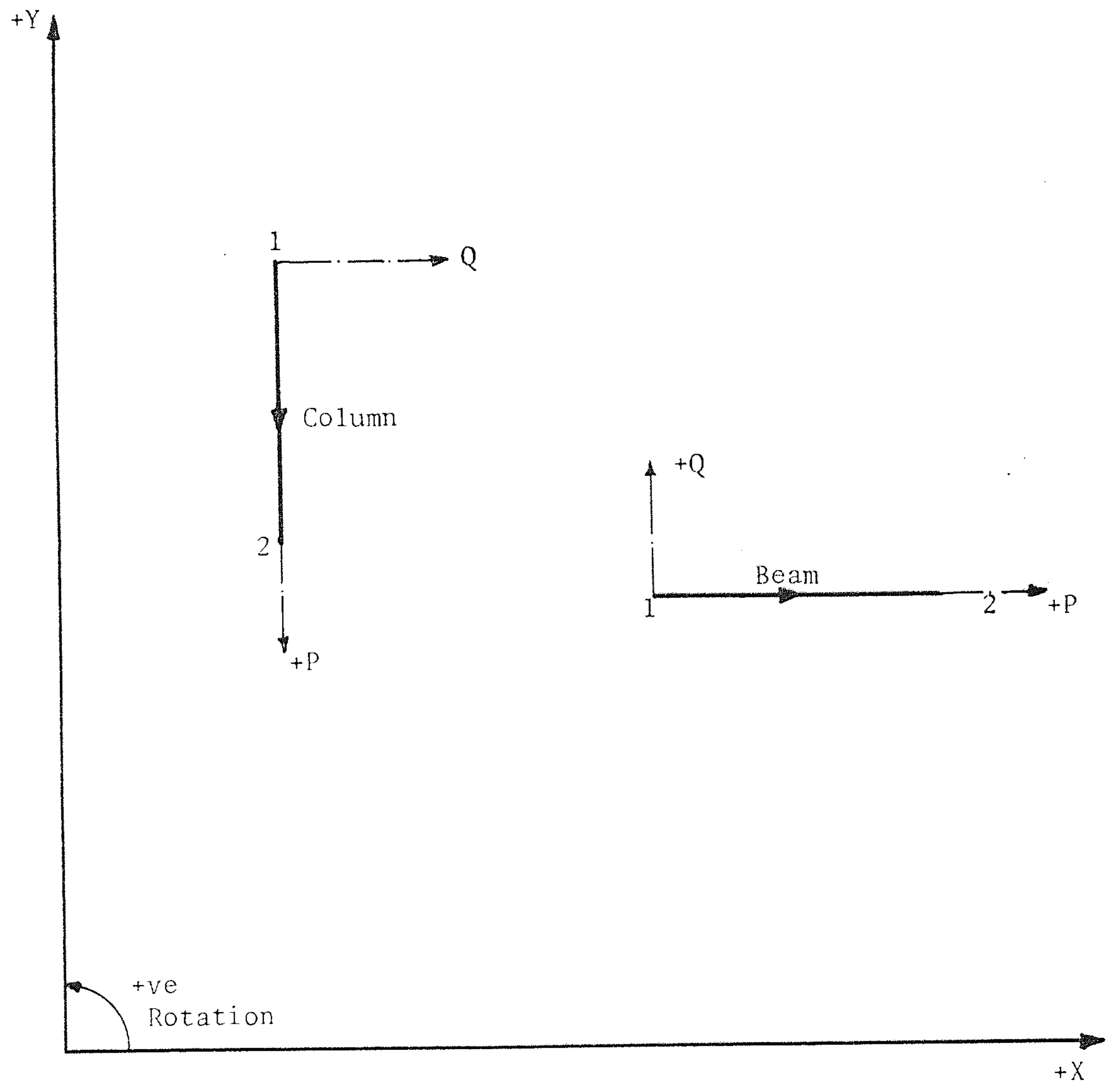


FIGURE 2.1: THE SIGN CONVENTION AND THE POSITIVE DIRECTION OF THE LOCAL AND GLOBAL AXES

$$
\begin{bmatrix} H_i \\ V_i \\ M_i \\ \vdots \\ H_j \\ V_j \\ M_j \end{bmatrix}
=
\begin{array}{c} \text{first} \\ \text{end} \\ \\ \\ \text{second} \\ \text{end} \end{array}
\begin{array}{c}
\begin{array}{cc} \text{First end} & \text{Second end} \end{array} \\
\left[\begin{array}{ccc:ccc}
a & 0 & 0 & -a & 0 & 0 \\
0 & b & -d & 0 & -b & -d \\
0 & -d & e & 0 & d & f \\
\hdashline
-a & 0 & 0 & a & 0 & 0 \\
0 & -b & d & 0 & b & d \\
0 & -d & f & 0 & d & e
\end{array}\right]
\end{array}
\begin{bmatrix} x_i \\ y_i \\ r_i \\ \vdots \\ x_j \\ y_j \\ r_j \end{bmatrix}
\qquad (2.4)
$$

where $\underline{L} = \{H_i \; V_i \; M_i \; .... \; H_j \; V_j \; M_j\}$ is the load matrix, and $\{X\} = \{x_i \; y_i \; r_i \; x_j \; y_j \; r_j\}$ is the joint displacement vector. Similarly for a column:

$$
\underline{L} =
\begin{array}{c} \text{First} \\ \text{end} \\ \\ \\ \text{Second} \\ \text{end} \end{array}
\begin{array}{c}
\begin{array}{cc} \text{First end} & \text{Second end} \end{array} \\
\left[\begin{array}{ccc:ccc}
b & 0 & -d & -b & 0 & -d \\
0 & a & 0 & 0 & -a & 0 \\
-d & 0 & e & d & 0 & f \\
\hdashline
-b & 0 & d & b & 0 & d \\
0 & -a & 0 & 0 & a & 0 \\
-d & 0 & f & d & 0 & e
\end{array}\right]
\end{array}
\begin{bmatrix} x_i \\ y_i \\ r_i \\ \vdots \\ x_j \\ y_j \\ r_j \end{bmatrix}
\qquad (2.5)
$$

If the axial stiffness (EA/L) of a member is neglected, equations (2.4) are simplified further by removing the first and the fourth columns and rows from the stiffness matrix. Equations (2.5) are also simplified by removing the second and the fifth columns and rows of $\underline{K}$. In this manner, the contributions for a beam and a column, to the stiffness matrix become the same. This is provided that the sign convention and the direction of the axes of the members are kept as specified. Thus the contributions of a member to

$\{L\} = \underline{K}\ \{X\}$ becomes:

$$
\begin{array}{ccccc}
 & & & \text{First end} \qquad \text{Second end} & \\
\begin{array}{c}\text{First}\\ \text{end}\\ \\ \text{Second}\\ \text{end}\end{array} & \begin{bmatrix} V_i \\ M_i \\ \vdots \\ V_j \\ M_j \end{bmatrix} & = & \left[\begin{array}{cc:cc} b & -d & -b & -d \\ -d & e & d & f \\ \hdashline -b & d & b & d \\ -d & f & d & e \end{array}\right] & \begin{bmatrix} y_i \\ r_i \\ \vdots \\ y_j \\ r_j \end{bmatrix} \text{(beam)}
\end{array} \qquad (2.6)
$$

Notice that suffixes i and j refer to the ends of the member.

$$
\begin{array}{ccccc}
 & & & \text{First end} \qquad \text{Second end} & \\
\begin{array}{c}\text{First}\\ \text{end}\\ \\ \text{Second}\\ \text{end}\end{array} & \begin{bmatrix} H_i \\ M_i \\ \vdots \\ H_j \\ M_j \end{bmatrix} & = & \left[\begin{array}{cc:cc} b & -d & -b & -d \\ -d & e & d & f \\ \hdashline -b & d & b & d \\ -d & f & d & e \end{array}\right] & \begin{bmatrix} x_i \\ r_i \\ \vdots \\ x_j \\ r_j \end{bmatrix} \text{(column)}
\end{array} \qquad (2.7)
$$

2.3 MEMBER AND JOINT SPECIFICATION

In this thesis, only orthogonal frames with horizontal beams and vertical columns are considered. This simplifies the numbering of the joints and the members and, thus the data preparation can be reduced to a minimum. In Figure 2.2 a frame is shown with joints and members numbered. Joints in the beams are numbered first. This is done starting at the left end of the top storey. The numbers are consecutive and are from left to right. The columns joints are then numbered in the same way. A member is given the same number as the joint at its first end, which is specified by the tail of an arrow placed on the member. This arrow also specifies the positive direction of the longitudinal p axis of the member. Thus on the second floor, left bay of the frame shown in Figure 2.2, the beam connecting

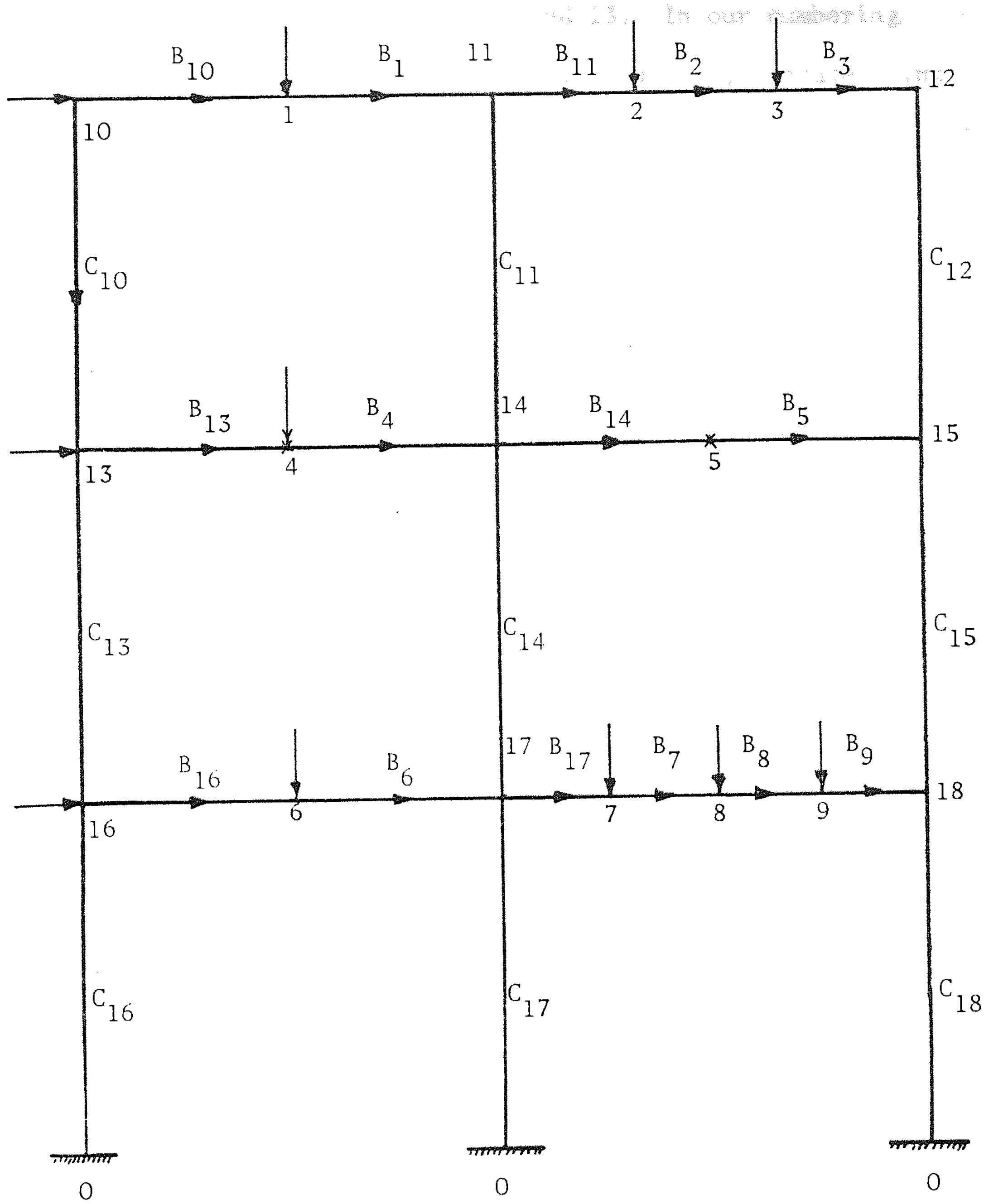


FIGURE 2.2: NUMBERING OF MEMBERS AND JOINTS OF A FRAME

the two columns carries a load along its span. This is at joint 4. The joint to the left of this is numbered 13. In our numbering system, the beam to the left of joint 4 is beam B_{13}, because joint 13 is at the first end of this beam. The beam to the right of joint 4 is beam B_4 because joint 4 is at its first end. The left side outer column at the second storey is C_{13} because joint 13 is at its first end. All supports are numbered as zeroes.

2.4 STIFFNESS EQUATIONS FOR A GENERAL JOINT IN THE FRAME

A column joint is at the end of a column while a beam joint lies somewhere along a beam. Figure 2.3 shows a general beam which may be carrying a load at joint j and other loads at i or k. Alternatively, i or k may be joints at the head of columns on either end of the beam. In our specification, joint i is to the left of j, and k is to its right. The beam joining i to j is beam B_i with its second moment of area I_{Bi}. The beam to the right of j joining j to k is beam B_j with its second moment of area I_{Bj}. For any column joint, there are six possible configurations. These are shown in Figure 2.4. Of these, type 6 is the general case where two columns and two beams meet. This type is also shown in Figure 2.5 where joint j is shown together with its neighbouring joints. The second moments of area for the members connected to j are also specified in this figure. All the other types 1 to 5 can be obtained from this configuration. For this reason joint type 6 is used to derive the relevant equations for the design of a column.

2.4.1 Stiffness Equations for a Beam Joint

The relationship between the external loads $\underline{L}$ and the vectorially equivalent joint displacements $\underline{X}$, in terms of the system coordinates XY, is expressed by the equation

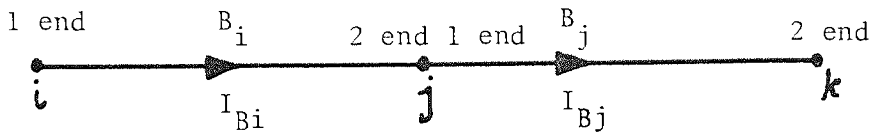


FIGURE 2.3: SPECIFICATIONS FOR A JOINT IN A BEAM

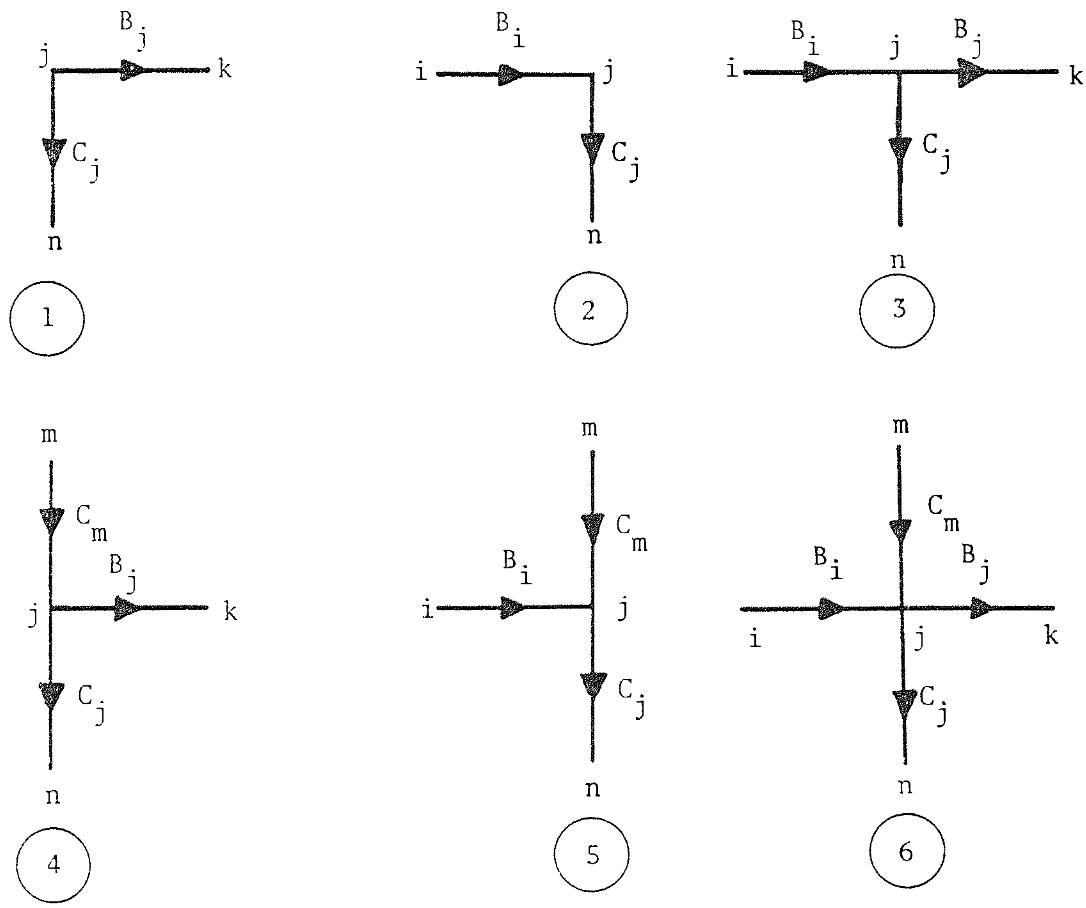


FIGURE 2.4: CONFIGURATIONS OF THE MEMBERS AT A COLUMN JOINT

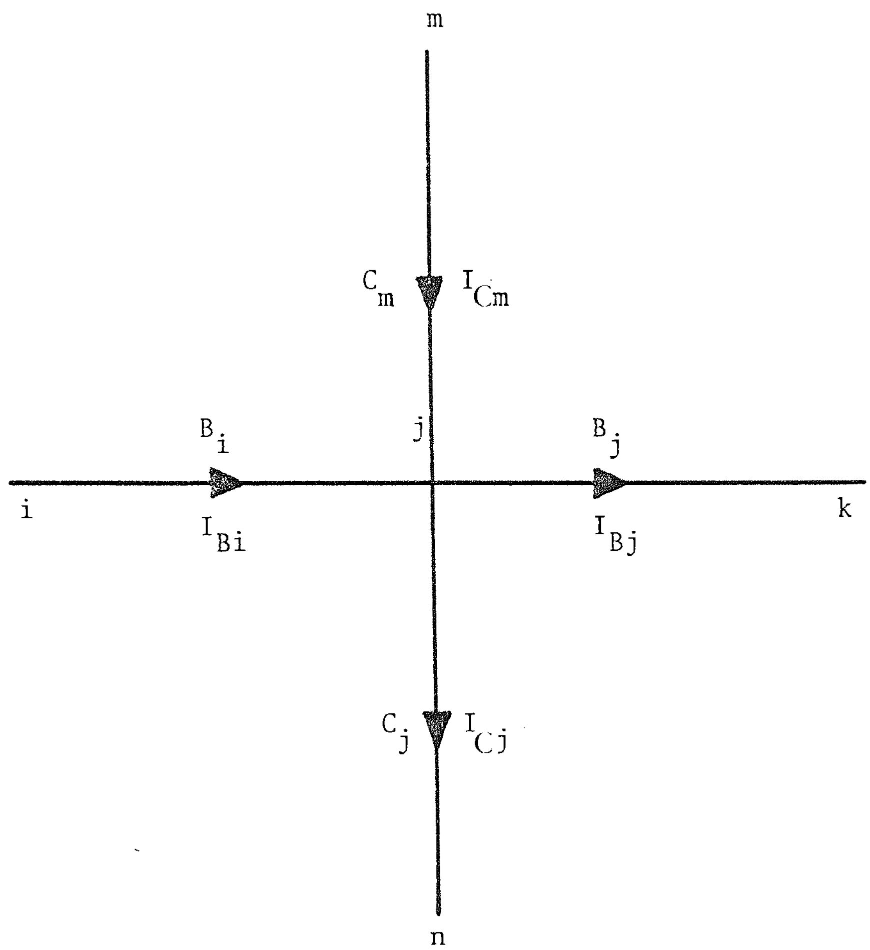


FIGURE 2.5: DEFINITION OF THE NOTATION RELATING THE NEIGHBOURING JOINTS TO THE JOINT UNDER CONSIDERATION

$$\underline{L} = \underline{K}\ \underline{X} \tag{2.8}$$

This is used to evaluate the deflections $\underline{X}$, thus

$$\underline{X} = \underline{K}^{-1}\ \underline{L} \tag{2.9}$$

For a beam joint j, the vector $\underline{X}$ is $\{x_j, y_j, r_j\}$. Here x_j is the horizontal displacement; y_j is the vertical displacement; and r_j is the rotation of the joint. If the axial stiffness (EA/L) in a beam is neglected, the displacement vector becomes $\underline{X} = \{y_j, r_j\}$, thus two equations are needed for defining y_j and r_j. These are obtained from the contribution of the beams B_i and B_j connected to joint j, (Figure 2.3). The stiffness coefficients of beam B_j are:

$$\begin{aligned} b_j &= 12EI_{Bj}/L_{Bj}^3 \\ d_j &= -6EI_{Bj}/L_{Bj}^2 \\ e_j &= 4EI_{Bj}/L_{Bj} \\ f_j &= 2EI_{Bj}/L_{Bj} \end{aligned} \tag{2.10}$$

where L_{Bj} is the length of the beam. Similarly for B_i the stiffness coefficients are:

$$\begin{aligned} b_i &= 12EI_{Bi}/L_{Bi}^3 \\ d_i &= -6EI_{Bi}/L_{Bi}^2 \\ e_i &= 4EI_{Bi}/L_{Bi} \\ f_i &= 2EI_{Bi}/L_{Bi} \end{aligned} \tag{2.11}$$

where L_{Bi} is the length of B_i.

2.4.1.1 The Vertical Deflection y_j

The vertical deflection y_j for a joint in a beam is obtained from the stiffness equations as follows:

i) The contributions of the beam B_j to the vertical deflection of joint j, which is at its first end, are calculated from the first row of equations (2.6), thus

$$V_j = b_j y_j - d_j r_j - b_j y_k - d_j r_k \tag{2.12}$$

Rearrange:

$$y_j = (V_j + d_j r_j + b_j y_k + d_j r_k)/b_j \tag{2.12a}$$

ii) The contributions of the beam B_i to joint j, which is at its second end of the beam, are calculated from the third row of equations (2.6), thus

$$0 = -b_i y_i + d_i r_i + b_i y_j + d_i r_j \tag{2.13}$$

The vertical load is considered to be acting at end j of B_j. This is why in equation (2.13), the left hand side is zero indicating that no load is acting at the second end of B_i. Rearranging equation (2.13) for y_j gives:

$$y_j = (+b_i y_i - d_i r_i - d_i r_j)/b_i \tag{2.13a}$$

iii) Adding the contributions of B_i and B_j, given by equations (2.12a) and (2.13a) results in the following equation for y_j:

$$y_j = (V_j + d_j r_j + b_j y_k + d_j r_k + b_i y_i - d_i r_i - d_i r_j)/(b_j + b_i) \tag{2.14}$$

2.4.1.2 The Rotation r_j

Similarly the rotation r_j for a beam joint is obtained as follows:

i) The second row of equations (2.6) gives the contributions of the beam B_j to the rotation of joint j as:

$$M_j = -d_j y_j + e_j r_j + d_j y_k + f_j r_k \tag{2.15}$$

Thus $$r_j = (M_j + d_j y_j - d_j y_k - f_j r_k)/e_j \tag{2.15a}$$

ii) The fourth row of equations (2.6) gives the contributions of beam B_i as:

$$0 = d_i y_i + f_i r_i + d_i y_j + e_i r_j \tag{2.16}$$

or $$r_j = (+ d_i y_i - f_i r_i - d_i y_j)/e_i \tag{2.16a}$$

It should be noted that M_j is taken into account when considering the contributions of B_j. As a result, in equation (2.16) and (2.16a), M_j is not reconsidered.

iii) The rotation of joint j is obtained by adding equations (2.15a) and (2.16a). This gives:

$$r_j = (M_j + d_j y_j - d_j y_k - f_j r_k + d_i y_i - f_i r_i - d_i y_j)/(e_j + e_i) \tag{2.17}$$

2.4.2 Stiffness Equations for a Column Joint

The displacements vector for a column joint, is similar to that of a beam, except that the axial deformation of the column must be considered. Thus, three equations are needed to find x_j, y_j and r_j. These equations are obtained from the contributions of the beams B_j and B_i, and the columns C_j and C_m which are connected to joint j, see Figure 2.5. The column C_j which connects joint j to n is defined by the joint number j at its first end. Its second moment of area is I_{Cj}. Members B_i, B_j and C_m are defined similarly by the joint at their first end. The second moments of area of these members are I_{Bi}, I_{Bj} and I_{Cm}.

The member stiffness coefficients a, b, d, f and e defined in equations (2.3) are found for each member connected to joint j. For column C_j these coefficients are b_{Cj}, d_{Cj}, e_{Cj} and f_{Cj}, while for column C_m they are b_{Cm}, d_{Cm}, e_{Cm} and f_{Cm}. These coefficients for the beams B_i and B_j have been given by equations (2.10) and (2.11). The axial stiffness for column C_j is:

$$a_{Cj} = EA_{Cj}/L_{Cj} \tag{2.18}$$

where A_{Cj} is the area of the column while the axial stiffness for the column C_m is:

$$a_{Cm} = EA_{Cm}/L_{Cm} \tag{2.19}$$

2.4.2.1 The Horizontal Deflection x_j

The expression for x_j (the horizontal displacement of joint j) is obtained from the stiffness equations as follows:

i) The contributions of the column C_j to the sway are taken from the first row of equations (2.7), which is

$$H_j = b_{Cj}x_j - d_{Cj}r_j - b_{Cj}x_n - d_{Cj}r_n \tag{2.18}$$

Thus $$x_j = (H_j + d_{Cj}r_j + b_{Cj}x_n + d_{Cj}r_n)/b_{Cj} \tag{2.18a}$$

ii) For column C_m, joint j is at its second end, so its contributions to the horizontal deflection x_j are obtained from the third row of equations (2.7), which gives:

$$0 = -b_{Cm}x_m + d_{Cm}r_m + b_{Cm}x_j + d_{Cm}r_j \tag{2.19}$$

and thus:

$$x_j = (+ b_{Cm}x_m - d_{Cm}r_m - d_{Cm}r_j)/b_{Cm} \tag{2.19a}$$

iii) There are no contributions from the beams to the horizontal deflection x_j at j because the axial stiffness of these is ignored.

iv) The total deflection x_j at j is thus obtained by adding equations (2.18a) and (2.19a). Hence

$$x_j = (H_j + d_{Cj}r_j + b_{Cj}x_n + d_{Cj}r_n + b_{Cm}x_m - d_{Cm}r_m - d_{Cm}r_j)/(b_{Cj} + b_{Cm}) \tag{2.20}$$

2.4.2.2 <u>The Vertical Deflection y_j</u>

The vertical displacement of joint j is obtained from the contributions of all the members connected to it as follows:

i) The contributions of the beams B_i and B_j to y_j are given in equations (2.12a) and (2.13a) for a beam joint. These remain the same for a column joint.

ii) The contributions of a column C_j to the vertical deflection y_j are obtained by considering the axial deformation of that column. This is given by the second row of equations (2.5). Thus

$$0 = a_{Cj}y_j - a_{Cj}y_n \tag{2.21}$$

That is $$y_j = a_{Cj}y_n/a_{Cj} \tag{2.21a}$$

iii) Joint j is at the second end of column C_m. Thus the contribution of this column to the vertical deflection at j is given by the fifth row of equations (2.5), thus:

$$0 = -a_{Cm}y_m + a_{Cm}y_j \tag{2.22}$$

or $$y_j = a_{Cm}y_m/a_{Cm} \tag{2.22a}$$

iv) The total value of y_j is then obtained by adding equations (2.12a), (2.13a), (2.21a) and (2.22a) to give:

$$y_j = (V_j + d_j r_j + b_j y_k + d_j r_k + b_i y_i - d_i r_i - d_i r_j$$
$$+ a_{Cj}y_n + a_{Cm}y_m)/(b_j + b_i + a_{Cj} + a_{Cm}) \tag{2.23}$$

Notice that the external load applied at a joint is only considered once with the contribution of any member connected to that joint. In the case of y_j, the external load at j was considered when y_j for the beam B_j was derived.

2.4.2.3 The Rotation r_j

The rotation r_j of a column joint j is obtained as follows:

i) The contributions of the beams B_i and B_j to r_j are already derived and are given by equations (2.15a) and (2.16a).

ii) The contributions of columns C_j to r_j are obtained from the second row of equations (2.7). Joint j is at the first end of this column. Thus:

$$0 = -d_{Cj}x_j + e_{Cj}r_j + d_{Cj}x_n + f_{Cj}r_n \tag{2.24}$$

i.e. $$r_j = (+d_{Cj}x_j - d_{Cj}x_n - f_{Cj}r_n)/e_{cj} \tag{2.24a}$$

iii) Joint j is at the second end of column C_m. Hence the fourth row of equations (2.7) gives the contribution of this member to r_j. Thus:

$$0 = -d_{Cm}x_m + f_{Cm}r_m + d_{Cm}x_j + e_{Cm}r_j \tag{2.25}$$

$$\text{i.e.} \quad r_j = (+ d_{Cm}x_m - f_{Cm}r_m - d_{Cm}x_j)/e_{Cm} \tag{2.25a}$$

iv) Adding equations (2.15a), (2.16a), (2.24a) and (2.25a) together, gives the final rotation of joint j as:

$$r_j = (M_j + d_jy_j - d_jy_k - f_jr_k + d_iy_i - f_ir_i - d_iy_j + d_{Cj}x_j - d_{Cj}x_n - f_{Cj}r_n + d_{Cm}x_m - f_{Cm}r_m - d_{Cm}x_j / (e_j + e_i + e_{Cj} + e_{Cm}) \tag{2.26}$$

Equations (2.20), (2.23) and (2.26) express the joint displacements for a member with configuration 6. Figure 2.4 shows this configuration, which is that of a general case. To obtain similar equations for configurations 1 to 5, terms contributed by the missing members should be omitted. E.g. for configuration 3, terms contributed by column C_m to all three equations should be omitted.

In the orthodox stiffness method of analysis, to calculate the sway of a complete storey, one adds the rows and the columns in the stiffness matrix corresponding to x_j for all the joints in the storey. Thus equation (2.20), which calculates the sway of a column joint, can be extended to calculate the sway of a complete storey by adding up the terms in the equation to include all the joints in the storey. The numerator SN_j of equation (2.20) is

$$SN_j = H_j + d_{Cj}r_j + b_{Cj}x_n + d_{Cj}r_n + b_{Cm}x_m - d_{Cm}r_m - d_{Cm}r_j \tag{2.27}$$

while the denominator is:

$$SD_j = b_{Cj} + b_{Cm} \tag{2.28}$$

The sway x_s, for the complete storey, is thus:

$$x_s = \sum_{j=J_1}^{j=J_S} SN_j / \sum_{j=J_1}^{j=J_S} SD_j \tag{2.29}$$

where J_1 is the first joint on the far left of the storey under consideration, and J_s is the joint on the far right of the storey.

2.4.3 Design Equations for a Column

If the sway of a storey x_s or of a column x_j is specified, then equations (2.20) and (2.29) can be used to find the second moment of area of the column. To do this the stiffness coefficients for column C_j are written as factors of I_{Cj}, thus:

$$\begin{aligned} b_{Cj1} &= 12E/L^3_{Cj} \\ d_{Cj1} &= -6E/L^2_{Cj} \\ e_{Cj1} &= 4E/L_{Cj} \\ f_{Cj1} &= 2E/L_{Cj} \end{aligned} \tag{2.30}$$

Equation (2.20) is rearranged to calculate I_{Cj} instead of x_j. This gives:

$$\begin{aligned} I_{Cj} = (H_j - b_{Cm}x_j + b_{Cm}x_m - d_{Cm}r_m \\ - d_{Cm}r_j)/(b_{Cj1}x_j - d_{Cj1}r_j - b_{Cj1}x_n - d_{Cj1}r_n) \end{aligned} \tag{2.31}$$

Figure (2.6) shows an intermediate storey of height h in a frame of non-uniform bay width. The simple "portal method" of design assumes that the horizontal shear at any level is divided between the bays in proportion to their relative width (Allwood, et al, 1966), (Anderson and Islam 1979). Hence, for the columns,

$$\frac{I_1}{L_1} = \frac{I_2}{L_1 + L_2} = \frac{I_3}{L_2 + L_3} \ldots\ldots\ldots = \frac{I_{nb}}{L_{nb}} \tag{2.32}$$

where nb is the number of bays. The ratio I_2/I_1 is referred to as the internal to external column ratio, and it is denoted by ρ. For the first column joint in the storey (that is the one on the immediate left) ρ_1 is equal to 1. Hence for the rest of the joints of the columns ρ is defined as:

$$\rho_1 = I_1/I_1, \quad \rho_2 = I_2/I_1, \quad \rho_3 = I_3/I_1, \quad \rho_j = I_j/I_1$$

$$\ldots\ldots\ldots, \quad \rho_n = I_{(nb+1)}/I_1 \tag{2.33}$$

Using equations (2.32):

$$\rho_1 = 1.0, \ \rho_2 = (L_1 + L_2)/L_1, \ \rho_3 = (L_2 + L_3)/L_1,$$

$$\ldots\ldots, \ \rho_n = L_{nb}/L_1 \tag{2.34}$$

If the sway of the complete storey is specified, then equation (2.29) can be rearranged to calculate I_1, in terms of the sway x_s, and the ratios ρ_j. The second moments of area for the remaining columns in the storey are then calculated from equation (2.33).

It should be stressed that the ratios ρ for the second moment of area of internal and external column has to be specified before the above equations are used for the design of the columns. However, it should be stressed that the portal method described above is not a necessary part of the design method. The portal method is only used to begin with to specify some initial ratios. Later, as the design of the frame proceeds, the value of ρ changes continuously so that the final design has the most economic value of ρ for each storey.

2.4.3.1 Equation for Second Moment of Area of the External Left Column I_1

i) In the design procedure the contributions of each column in the storey to I_1 should be found first. These are calculated from equation (2.31), which is rewritten in terms of ρ_j and I_1. Thus, the contributions of columns C_j to I_1 are:

$$I_1 = (H_j - b_{Cm}x_j + b_{Cm}x_m - d_{Cm}r_m - d_{Cm}r_j)/ \left(\rho_j\ (b_{Cj1}x_j - d_{Cj1}r_j - b_{Cj1}x_n - d_{Cj1}r_n)\right) \tag{2.35}$$

ii) Terms of equation (2.35) are added for all the columns in the storey. The numerator of equation (2.35) is SN'_j given by:

$$SN'_j = (H_j - b_{Cm}x_j + b_{Cm}x_m - d_{Cm}r_{Cm} - d_{Cm}r_j) \tag{2.36}$$

while the denominator is SD'_j

$$SD'_j = \rho_j\ (b_{Cj1}x_j - d_{Cj1}r_j - b_{Cj1}x_n - d_{Cj1}r_n) \tag{2.37}$$

I_1 is thus given by:

$$I_1 = \sum_{j=J_1}^{j=J_s} SN'_j / \sum_{j=J_1}^{j=J_s} SD'_j \tag{2.38}$$

Where J_1 and J_s are as given before in equation (2.29).

2.4.4 An Example on the Derivation of the Stiffness Equations

The above equations are best explained by deriving them for a single-storey single-bay frame, shown in Figure 2.7. To keep the problem as simple as possible the axial deflection of the columns is disregarded at this stage. The stiffness equation $\underline{L} = \underline{K}\ \underline{X}$ for the frame in Figure 2.7 are:

$$\begin{bmatrix} Q \\ 0 \\ P \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{12EI_1}{h^3} + \frac{12EI_1}{h^3} & \frac{-6EI_1}{h^2} & 0 & 0 & \frac{-6EI_1}{h^2} \\ \frac{-6EI_1}{h^2} & \frac{4EI_1}{h} + \frac{4EI_2}{L/2} & \frac{-6EI_2}{(L/2)^2} & \frac{2EI_2}{L/2} & 0 \\ 0 & \frac{-6EI_2}{(L/2)^2} & \frac{12EI_2}{(L/2)^3} + \frac{12EI_2}{(L/2)^3} & 0 & \frac{6EI_2}{(L/2)^2} \\ 0 & \frac{2EI_2}{L/2} & 0 & \frac{4EI_2}{L/2} + \frac{4EI_2}{L/2} & \frac{2EI_2}{L/2} \\ \frac{-6EI_1}{h^2} & 0 & \frac{6EI_2}{(L/2)^2} & \frac{2EI_2}{L/2} & \frac{4EI_1}{h} + \frac{4EI_2}{L/2} \end{bmatrix} \begin{bmatrix} x \\ r_B \\ y_F \\ r_F \\ r_C \end{bmatrix} \qquad (2.39)$$

where r_B, r_F and r_C are the rotations of joints B, F and C respectively, x is the horizontal sway, and y_F is the vertical sway of joint F. The second moment of area for the column and beam are I_1 and I_2.

For the beam joint F, equation (2.14), or the third row of equation (2.39), gives:

$$y_F = \frac{P + \dfrac{6EI_2}{(L/2)^2}\, r_B - \dfrac{6EI_2}{(L/2)^2}\, r_C}{\dfrac{12EI_2}{(L/2)^3} + \dfrac{12EI_2}{(L/2)^3}} \qquad (2.40)$$

while equation (2.17) or the fourth row of equation (2.39) gives

$$r_F = \frac{0 - \frac{2EI_2}{L/2} r_B - \frac{2EI_2}{L/2} r_C}{\frac{4EI_2}{L/2} + \frac{4EI_2}{L/2}} \tag{2.41}$$

For the column joint B, the sway x can be obtained from equation (2.29), or from the first row of equation (2.39); thus

$$x = \frac{Q + \frac{6EI_1}{h^2} r_B + \frac{6EI_1}{h^2} r_c}{\frac{12EI_1}{h^3} + \frac{12EI_1}{h^3}} \tag{2.42}$$

If, x is specified (e.g. x = h/300) I_1 can be obtained from equation (2.42), thus

$$I_1 = \frac{Q}{(\frac{12E}{h^3} + \frac{12E}{h^3}) x - \frac{6E}{h^2} r_B - \frac{6E}{h^2} r_C} \tag{2.43}$$

Application of equation (2.38) gives the same result.

The rotation r_B is found from equation (2.26), which is the same as the second row of the stiffness matrix. Both give:

$$r_B = \frac{0 + \frac{6EI_1}{h^2} x + \frac{6EI_2}{(L/2)^2} y_F - \frac{2EI_2}{L/2} r_F}{\frac{4EI_1}{h} + \frac{4EI_2}{L/2}} \tag{2.44}$$

similarly the rotation r_C is given by:

$$r_C = \frac{0 + \frac{6EI_1}{h^2} x - \frac{6EI_2}{(L/2)^2} y_F - \frac{2EI_2}{L/2} r_F}{\frac{4EI_1}{h} + \frac{4EI_2}{L/2}} \tag{2.45}$$

2.5 PRELIMINARY BEAM SECTIONS

In the design process the beam sections should be selected to satisfy the stress constraints. First of all, a lower bound is obtained for a beam by considering the vertical load on it and deriving the equation of beam mechanism. In most cases, the section obtained in this manner is not adequate for resisting the side-sway except for the top one or two storeys. Nevertheless, no beam section is allowed to be reduced below that obtained by beam mechanism. Under wind-loads, preliminary beam sections could be obtained to resist the sway deflection by a method due to Anderson and Islam 1979) as described in Chapter 1. In some cases, sections obtained in this way are found to be totally unsatisfactory. However, Anderson and Islam's approach can still be utilized provided that, in this preliminary calculation, the allowable sway is severely restricted for the time being. It has been found that equations given by Anderson and Islam underestimate the beam sections considerably.

Another method, which could be used to select an initial section for a beam, is to use the simple design methods of The Steel Designers' Manual (Allwood, et al, 1972). In these methods a point of contraflexure is taken to exist at the midheight of each column and at the midlength of each beam. In most cases, sections selected by this method to satisfy strength requirements are found to be unsatisfactory for resisting the sway deflection. However, for a preliminary trial, these sections can be magnified by a certain factor and obtain an upper bound for each beam. This factor might change from the beams at the top storey to those at the bottom and it can have any value between 1.5 and 2 at the top and 3 to 5 at the bottom, depending on the height of the frame.

2.6 SOLUTION OF THE MODIFIED STIFFNESS EQUATIONS BY INERATIVE TECHNIQUE

The modified stiffness equations given for a general joint section (2.4.1, 2.4.2 and 2.4.3) are applied to each joint of the frame to obtain a sufficient number of equations for the evaluation of the joint displacement and the column second moments of area. This is provided that sections have already been selected for the beams.

Although these equations are non-linear, an iterative technique, which is a significant part of this thesis, proved to be a powerful method for solving them. This technique starts by assuming estimated values for the unknowns which, in the absence of a better value, may be taken as zero or one. A second revised estimate may then be found using the modified stiffness equations one at a time. The current value of a variable, at each stage of the iteration is used as soon as it is available to calculate a new value for the next variable in the iteration chain.

It must be emphasised here, that only an unknown related to the leading diagonal element of the stiffness matrix is calculated from each equation. The obvious reason is that the coefficients of the leading diagonal element is much larger than the others (Lennox, et al, 1974, Cohen, et al, 1973). This prevents divergence and also accelerates the convergence of the process.

2.7 AN EXAMPLE ON THE SOLUTION OF THE MODIFIED STIFFNESS EQUATIONS

The solution method is best explained by applying it to the single bay frame dealt with in section 2.4.4. Assuming $r_B = r_C = r_F = 0$, $y_F = -1$, $x/h = 1/360$, and I_2 is calculated from the beam mechanism, which gives $I_2 = 21345 \times 10^4\ mm^4$. Equation (2.40) gives

$y_F = -11.92$ mm. For the rest of this cycle, this new value of y_F is used for the calculation of all the other variables. Equation (2.41) gives: $r_F = 0.0$, and equation (2.43) gives the first value for the column section as $I_1 = 3106 \times 10^4$ mm^4. With these new values for y_F, r_F and I_1, equation (2.44) gives $r_B = -0.00395$. Thus r_C is calculated from equation (2.45) as 0.00267. This procedure is repeated until all the variables converge to stable results. To test this convergence, a tolerance is introduced, and a variable is said to have become stable if its:

$$\left| \frac{\text{new value - old value}}{\text{new value}} \right| < \text{The tolerance}$$

Table 2.1 shows the results of the iteration process, with a tolerance of 0.01. It can be noticed that 13 iterations are needed to converge to the final design values, which include the second moments of area.

2.8 AN OUTLINE OF DESIGN PROCEDURE

At this stage it is useful to give an outline of the design procedure. This consists of the following steps.

Step 1 - Select an initial set of beam sections by the beam mechanism method under vertical loads.

Step 2 - Specify the horizontal deflection at each storey level

Step 3 - Define the ratio of the second moments of area of an internal and an external column. Any ratio is acceptable but it was decided to start with the ratios calculated above in section 2.4.3.

Step 4 - Assume the second moment of area of each column, the deflection and the rotation of all the joints. Infeasible values such as $I = 1$ mm^4, $y = 1$ and $r = 0.0$ are sufficient for the purpose of starting the iteration process.

Step 5 - Using the modified stiffness equations given above, calculate the deflections of the beam joints.

Step 6 - Using these values of deflections calculate the first unknown sectional property I_C of one of the column. Using this value of I_C calculate the vertical deflection and the rotation of that column joint.

Step 7 - Continue with iteration until a new value is obtained for each unknown and all the modified stiffness equations for all the joints are used

Step 8 - Repeat steps 5, 6 and 7 until convergence is achieved. At this stage one design cycle is completed and a set of sections become available for columns. All the joint deflections and rotations also become available. Notice that the beam sections and the horizontal deflection of the joints are specified but the other unknowns are calculated. The column sections and the joint deflections calculated fully satisfy the stiffness equations and the design obtained is feasible and most structures designed in this manner are ready to be constructed. However, the design process continues for the following reasons:

a - To include the effect of axial forces in the columns.

b - To achieve economy by altering the ratios between internal and external columns.

c - To achieve economy by repeating the operation with a different set of beam sections. This is provided no beam section is reduced below those given by the beam mechanism.

d - To check that while deflection and sway requirements are satisfied, the stress (plastic hinge moments) are satisfactory in all the members.

Step 9 - Modify the stiffness equations by including stability functions, see later.

Step 10 - Alter some of the beam sections if this is economical, see later.

Step 11 - Repeat the iteration process, i.e. steps, 5, 6 and 7 until convergence is obtained once again. Notice that when entering the iteration this time, the current values of column sections and joint rotations are employed. These values are more realistic than the initial infeasible ($I = 1$, $y = -1$ and $r = 0$) values. For this reason the second round of iteration converges very quickly.

Step 12 - Alter the ratio between the internal and external columns, if this is economical, see later.

Step 13 - Repeat the iteration, i.e. steps 5, 6 and 7 until convergence is achieved. Once again the iteration is initiated with current available values and convergence is very fast.

Step 14 - Repeat steps 9 to 13 until no economy is achieved either for altering the beam sections or for altering the ratios of internal to external columns. At the end of this step the frame stability is satisfied and the deflections and sectional properties calculated fully satisfy the stiffness equations.

Step 15 - Check the stress requirements in the frame. This is done by:

a - checking that the plastic modulus of the beams under combined vertical and wind loads are satisfactory. With the unit load factor $\lambda = 1$ no plastic hinge should develop in any beam and the entire frame must be elastic under such a load factor. See details later.

b - Checking that under factored loads, $\lambda > 1$ (say $\lambda = 1.29$ as in the recent codes), no hinge developsin any column. See details later.

If the plastic modulus of all the sections are satisfactory, which is often the case, the design process is complete. If not, select new beam and column sections for the members and repeat steps 5, 6 and 7. Notice that if I_C of the columns are known the iteration at this stage calculates the horizontal joint deflections instead. Thus during each round of iteration either column sections are unknown and calculated with given specified values of the horizontal storey deflection or these deflections are calculated using column sections selected to satisfy strength requirements.

It is evident that the values of the horizontal deflection at each storey can be specified in more than one way. In the following sections a method is proposed for this purpose so that the recent suggestions made by the British Standard documents are satisfied.

2.9 GENERAL EQUATION FOR THE SWAY DEFLECTION CONSTRAINTS

The draft of the British Standards Institution B/20 document 77/13908 DC restricts the differential deflection between the two ends of any column to not more than a constant value. This constant may be taken as $\alpha H = H/300$, where H is the length of the column. Excessive differential deflection causes cracks in the walls, cracks in the cladding and may also damage windows. If the sway in each storey is the maximum of H/300 then the deflection profile of a multistorey frame is a straight line such as OA in Figure 2.8. In practical frames, the sway in the lower columns is less than those in more flexible upper columns and thus a linear profile cannot take place. It is therefore necessary to derive a non-linear deflection profile for the frame which represents the deformation of the frame more realistically. Measuring x from the top of the frame, let the deflection y at a distance x from the top be given by the deflection function.

$$y = a_1 + a_2 x + a_3 x^2 + a_4 x^3 + a_5 x^n \tag{2.46}$$

where a_1, a_2, a_3, a_4 and a_5 are constants and must satisfy the boundary conditions at the top and the bottom of the frame and n is a variable which for economic reasons should be numerically as high as possible. As n increases, the frame becomes more flexible and the non-linear profile of the frame approaches the linear upper bound OA. The constants a_1 to a_5 are found as follows:

1 - Since in a practical frame each floor is required to be stiffer than or as stiff as the floor above, it follows that the sway in each storey must not be more than the sway in the storey above. In this thesis it is proposed that a sway of H/300 should be allowed only in the top storeys. Thus the slope of the deflection profile, curve OB in Figure 2.8, is made equal to α i.e. when $x = 0$ $dy/dx = \alpha$.

Now:

$$dy/dx = a_2 + 2a_3 x + 3a_4 x^2 + na_5 x^{n-1}$$

and when $x = 0$, $dy/dx = \alpha$

$\therefore \quad \alpha = a_2 + 0$

and $a_2 = \alpha = 1/300$

2 - The top end of the frame is free and considering the whole frame as a single element, the bending moment at this free end will be zero. Thus when $x = 0$ $d^2y/dx^2 = 0$. Now:

$d^2y/dx^2 = 2a_3 + 6a_4x + n(n-1)\ a_5x^{n-2}$

and when $x = 0 \quad d^2y/dx^2 = 0$

$\therefore \quad a_3 = 0$

3 - At the free end, at the top of the frame, the shear force is also equal to zero. Thus when $x = 0$ $d^3y/dx^3 = 0$ and thus $a_4 = 0$.

4 - Because of the fixed end condition at the ground level, we have when $x = L$, $y = 0.0$. Thus equation (2.46) gives:

$$0.0 = a_1 + \alpha L + a_5L^n \tag{2.47}$$

where L is the total height of the frame.

5 - Furthermore, for the condition of fixity at the base, when $x = L$, $dy/dx = 0.0$. This leads to

$$0.0 = \alpha + na_5L^{n-1} \tag{2.48}$$

Solving equations (2.47), and (2.48) gives the value of a_1 and a_5 as:

$$a_1 = -\alpha L\left(\frac{n-1}{n}\right) \tag{2.49}$$

$$a_5 = -\frac{\alpha}{n\ L^{n-1}} \tag{2.50}$$

Equation (2.46) now becomes:

$$y = -\frac{\alpha}{L^{(k+3)}}\left[\frac{x^{(k+4)}}{k+4} - L^{(k+3)}\ x + \frac{k+3}{k+4}\ L^{(k+4)}\right] \tag{2.51}$$

where n = k + 4. As k tends to infinity, equation (2.51) tends to the ultimate straight line deflection, OA in Figure 2.8. In practice, k is taken as large as possible. Provided that x^{k+4} does not lead to an exponential overflow. During the calculation of the deflection for each storey using equation (2.51). The value of k at which the overflow occurs changes from one frame to another. For each frame therefore the value of k is taken as a number which is smaller than that which causes an exponential overflow. Equation (2.51) ensures that the sway in each storey will be less than the sway in the storey above and this sway reaches H/300 only at the top floor. For a large value of k, the profile OB is nearly parallel to OA except for the bottom part of OB.

In the special case, where m storeys of the frame are of equal height h, equation (2.51) can be written in terms of the storey level r Figure 2.8. This gives:

$$y_r = -\frac{\alpha h}{m^{(k+3)}}\left[\frac{(m-r)^{k+4}}{k+4} - m^{k+3}\ (m-r) + \frac{(k+3)}{(k+4)}\ m^{k+4}\right] \tag{2.52}$$

where y_r is the sway of the storey at level r.

2.10 UNIVERSAL BEAM AND COLUMN SECTIONS USED IN THE DESIGN

In multi-storey steel frames it is normal practice to use the available rolled sections unless, in special cases, built-up sections are found to be more economical. The computer program written for the design of sway frames selects universal column sections for column members, and universal beam sections for beam members. Because of local instability problems, certain sections, depending on the grade of the steel used are not suitable (Horne, 1964) and are not included in the final selection. For mild steel (grade 43) and high yield steel grade (50) the universal column and beam sections (BCSA, CONSTRADO, 1971) are reorganized in the order of their cost. The high yield steel (grade 55) is not used, because of the high probability of local buckling. Furthermore, in multi-storey work, Grade 50 is frequently the most economic (Needham, 1977). Expensive sections are omitted from the list. The prices of the universal column sections and beam sections are taken from the price list of steel sections (British Steel Corporation, 1977). A section is considered expensive if there is another less costly section with an equal or larger second moment of area. The sections with the minimum price-inertia ratio are selected. Tables 2.2 and 2.3 show the economical beam and column sections respectively, for mild steel grade 43; and Tables 2.4 and 2.5 are for high yield steel grade 50.

For high-rise multi-storey frames, universal sections are not adequate. For this reason a few built-up sections are added. These are fabricated from the largest universal section with two reinforced steel plates.

To satisfy the deflection requirements the beam sections (calculated in section 2.5) are selected from the list before designing the column sections. Thus the discrete available beam sections are made full use of during the process of satisfying the deflection requirements.

2.11 ALTERING THE BEAM SECTIONS

The selected beam sections (Section 2.5) do not necessarily result in the most economic design for the complete frame. For this reason, a linear extrapolation is used to alter the beam sections in an effort to reduce the cost of the frame. To apply this extrapolation, two preliminary designs are needed, and initially two sets of beam sections must be provided for these two designs. The beam sections selected in Section (2.5) upgraded by one provide the first set of beams. The second set of beam sections are provided by those obtained in Section (2.5). The iteration technique is entered with the first set of beams to obtain the sectional properties of the column, deflections and rotations, so producing the first cycle in the design process. For the second design the iteration technique is entered again with the second set of beams. The cost of the material for the frame in the first and second cycles is found, and used in an extrapolation process to reduce cost. In this process I_{B1} and I_{C1} are the second moment of area for the beam and the column in a given storey for the first cycle. I_{B2} and I_{C2} are those for the second cycle. If I_{B3} is smaller than I_{B2} by one section, then Figure 2.9 shows that the second moment of area I_{C3} for the column section is given by:

$$(I_{C3} - I_{C1})/(I_{C2} - I_{C1}) = (I_{B1} - I_{B3})/(I_{B1} - I_{B2}) \qquad (2.53)$$

i.e.

$$I_{C3} = I_{C1} + (I_{C2} - I_{C1})\ (I_{B1} - I_{B3})/(I_{B1} - I_{B2}) \qquad (2.54)$$

Similarly, this equation can also be used if the beam section for each storey is upgraded by one section to find the expected column section.

It must be emphasised that the extrapolation is introduced to avoid solving the modified stiffness equations when the beam section is increased or decreased. The value obtained for the second moment of area of a column is approximate and is used only for costing purposes. However, it was found that the value of the second moment of area for a column obtained using equation (2.54) is very near the value obtained by solving the modified stiffness equations.

The approximate cost of the material at each storey (that is the cost of the beam and the columns supporting it) is calculated by extrapolation for the following:

1 - for the first design cycle;

2 - for the second design cycle;

3 - for the case when the beam is downgraded by one section;

4 - for the case when the beam is upgraded by one section.

The beam section which gives the lowest cost is chosen for the third design cycle. This process is continued for each storey until successive design cycles will show that a decrease or an increase in any beam section would not lead to a cheaper design of the frame.

To avoid reverse column taper, a check is made in each design cycle to ensure that each column section is less than or equal to the column below it. If a current design cycle results in reverse column taper in a given storey, the beam section of the previous cycle is adopted, and no reduction on this beam size is allowed.

2.11.1 Example on Reducing Beam Sections

The four storey single bay frame shown in Figure 2.10a is designed as an example. This frame has been designed by (Majid, 1972) using the elastic plastic design method for sway frames.

This showed that the sway deflection of the frame was somewhat excessive. This is in spite of the fact that the frame itself had a reasonable reserve of strength over and above that required by (BS 449,1969). The result of this design is summarized in Figure 2.10b which shows the second moment of area for each member after selecting sections from the list of rolled sections, and in Figure 2.10c which shows the order of the formation of hinges under combined loading. It can be seen that collapse takes place at a load factor of 1.49, which is above an adopted permissible value of 1.4. However, the maximum deflection for this design is 53.8 mm which is more than allowable. This frame is used here for an extensive investigation into the design method proposed in this thesis. Although the reduction in the cost of the material is considered in the actual computer program, a reduction in weight is used for this example to compare the results with those given by an optimum design program based on weight optimisation (Saka, 1975). The axial force effect and the axial stiffness are not considered in this example for similar reasons.

Application of equation 2.52 for various values of k is shown in Figure 2.11. The value of k just before an overflow occurred was 56; the deflection profile for this value of k is used to specify the horizontal deflection at each beam level. It can be seen in Figure 2.11 that for k = 56 the non-linear and the linear profile are nearly the same. Any difference between the adopted deflections and those obtained by the straight line are in the bottom storeys.

i) <u>First Design cycle (Figure 2.12)</u>

The second moment of area of the beam sections for the first design cycle are found as follows:

1) The second moment of area for the sections given by beam mechanism are found with a load factor of 1.75. This gives I = 21345 cm^4.

2) The values obtained in (1) above are multiplied by a magnification factor. This factor can take any value between 1.5 and 2. In this example this factor is taken equal to 1.75 for all beams (i.e. 1.75 × 21345 = 37354).

3) Select sections from the universal beam sections this gives I = 40414 cm^4.

4) The sections obtained in (3) above are upgraded by one section which gives I = 55779 cm^4

These values of I are used in the iteration using the modified stiffness equations. The result is summarized in Figure 2.12. The second moment of area for the beams (chosen from the list) is shown on the top of each beam. The calculated and the selected second moments of area for the columns are shown next to the columns.

ii) Second Design Cycle (Figure 2.13 and 2.14)

The second moment of area of the beam sections for this design cycle are those of the first cycle downgraded by one section (i.e. I = 40414 cm^4). The result of the iteration using the modified stiffness equations is shown in Figure 2.13. This shows that columns number 9 and 10 have a section larger than that of columns 11 and 12 in the bottom storey. For this reason this design is unacceptable. To improve it, beam 3 was increased to the next larger size in the table with I = 55779 cm^4. The iteration gives columns 9 and 10 again larger than the columns below. Therefore, beam 3 is increased once more to the next larger size which has $I = 75549\ cm^4$. Figure 2.14 shows the results with beam 3 having $I = 75549\ cm^4$ and all other beams having $I = 40414\ cm^4$. This modified design replaces the one in Figure 2.13. It can be seen that the design in Figure 2.14 is lighter than the design in Figure 2.12. That in Figure 2.14 is used to start the extrapolation for the third design cycle.

iii) Third Design Cycle (Figure 2.15)

To obtain the second moments of area for the beam sections the procedure of section 2.11 is applied. Each beam section (except beam 3) is upgraded and downgraded by one section and the weight of each storey is found with the column sections obtained from equation 2.54. In this equation I_{B1} and I_{C1} are those of Figure 2.12, while I_{B2} and I_{C2} are taken from Figure 2.14. The beam which gives the least weight in each storey is used in the iteration. The result is shown in Figure 2.15. The design of the top storey is altered to avoid beam mechanism collapse, due to the formation of hinges at the top ends of the columns. The top columns are given the same sections as the columns below, and the beam is designed to satisfy strength requirements. This gives $I = 25464\ cm^4$, no further reduction on this beam section is possible.

vi) Fourth Design Cycle (Figures 2.16 and 2.17)

Further reduction on the beam sections of the third cycle gives the design shown in Figure 2.16. This is unacceptable, because the I values for columns 9 and 10 are larger than those for columns 11 and 12. To avoid reverse column taper the I value for beam 4 in the fourth design cycle is kept as it is in the third. The fourth design cycle is shown in Figure 2.17.

v) Fifth Design Cycle (Figure 2.18)

This is the final design cycle, the result is shown in Figure 2.18. No further reduction in beam sections could be made. The reason is that any reduction in the bottom two beams will result in reverse taper column, while any change in the top two beams will violate the strength requirements for vertical load.

Figure 2.19 shows the result obtained by Saka (1975), using an optimisation method. The second moments of area for the beam sections obtained by Saka are shown above each beam, while the available second moment of area for the sections selected from the list is given below the beam. To calculate the weight of the frame the length L of each member is multiplied by the second moment of area I, and by the mass per unit length m.

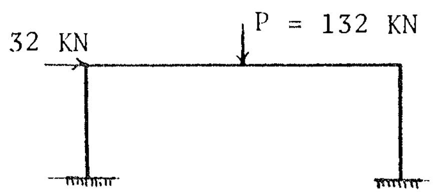
The sums ΣIL and ΣmL are calculated. It was found that ΣIL for the final design proposed in this thesis is 1.9032×10^{13} mm^5, while for the optimum design is 1.8835×10^{13} mm^5. The percentage difference being 1.03%. However, using the list of available sections for both designs it was found that the total mass for the author's final design was 5349.0 kg which is less than that for the optimum design (5497.3 kg) by 2.77%. This difference is solely due to the discrete nature of the beam sections available. The computer time needed to complete the design by the present method was 1/20 of that needed for the optimum design. The storage needed was 1/10 of that needed by the optimum design.

Finally, to summarize the design cycles given in Figure 2.12 to 2.18 a graph is drawn for the total mass of each design cycle in Figure 2.21. This shows the reduction in mass in each step. The increase in the total mass in the third design cycle is due to the increase in the top column sections to avoid the formation of plastic hinges at the top ends of these columns.

2.12 SELECTING THE INTERNAL TO EXTERNAL COLUMNS RATIO

Equation (2.34) gives the ratio of the second moments of area of the internal and external columns as might be obtained using a simple design method. This ratio is not the most economical one. A procedure to obtain a more economical distribution of inertia between internal and external columns is now considered.

In the first two preliminary design cycles, equation 2.34 gives the ratio between the internal and the external second moment of area of the columns. After calculating the columns inertia and then selecting a section for each column from the list, the selected sections will have a ratio different from the one obtained from equation 2.34. The new ratio between the selected sections might be more economical to use. Therefore, a new design is examined for cost, the design having the same set of beams, but with the new ratio. The flow diagram shown in Figure 2.21 summarizes the procedure for economy in selecting beam section, and internal to external column ratio.



$E = 207\ kN/mm^2$

Iter-ation	Initial Values	1	2	3	4	5	6	7	8	9	10	11	12	13
y_F (mm)	-1.0	-11.92	-19.5	-24.0	-26.8	-28.4	-29.4	-30.0	-30.4	-30.6	-30.8	-30.9	-30.9	-30.9
r_F	0.0	0.0	0.00032	.00042	.00045	.00047	.00047	.00047	.00047	.00047	.00047	.00047	.00047	.00047
I_1 (cm^4)	1.0	3106	3512	3662	3714	3732	3737	3739	3740	3740	3740	3740	3740	3740
r_B	0.0	-0.00395	-.00614	-.00740	-.00814	-.00858	-.00885	-.00901	-.00911	-.00918	-.00921	-.00924	-.00925	-.00926
r_C	0.0	.00267	.00445	.00558	.00628	.00671	.00697	.00713	.00723	.00729	.00733	.00735	.00737	.00738

TABLE 2.1: SHOWS THE CONVERGENCE OF THE DEFLECTIONS AND SECOND MOMENTS OF AREA OF THE COLUMNS TO THE DESIGN VALUES, WITH A TOLERANCE OF 0.01

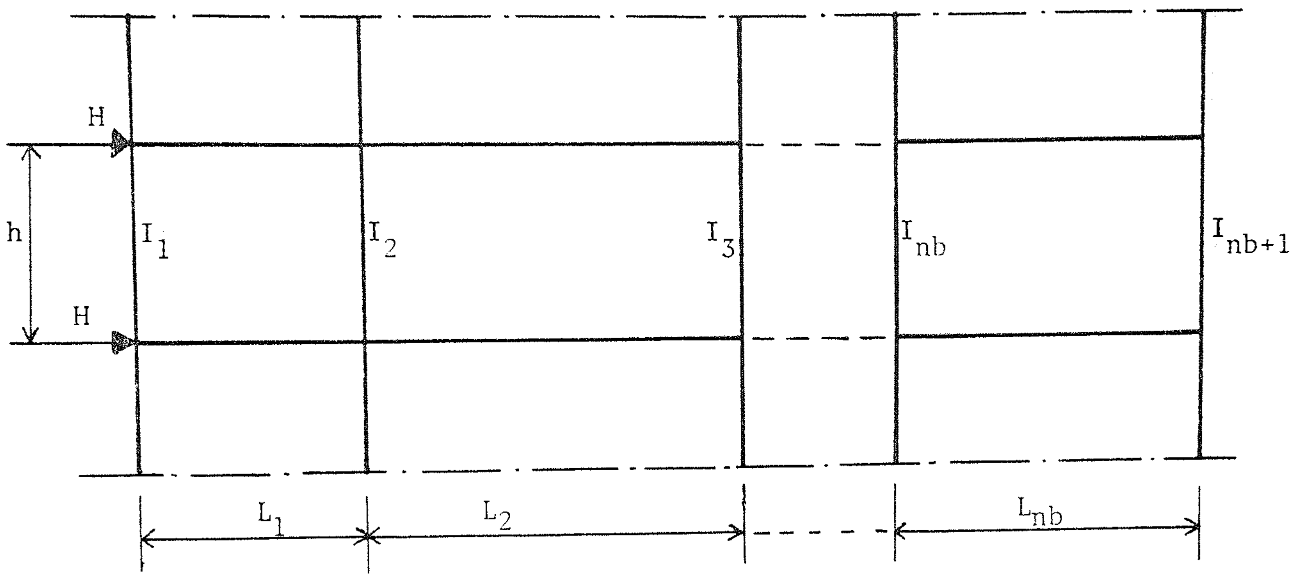


FIGURE 2.6: INTERMEDIATE STOREY IN A RECTANGULAR FRAME OF NON-UNIFORM BAY WIDTH

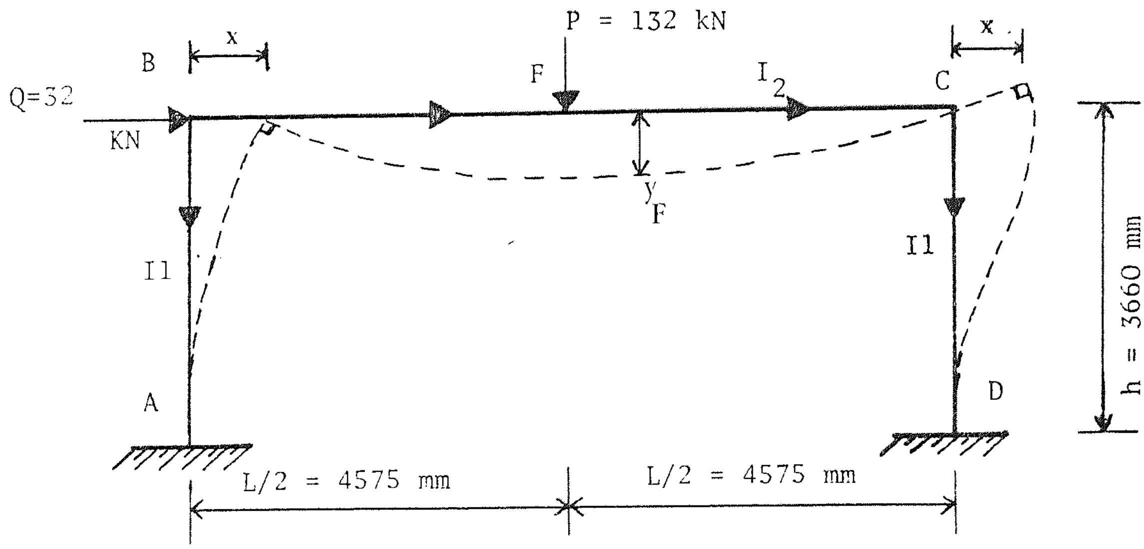


FIGURE 2.7: SINGLE-STOREY SINGLE-BAY FRAME

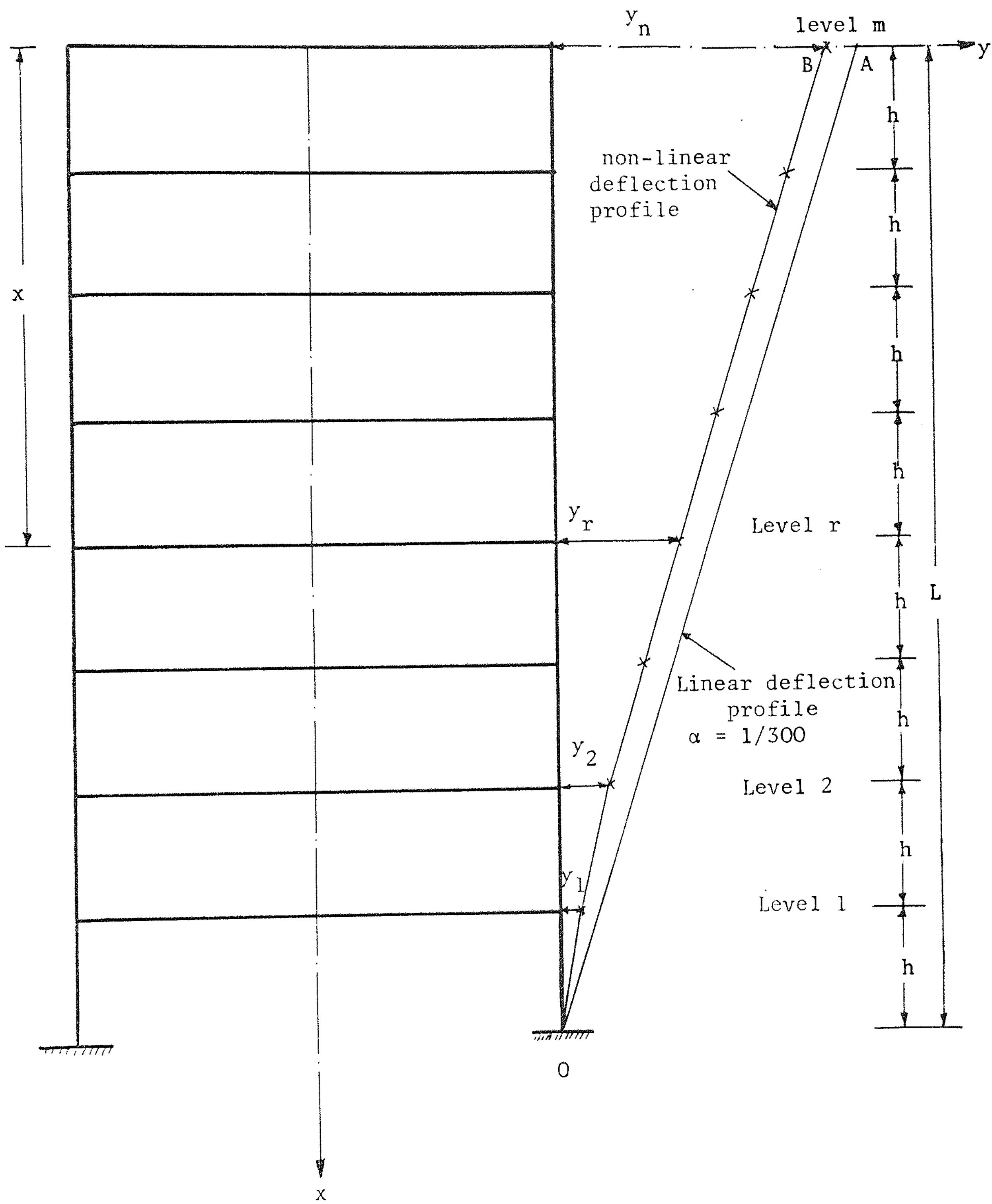


FIGURE 2.8: LINEAR AND NON-LINEAR DEFLECTION PROFILES

UB sections	Second moment of area cm^4	Area cm^2	Price £ per m
914 × 419 × 388 + 2 - 60 mm plates	1911319	996.7	185.54
914 × 419 × 388 + 2 - 40 mm plates	1480449	829.1	156.00
914 × 419 × 388 + 2 - 20 mm plates	1082870	661.5	126.45
914 × 419 × 388	717325	493.9	86.91
914 × 419 × 343	623866	436.9	76.83
914 × 305 × 289	503781	368.5	64.30
915 × 305 × 253	435796	322.5	56.29
914 × 305 × 224	375111	284.9	49.84
914 × 305 × 201	324715	256.1	44.72
838 × 292 × 194	278833	246.9	42.68
838 × 292 × 176	254412	223.8	38.72
762 × 267 × 173	204747	220.2	37.80
762 × 267 × 147	168535	187.8	32.12
686 × 254 × 140	135972	178.4	30.59
686 × 254 × 125	117700	159.4	27.31
610 × 229 × 113	87260	144.3	24.46
610 × 229 × 101	75549	129.0	21.87
533 × 210 × 92	55225	117.6	19.64
533 × 210 × 82	47363	104.3	17.51
457 × 191 × 82	37039	104.3	16.56
457 × 191 × 74	33324	94.9	14.95
457 × 191 × 67	29337	85.4	13.53
457 × 152 × 60	25464	75.9	12.93
457 × 152 × 52	21345	66.5	11.21
406 × 178 × 54	18576	68.3	10.99
406 × 140 × 46	15603	58.9	9.98
406 × 140 × 39	12408	49.3	8.46
356 × 127 × 33	8167	41.7	7.26
305 × 102 × 28	5415	36.3	6.22
305 × 102 × 25	4381	31.4	5.55
254 × 102 × 22	2863	28.4	4.86

TABLE 2.2 UNIVERSAL BEAM SECTIONS FOR GRADE 43 STEEL

UC sections	Second moment of area cm^4	Area cm^2	Price £ per m
356 × 406 × 634 + 2 - 60 mm plates	639385	1316.9	241.74
356 × 406 × 634 + 2 - 40 mm plates	499630	1147.3	211.86
356 × 406 × 634 + 2 - 20 mm plates	378668	977.7	181.95
356 × 406 × 634	275140	808.1	142.02
356 × 406 × 551	227023	701.8	123.42
356 × 406 × 467	183118	595.5	104.61
356 × 406 × 393	146765	500.9	88.03
356 × 406 × 340	122474	432.7	76.16
356 × 406 × 287	99994	366.0	64.29
356 × 406 × 235	79110	299.8	52.64
356 × 368 × 202	66307	257.9	44.95
356 × 368 × 177	57153	225.7	39.38
356 × 368 × 153	48525	195.2	34.04
305 × 305 × 158	38740	201.2	32.86
305 × 305 × 137	32838	174.6	28.50
305 × 305 × 118	27601	149.8	24.54
254 × 254 × 107	17510	136.6	22.26
254 × 254 × 89	14307	114.0	18.51
254 × 254 × 73	11360	92.9	15.18
203 × 203 × 71	7647	91.1	14.91
203 × 203 × 60	6088	75.8	12.60
203 × 203 × 52	5263	66.4	10.92
152 × 152 × 37	2218	47.4	7.97
152 × 152 × 30	1742	38.2	6.47

TABLE 2.3: UNIVERSAL COLUMN SECTIONS (GRADE 43)

UB Sections	Second moment of area cm^4	Area cm^2	Price £ per m
As in Table 2.2 for steel grade 43			
838 × 292 × 194	278833	246.9	42.68
762 × 267 × 173	204747	220.2	37.80
762 × 267 × 147	168535	187.8	32.12
686 × 254 × 140	135972	178.4	30.59
610 × 229 × 140	111673	178.2	30.31
610 × 229 × 125	98408	159.4	27.06
610 × 229 × 113	87260	144.3	24.46
533 × 210 × 109	66610	138.4	23.27
533 × 210 × 101	61530	129.1	21.56
533 × 210 × 92	55225	117.6	19.64
457 × 191 × 89	40956	113.8	17.98
457 × 191 × 82	37039	104.4	16.56
457 × 191 × 74	33324	94.9	14.95
457 × 191 × 67	29337	85.4	13.53
457 × 152 × 60	25464	75.9	12.93
457 × 152 × 52	21345	66.5	11.21
406 × 140 × 46	15603	58.9	9.98
356 × 127 × 39	10054	49.3	8.58
356 × 127 × 33	8167	41.7	7.26
305 × 102 × 28	5415	36.3	6.22
305 × 102 × 25	4381	31.4	5.55
305 × 102 × 22	2863	28.4	4.86

TABLE 2.4: UNIVERSAL BEAM SECTIONS FOR GRADE 50 STEEL

UC Sections	Second moment of area cm^4	Area cm^2	Price £ per m
As in Table 2.3 for steel grade 43			
356 × 368 × 202	66307	257.9	44.95
305 × 305 × 198	50832	252.3	41.18
305 × 305 × 158	38740	201.2	32.86
305 × 305 × 137	32838	174.6	28.50
254 × 254 × 132	22416	167.7	27.46
254 × 254 × 107	17510	136.6	22.26
254 × 254 × 89	14307	114.0	18.51
203 × 203 × 86	9462	110.1	18.06
203 × 203 × 71	7647	91.1	14.91
203 × 203 × 60	6088	75.8	12.60

TABLE 2.5: UNIVERSAL COLUMN SECTIONS FOR GRADE 50 STEEL

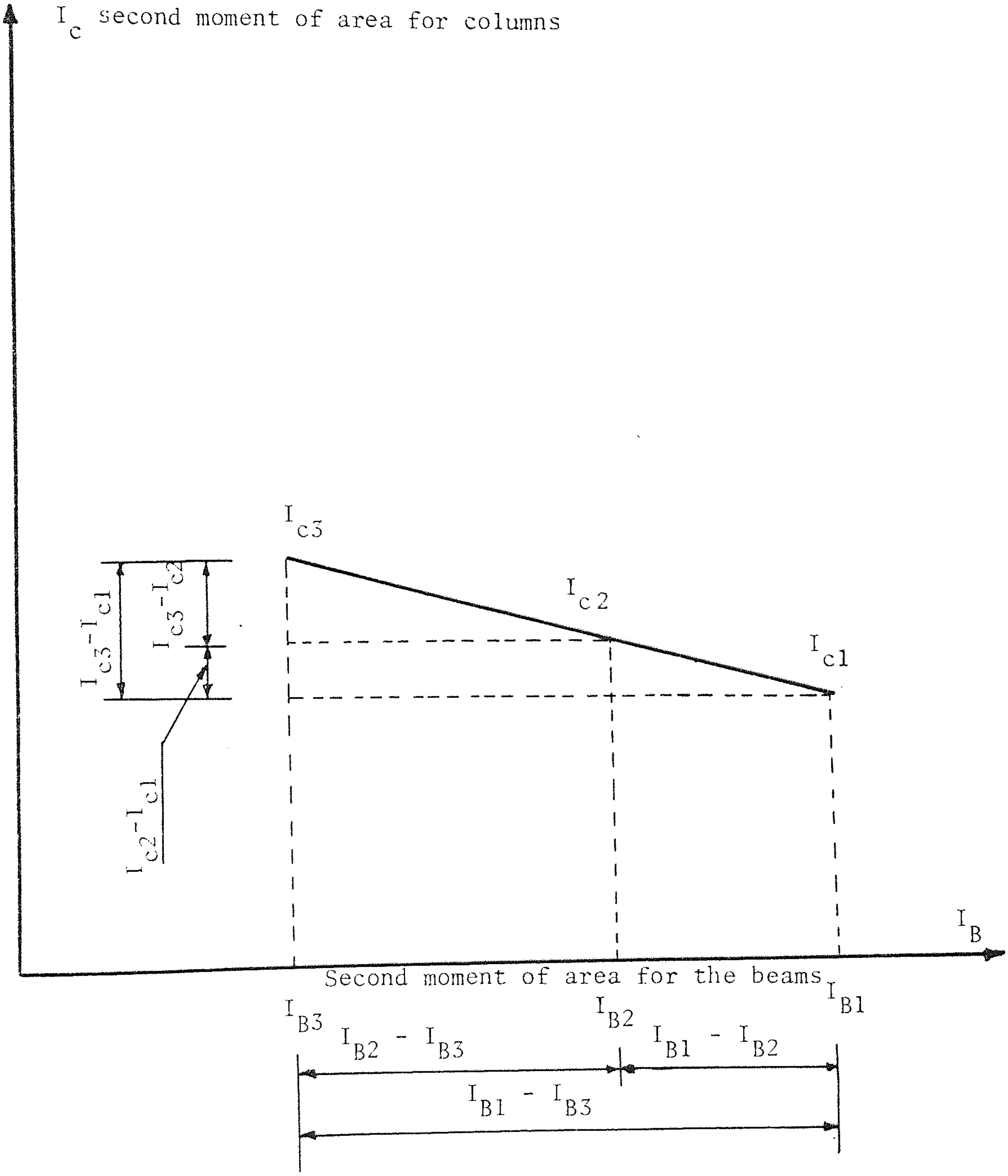
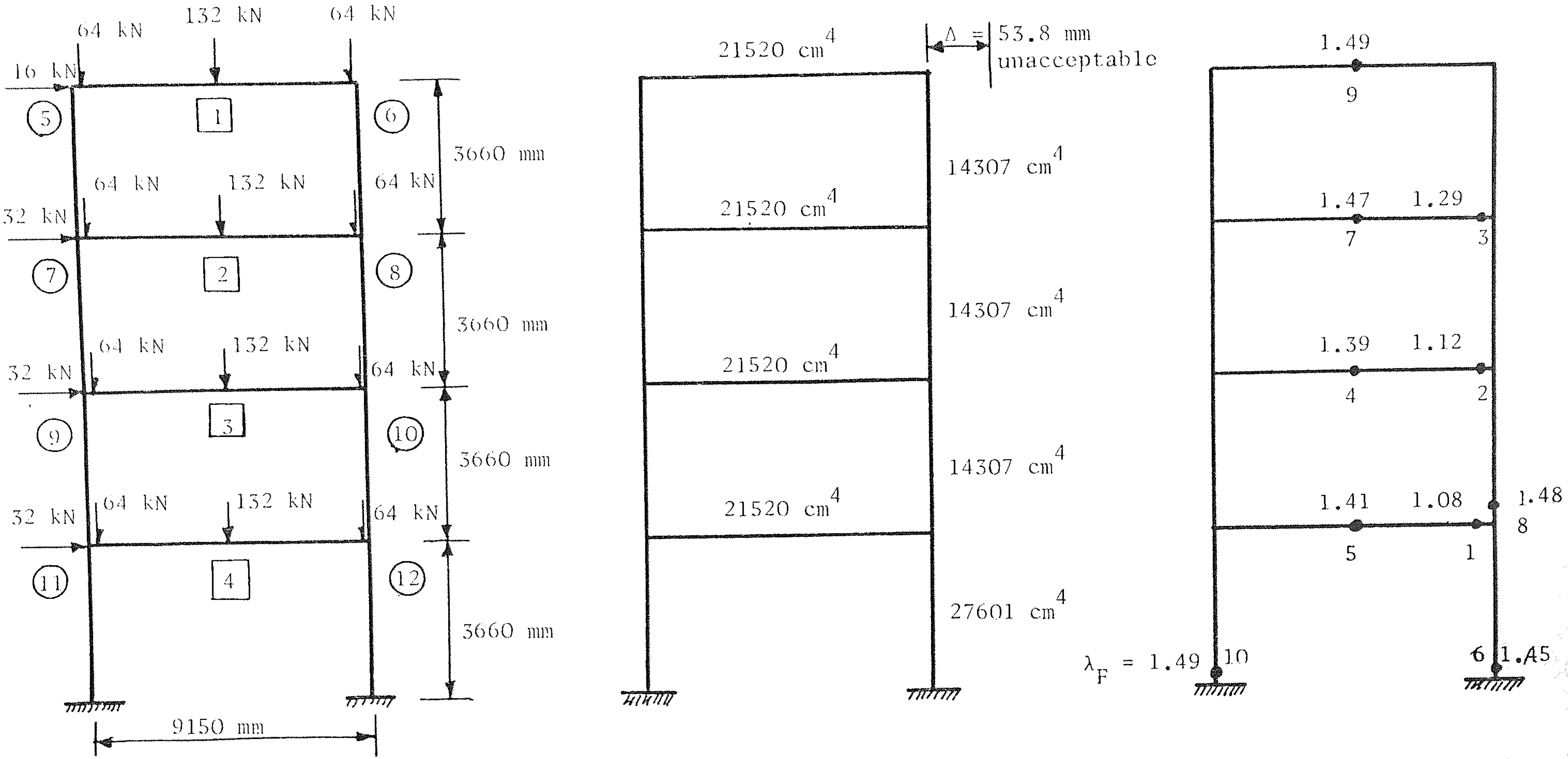


FIGURE 2.9 LINEAR EXTRAPOLATION TO FIND THE SECOND MOMENT OF AREA FOR THE COLUMNS



(a) Load and dimensions of the four storey single bay frame
□ Beam joint
○ Column joint

(b) The second moment of area of the sections selected from the list. For elastic-plastic design total mass 5015 kg.

(c) Order of formation of hinges in the elastic-plastic design for combined loading

FIGURE 2.10 THE 4 STOREY SINGLE BAY FRAME

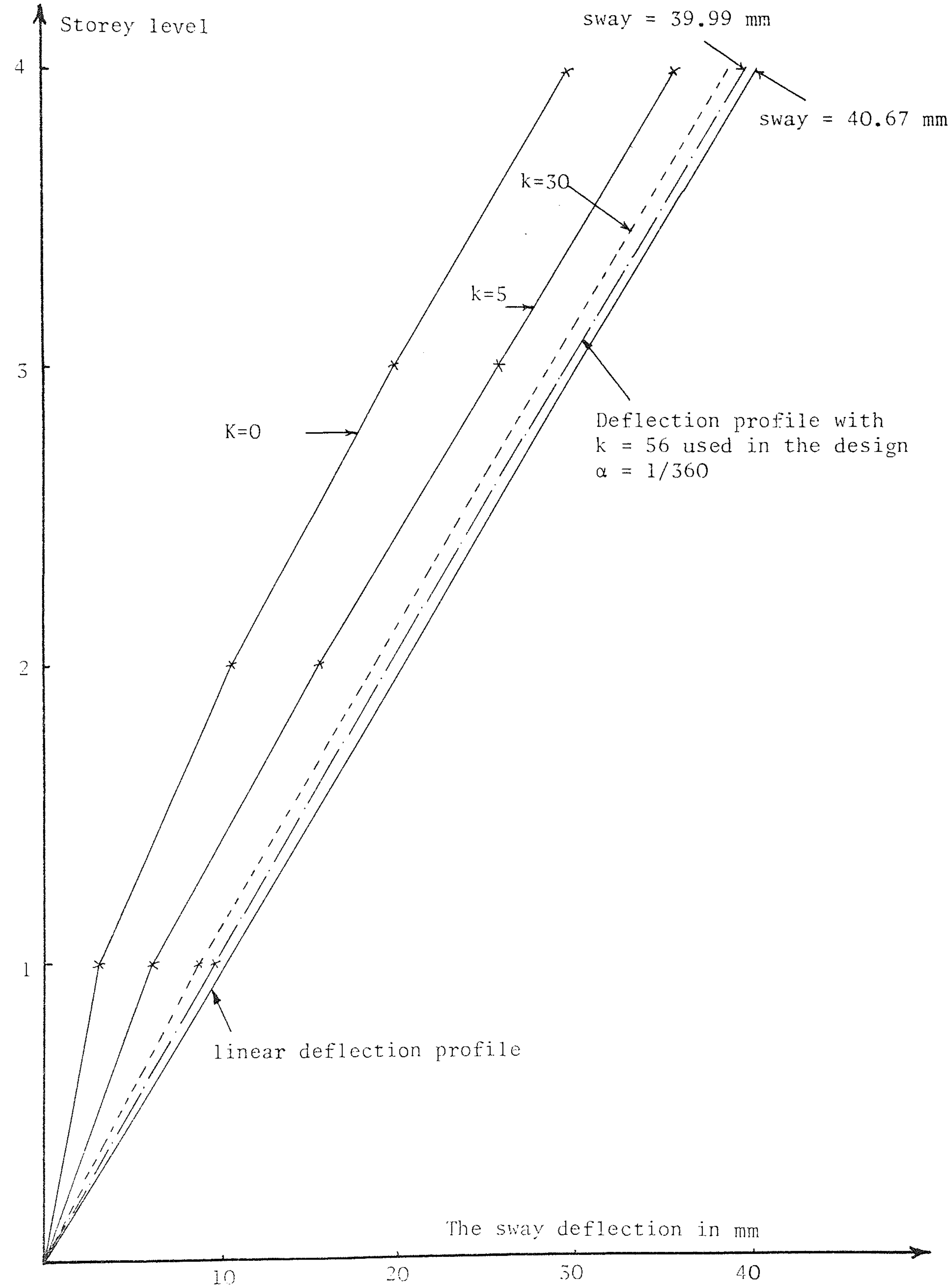
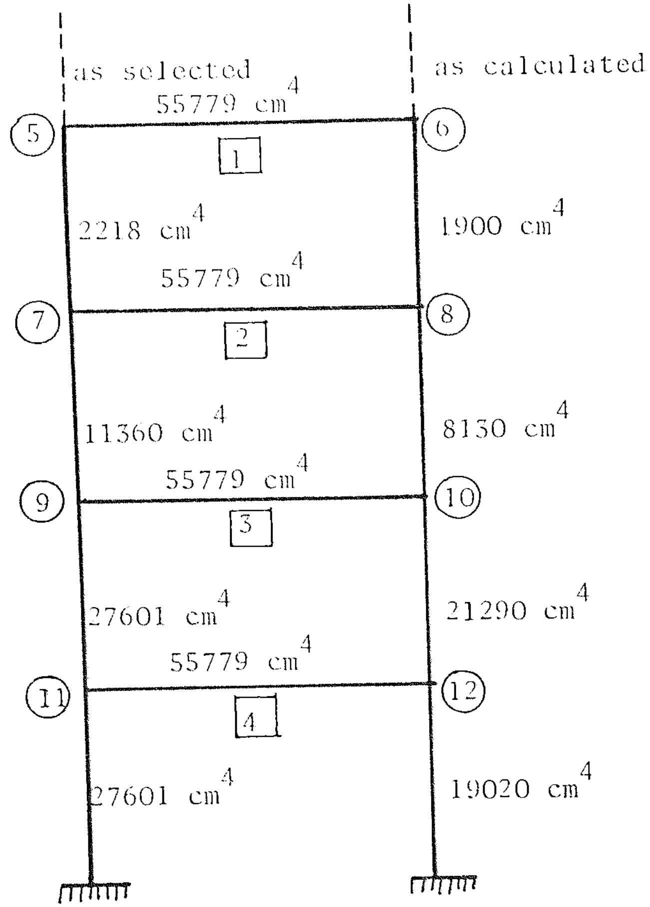
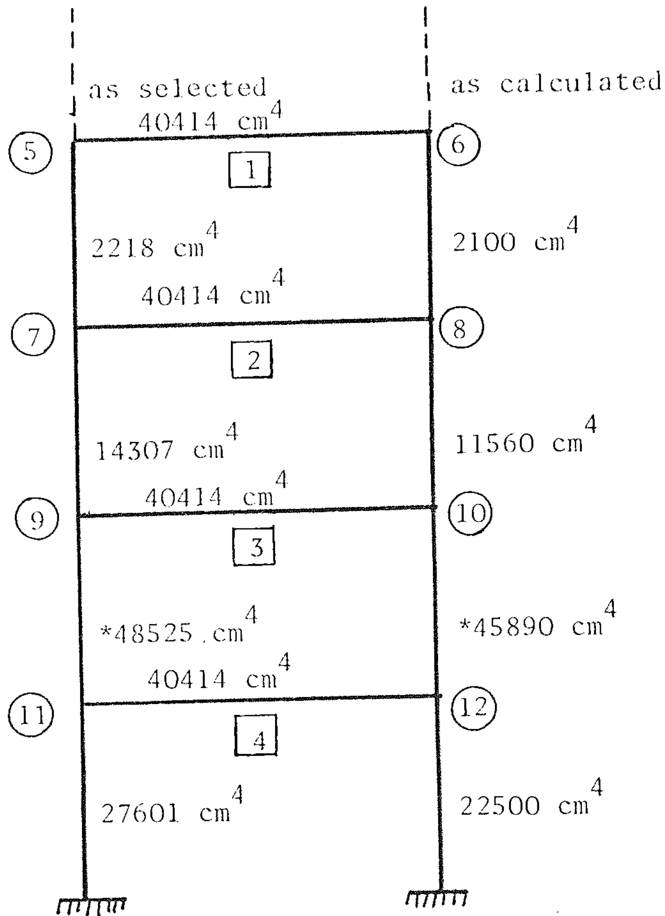


FIGURE 2.11 DEFLECTION CURVES FOR VARIOUS VALUES OF k FOR THE FOUR STOREY SINGLE BAY FRAME



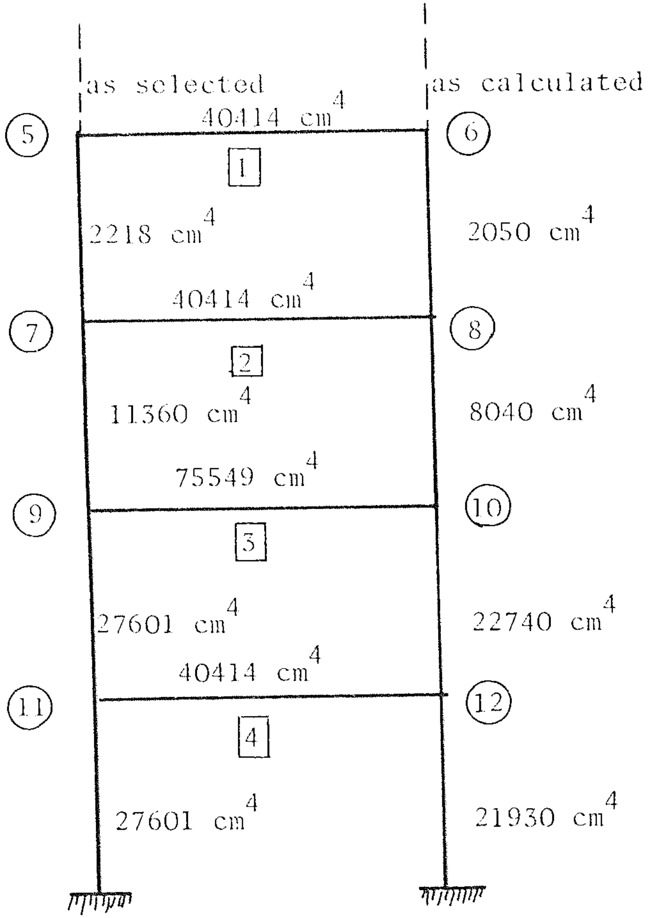
Mass of beams 3001 kg
Mass of columns 2533 kg
Total mass 5534 kg

FIGURE 2.12: FIRST DESIGN CYCLE



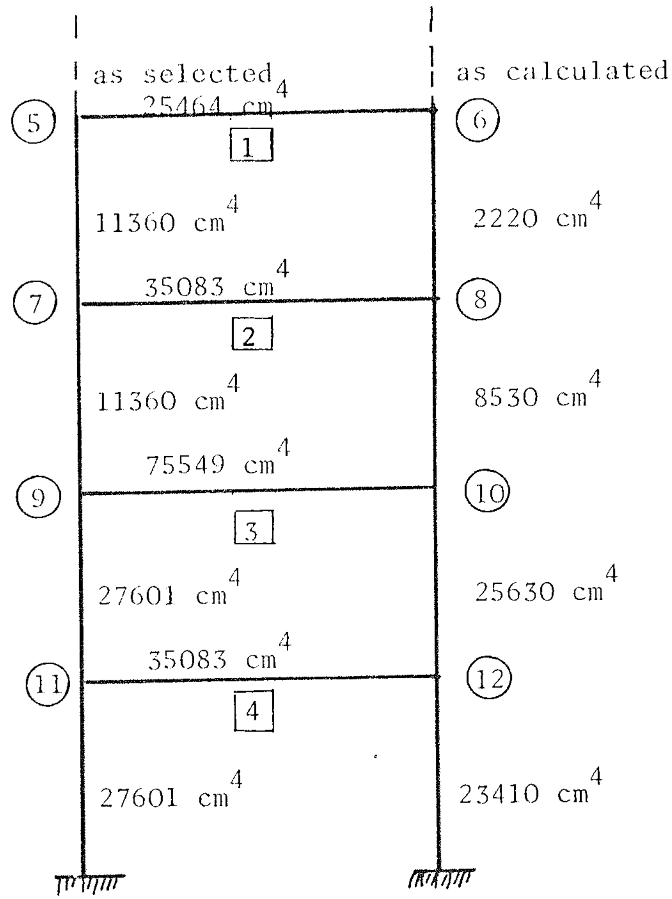
* Unacceptable
Mass of beams 2672 kg
Mass of columns 2906 kg
Total mass 5578 kg

FIGURE 2.13 SECOND DESIGN CYCLE



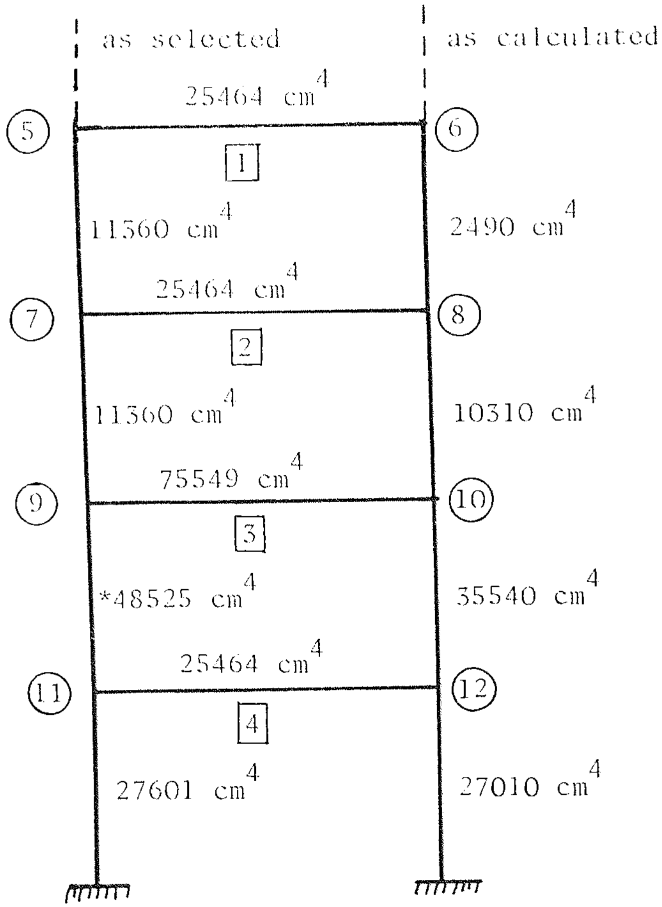
Mass of beams 2928 kg
Mass of columns 2533 kg
Total mass 5461 kg

FIGURE 2.14: MODIFIED SECOND DESIGN CYCLE



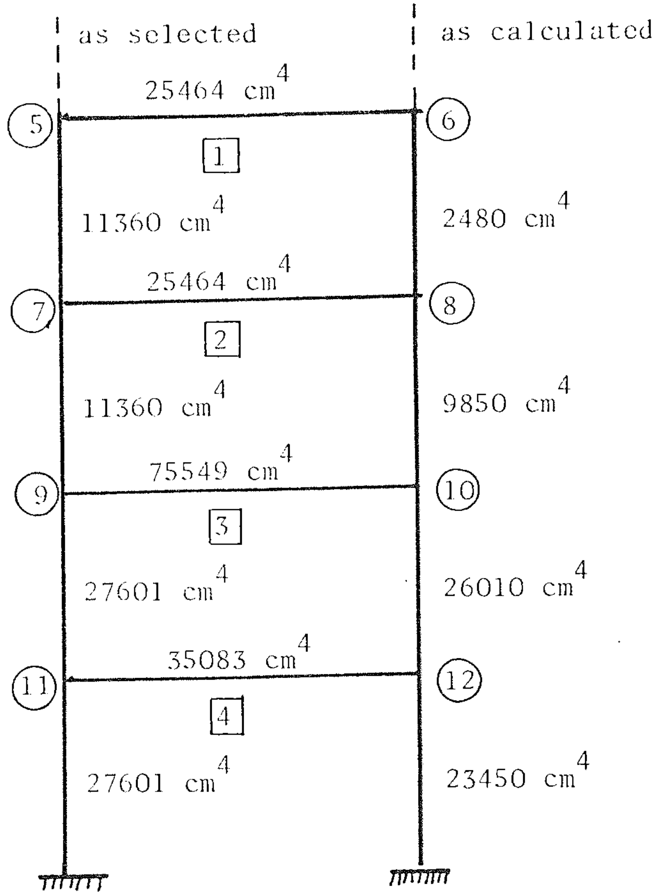
Mass of beams 2681 kg
Mass of columns 2796 kg
Total mass 5477 kg

FIGURE 2.15: THIRD DESIGN CYCLE



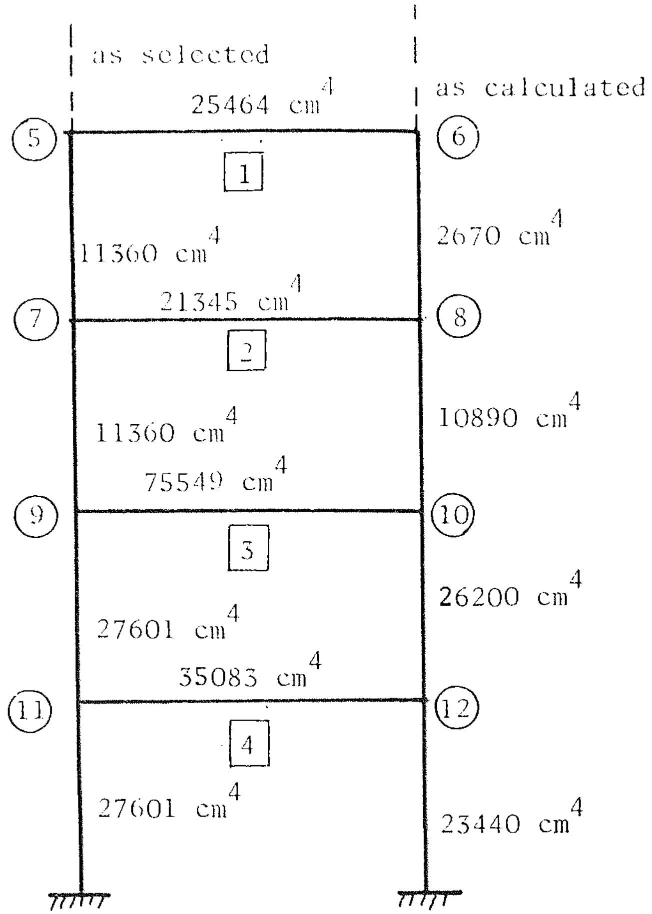
* Unacceptable

FIGURE 2.16: RESULT OF THE REDUCTION OF THE THIRD CYCLE BEAMS



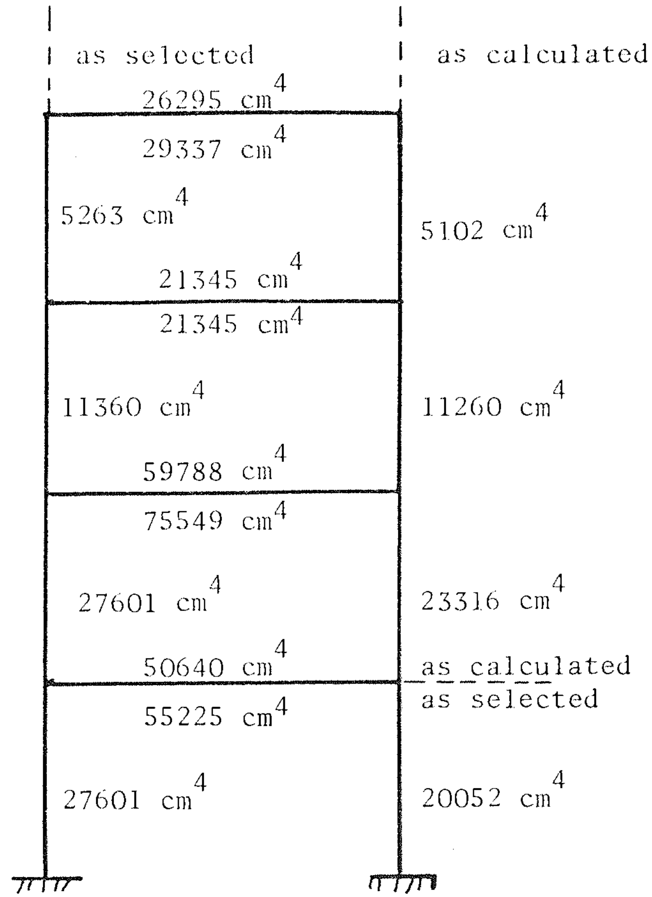
Mass of beams 262 kg
Mass of columns 2796 kg
Total mass 5422 kg

FIGURE 2.17: FOURTH DESIGN CYCLE



$\Sigma IL = 1.9032 \times 10^{13}$ mm^5 (using I as calculated)
Mass of beams 2552.8 kg
Mass of columns 2796.2 kg
Total mass 5349.0 kg

FIGURE 2.18: FINAL DESIGN



$\Sigma IL = 1.8835 \times 10^{13}$ mm^5 (using I as calculated)
Mass of beams 2854.8 kg
Mass of columns 2642.5 kg
Total mass 5497.3 kg

FIGURE 2.19: OPTIMUM DESIGN OF THE FRAME

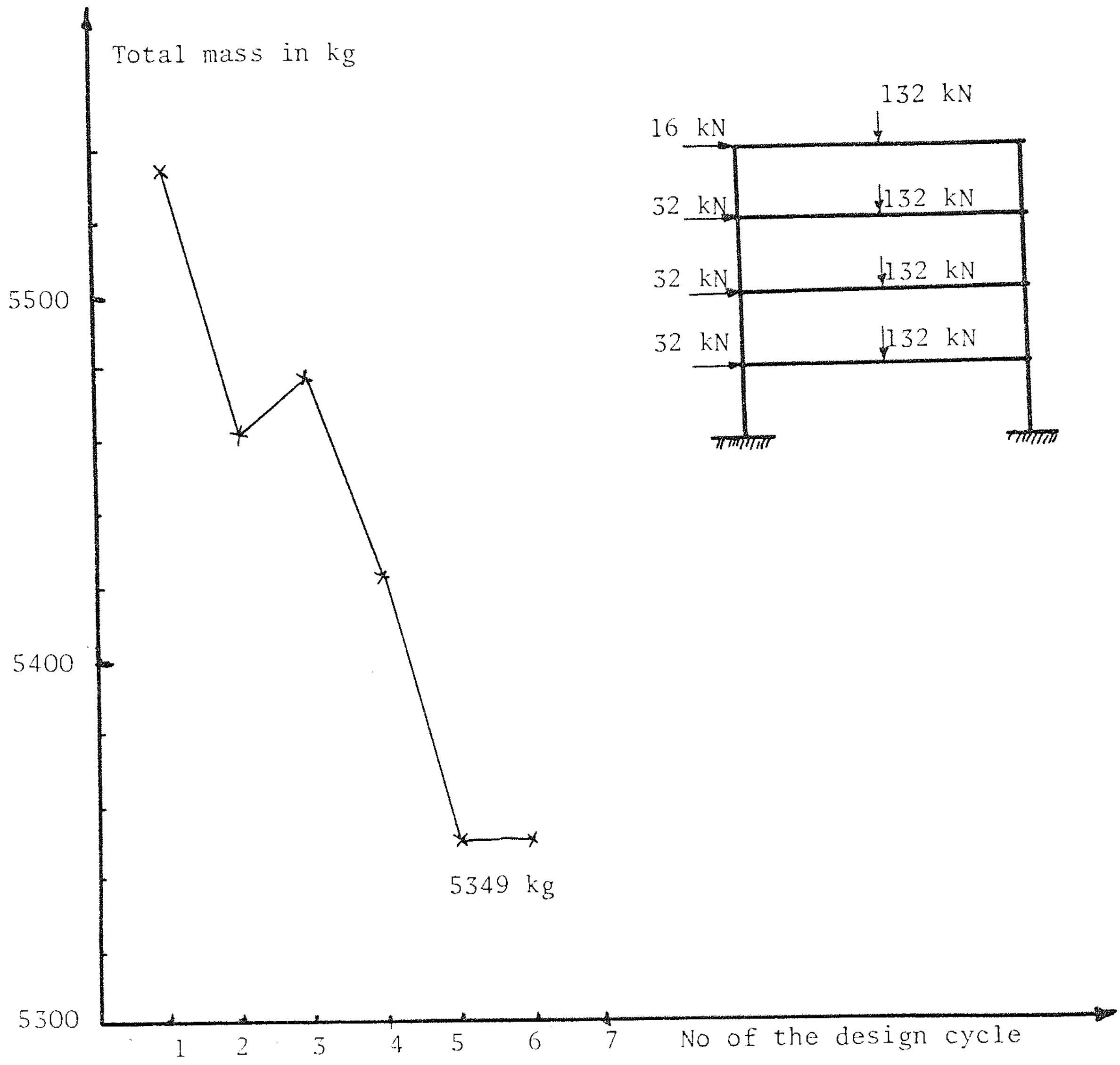


FIGURE 2.20: TOTAL MASS FOR EACH DESIGN CYCLE FOR THE FOUR-STOREY SINGLE BAY FRAME

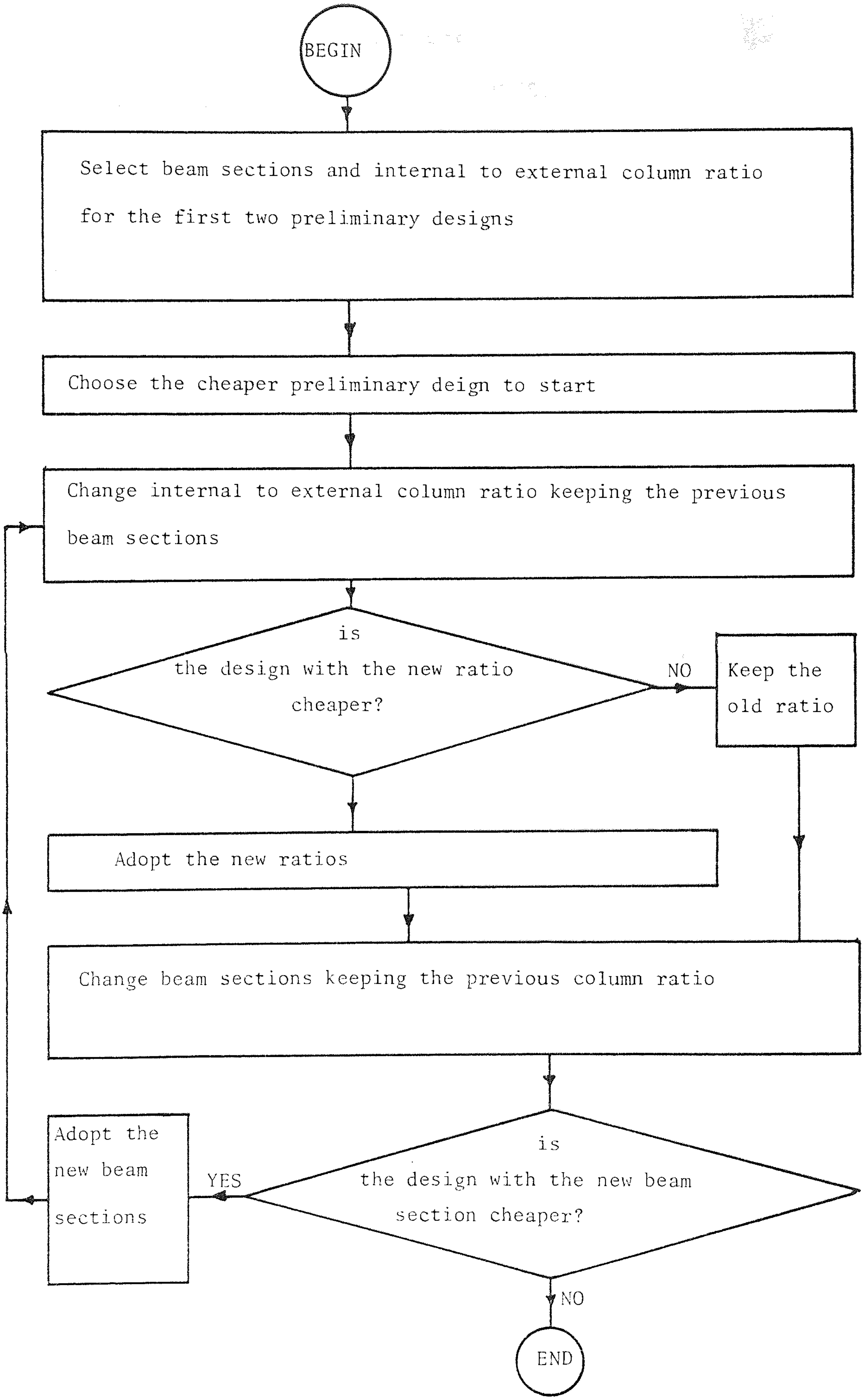


FIGURE 2.21 THE PROCEDURE FOR ECONOMY

CHAPTER THREE

REFINING THE DESIGN PROCESS

3.1 REDUCTION IN THE BENDING STIFFNESS OF THE COLUMNS

In the presence of high member axial forces the bending stiffness of a column is reduced by including the "stability functions" in the stiffness equations of the column. If Livesley's ϕ stability functions are used, equations (2.5) are altered to become:

$$\begin{array}{l} \\ \text{first end} \\ \\ \underline{L} = \\ \\ \text{second end} \\ \\ \end{array} \left[\begin{array}{ccc|ccc} b\phi_5 & 0 & -d\phi_2 & -b\phi_5 & 0 & -d\phi_2 \\ 0 & a & 0 & 0 & -a & 0 \\ -d\phi_2 & 0 & e\phi_3 & d\phi_2 & 0 & f\phi_4 \\ \hline -b\phi_5 & 0 & d\phi_2 & b\phi_5 & 0 & d\phi_2 \\ 0 & -a & 0 & 0 & a & 0 \\ -d\phi_2 & 0 & f\phi_4 & d\phi_2 & 0 & e\phi_3 \end{array}\right] \left[\begin{array}{c} x_i \\ y_i \\ r_i \\ \hline x_j \\ y_j \\ r_j \end{array}\right] \tag{3.1}$$

Accordingly, the modified stiffness equation for a joint in a column (given by equations 2.20, 2.23, 2.26, 2.29, 2.31 and 2.38) are altered. For example, equation (2.20) becomes:

$$\begin{aligned} x_j = (H_j &+ d_{cj}\,\phi_{2cj}\,r_j + b_{cj}\,\phi_{5cj}\,x_n + \\ & d_{cj}\,\phi_{2cj}\,r_n + b_{cm}\,\phi_{5cm}\,x_m - d_{cm}\,\phi_{2cm}\,r_m \\ & -d_{cm}\,\phi_{2cm}\,r_j)/(b_{cj}\,\phi_{5cj} + b_{cm}\,\phi_{5cm}) \end{aligned} \tag{3.2}$$

where j and m refer to member numbers.

Obviously, the values of the ϕ functions are dependent on the second moment of area of the column but this is not known before the iteration starts. Furthermore, during the first design cycle the values of I for the column change rapidly as the iteration is started with unacceptably small values of I. For these reasons, and since the first design cycle is merely a procedure to obtain approximate sections, the effects of the axial forces are neglected in this cycle. Thus $\phi_1 = \phi_2 = \phi_3 = \phi_4 = \phi_5 = 1$.

At the beginning of a subsequent design cycle an approximate value of I is available for each column. This is used for calculating the ϕ function in the first iteration. At the end of each iteration, a new I is obtained for each column which is then used in the next iteration to calculate a new and revised set of stability functions.

3.2 CALCULATION OF THE AXIAL FORCES

The values of the stability functions are dependent on the axial force in the column which is also not known before the iteration starts. However, to assess the axial forces it is found that it is sufficiently accurate to proportion the vertical loads between the columns so that the external columns equally carry half the load on the external bays above. On the other hand, internal columns proportionally share the rest. The use of this approximation in assessing the axial forces instead of calculating the exact ones simplify the design procedure and reduce the computer time. Alternatively, it is possible, using a computer, to calculate the axial forces exactly from:

$$\{p\} = \underline{k} \cdot \underline{A} \cdot \{x\} \tag{3.3}$$

where $\{p\}$ is the vector of member forces, $\underline{k}$ is the member stiffness matrix, $\underline{A}$ is the displacement transformation matrix of the member, and {x} is the vector of joint displacements.

To obtain the exact axial forces for each design cycle the iteration must be used first to find the joint displacements. The internal forces are then calculated using equations (3.3). This process must be repeated until the difference between two successive sets of axial loads is smaller than a specific tolerance. Such a calculation is lengthy and it can be shown that the difference between the approximate and the exact values of the axial forces is small. Their effect on the outcome of the design is even smaller.

3.3 EXAMPLE ON THE AXIAL LOAD EFFECT

The five-storey, three unequal-bay, frame shown in Figure 3.1 was designed using the deflection profile shown in the figure. This frame is used here for an extensive investigation into the effect of the axial load in the design of sway frames. Two designs were obtained. The first design procedure initially neglected the effect of axial forces in the columns, until a reduced cost design was obtained. The beams of this design were kept unaltered but the columns were subsequently altered to allow for the effect of axial forces. These are calculated exactly using equations (3.3), with a tolerance of 0.001. Second column in Table 3.1 shows the reduced cost design with the axial forces neglected, while the third column in the table shows the design with the axial load effect included. In both designs the second moments of area after selecting sections from the list are shown. It can be seen that some column sections are increased due to the effect of the application of the axial load. These sections are marked by an asterisk in Table 3.1. Including the axial load effects increases the material cost by 4.09%.

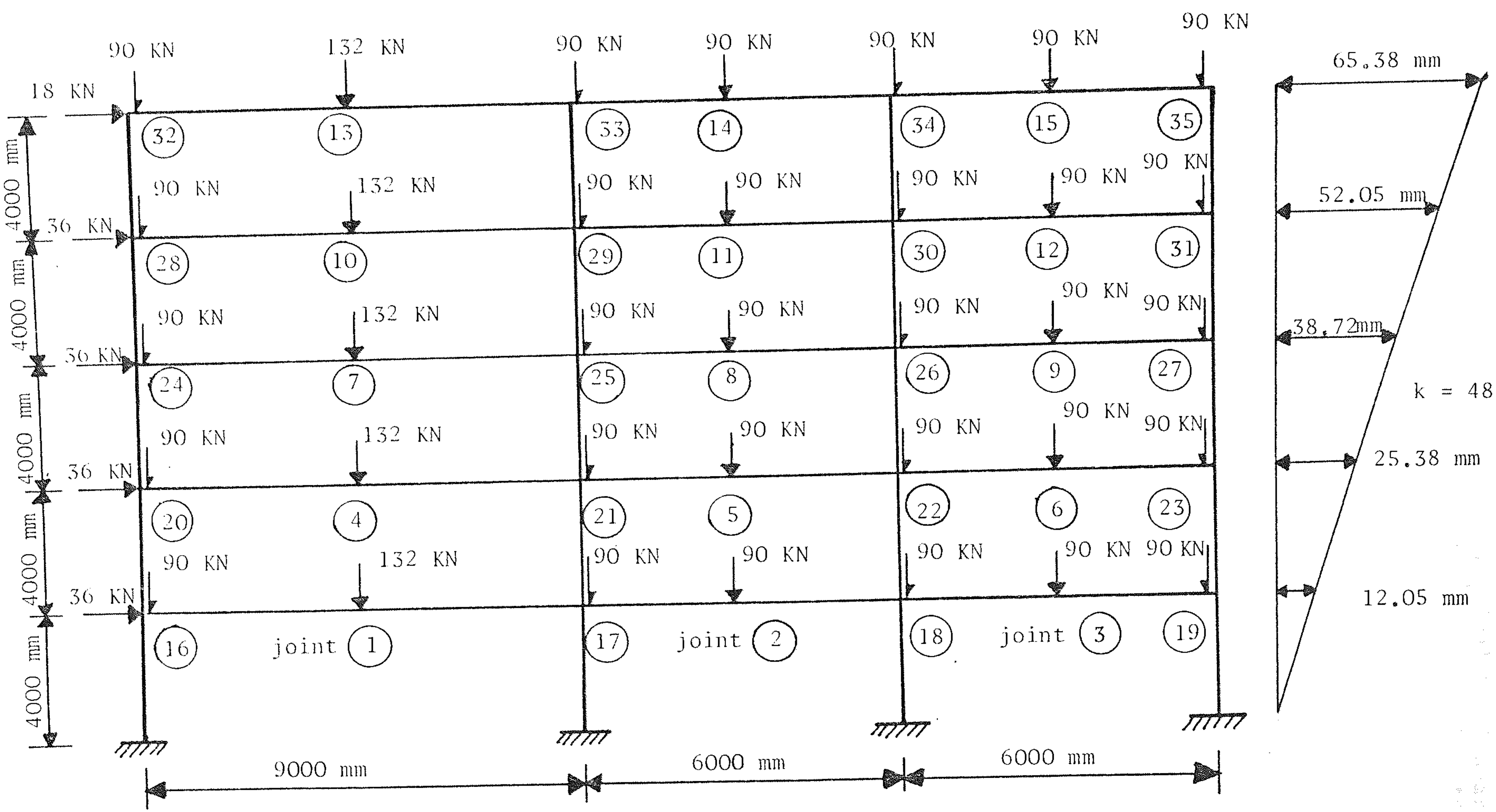


FIGURE 3.1 FIVE-STOREY THREE UNEQUAL-BAY FRAME, LOADS AND DIMENSIONS OF THE FRAME, THE NON-LINEAR SWAY CURVE FOR α = 1/300 IS SHOWN ON THE RIGHT

Member number (taken as joint number	Final linear design axial load effect is neglected	Final Non-linear design. The linear design is modified by considering the exact axial load I in cm^4	Final Non-linear design. The effect of the approximate axial load is considered from first design cycle I in cm^4
1 - 3	47363	47363	33324
4 - 6	47363	47363	47363
7 - 9	21345	21345	29337
10 - 12	21345	21345	18576
13 - 15	18576	18576	18576
16	11360	11360	11360
17	14307	17510*	17510
18	11360	14307*	14307
19	6088	7647*	7647
20	11360	11360	11360
21	14307	17510*	17510
22	11360	14307*	14307
23	6088	6088	7647
24	5263	6088*	7647
25	11360	14307*	11360
26	5263	6088*	11360
27	5263	6088*	5263
28	5263	6088*	5263
29	7647	7647	7647
30	5263	6088*	6088
31	5263	5263	5263
32 - 35	5263	5263	5263
Total cost of material in £	£2485.95	£2592.03	£2591.13
Total mass in kg	11778 kg	12286 kg	12519 kg

* marks sections change due to the effect of exact axial force

TABLE 3.1: THE DESIGNS OF THE FIVE STOREY FRAME

In the second design procedure the axial load effect is introduced from second design cycle as described in section 3.2. The axial forces for this design are calculated approximately as explained in section 3.3. The design obtained in this manner is shown in column four of Table 3.1. Although this design is different from that obtained by considering the exact axial force, the cost of the material for both designs is almost exactly the same. However the total mass is different by 1.9%. This shows that a consideration of a cost reduction in the design procedure is much more realistic than a consideration of a weight reduction.

When considering the design with approximate axial forces, column 25 and 27 are found to have smaller sections than those obtained when the axial forces are calculated exactly. On the other hand beams 7, 8 and 9 have larger sections, see Table 3.1.

From this example one could conclude the following two points

1 - Taking axial load effect into consideration increases the initially selected column sections

2 - Calculation of the axial loads approximately, simplifies the design procedure but produces feasible designs which are not much different from those obtained using an exact procedure.

3.4 ECONOMY IN COMPUTER TIME

Normally between five and twelve design cycles are needed to obtain the final set of sections for a frame. Although the iteration process in each cycle converges quickly it is possible to reduce the computer time in two different ways. These are:

(a) Once the iteration in a design cycle is complete, the joint deflections, the axial forces and the second moment of area of each column becomes known. Before entering the next design cycle, a set of beam sections are selected in an effort to reduce cost. This set is only slightly different from the one used in the previous cycle as each beam is only up or down graded by one section. Such a change does not alter the member axial forces and joint deflections drastically. Rather than starting the iteration, in the new design cycle, from unrealistic initial values for all the variables in the modified stiffness equations, it was decided to use the old values of these variables as given in the previous design cycle. Starting the iteration using these realistic initial values, speeded up the subsequent design cycles considerably.

For example, eight design cycles were needed to obtain the final reduced cost design of Table 3.1, column four. The number of iterations for each trial is shown in Figure 3.2, where it can be seen that the number of iterations in each cycle is reduced drastically after the second design trial. The reason is that, the results of one design cycle are used as the starting point for the next design. The large number of iterations in the second design trial is due to the introduction of the stability functions at this stage. The total number of iterations for the last six trials is less than that of the first or the second trial.

(b) As an iterative technique is being used for solving the modified stiffness equation, the tolerance control of this technique has a vital influence on the speed of convergence. A small tolerance increases the number of iterations and results in a considerable increase in computer time. This is not only unnecessary but also can consume all the available computer time before convergence is achieved. A large number of tests showed that a tolerance of 0.01 was adequate for relatively small frames (up to twelve storeys), and also for medium size frames (up to twenty-four storeys) which

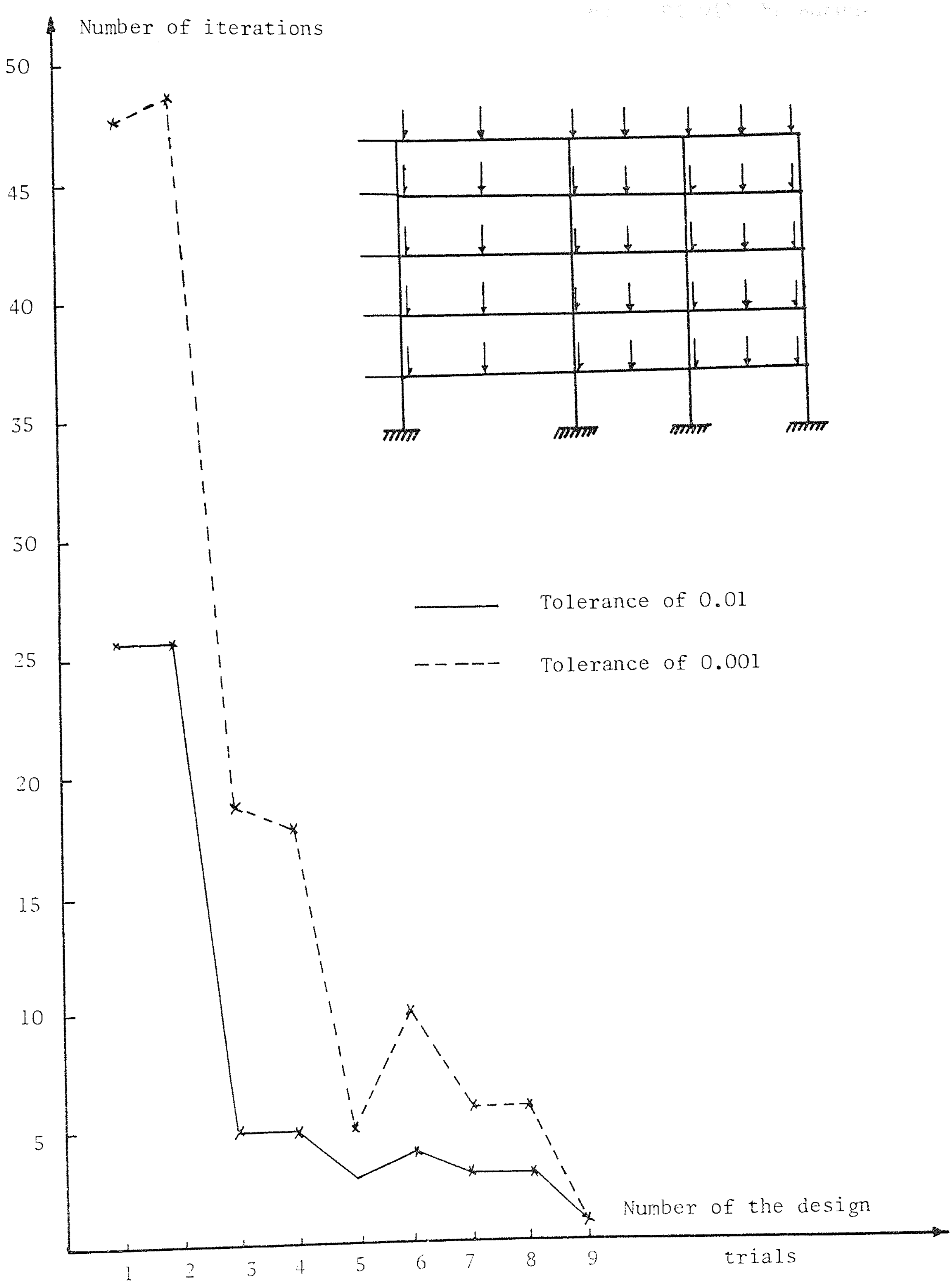


FIGURE 3.2 NUMBER OF ITERATIONS FOR EACH DESIGN TRIAL FOR THE FIVE-STOREY THREE-UNEQUAL BAY FRAME

had equal or nearly equal bays. For medium irregular frames, with unequal bays, it was found that a tolerance of 0.001 will be adequate. This is also sufficient for larger frames. To demonstrate the effect of the tolerance on the design outcome, the five-storey three unequal bay frame of Figure 3.1 was first designed with a tolerance of 0.001. The results of this design are shown in Table 3.1, column four. This same frame was then designed using a tolerance of 0.01 and resulted in the same final sections. To compare the displacements of the joint, this design was analysed by an existing computer program which solves the stiffness equations explicitly (Celik, 1977). Comparison between sway deflections is not possible because these are constants in the design whatever the tolerance. For this reason the vertical displacements are compared . For joints 1, 2 and 3, see Figure 3.1, these are listed in Table 3.2.

Kinds of Solutions	y for joint 1	y for joint 2	y for joint 3
Direct Solution	-12.42 mm	-2.61 mm	-3.78 mm
Iterative Solution with tolerance of 0.001	-12.42 mm	-2.61 mm	-3.78 mm
Iterative solution with tolerance of 0.01	-12.40 mm	-2.64 mm	-3.79 mm

TABLE 3.2 VERTICAL DISPLACEMENTS OF JOINTS 1, 2 AND 3 OF FIGUTE 3.1

It can be seen from the table that the direct solution of the stiffness equations leads to the same answer as that obtained by the iterative technique with a tolerance of 0.001. For the tolerance of 0.01, the displacements are nearly the same, and the difference does not exceed 0.05 mm.

The computer time needed, with a tolerance of 0.01, using Aston Computer (ICL 1904s) to complete the eight design trials was only 26 seconds. This included the time to check the strength of the final design. The efficiency of the design method is realised when it is noticed that a single explicit non-linear analysis of this frame, using a compact storage scheme, needed as much as 22 seconds. The dotted graph in Figure 3.2 shows the number of iterations for each trial with a tolerance of 0.001. As expected, the number of iterations for each trial increased, and the computer time increased to 46 seconds. However, the final reduced cost design is the same. Thus, in this example, no benefit is gained using a small tolerance. Even then the computer cost of a complete design is only twice as much as an explicit analysis solving $\underline{L} = \underline{K}\ \underline{X}$ directly.

3.5 ECONOMIC USE OF COMPUTER STORAGE

As stated in Chapter 2, sections 2.4, 2.4.1 and 2.4.2 the modified stiffness equations are formulated and solved iteratively one joint at a time. Thus no matter how large the frame is, the storage requirements is that needed by a single joint. The computer program written for the design method does not therefore require the use of any backing store as is often the case in the analysis of large frames. Five one-dimensional arrays are needed in the core store. These keep the current values of areas and the second moments of area of the sections, the current ratio of internal to external columns, the current rotations of the joints, and their vertical displacements. A further, five similar arrays are also needed to store the previous values of these variables. Thus the storage requirement of the design method is very small indeed. For a total of n joints in a frame, the iteration process only required 10 n locations. For instance, the five storey, three bay frame shown in Figure 3.1 used only 8050 words (approximately 8 k). This included storing the computer program itself.

3.6 THE DESIGN CRITERIA

Two categories of design criteria are adopted. The first category includes the deflection requirements, while the second is concerned with the strength requirements. For deflection requirements, the existing BS 449 does not recommend any specific value as the limit for the horizontal sway in the columns of multi-storey frames. On the other hand the B/20 Draft (Document 77/13908DC, 1977) which will replace BS 449 recommends that:

> "The sway of the column should not exceed the limit of the height of the storey/300".

This design criterion dominates the design of the sway frame when the wind load is high. In reality the wind load is calculated according to CP3, Chapter five (British Standard 1972). If the frame is subjected to such a wind load, and is made from grade 50 steel, then the horizontal sway in the columns usually governs its design. Frames made from grade 43 steel are also dominated by the column sway in many cases. Thus one criterion is

i) The sway of the storey does not exceed its height/300.

The second category of the design criteria caters for strength requirements. For this B/20 Draft Specification in Chapter 12, clause 12.5.4.1 recommends the following:

"In a multi-storey rigid jointed frame which is not effectively braced against sidesway in its own plane, plastic design shall be used only when proper allowance is made for frame-instability effects. This may be done either by carrying out a full elastic-plastic sway analysis using methods which make proper allowance for frame instability or alternatively, in appropriate cases by the simple method specified in 12.5.4.3".

The method of Majid (1972), (Majid and Anderson 1969), and (Anderson, 1969) is one of these methods which makes proper allowance for frame instability. The overall instability of the frame due to gradual deterioration of its stiffness is fully considered. For this reason, most of their design criteria are adopted; these ensure that the frame has adequate load factors against collapse under various loading cases. The aim is to ensure that the two following conditions are satisfied.

(ii) Under combined vertical and wind load from either side, the frame does not collapse until the load factor exceeds λ_1 (taken as 1.29 in this thesis).

(iii) Under dead load and vertical load the frame does not collapse until the load factor exceeds λ_2 (taken as 1.75).

Analysis of a large number of frames by Majid (1972), (Majid and Anderson 1969) and (Anderson, 1969) shows that if certain restrictions are made on the formation of the plastic hinges, then in most cases the above two conditions are not violated. These restrictions are:

(iv) No plastic hinge should develop in a beam below the load factor of unity and the frame should be entirely elastic under the specified unfactored loads.

(v) No plastic hinge shall develop in a column until the load factor exceeds a permissible value λ_1 under combined loading or λ_2 under vertical loading.

The condition (iii) and the restrictions on the formation of the plastic hinges (iv) and (v) are used as design criteria for this work. Condition (ii) is excluded because it is necessary to perform an elastic-plastic analysis, which follows the successive development of plastic hinges. This kind of analysis needs considerable computer storage and time. However, the restriction on the formation of plastic hinges is a safeguard for this criterion. Furthermore, the instability of the frame increases rapidly with the increase of the sway of the frame. For this reason the restriction on sway imposed by criterion (i) will provide the proper allowance for frame instability proposed in Clauses 12.5.4.1 of B.20.

To ensure that the above criteria are all satisfactory, the final design is analysed at a load factor λ_1 = 1.29 under combined loading and taking the effect of axial loads into consideration. The maximum bending moment in each column obtained by this analysis, is compared with the reduced plastic hinge moment of the member cross-section. If the section is unsatisfactory, a new satisfactory section is selected for that member.

Table 5.2.1 of B/20 specifies the factors λ_1 as 1.2 and λ_2 as 1.6. This is equivalent to λ_1 = 1.29 and λ_2 = 1.75 used in this thesis if the minimum yield stress is used instead of the design strength. B/20 clause 5.7.1 specifies the design strength as follows:

> "p_y may be taken as $0.93Y_s$ .... where Y_s is the minimum yield stress specified in BS 4360".

In this design method the minimum yield stress is used as the design strength to account for the increased in the design strength up to the yield stress. The values of λ_1 and λ_2 are therefore taken as 1.29 and 1.75 respectively. The reason for taking these values is that, if M_p is the total plastic moment of the section, and Z_p is the plastic modulus, the design strength is given by $P_y = 1.2\ M_p/Z_p$ or alternatively $0.93\ y_s = 1.2\ M_p/Z_p$. This gives $y_s = 1.29\ M_p/Z_p$. For similar reasons a load factor of 1.75, taken in this thesis, is equivalent to 1.6 specified in Table 5.2.1 of B/20.

It should be noted that the design program has been written in general terms, and the permissible load factors may therefore be assigned any values. However, for the examples in this thesis, the values given above are used.

Notes:

(1) Sections must be selected for columns so that reverse column taper does not occur. If it does, then the beam section of the storey is increased to the next larger size until the same column section could be used for two successive storeys. It should be noted that a redesign is needed for each increase in a beam section.

(2) Certain sections which are not suitable for plastic actions are omitted from the sections list. This is because failure due to local buckling for these sections might occur before they reach their full plastic moments.

(3) This thesis only deals with plane rigidly jointed frames and assumes that satisfactory restraints are available to prevent buckling out of the plane of the frame.

3.7 STRENGTH REQUIREMENTS

Once a set of sections are selected which satisfy the sway requirements, the member forces are checked and the task becomes that of satisfying the strength requirements if these are violated.

3.7.1 Forces in a Member

The axial force, shear force and bending moment of a member are obtained from the equation

$$\{p\} = \underline{k}\ \underline{A}\ \{x\} \tag{3.4}$$

where $\{p\}$, $\underline{k}\ \underline{A}$ and $\{x\}$ are as defined before in Section 3.2. For a member linking joint i and j the member forces are calculated from:

$$\begin{bmatrix} P \\ S \\ M_1 \\ M_2 \end{bmatrix} = \begin{array}{c} \quad\text{first end} \qquad\qquad\qquad \text{second end} \\ \left[\begin{array}{ccc:ccc} -al_p & -am_p & 0 & al_p & am_p & 0 \\ b\phi_5 m_p & -b\phi_5 l_p & d\phi_2 & -b\phi_5 m_p & b\phi_5 l_p & d\phi_2 \\ d\phi_2 m_p & -d\phi_2 l_p & e\phi_3 & -d\phi_2 m_p & d\phi_2 l_p & f\phi_4 \\ d\phi_2 m_p & -d\phi_2 l_p & f\phi_4 & -d\phi_2 m_p & d\phi_2 l_p & e\phi_3 \end{array}\right] \end{array} \begin{bmatrix} x_i \\ y_i \\ \theta_i \\ \hdashline x_j \\ y_j \\ \theta_j \end{bmatrix} \tag{3.5}$$

Notice that joint i is the first end of the member, and joint j is its second end. For a beam member with axes as shown in Figure 2.1 l_p is equal to 1 and m_p is 0. If the axial stiffness, EA/L, is ignored, then the first row of equation (3.5) should be disregarded. Furthermore, if the axial load effect is not considered, then ϕ_1, ϕ_2, ϕ_3, ϕ_4 and ϕ_5 should be equal to 1. Thus the equation which calculates the member force in a beam is given by:

$$\begin{bmatrix} S \\ M_1 \\ M_2 \end{bmatrix} = \begin{array}{c} \text{First end} \qquad \text{Second end} \\ \left[\begin{array}{cc:cc} -b & d & b & d \\ -d & e & d & f \\ -d & f & d & e \end{array}\right] \end{array} \begin{bmatrix} y_i \\ \theta_i \\ \hdashline y_j \\ \theta_j \end{bmatrix} \tag{3.6}$$

For a column member with axes as shown in Figure 2.1, $l_p = 0$ and $m_p = -1$. Because the axial load is estimated as in Section 3.2, the first row of equations (3.5) is ignored. Hence the column forces are given by:

$$\begin{bmatrix} S \\ M_1 \\ M_2 \end{bmatrix} = \left[\begin{array}{cc|cc} -b\phi_5 & d\phi_2 & b\phi_5 & d\phi_2 \\ -d\phi_2 & e\phi_3 & d\phi_2 & f\phi_4 \\ -d\phi_2 & f\phi_4 & d\phi_2 & e\phi_3 \end{array}\right] \left[\begin{array}{c} x_i \\ \theta_i \\ \hline x_j \\ \theta_j \end{array}\right] \qquad (3.7)$$

The joint displacements x_i, y_i and θ_j at the first end of the member, and x_j, y_j, θ_j at the second end are calculated by the iteration method given earlier. This also gives the second moment of area of the columns. These are obtained under unfactored load. Thus, the shear force and bending moments at both ends of a member are found for working load conditions. To obtain the internal forces under factorized loading, the values of these forces are multiplied by the appropriate factor.

3.7.2 Checking the Beam Sections

It was stated in Chapter 2, section 2.5 that under vertical loading each beam is prevented from collapsing by a beam mechanism at a load factor below λ_2. This restriction on beam sections is applied when selecting the beam section even before considering the deflection requirements.

Later at the end of the design cycles under vertical and wind loading, the bending moments at the ends of a beam and at mid-span are calculated from equation (3.6) under specified loads. For a unit load factor none of these is allowed to exceed the full plastic moment of the section as selected for the reduced cost design. If it is, then a larger section is selected, for the beam. This is done using:

$$Z_p = M_{max}/Y_s$$

where M_{max} is the maximum bending moment in the member, Y_s is the minimum yield stress, and Z_p is the plastic modulus of the section. If we consider the deflection requirements, an increase in the beam section will reduce the column sections. For this reason, if a beam section is changed, the iterative technique is required to be applied again to obtain new column sections. These may turn out to be less than the previous ones. The joint displacements will also change. These new values are used to calculate new bending moments. This process is repeated until no change in beam sections is required.

3.7.3 Checking the Column Section for Strength

At the end of the design cycles, the bending moment at each end of a column is calculated from equation (3.7) at a unit load factor for combined loading. The member forces including the axial forces are then increased by the load factor λ_1 which is taken as 1.29. The factorized maximum bending moment $\lambda_1 M_{max}$ is then compared with the reduced plastic moment of the column. If the reduced plastic moment is less than the $\lambda_1 M_{max}$ in the column, then a larger column section is needed. The reduced plastic modulus of this is given by:

$$Z'_p = \lambda_1 M_{max}/Y_s \tag{3.8}$$

A new universal column is then selected from the tables so that it has a reduced plastic section modulus larger than Z'_p. The iteration process is then repeated but this time the object is to calculate the actual sway in each storey after the column sections are modified. Naturally, because the strength requirements dominate, the sway in the storeys will be less than those permitted. The process of altering the column section and finding the internal forces is repeated until no change in any column section is required.

CHAPTER FOUR

COMPUTER PROGRAM

4.1 INTRODUCTION

This chapter presents a computer program for the design of two dimensional, rectangular, steel, sway-frames. The program makes use of the design procedure described in the previous chaters. It is written in Fortran and runs on the ICL 1905 computer at the University of Aston.

The program consists of a master segment calling a number of subroutines, the functions of which are illustrated by the flow diagram shown in Figures 5.1, 5.2 and 5.3. There are nine subroutines in the program. These are:

I - Subroutine ITERATE: This constructs the modified stiffness equations and solves them.

II - Subroutine REDUCE: This calculates the cost of the material for each storey of a frame. It finds the best set of beams which should be used in the next design cycle. It also decided the ratio for the second moment of area of an iternal column to that for an external column.

III - The Subroutine INTERNAL FORCES: This calculates the bending moments and shear forces in each member of the frame from the joint displacements obtained by subroutine ITERATE.

IV - Subroutine COLUMN-I: This chooses a section for a column, from the universal column sections, after the value of I is calculated by the iteration process.

V - Subroutine BEAM-I: This selects a section for a beam from the universal beam sections.

VI - Subroutine COLUMN-ZP; This chooses a section for a column from the universal column sections, once the plastic modulus of the column is known.

VII - Subroutine BEAM-ZP: This chooses a section for a beam from the universal beam sections, once the plastic modulus of the beam is known.

VIII - Subroutine REDUCED-ZP: Calculates the reduced plastic modulus for a column.

IX - The subroutine STABILITY-F: Calculates the stability functions of a column member.

The flowchart of the master segment shown in Figures 5.1, 5.2 and 5.3 is divided into 16 steps. The number of each step appears in brackets to the right of the chart. These steps are explained in some detail in the next three sections of this chapter.

4.2 INPUT DATA (Steps 1 and 2)

STEP 1: The method of numbering the joints was described in section 2.3. The program is written to expect at least one joint in each beam, whether there is a load applied on the span or not. The data consists of four groups these are:

a - preliminary data - one card

b - Beam-joints data - one card per joint

c - Column-joints data - two cards per joint

d - Frame data - two cards.

An example of the data preparation is given in Appendix A of this thesis.

The data preparation can be tedious especially for large frames, and might involve human errors. For these reasons, a

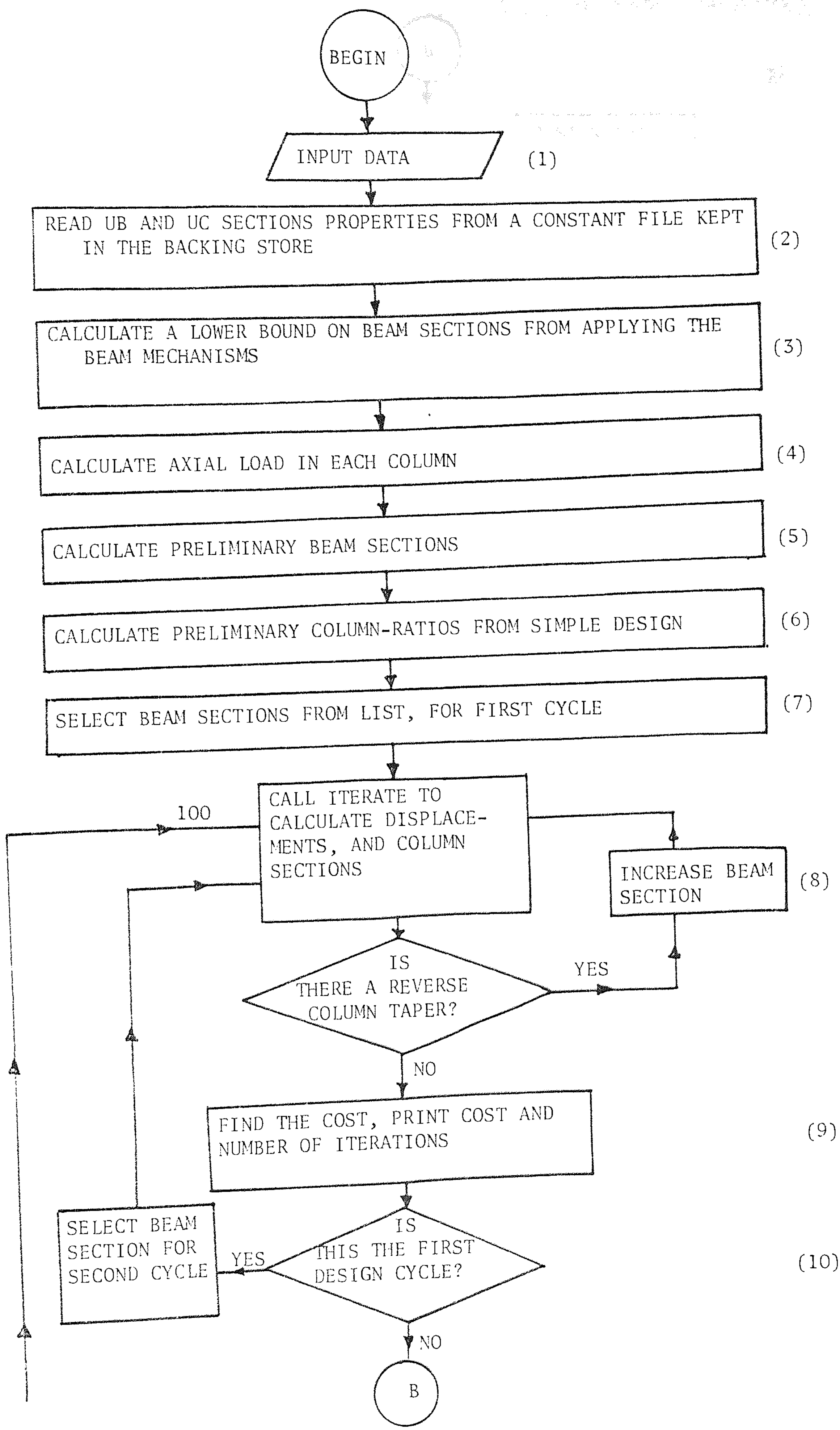


FIGURE 5.1: MASTER PROGRAM

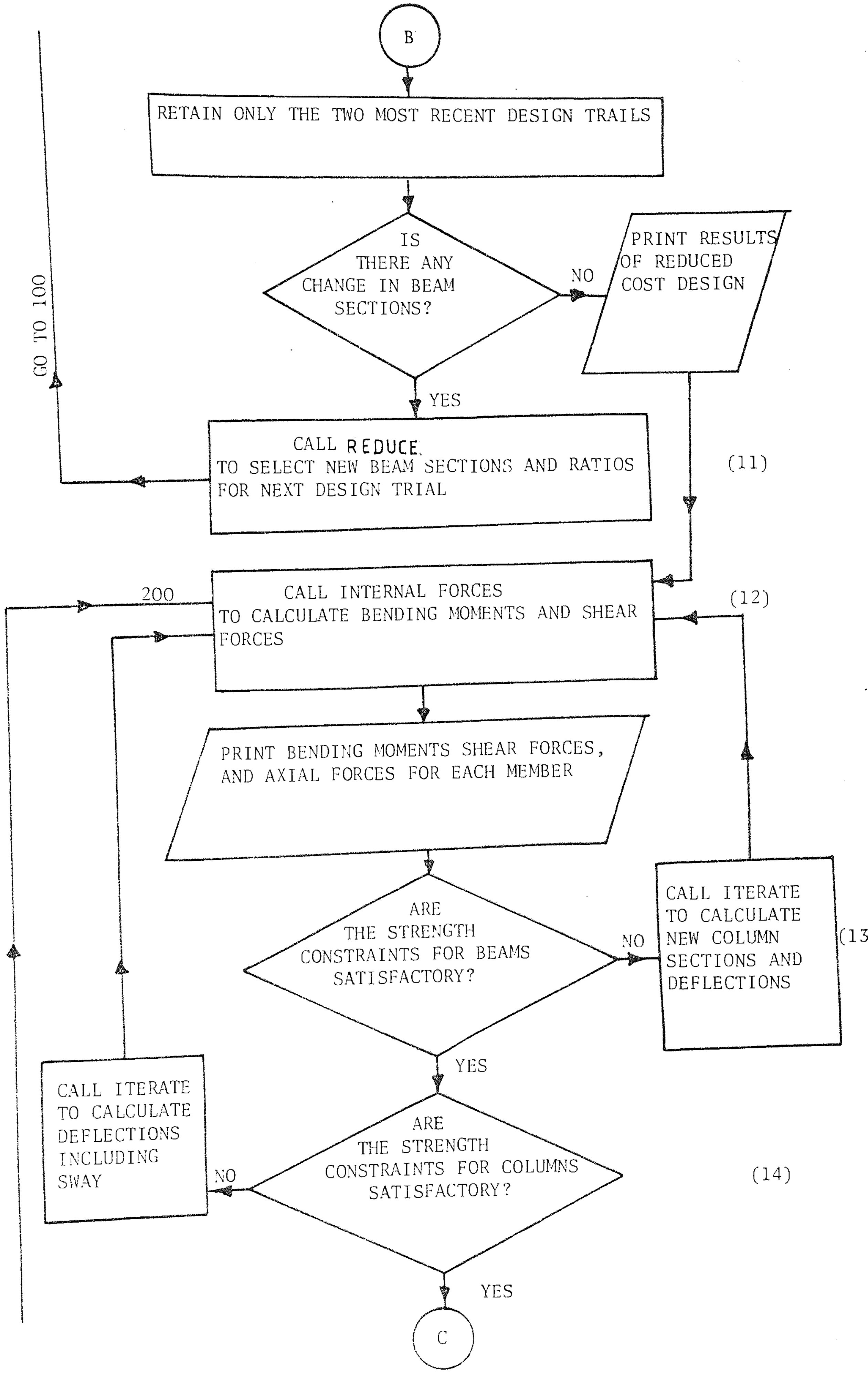


FIGURE 5.2: MASTER PROGRAM (Continued)

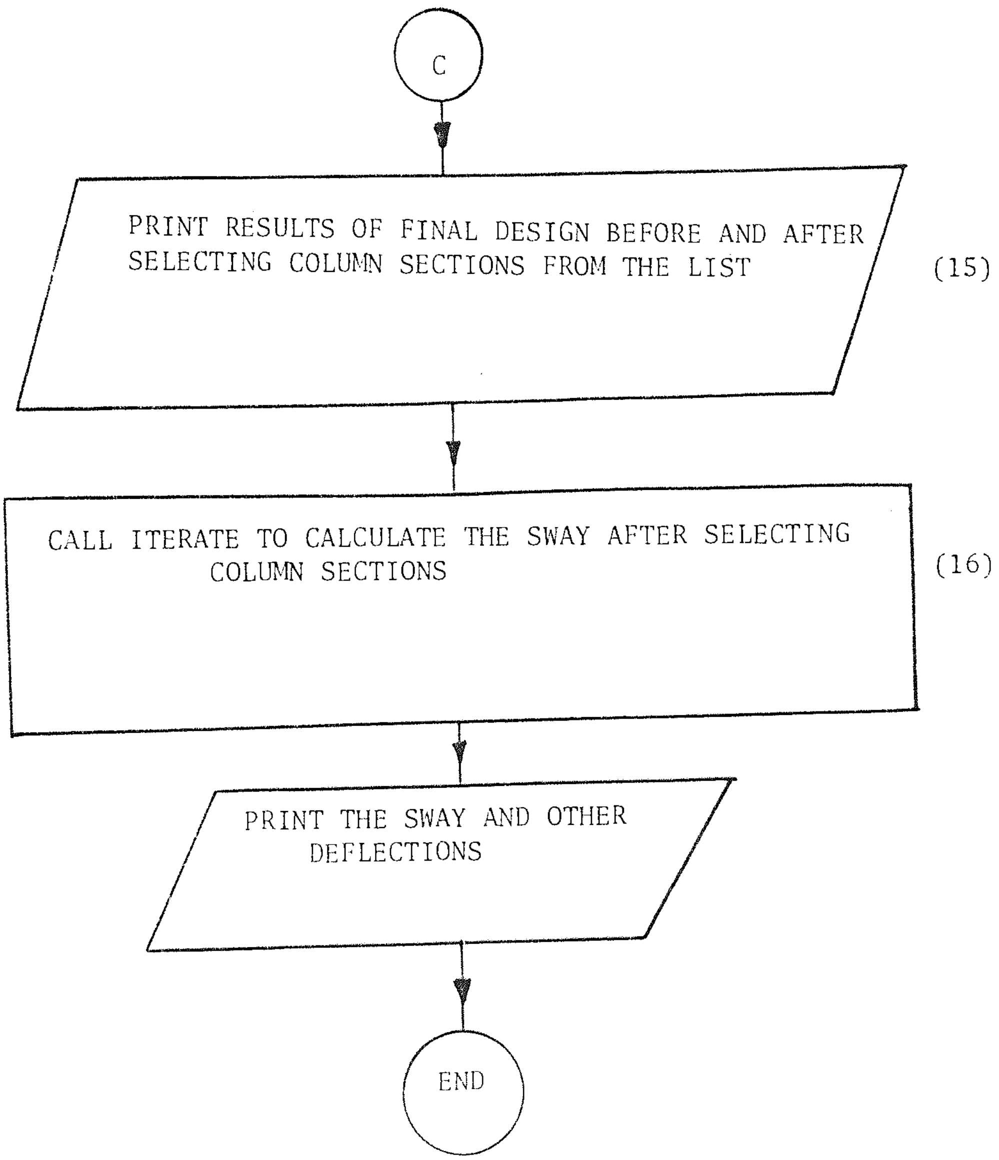


FIGURE 5.3: MASTER PROGRAM (Continued)

reduction in the data is always useful. For regular frames a program is written to prepare data for the frame. The program creates a data file for the original design program. The only data needed in this case is the preliminary data, the frame data, the height of the storey, and the loads applied on the frame.

STEP 2: The properties of the universal columns and beams must be given to the program as data. To avoid punching these for each frame designed, they are kept in a data file on the backing store. This file is called each time a frame is to be designed. The properties kept in this file are the sizes of the section, the current price, the area, the second moment of area, the weight, and the plastic modulus of the section.

4.3 DESIGN FOR DEFLECTION REQUIREMENTS (Steps 3 to 11)

STEP 3: A set of lower-bound sections is selected for the beams. To avoid collapse under vertical loading at a load factor λ_2 (1.75), the lower-bound beam sections are selected so as to avoid collapse by a rigid-plastic beam type mechanism. The sections obtained in this manner are retained, and each time a beam is redesigned, the section is compared with its lower-bound beam section.

STEP 4: The axial loads are calculated approximately as described in section 3.3.

STEP 5: Preliminary second moments of area for beam sections are calculated according to section 2.5, either by using the method of Anderson and Islame, or by using the portal method.

STEP 6: The simple portal method of design is applied to calculate the preliminary ratios of internal to external column second moments of area. Equations (2.34) give these ratios using the portal frame method.

STEP 7: For each beam section the second moment of area is calculated in Step 5, a universal beam is chosen from the list by calling subroutine BEAM-I.

STEP 8: The second moments of area of the columns, and the joint displacements are calculated for the first set of beams. The subroutine ITERATE for doing this will be discussed later in this chapter. The second moments of area of the columns resulting from this subroutine might form a reverse column taper. In this case the appropriate beam section is increased to the next larger size on the list. Subroutine ITERATE is called again to calculate the new second moments of area of the columns and the joint displacements. This process is repeated until no reverse column taper occurs.

STEP 9: The cost of the material and the number of iterations needed for a design cycle are printed.

STEP 10: The beam sections are reduced to the next smaller size on the list, and the procedure of steps 8 and 9 is repeated for the second design cycle.

STEP 11: The results of the two most recent design cycles are kept in the memory to be used in the extrapolation for reducing the cost of the material of the frame. This is done by subroutine REDUCE, which calculates the cost of the material for each storey for the most recent two designs. The subroutine reduces the sections of the beams to the next smaller sizes in the list of universal beam sections, finds the cost of the material, and decides which beam section is most suitable to use in the next design cycle. It also decides the best internal to external column ratio. This process of reducing the material cost is repeated until any change in a beam section results in a higher cost, or in a beam section smaller than that specified as a lower bound in step 3. The sections of the final reduced cost design and their properties are printed. Also the joint displacements are printed in addition to the cost of the material of the frame, the total weight, and the number of iterations.

4.4 DESIGN FOR STRENGTH REQUIREMENTS (Steps 12 to 17)

STEP 12: The final reduced cost design should be checked for strength. To do so, the joint displacements, obtained for the last design cycle are used in calculating the internal forces in the frame. This is done by subroutine INTERNAL FORCES. The resulting bending moment at both ends of a member, the shear force, and the approximate axial force are printed in IS units.

STEP 13: The procedure of paragraph 3.5.2 is applied to check the beam sections. If the strength requirements in one of the beams are not satisfied, then that beam section must be increased. A new plastic modulus for the section is obtained by using $Z_p = M_{max}/Y_s$. A universal beam section, in turn, is obtained by calling subroutine BEAM-ZP. An increase in a beam section may result in smaller column sections. For this reason, subroutine ITERATE is called to obtain new column sections and joint displacements.

STEP 14: The procedure of paragraph 3.5.3 is applied to check the strength of the column sections. The reduced plastic modulus of a column section should be more than that required by $Z'_p = \lambda_1 M_{max}/Y_s$. The reduced plastic modulus is obtained by calling subroutine REDUCED-ZP. If a column section is not satisfactory, then the section is increased and a new universal column section is found by calling subroutine COLUMN-ZP. This increase in the column section results in a decrease of the sway below the allowable. For this reason subroutint ITERATE is called. This time to obtain the sway and the rest of the joint displacements.

The procedure described in steps 13 to 14 above is repeated until all the sections satisfy the strength requirements.

STEP 15: The results of the final design which satisfy deflection, strength, and reduced cost requirements are printed, before and after selecting universal sections for the columns.

STEP 16: Selecting a universal section for a column reduces the sway in a storey below the allowable. To examine the new sway deflections, the subrouting ITERATE is called to calculate the resulting sway and other joint displacements. These are due to the difference between the actual second moment of area of a column and that of the selected universal section.

4.5 SUBROUTINE ITERATE

This subroutine is used to calculate the second moments of area of the columns if the sway is specified, or to calculate the joint deflections if the second moments of area of the columns are known. In both cases the vertical displacements and the rotations of the joints are also calculated. A comprehensive flowchart for this subroutine is given in Figures 5.5, 5.6, 5.7 and 5.8, in which the integer NIR is used to count the number of iterations as they are carried out. JB and JC are used to count the joints of beams and columns as they are handled.

The subroutine starts by setting the values of the joint displacements x, y and R, and the second moment of area of the column I to their values in the previous design. Obviously for the first design cycle these are set to their preliminary values (i.e. I = 1, y = -1 and R = 0). Level 600 of Figure 5.5 in the chart marks the

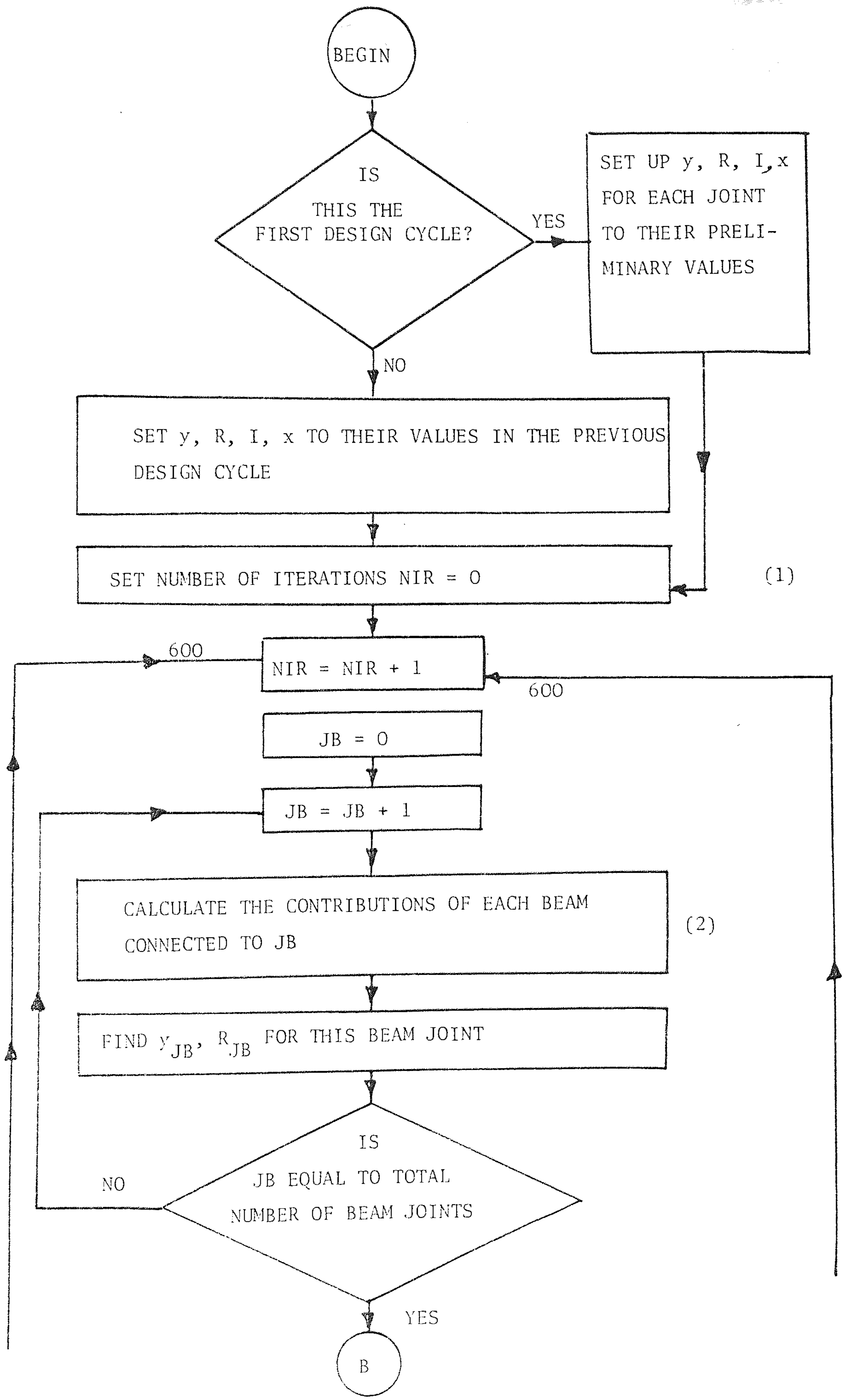


FIGURE 5.4: SUBROUTINE ITERATE

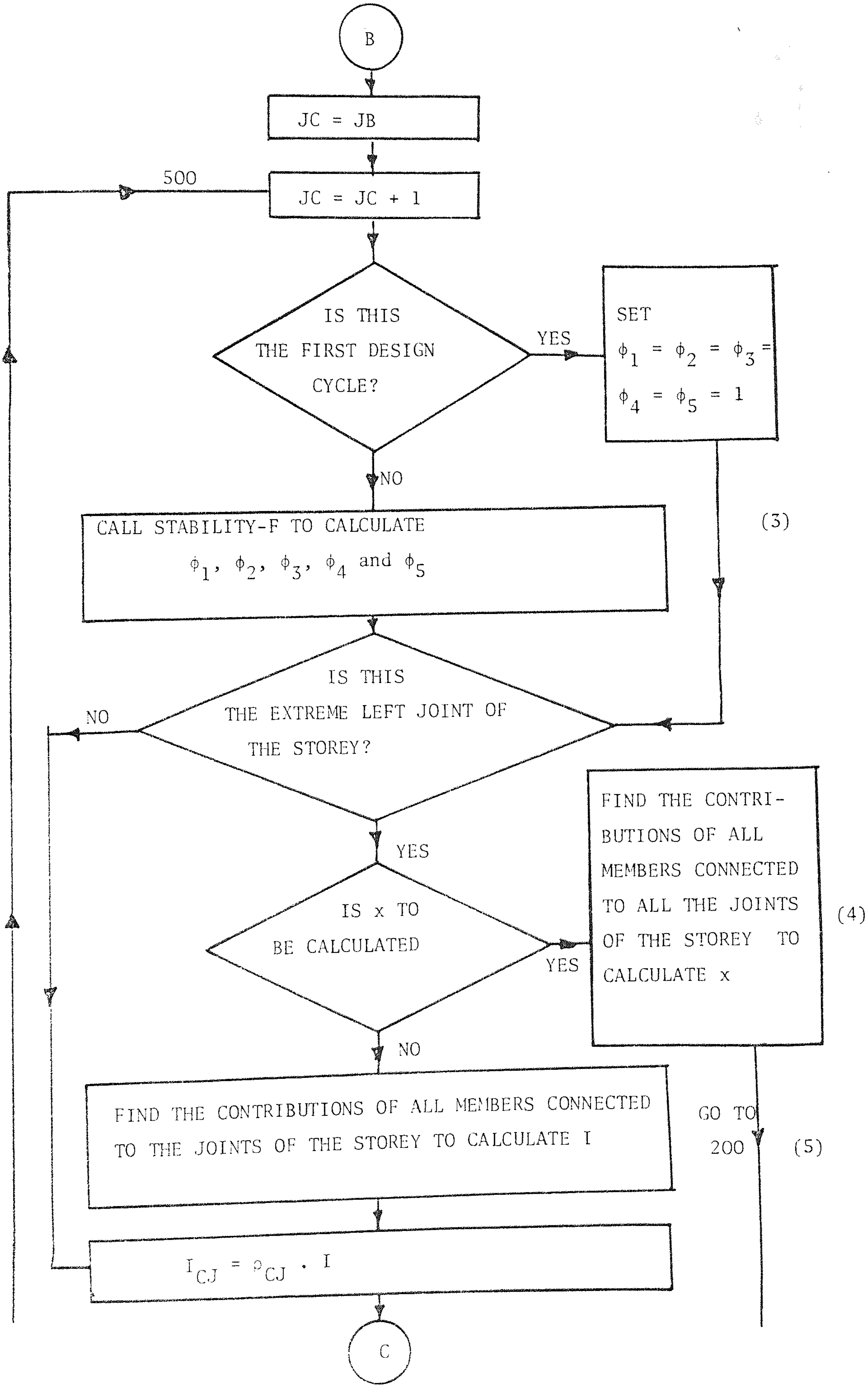


FIGURE 5.5: SUBROUTINE ITERATE (Continued)

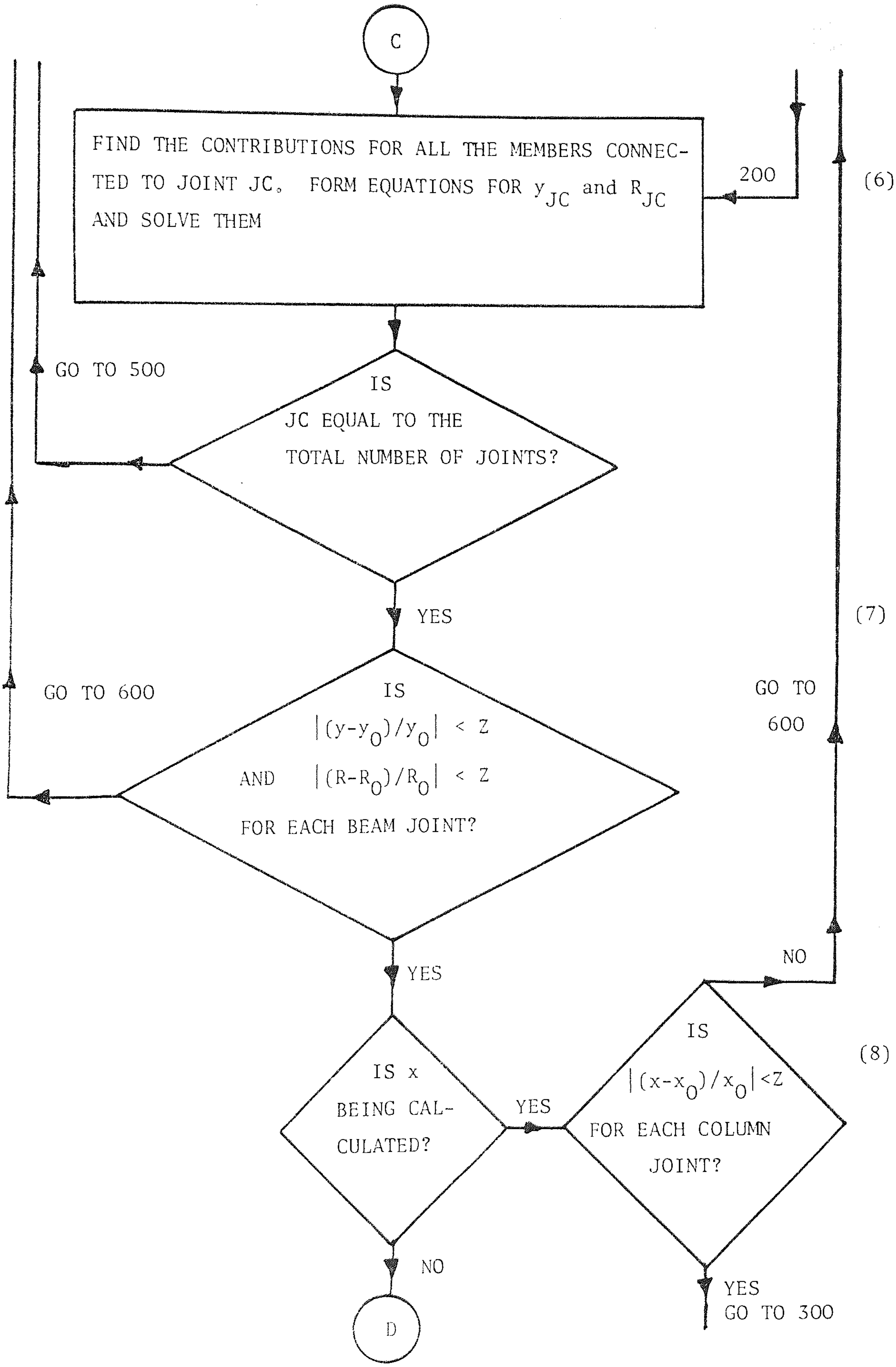


FIGURE 5.6: SUBROUTINE ITERATE (Continued)

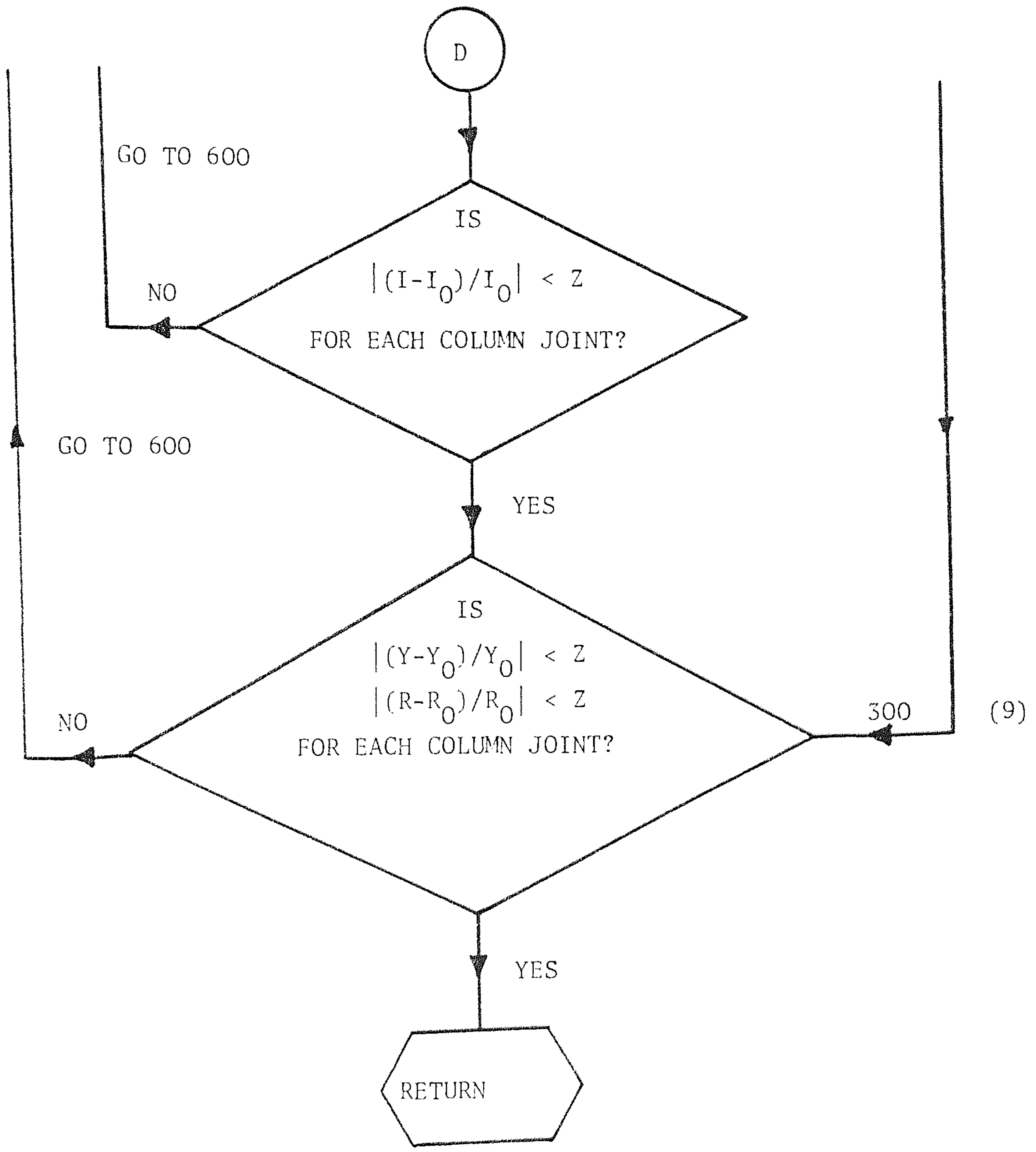


FIGURE 5.8: SUBROUTINE ITERATE (continued)

start of one iteration. The operations between this level and level 300 in Figure 5.8 represents one iteration. The process of calculation for an iteration is divided into nine steps. The number of each step appears in brackets at the right of the chart.

STEP 1: The number of iteration NIR is set to zero and is increased by one for each iteration.

STEP 2: For each beam joint the contributions of the beams connected to the joint are calculated. Then the vertical displacement of the joint y_{JB} and the rotation R_{JB} are found, using equations (2.14) and (2.17) respectively.

STEP 3: For each column joint the ϕ functions are calculated for columns C_j and C_m connected to joint JC. However, for the first design cycle the stability functions are set to 1, as the axial load effect is neglected in this cycle.

STEP 4: The second moment of area of a column needs to be calculated once in each storey. The rest of the column second moments of area are then found by proportion. The calculation is carried out for the extreme left joint of the storey, and the value of I is obtained from equation (2.38). However, if the second moment of area of the column is known and the sway is wanted, then equation (2.29) is used instead to calculate the sway for the complete storey.

STEP 5: The second moments of area of the other columns in a storey are found from the internal to external columns ratios by multiplying I by the appropriate ratio ρ.

STEP 6: The vertical displacement of joint JC and the joint rotation must be found for each column joint. This is done by solving equations (2.23) and (2.26) respectively.

STEP 7: To determine whether a desired degree of accuracy (Z) is acquired or not. The vertical displacement y and the joint rotation R are examined using the inequality

$$\frac{\text{The current value} - \text{The Previous Value}}{\text{The Previous Value}} < Z$$

where Z is the required accuracy. If the above condition is not satisfied, then another iteration will be needed starting from level 600.

STEP 8: For each column joint, the above inequality is used to establish whether the value of I or $\bar{x}$ achieves the required accuracy or not.

STEP 9: For each column joint the test is also applied on the value of the vertical displacement y and the rotation of the joint R.

In both steps 8 and 9 the control is transferred to level 600 to start a new iteration if the above inequality is not satisfied.

CHAPTER FIVE

DESIGN EXAMPLES

5.1 A 12 STOREY-SINGLE BAY FRAME

Most of the existing design methods for deflection limitation do not consider the axial stiffness and the axial load effect (see Chapter One). In order to show these effects, the 12 storey single bay frame of Figure 5.1 was first designed disregarding the axial stiffness EA/L and the stability functions. The deflection profile used in the design is shown in Figure 5.2. The value of k used in the deflection curve was 30.

The second column in Table 5.1 shows the second moments of area of the initial beam sections. These are obtained as follows:

1 - The beam sections are designed using the simple portal frame method of the steel designer's manual (Allwood, et al, 1966). It should be noted that no beam section is allowed to be below that obtained by a beam mechanism.

2 - The second moment of area of the beam sections obtained in step (1) are magnified by a factor. This is taken as 2 at the top storey, and 4 at the bottom. From this initial design, eleven cycles are needed to obtain the final linear reduced cost design.

Column three of Table 5.1 shows the second moments of area for this design after sections have been selected from the list. The number of iterations per design cycle is represented in Figure 5.3. This shows the reduction in the number of iterations after the first design cycle. Figure 5.4 is for the reduction in material cost in each design cycle. Trial 6 shows an increase in the material cost, due to the increase in some beam sections to prevent reverse column taper. The computer time needed to complete the design, with a tolerance of 0.01, was 27 second on Aston 1904s Computer (at a cost of £2.37). This is less than the 29 second needed for one explicit analysis solving $\underline{L} = \underline{K}\ \underline{X}$ directly.

In the second stage, the beams for the final linear design were kept unaltered, but the columns were subsequently altered to allow for the effect of axial forces. These were calculated exactly using equations (3.3), with a tolerance of 0.01. The design with the axial load effect included is shown in the fourth column in Table 5.1 This gives the second moments of area of the sections selected from a list for universal beams and columns. The time needed to obtain the final non-linear design was 82 seconds. This includes the time to check the strength of the final non-linear design. Thus, when considering the axial load effect the computer time increases considerably, but even then the computer cost of a complete design is only £7.20.

The effect of the axial stiffness and the stability functions on the design of the column sections is obvious. A glance at the third and fourth columns in Table 5.1 shows that, except for the top two columns, all the sections are increased when the effect of axial load is considered. This will cause the total material cost of the frame to increase by 12.24% from £5608.18 to £6390.69. The total mass increases by 11.18% from 25832.28 kg to 29082.36 kg.

To check the safety of the final design, a non-linear elastic-plastic analysis was carried out for this frame by an existing computer program (Majid and Anderson, 1967). This showed that under combined loads the frame remained elastic at a load factor of 1.29. No plastic hinge occurred in any member of the frame at this load factor.

5.2 A 24 STOREY FRAME WITH THREE EQUAL BAYS

The computer program described in Chapter Four was used to design the 24 storey frame shown in Figure 5.5. This frame belongs to a structure consisting of parallel frames which are 4.5 m apart. The vertical and the wind loads are calculated according to CP3 Parts 1 and 2. The wind load is calculated for a frame to be built on a surface with large and frequent obstructions. Each frame is symmetrical and is loaded symmetrically.

The deflection curve used in the design and the maximum allowable sway in each storey are shown in Figure 5.6. The value of k used in the deflection curve is 26. The results of the design in which the axial load effect is included are shown in Table 5.2. This gives the second moments of area of the selected sections, in which steel grade 50 is used.

The design was carried out with a tolerance of 0.001. However, the same frame was designed with a tolerance of 0.01 and the same sections were obtained with considerable saving in computer time. The time needed for designing the frame with a tolerance of 0.001 is 1020 seconds, which means that the cost of the design for a user from outside the university would be £88. The cost reduces to £23.76, if a tolerance of 0.01 is used. The computer time needed in this case is 267 seconds.

To start with, the beam sections were taken from Anderson and Islam's design described in Chapter One, using the linear deflection curve. The sway between the two ends of a column in their design was taken to be H/300, where H is the column's length. The results of the design using their method is shown in Table 5.3, which gives the second moments of area of the selected sections.

Anderson and Islam's design does not consider the effect of the vertical load on the sway. Therefore, to compare the results, a third design should be obtained. This is done by using the computer program described in Chapter Four, but neglecting the axial stiffness of the columns and the axial load effects. Table 5.4 shows the second moments of area for the selected sections.

The price of the material and the total mass of the frames are given in Table 5.5 for the three designs. It can be seen that the design obtained by this method (neglecting the axial stiffness and the axial load effect) is 1.22% cheaper than that obtained by Anderson and Islam's method. It is also noticed that if the axial load effects are included in the design of this frame then the material cost will increase by 11.45%.

In order to justify this increase and to examine the efficiency of Anderson and Islam's design, each frame was analysed using an existing computer program. The differential sway between the ends of each column, obtained by each analysis, is shown in Figure 5.7. The vertical line shows the h/300 limit on the differential sway. Curve 1 shows sway in each column for the final design. None of the points of this curve is to the right of the vertical limiting line. Curves 2 and 3 show the differential sway between the ends of each column for the frame designed by Anderson and Islam's method, given in Table 5.3. Curve 2 is for wind load only, excluding axial load effect. Curve 3 is for wind and vertical loads, including axial load effects. Because this frame is symmetrical and loaded symmetrically, the difference between curves 2 and 3 is solely due to the effect of axial loads. It can be seen that this difference is significant and cannot be neglected. In fact an increase of 11.45% in the material cost is needed if the axial load effects are considered in the final design.

Twelve design trials are needed to reduce the cost of the material from the initial beams, obtained from Anderson and Islam's design. The reduction in the material cost in each trial is shown in Figure 5.8. Trials 8 and 9 show an increase in the material cost, due to the increase in some beam sections to prevent reverse column taper. The number of iterations used in each design trial for a tolerance of 0.001 is plotted in Figure 5.9. This shows a considerable reduction in the number of iterations after the second trial. This is due to the use of the previous design results as initial values for the current ones. The relatively large number of iterations in the second design trial is due to the introduction of the axial load effect at this stage.

The final design shown in Table 5.2 was obtained by applying the deflection requirements only. However, the design was checked for strength requirements as described in Chapter Three sections (3.5, 3.5.1, 3. 5.2 and 3.5.3). This showed that all the sections were satisfactory. Furthermore, a non-linear elastic-plastic analysis of the frame with the final sections was carried out using an existing computer program (Majid and Anderson, 1967). This analysis showed that under combined loads the frame remained elastic at a load factor of 1.29. It should be noted that, for strength requirements a beam was allowed to develop a plastic hinge below a load factor of 1.29, but because of the deflection requirements governing the design, it was found that all the beams remained elastic at this load factor

5.3 A 35 STOREY - TWO UNEQUAL BAY FRAME

The 35 storey - two unequal bay frame shown in Figure 5.10

was designed from steel grade 50. In the out-of-plane direction, the frames were assumed to be 4.5 m apart. The vertical and the wind loads were calculated according to CP3 Parts 1 and 2. The wind load was calculated from a frame to be built on a surface with large and frequent obstruction. Because the frame is assymmetrical, both vertical and wind loads affect the deflection. The linear and the non-linear profiles of the frame are shown in Figure 5.11. For practical reasons, the left and the right spans in each storey are to be built out of the same beam section.

The frame was first designed using Anderson and Islam's equations. The sway between the ends of the column was taken as height/300. The results of the design using their equations with the linear deflection curve are shown in Table 5.6. This shows the second moment of area of the members after selecting sections from the list of universal beams and columns. The design represented in the table was analysed using an existing computer program (Celik, 1977). Four different load cases were considered, and the sway between the ends of each column is shown in Figures 5.12 and 5.13.

(1) Curve 1 of Figure 5.12 is for wind load only with the wind acting from the left; and neglecting the axial load effects.

(2) Curve 2 in Figure 5.12 is for the combined loading, with the wind acting from left; and considering the axial load effects.

(3) Curve 1 of Figure 5.13 is for wind load only, with the wind acting from the right; and neglecting the axial load effects.

(4) Curve 2 of Figure 5.13 is for combined loading, with the wind acting from the right; and considering the axial load effects.

In both Figure 5.12 and 5.13 the vertical line shows the permissible limit on sway. A point to the left of this vertical line means that the sway at a storey is acceptable, while a point to the right means it is not. It is obvious from Curves 1 and 2 in both figures that the sway exceed, the limit of H/300 by a considerable margin. In fact for the combined load case with the wind acting from the right, the sway of the 33rd floor is equal to H/120. This is 2.5 times larger than the limit.

The design obtained from Anderson and Islam's equations was therefore found to be totally unsatisfactory. The beams obtained from their equations with the sway restriction of H/300 cannot be used as a starting point for the iteration technique. However, Anderson and Islam's approach can still be utilized to select trial initial beams provided that the allowable sway is severely restricted. It must be emphasized that this restriction on sway only applies when selecting beam sections from Anderson and Islam's equations. But when using the design method proposed by the author, the code limitation of H/300 is observed. In other words Anderson and Islam's approach is only used to select an initial trial set of sections.

In the design procedure described in Chapter Two, economy in the cost of the material of the frame was achieved by reducing each beam section to the next available in the list of universal sections. For large frames such a policy can increase the number of design cycles. The design procedure can be speeded up by drawing a curve relating the cost to the beam sections as follows:

(i) Anderson and Islam's equation are used to calculate several sets of beam sections under different restrictions of the sway between the two ends of the column. Some of these, however, may be too weak to resist failure by a beam type mechanism. To avoid such a failure, these initial trial sections are increased substantially.

(ii) For each initial set of beams thus obtained, the columns are designed using the method developed in this thesis to satisfy the deflection profile given by curve b of Figure 5.11. The axial load effects are included when designing these columns. A graph is drawn to show the cost of material for different designs as obtained from a number of trial sets of beams. Figure 5.14 shows the cost/beam-section curve for the 35 storey frame. It can be seen that six trials are needed to draw the curve. This is considerably less than the 12 trials, which was needed to obtain the reduced cost design of the 24 storey frame shown in Figure 5.5 Figure 5.14 shows that the most economic design is obtained when the sway in Anderson and Islam's equations are restricted to H/800. A subsequent analysis of this design showed that plastic hinges developed in some of the columns below a load factor of 1.29. This is perfectly acceptable provided that failure takes place above such a load factor. However to avoid the formation of such hinges two modifications can be made:

(1) The column sections where hinges occur could be increased, until an analysis shows that no hinge is developing below a load factor of 1.29. The beam sections are kept unchanged.

(2) A smaller set of beams could be used to satisfy sway requirements so as to obtain larger columns from the iteration.

In fact the second solution was found to be more economical.

In order to produce a design which satisfies both sway and strength requirements, the second approach described above was therefore used. First, the iteration technique using a set of beams obtained from Anderson and Islam's equations with a restriction of H/700 was analysed. This was found to be similar to that of H/800. Then, the design using a set of beams with sway restrictions of H/600 was analysed. The beams in this case were relatively small and the resulting columns were relatively large. The analysis of the frame designed in this manner under combined loading showed that no hinge developed below a load factor of 1.29, and the frame was completely elastic at this load factor. Although hinges were allowed to develop in the beams below a load factor of 1.29 (for combined loading), it was found that these too remained elastic.

The designs of the frame obtained using the computer program described in Chapter Four are summarized in Table 5.7 and 5.8. These show the second moment of area for the selected sections from combined loading, with the wind acting from either side. The second moment of area obtained by using Anderson and Islam's equations, with the restriction of H/600 on sway, plus the second moment of area for a section that satisfies a beam mechanism provides the actual beam section used in the iteration technique. The deflection profile used is that of Figure 5.11 (the non-linear curve b). The design was carried out with a tolerance of 0.001, and when a tolerance of 0.01 was used a premature convergence was noticed. It is obvious from Table 5.7 and 5.8 that the critical design is that for wind acting from the right. This design is considered as the final design because the column sections obtained are larger than those resulting from the wind acting from the left.

It should be noticed that in this example, only two design cycles were sufficient for each initial set of beams. In the first design cycle the axial load effect and the axial stiffness were neglected. In the second design cycle the results of the first cycle were used to introduce these effects.

The number of iterations needed for the first design cycle is 565, while 493 iterations are needed for the second design cycle. Comparing these with the number of iterations needed for other frames in the first two design cycles shows that the number of iterations increases rapidly for large frames. However, the computer time needed, to obtain the design of Table 5.7 , 1076 second. This includes the time to check the strength of the final design, which means that the cost of the design for a user from outside the university would be £92.05. The store needed was 3120 words (approximately 32 k). This included storing the computer program itself.

To ensure that the sway is under the limit and to examine the accuracy of the program, the design of Table 5.7 was analysed considering the axial load effects. The sway between the ends of each column is shown by curve 3 of Figure 5.12, for combined loading with the wind acting from the left. This shows that all the points are within the permissable limit on sway represented by the vertical line. For the case of wind acting from the right, the sway of each storey is shown by curve 3 of Figure 5.13. It can be seen that for the case of combined loading with wind acting from the right, some of the point are slightly out of the limit. The reasons for this are:

(1) The approximation made in calculating the axial loads in the iteration method.

(2) The change of the second moment of area of the column due to a selection of a column section from the list of universal columns.

5.4 CONCLUSIONS

From the examples given above, it is possible to conclude the following:

1 - The examples indicate that the iteration technique can be used. Successfully for designing different sizes of frames. In all the frames designed, the iteration converged to a set of acceptable column sections.

2 - For larger frames a small tolerance such as 0.001 is required to obtain a complete convergence. The 35 storey frame showed that a premature design was obtained when a tolerance of 0.01 was used. However, the latter was used successfully for all the other frames. Obviously, a larger number of iterations and design cycles are needed for a larger frame.

3 - A frame which is designed to satisfy deflection requirements results in the frame having an overall strength which is greater than that required by the elastic-plastic design methods. In fact, for all the above designs the non-linear elastic plastic analysis under combined loading shows that the frames remain elastic at a load factor of 1.29.

4 - The cost of the computer time and storage is negligible compared to the material cost of the frame.

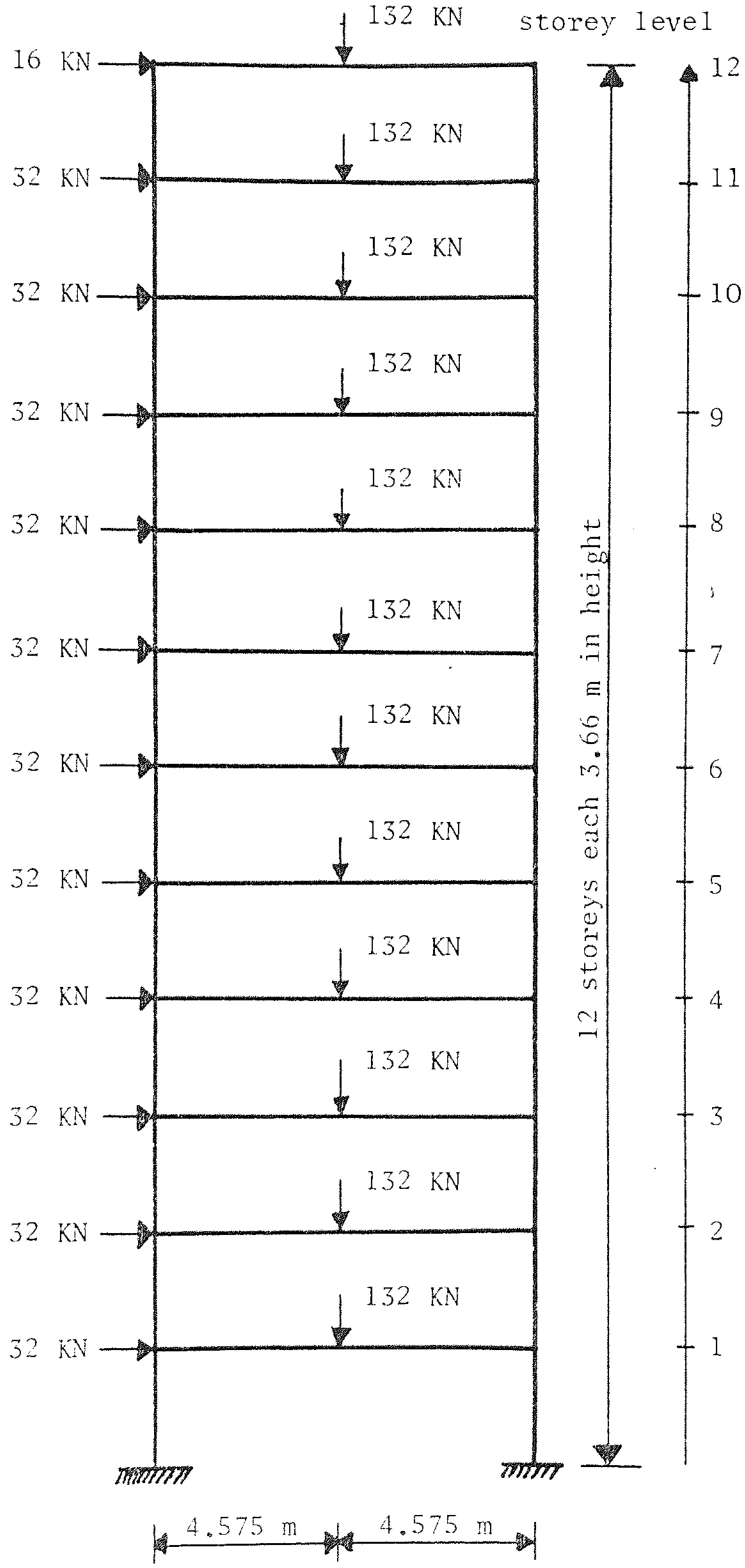


FIGURE 5.1 12 STOREY - SINGLE BAY FRAME - DIMENSIONS AND LOADS

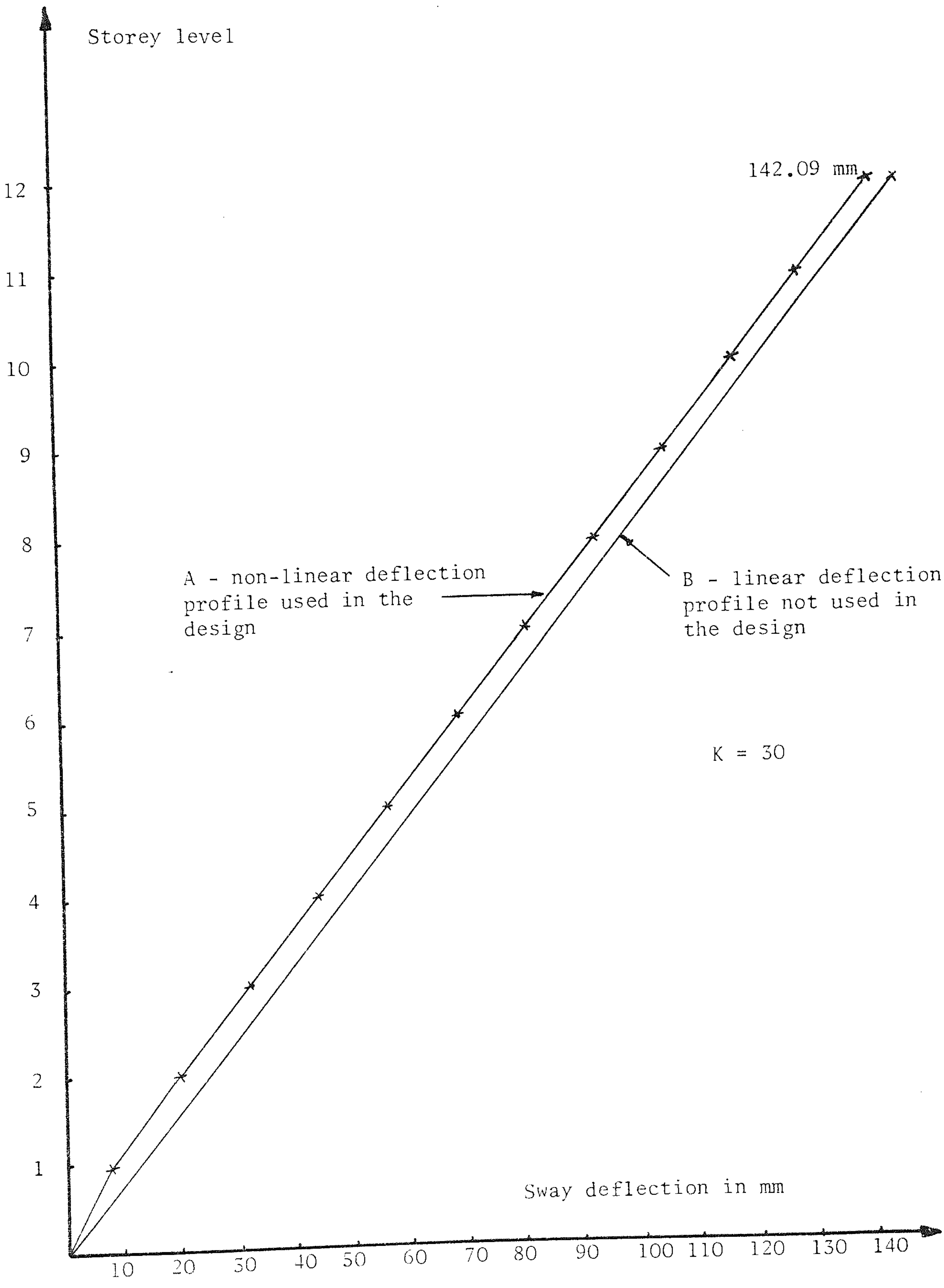


FIGURE 5.2 THE 12 STOREY - SINGLE BAY FRAME DEFLECTION PROFILES

Level	Initial design I in cm^4	Linear design I in cm^4	Non-Linear design I in cm^4
BEAMS			
Level 1	503781	324715	324715
Level 2	435796	204747	204747
Level 3	375110	245412	245412
Level 4	375110	168535	168535
Level 5	245412	168535	168535
Level 6	245412	135972	135972
Level 7	168535	117700	117700
Level 8	135973	75549	75549
Level 9	75549	55225	55225
Level 10	75549	47363	47363
Level 11-12	75549	21345	21345
COLUMNS			
Level 1-2	-	66307	79110
Level 3	-	57153	79110
Level 4-5	-	48525	66307
Level 6	-	38740	57153
Level 7-8	-	32838	57153
Level 9	-	27601	57153
Level 10	-	17510	32838
Level 11	-	11360	17510
Level 12	-	5263	5263
Total Mass		25832.28 kg	29082.36 kg
Total cost of material		£5608.18	£6390.69

TABLE 5.1: DESIGNS OF THE 12 STOREY FRAME

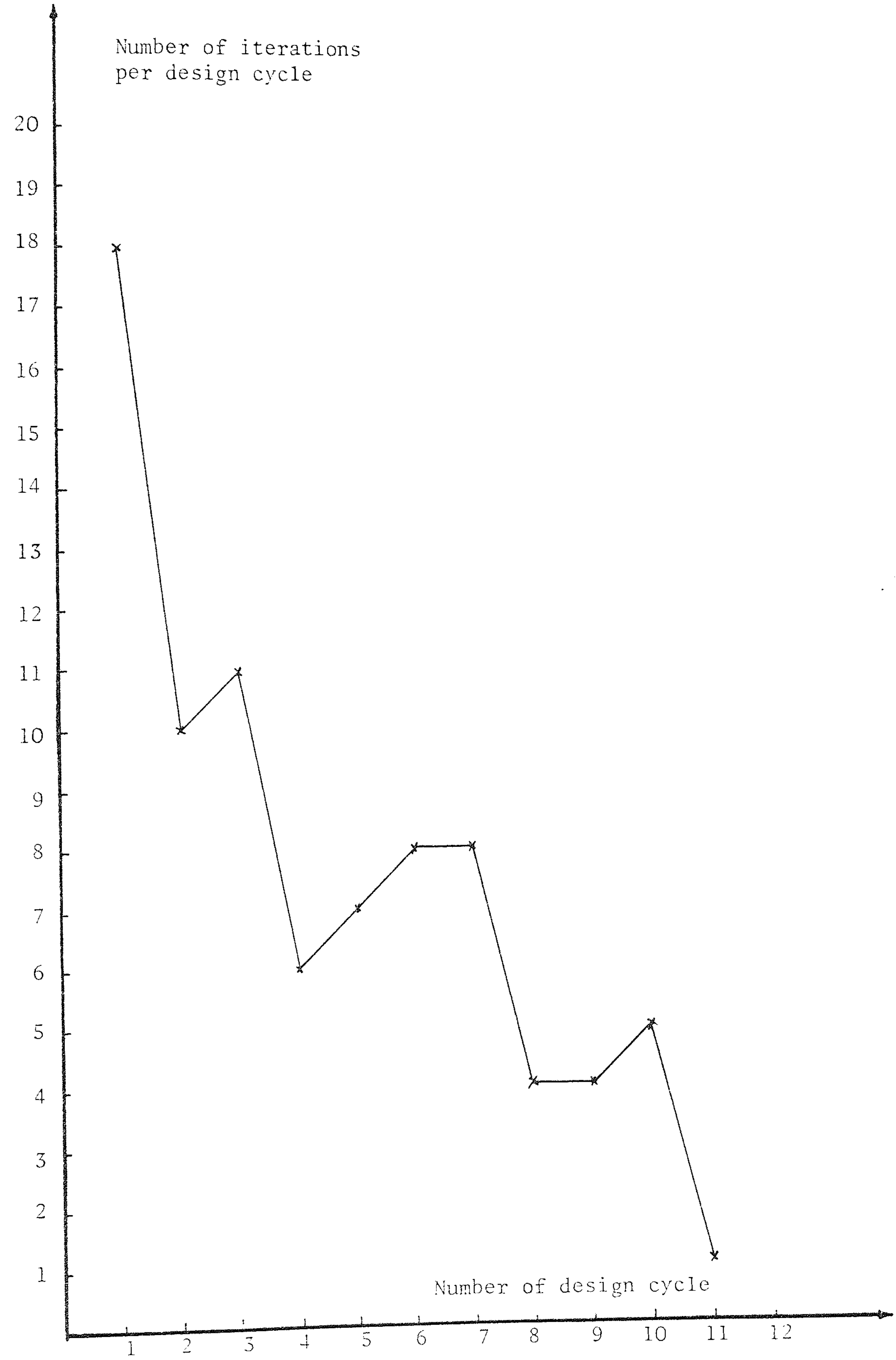


FIGURE 5.3 12 STOREY - SINGLE BAY FRAME NUMBER OF ITERATIONS PER DESIGN CYCLE

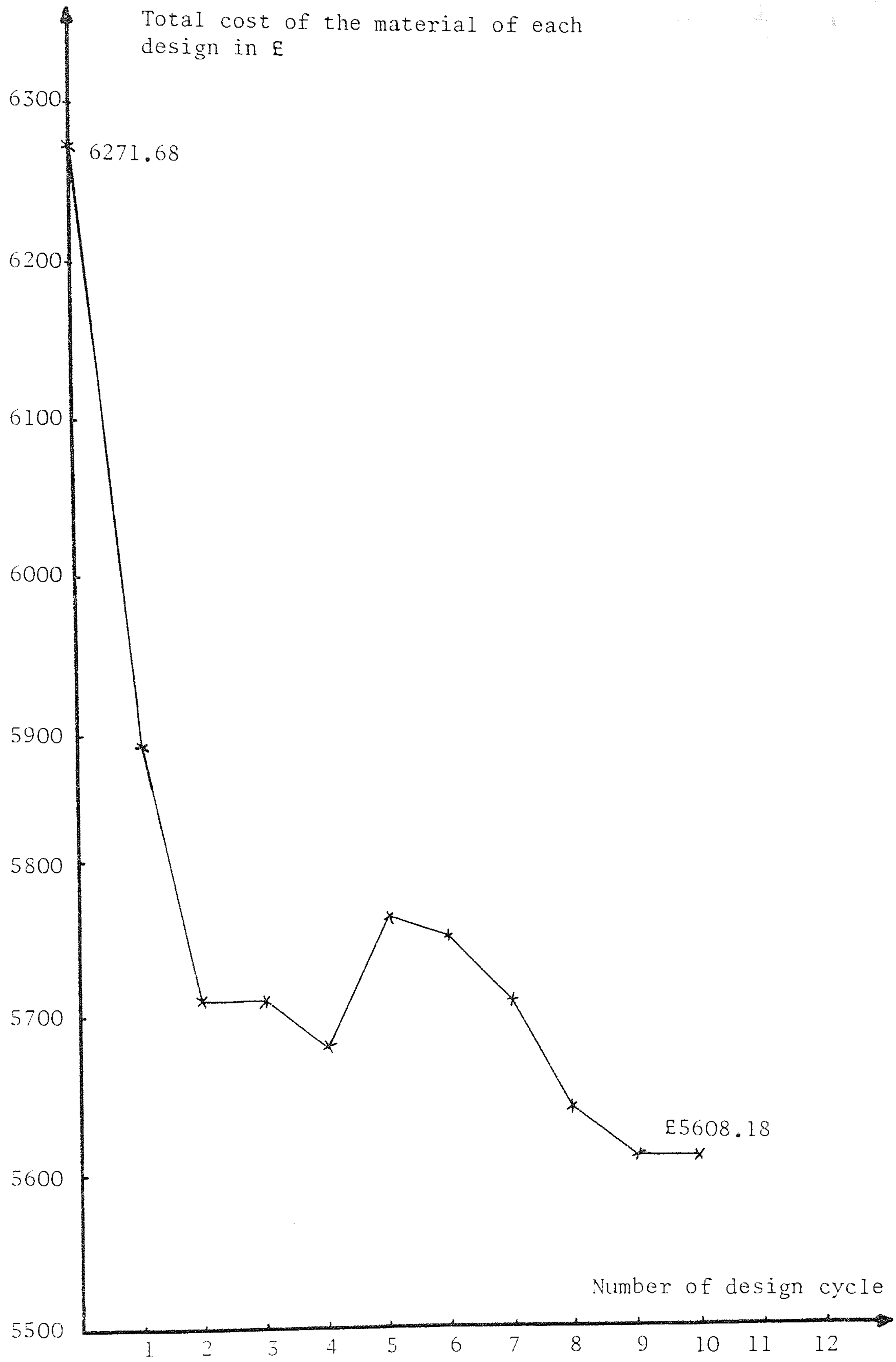


FIGURE 5.4 THE 12 STOREY - SINGLE BAY FRAME MATERIAL COST FOR EACH DESIGN

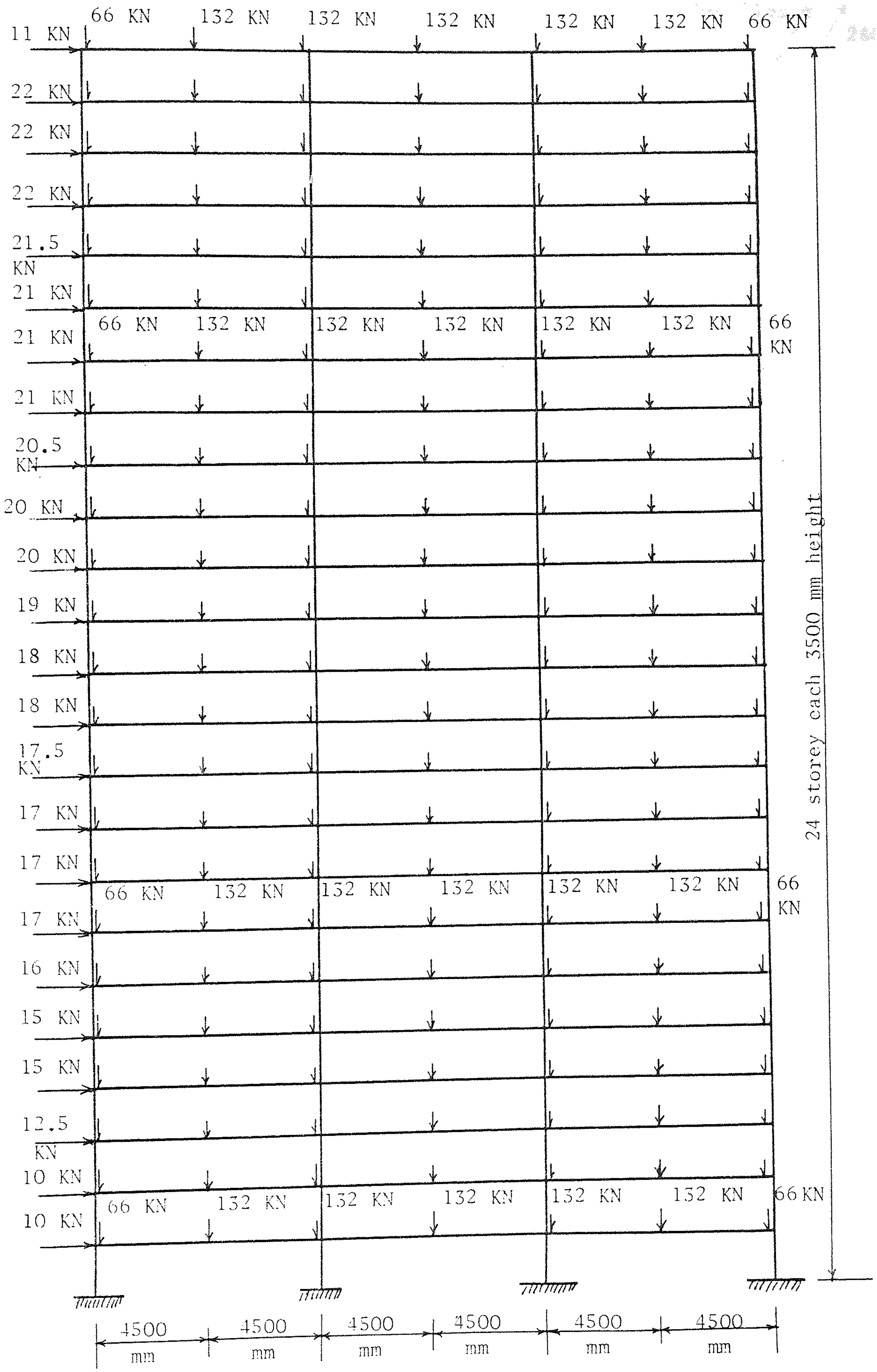


FIGURE 5.5 24 STOREY - 3 EQUAL BAY FRAME DIMENSIONS AND LOADS

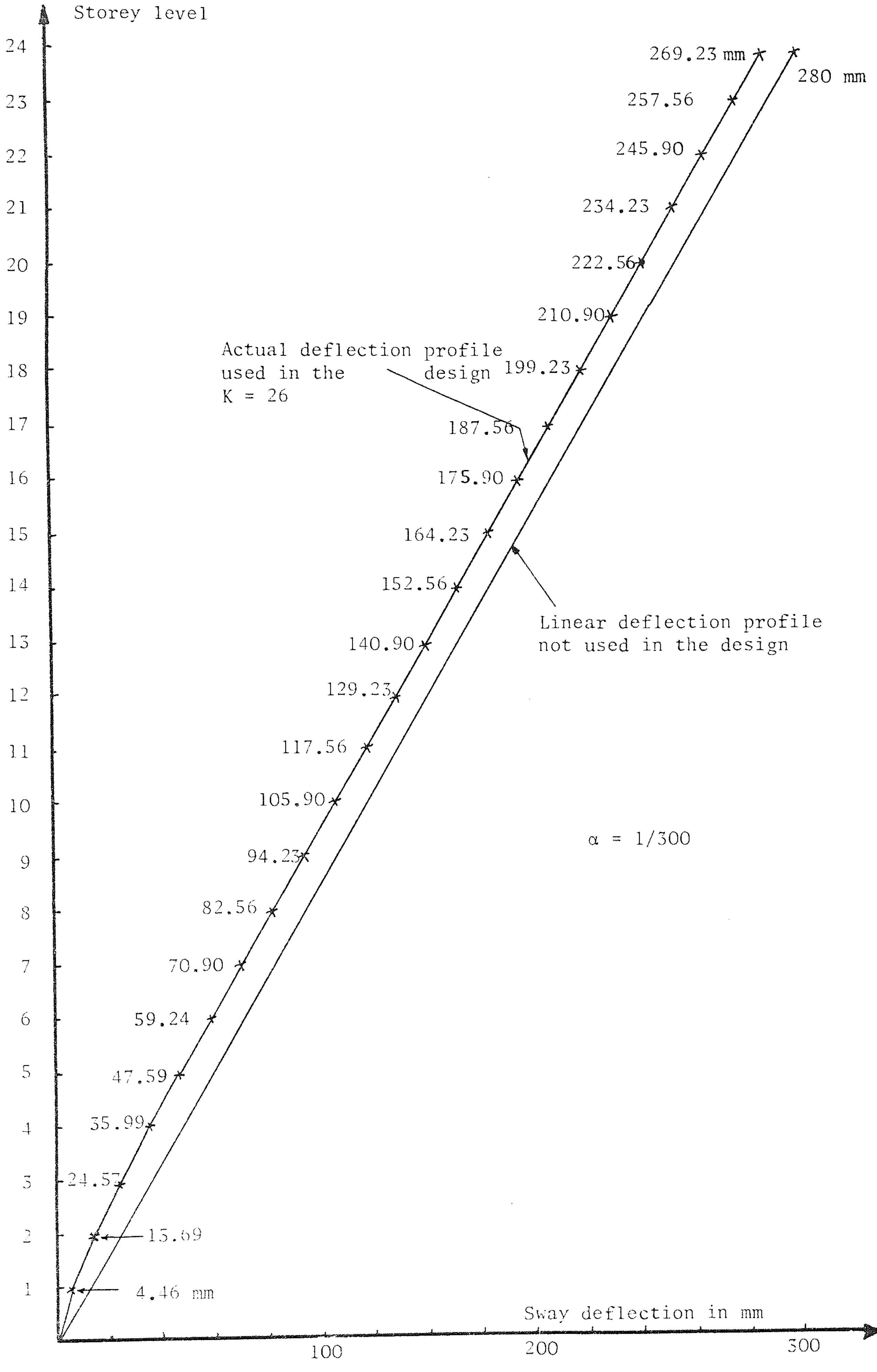


FIGURE 5.6: THE DEFLECTION CURVES FOR THE DESIGN OF THE 24 STOREY FRAME SHOWN IN FIGURE 5.3.

Second moments of area x 10^4			
Storey Level	Beam mm^4	External Column mm^4	Internal Column mm^4
Level 1	87260	99994	183118
Level 2	87260	66307	122474
Level 3	87260	50832	99994
Level 4	87260	38740	79110
Level 5	87260	32838	66307
Level 6	87260	32838	66307
Level 7	87260	32838	66307
Level 8	87260	32838	50832
Level 9	87260	22416	50832
Level 10	87260	22416	50832
Level 11	66610	22416	50832
Level 12	66610	22416	50832
Level 13	55225	22416	50832
Level 14	55225	22416	50832
Level 15	40956	22416	50832
Level 16	40956	22416	50832
Level 17	33324	22416	38740
Level 18	29337	17510	38740
Level 19	21345	17510	32838
Level 20	21345	14307	32838
Level 21	15603	9462	17510
Level 22	10054	6088	14307
Level 23 and 24	10054	6088	6088
Total mass	111597.0 kg	Total cost	£23817.19

TABLE 5.2: DESIGN OF THE 24 STOREY - 3 EQUAL BAY FRAME

Second moments of area x 10^4			
Storey Level	Beam mm^4	External Column mm^4	Internal Column mm^4
Level 1	87260	38740	79110
Level 2	87260	38740	79110
Level 3	87260	38740	79110
Level 4	87260	38740	79110
Level 5	66610	38740	66307
Level 6	66610	32838	66307
Level 7	61530	32838	66307
Level 8	61530	32838	66307
Level 9	55225	32838	66307
Level 10	55225	32838	66307
Level 11	55225	32838	50832
Level 12	55225	32838	50832
Level 13	40956	22416	50832
Level 14	40956	22416	50832
Level 15	37039	22416	38740
Level 16	33324	17510	32838
Level 17	29337	14307	32838
Level 18	25464	14307	32838
Level 19	21345	14307	22416
Level 20	15603	9462	17510
Level 21	15603	7647	14307
Level 22	8167	6088	14307
Level 23 and 24	4381	6088	6088
Total mass	100398.0 kg	Total cost	£21347.18

TABLE 5.3: ANDERSON AND ISLAM'S DESIGN OF THE 24 STOREY - 3 EQUAL BAY FRAME

Second moments of area x 10^4			
Storey Level	Beam mm^4	External Column mm^4	Internal Column mm^4
Level 1	87260	79110	146765
Level 2	98408	38740	79110
Level 3	61530	38740	79110
Level 4	87260	32838	66307
Level 5	66610	32838	50832
Level 6	87260	32838	50832
Level 7	87260	22416	38740
Level 8	87260	17510	32838
Level 9	87260	14307	32838
Level 10	87260	14307	32838
Level 11	66610	14307	32838
Level 12	55225	14307	32838
Level 13	55225	14307	32838
Level 14	66610	14307	22416
Level 15	37039	14307	22416
Level 16	55225	9462	22416
Level 17	29337	7647	17510
Level 18	37039	7647	17510
Level 19	21345	7647	14307
Level 20	29337	7647	9462
Level 21	15603	6088	6088
Level 22	15603	6088	6088
Level 23 and 24	10054	6088	6088
Total mass	99080.0 kg	Total cost	£21088.36

TABLE 5.4: DESIGN OF 24 STOREY - 3 BAY FRAME NEGLECTING AXIAL STIFFNESS AND AXIAL LOAD EFFECT

Type of Design	Cost of the material	Total mass of the frame in Kg
The final design includes the axial stiffness and the axial load effect	23817.19	111597.0
Design neglecting axial stiffness and axial load effect	21088.36	99080.0
Anderson and Islam's Design	21347.18	100398.0

TABLE 5.5: COST AND MASS OF THE 24 STOREY - 3 EQUAL BAY FRAME DESIGNED BY VARIOUS METHODS

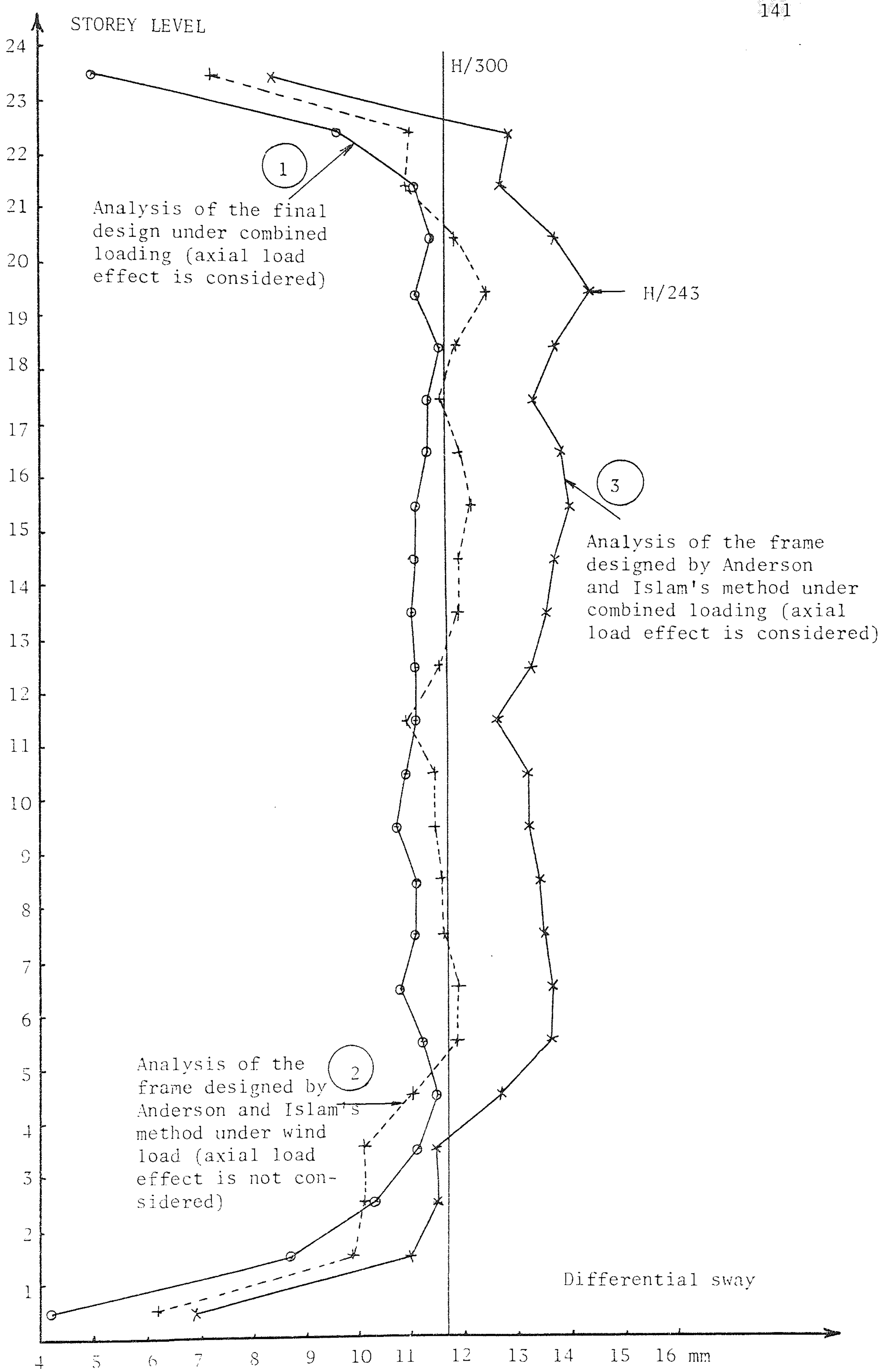


FIGURE 5.7 ANALYSIS OF DIFFERENT DESIGNS OF THE 24 STOREY - 3 EQUAL BAY FRAME

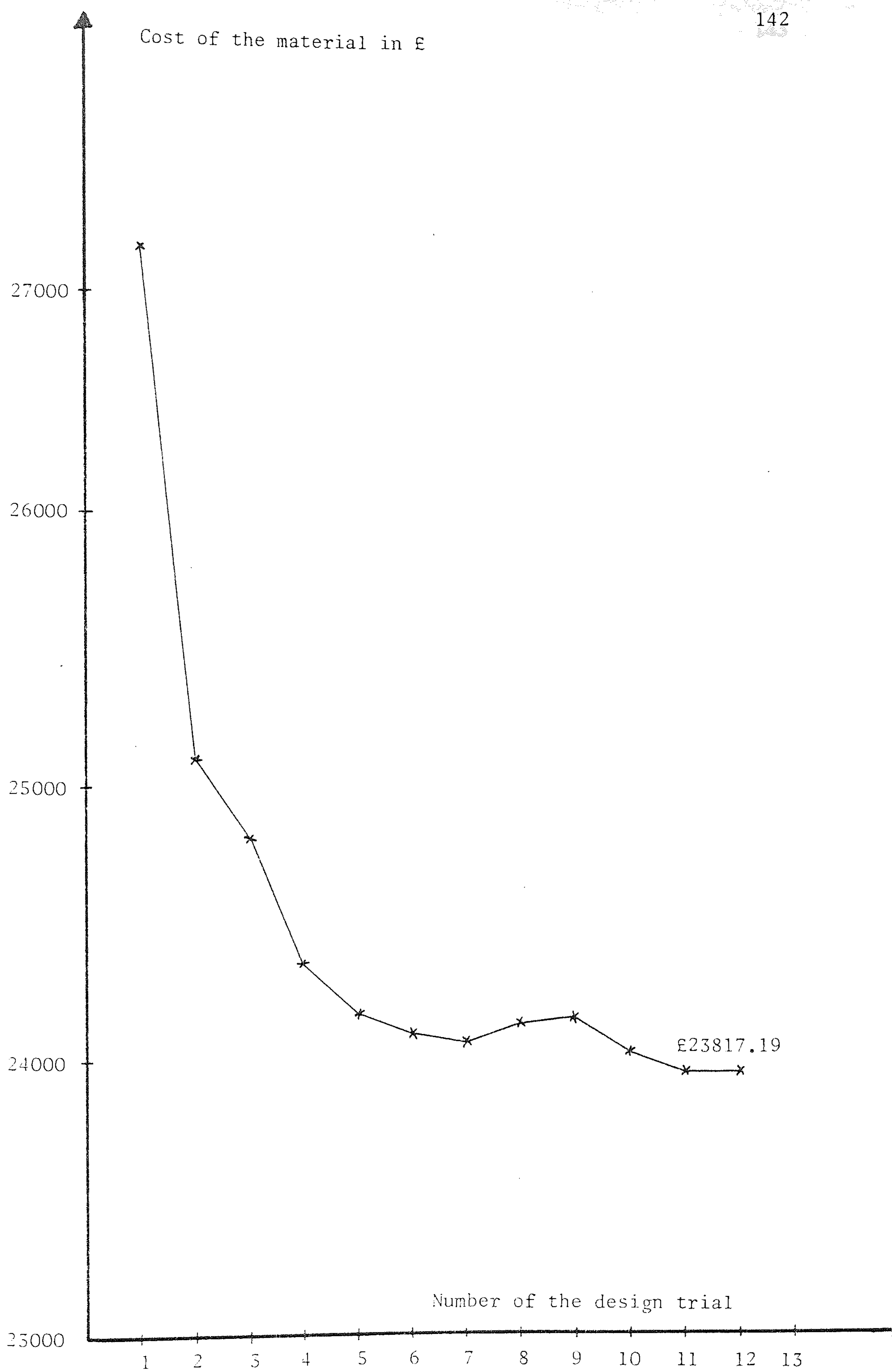


FIGURE 5.8 REDUCTION IN THE MATERIAL COST OF 24 STOREY 3 EQUAL BAY FRAME

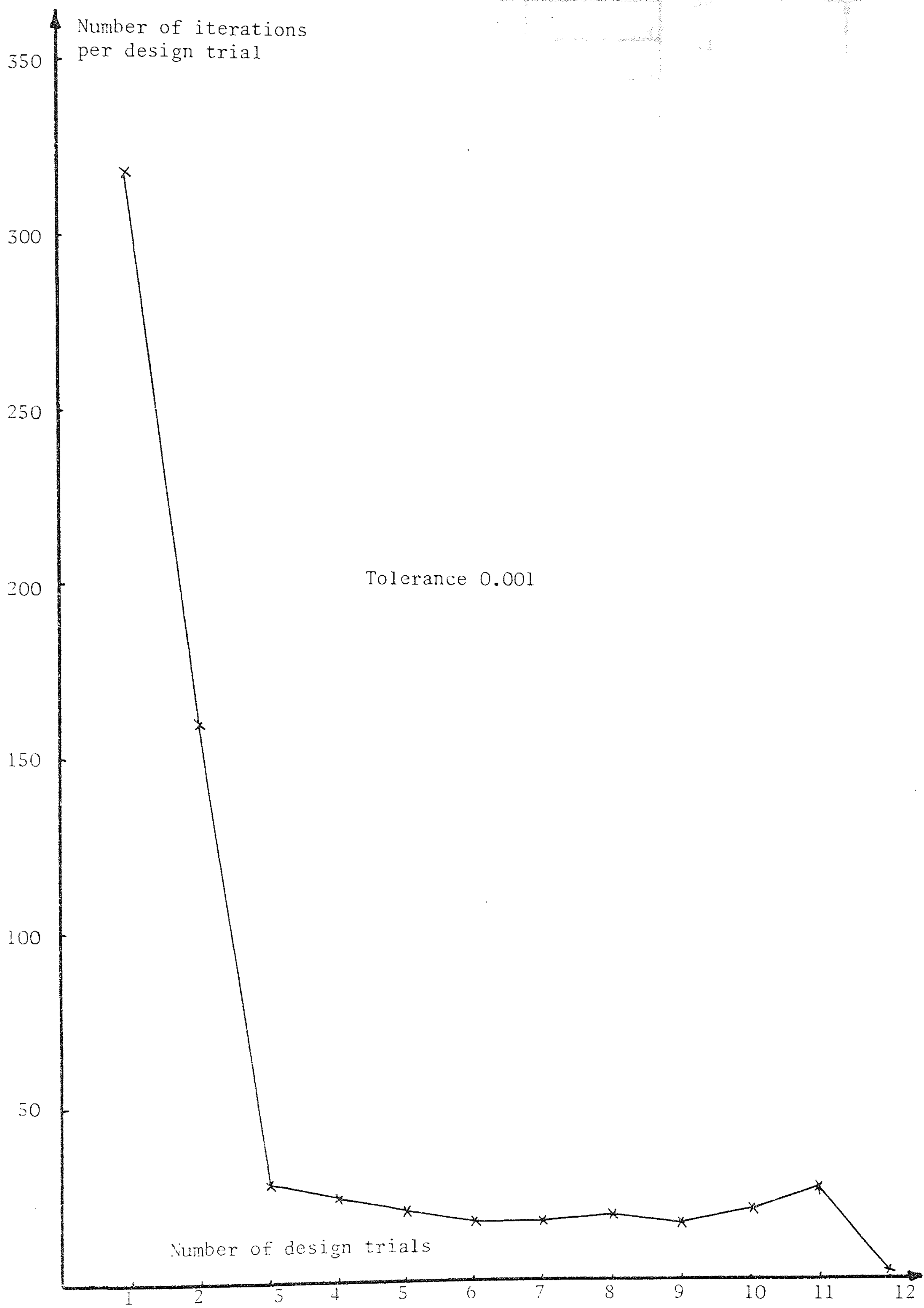


FIGURE 5.9: ITERATION IN EACH DEISGN TRIAL FOR THE 24 STOREY - 3 EQUAL BAY FRAME

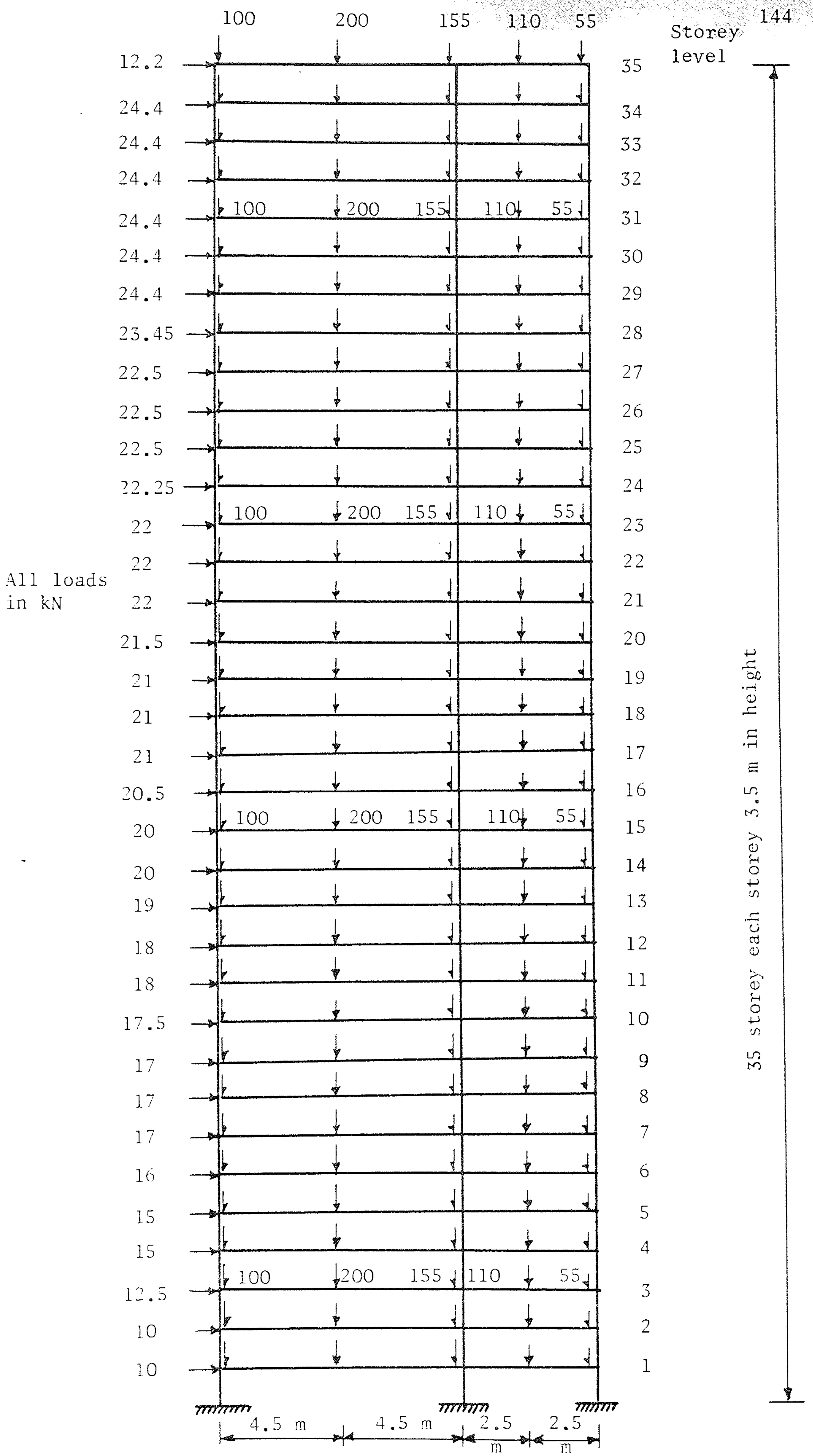


FIGURE 5.10 35 STOREY - 2 BAY FRAME

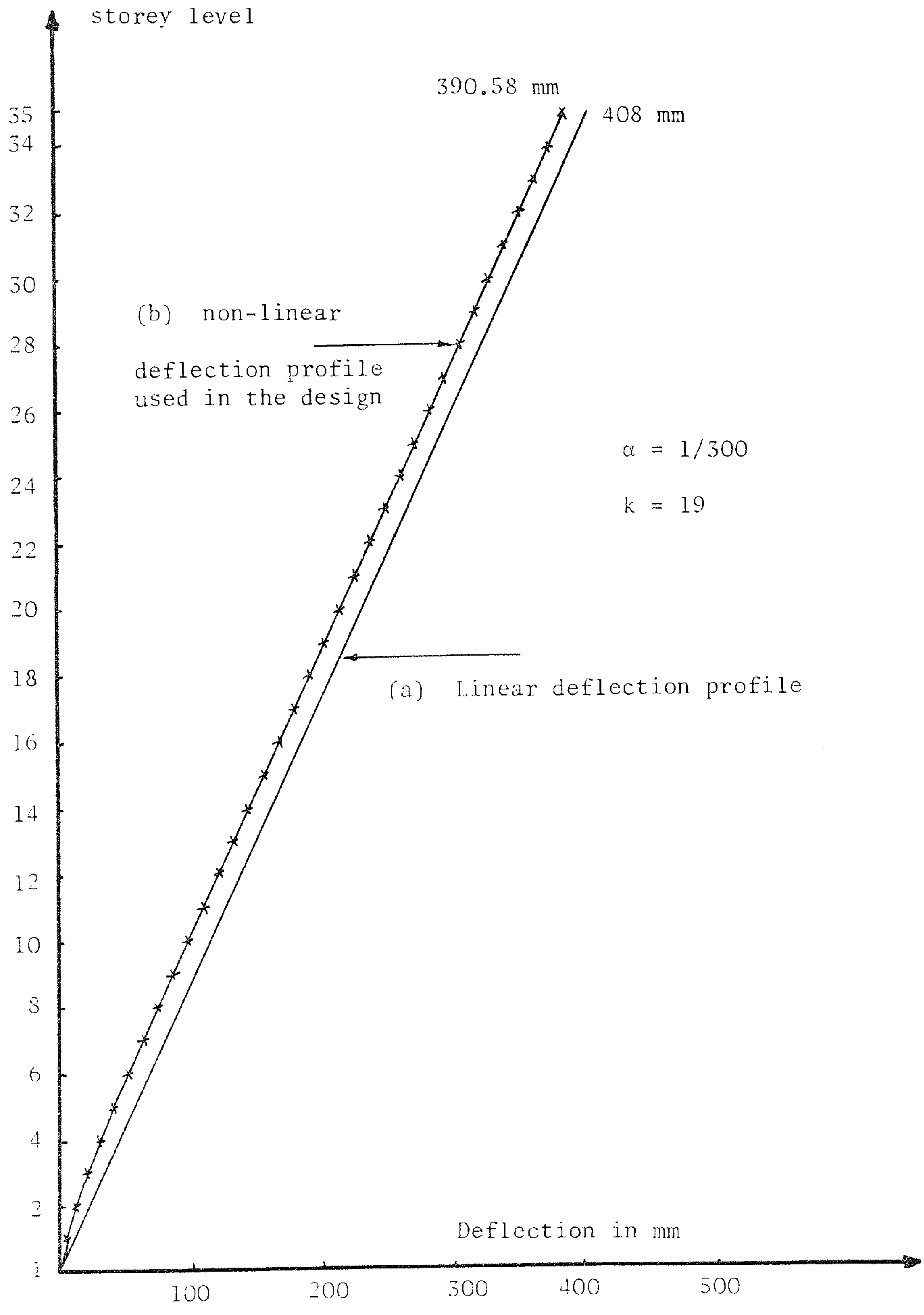


FIGURE 5.11: DEFLECTION CURVE FOR THE 35 STOREY - 2 BAY FRAME

Second moments of area for selected sections in $x10^4$ mm^4					
Storey Level	Left Beam	Right Beam	Left Column	Internal Column	Right Column
Level 1-4	278833	87260	122474	183118	66307
Level 5-6	278833	87260	99994	183118	66303
Level 7	278833	66610	99994	146765	66303
Level 8	278833	66610	99994	146765	50832
Level 9	204747	61530	99994	146765	50832
Level 10-12	204747	55225	99994	146765	50832
Level 13	204747	55225	79110	122474	50832
Level 14-15	168535	55225	79110	122474	50832
Level 16	168535	55225	79110	122474	38740
Level 17	168535	55225	66307	122474	38740
Level 18	168535	55225	66307	99994	38740
Level 19-20	135972	40956	66307	99994	32838
Level 21	135972	37039	66307	99994	32838
Level 22-23	111673	33324	50832	79110	32838
Level 24	98408	29337	50832	66307	32838
Level 25	87260	29337	38740	66307	22416
Level 26	87260	25464	38740	66307	22416
Level 27	87260	21345	32838	50832	17510
Level 28	61530	21345	32838	50832	17510
Level 29	55225	21345	32838	38740	14307
Level 30	55225	15603	22416	32838	14307
Level 31	37039	15603	17510	32838	9462
Level 32	29337	8167	14307	22416	7647
Level 33	21345	5415	9462	17510	6088
Level 34-35	10054	4381	6088	9462	6088
Total mass 138773.50 kg			Total cost £30253.82		

TABLE 5.6: TRIAL INITIAL SECTIONS FOR A 35 STOREY - 2 UNEQUAL BAY FRAME OBTAINED BY ANDERSON AND ISLAM'S EQUATIONS USING THE LINEAR DEFLECTION CURVE WITH $\alpha=1/300$

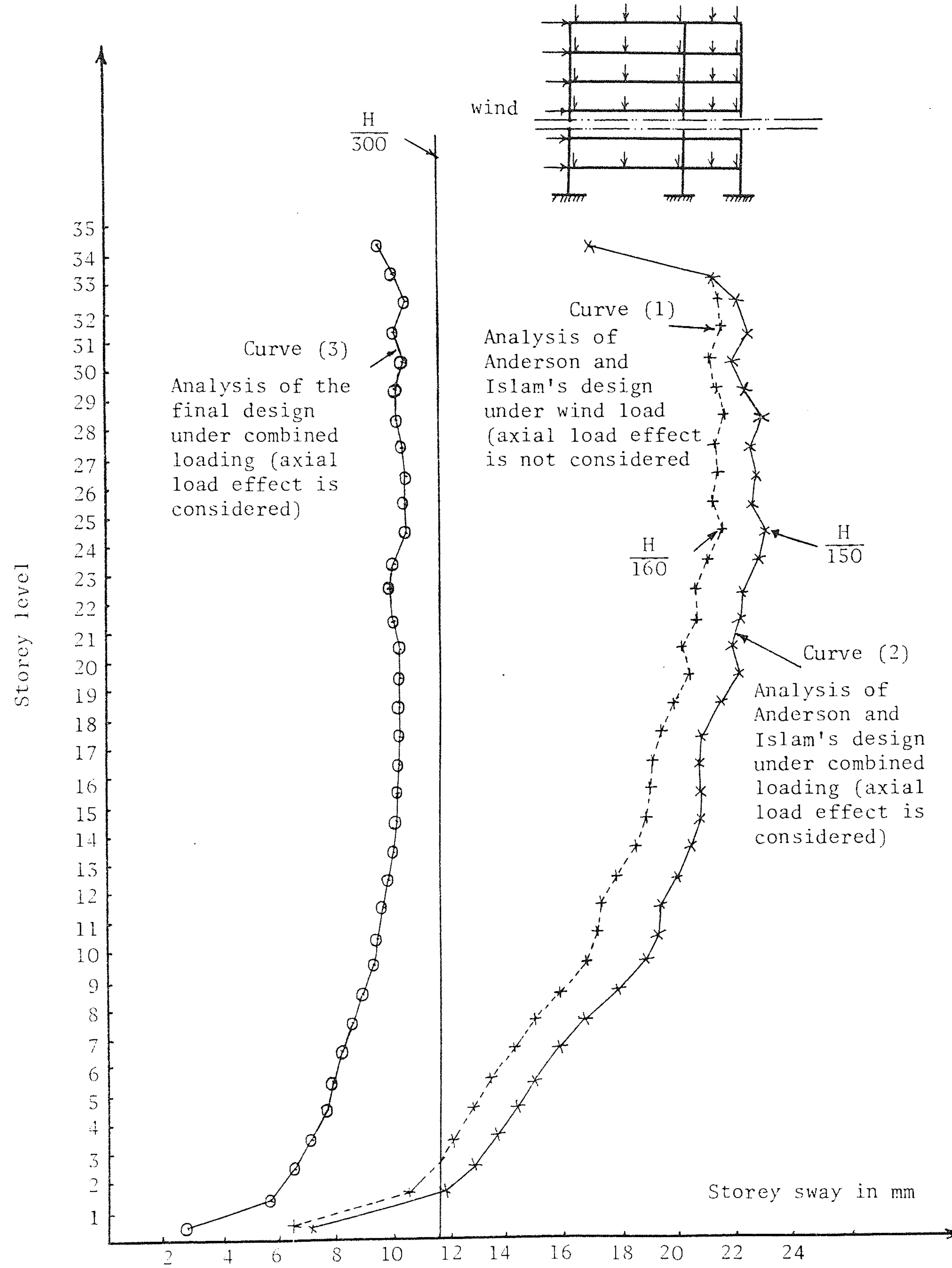


FIGURE 5.12 COLUMN SWAY OF THE 35 STOREY - 2 UNEQUAL BAY FRAME, WITH WIND FROM THE LEFT FOR VARIOUS DESIGNS

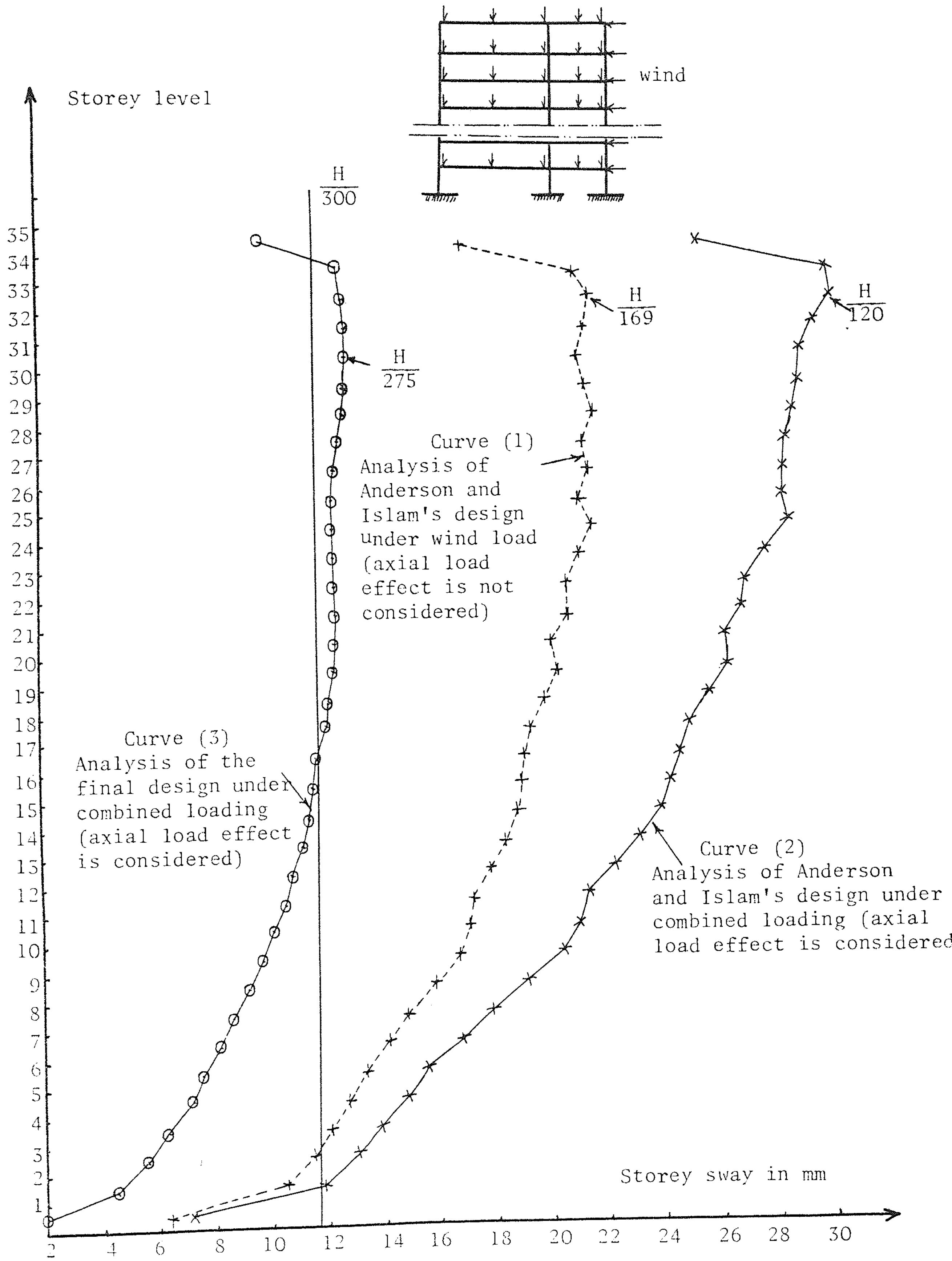


FIGURE 5.13 ANALYSIS OF DIFFERENT DESIGNS OF THE 35 STOREY - 2 UNEQUAL BAY FRAME, WIND FROM RIGHT

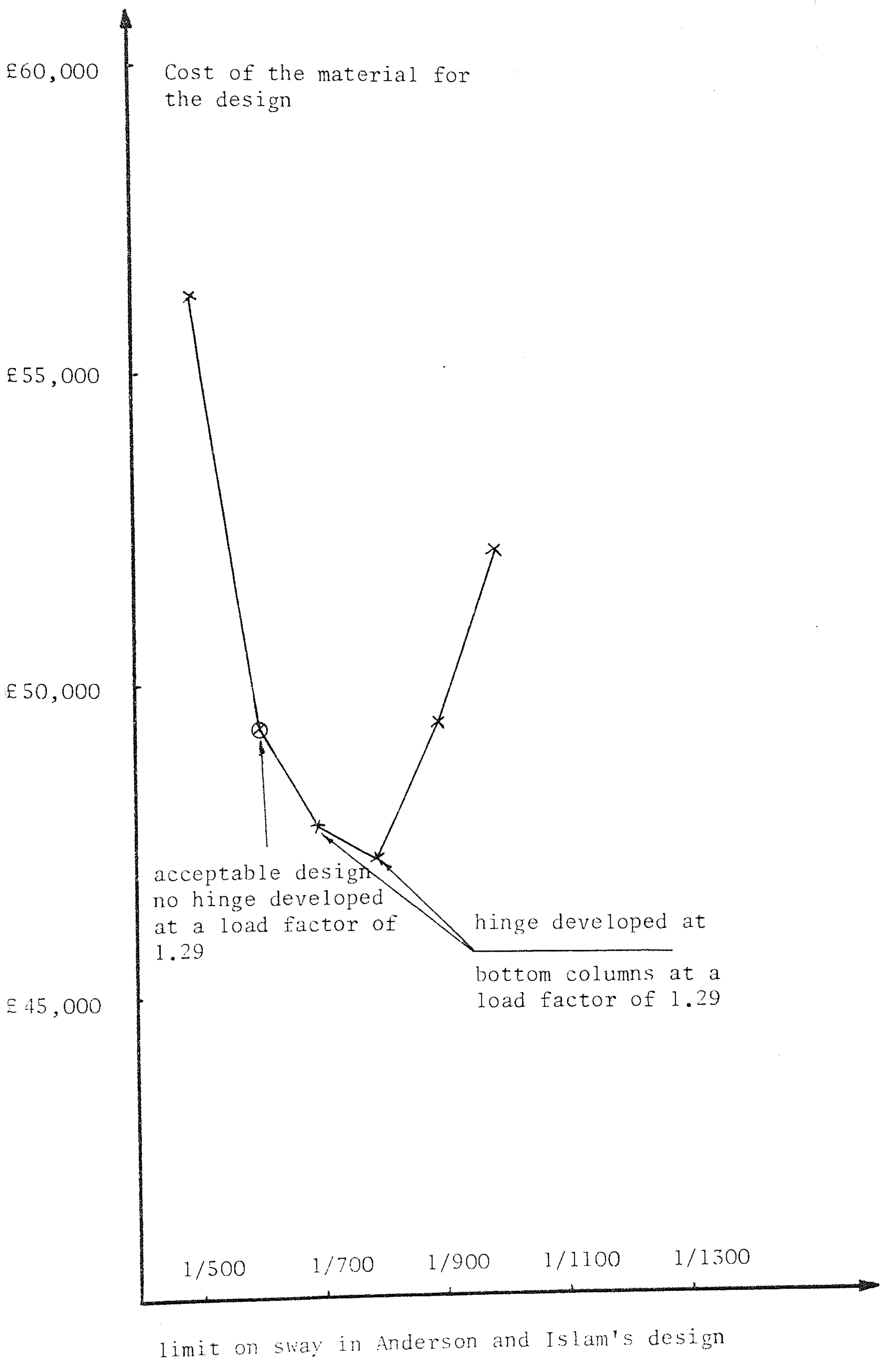


FIGURE 5.14: COST OF THE MATERIAL FOR FINAL DESIGNS STARTING WITH DIFFERENT SETS OF BEAMS OBTAINED FROM ANDERSON AND ISLAM'S DESIGN

Second moments of area of the selected sections				
Storey Levels	Beam $mm^4 \times 10^4$	right column $mm^4 \times 10^4$	Internal column $mm^4 \times 10^4$	left column $mm^4 \times 10^4$
Level 1	503781	146765	378668	275140
Level 2-4	503781	99994	227023	146765
Level 5-8	503781	79110	227023	146765
Level 9-12	435796	79110	227023	146765
Level 13-16	375110	79110	227023	146765
Level 17-19	324715	79110	227023	146765
Level 20-21	278833	79110	227023	146765
Level 22	278833	66307	227023	146765
Level 23	278833	50832	227023	146765
Level 24	278833	50832	183118	122474
Level 25	204747	38740	146765	99994
Level 26	204747	32838	146765	99994
Level 27	168535	32838	122474	79110
Level 28	168535	32838	99994	66307
Level 29	135972	22416	99994	66307
Level 30	135972	17510	79110	50832
Level 31	98408	14307	50832	38740
Level 32	87260	9462	50832	32838
Level 33	61530	6088	32838	22416
Level 34	55225	6088	14307	7647
Level 35	25464	6088	6088	6088
Total mass 221865.17 kg		Total cost £49275.87		

TABLE 5.7: DESIGN OF THE 35 STOREY - 2 UNEQUAL BAY FRAME, COMBINED LOADING, WIND FROM RIGHT

Second moments of area of the selected sections				
Storey Level	Beam mm^4 x 10^4	right column mm^4 x 10^4	Internal column mm^4 x 10^4	left column mm^4 x 10^4
Level 1	503781	146765	378668	275140
Level 2-8	503781	99994	227023	146765
Level 9-12	435796	79110	227023	146765
Level 13-16	375111	79110	227023	146765
Level 17-19	324715	79110	227023	146765
Level 20-21	278833	79110	227023	146765
Level 22	278833	66307	183118	122474
Level 23	278833	50832	146765	99994
Level 24	278833	50832	122474	79110
Level 25	204747	38740	122474	66307
Level 26	204747	32838	99994	66307
Level 27	168535	32838	79307	50832
Level 28	168535	32838	66307	50832
Level 29	135972	22416	66307	38740
Level 30	135972	17510	50832	32838
Level 31	98408	14307	32838	22416
Level 32	87260	9462	32838	17510
Level 33	61530	6088	17510	14307
Level 34	55225	6088	9462	6088
Level 35	25464	6088	6088	6088
Total mass 215414.50 kg		Total cost £47929.49		

TABLE 5.8: DESIGN OF THE 35 STOREY - 2 UNEQUAL BAY FRAME FOR COMBINED LOADING WIND FROM LEFT

CHAPTER SIX

DESIGN OF REINFORCED CONCRETE FRAMES

6.1 INTRODUCTION

The design procedure given in Chapters 2-5 for the design of steel frames is used in this chapter for the design of reinforced concrete frames. The CP110:1972 Code of Practice for The Structural Use of Concrete is based on the principles of Limit State Design. The object of limit state design is to achieve an acceptable probability that a structure will not become unservicable in its lifetime. It, therefore, sets out to examine all the ways in which a structure may become unfit for use.

The condition of a structure when it becomes unserviceable is called a "limit state". The most important of these limit states which must be examined when designing a frame are:

(a) the ultimate limit state which requires that neither the whole structure nor any part of it should collapse under foreseable overload (strength requirements);

(b) the serviceability limit state of deflection which requires that the deflection of the structure should not adversely affect its appearance and;

(c) the serviceability limit state of cracking.

The usual approach is to design a reinforced concrete frame on the basis of the most critical limit state and then check that the remaining limit state requirements are not violated. In general, engineers often try to satisfy the ultimate limit state (strength requirement) first and then check that the deflections of same joints or the sway in the members within acceptable limits. They may also check that any crack that may develop is within acceptable limits. However, for sway frames, especially for tall slender frames the sway in the columns can often be more critical than strength. For this reason, it is believed that the rational approach for the design of such frames will be to consider the limit state of deflection first and then proceed to check for strength and crack development.

Clause 2.2.3.1 on deflection in CP110 recommends the following:

"The deflection of the structure or any part of the structure should not adversely affect the appearance or efficiency of the structure.

The engineer must satisfy himself that deflections are not excessive having regard to the requirements of the particular structure ....

The effect of lateral deflection should be considered, particularly for tall slender structures ...".

The clause does not recommend any specific value as the limit of the horizontal deflection for multistorey frames. However, the original draft did consider a severe limit of height /500 (Allen, 1977). In the absence of a better limit, and as a large sway might produce excessive cracks, the limit of height/500 is adopted for the design of concrete frames in this thesis.

6.2 THE PRINCIPLES OF DESIGN

The design method described in Chapters 2-5 is also applied here to generate the values of the required second moments of area for the columns of a multistorey sway frame. This is done to satisfy the horizontal sway limitations imposed by the serviceability limit state of deflection. As before, this is conditional on that the values of the second moments of area for the beams being known before starting the iteration. The second moment of area of each column is then used to obtain the minimum column depth required so that the sway does not exceed the limit.

It was found, when designing several frames, that the actual column depth obtained by this method is considerably greater than the minimum imposed by the code.

The code suggests in clause 3.5.8 that if the depth of a column is more than its effective height/30 then the sway requirement may be satisfied without further calculation. However, the author found that this underestimates the column depth needed for multi-storey frames and particularly the depth of the columns at the lower storeys.

The 1972, CP110 also requires that the frame should be designed for the serviceability limit state of deflection for different load cases. To produce a design which satisfies all the load cases, the column sizes obtained, when designing for the first loading case, are used as lower bounds for the second loading case, and so on. The application of lower bounds obtained when designing for a previous load case ensures that each load condition is satisfied.

In the proposed design method, the sections obtained from the final deflection limitation design are checked for strength requirements. Again the frame has to be checked for all the ultimate limit state load cases. For this purpose, an elastic analysis using the iteration method given in Chapter 2 is carried out for each load case. Once again the axial load effect and the axial stiffness (EA/L) are taken into consideration. The maximum bending moments, the shear force, and the axial force from all the load cases are obtained, and the sections are checked for these ultimate values.

6.3 DESIGN ASSUMPTIONS

When iteratively analysing a reinforced concrete frame for the ultimate limit state, the relative stiffness of the members may be based on any one of the following (CP110, clause 2.4.3.1):

(a) The concrete section: the entire concrete cross-section, ignoring the reinforcement.

(b) The gross section: The entire cross-section, including the reinforcement on the basis of modular ratio. The second moment of area for this section is called the uncracked second moment of area as all the area of the concrete is considered.

(c) The transformed section: The compression area of the concrete cross-section combined with the reinforcement on the basis of modular ratio. The second moment of area for this section is called the cracked second moment of area as the tension area of the concrete cross-section is ignored.

The work of Beeby and Taylor (1978) shows that the results of an analysis vary considerably with the assumption made in choosing the properties of the sections. Their analysis of a portal frame using different assumptions showed that the maximum bending moment was obtained when the cracked second moment of area for the beam section was used and the uncracked one for the column section.

In a multistorey frames a column should be designed to resist the axial load and the bending moment. Usually the axial force in a column dominates the design of the section (except for the top one or two storeys where small axial forces may occur). If the moment is small and the axial load is comparatively large, then the position of the neutral axis would be outside the section. In this case the column area would be under compression and the section uncracked. For the reasons described above, it is logical to assume that:

i) The relative stiffness of a column section is based on assumption (b), and the uncracked second moment of area is used.

ii) For a beam section the bending moment would dominate the design of the section and the moment would be large enough to make the major parts of the beam crack. Therefore, it is reasonable to assume that the relative stiffness of a beam section is based on assumption (c), and the cracked second moment of area of a rectangular section is used.

Assumptions (i) and (ii) are used in this chapter for evaluating the relative stiffness for both ultimate limit state and serviceability limit state of deflection. In fact CP110:1972 Appendix A, clause 1, states that greater accuracy may be achieved in the deflection calculation if the relative stiffness of the cracked beam is used.

iii) The effects on deflection of temperature, creep and shrinkage are ignored in this thesis.

iv) The bar spacing rules should be satisfied so that no excessive cracking occurs. In this respect, it is assumed that the reinforcement will be distributed within the section so that the distance between two bars does not infringe the maximum and minimum limits. Furthermore, the cover for the reinforcement is chosen according to the characteristic strength of the materials to prevent the influence of weather on reinforcement. Minimum concrete covers for the reinforcement are given in clause 3.11.2, and Table 19 of CP110. In the design examples given in this chapter these recommendations are taken into consideration.

v) For practical purposes the overall depth of a beam or a column is rounded to the nearest 50 mm.

6.4 REINFORCEMENT REQUIREMENTS

Requirements regarding areas of reinforcement in a beam are set out in CP110 as follows:

(1) Clause 3.11.4.1. The area of tensile reinforcement should not be less than: 0.15% $b_t d$ for high yield reinforcement and 0.25% $b_t d$ for mild steel. Here b_t is the width of the section and d is the effective depth. For a T-section, b_t should be taken as the average breadth of the concrete below the upper flange.

(2) Clause 3.11.5. The maximum area of reinforcement in either tension or compression should not exceed 4% of the gross cross-sectional area of the concrete.

Requirements regarding areas of reinforcement in the columns are defined in clause 3.11.4.1; viz

(a) The minimum area of main longitudinal bars is 1% of the gross area of the column

(b) The minimum number of bars in a rectangular column is 4 and their diameter should not be less than 12 mm.

The maximum area of longitudinal reinforcement is given in clause 3.11.5, CP110. This should not exceed 6% of the gross cross-section in vertically cast columns.

6.5 DESIGN LOADS

Unlike the Steel Code of Practice, factored loading is not allowed in CP110;1972. Furthermore, the design loads for the serviceability limit states of deflection are not just the working loads.

The characteristic loads are defined in clause 2.3.1 as:

(a) Characteristic dead load G_k which is the weight of the complete structure.

(b) Characteristic imposed load Q_k which depends on the use of the building. These loads are given in CP3 Chapter V, Part 1, for various buildings.

(c) Characteristic wind load W_k. This is defined and calculated in accordance with CP3(1970) Chapter V, Part 2.

The design load is obtained by multiplying the characteristic load by the partial factor of safety γ_f which takes account of (1) possible overloads; (2) inaccurate assessment of the effects of loading and unforeseen stress redistribution within the structure; (3) variations in dimensional accuracy; and (4) the limit state being considered. The value of γ_f varies for the different limit states as set out in Tables 6.1 and 6.2.

Load Combination	Dead Load	Imposed Load	Wind Load
1 - Dead and imposed load	1.0	1.0	0
2 - Dead and wind load	1.0	0	1.0
3 - Dead, imposed and wind load	1.0	0.8	0.8

TABLE 6.1: VALUES OF γ_f, SERVICEABILITY LIMIT STATE OF DEFLECTION

6.5.1 Design Loads for Serviceability Limit

For the serviceability limit state of deflection there are three load combinations. The first load combination concerns vertical loads only and as sway is mostly due to wind load, the first load case is disregarded in this thesis. The second load combination gives $1.0\ G_k + 1.0\ W_k$ and there is no need to consider any change in the load arrangements. The third load combination gives $1.0\ G_k + 0.8\ Q_k + 0.8\ W_k$.

6.5.2 Design Load for Ultimate Limit State

With regard to ultimate limit state, the arrangement of loads should cause the most severe forces. Under load combination (1) on the loaded spans there should be $1.4\ G_k + 1.6\ Q_k$, but only $1.0\ G_k$ on the unloaded span. For example, for a three bay frame, four arrangements should be considered. Figure 6.1 shows the different loading patterns for individual floors. The frame should be analysed four times, with the appropriate loading pattern applied on all floors.

Load Combination	Dead Load Max.	Dead Load Min.	Imposed Load Max.	Imposed Load Min.	Wind Load
1 - Dead and imposed load	1.4	1.0	1.6	0	0
2 - Dead and wind load	1.4	0.9	0		1.4
3 - Dead, imposed and wind load	1.2		1.2		1.2

TABLE 6.2: VALUES OF γ_f, ULTIMATE LIMIT STATE

Under load combination (2), the most critical condition will have to be considered. This may arise when moments due to 1.4 G_k on some parts of the structure are added to the wind moments, and moment due to 0.9 G_k on other parts of the structure from the restoring moment. For the three-bay frame, another four load cases should be considered. Figure 6.2 shows the different loading patterns for individual floors. It should be noted that the wind load for the four arrangements remains the same, i.e. 1.4 W_k.

Under load combination (3) a factor of 1.2 on all loads is used throughout the structure, with no variations for loaded and unloaded spans. This load case is similar to the one considered in steel design and the two codes CP110 and the draft B/20 agree on the load factor of 1.2.

It can be seen that for the three-bay frame, nine load cases should be considered, to obtain the maximum internal forces in the frame. The number of load cases obtained from the three load combinations depends on the number of bays in the frame, but at least four load cases are needed for a single bay frame.

6.6 EQUATION FOR THE MINIMUM COLUMN DEPTH

The iteration technique described in Chapter 2 is used to obtain the second moments of area for the columns so that the sway deflections are satisfied. These are then used to obtain a lower bound for the overall depth of each column.

For a rectangular section Figure 6.3, the uncracked second moment of area of the section is given by Reynolds and Steedman (1976) as:

$$I_{xx} = \frac{1}{3} b \, [x^3 + (h-x)^3] + (\alpha_e - 1) \\ [A_s \, (d-x)^2 + A'_s \, (x-d')^2] \qquad (6.1)$$

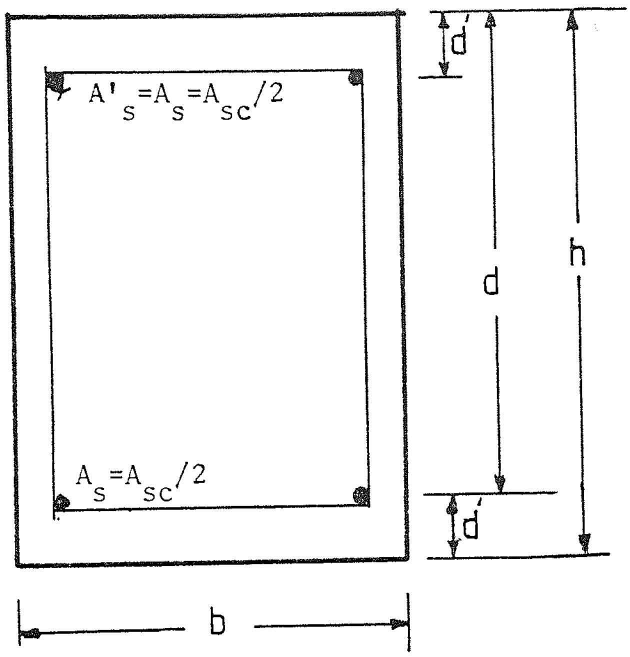


FIGURE 6.3: A TYPICAL COLUMN SECTION

where x is the distance of the neutral axis below the top edge, which is

$$x = \frac{1}{A_{tr}} \left[\frac{1}{2} bh^2 + (\alpha_e - 1)(A_s d + A_s' d')\right] \tag{6.2}$$

Here A_{tr} is the effective area given by:

$$A_{tr} = bh + (\alpha_e - 1)(A_s + A_s') \tag{6.3}$$

and α_e is the modular ratio, thus

$$\alpha_e = E_s/E_c \tag{6.4}$$

where E_s is the modulus of elasticity of steel and

E_c is the modulus of elasticity of concrete. Values for E_c are obtained from Clause 2.4.2.2 of CP110 for short term loading. In these expressions A_s is the area of tensile reinforcements, A_s' is the area of compression reinforcements h is the overall depth of the column section, b is the width of the section, d is the effective depth to tensile reinforcement and d' is the depth to compression reinforcement.

When designing the column section, it is common practice to take the compression reinforcment equal to the tensile reinforcement (i.e. $A_s = A'_s$). In fact, all the design charts for rectangular column cross-section in CP110:Part 2:1972 have been derived using $A_s = A'_s$. If this is the case, then equation (6.2) is simplified to $x = h/2$, and equation (6.1) becomes

$$I_{xx} = bh^3/12 + 2A_s\,(\alpha_e - 1)\,(h/2 - d')^2 \tag{6.5}$$

Equation (6.5) could be written in terms of the percentage of the reinforcement ρ, thus

$$I_{xx} = bh^3/12 + \frac{\rho bh}{100}\,(\alpha_e - 1)\,\left(\frac{h^2}{4} - hd' + d'^2\right) \tag{6.6}$$

where $$\rho = (100 \times 2A_s)/(bh) \tag{6.7}$$

Let

$$C_1 = \rho\,(\alpha_e - 1)/100 \tag{6.8}$$

and with $E_s > E_c$ C_1 is always positive. Equation (6.6) can be written in terms of C_1, thus:

$$\frac{bh^3}{12}\,(1 + 3C_1) - C_1 d'bh^2 + C_1 d'^2\,bh - I_{xx} = 0 \tag{6.9}$$

Now with $$C_2 = b(1 + 3C_1)/12 \tag{6.10}$$

which is always positive, equation (6.9) becomes:

$$h^3 - \frac{C_1 d'b}{C_2}\,h^2 + \frac{C_1 d'\,b}{C_2}\,h - I_{xx}/C_2 = 0.0 \tag{6.11}$$

which can be written in the form

$$h^3 + A_1 h^2 + A_2 h + A_3 = 0.0 \tag{6.12}$$

where

$$A_1 = -C_1 d' b / C_2 \tag{6.13}$$

$$A_2 = C_1 d'^2 b / C_2 \tag{6.14}$$

$$A_3 = - I_{xx} / C_2 \tag{6.15}$$

An analytical solution for equation (6.12) is available (Spiegel, 1968), and it depends on the value of the discriminant. This is defined as:

$$D = Q^3 + R^2 \tag{6.16}$$

where
$$Q = (3A_2 - A_1^2)/9 \tag{6.17}$$

and
$$R = (9A_1A_2 - 27A_3 - 2A_1^3)/54 \tag{6.18}$$

It can be proved that D in this case is always a positive number. For a positive discriminant equation (6.12) has only one real root, and two other complex conjugates. This means that there is only one real solution for equation (6.12) which is

$$h = S + T - 1/3\ A_1 \tag{6.19}$$

where $S = \sqrt[3]{R + \sqrt{D}}$ (6.20)

and $T = \sqrt[3]{R - \sqrt{D}}$ (6.21)

Here R and D are as defined by equations (6.18) and (6.16) above. It should be noticed that to be able to calculate h the values of ρ, b and d' should be known. These are given as data in the computer program written for this purpose.

6.7 DESIGN OF A BEAM SECTION

The second moments of area for a beam section should be known before the iteration technique can be applied to obtain the column second moments of area. The width b of the beam section is first assumed, and then the effective depth d is taken as the maximum obtained by the following two methods:

1) The second moment of area I of the beam is obtained from Islam (1978) and Anderson and Islam (1979). The plain concrete section is then used, only to obtain a preliminary beam depth from $I = bh^3/12$ or $h = \sqrt[3]{12I/b}$.

2) The effective depth is obtained from the limitation of the span/effective depth ratio. The allowable value for span/effective depth ratio depends on:

   a) The span and the support conditions;

   b) The amount of tensile steel and its stress;

   c) The amount of compression steel; and

   d) The type of beam.

The basic span/effective depth ratios for rectangular beams are given in table 8, CP110. These are based on a beam with 1% tension reinforcement with a characteristic strength of 410 N/mm^2. The limit on deflection is span/250. The values given depend on the support conditions and apply to beams up to 10 m span. The basic span/effective depth ratio from Table 8, CP110 is 20 for a simply supported beam. To obtain a safe preliminary beam depth, this ratio is taken to be 20 for the beams of the frame. For a steel with a characteristic strength of 250 N/mm^2, the span/effective depth ratio is taken to be 29 according to Table 10, CP110.

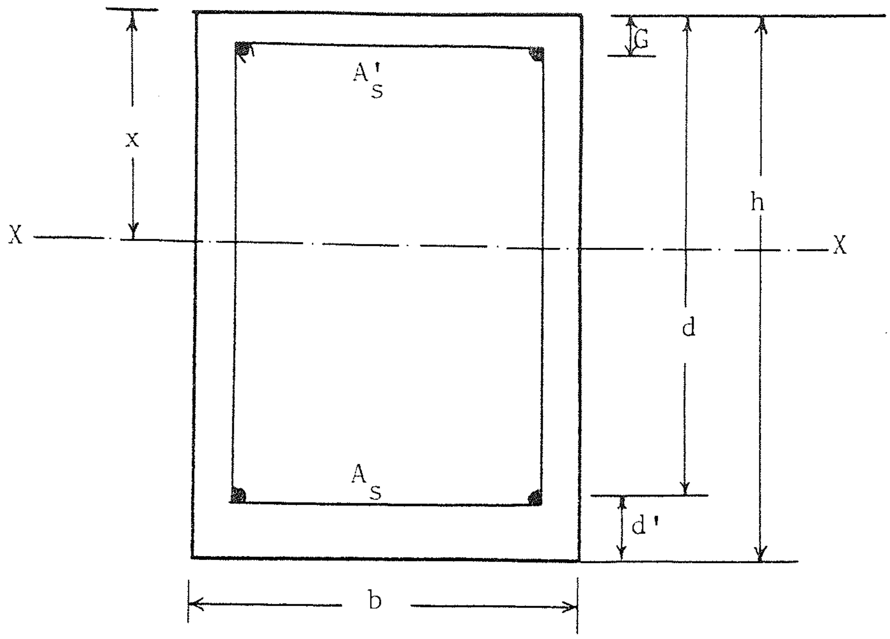


FIGURE 6.4 A TYPICAL RECTANGULAR BEAM SECTION

The larger value for the beam depth obtained from (1) or (2) above is used with a reinforcement of 1% to obtain the cracked second moment of area of the rectangular section. This is then used in the iteration technique to find the second moments of area of the columns so that the sway in each column is satisfied.

6.7.1 Design of the Beam Section for Ultimate Limit State

When checking the beam section for strength requirements (ultimate limit state), the maximum sagging and hogging bending moments are needed. These are obtained from applying all the ultimate limit state load cases and then choosing the most severe bending moments. These are then used to obtain the reinforcements for the beam section. For the maximum hogging moment a rectangular cross-section (Figure 6.4) is used with width b, and effective depth d obtained by (1) or (2) above in section 6.7. For the

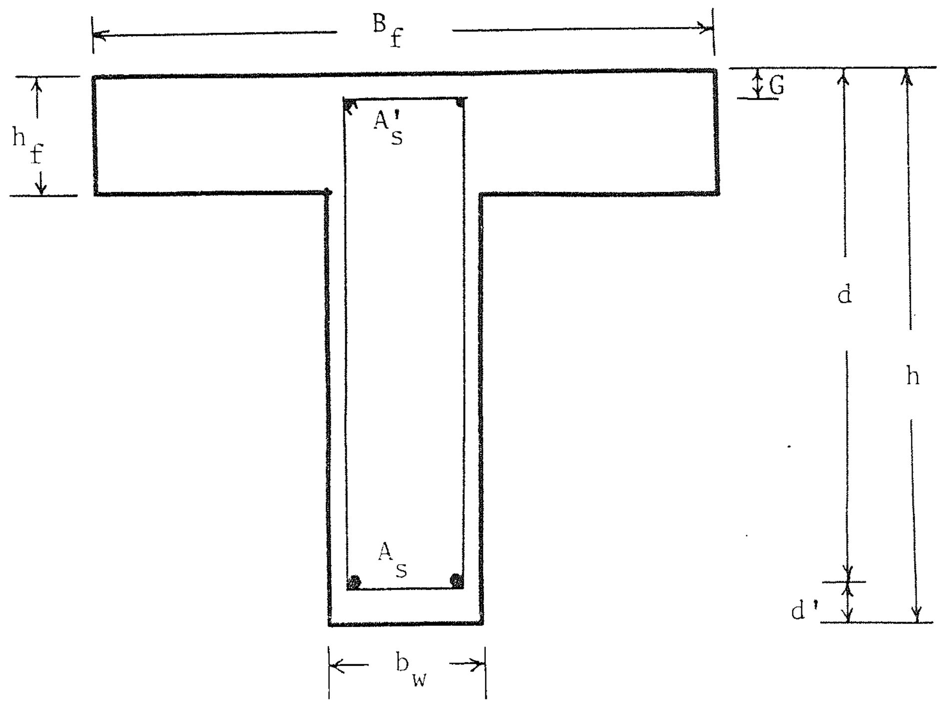


FIGURE 6.5 A TYPICAL T-BEAM SECTION AT MID-SPAN

maximum sagging moment a T section is used, where the beams are integrally cast and support a continuous floor slab. Part of the slab adjacent to the beam is thus regarded as the compression flange, see (Figure 6.5). In this figure B_f is the effective width of the compression flange, b_w is the width of the web and h_f is the thickness of the floor slab. The effective width B_f of the compression flange is specified in Clause 3.3.1.2 of CP110.

For both rectangular and T-sections, the tensile reinforcements are considered first. However, if the concrete in compression is overstressed, compression reinforcements is then added to assist the concrete to carry the extra load.

Design charts for singly reinforced beams and doubly reinforced beams are given in CP110:Part 2:1972. All the charts have been derived using the assumptions given in Clause 3.3.5.1 of CP110:Part 1:1972 for the analysis of cross-sections. A computer subroutine was written by MacGinley (1978) to derive these charts. This however was limited to rectangular sections, with a constant beam depth. For this reason, MacGinley's flow-chart is altered in this thesis to design T-sections, and to increase the beam depth for cases when the maximum reinforcement in tension and compression is exceeded.

6.8 ULTIMATE LIMIT STATE FOR THE COLUMNS

Once the columns are designed for deflection limit state, the overall calculated depth h_j of a section and the area of reinforcement A_{sd} must be checked for the ultimate limit state. To do this, it is necessary to calculate the maximum bending moment M_{max} and the maximum axial load N_{max} for all the load cases. Checking is carried out by recalculating the area of reinforcement A_{sc} for a column section of depth h_j to provide the section with an ultimate resistance to M_{max} and N_{max}. The area A_{sc} is then adopted if $A_{sc} > A_{sd}$. This is provided that the area of the reinforcement is less than the maximum allowable 6%. If not, the computer programme increases the depth h_j in increments of 50 mm and the area A_{sc} is recalculated until it is below 6%.

The reinforcement for columns resisting moments and axial forces can be either calculated using Clause 3.5.5.3 of CP110 or obtained using charts given in part 2 of the code. The charts are for rectangular parabolic stress distribution for concrete and a trilinear stress-strain curve for the steel reinforcements, as for beams (Clause 3.3.5.1, CP110). A computer subroutine is written to derive these charts. This makes use of certain significant points on the chart to accelerate the trial and error procedure needed for the design. To show how this subroutine works an explanation is given first of how the design charts are constructed and what properties they have. More information about the use of these charts is given in CP110:Part 2:1972, (Allen 1977) and (MacGinley 1978).

6.8.1 Construction of Design Chart in CP110

A symmetrically reinforced column section subjected to the ultimate axial load N_{max} and the ultimate moment M_{max} is considered. The moment is equivalent to the axial load acting at an eccentricity $e = M_{max}/N_{max}$. Depending on the relative values of N_{max} and M_{max}, two cases occur for analysis.

i) Compression over the whole section where the neutral axis is outside the section.

ii) The section is divided into two parts by the neutral axis, in one part the steel is under tension while in the other part both steel and concrete are in compression.

For a given location of the neutral axis, and for a given reinforcement, the strains and stresses in both the concrete and the steel can be determined, and from these the values for the internal forces can be found. The resultant internal axial force and resistance moment can then be evaluated. These should be equal to N_{max} and M_{max}. Consider a section subject to an ultimate load N_{max} which is less than the capacity of the section under pure axial load. This section is able to support N_{max} and a determinable maximum ultimate moment M_{max} when the concrete is at its maximum strain and design strength. The stresses in the steel depends on the location of the neutral axis.

It can be noticed from the above that for a given percentage of reinforcement the value of the resisting moment and axial force for the section depends on the location of the neutral axis. This means that successive trials would be required to find the solution. To assist the engineer the different trials are readily calculated and plotted in the form of a design curve in part 2 of CP110.

For a selected grade of concrete and reinforcement, the design curve for a section with a given percentage of reinforcement 100 A_{sc}/bh and its location d/h symmetrically placed about the centre line of the section is formed. This is done by plotting values of N/bh against M/bh^2 for various positions of the neutral axis. It should be noticed that A_{sc} is the total area of reinforcement and that this is divided equally between the faces parallel to the axis of bending. (e.g. in Figure 6.3 $A_s = A_s' = A_{sc}/2$). A typical design curve is shown in Figure 6.6 with the stress strain curve for the reinforcement shown in Figure 6.7, other curves can be constructed for percentages of steel ranging from zero to a maximum of 8%. The family of curves forms the design chart for a particular combination of materials and a typical chart is shown in Figure 6.8.

In Figure 6.6, the specific points correspond to those in the stress-strain curve for steel as shown in Figure 6.7. For example, point A in the design curve corresponds to the final yield in the stress-strain curve.

The strain in the steel is obtained from the stress strain curve (e.g. for mild steel the strain is 0.00309). While the strain in concrete is always assumed to be at its maximum value of 0.0035 as specified in the code. The depth x of the neutral axis is calculated assuming a linear strain distribution as shown in Figure (6.9b). From this:

$$\frac{0.0035}{x} = \frac{0.00309}{0.8h-x} \quad \text{or} \quad x = 0.425h$$

Similarly, for point B in the design curve which corresponds to a strain of 0.00087 at first yield, the strain diagram shown in Figure(6.9 c) gives the position of the neutral axis as:

$$\frac{0.0035}{x} = \frac{0.00087}{0.8h-x}$$

$$x = 0.646h$$

Point c in the design curve corresponds to zero strain in tensile steel (Figure 6.9d). Thus the neutral axis is at the centre of the tensile steel with zero stress. For the part CDE of the design curve in Figure 6.6 the tensile steel is in compression. For the part FABC of the curve the steel is in tension. Point F of the curve is for x = h - d. This value of x is the minimum allowable in the code. No solutions are given for x less than h-d. The design curve is flattened off at part DE to ensure that a minimum allowable moment of 0.05 N_{max} h is automatically included.

From the above description of the design curve, it could be concluded that five significant points can be recognised. These are:

(1) Point F where $x = h - d = d'$. This is the lowest allowable value of x

(2) Point A where x is obtained for final yield of the tensile steel

(3) Point B where x is obtained for first yield of the tension steel

(4) Point C where $x = d$, and the stress in the tensile steel is zero.

(5) Point D where x is obtained for the section resisting a minimum moment of 0.05 Nh.

The design curve FABCDE is for a specific percentage of steel as 4%. Other design curves for the steel percentages of 0, 1, 3, 5, 6, 7 and 8 may be plotted. These form the design chart shown in Figure 6.8. The various zones which depend on the location of the neutral axis are marked on the chart. The significant points on the design curves are marked, and equivalent points are connected by the dotted lines to form these zones.

6.8.2 Computer Subroutine

A computer subroutine is written in Fortran to determine the amount of reinforcement required in a symmetrically reinforced section to resist a given axial force N_{max} and moment M_{max}. If $M_{max} < 0.05\ N_{max}h$ then M_{max} is taken as equal to $0.05\ N_{max}$. A process of successive trials making use of the significant points on a design chart is used. The process is general, but it would be simpler to explain by the following example.

6.8.3 Example

Design the reinforcement of a column section subjected to an ultimate load of N_{max}/bh of 10.5 N/mm^2 an ultimate moment of

M_{max}/bh^2 of 3.5 N/mm^2. The materials are grade 25 concrete and mild steel reinforcement. The location of the steel bars is given by d/h = 0.8.

The design chart for the materials specified is shown in Figure 6.8, and the subroutine is searching for point R which is located as follows:

(1) Set the percentage of reinforcement at the minimum value of 1%.

(2) Set the depth of the neutral axis to correspond to point A in the design chart with x = 0.425 h.

(3) Calculate the value of the resisting moment and axial force Q and W.

(4) If $Q < M_{max}$ and $W < N_{max}$, then increase the percentage of reinforcement by 0.25%.

(4a) Repeat step 4 until $Q \geq M_{max}$ or $W \geq N_{max}$, or the percentage of steel is starting to become more than 6%.

(5) If the percentage of reinforcement exceeds 6% and Q is less than M_{max} and W is less than N_{max}, then increase column depth by an increment of 50 mm. Steps 1 to 5 are repeated.

(6) Depending on the values of Q and W one of these two cases occurs first.

a) $Q > M_{max}$ but $W < N_{max}$. This means that point R is above the dotted line AA' on the design chart.

b) $Q < M_{max}$ but $W > N_{max}$. This means that point R is below the line AA'. For the example under consideration, case (a) occurs. In Figure 6.8, as soon as the steel percentage is increased from 3.50% to 3.75%, Q/bh^2 will be equal to 3.6 and W/bh = 3.8. Thus $Q > M_{max}$ and $W < N_{max}$. This means that point R for this example is above the line AA' and the depth of the neutral axis is more than 0.425h.

From the above steps it is only possible to know whether the neutral axis is below or above one of the dotted lines shown in Figure 6.8. Repeating the steps with points B and C it is possible to specify upper and lower boundaries for the depth of the neutral axis. Once this is done, a new trial position of the neutral axis is obtained by:

$$x_3 = (x_1 + x_2)/2 \tag{6.22}$$

where x_1 and x_2 are the upper and lower boundaries respectively. For point R for instance, $x_1 = 0.8h$, and $x_2 = 0.646h$. Thus for the first trial $x_3 = (0.8h + 0.646h)/2 = 0.723h$ steps 1 to 6 are now repeated to determine whether x_3 is a new lower or upper bound. Equation (6.22) is then applied with this value of x_3 replacing x_2 to calculate a new value for x_3 and the process is repeated until two successive values of x_3 are within a specified tolerance.

The amount of reinforcement is obtained by starting with A_{sc}/bd as 1% and then increasing it in steps of 0.25% until $Q = M_{max}$, and $W = N_{max}$. At this point the actual neutral axis is known as well as percentage of steel. A large number of columns were designed by the above computer subroutine and it was found to be faster than that used by MacGinley (1978). Usually five to seven trials are needed to determine the actual position of the neutral axis.

6.9 COMPUTER PROGRAM FOR THE DESIGN OF REINFORCED CONCRETE FRAMES

A program for the design of two dimensional, rectangular reinforced concrete, sway frames was written in Fortran and run on the ICL 1904S computer at the University of Aston. This program makes use of the design procedure described in this chapter. The program consists of a master segment calling a number of segments, the functions of which are similar to that of the steel program described in Chapter 4. However, in the program for the design of reinforced concrete frames, the beam sections are kept unaltered. The ratio between the second moment of area for internal and external columns is also kept constant. This is because there is more flexibility in the procedure for the design of reinforced concrete columns. For instance, the design for deflection limit state is carried out with 1% reinforcement and rather than altering beam sections, the ultimate limit state design is obtained by increasing this percentage.

6.9.1 The Program Segments for the Design of Reinforced Concrete Frames

This consists of the master segment calling eight other segments these are:

1 - ITERATE

2 - INTERNAL FORCES as in the steel program

3 - STABILITY-F

4 - The function CDEPTH: This calculates the depth of the column provided that the second moment of area of the column section and the reinforcement ratio are known. This segment makes use of equation (6.19).

5 - The function SMAB: This calculates the cracked second moment of area for a beam section provided that the beam depth is known. 1% reinforcement is assumed. The beam depth is calculated as

described in section 6.7.

6 - The subroutine U-BEAM: This designs the reinforcement for a beam section for the ultimate limit state making use of section 6.7.1.

7 - The subroutine U-COLUMN: This designs the reinforcement for a symmetrically reinforced column section for the ultimate limit state making use of section 6.8.2. This is done after the column depth is calculated from the limit state for deflection.

8 - The subroutine SSSC: This is called by subroutine U-COLUMN to calculate the steel stresses from the stress-strain curve. The strain in the steel is calculated according to the position of the neutral axis.

6.10 A 15 STOREY REINFORCED CONCRETE FRAME WITH THREE UNEQUAL BAYS

The computer program described was used to design the 15 storey - 3 unequal bay frame shown in Figure 6.10. The frame is to be built of reinforced concrete. The characteristic strength of the concrete and the steel are 40 N/mm^2 and 410 N/mm^2 respectively. The applied loads on the frame are shown in Figure 6.10. The design loads are obtained by making use of sections (6.5, 6.5.1 and 6.5.2). Hence two design load cases for deflection limit state, and nine others for the ultimate limit state are obtained.

The limit on the sway is taken to be equal to the height of the column/500. The deflection profile used in the design is shown in Figure 6.11. The width of the beam and column sections is taken equal to 200 mm for all storeys. The flange depth h_f for T-beams is taken as 120 mm, while the flange width B_f equals 830 mm.

The covers of tensile steel (d') and compression steel (G) are 33 mm, and 28 mm respectively. The percentage of the reinforcement assumed to resist the sway deflection is 2% for the columns at levels 1 to 5. 1.5% for the levels 6-10, and 1.0% for the columns of the top five storeys. The cover for this reinforcement d' is 36 mm for all columns.

A typical storey is represented in Figure 6.12 in which the sections at which the reinforcments are calculated are shown. In each beam three sections are considered. These are A, B and D. For each column one section is considered this is the section with the maximum bending moment, which is at one of the column ends.

The design of the beam sections for the limit state for deflection is shown in Table 6.3. All beams have the same section and reinforcement. This is because the beam depth obtained from the limitation of the span/effective depth ratio dominates the

Span under consideration	H mm	B mm	Reinforcement at A, B and D	
			A_s mm^2	A'_s mm^2
Left-span beam sections	500	200	934	233
Mid-span beam sections	300	200	534	133
Right-span beam sections	400	200	734	183

TABLE 6.3: 15 STOREY FRAME - THE DESIGN OF THE BEAM SECTIONS FOR THE LIMIT STATE OF DEFLECTION

design. For all the beams, the limitation gives larger beam depths than those obtained using Anderson and Islam's equation. The percentage of the reinforcement (A_s/bd) is assumed to be 1% for all the beams. This is the basic percentage referred to in CP110 (1972), at which the span/effective depth equals 20 for a simply supported beam. The beam sections for the final design which satisfy the ultimate state as well as the limit state for deflection and cracking remain unchanged. However the steel area is increased for most of the beam sections. The steel area for beam sections A, B and D of the final design are shown in Table 6.4. It can be seen that some reinforcement area are determined by the limit state of deflection. These are marked by an asterisk in Table 6.4.

The design of the column sections is shown in Table 6.5 for deflection limit state and the ultimate limit state. It can be seen from the table that the ultimate limit state is governing the design and most of the column depths obtained from this state are more than those obtained from the limit state for deflection. Furthermore the reinforcement is also increased considerably.

It should be noticed, that the column depth obtained for deflection limit state is considerably greater than the column depth specified in CP110:1972. The code considers a value of effective height/30 to be acceptable. This underestimates the column depth needed to satisfy the limit state for deflection. For example in this frame, if the effective height is taken equal to the actual height (i.e. 3500), then the minimum column depth specified in the code is 3500/30 = 150 mm (rounded to the nearest 50 mm) and except for the top three storeys this is much less than the column depth proposed by this new design method.

6.11 A 24 STOREY REINFORCED CONCRETE FRAME WITH TWO UNEQUAL BAYS

The 24 storey - two unequal bay frames shown in Figure 6.13 was designed using the computer program described in section 6.9. The frame is to be built of reinforced concrete. The characteristic strength of the concrete and the steel are 40 N/mm^2 and 410 N/mm^2 respectively. The applied load on the frame are shown in Figure 6.13. The design loads are obtained as described in sections (6.5, 6.5.1 and 6.5.2). This gives two design load cases for the limit state for deflection, and five other cases for the ultimate limit state.

The non-linear deflection profile (used in the design) and the linear profile are shown in Figure 6.14, in which the sway is restricted to h/500. However, for selecting the beam sections, Anderson and Islam's equations are used with sway restricted to h/1000. It should be stressed that this latter restriction is only applied for the purpose of selecting the beam sections since Anderson and Islam's equations underestimate these.

To calculate the column depths, and the reinforcement of the beams and the columns certain sectional properties need to be supplied. These are:

1) The covers to the reinforcement, which are given in Table 6.6 making use of the recommendation given in CP110:1972, clause 3.11.2. In this table d' and G are those shown in Figures 6.3 and 6.4.
2) The width of the beam and column sections, which is taken equal to 250 mm for storey levels 1-18 inclusive. This is reduced to 200 mm for the rest of the frame.

3) For T-beam sections the flange depth h_f (see Figure 6.5) is taken equal to 120 mm for all the storeys. The flange width B_f is 880 mm for storey levels 1-18, but is reduced to 830 mm for the remaining storeys.

4) The percentage of reinforcment ($100A_{sc}/bh$) assumed to resist the sway of the column. These percentage are given in Table 6.7.

A typical storey is represented in Figure 6.15 in which the sections at which the reinforcements are calculated are indicated. There are three sections in each beam A, B and D, and one section in each column.

The design of the beam sections for the limit state for deflection is shown in Table 6.8. It is noticed that (except for the top three storeys) the beam depth designed by Anderson and Islam's equations is larger than that obtained from the limitation of the span/effective depth ratio. The percentage of reinforcement (A_s/bd) assumed to be 1% for all the beams. The steel areas obtained using this percentage are given in Table 6.8. For the compression steel the minimum percentage of 0.25% is used.

The beam sections for the final design which satisfy the ultimate limit state as well as the limit state for deflection and cracking remain unchanged. The area of the tensile steel for this design is shown in Table 6.9. Sections A, B and D in the table refer to those of Figure 6.14. The area of the compression steel remains nominal at the minimum of 0.25%. It can be seen that many tensile steel areas are determined using the deflection limit state. These are marked by asterisks in Table 6.9.

The design for the column sections for the deflection limit state is shown in Table 6.10. This also shows the final design, in which the ultimate limit state as well as the limit states for deflection and cracking are satisfied. Unlike the 15 storey frame, most of the column depths obtained by the limit state for deflection are adequate for the ultimate limit state, and no increase in the column depth is required. The unchanged column depths in the final design are enclosed in brackets in (Table 6.10). For some sections the deflection limit state governs the design, where the column depth and the steel area are determined by this state. For example, column C_1 at level 14 (see Table 6.10) has a column depth of 450 mm. The area of reinforcment resisting the sway is 2250 mm^2, which is more than that required for the ultimate limit state. This means that the final design should have an area of steel equal to 2250 mm^2 to satisfy deflection as well as strength requirements. The steel areas determined by the limit state for deflection and not by the ultimate limit state are marked by asterisks in Table 6.10. However, most of the sections need a larger steel area for the ultimate limit state.

It is noticed, that the column depth obtained for the limit state for deflection is considerably greater than the column depth specified in CP110:1972, clause 3.5.8, which gives a column depth of 150 mm which is adequate only at the top storey.

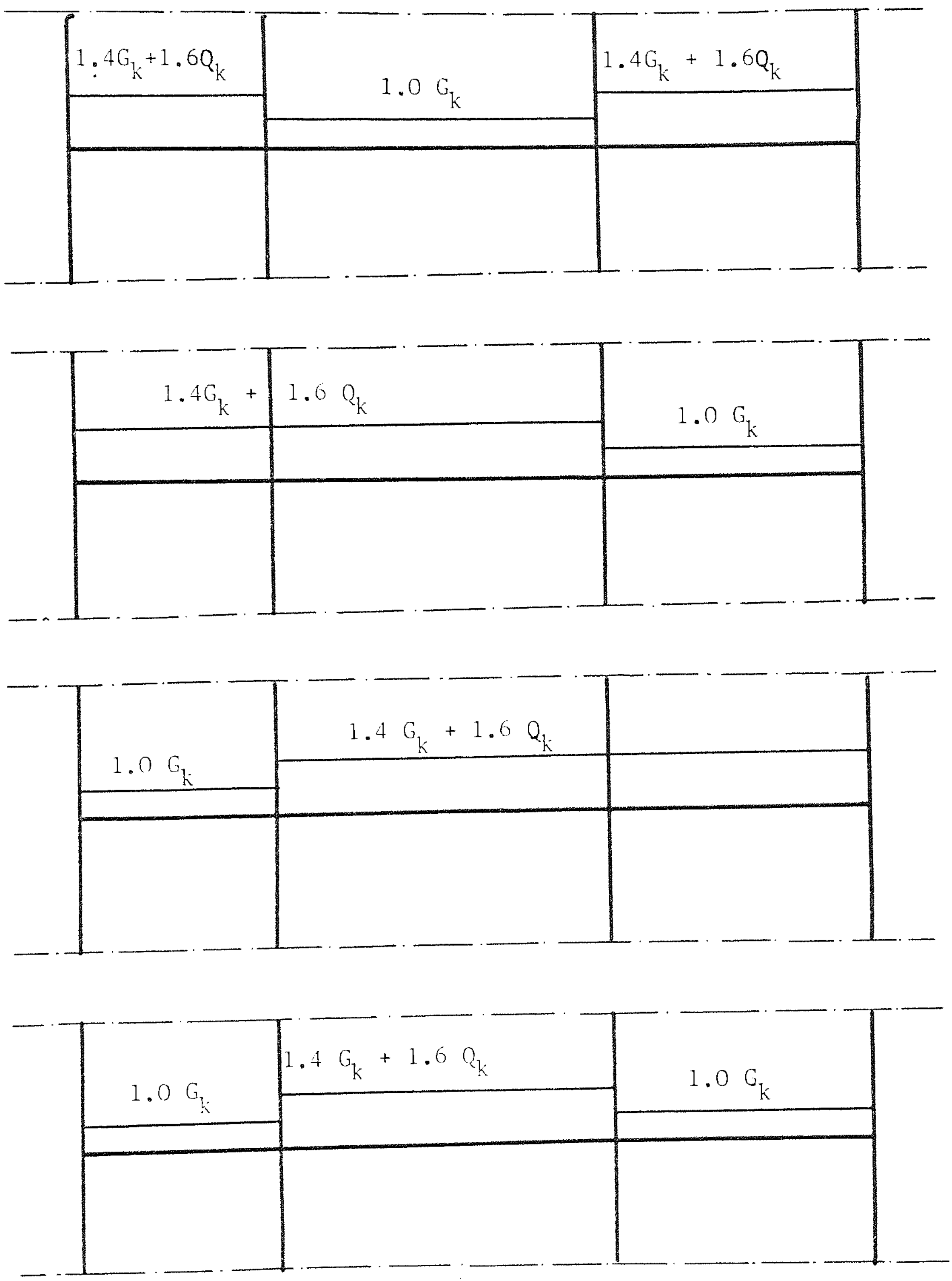


FIGURE 6.1: LOAD COMBINATION (1), DIFFERENT LOAD ARRANGEMENTS FOR ULTIMATE LIMIT STATE

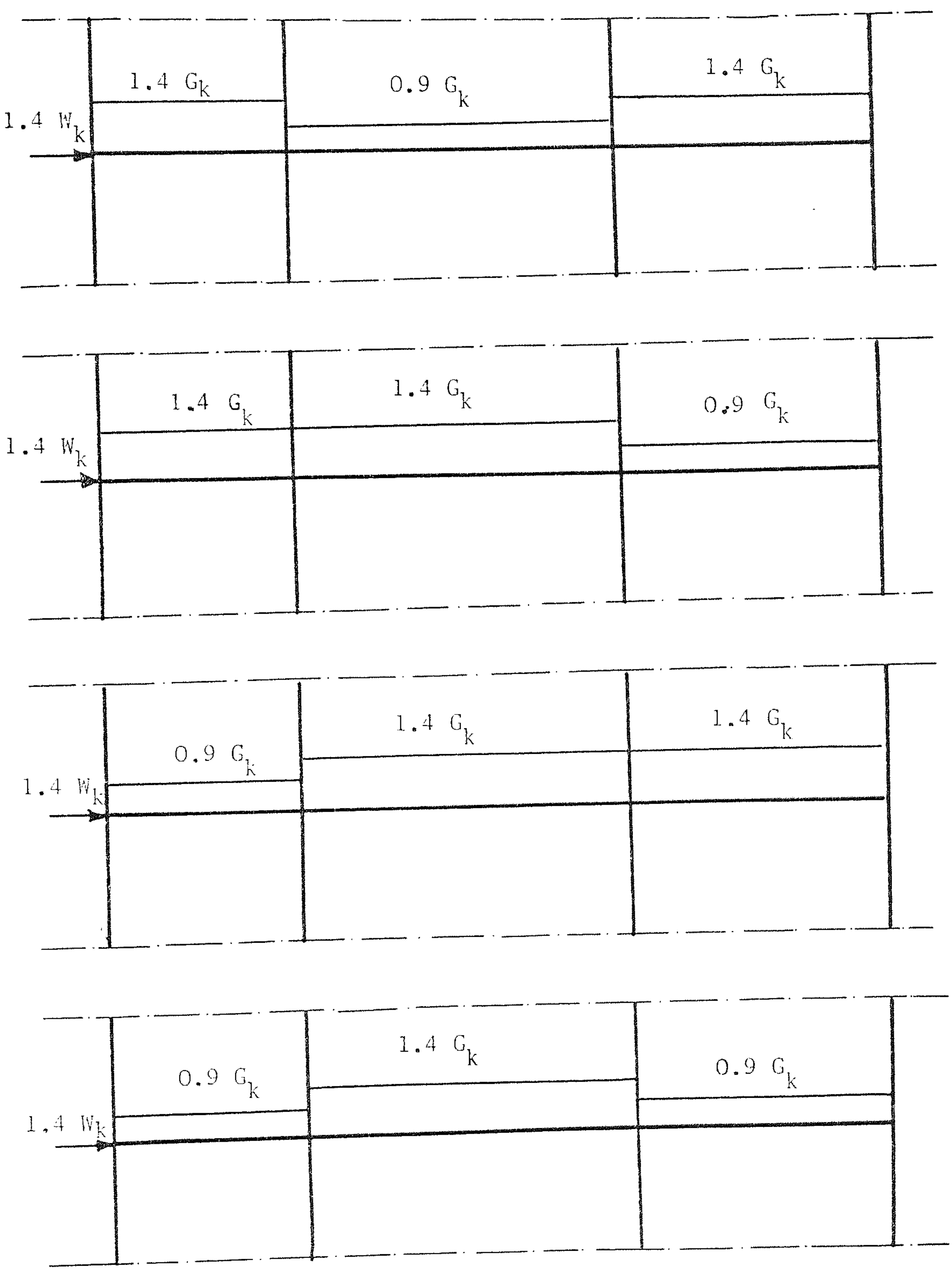


FIGURE 6.2 LOAD COMBINATION (2), DIFFERENT LOAD ARRANGEMENTS FOR ULTIMATE LIMIT STATE

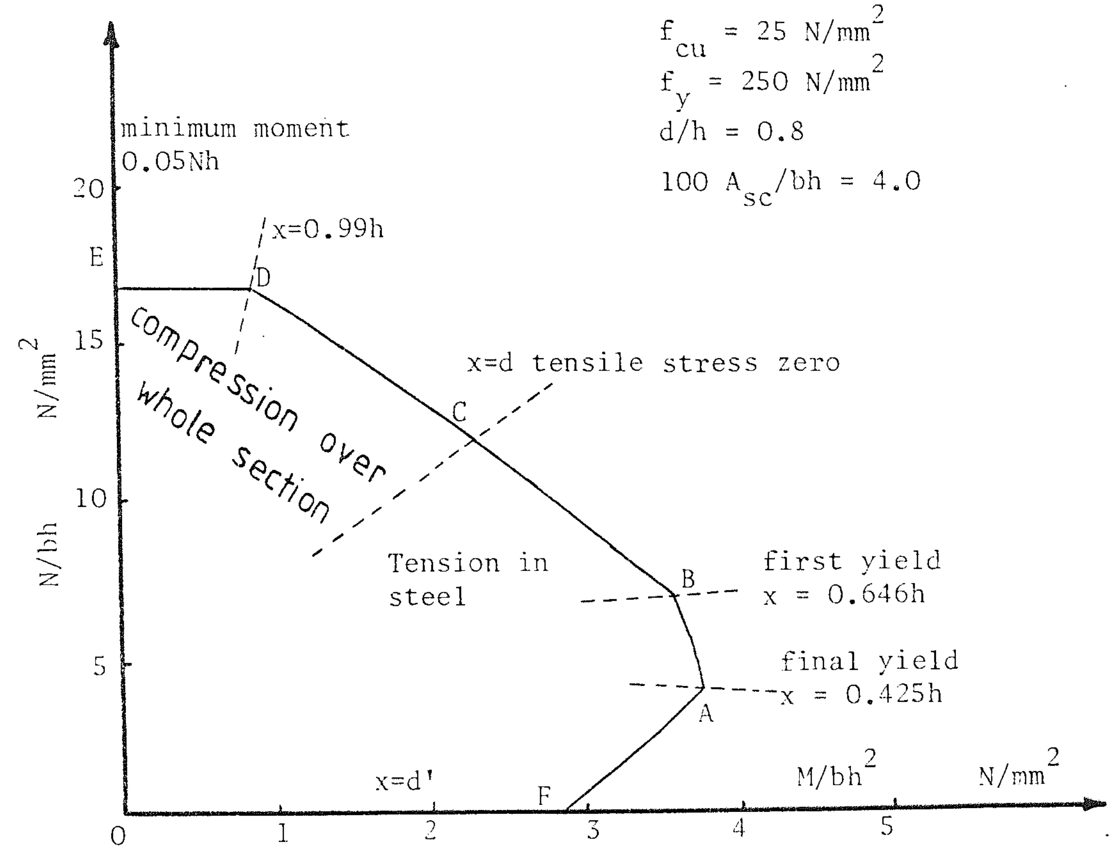


FIGURE 6.6 A TYPICAL DESIGN CURVE FOR RECTANGULAR COLUMN

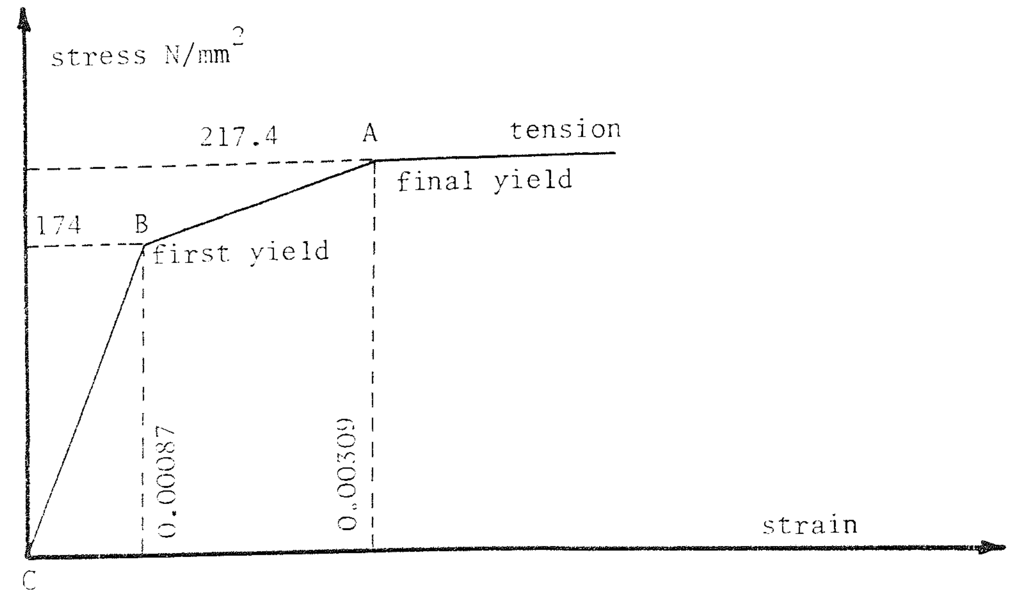


FIGURE 6.7 DESIGN STRESS-STRAIN CURVE FOR REINFORCEMENT IN TENSION

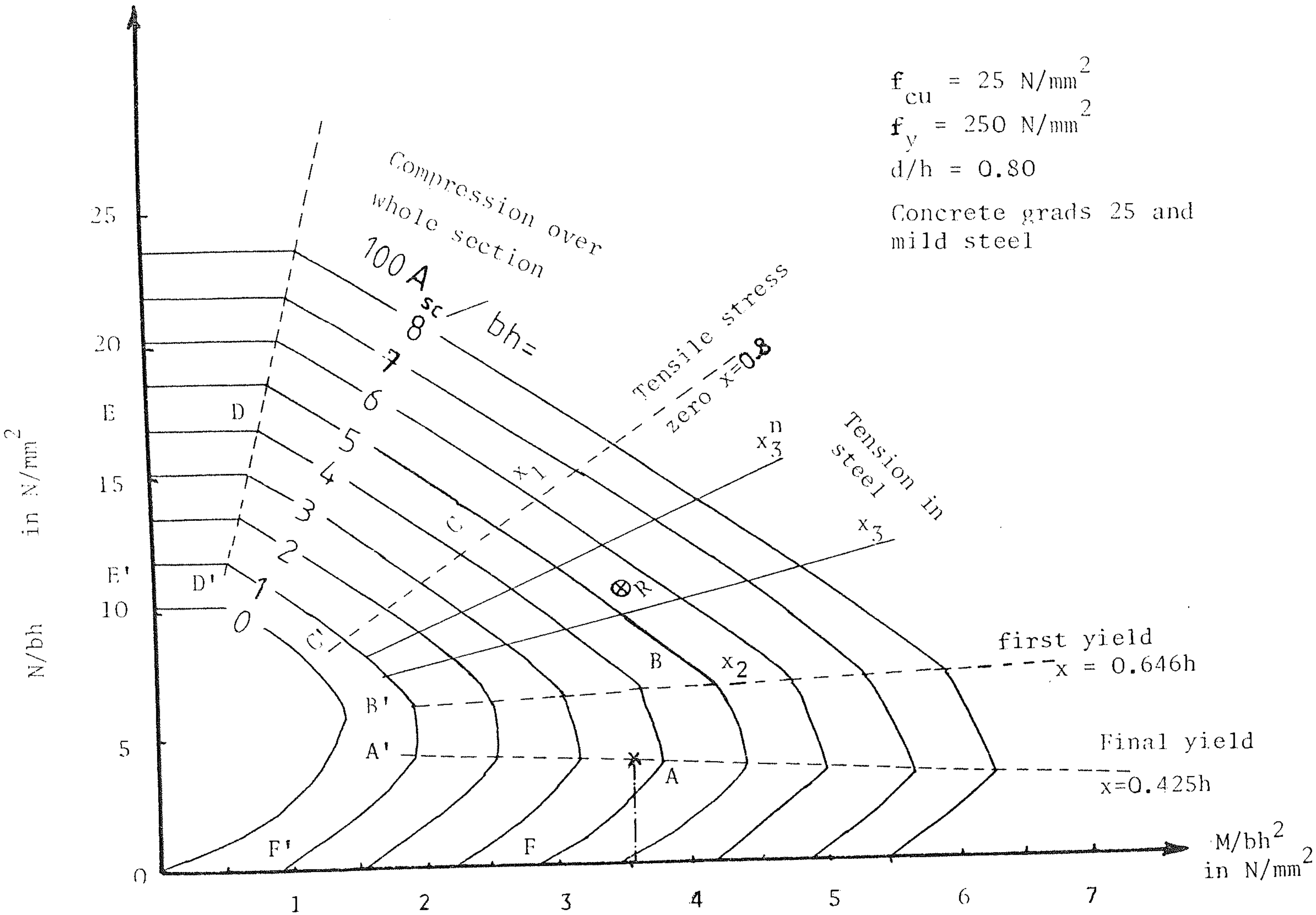
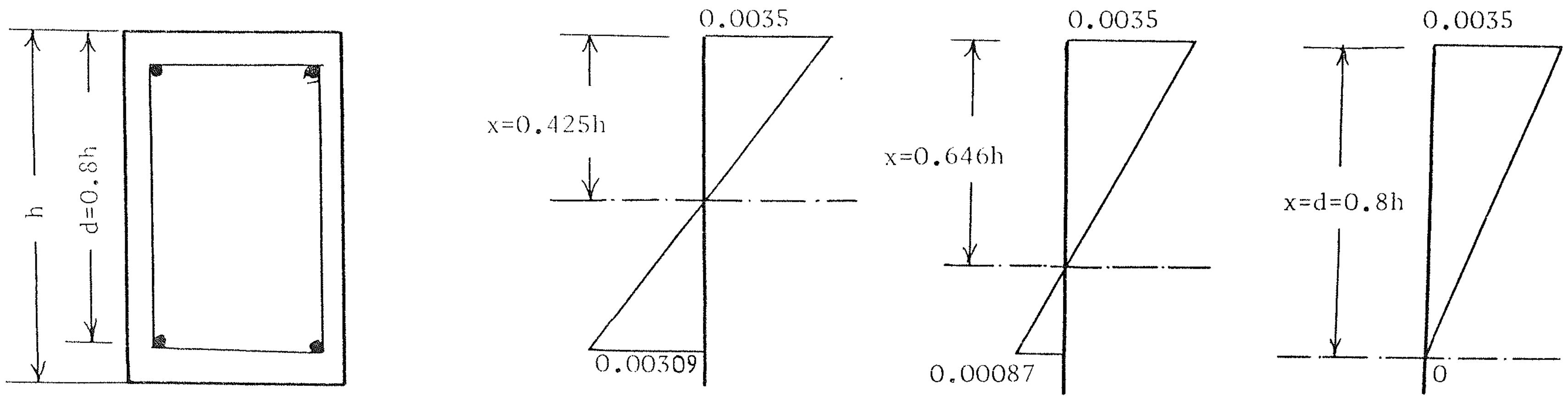


FIGURE 6.8 A TYPICAL DESIGN CHART FOR RECTANGULAR REINFORCED CONCRETE COLUMN SECTION



(a) Section

(b) Strain diagram for final yield of the tensile steel

(c) Strain diagram for first yield of the tensile steel

(d) Strain diagram for zero strain at the tensile steel

FIGURE 6.9 STRAIN DIAGRAMS FOR SOME SIGNIFICANT POINTS

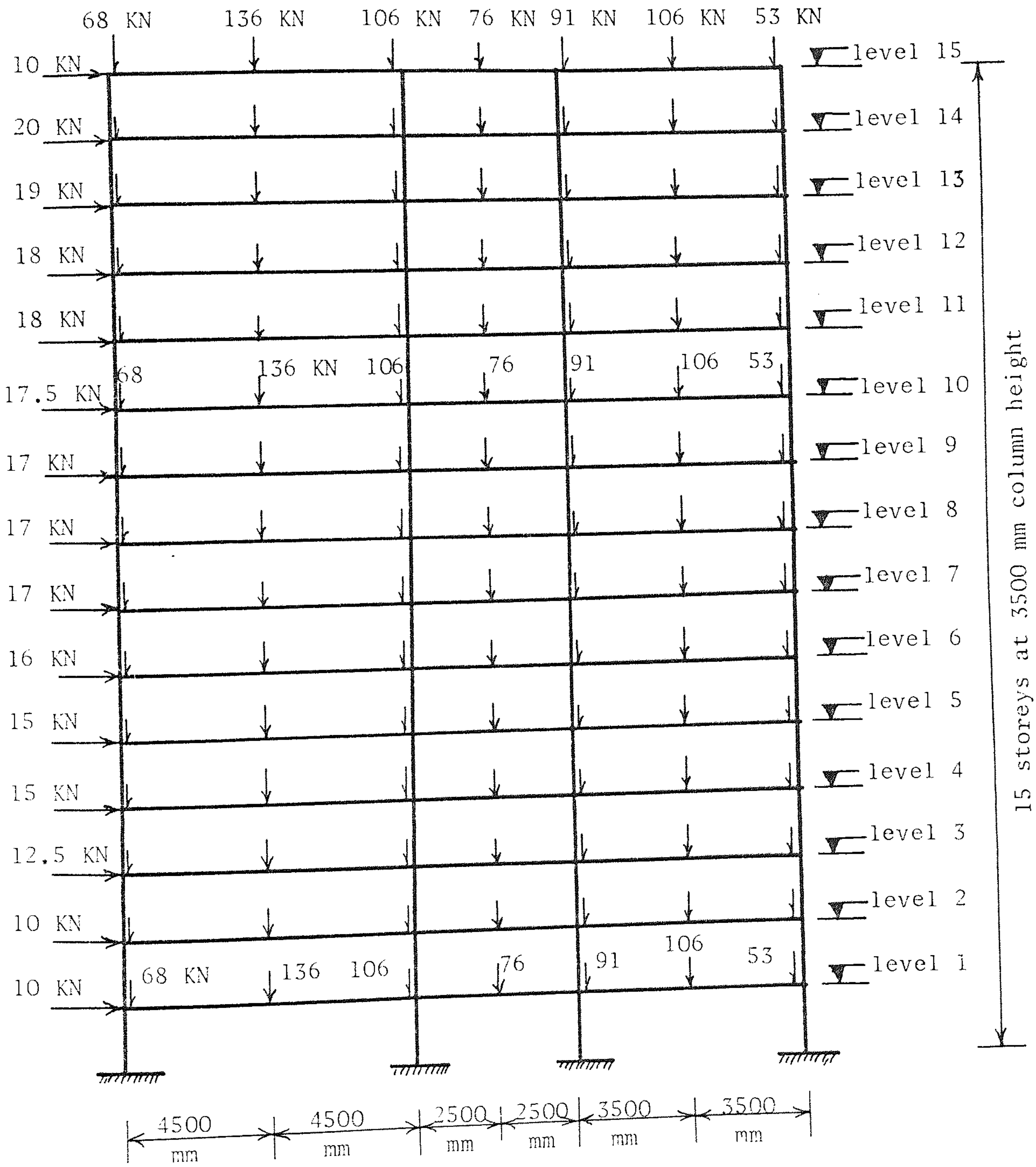


FIGURE 6.10 THE 15 STOREY - 3 UNEQUAL BAY FRAME. DIMENSIONS AND APPLIED LOADS

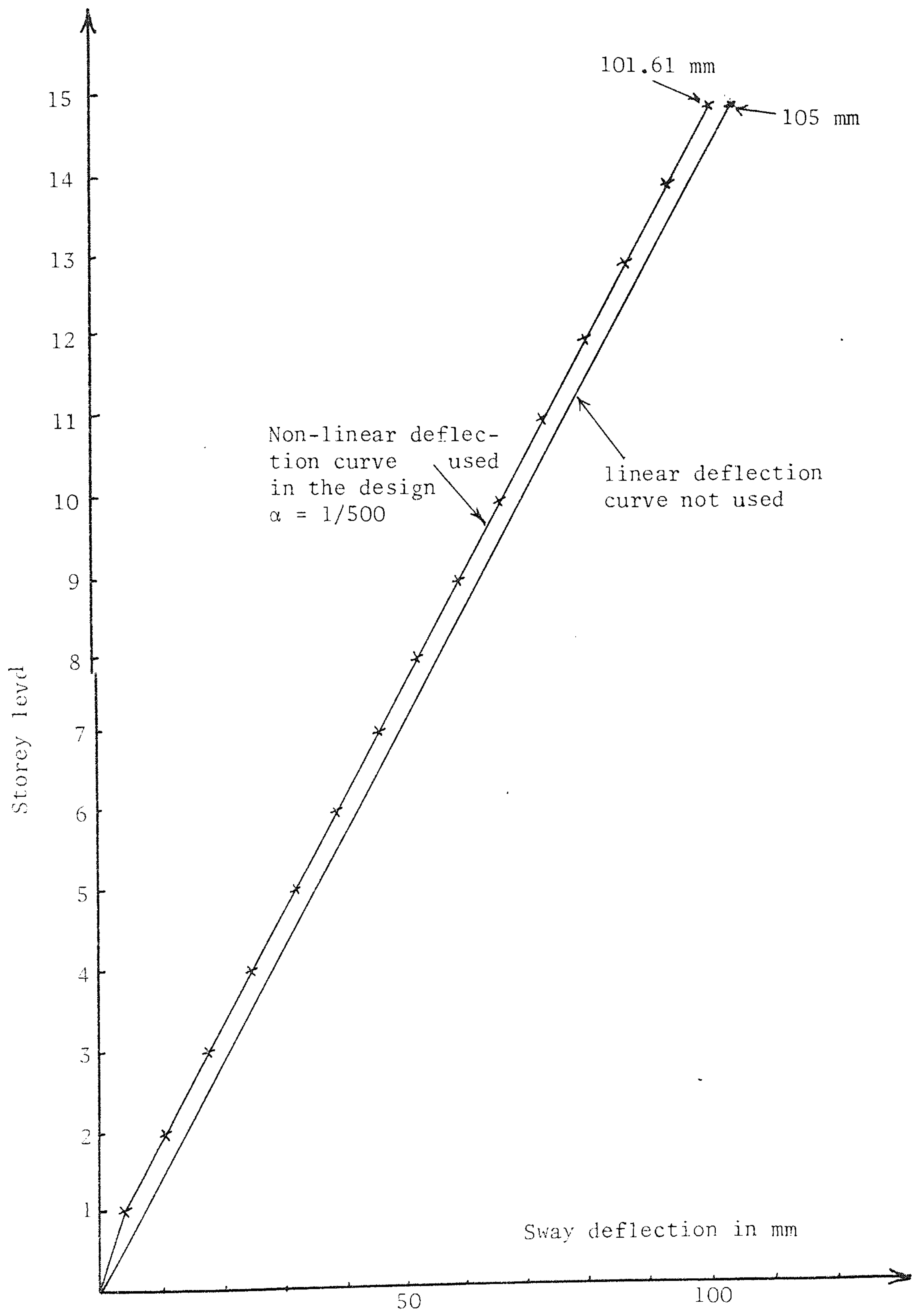


FIGURE 6.11 SWAY DEFLECTION CURVE FOR THE 15 STOREY FRAME SHOWN IN FIGURE 6.10

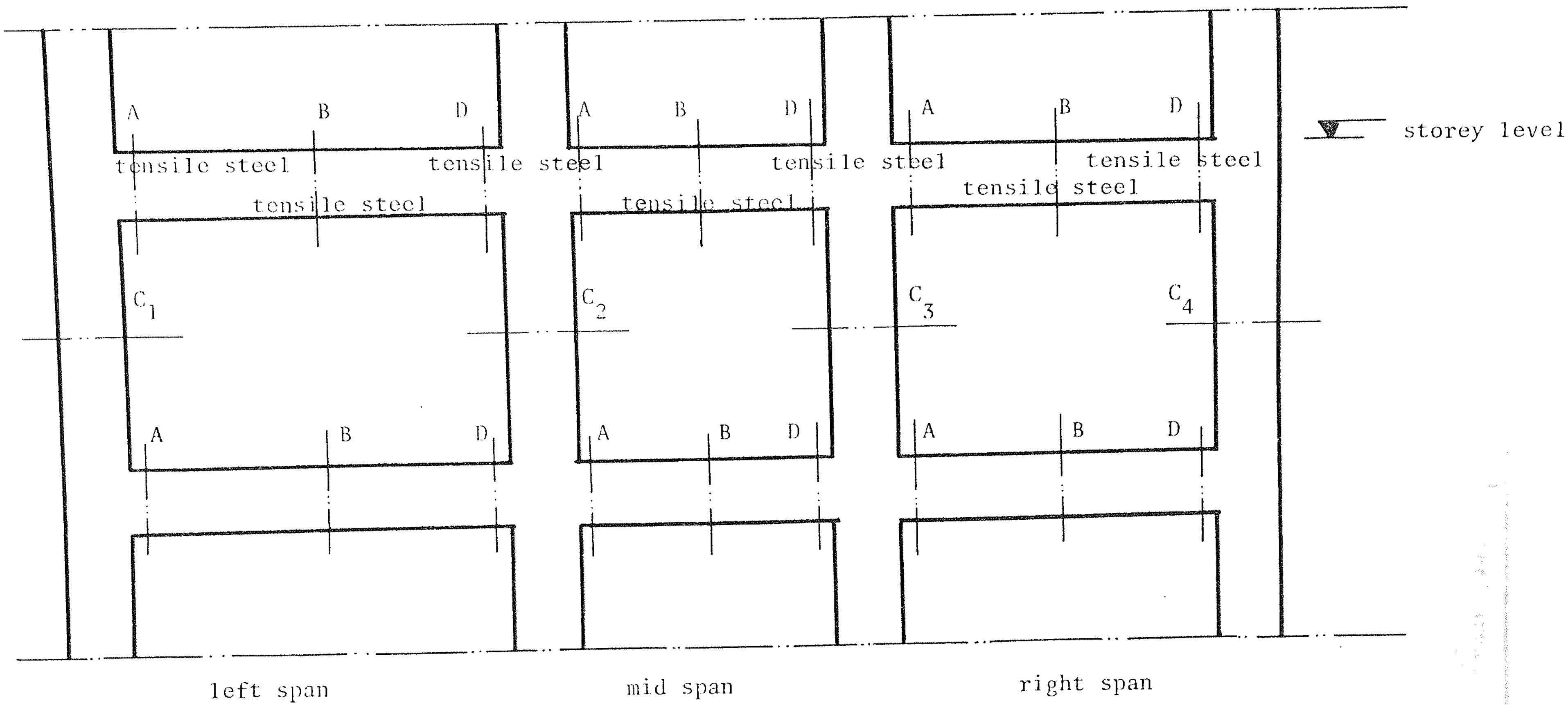


FIGURE 6.12 TYPICAL STOREY REPRESENTATION FOR THE 15 STOREY FRAME

Storey level	Span	Steel area Section A mm^2		Steel area Section B mm^2		Steel area Section D mm^2	
		A_s	A'_s	A_s	A'_s	A_s	A'_s
1	Left Mid Right	1214 854 1159	233 133 183	1459 763 1117	233 133 183	2891 1696 2258	883 555 678
2	Left Mid Right	1282 847 1115	233 133 183	1426 764 1096	233 133 183	3089 1824 2465	1110 701 916
3	Left Mid Right	1329 867 1107	233 133 183	1428 764 1097	233 133 183	3093 1842 2484	1115 722 937
4	Left Mid Right	1385 895 1112	233 133 183	1435 764 1102	233 133 183	3037 1820 2445	1051 697 893
5	Left Mid Right	1417 927 1126	233 133 183	1462 765 1118	233 133 183	2925 1766 2346	921 635 799
6	Left Mid Right	1413 952 1147	233 133 183	1503 765 1142	233 133 183	2772 1696 2201	746 554 612
7	Left Mid Right	1358 981 1151	233 133 183	1557 765 1174	233 133 183	2609 1613 2041	558 459 428
8	Left Mid Right	1270 996 1157	233 133 183	1619 765 1211	233 133 183	2444 1538 1870	369 373 231
9	Left Mid Right	1163 1008 1158	233 133 183	1690 764 1254	233 133 183	2280 1467 1695	233 291 183
10	Left Mid Right	1034 1016 1155	233 133 183	1773 763 1306	233 133 183	2100 1396 1389	233 210 183
11	Left Mid Right	*934 1078 1141	233 133 183	1873 760 1371	233 133 183	1783 1327 1120	233 133 183
12	Left Mid Right	*934 1227 1122	233 133 183	1995 755 1452	233 133 183	1519 1263 857	233 133 183

* marks steel area determined by the limit state of deflection

TABLE 6.4 15 STOREY FRAME - THE STEEL AREA FOR BEAM SECTIONS OF THE FINAL DESIGN (Cont)

Storey level	Span	Steel area Section A mm^2		Steel area Section B mm^2		Steel area Section D mm^2	
		A_s	A'_s	A_s	A'_s	A_s	A'_s
13	Left Mid Right	*934 1330 1084	233 133 183	2148 747 1559	233 133 183	1248 1192 *734	233 133 183
14	Left Mid Right	*934 1472 1020	233 133 183	2355 732 1713	233 133 183	997 1260 *734	233 133 183
15	Left Mid Right	*934 1610 946	233 455 183	2558 715 1874	233 198 183	*934 1373 *734	233 133. 183

TABLE 6.4: 15 STOREY FRAME - THE STEEL AREA FOR BEAM SECTIONS OF THE FINAL DESIGN

Storey level	Column section	Limit state of deflection		Ultimate limit state	
		h_{mm}	A_{sc} mm^2	h_{mm}	A_{sc} mm^2
1	C_1	550	2200	600	3575
	C_2	600	2400	900	6400
	C_3	600	2400	750	5525
	C_4	500	2000	500	3250
2	C_1	550	2200	550	4675
	C_2	600	2400	850	7225
	C_3	600	2400	700	6300
	C_4	500	2000	500	4500
3	C_1	550	2200	550	5700
	C_2	600	2400	800	7650
	C_3	600	2400	650	7500
	C_4	500	2000	500	4750
4	C_1	500	2000	500	3825
	C_2	600	2400	700	6500
	C_3	550	2200	600	4950
	C_4	450	1800	450	3800
5	C_1	400	1600	450	4000
	C_2	500	2000	650	6000
	C_3	450	1800	550	4750
	C_4	400	1600	400	4200
6	C_1	350	1050	400	3400
	C_2	400	1200	600	5500
	C_3	400	1200	500	4500
	C_4	300	900	350	3325
7	C_1	300	900	400	3400
	C_2	350	1050	550	5500
	C_3	300	900	450	4500
	C_4	300	900	350	3325
8	C_1	250	750	350	3500
	C_2	300	900	500	5000
	C_3	300	900	400	4200
	C_4	250	750	350	2625
9	C_1	250	750	400	1800
	C_2	250	750	450	4725
	C_3	250	750	350	4025
	C_4	200	600	300	2850

TABLE 6.5: 15 STOREY FRAME - DESIGN OF THE COLUMN SECTIONS FOR DEFLECTION LIMIT STATE AND ULTIMATE LIMIT STATE

Storey level	Column section	Limit state of deflection		Ultimate limit state	
		h mm	A_{sc} mm^2	h mm	As_c mm^2
10	C_1 C_2 C_3 C_4	200 250 250 200	600 750 750 600	300 400 350 300	3000 4200 2800 2100
11	C_1 C_2 C_3 C_4	200 200 200 200	400 400 400 400	300 400 300 250	2100 2800 2550 2375
12	C_1 C_2 C_3 C_4	150 200 200 150	300 400 400 300	250 300 250 250	2125 3450 2125 1500
13	C_1 C_2 C_3 C_4	150 150 150 150	300 300 300 300	200 250 200 200	2200 3000 2100 1600
14	C_1 C_2 C_3 C_4	150 150 150 150	300 300 300 300	200 200 200 150	800 2200 800 1800
15	C_1 C_2 C_3 C_4	150 150 150 150	300 300 300 300	150 150 150 150	300 825 300 300

TABLE 6.5: 15 STOREY FRAME - DESIGN OF THE COLUMN SECTIONS FOR DEFLECTION LIMIT STATE AND ULTIMATE LIMIT STATE (Cont)

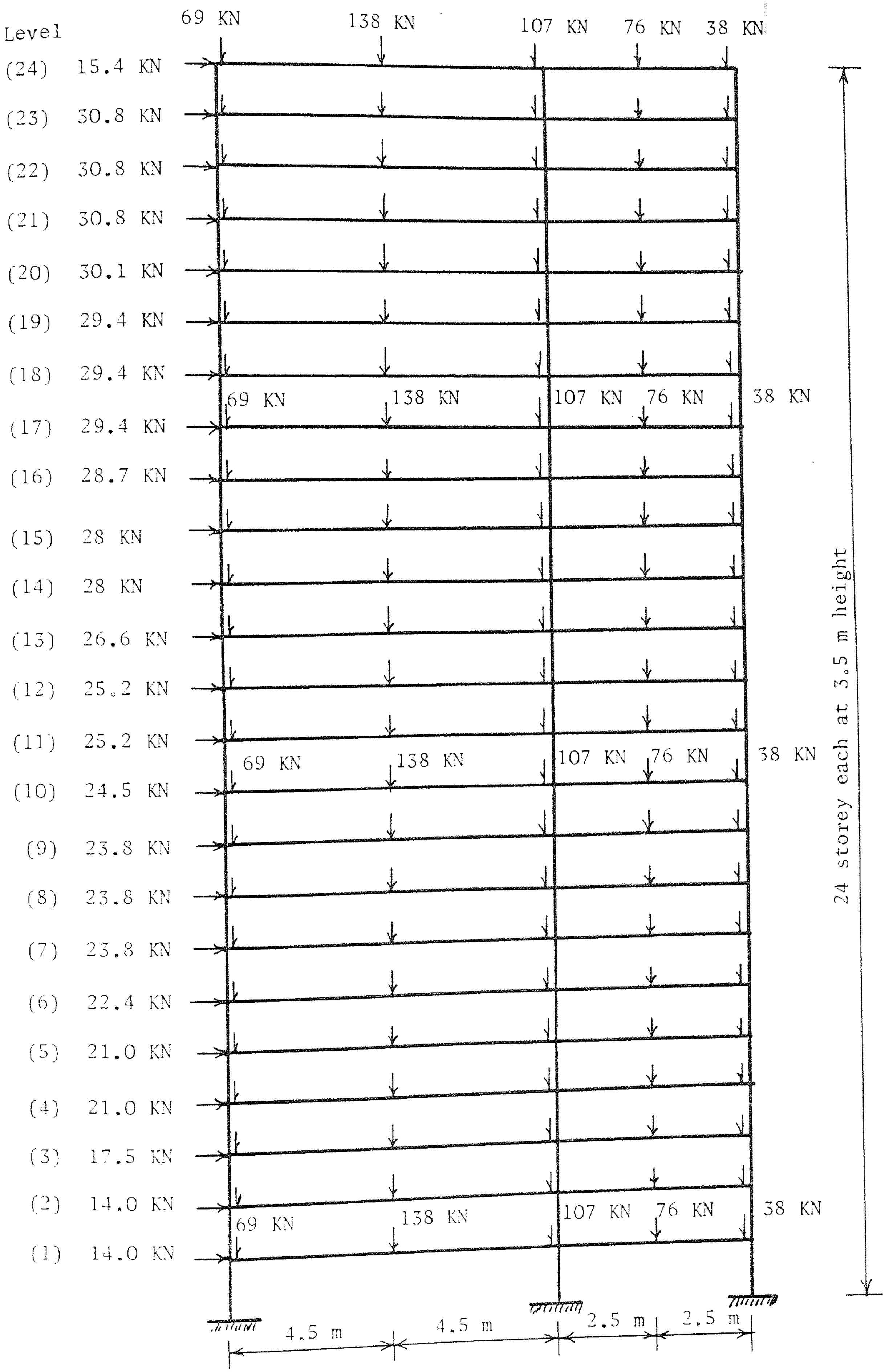


FIGURE 6.13 24 STOREY CONCRETE FRAME

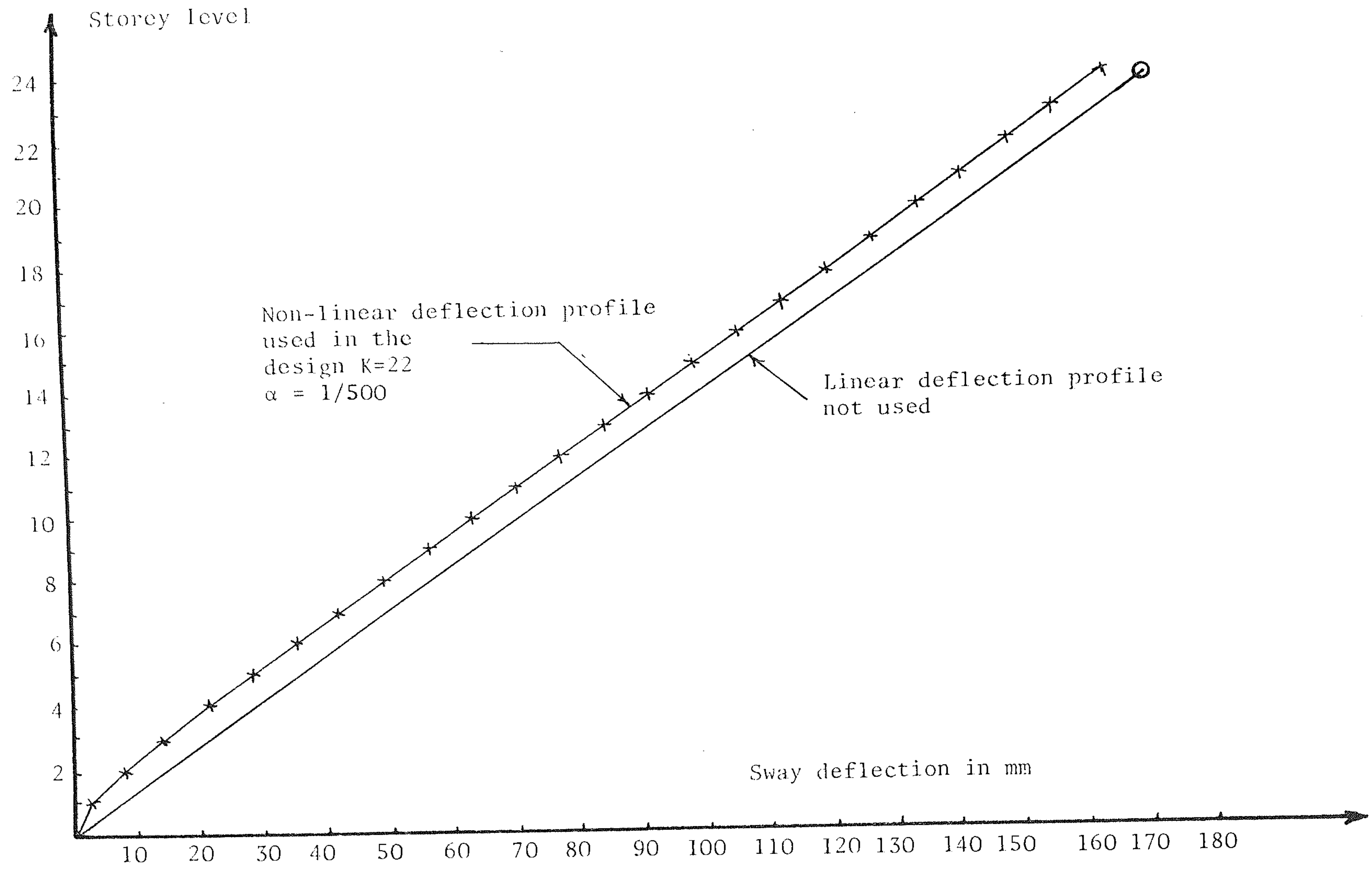


FIGURE 6.14 24 STOREY CONCRETE FRAME - SWAY DEFLECTION PROFILES

Storey level	Covers for beam reinforcement		Covers for column reinforcement
	d'	G	d'
1 to 12	36 mm	30 mm	40 mm
13 to 24	33 mm	28 mm	36 mm

TABLE 6.6: 24 STOREY FRAME - REINFORCEMENT COVERS

Storey level	Reinforcement ratio of the column sections (100 A_{sc}/bh)
1 to 18	2.0
19 - 21	1.5
22 - 24	1.0

TABLE 6.7: 24 STOREY FRAME-REINFORCEMENT RATIOS ASSUMED FOR THE LIMIT STATE OF DEFLECTION

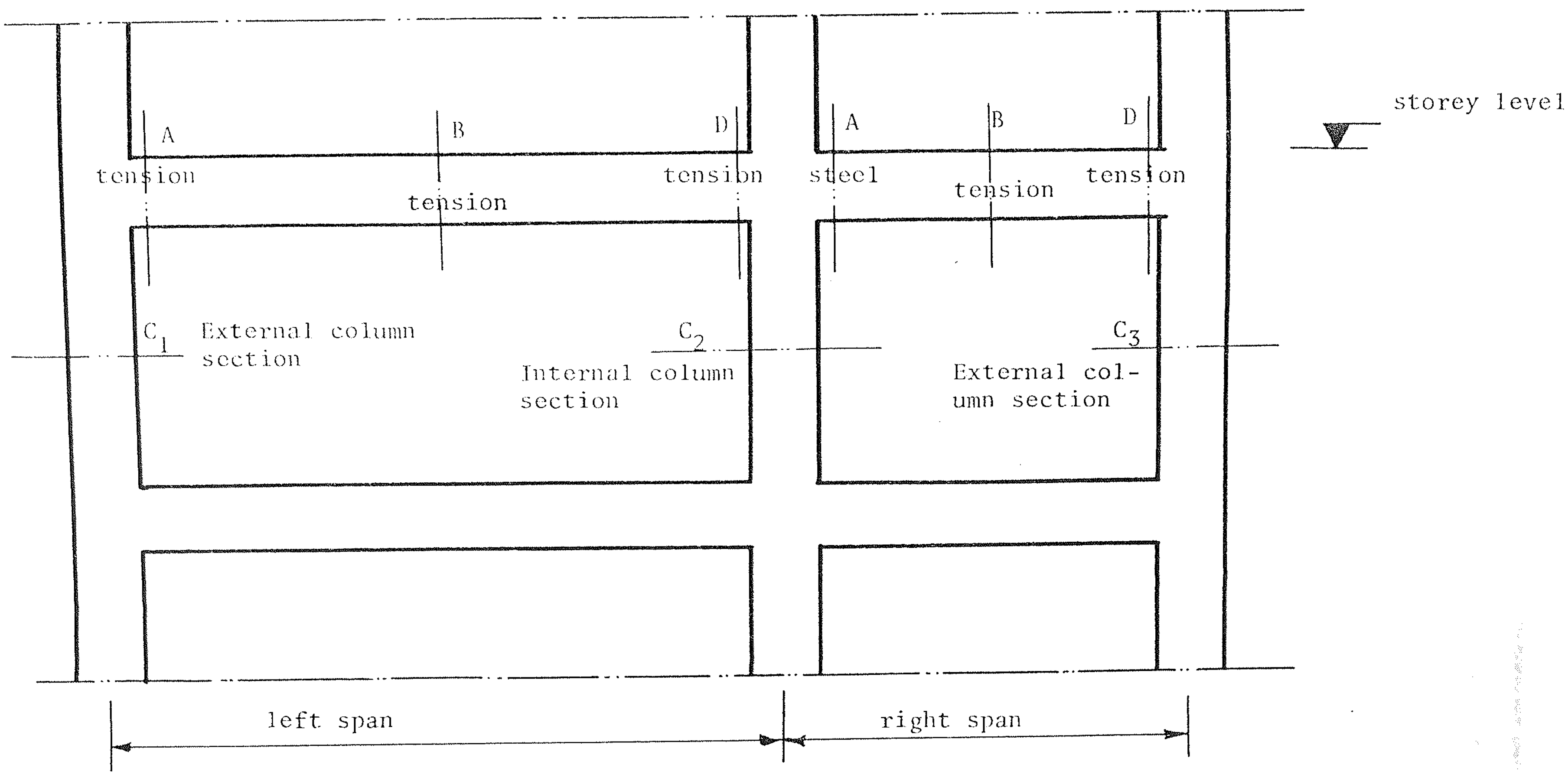


FIGURE 6.15 24 STOREY CONCRETE FRAME - TYPICAL STOREY REPRESENTATION

Storey level	Span under Consideration	Beam Depth h mm	Reinforcement at A, B and D	
			A_s mm^2	A'_s mm^2
1 - 4	Left Right	750 500	1785 1160	446 290
5 - 9	Left Right	700 500	1660 1160	415 290
10 - 12	Left Right	650 450	1535 1035	383 258
13	Left Right	650 450	1542 1042	385 260
14	Left Right	600 450	1417 1042	354 260
15	Left Right	600 400	1417 917	354 229
16 - 18	Left Right	550 400	1292 917	323 229
19	Left Right	500 400	1034 734	258 183
20 - 21	Left Right	500 350	934 634	233 158
22 - 24	Left Right	500 300	934 534	233 133

TABLE 6.8: 24 STOREY FRAME - THE DESIGN OF THE BEAM SECTIONS FOR THE LIMIT STATE FOR DEFLECTION

Storey level	Span	A_s at section A mm	A_s at section B mm^2	A_s at section D mm^2
1	Left Right	*1785 1213	*1785 *1160	2960 1918
2	Left Right	2130 1801	*1785 *1160	3817 2684
3	Left Right	2240 1967	*1785 *1160	4091 2809
4	Left Right	2196 2049	*1785 *1160	4112 2878
5	Left Right	1916 2250	*1660 *1160	3946 2978
6	Left Right	1822 2260	*1660 *1160	3891 2981
7	Left Right	1699 2176	*1660 *1160	3799 2915
8	Left Right	*1660 2102	*1660 *1160	3543 2856
9	Left Right	*1660 2113	*1660 *1160	3447 2849
10	Left Right	*1535 1841	*1660 *1160	3526 2674
11	Left Right	*1535 1735	*1535 *1035	3251 2601
12	Left Right	*1535 1554	*1535 *1035	2932 2447
13	Left Right	*1542 1406	*1542 *1042	2621 2246
14	Left Right	*1417 1350	*1417 *1040	2473 2120
15	Left Right	*1417 962	*1417 * 917	2352 2081
16	Left Right	*1292 947	1331 * 917	2387 2124
17	Left Right	*1292 935	1356 *917	2357 2088

* Marks steel area determined by the limit state for deflection.

TABLE 6.9: 24 STOREY FRAME - THE STEEL AREA FOR BEAM SECTIONS OF THE FINAL DESIGN (Cont.)

Storey level	Span	A_s at section A mm^2	A_s at section B mm^2	A_s at section D mm^2
18	Left Right	*1292 *917	1400 *917	2272 1864
19	Left Right	1090 *734	1390 *734	2295 1750
20	Left Right	1405 *634	1534 661	2361 1652
21	Left Right	1359 *634	1608 681	2237 1443
22	Left Right	1104 841	1772 831	1925 1065
23	Left Right	*934 1091	2073 887	1398 *534
24	Left Right	*934 1455	2446 943	957 *534

* Marks steel area determined by the limit state for deflection

TABLE 6.9: 24 STOREY FRAME - THE STEEL AREA FOR BEAM SECTIONS OF THE FINAL DESIGN

Storey level	Column section	The design for the limit state of deflection		Final design	
		h mm	A_{sc} mm^2	h mm	A_{sc} mm^2
1	C_1	600	3000	750	10781
	C_2	650	3250	950	13656
	C_3	500	2500	(500)	6875
2	C_1	550	2750	700	8750
	C_2	600	3000	900	11813
	C_3	450	2250	(450)	5906
3	C_1	500	2500	700	7438
	C_2	600	3000	850	11156
	C_3	400	2000	450	5063
4	C_1	500	2500	650	7719
	C_2	600	3000	800	11000
	C_3	400	2000	(400)	5750
5	C_1	500	2500	650	6906
	C_2	600	3000	800	10500
	C_3	400	2000	450	4219
6	C_1	500	2500	600	6750
	C_2	600	3000	750	10313
	C_3	400	2000	(400)	5250
7	C_1	500	2500	550	6875
	C_2	600	3000	700	10500
	C_3	400	2000	(400)	5000
8	C_1	500	2500	550	5844
	C_2	600	3000	700	9625
	C_3	400	2000	(400)	4500
9	C_1	500	2500	(500)	5938
	C_2	600	3000	650	9344
	C_3	400	2000	(400)	4000
10	C_1	500	2500	(500)	5313
	C_2	600	3000	650	8531
	C_3	400	2000	(400)	3500

Unchanged column depths are enclosed in brackets

TABLE 6.10: 24 STOREY FRAME - DESIGN OF THE COLUMN SECTIONS FOR LIMIT STATE OF DEFLECTION AND ULTIMATE LIMIT STATE (Cont.)

Storey level	Column section	The design for the limit state of deflection		Final design	
		h mm	A_{sc} mm^2	h mm	A_{sc} mm^2
11	C_1 C_2 C_3	500 600 400	2500 3000 2000	(500) (600) (400)	4063 8525 2750
12	C_1 C_2 C_3	450 550 400	2250 2750 2000	(450) 600 (400)	4219 7500 2250
13	C_1 C_2 C_3	450 500 350	2250 2500 1750	(450) 600 (350)	3375 6000 2844
14	C_1 C_2 C_3	450 500 350	2250 2500 1750	(450) (500) (350)	*2250 6563 2406
15	C_1 C_2 C_3	450 500 350	2250 2500 1750	(450) (500) (350)	*2250 5625 1969
16	C_1 C_2 C_3	450 500 350	2250 2500 1750	(450) (500) (350)	*2250 4688 *1750
17	C_1 C_2 C_3	400 450 350	2000 2250 1750	(400) (450) (350)	2500 4781 *1750
18	C_1 C_2 C_3	350 400 300	1750 2000 1500	(350) (400) (300)	*1750 5000 1875
19	C_1 C_2 C_3	350 400 300	1050 1200 900	(350) (400) (300)	2100 4800 1800
20	C_1 C_2 C_3	350 400 300	1050 1200 900	(350) (400) (300)	2100 4000 1500

* marks steel area determined by the limit state of deflection and not by ultimate limit state

TABLE 6.10: 24 STOREY FRAME - DESIGN OF THE COLUMN SECTIONS FOR LIMIT STATE OF DEFLECTION AND ULTIMATE LIMIT STATE (Cont.)

Storey level	Column section	The design for the limit state of deflection		Final design	
		h mm	A_{sc} mm^2	h mm	A_{sc} mm^2
21	C_1	300	900	(300)	2700
	C_2	350	1050	(350)	4025
	C_3	250	750	(250)	1875
22	C_1	250	500	300	1800
	C_2	250	500	300	3300
	C_3	250	400	200	1000
23	C_1	200	400	(200)	500
	C_2	200	400	250	2750
	C_3	150	300	200	400
24	C_1	150	300	(150)	1725
	C_2	150	300	(150)	900
	C_3	150	300	(150)	*300

TABLE 6.10: 24 STOREY FRAME - DESIGN OF THE COLUMN SECTIONS FOR LIMIT STATE OF DEFLECTION AND ULTIMATE LIMIT STATE

CHAPTER SEVEN

DESIGN OF SPACE FRAMES

7.1 INTRODUCTION

The iteration technique given in Chapter 2 can in fact be generalised for the design of space frames. In this chapter, it is used to design pin-jointed space frames in which the deflections are specified at certain joints in these frames. The aim here is to select member areas so that the stress requirements are satisfied while the deflection at the specified joints are allowed to take place.

In general the deflection, usually at the middle of certain spans, are specified by codes of practice. For instance, the British Standard draft B/20 restricts the deflection in the middle span of a roof to the span/360. For such a frame, the procedure is to calculate the member areas so that the deflection in the middle span will be equal to the specified limit. Naturally, the strength requirements in some of the members will not be satisfied. The cross sectional areas of these are therefore increased and the interaction is repeated so that the areas of other members are reduced. The process is repeated until the deflection as well as the strength requirements are satisfied throughout.

7.2 The Contributions of a Member to the Overall Stiffness Matrix

In a given pin-jointed space frame, let a general member i be connected to joint j at its first end and k at its second end. This member is shown in Figure 7.1, in which the arrow on the member specifies its positive longitudinal P axis. The system

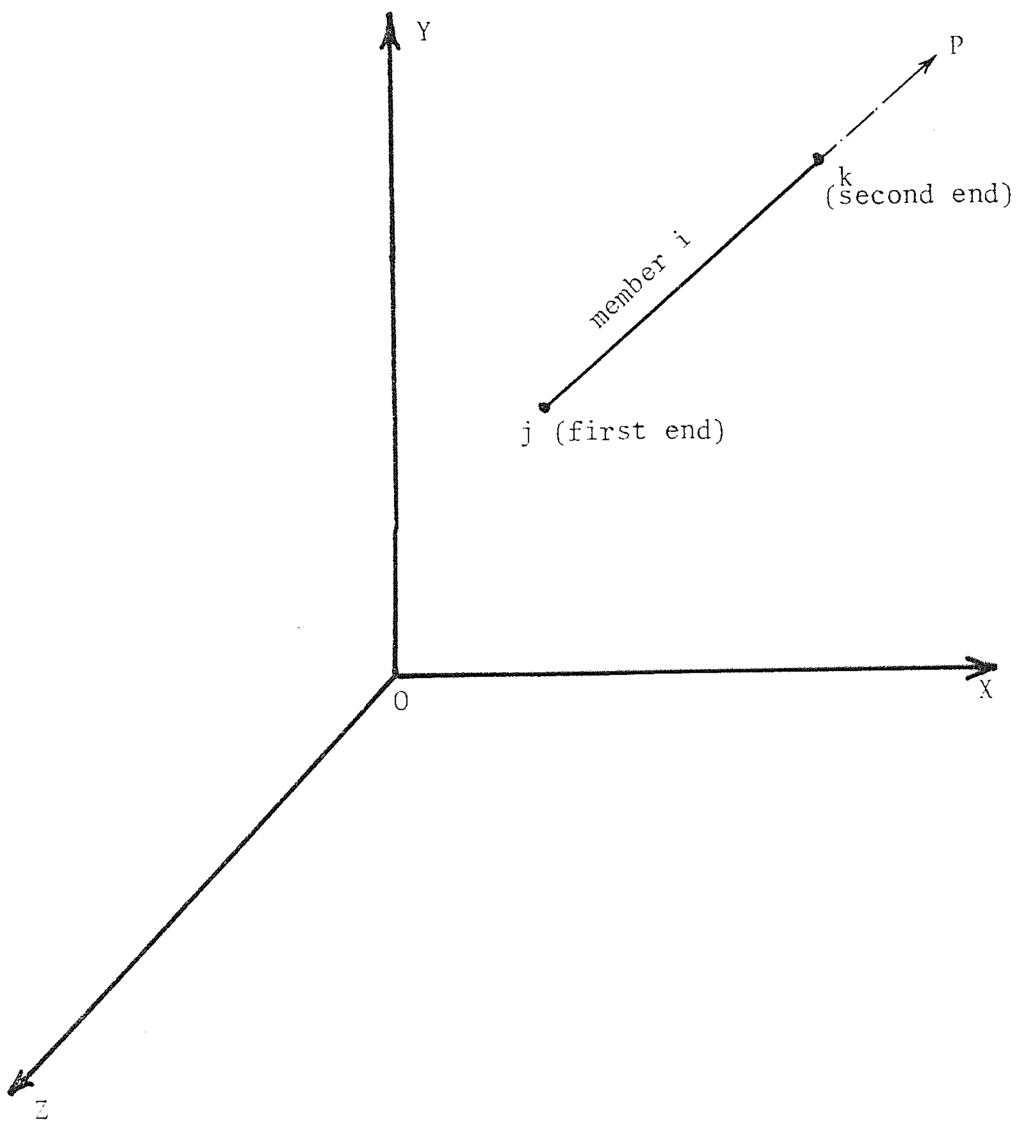


FIGURE 7.1 A MEMBER IN A PIN-ENDED SPACE FRAME

coordinates XYZ are also shown in the figure. A right handed system is used for the X, Y and Z axes.

The contributions of member i to the overall stiffness matrix [K] of the space frame is given below by equation (7.1) on the next page (Majid, 1978). In equation (7.1) $\{w_{xj}\ w_{yj}\ w_{xj} \ldots\ldots w_{xk}\ w_{jk}\ w_{zk}\}$ is the vector of the external loads applied to the joints. Each load w has two subscripts, the first denoting the direction of the force, and the second, either j or k, denoting the joint at the end of the member being considered. The direction cosines of the member are ℓ_p, m_p and n_p. It is noticed that only the direction cosines of the P axis relative to the X axis are involved. In equation(1.7) a= EA/L is the axial stiffness of the member. Where A is the area of the member, L is the length, and E is the elastic modules. The column vector $\{x_j\ y_j\ z_j \ldots\ldots z_k\ y_k\ z_k\}$ is the joint displacements vector.

It is useful to note that equation (7.1) does not alter if the arrow on the member is reversed in Figure 7.1 so that j becomes the second end of the member. The direction cosines change their sign but the signs of ℓ_p^2, m_p^2, $\ell_m m_p$ etc do not change. This observation is useful when deriving the modified stiffness equations for the structure.

7.3 MODIFIED STIFFNESS EQUATIONS FOR A GENERAL JOINT IN THE PIN JOINTED SPACE FRAME

The relationship between the external loads $\underline{W}$ and the vectorically equivalent joint displacements $\underline{\Delta}$, in terms of the system coordinates XYZ, is expressed by the equation

$$\underline{W} = \underline{K}\ \underline{\Delta} \tag{7.2}$$

The vector $\underline{\Delta}$ has x_j, y_j and z_j as three displacements in X, Y and Z directions of a general joint j. Thus, three equations are

$$\begin{bmatrix} W_{xj} \\ W_{yj} \\ W_{zj} \\ \hdashline W_{xk} \\ W_{yk} \\ W_{zk} \end{bmatrix} = \begin{array}{c} \begin{array}{cc} \text{end (1) at joint } j \qquad\qquad & \qquad\qquad \text{end (2) at joint } k \end{array} \\ \left[\begin{array}{ccc:ccc} a\ell_p^2 & a\ell_p m_p & a\ell_p n_p & -a\ell_p^2 & -a\ell_p m_p & -a\ell_p n_p \\ a\ell_p m_p & am_p^2 & an_p m_p & -a\ell_p m_p & -am_p^2 & -an_p m_p \\ a\ell_p n_p & an_p m_p & an_p^2 & -a\ell_p n_p & -am_p n_p & -an_p^2 \\ \hdashline -a\ell_p^2 & -a\ell_p m_p & -a\ell_p n_p & a\ell_p^2 & a\ell_p m_p & a\ell_p n_p \\ -a\ell_p m_p & -am_p^2 & -am_p n_p & a\ell_p m_p & am_p^2 & an_p m_p \\ -a\ell_p n_p & -an_p m_p & -an_p^2 & a\ell_p n_p & an_p m_p & an_p^2 \end{array}\right] \end{array} \times \begin{bmatrix} x_j \\ y_j \\ z_j \\ \hdashline x_k \\ y_k \\ z_k \end{bmatrix} \begin{array}{l} \text{end (1) at joint } j \\ \\ \\ \text{end (2) at joint } k \end{array} \qquad (7.1)$$

THE CONTRIBUTIONS OF A SPACE FRAME MEMBER TO THE OVERALL STIFFNESS MATRIX needed to define the displacements of each joint.

In the design problem, the deflection at a joint in a given direction may be specified. The corresponding stiffness equation is then modified so that it is used to calculate the areas of the members connected to that joint. The modified stiffness equation for x_j is obtained as follows:

The contributions of a member i to the deflection x_j are calculated from the first row of equations (7.1), thus

$$\begin{aligned} a_i \ell_{pi}^2 x_j + a_i \ell_{pi} m_{pi} y_j + a_i \ell_{pi} n_{pi} z_j - a_i \ell_{pi}^2 x_k \\ - a_i \ell_{pi} m_{pi} y_k - a_i \ell_{pi} n_{pi} z_k = w_{xj} \end{aligned} \tag{7.3}$$

Rearranging equation (7.3) for x_j we obtain

$$\begin{aligned} x_j = (-w_{xj} + a_i \ell_{pi} m_{pi} y_j + a_i \ell_{pi} n_{pi} z_j - a_i \ell_{pi}^2 x_k \\ - a_i \ell_{pi} m_{pi} y_k - a_i \ell_{pi} n_{pi} z_k)/(-a_i \ell_{pi}^2) \end{aligned} \tag{7.4}$$

Equation (7.4) gives the deflection x_j when one member only is connected to the joint. If R members are connected to j then each will have the same pattern of contributions, and the modified stiffness equations for x_j becomes:

$$\begin{aligned} x_j = [-w_{xj} + \sum_{i=1}^{i=R} (a_i \ell_{pi} m_{pi} y_j + a_i \ell_{pi} n_{pi} z_j - a_i \ell_{pi}^2 x_k \\ - a_i \ell_{pi} m_{pi} y_k - a_i \ell_{pi} n_{pi} z_k)] / [-\sum_{i=1}^{i=R} (a_i \ell_{pi}^2)] \end{aligned} \tag{7.5}$$

Similarly, the modified stiffness equations for y_j and z_j are:

$$\begin{aligned} y_j = [-w_{yj} + \sum_{i=1}^{i=R} (a_i \ell_{pi} m_{pi} x_j + a_i n_{pi} m_{pi} z_j - a_i \ell_{pi} m_{pi} x_k \\ - a_i m_{pi}^2 y_k - a_i m_{pi} n_{pi} z_k)]/[-\sum_{i=1}^{i=R} (a_i m_{pi}^2)] \end{aligned} \tag{7.6}$$

and

$$z_j = [-w_{zj} + \sum_{i=1}^{i=R} (a_i \ell_{pi} n_{pi} x_j + a_i m_{pi} n_{pi} y_j - a_i \ell_{pi} n_{pi} x_k$$

$$- a_i m_{pi} n_{pi} y_k - a_i n_{pi}^2 z_k)] / [- \sum_{i=1}^{i=R} (a_i n_{pi}^2) \qquad (7.7)$$

7.4 DESIGN EQUATION FOR MEMBER AREAS

If one of the deflections x_j, y_j or z_j is specified, then equations (7.5), (7.6) or (7.7) can be used to find the member areas. To do this, the axial stiffness of member i is written as a factor of the area A_i, thus

$$a_i = A_i b_i \qquad (7.8)$$

where b_i is equal to E/L_i.

In practice, for construction purposes the members are grouped together so that the members of a group have the same area A_g. For a total of n groups with areas A_{g1}, A_{g2}, ... A_{gr}, ... A_{gn}, let

$$\alpha_1 = A_{g1}/A_{g1} = 1,\ \alpha_2 = A_{g2}/A_{g1},\ \ldots\ldots,\ \alpha_r = A_{gr}/A_{g1},\ \ldots\ldots,$$
$$\alpha_n = A_{gn}/A_{g1} \qquad (7.9)$$

Hence, the axial stiffness of member i given in equation (7.8) can be written in terms of the area of the first group of members as:

$$a_i = \alpha_{ir} A_{g1} b_i \qquad (7.10)$$

where the area A_i of member i, which is in group r, is equal to $\alpha_{ir} A_{g1}$. Each ratio α has two subscripts, the first i denoting the member number, and the second r, denoting the group to which member i belongs.

For a specified deflection in a given direction the design equations are written to calculate the area of the member in the first group. The areas of the other groups are then obtained from equation (7.9).

7.4.1 Calculation of A_{g1}

If the deflection x_j in the direction of X axis is specified, then equations (7.4), and (7.5) can be used to find the area of the members in the first group. Equation (7.4) is rewritten in terms of A_{g1} and α_{ir} as:

$$A_{g1} = w_{xj} \Big/ (\alpha_{ir}\, b_i\, \ell_{pi}^2\, x_j + \alpha_{ir}\, b_i\, \ell_{pi}\, m_{pi}\, y_j + \alpha_{ir} b_i \ell_{pi} n_{pi} z_j - \alpha_{ir} b_i \ell_{pi}^2 x_k - \alpha_{ir} b_i \ell_{pi} m_{pi} y_k - \alpha_{ir} b_i \ell_{pi} n_{pi} z_k) \tag{7.11}$$

When R members are connected to joint j then equation (7.11) becomes:

$$A_{g1} = w_{xj} \Big/ [\sum_{i=1}^{i=R} (\alpha_{ir} b_i \ell_{pi}^2 x_j + \alpha_{ir} b_i \ell_{pi} m_{pi} y_j + \alpha_{ir} b_i \ell_{pi} n_{pi} z_j - \alpha_{ir} b_i \ell_{pi}^2 x_k - \alpha_{ir} b_i \ell_{pi} m_{pi} y_k - \alpha_{ir} b_i \ell_{pi} n_{pi} z_k)] \tag{7.12}$$

Similarly equations (7.6) and (7.7) become:

$$A_{g1} = w_{yj} \Big/ \sum_{i=1}^{i=R} (\alpha_{ir} b_i \ell_{pi} m_{pi} x_j + \alpha_{ir} b_i m_{pi}^2 y_j + \alpha_{ir} b_i n_{pi} m_{pi} z_j - \alpha_{ir} b_i \ell_{pi} m_{pi} x_k - \alpha_{ir} b_i m_{pi}^2 y_k - \alpha_{ir} b_i n_{pi} m_{pi} z_k) \tag{7.13}$$

and

$$A_{g1} = w_{zj} \Big/ \sum_{i=1}^{i=R} (\alpha_{ir} b_i \ell_{pi} n_{pi} x_j + \alpha_{ir} b_i m_{pi} n_{pi} y_j + \alpha_{ir} b_i n_{pi}^2 z_j - \alpha_{ir} b_i \ell_{pi} n_{pi} x_k - \alpha_{ir} b_i m_{pi} n_{pi} y_k - \alpha_{ir} b_i n_{pi}^2 z_k) \tag{7.14}$$

It should be noticed, that in equations (7.12), (7.13) and (7.14) the area of the members of the first group A_{g1} cannot be calculated if joint j is unloaded in the direction of the specified deflection. However, it is unlikely for a joint to have a critical deflection in the unloaded direction.

The ratios α_1, α_2, ..., α_r, ..., α_n are not known in the first design cycle. For this reason, in this cycle, it is assumed that $\alpha_1 = \alpha_2 = \ldots = \alpha_r = \ldots = \alpha_n = 1$. Obviously, this means that for the first design cycle all the member areas are equal and the frame is constructed from the same group. The ratios α are changed in the subsequent design cycles.

7.5 DESIGN PROCEDURE

The design procedure for a pin-jointed space frame is to some extent similar to that of a steel sway frame, described in section 2.8. This consists of the following steps:

STEP 1 - Specify the joint at which the deflection is expected to be critical.

STEP 2 - Specify the direction (X, Y or Z) and the value of this deflection.

STEP 3 - Set each α to unity to initiate the first design cycle.

STEP 4 - Assume the area of the members in the first group, and the deflection of all the joints. Infeasible values for the deflections such as $x_j = y_j = z_j = 1$ etc. are acceptable for the purpose of starting the iteration process. The area A_{g1} could have an infeasible value too, but it was found that convergence is achieved more rapidly with a large A_{g1}, e.g. 10^5 mm^2.

STEP 5 - Using the modified stiffness equations, calculate the joint displacements for all the joint up to that in which the deflection is specified.

STEP 6 - Using the appropriate design equation, now calculate the first new value of A_{g1} and hence calculate the new member areas in all the other groups using equation (7.9).

STEP 7 - Continue with the iteration and calculate a new value for each unknown joint displacement. At this stage all the modified stiffness equations for all the joints have been utilized.

STEP 8 - Repeat steps 5, 6 and 7 until convergence is achieved. This happens when (the new value of A_{g1} - the previous value of A_{g1})/(the New value of A_{g1}) is less than a specified tolerance. Notice that the convergence test could also be applied to the joint displacements. At this stage of the design procedure the first design cycle is complete and a new area is provided for each member. All the joint deflections are also found. Notice that for this first design cycle all the calculated member areas have the same value. The member areas and the joint deflections calculated fully satisfy the stiffness equations and the design obtained in this manner is feasible. However, the design process continues for the following reason:

a) To check that while deflection requirements are satisfied, the stresses are satisfactory in all the members

b) To achieve economy by altering the ratios between the areas of the groups.

STEP 9 - Check that the deflection of each joint in the frame is less than that specified in Steps (1) and (2) for a given joint. If the maximum deflection appears in a joint other than the one specified in step 1, then repeat steps 1 to 8 with the deflection limitation applied to this new joint.

STEP 10 - Using the joint displacements obtained in steps 8 and 9 calculate the internal forces in the members of the pin jointed frame. The force p_i in a member i is expressed in terms of the displacements of the joints to which it is connected as

$$p_i = (EA_i/L_i)\ (-x_j\ell_{pi} - y_j m_{pi} - z_j n_{pi} + x_k\ell_{pi} + y_k m_{pi} + z_k n_{pi}) \tag{7.15}$$

STEP 11 - Design the member areas to meet the stress requirements. The design stresses are those given in British Standard Draft B/20. Notice that two design stresses are used. The first is for a member under tension, and the second is for compression. As a result new areas for the members in each group are obtained, and the new ratios for each group are calculated.

STEP 12 - Repeat the iteration process, i.e. steps 5, 6 and 7 until convergence is obtained once again. Notice that when entering the iteration this time, the current values of the joint displacements and member areas are employed. These values are more realistic than the initial infeasible ones.

STEP 13 - Repeat steps 10 to 12 until the values of α_1, α_2, ... , α_r, ..., α_n become stable. This happens when:

$$\left| \frac{\text{new value of } \alpha - \text{old value of } \alpha}{\text{New value of } \alpha} \right| < \text{The tolerance}$$

At this stage the design is complete. The final design obtained in this manner satisfies:

a - The deflection requirement at the critical joint;

b - The strength requirement, as the stresses in all the members are less than or equal to the design stresses;

c - Design economy is achieved, as this design satisfies the deflection requirements, and at the same time the cross sectional areas of the members are distributed according to their stress requirements. In this manner the member areas are distributed more efficiently.

7.6 COMPUTER PROGRAM

A computer program based on the design procedure just described was written in FORTRAN and run on the ICL 1904S computer. The program consists of three segments. These are:

1 - The MASTER segment: This follows the steps of the design procedure of Section 7.5.

2 - The subroutine ITERATE. This calculates the area of the members in the first group and the joint displacements. A flow-chart of this subroutine is given in Figure 7.2 and it will be explained in more detail later.

3 - The subroutine INTERNAL-FORCES: This calculates the force in each member using equation (7.15). A positive force means that the member is in tension.

The data preparation for this program is described in Appendix C of this thesis. The data given is similar to that used by the PAVO ANALYSIS SYSTEM, Space Structures Research Centre, University of Surrey. Obviously, extra data should be supplied to specify the joint at which the critical deflection is expected to occur and the direction of this deflection.

7.6.1 Subroutine Iterate

Figure 7.2 shows a flow chart for this subroutine. This consists of three loops. The inner loop, starts at level 10 and counts the number of members, in the structure. The middle loop starts at level 100 and counts the number of joints; and the outside loop starts at level 1000 and counts the number of iterations NIR needed for each design. The integers I and J are used to count the members, and the joints as they are handled. M and N are the total number of members and joints respectively.

For a particular joint J, the contributions to the modified stiffness equations for each member connected to the joint are calculated. These are given by equations (7.5), (7.6) and (7.7). If the deflection at joint J is specified, in one of the direction, X, Y or Z, then the contributions to the appropriate design equation for each member connected to that joint are found instead.

The above procedure is carried out for each joint in the space frame, calculating each joint deflection, and the area of the members A_{g1} in the first group. Several iterations will be needed to obtain a complete convergence.

7.7 DESIGN EXAMPLE 1 : A SIX MEMBER ISOSTATIC SPACE FRAME

The isostatic pin-jointed space frame shown in Figure 7.3 was designed by the computer program described above. This frame was also designed by hand and the results were compared with those obtained by the program. No difference between the two results was observed.

The members were allowed to have any area and were not specifically grouped. However, for the first design cycle all the member areas were assumed to be equal, thus $\alpha_1 = \alpha_2 = \ldots\ldots = \alpha_6 = 1$.

The deflection at joint (1) in the positive y direction was specified to be 11.11 mm.

Using a small tolerance of 0.001, the area of the members obtained by the iteration technique was 402.01 mm^2. This is accurate to two decimal places. Thirty five iterations were needed to achieve convergence. The result of each iteration is shown by the graph in Figure 7.4. The total volume of the frame was 10951853.8 mm^3. As a result of this design cycle the joint displacements were also obtained, from which the internal forces in the members were calculated using equation (7.15). The forces in the members and the design for the stress requirements are given in Table 7.1. The design stresses used were 0.200 KN/mm^2 and 0.250 kN/mm^2. These values are taken from Table 6.2.2/5.5 of the B/20 draft specification for a slenderness $\lambda = 50$ and for a Robertson constant a = 5.5. Table 7.1 also shows the new ratios of the areas of the members, It can be seen from this that the area of members 2, 3 and 4 have to be increased, while the rest have to be reduced.

The second design cycle was carried out with these values of α, and using the iteration technique. A total of only 9 iteration was sufficient to obtain a new set of member areas. This was equal to 125.95 mm^2 for member 1. Multiplying this value by each α, the new member areas were calculated. These are given at the bottom of Table 7.1. The total volume for this design which satisfies both deflection and stress requirements was 6020203.3 mm^3. This is considerably less than the total volume obtained for equal member areas in the first design cycle.

No further design cycles could be carried out, because the frame is isostatic and the member forces are independent of the frame stiffness.

7.8 DESIGN EXAMPLE 2 - AN 8 MEMBER HYPERSTATIC SPACE FRAME

To obtain a hyperstatic space frame, two members (7 and 8) are added to the structure designed above. Member 7 connects joint (5) to (1) and member 8 connects joint (6) to (1). The resulting frame, is shown in Figure 7.5. The dimensions, design stress, loading and deflection specifications are kept as in example 1. The frame is used here for an extensive investigation into the proposed design method. The members are kept ungrouped and each member is allowed to have a separate area.

At the first design cycle the member areas were assumed to be equal to each other. For a tolerance of 0.01, fifteen iterations were needed to obtain the joint displacements and the member areas. These were found to be equal to 187.61 mm^2.

For the subsequent design cycles the α ratios were changed according to the designs for the stress requirements. The values of these ratios are given in Table 7.2 for various design cycles. This shows that α_5 and α_7 converge to zero, which means that for the final design, members 5 and 7 are to be removed from the structure. Eleven cycles were needed to obtain the final design, the member areas are given in Table 7.3. This also shows the total volume of the structure. The change in volume at each design cycle is represented in Figure 7.6, which shows the reduction in the total volume of the frame due to the distribution of the member areas according to the stress requirements. The member areas needed to satisfy the stress requirements are given in Table 7.4. From which, it can be seen that these are less than those needed for the deflection requirements. Thus, for the final design the member areas given in Table 7.3 should be used. Figure 7.7 shows the number of iterations per design cycle. This number reduces considerably after the first cycle.

It can be noticed from the above example, that the hyperstatic space frame specified initially was reduced to its basic isostatic shape at the final design stage. To examine whether this was accidental or not, the frame was redesigned, but this time with joint 2 unloaded. All the members and the other loads remain the same. Again the members were allowed to have any area and no grouping was imposed. After 12 design cycles the final design was obtained, in which members 1, 2, 5, 6 and 7 were removed from the space frame. The remaining members had areas:

$$A_3 = 337.5 \text{ mm}^2; \; A_4 = 198.0 \text{ mm}^2, \; A_8 = 90.1 \text{ mm}^2$$

The total volume of the structure at the first design cycle was 6726062.7 mm^3, while at the final design this was reduced to 2930511.0 mm^3, a reduction of more than 50%.

From the final design obtained above with joint (2) unloaded, the space frame was also seen to be reduced to its basic isostatic shape. The members connecting joint (2) to the supports and to joint (1) need not be constructed to resist the specified y_1 deflection at joint (1). Furthermore, the final design obtained for the deflection limitation was found to be satisfactory for the stress requirements as well. The areas needed to resist the internal forces are:

$$A_3 = 250.0 \text{ mm}^2; \; A_4 = 146.6 \text{ mm}^2 \text{ and } A_8 = 66.7 \text{ mm}^2$$

These are less than those given above for the deflection limitation.

The same frame with joints 1 and 2 loaded as in Figure 7.5 was also designed, but this time grouping the members into three groups. The first contained members 1 and 4, member 2, 5 and 6 were in group 2, while the third group consisted of members 3, 7 and 8.

In the final design stage none of the members were removed. The member areas in each group, for the design which satisfied both deflection and stress requirements, are:

$$A_{g1} = 107.9 \text{ mm}^2; \; A_{g2} = 196.1 \text{ mm}^2 \; A_{g3} = 216.8 \text{ mm}^2$$

with a volume of 7089646.9 mm^3. This is considerably greater than the design without grouping the members shown in Table 7.3.

7.9 DESIGN EXAMPLE 3.

The hyperstatic pin-jointed space frames (a), (b) and (c) shown in Figures (7.8), (7.9) and (7.10) were also designed using the computer program described in Section 7.6. The space frames (a), (b) and (c) were made hyperstatic by adding one, two, and three extra members respectively to their basic isostatic frame. The joint coordinates, the external applied loads, and the deflection specifications were kept the same for the three frames. These are given in Table 7.5 in which joints, 3, 4, 5 and 6 are the supports. Each of the other joints have three degrees of freedom. The deflection at joint (7) in the positive y direction was specified to be 11.11 mm. The members were not grouped. The design stresses were taken as in example 1 and 2 above.

The member areas are given in Table 7.6 for the final designs of the three frames. Although, these areas are calculated to satisfy deflection requirements it is found that they satisfy stress requirements as well. In each case member 3 was removed. Member

10 was also removed from frames (b) and (c) while member 5 was omitted from frame (c). The members which have to be omitted from each frame are denoted by asterisks in Table 7.6.

At the final design stage, all the hyperstatic space frames specified initially were reduced to their basic isostatic shapes. Table 7.6 gives the volumes for the frames from which it can be seen that frame (b) has the smallest volume of the three. Curves I and II in Figure 7.11 show the change of volume of frames (a) and (b). Curve III in Figure 7.12 shows the same information for frame (c).

7.10 DESIGN EXAMPLE 4 - A PYRAMID SHAPE SPACE FRAME

The four sided pyramid shape space frame shown in Figure 7.13 was designed next. Members 9 to 14 are in a plane parallel to the Y-Z plane. The joint coordinates and the external loads are given in Table 7.7. The deflection of joint (1) at the apex of the pyramid in +y direction was specified to be equal to 13.33 mm.

The member areas at the final cycle are summarised in Table 7.8. The first column shows the member numbers the second gives the area of each member obtained to satisfy deflection requirements. Each member has a different area, since members were not grouped. Columns 3 and 4 of Table 7.8 give the internal forces in the members and the member areas obtained by designing for these forces. The design stresses used were 0.250 kN/mm^2 in tension and 0.200 kN/mm^2 in compression. These were taken from Table 6.2.2/5.5 of the B/20 draft specification for $\lambda = 50$ and $a = 5.5$. The table also gives the values of α at the final design cycle.

It can be seen from Table 7.8 that the member areas needed to satisfy deflection requirements are larger than those needed for stress requirements. Thus, in the final design the member areas (which are given in the second column of Table 7.8) needed to satisfy the deflection requirement should be used. Notice that members 4, 13 and 14 are removed and the final frame is statically determinate.

At the first design cycle the total volume of the frame was 4987942 mm^3. This was reduced to 3322268 mm^3 at the final design, a reduction of 50.14%.

7.11 DESIGN EXAMPLE 5 - A SPACE FRAME WITH 781 MEMBERS

The practical pin-jointed space frame shown in Figure 7.14 has 781 members and 242 joints and was designed using the computer program described in Section 7.6. In the figure, the structure is cut by planes parallel to the X-Y and the Y-Z planes to show how the members are connected. Figure 7.15 shows an XZ section at level 0 (i.e. Y = 0). The small circles in the figure represent the supports. There are 40 supports and each one is prevented from movement in the X, Y and Z directions. The other joints have three degrees of freedom each. The distance between two neighbouring joints in XY, ZY or XZ plane is 2000.0 mm. Figure 7.16 shows the top net of members in plane XZ at level y = 2000.0 mm. Various other sections in the XY and YZ planes are shown in Figure 7.17. It can be seen that two inclined members are used at the edge sections, but only one at other sections.

The structure is subjected to a dead load of 10 kN at each joint and a concentrated imposed load of 140 kN at joint A which is at the midspan (see figure 7.15). The vertical deflection at A was limited to 20000/360 = 55.55 mm to satisfy the requirements specified in the draft code B/20.

For easy construction, the members are grouped into four categories as is common practice. The first group consists of all the inclined members, the vertical members are all in group 2 and the horizontal members are in groups 3 and 4. These are shown in figures 7.15 and 7.16 respectively and are for the members in the bottom and the top XZ planes.

The frame is to be built of high yield. This has a tensile design strength of 400 N/mm^2 and a compressive design strength specified in Table 6.2.2/5.5 of B/20 Draft as 304 N/mm^2. This means that the slenderness λ of a member should not exceed 50.

Figure 7.18 shows the change of volume of the frame at each design cycle. Curve (I) shows this change when the frame was subject only to deflection constraint at A. Curve (II) gives the same information but with the frame subject to stress constraints.

It can be seen from this figure that, each point on curve (I) requires larger volume than a point on curve (II), which means that the deflection requirements govern the design of this frame. It should be stressed that, although curve (I) was obtained by imposing the deflection requirements only, the designs obtained were found satisfactory when stress requirements were checked for each and every member.

Twelve design cycles were required to obtain a design in which the ratios α, between the member areas in each group and those in the first group, became stable. The values of α for each design cycle and the area of the members in group (1) are given in Table 7.9. A tolerance of 0.005 was adopted.

It can be seen from Figure 7.18 that the frame has the lowest volume at point C obtained in the second cycle of the design process.

The member areas and the total volume for this design are given by the first row of Table 7.10. The corresponding member areas required to satisfy stress requirements are given in the second row of this table. These are less than or equal to those required for deflection requirements. Thus, the design given in the first row is safe and serviceable.

The same frame was designed, but this time using mild-steel. For a slenderness ratio λ not more than 50, Table 6.2.2/5.5 of B/20 Draft specifies the tensile and compressive design strengths for this steel as 250 N/mm^2, and 200 N/mm^2 respectively.

Seven design cycles were needed this time to stabilise the values of α. Table 7.11 shows the change in these ratios for all the design cycles. This table also gives the area of the members in the first group.

The change in the volume of the structure at each cycle is shown in Figure 7.19. Curve (I) shows this change when deflection constraints were applied, while curve (II) shows it when stress requirements were applied.

From this figure it can be seen that stress requirements govern the design in the case. The design with the lowest volume was again obtained in the second design cycle.

The area of the members at the second cycle are:

A_{g1} = 497.6 mm^2; A_{g2} = 295.7 mm^2, A_{g3} = 682.2 mm^2, A_{g4} = 1158.1 mm^2

the total volume for this design is 119.2639 x 10^7 mm^3. When the deflection requirements only were imposed, then the member areas were:

A_{g1} = 411.3 mm^2; A_{g2} = 256.3 mm^2, A_{g3} = 596.1 mm^2; A_{g4} = 915.5 mm^2 with total volume of 98.198 x 10^7 mm^3. These areas are less than those given above. Thus, stress requirements govern the design in this case and the first set of areas should be used to obtain a safe and serviceable design.

The number of iterations per design cycle is represented in Figure 7.20 for a tolerance of 0.005.

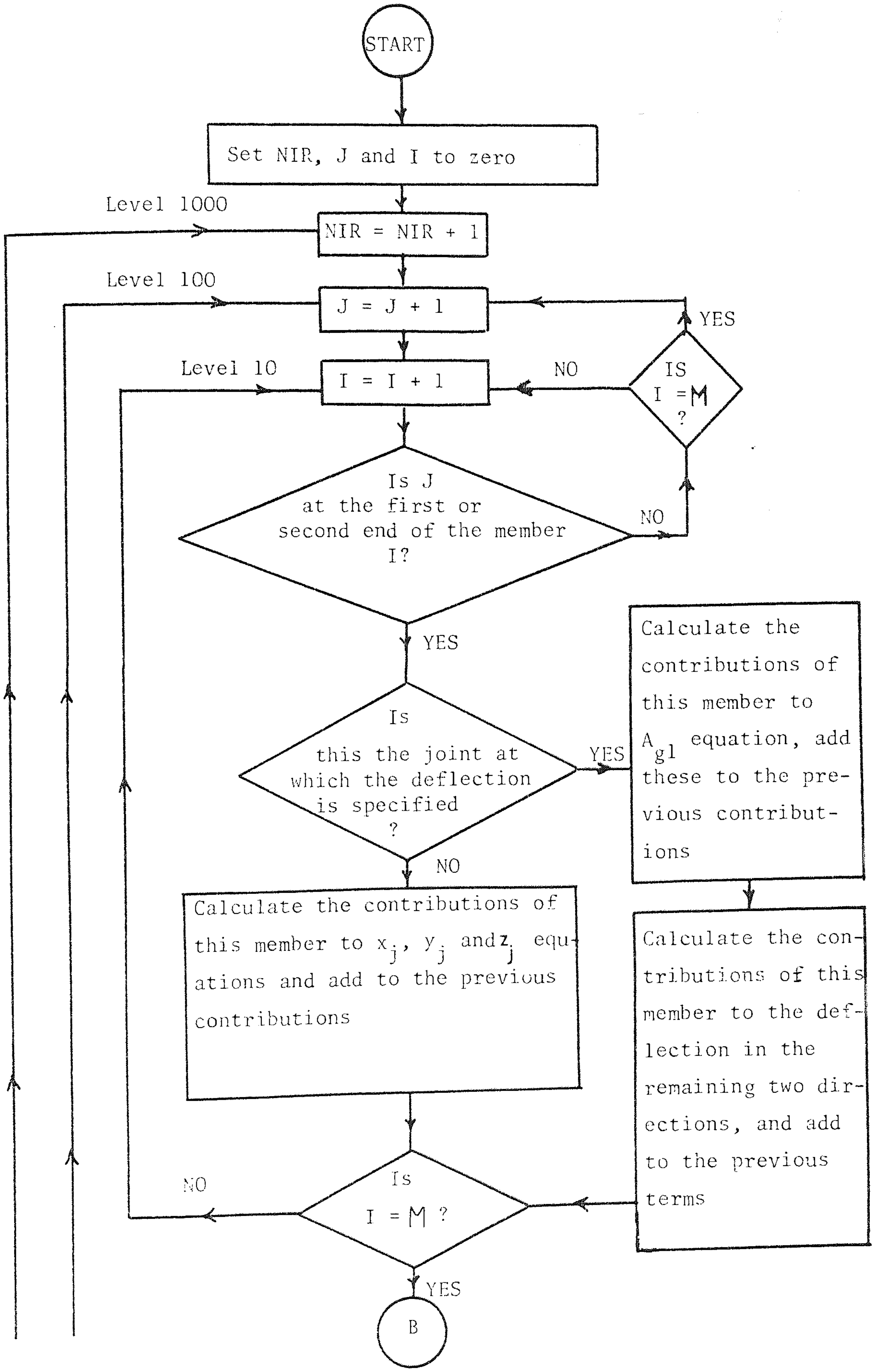


Figure 7.2 PIN-JOINTED SPACE FRAME, FLOW CHART FOR SUBROUTINE ITERATE

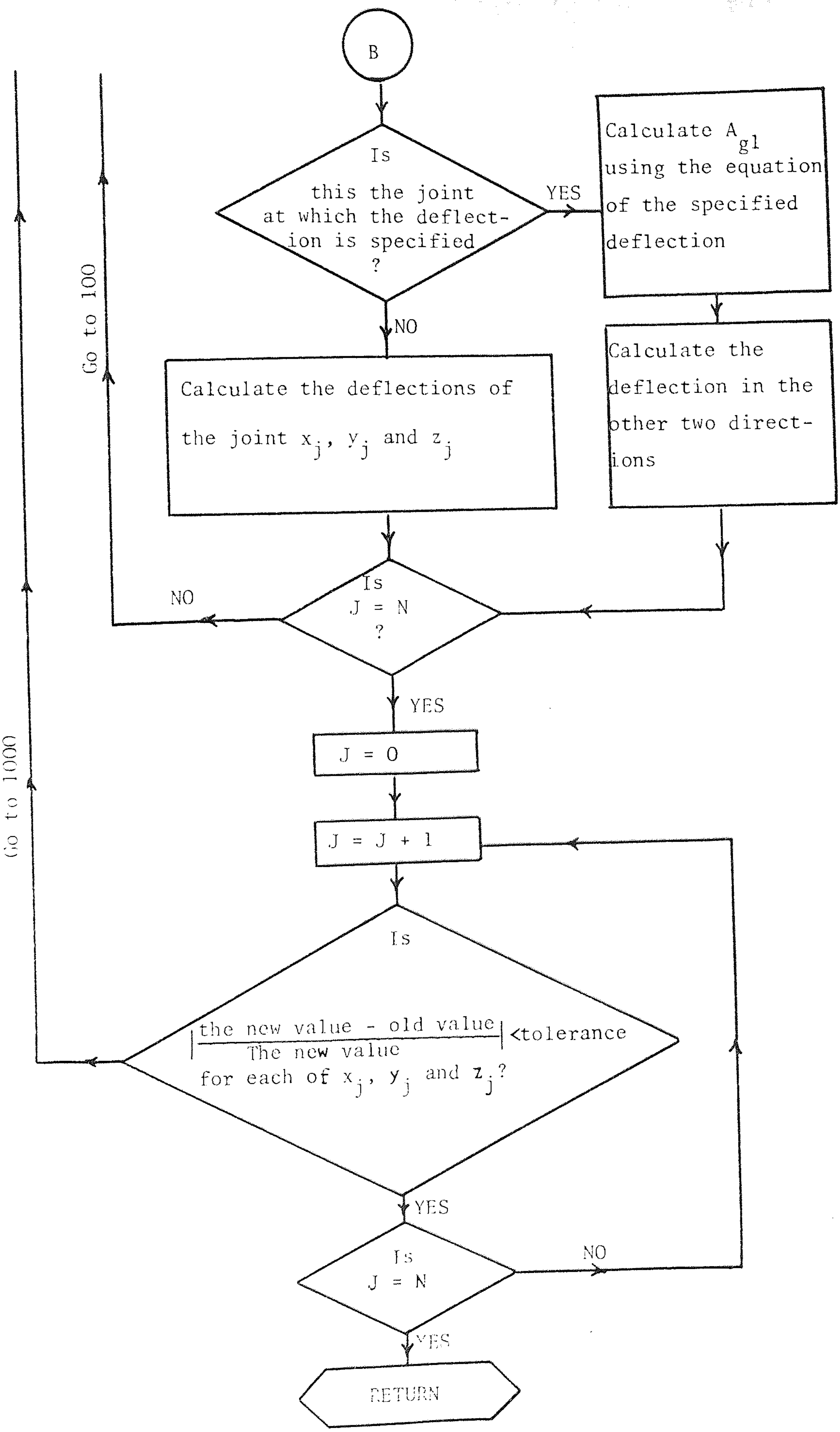


FIGURE 7.2 continued

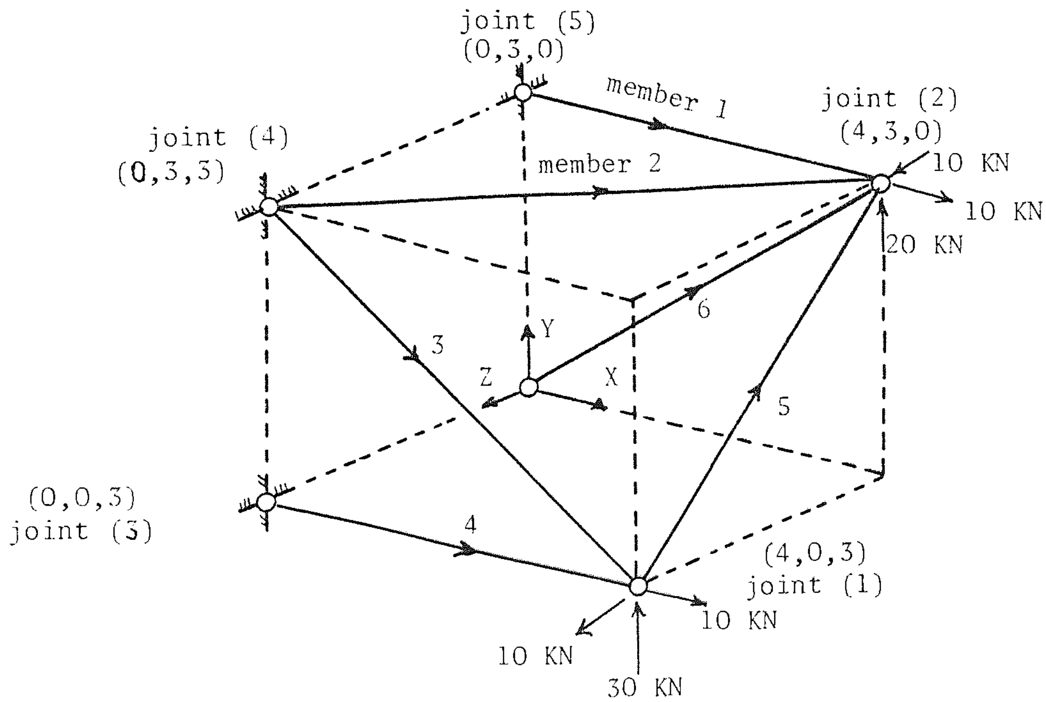


FIGURE 7.3 6 MEMBER - ISOSTATIC SPACE FRAME

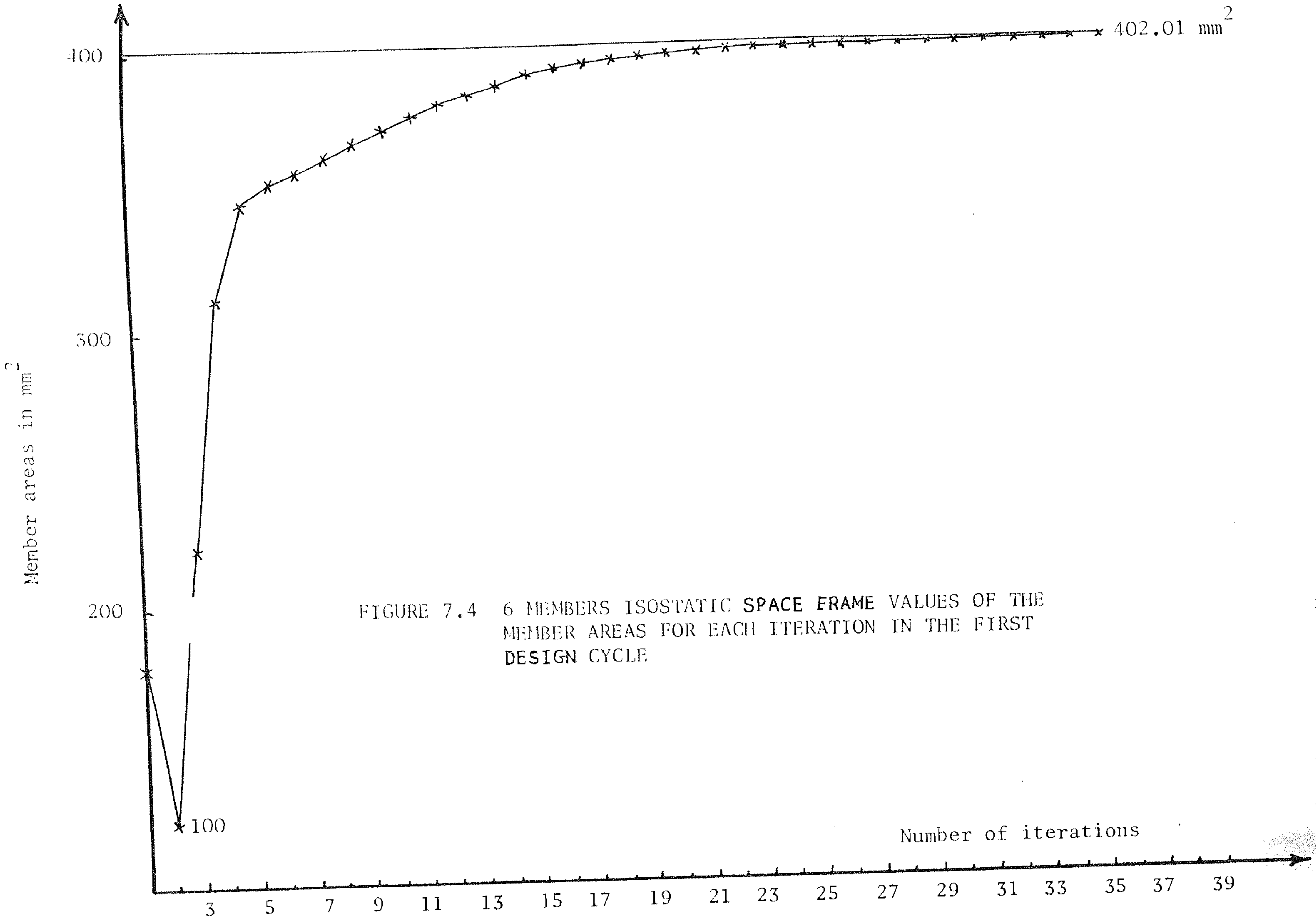


FIGURE 7.4 6 MEMBERS ISOSTATIC **SPACE FRAME** VALUES OF THE MEMBER AREAS FOR EACH ITERATION IN THE FIRST **DESIGN** CYCLE

Member areas obtained in the first design cycle in mm^2	$A_1 = A_2 = A_3 = A_4 = A_5 = A_6 = 402.01$		
Member forces in kN	$p_1 = 23.33$	$p_2 = -33.33$	$p_3 = -66.66$
	$p_4 = 63.33$	$p_5 = 14.14$	$p_6 = 16.67$
* Design of the member for stress requirements, area in mm^2	$A_1 = 93.3$	$A_2 = 166.6$	$A_3 = 333.3$
	$A_4 = 253.3$	$A_5 = 56.6$	$A_6 = 66.7$
The new ratios between the member areas	$\alpha_1 = 1.0$	$\alpha_2 = 1.786$	$\alpha_3 = 3.573$
	$\alpha_4 = 2.715$	$\alpha_5 = 0.606$	$\alpha_6 = 0.714$
Final design, which satisfy strength and deflection requirements area of the members in mm^2	$A_1 = 125.954$	$A_2 = 225.0$	$A_3 = 450.0$
	$A_4 = 342.0$	$A_5 = 76.3$	$A_6 = 90.0$
Total volume obtained in the first cycle	$10951853.8\ mm^3$		
Total volume obtained in the second cycle	$6020203.3\ mm^3$		
Percentage reduction	81.92%		

Table 7.1: 6 MEMBERS ISOSTATIC SPACE FRAME
Various design values

* design stress in tension = $0.250\ KN/mm^2$
design stress in compression = $0.200\ KN/mm^2$

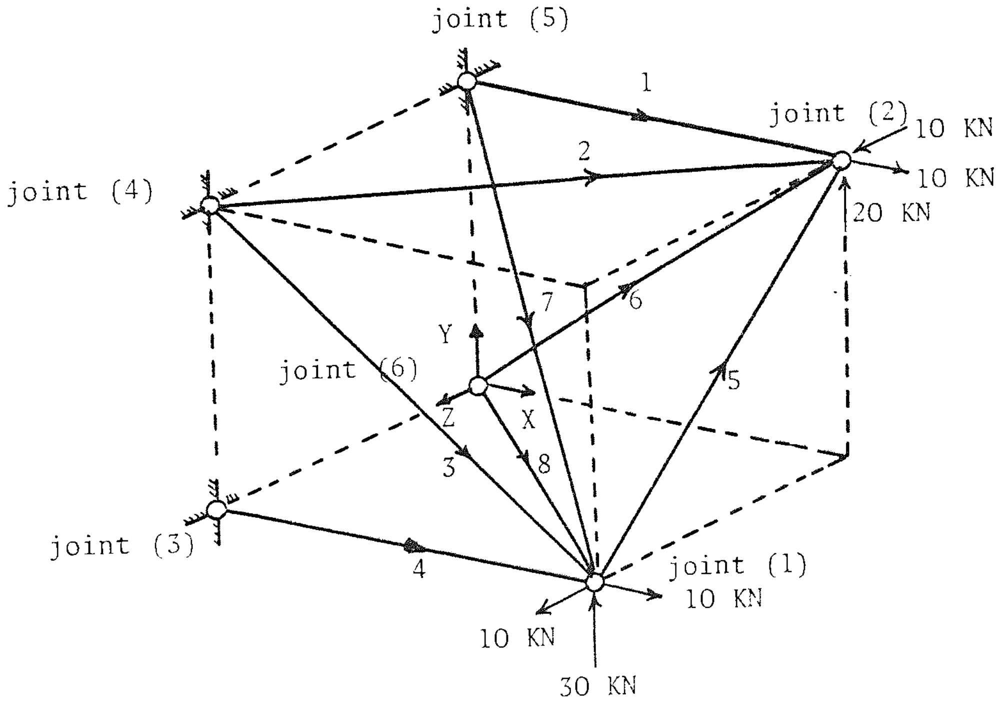


FIGURE 7.5 8 MEMBER HYPERSTATIC SPACE FRAME

Design cycle	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	Area of the first member in mm^2
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	187.61
2	1.0	1.817	4.603	2.389	0.299	3.750	1.805	3.247	58.89
3	1.0	2.155	5.862	3.151	0.269	4.210	1.444	3.223	50.20
4	1.0	2.631	7.469	4.145	0.255	4.847	1.114	3.294	41.75
5	1.0	3.276	9.521	5.411	0.165	5.711	0.813	3.492	34.05
6	1.0	4.073	11.975	6.919	0.095	6.788	0.547	3.816	27.71
7	1.0	4.783	14.145	8.259	0.029	7.736	0.319	4.101	23.70
8	1.0	5.098	15.161	8.888	0.000	8.156	0.154	4.183	22.18
9	1.0	5.094	15.226	8.929	0.000	8.150	0.065	4.120	22.14
10	1.0	5.078	15.207	8.918	0.000	8.125	0.006	4.081	22.20
11	1.0	5.076	15.204	8.919	0.000	8.121	0.000	4.079	22.19

Table 7.2: 8 MEMBERS-HYPERSTATIC SPACE FRAME, RATIOS BETWEEN THE MEMBER AREAS AND THE AREA OF MEMBER (1) FOR VARIOUS DESIGN CYCLES

$A_1 = 22.2\ mm^2$	$A_2 = 112.6\ mm^2$	$A_3 = 337.4\ mm^2$	$A_4 = 197.9\ mm^2$	Total volume
$A_5 = 0.0$	$A_6 = 180.2\ mm^2$	$A_7 = 0.0$	$A_8 = 90.5\ mm^2$	4,487,498 mm^3

Table 7.3: 8 MEMBER-HYPERSTATIC SPACE FRAME. MEMBER AREAS IN THE FINAL DESIGN. BOTH STRESS AND DEFLECTION REQUIREMENTS ARE SATISFIED

Member forces in kN	$p_1 = -3.30$	$p_2 = -16.67$	$p_3 = -49.97$	$p_4 = 36.65$	Total volume
	$p_5 = 0.00$	$p_6 = 33.33$	$p_7 = 0.00$	$p_8 = 16.70$	
Member areas in mm^2	$A_1 = 16.5$	$A_2 = 83.3$	$A_3 = 249.8$	$A_4 = 146.6$	3,320,001 mm^3
	$A_5 = 0.00$	$A_6 = 133.3$	$A_7 = 0.0$	$A_8 = 66.8$	

Table 7.4: 8 MEMBER-HYPERSTATIC SPACE FRAME MEMBER FORCES, AND MEMBER AREAS WHEN ONLY STRESS REQUIREMENTS ARE SATISFIED

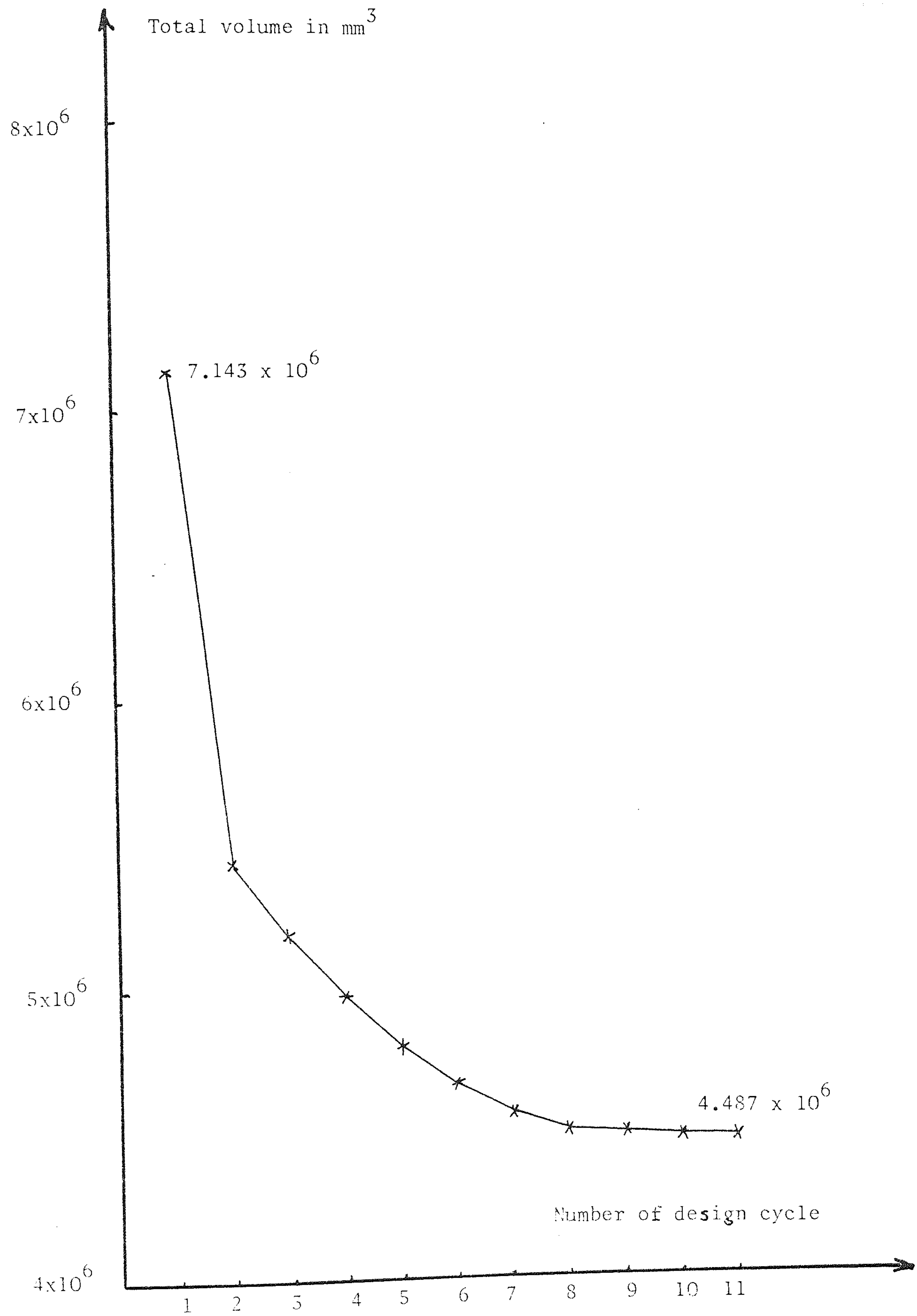


FIGURE 7.6 8 MEMBERS - HYPERSTATIC SPACE FRAME, TOTAL VOLUME AT EACH DESIGN CYCLE

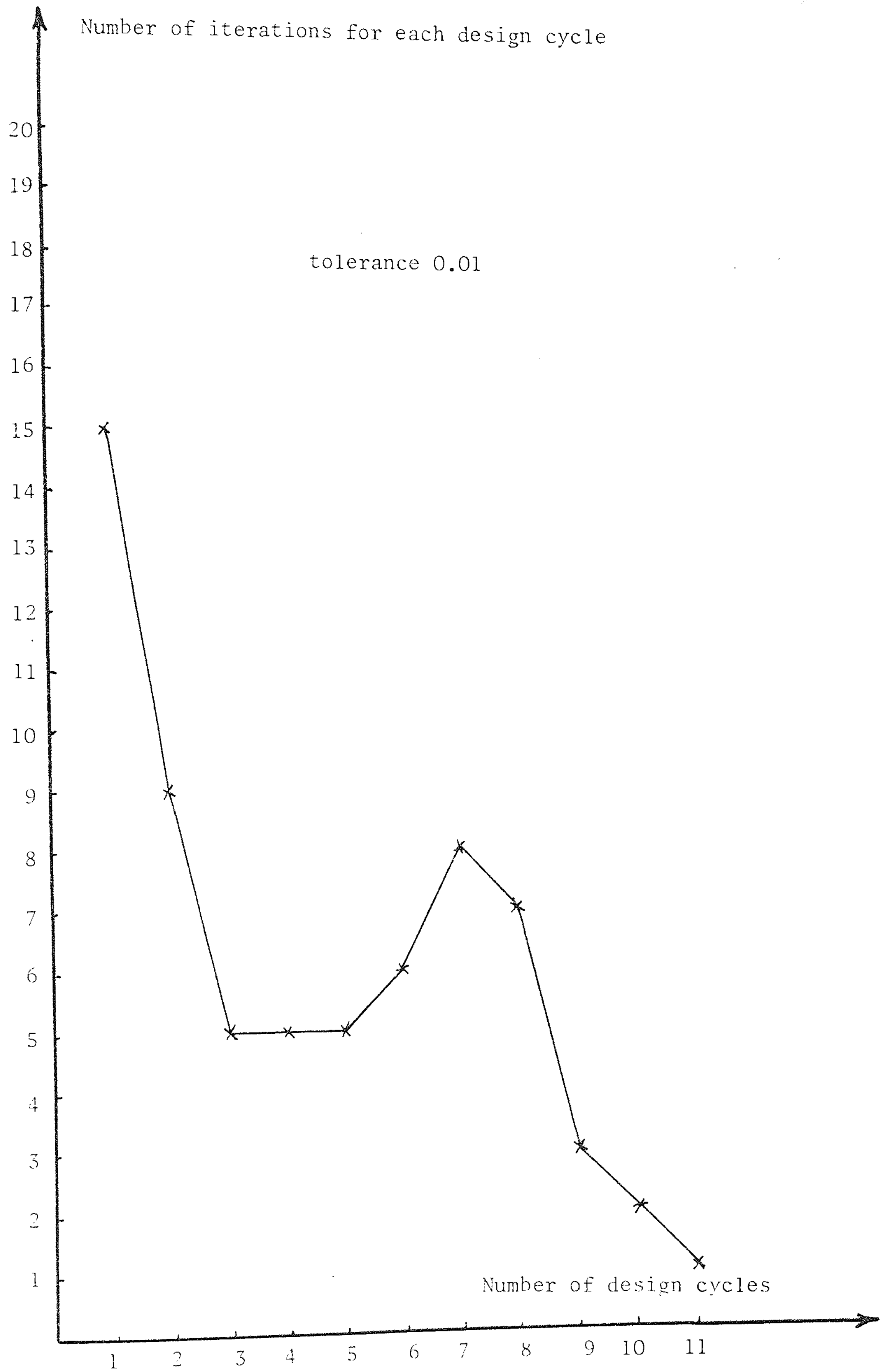


FIGURE 7.7 8 MEMBERS - HYPERSTATIC SPACE FRAME NUMBER OF ITERATIONS PER DESIGN CYCLE

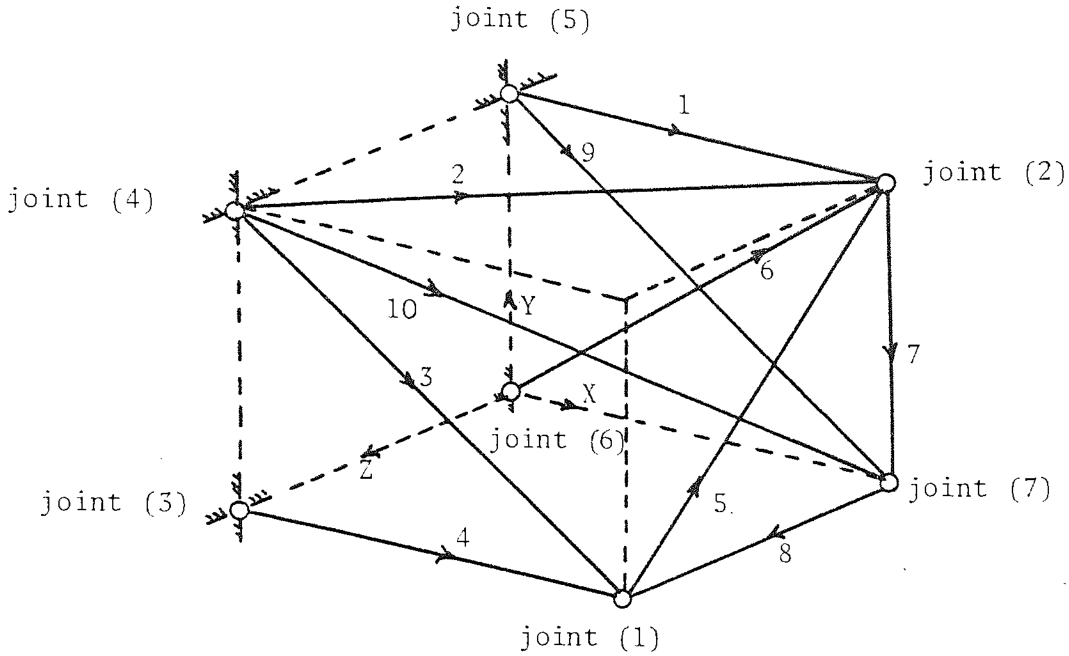


FIGURE 7.8 SPACE FRAME (a) - MEMBER AND JOINT NUMBERS

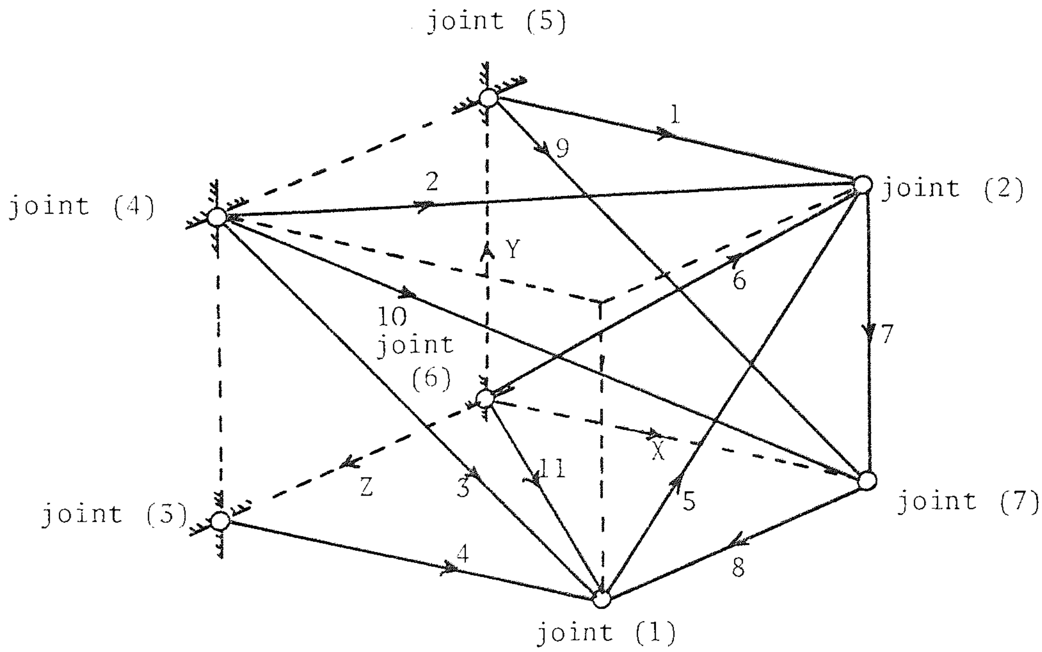


FIGURE 7.9 SPACE FRAME (b) - MEMBER AND JOINT NUMBERS

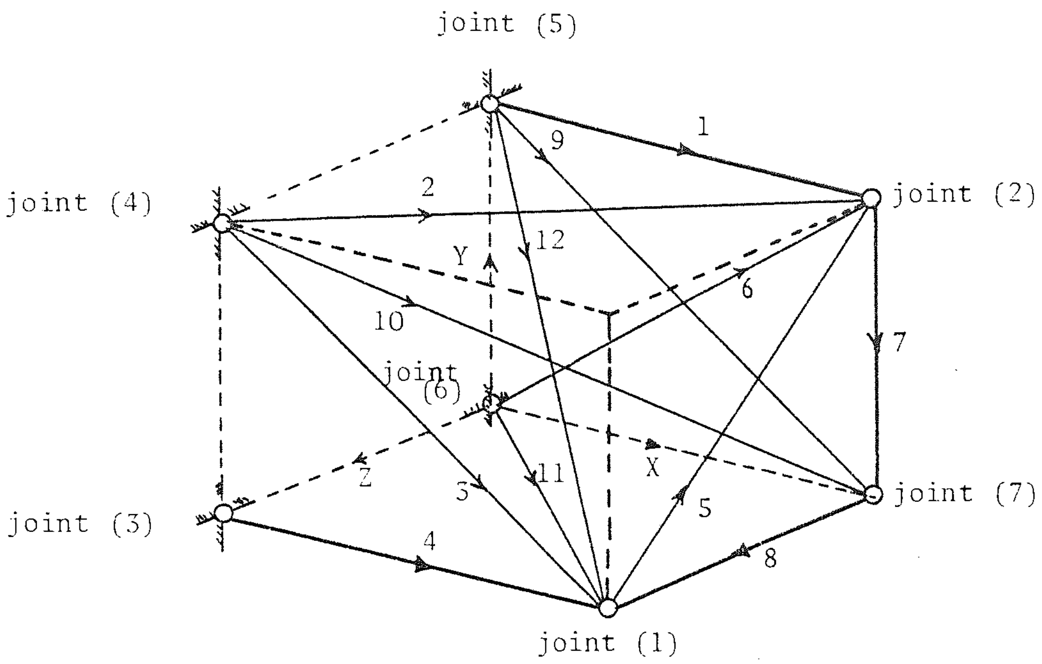


FIGURE 7.10 SPACE FRAME (c) - MEMBER AND JOINT NUMBER

joint number	Coordinates in m			Loads in kN		
	X	Y	Z	w_x	w_y	w_z
1	4.0	0.0	3.0	5.0	5.0	5.0
2	4.0	3.0	0.0	10.0	10.0	10.0
3	0.0	0.0	3.0	0.0	0.0	0.0
4	0.0	3.0	3.0	0.0	0.0	0.0
5	0.0	3.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0
7	4.0	0.0	0.0	-10.0	40.0	-17.0

Table 7.5: JOINT COORDINATES AND EXTERNAL APPLIED LOADS FOR FRAMES (a), (b) AND (c) OF FIGURES (7.8), (7.9) AND (7.10)

Member number	Design of Frame (a) area in mm^2	Design of Frame (b) area in mm^2	Design of Frame (c) area in mm^2
1	394.8	393.8	281.3
2	69.5	70.3	140.6
3	0.0*	0.0*	0.0*
4	33.2	96.7	141.7
5	60.3	59.7	0.0*
6	534.8	534.4	478.2
7	274.1	274.2	274.2
8	67.8	114.8	114.8
9	203.1	105.5	105.5
10	91.1	0.0*	0.0*
11	-	98.4	98.4
12	-	-	82.0
Total volume in mm^3	7564876 mm^3	7425600 mm^3	7450113 mm^3

* Denote the members which has been removed from the final design

TABLE 7.6: FINAL DESIGNS (IN WHICH DEFLECTION AND STRESS REQUIREMENTS ARE SATISFIED) FOR FRAMES (a), (b) AND (c) SHOWN IN FIGURES (7.8), (7.9) AND (7.10)

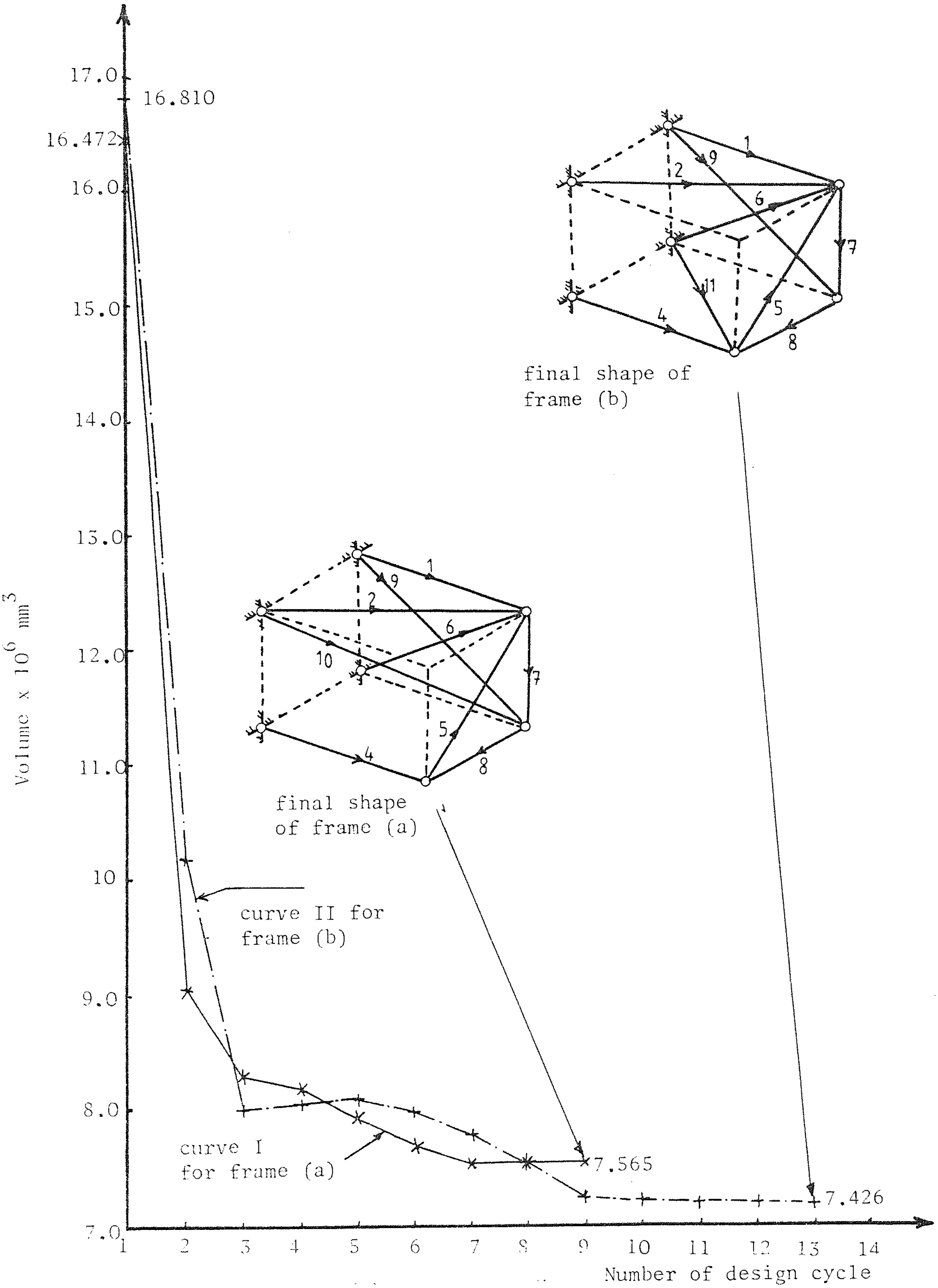


FIGURE 7.11 FRAMES (a) AND (b) OF FIGURES 7.8 and 7.9, TOTAL VOLUME AT EACH DESIGN CYCLE

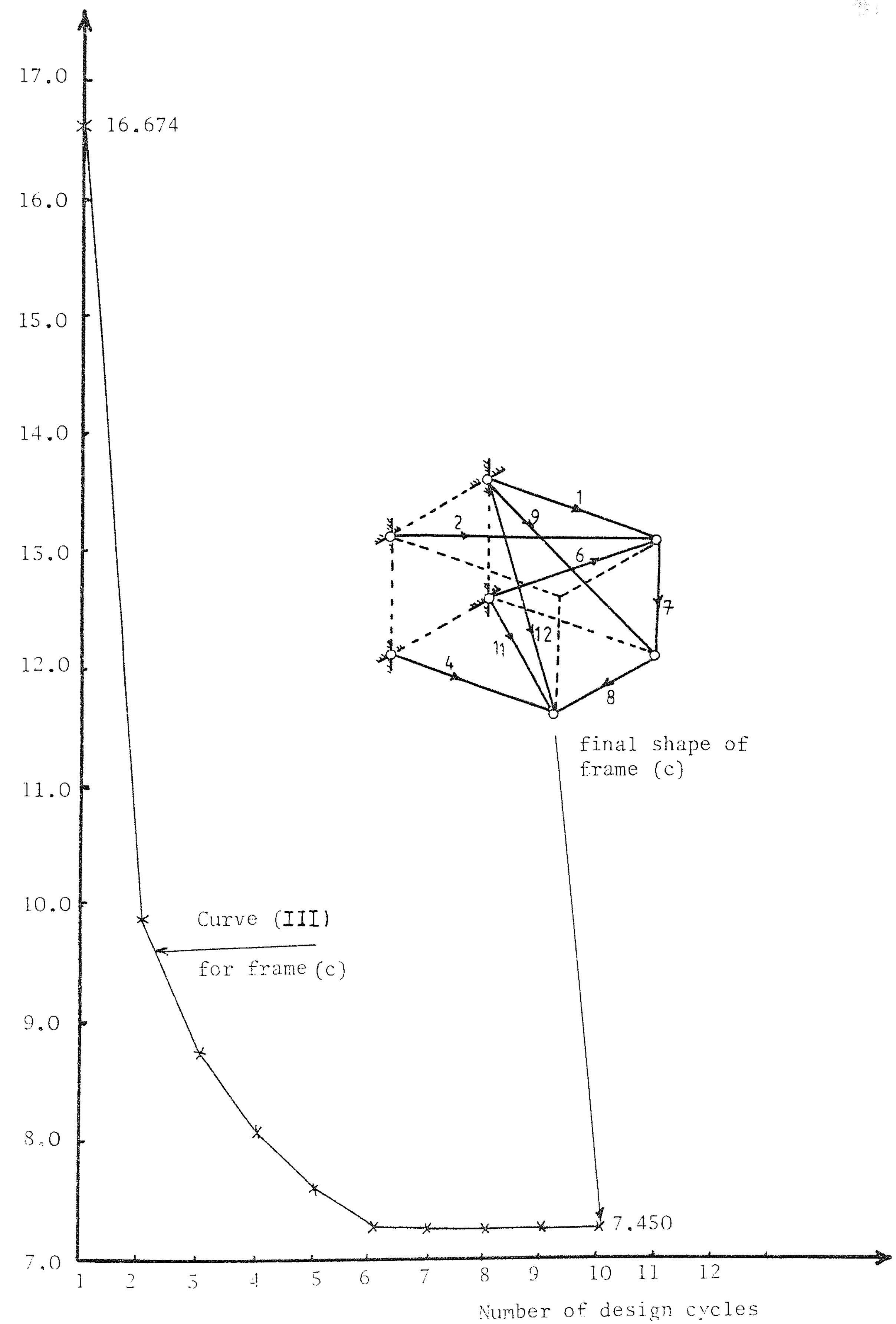
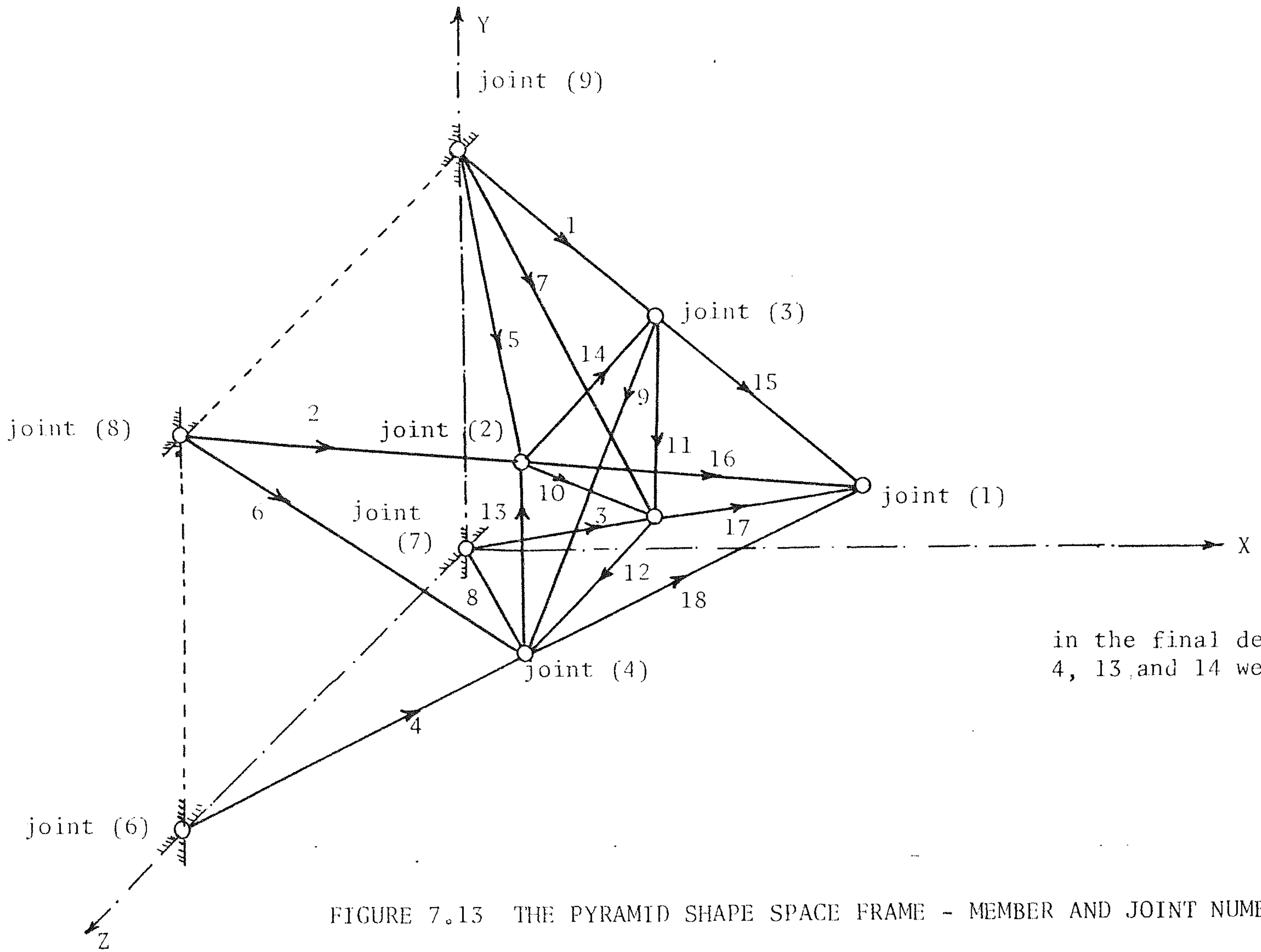


FIGURE 7.12 FRAME (c) OF FIGURE 7.10 TOTAL VOLUME AT EACH DESIGN CYCLE



in the final design members
4, 13 and 14 were omitted

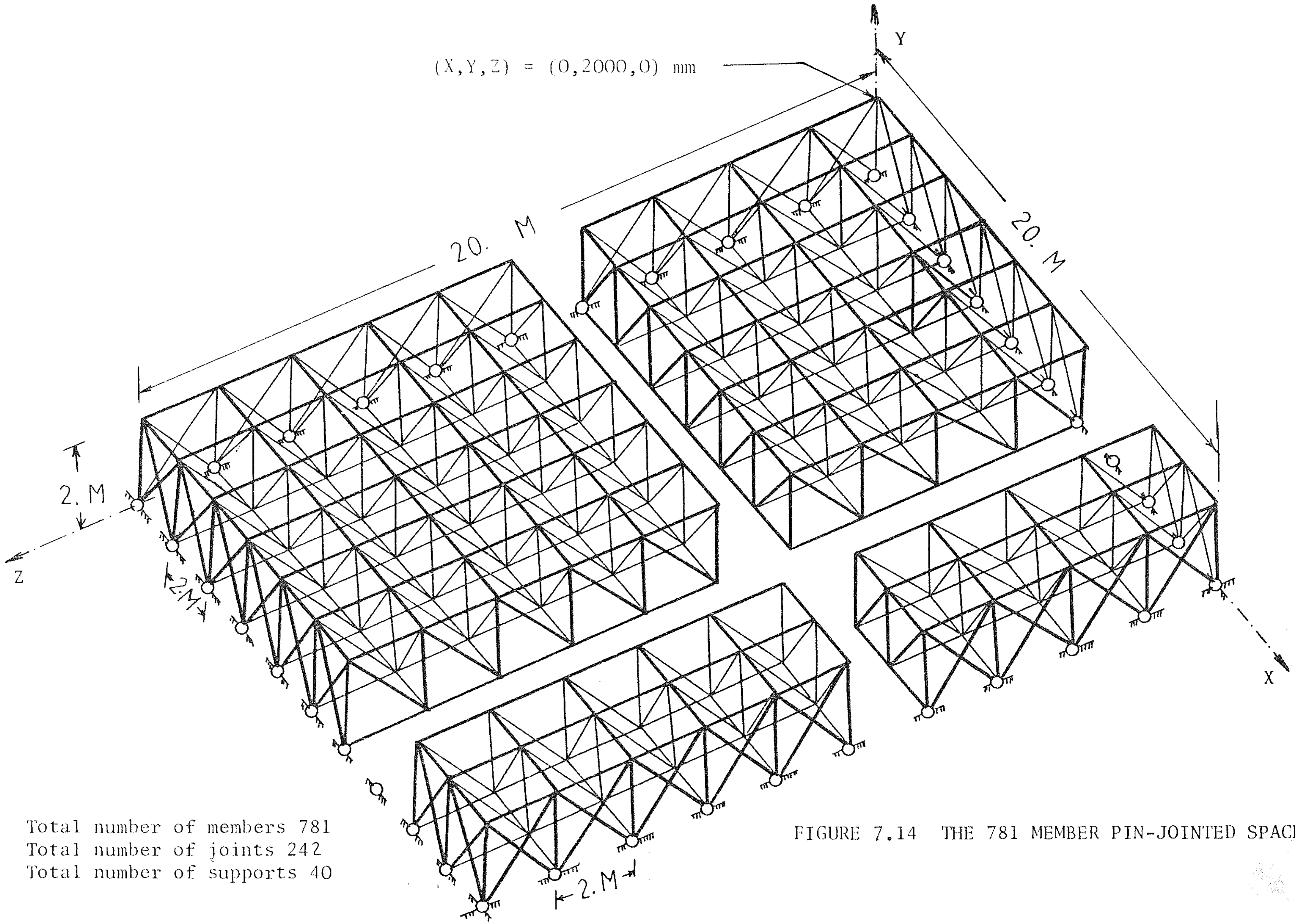
FIGURE 7.13 THE PYRAMID SHAPE SPACE FRAME - MEMBER AND JOINT NUMBERS

Joint number	Coordinates in m			Load in kN		
	X	Y	Z	w_x	w_y	w_z
1	4.00	1.50	1.50	-30.0	30.0	5.0
2	2.0	2.25	2.25	-5.0	5.0	10.0
3	2.0	2.25	0.75	-5.0	5.0	5.0
4	2.0	0.75	2.25	-10.0	5.0	5.0
5	2.0	0.75	0.75	-5.0	5.0	5.0
6	0.0	0.0	3.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	3.0	3.0	0.0	0.0	0.0
9	0.0	3.0	0.0	0.0	0.0	0.0

Table 7.7: THE PYRAMID SHAPE SPACE FRAME-JOINT COORDINATES AND EXTERNAL APPLIED LOADS

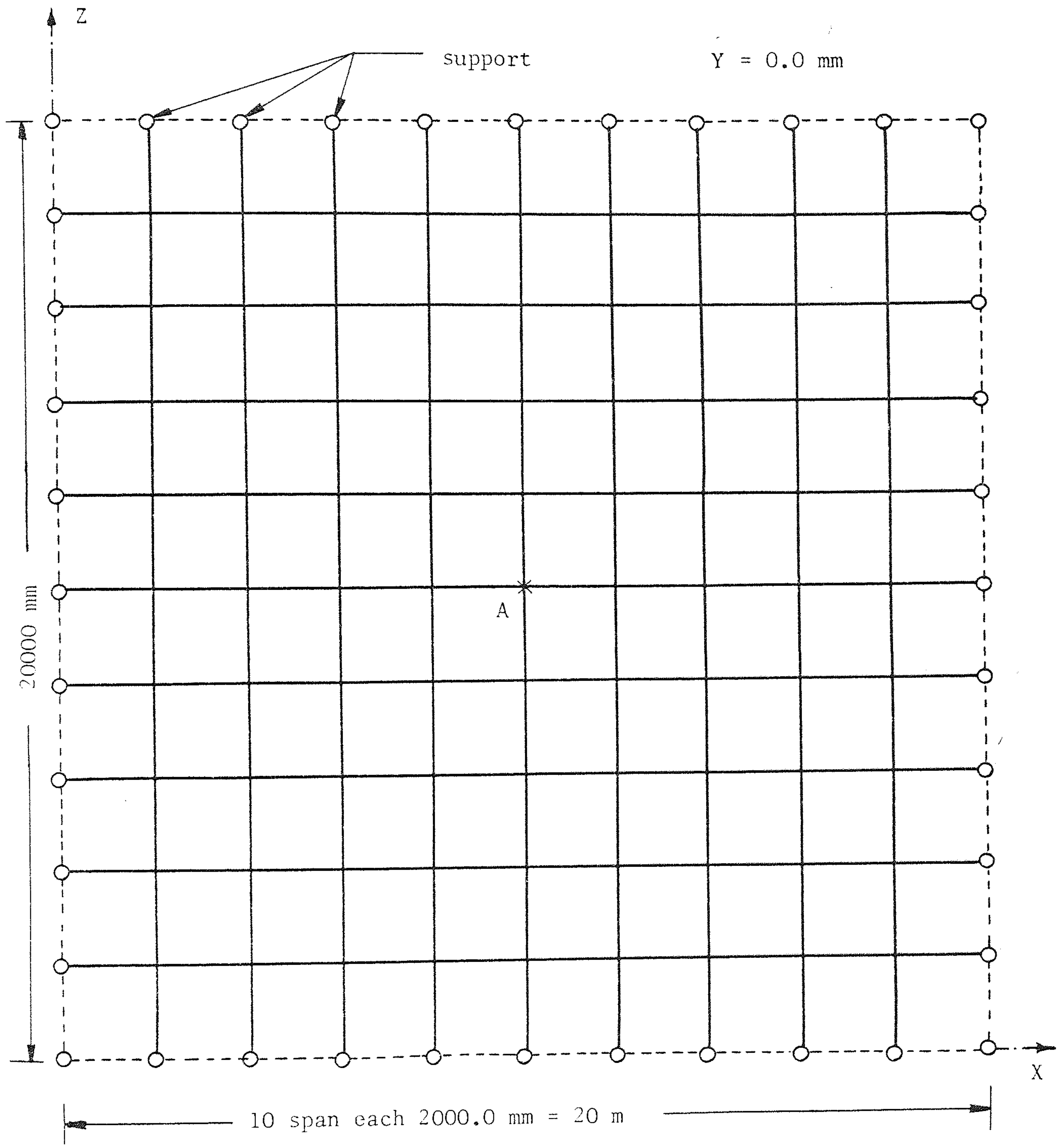
Member number	Area needed for deflection requirements mm^2	Internal forces in the members in kN	Area needed for stress requirement only in mm^2	Value of α
1	114.4	-19.863	99.3	1.0
2	330.8	-57.536	287.7	2.897
3	65.2	14.148	56.6	0.570
4	0.0*	0.0*	0.0*	0.0*
5	24.2	5.248	21.0	0.211
6	11.7	-2.015	10.1	0.102
7	118.4	-20.612	103.1	1.038
8	85.8	18.674	74.7	0.752
9	55.6	-9.680	48.4	0.487
10	20.0	4.363	17.5	0.176
11	45.8	9.971	39.9	0.402
12	57.2	-9.961	49.8	0.502
13	0.0*	0.0*	0.0*	0.0*
14	0.0*	0.0*	0.0*	0.0*
15	81.8	-14.201	71.0	0.715
16	276.2	-48.056	240.3	2.420
17	22.1	4.768	19.1	0.192
18	108.1	23.529	94.1	0.948
Total volume in mm^3	3,322,268		2,888,220	

Table 7.8: PYRAMID SHAPE SPACE FRAME - DESIGN VALUES AT THE FINAL CYCLE



Total number of members 781
Total number of joints 242
Total number of supports 40

FIGURE 7.14 THE 781 MEMBER PIN-JOINTED SPACE FRAME



The deflection at joint A was specified to be 55.55 mm in Y direction

FIGURE 7.15 MEMBERS IN GROUP 3 IN THE X-Z PLANE AT Y = 0.0 mm

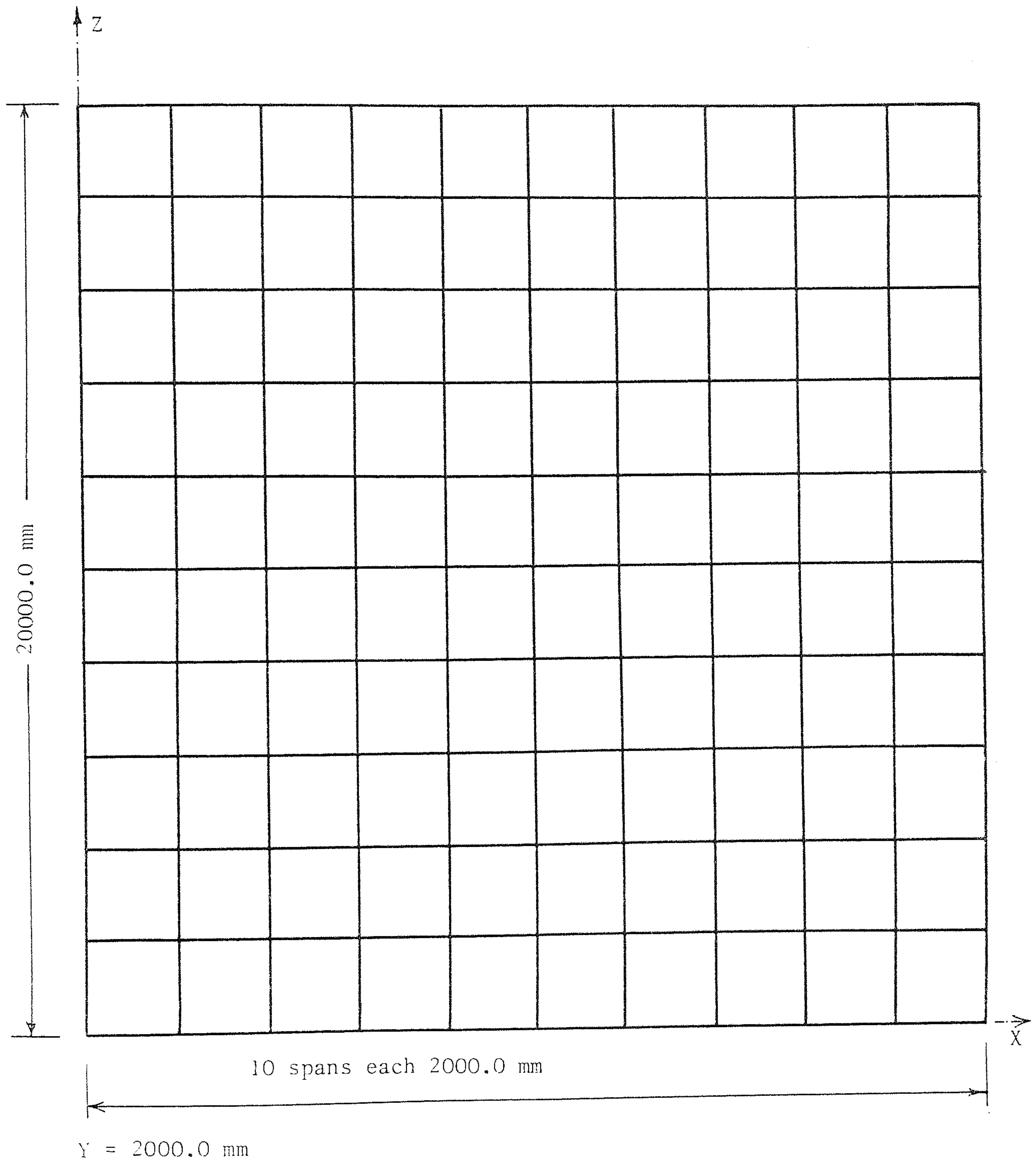
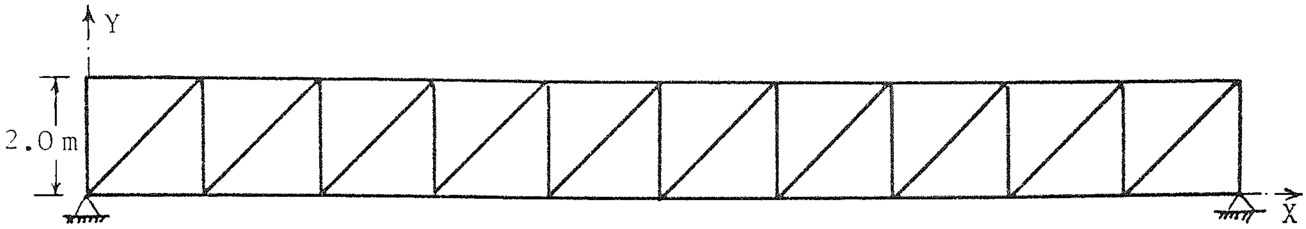
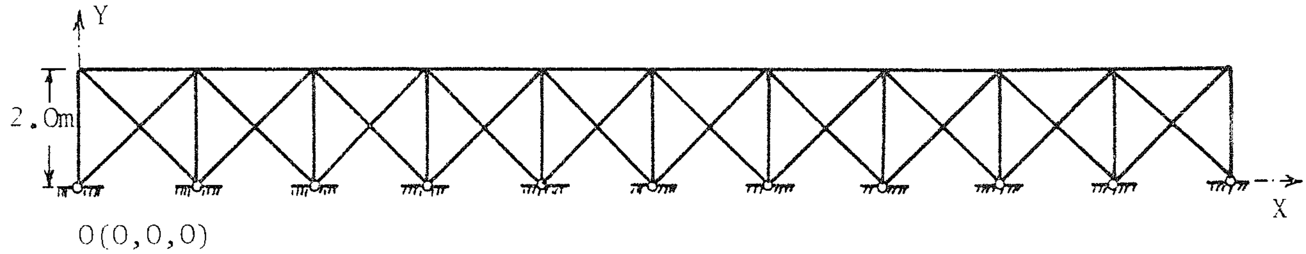


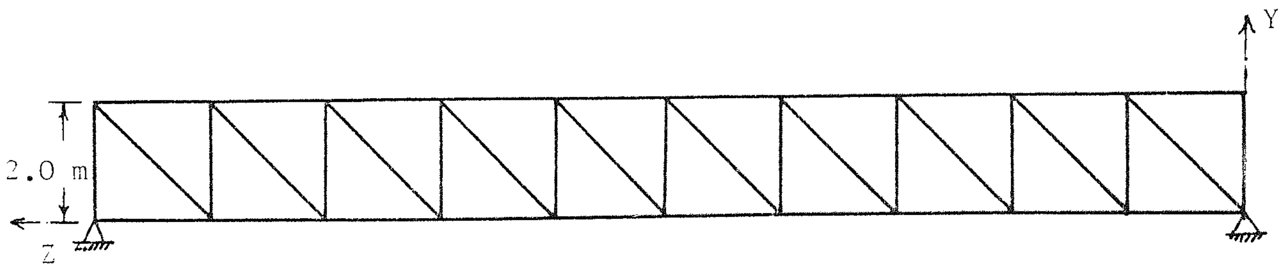
FIGURE 7.16 MEMBERS IN GROUP 4 IN THE X-Z PLANE AT Y = 2000.0 mm



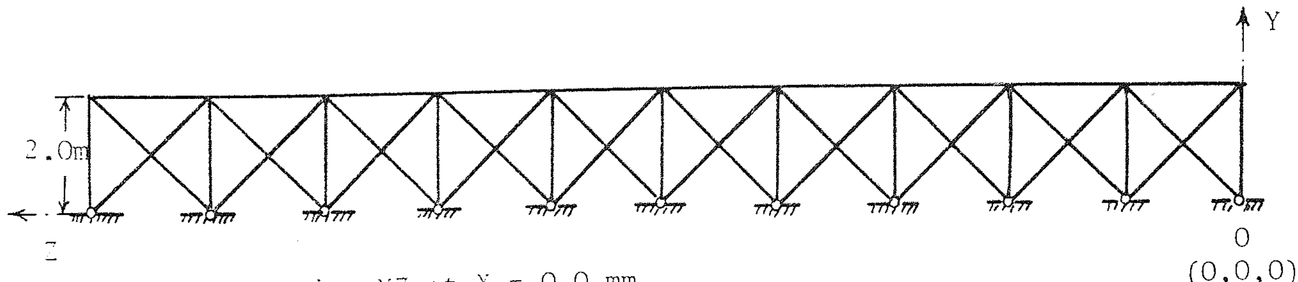
(a) Section XY at Z = 10000.0 mm



(b) Section XY at Z = 0.0 mm



(c) Section YZ at X = 10000.0 mm



(d) Section YZ at X = 0.0 mm

FIGURE 7.17 THE 781 MEMBER SPACE FRAME - VARIOUS SECTIONS IN THE STRUCTURE

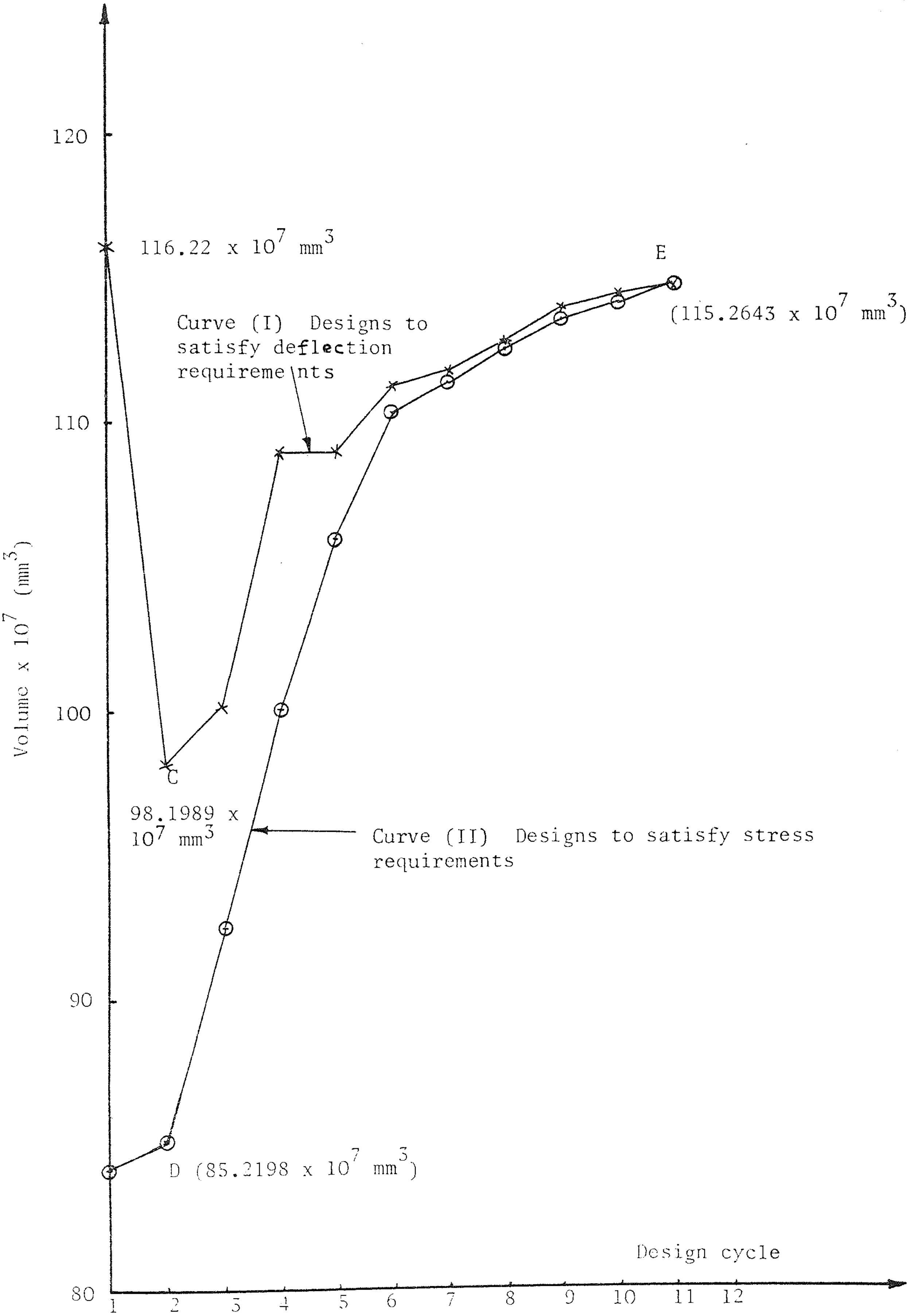


FIGURE 7.18 THE 781 MEMBER SPACE FRAME CHANGE IN VOLUME FOR EACH DESIGN CYCLE - HIGH YIELD STEEL USED

Design cycle	α_1	α_2	α_3	α_4	Area of the members in groups (1) mm^2
1	1.0	1.0	1.0	1.0	653.86
2	1.0	0.62287	1.44867	2.22499	411.45
3	1.0	0.59418	1.37098	2.79649	389.34
4	1.0	0.58592	1.30943	3.28939	389.96
5	1.0	0.577994	1.25918	3.84170	361.13
6	1.0	0.57470	1.27180	4.11295	360.70
7	1.0	0.57253	1.28253	4.38785	342.56
8	1.0	0.57471	1.28683	4.53106	338.89
9	1.0	0.57585	1.29028	4.65067	337.21
10	1.0	0.57626	1.29236	4.75762	333.89
11	1.0	0.57652	1.29200	4.83382	332.47
12	1.0	0.57644	1.29209	4.83555	332.46

Table 7.9: THE 781 MEMBER SPACE-FRAME HIGH YIELD STEEL - MEMBER AREAS IN THE FIRST GROUP AND VALUES OF α FOR EACH DESIGN CYCLE

Design cycle		Areas in group (1) mm^2	Areas in group (2) mm^2	Areas in group (3) mm^2	Areas in group (4) mm^2	Total volume $\times 10^7$ mm^3
Second Design Cycle	Area corresponding to point C in figure 7.18	411.45	256.3	596.1	915.5	98.1989
	Area corresponding to point D in figure 7.18	327.4	194.5	448.8	915.5	85.2198
Final Design Cycle	Area corresponding to point E in figure 7.18	332.5	191.7	429.6	1607.1	115.2643

Table 7.10: THE 781 MEMBERS SPACE FRAME - VARIOUS DESIGNS HIGH-YIELD STEEL USED

Design cycle	α_1	α_2	α_3	α_4	Area of the members in groups (1) mm^2
1	1.0	1.0	1.0	1.0	653.86
2	1.0	0.6229	1.4487	2.2250	411.45
3	1.0	0.5942	1.3710	2.3274	412.34
4	1.0	0.5861	1.324	2.3925	413.34
5	1.0	0.5828	1.2916	2.4370	412.77
6	1.0	0.5827	1.2789	2.4575	412.06
7	1.0	0.5822	1.2738	2.4548	411.75

Table 7.11: VALUES OF α FOR THE MILD STEEL SPACE FRAME

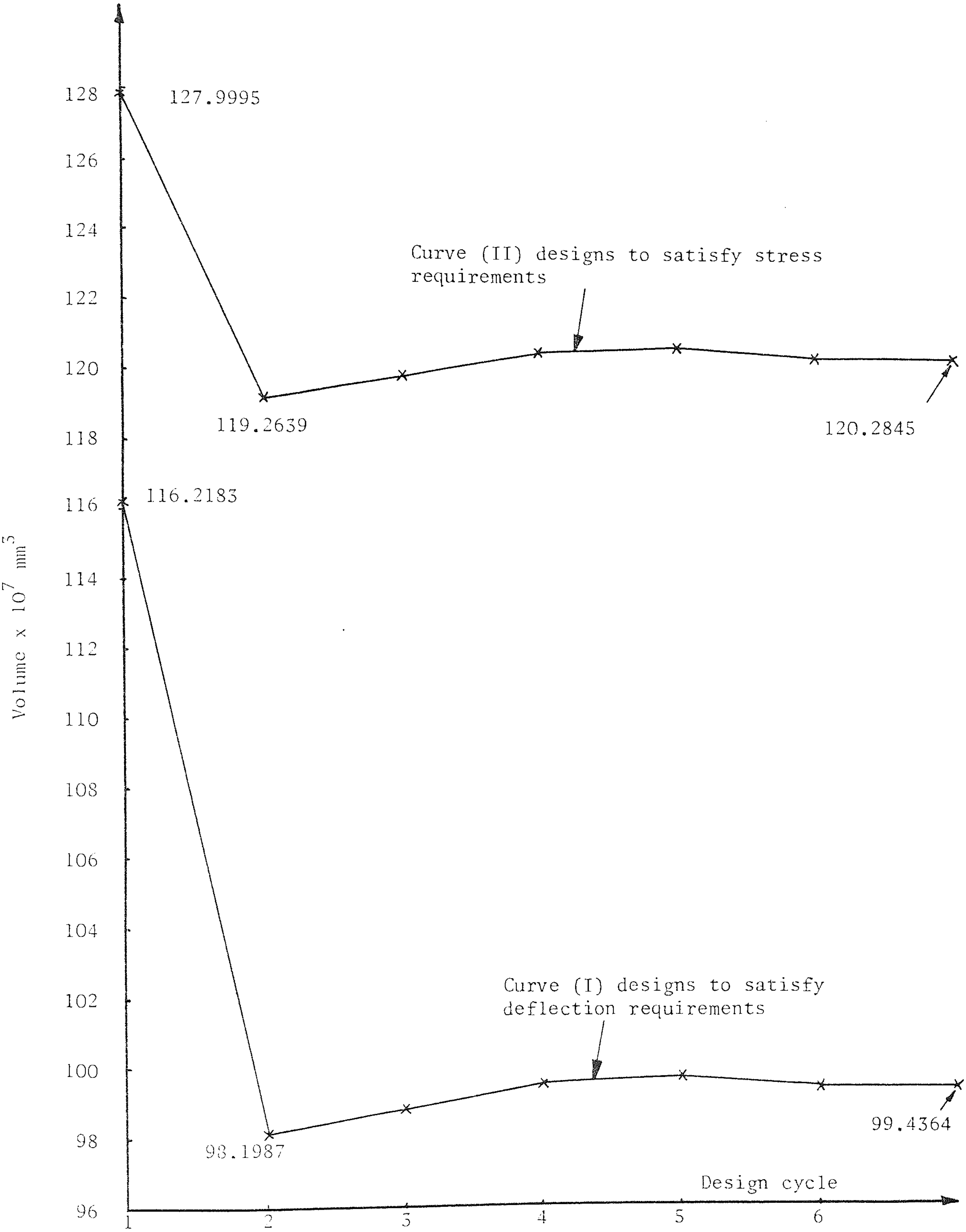


FIGURE 7.19 THE 781 MEMBER SPACE FRAME - MILD STEEL USED. VOLUME OF THE STRUCTURE AT EACH DESIGN CYCLE

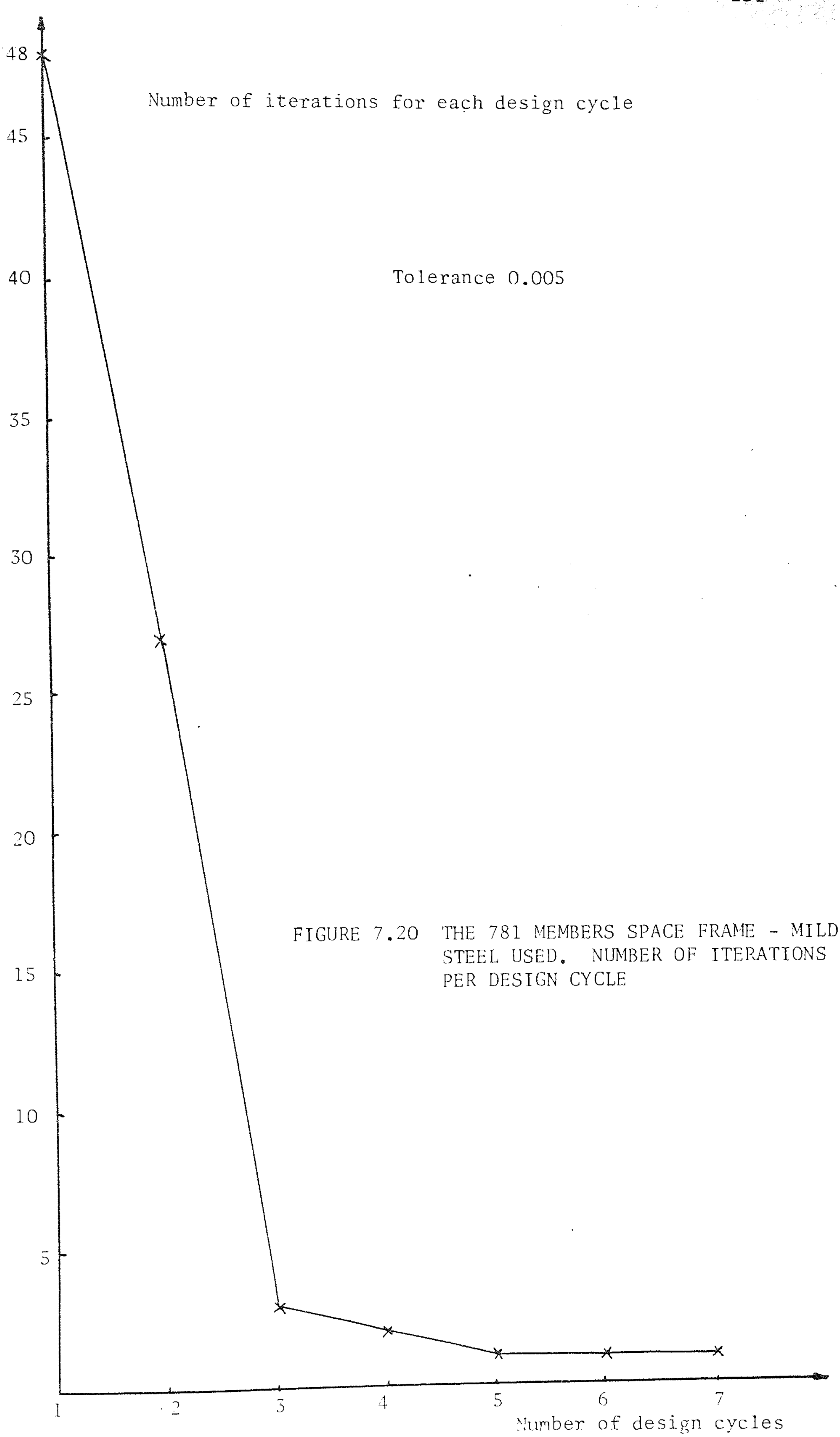


FIGURE 7.20 THE 781 MEMBERS SPACE FRAME - MILD STEEL USED. NUMBER OF ITERATIONS PER DESIGN CYCLE

CHAPTER EIGHT

OPTIMUM DESIGN OF RIGIDLY JOINTED STEEL FRAMES

8.1 INTRODUCTION

In Chapter 2, once a design was obtained, an extrapolation technique was then used to change the beam sections of rigidly jointed steel frames. Although this is an approximate method for reducing the material cost of the frame, it was found to be satisfactory from the practical point of view. In this chapter a more mathematical approach is presented using a classical method of optimisation by calculus. The constraints involved are all equalities and the method of Lagrange multipliers (Hadley 1970, and Majid, 1980) can be used to formulate the design problem. Such an approach may be more efficient than using linear programming because in the latter case each equality requires the introduction of an artificial variable which is often difficult to remove from the simplex table (Majid, 1974 and Saka, 1975).

Using the second moment of area of the members as the design variables, the objective function Z for a minimum weight design may be expressed as:

$$Z = f(I) = \sum_{u=1}^{u=M} I_u L_u \qquad (8.1)$$

where M is the total number of members, I_u is the second moment of area of member u, which has length L_u. If the effects of the axial loads in the members are neglected then the number of the stiffness constraints can be reduced to 2N, where N is the number of joints in the frame. These constraints are:

$$g_v(X) = b_v \quad v = 1, \ldots\ldots, 2N \qquad (8.2)$$

Each of the above equations represents a row in the stiffness equations $\underline{L} = \underline{K}\ \underline{X}$.

To apply the Langrange multiplier technique (see Chapter 1, section 1.4) equations (8.2) are first converted to the form:

$$G_v = \lambda_v\ [b_v - g_v\ (X)] \quad v = 1,\ \ldots\ldots, 2N \tag{8.3}$$

where the variable λ_v is known as the Lagrange multiplier. The constraints G_v are then added to the objective function Z to form

$$F\ (Y,\ \lambda) = f(I) + \sum_{v=1}^{2N} \lambda_v [b_v - g_v(X)] \tag{8.4}$$

F (Y, λ) is the unconstrained Lagrangian function which when minimum, ensures that Z is also minimum. The difference between F(Y, λ) and f(I) given by (8.1) is that F(Y, λ) has more variables than f(I). For example F(Y, λ) contains the variables I, as well as the stiffness constrains $g_v\ (X) = b_v$, and the Lagrange multipliers λ. Notice that the turning points of F(Y, λ) also include those of f(I).

To obtain the point of Absolute Minimum for the new unconstrained objective function (8.4), its partial derivatives with respect to each of the M+2N variables are set to zero. Thus:

$$\frac{\partial F}{\partial I_u} = \frac{\partial f}{\partial I_u} - \sum_{v=1}^{2N} \lambda_v \frac{\partial g_v}{\partial I_u} = 0, \quad u = 1, \ldots\ldots, M \tag{8.5}$$

$$\frac{\partial F}{\partial X_v} = \frac{\partial f}{\partial X_v} - \sum_{v=1}^{2N} \lambda\ v \ \frac{\partial g_v}{\partial X_v} = 0, \quad v = 1,\ \ldots\ldots,\ 2N \tag{8.6}$$

$$\frac{\partial F}{\partial \lambda_v} = b_v - g_v(X) = 0, \qquad v = 1,\ \ldots\ldots,\ 2N \tag{8.7}$$

These equations are solved iteratively to obtain the unknown second moments of area I, the joint displacements $\underline{X}$ and the Lagrangian multipliers λ. Notice that equations (8.7) simply state that $b_v = g_v(X)$ and are in fact the stiffness equations of the frame. These were, on their own, used in Chapter 2, to calculate the joint displacements and the second moments of area of the columns. Thus part of the new iteration process is merely a repeat of the process given in Chapter 2. Some of the equations in (8.6) are partial derivatives of F with respect to the joint rotations at the ends of the columns. These give expressions containing the second moments of area I_B of the beams and are thus used to calculate I_B. The rest of equations (8.6) and all the equations (8.5) are used to calculate the unknown values of λ.

8.2 A DESIGN EXAMPLE

To explain the use of equations (8.5) to (8.7) they will be formulated for the frame shown in Figure 8.1 and then used to demonstrate the steps of the iteration process. The frame was used in Chapter 2, section 2.4.4 to explain the derivation of the modified stiffness equations. The stiffness equations $\underline{L} = \underline{K}\ \underline{X}$ for this frame were given by equations (2.39). These are:

$$\begin{bmatrix} Q \\ 0 \\ P \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{12EI_1}{h^3} + \frac{12EI_1}{h^3} & -\frac{6EI_1}{h^2} & 0 & 0 & \frac{6EI_1}{h^2} \\ -\frac{6EI_1}{h^2} & \frac{4EI_1}{h} + \frac{4EI_2}{L/2} & -\frac{6EI_2}{(L/2)^2} & \frac{2EI_2}{L/2} & 0 \\ 0 & -\frac{6EI_2}{(L/2)^2} & \frac{12EI_2}{(L/2)^3} + \frac{12EI_2}{(L/2)^3} & 0 & \frac{6EI_2}{(L/2)^2} \\ 0 & \frac{2EI_2}{L/2} & 0 & \frac{4EI_2}{L/2} + \frac{4EI_2}{L/2} & \frac{2EI_2}{L/2} \\ -\frac{6EI_1}{h^2} & 0 & \frac{6EI_2}{(L/2)^2} & \frac{2EI_2}{L/2} & \frac{4EI_1}{h} + \frac{4EI_2}{L/2} \end{bmatrix} \begin{bmatrix} x_2 \\ r_2 \\ y_1 \\ r_1 \\ r_3 \end{bmatrix} \tag{8.8}$$

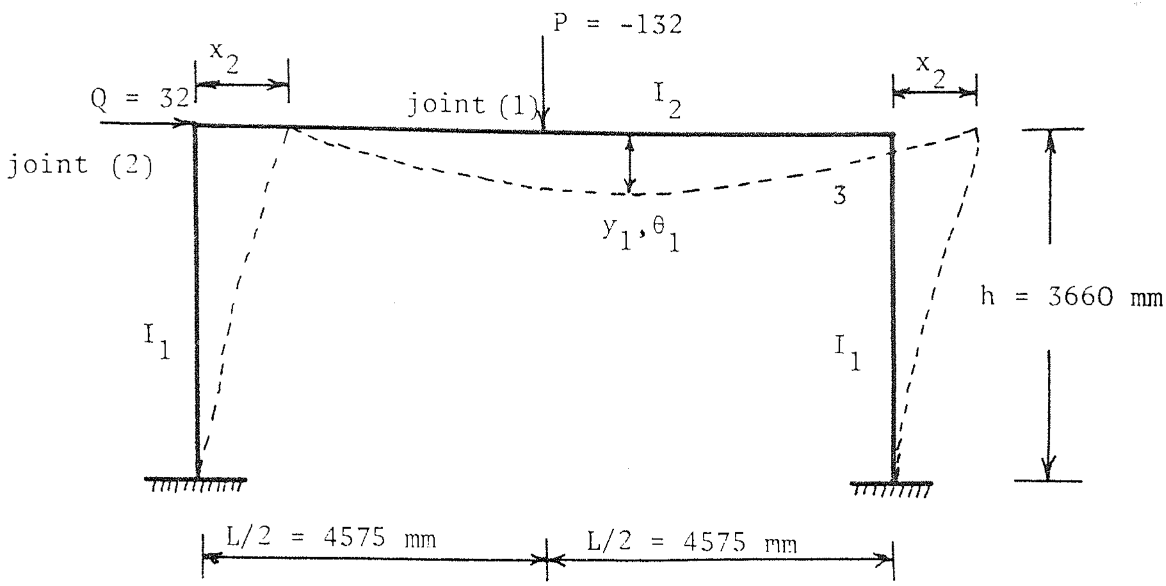


FIGURE 8.1 SINGLE STOREY - SINGLE BAY FRAME

where r_1, r_2, r_3 are the rotations of joints 1, 2 and 3 respectively, x_2 is the horizontal sway and y_1 is the vertical sway of joint 1. The second moment of area for the columns and the beam are I_1 and I_2 respectively. The modulus of elasticity E for the steel may be taken as 207 kN/mm^2.

The problem is to minimise the objective function Z given by:

$$Z = f(I) = 2I_1h + I_2L \tag{8.9}$$

To apply the Lagrange multiplier technique the stiffness equations (8.8) are first converted to the form $\underline{L} = \underline{S}_a \underline{X}$ or equation (8.10) below.

In equations (8.10) each Lagrange multiplier has two suffixes. The first denotes the joint number 1, 2 or 3, and the second suffix, which is either 1 or 2, denotes the type of deflection being considered. One specifies a vertical translation (e.g. y_1, or x_2) while two refers to a joint rotation.

It should be noticed that, equations (8.10) are the matrix form of equations (8.3). To obtain the Lagrangian function $F(Y, \lambda)$ of equation (8.4) each row of equations (8.10) is added to the

$$
\begin{bmatrix} \lambda_{21}Q \\ \lambda_{22}\times 0 \\ \lambda_{11}P \\ \lambda_{12}\times 0 \\ \lambda_{32}\times 0 \end{bmatrix}
=
\begin{array}{c}
\begin{array}{ccccc} \quad x_2 \quad & \quad r_2 \quad & \quad y_1 \quad & \quad r_1 \quad & \quad r_3 \quad \end{array} \\
\begin{bmatrix}
\lambda_{21}\left(\frac{24EI_1}{h^3}\right) & -\lambda_{21}\left(\frac{6EI_1}{h^2}\right) & 0 & 0 & -\lambda_{21}\left(\frac{6EI_1}{h^2}\right) \\
-\lambda_{22}\,\frac{6EI_1}{h^2} & \lambda_{22}\left(\frac{4EI_1}{h}+\frac{4EI_2}{L/2}\right) & -\lambda_{22}\left(\frac{6EI_2}{(L/2)^2}\right) & \lambda_{22}\left(\frac{2EI_2}{L/2}\right) & 0 \\
0 & -\lambda_{11}\,\frac{6EI_2}{(L/2)^2} & \lambda_{11}\left(\frac{24EI_2}{(L/2)^3}\right) & 0 & \lambda_{11}\frac{6EI_2}{(L/2)^2} \\
0 & \lambda_{12}\left(\frac{2EI_2}{L/2}\right) & 0 & \lambda_{12}\left(\frac{8EI_2}{L/2}\right) & \lambda_{12}\left(\frac{2EI_2}{L/2}\right) \\
-\lambda_{32}\left(\frac{6EI_1}{h^2}\right) & 0 & \lambda_{32}\frac{6EI_2}{(L/2)^2} & \lambda_{32}\left(\frac{2EI_2}{L/2}\right) & \lambda_{32}\left(\frac{4EI_1}{h}+\frac{4EI_2}{L/2}\right)
\end{bmatrix}
\end{array}
\begin{bmatrix} x_2 \\ r_2 \\ y_1 \\ r_1 \\ r_3 \end{bmatrix}
\qquad (8.10)
$$

$$\underline{S}_a$$

objective function given by equation (8.9). This results in:

$$F(Y,\lambda) = 2I_1h + I_2L + \lambda_{21}\ (-Q + \frac{24EI_1}{h^3}\ x_2$$

$$- \frac{6EI_1}{h^2} r_2 - \frac{6EI_1}{h^2} r_3) + \lambda_{22}\ (-0 - \frac{6EI_1}{h^2} x_2$$

$$+ \frac{4EI_1}{h} r_2 + \frac{4EI_2}{L/2} r_2 - \frac{6EI_2}{(L/2)^2} y_1 + \frac{2EI_2}{L/2} r_1) + \lambda_{11}\ (-P -$$

$$- \frac{6EI_2}{(L/2)^2} r_2 + \frac{24EI_2}{(L/2)^3} y_1 + \frac{6EI_2}{(L/2)^2} r_3) + \lambda_{12}\ (-0 + \frac{2EI_2}{L/2} r_2$$

$$+ \frac{8EI_2}{L/2} r_1 + \frac{2EI_2}{L/2} r_3) + \lambda_{32}\ (-0 - \frac{6EI_1}{h^2}\ x_2$$

$$+ \frac{6EI_2}{(L/2)^2} y_1 + \frac{2EI_2}{L/2}\ r_1 + \frac{4EI_1}{h} r_3 + \frac{4EI_2}{L/2} r_3) \qquad (8.11)$$

The above function includes the turning points of the objective function Z = f(I) and once minimised we obtain the minimum Z. To do so, the partial derivatives with respect to each of the variables are set to zero. These derivatives are given in a general form by equations (8.5),(8.6),(8.7). Equations (8.5) means that the derivatives with respect to I should be equal to zero. Thus:

$$\partial F/\partial I_1 = 0.0, \quad \partial F/\partial I_2 = 0.0 \qquad (8.12)$$

Equations (8.6) gives:

$$\partial F/\partial x_2 = 0.0,\ \partial F/\partial r_2 = 0.0,\ \partial F/\partial y_1 = 0.0$$

$$\partial F/\partial r_1 = 0.0,\ \partial F/\partial r_3 = 0.0 \qquad (8.12a)$$

Notice that equations (8.7), for which $\partial F/\partial\lambda_{21} = 0.0$, $\partial F/\partial\lambda_{22} = 0.0$, $\partial F/\partial\lambda_{11} = 0.0$, $\partial F/\partial\lambda_{12} = 0.0$ and $\partial F/\partial\lambda_{32} = 0.0$, are in fact the stiffness equations of the frame. These are given by equations 8.8, and they are used on their own in Chapter 2 (equations 2.40, 2.41, 2.43, 2.44 and 2.45) to calculate the joint displacements and the second moment of area of the columns.

In the above expressions, x_2 is a specified constant (e.g. $x_2 = h/300$), and in the case of more than one storey frame the values of the sway are usually calculated from equation (2.51), section 2.9. However, a relevant Lagrange multiplier λ_{21} is used, and in this case the derivative of $F(Y,\lambda)$ with respect to x_2 should be found (Hadley, 1970) although x_2 is a constant.

The derivatives given by equation (8.12) and (8.12a) are calculated below in the same order as in the computer program. This is the same order in which the equations are solved during the iteration process.

1) $\underline{\partial F/\partial y_1}$: The differentiation of the function $F(Y,\lambda)$ given by equation (8.11) with respect to y_1 is:

$$-\lambda_{22}\frac{6EI_2}{(L/2)^2} + \lambda_{11}\frac{24EI_2}{(L/2)^3} + \lambda_{32}\frac{6EI_2}{(L/2)^2} = 0.0 \tag{8.13}$$

The same result could be obtained from the third column of $\underline{S}_a$ in equations (8.10). This is because, the elements of this column are all coefficients of y_1, and remain in the derivative of y_1. From equation (8.13), λ_{11} which is relevant to y_1 is calculated. Hence

$$\lambda_{11} = \frac{-\lambda_{22}\dfrac{6EI_2}{(L/2)^2} + \lambda_{32}\dfrac{6EI_2}{(L/2)^2}}{-\dfrac{24EI_2}{(L/2)^3}} \tag{8.14}$$

2) $\underline{\partial F/\partial r_1}$: This is obtained from equation (8.11) as:

$$\lambda_{22} \left(\frac{2EI_2}{L/2}\right) + \lambda_{12} \left(\frac{8EI_2}{L/2}\right) + \lambda_{32} \left(\frac{2EI_2}{L/2}\right) \tag{8.15}$$

The above equation can also be obtained from the fourth column of matrix $\underline{S}_a$ in equations (8.10).

From equations (8.15) λ_{12} which is relevant to the rotation r_1 is calculated. Rearranging equation 8.15 gives:

$$\lambda_{12} = \frac{\lambda_{22} \frac{2EI_2}{L/2} + \lambda_{32} \frac{2EI_2}{L/2}}{- \frac{8EI_2}{L/2}} \tag{8.16}$$

3) $\underline{\partial F/\partial I_1}$: This is obtained by differentiating the function given by (8.11), thus:

$$2h + \lambda_{21} \left(\frac{24E}{h^3} x_2 - \frac{6E}{h^2} r_2 - \frac{6E}{h^2} r_3\right) + \lambda_{22} \left(- \frac{6E}{h^2} x_2 + \frac{4E}{h} r_2\right)$$

$$+ \lambda_{32} \left(- \frac{6E}{h^2} x_2 \qquad + \frac{4E}{h} r_3\right) = 0.0 \tag{8.17}$$

The same equation can be obtained by considering the elements which involve I_1 in the matrix $\underline{S}_a$. The coefficients of I_2 disappear when differentiating with respect to I_1.

From the above equation the value of λ_{22} is calculated. Thus rearranging for λ_{22} gives:

$$\lambda_{22} = \frac{2h + \lambda_{21} \left(\frac{24E}{h^3} x_2 - \frac{6E}{h^2} r_2 \qquad - \frac{6E}{h^2} r_3\right) + \lambda_{32} \left(- \frac{6E}{h^2} x_2 + \frac{4E}{h} r_3\right)}{-\left(- \frac{6E}{h^2} x_2 + \frac{4E}{h} r_2\right)} \tag{8.18}$$

4) $\partial F/\partial r_3$: Differentiating the function $F(Y,\lambda)$ or the fifth column of matrix $\underline{S}_a$ gives:

$$- \lambda_{21} \frac{6EI_1}{h^2} + \lambda_{11} \frac{6EI_2}{(L/2)^2} + \lambda_{12} \frac{2EI_2}{L/2} + \lambda_{32} \left(\frac{4EI_1}{h} + \frac{4EI_2}{L/2}\right) = 0.0 \tag{8.19}$$

From this equation the value of λ_{32} is calculated, hence, rearranging equation (8.19):

$$\lambda_{32} = \frac{- \lambda_{21} \frac{6EI_1}{h^2} + \lambda_{11} \frac{6EI_2}{(L/2)^2} + \lambda_{12} \frac{2EI_2}{L/2}}{-\left(\frac{4EI_1}{h} + \frac{4EI_2}{L/2}\right)} \tag{8.20}$$

5) $\partial F/\partial x_2$: Differentiating the function $F(Y,\lambda)$ with respect to x_2, or the first column in the matrix $\underline{S}_a$:

$$\lambda_{21} \frac{24EI_1}{h^3} - \lambda_{22} \frac{6EI_1}{h^2} - \lambda_{32} \frac{6EI_1}{h^2} = 0.0 \tag{8.21}$$

from which λ_{21} is:

$$\lambda_{21} = \frac{+ \lambda_{22} \frac{6EI_1}{h^2} + \lambda_{32} \frac{6EI_1}{h^2}}{\frac{24EI_1}{h^3}} \tag{8.22}$$

6) $\partial F/\partial r_2$: This is obtained using equation (8.11) or from the elements of the second row of matrix $\underline{S}_a$ thus,

$$- \lambda_{21} \frac{6EI_1}{h^2} + \lambda_{22} \left(\frac{4EI_1}{h} + \frac{4EI_2}{L/2}\right) - \lambda_{11} \frac{6EI_2}{(L/2)^2} + \lambda_{12} \frac{2EI_2}{L/2} = 0.0 \tag{8.23}$$

In the design procedure during the iteration process, the values of λ_{21}, λ_{22}, λ_{11}, λ_{12} and I_1 will be calculated previous to the use of the above equation. Thus, this equation is used to calculate I_2. Rearranging:

$$I_2 = \frac{+\lambda_{21}\dfrac{6EI_1}{h^2} - \lambda_{22}\dfrac{4EI_1}{h}}{\lambda_{22}\dfrac{4E}{L/2} - \lambda_{11}\cdot\dfrac{6E}{(L/2)^2} + \lambda_{12}\dfrac{2E}{L/2}} \tag{8.24}$$

8.2.1 Solution of the Derivative Equations by the Iteration Technique

The modified stiffness equations for the above example were solved in section 2.7. Table 2.1 gave the value of the joint displacements and I_1 after convergence was obtained. Using these values a new iteration solves the derivative equations given by (8.14), (8.16), (8.18), (8.20), (8.22) and (8.24). This is assuming (as in section 2.7) that x_2/h = 10.17 mm and I_2 = 2.1345 x 10^8 mm^4. The initial values of λ_{11}, λ_{12}, λ_{22}, λ_{32}, λ_{21} are set to zero.

The first iteration cycle starts by solving equations (8.14), and (8.16) respectively. These give $\lambda_{11} = \lambda_{12} = 0.0$. Equations (8.18) is solved next, which gives $\lambda_{22} = 2.40962 \times 10^6$. For the rest of this cycle the new value of λ_{22} is used for the calculation of all the other variables. Equations (8.20) and (8.22) gives $\lambda_{32} = 0.0$, and $\lambda_{21} = 2.20480 \times 10^9$. The above values of λ are now used to calculate the first value for the beam section using equation (8.24). This gives $I_2 = 2.92193 \times 10^7$ mm^4. It can be seen that for the first cycle some of the values of λ are zero, but these change as the iteration continues. For instance, in the second cycle we will find that $\lambda_{11} = 2.75600 \times 10^9$, $\lambda_{12} = -6.02405 \times 10^5$, $\lambda_{22} = 3.28459 \times 10^6$, $\lambda_{32} = 3.41365 \times 10^6$, $\lambda_{21} = 6.12889 \times 10^9$, and $I_2 = 1.51731 \times 10^7\ mm^4$. The procedure described is repeated until all the variables converge to stable values . To test this convergence a tolerance is introduced, and a variable is said to have become stable if;

$$\left|\frac{\text{New value - old value}}{\text{new value}}\right| < \text{The tolerance}$$

In the next three sections general expressions for the derivative equations will be obtained.

8.3 THE DERVIATIVES OF THE LAGRANGIAN FUNCTION FOR A GENERAL JOINT IN A FRAME

The derivatives given in equations (8.5) and (8.6) can be written in a general form for a typical joint j in a frame. To do this, it is necessary to multiply the contributions of a member, to the overall stiffness matrix, by the Lagrange variable. Equations (2.6) gave these contributions for a beam member connecting joint T at its first end to joint D at its second end. These equations are altered to:

$$\begin{array}{ll} & \text{First end at T} \qquad \text{Second end at D} \\ \begin{array}{l}\text{First} \\ \text{end D} \\ \\ \text{Second} \\ \text{end D}\end{array} \begin{bmatrix} V_T \\ M_T \\ \hline V_D \\ M_D \end{bmatrix} = & \left[\begin{array}{cc|cc} \lambda_{T1}b & -\lambda_{T1}d & -\lambda_{T1}b & -\lambda_{T1}d \\ -\lambda_{T2}d & \lambda_{T2}e & \lambda_{T2}d & \lambda_{T2}f \\ \hline -\lambda_{D1}b & \lambda_{D1}d & \lambda_{D1}b & \lambda_{D1}d \\ -\lambda_{D2}d & \lambda_{D2}f & \lambda_{D2}d & \lambda_{D2}e \end{array}\right] \begin{bmatrix} y_T \\ r_T \\ \hline y_D \\ r_D \end{bmatrix} \end{array} \qquad (8.25)$$

or $\underline{L} = \underline{S}_b\ \underline{X}$ (8.25b)

In the above equations suffix T, and D refer to the first, and second end of the member respectively. Each λ has two suffixes. The first denotes the joint number at the end of the member and the second suffix which is either 1 or 2 denotes the type of deflection being considered. For a beam, one specifies a vertical translation while two refers to a joint rotation.

It should be noticed that, in equations (8.25) the matrix $\underline{S}_b$ is not symmetrical because each row of the original stiffness matrix is multiplied by a different λ. For example, the element in the first row of $\underline{S}_b$ in equations (8.25) are multiplied by λ_{T1}, since this row is used to calculate the vertical deflection y_T at the first end of the beam. The elements in the second row are multiplied by λ_{T2}, because this row is used to calculate the rotation r_T.

The contributions of a column member to the stiffness equations $\{L\} = \underline{K}\ \{X\}$ were given by equation (2.7). These are altered to:

$$
\begin{array}{l}
\\
\text{First end T} \\
\\
\text{Second end D}
\end{array}
\left[\begin{array}{c} H_T \\ M_T \\ \hline H_D \\ M_D \end{array}\right]
=
\begin{array}{c}
\begin{array}{cc} \text{first end at joint T} & \text{second end at joint D} \end{array} \\
\left[\begin{array}{cc|cc}
\lambda_{T1}b & -\lambda_{T1}d & -\lambda_{T1}b & -\lambda_{T1}d \\
-\lambda_{T2}d & \lambda_{T2}e & \lambda_{T2}d & \lambda_{T2}f \\ \hline
-\lambda_{D1}b & \lambda_{D1}d & \lambda_{D1}b & \lambda_{D1}d \\
-\lambda_{D2}d & \lambda_{D2}f & \lambda_{D2}d & \lambda_{D2}e
\end{array}\right]
\end{array}
\left[\begin{array}{c} x_T \\ r_T \\ \hline x_D \\ r_D \end{array}\right]
\qquad (8.26
$$

i.e. $\underline{L} = \underline{S}_c\ \underline{X}$

It should be noted, that in equations (8.25) and (8.26) the derivatives of the load vectors $\{V_T\ M_T\ \ldots\ldots\ldots\ V_D\ M_D\}$ and

$\{H_T\ M_T\ \ldots\ldots H_D\ M_D\}$ are zero because they are constants. For this reason they disappear by differentiation.

The member and joint specifications were stated in section 2.3, where a beam joint was distinguished from a column joint. In that section, it was also stated that a member is given the same number as the joint at its first end. If a beam joint j is considered, then three equations are needed to evaluate I_j, λ_{j1} and λ_{j2} for this joint. These equations are:

$$\frac{\partial F}{\partial I_{Bj}} = 0, \quad \frac{\partial F}{\partial y_j} = 0, \text{ and } \quad \frac{\partial F}{\partial r_j} = 0 \tag{8.27}$$

Notice that suffix j refers to a specific joint in the frame and equations (8.27) from part of the general equations (8.5) and (8.6) for the frame. For a column joint equations similar to (8.27) are written, thus

$$\frac{\partial F}{\partial I_{cj}} = 0, \quad \frac{\partial F}{\partial x_j} = 0, \text{ and } \quad \frac{\partial F}{\partial r_j} = 0 \tag{8.28}$$

In the next two sections, expressions are derived for I_j, λ_{j1}, and λ_{j2} as was done for the modified stiffness equations in section 2.4. For a beam joint the differentiation $\partial F/\partial I_j$ need not be calculated. This is because in the design procedure the value of I for a beam section is specified during a design cycle and altered only at the end of that cycle.

8.4 THE DERIVATIVES OF THE LAGRANGIAN FUNCTION FOR A BEAM JOINT

The specifications for a joint in a beam were shown in Figure 2.3 and given in Figure 8.2. The stiffness coefficients b_j, d_j, e_j and f_j for beam B_j, and b_i, d_i, e_i and f_i for beam B_i connected to joint j were given by equations (2.10) and (2.11) respectively. The same notation is used here to obtain $\partial F/\partial y_j$ and $\partial F/\partial r_j$.

a) $\partial F/\partial y_j$: The derivative of Lagrangian function $F(X, \lambda)$ with respect to the vertical deflection y_j is obtained from the first

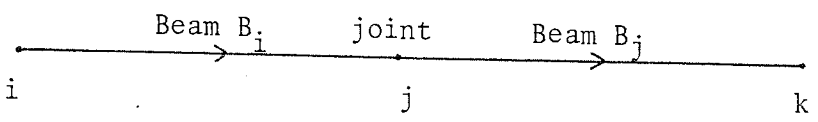


FIGURE 8.2 SPECIFICATIONS FOR A JOINT J IN A BEAM

column of $\underset{\rightharpoondown}{S}_b$ in equation (8.25).

The elements

$$\{\lambda_{T1}b \quad -\lambda_{T2}d \qquad -\lambda_{D1}d \qquad -\lambda_{D2}d\}$$

of this column are all coefficients of y_T and thus when differential with respect to y_T, they give:

$$\lambda_{j1}\, b_j - \lambda_{j2}d_j - \lambda_{k1}b_j - \lambda_{k2}d_j = 0 \tag{8.29}$$

Notice that the first end of beam B_j is connected to joint j and its second end is connected to joint k thus when constructing equation (8.29) from the first column of $\underset{\rightharpoondown}{S}_b$ we have replaced T by j and D by k.

Apart from beam B_j, the beam B_i is also connected to joint j. For this beam, i replaces T and j replaced D and the elements in the third column of $\underset{\rightharpoondown}{S}_b$ in (8.25), differentiation with respect to j, give:

$$-\lambda_{i1}b_i + \lambda_{i2}b_i + \lambda_{ji}b_i + \lambda_{j2}d_i = 0.0 \tag{8.30}$$

Adding the contributions of B_i and B_j, given by equations (8.29) and (8.30) results in the following equation for $\partial F/\partial y_j$:

$$\partial F/\partial y_j = \lambda_{j1}b_j - \lambda_{j2}d_j - \lambda_{k1}b_j - \lambda_{k2}d_j - \lambda_{i1}b_i$$

$$+ \lambda_{i2}d_i + \lambda_{j1}b_i + \lambda_{j2}d_i = 0 \qquad (8.31)$$

Hence

$$\lambda_{j1} = (+\lambda_{j2}d_j + \lambda_{k1}b_j + \lambda_{k2}d_j + \lambda_{i1}b_i - \lambda_{i2}d_i$$

$$- \lambda_{j2}d_i)/(b_j + b_i) \qquad (8.32)$$

Comparing this with equation (2.14) which was used to calculate y_j, it will be noticed that equation (8.32) can in fact be obtained from (2.14) by replacing each deflection in (2.14) by its corresponding λ. Thus λ_{k1} replaces the vertical deflection y_k for joint k, λ_{k2} replaces the rotation r_k and so on.

b) $\underline{\partial F/\partial r_j}$: The derivative of the Lagrangian function $F(X, \lambda)$ with respect to the rotation r_j is obtained as follows:

i) When differentiating with respect to r_j only the coefficients of r_j in the second column of $\underline{S}_b$ in equation (8.25) remain. All the other variables are kept constant and thus their coefficients in equation (8.25) disappear during the differentiation. Thus:

$$-\lambda_{j1}d_j + \lambda_{j2}e_j + \lambda_{k1}d_j + \lambda_{k2}f_j = 0.0 \qquad (8.33)$$

ii) The contributions of the beam B_i to $\partial F/\partial r_j$ are calculated from the fourth column of $\underline{S}_b$ in equations (8.25), as joint j is at the second end of beam B_i, thus

$$- \lambda_{i1}d_i + \lambda_{i2}f_i + \lambda_{j1}d_i + \lambda_{j2}e_i = 0.0 \qquad (8.34)$$

iii) The expression for the derivative $\partial F/\partial r_j$ is obtained by adding equations (8.33) and (8.34). Thus:

$$\partial F/\partial r_j = -\lambda_{j1}d_j + \lambda_{j2}e_j + \lambda_{k1}d_j + \lambda_{k2}f_j - \lambda_{i1}d_i \qquad (8.35)$$

$$+ \lambda_{i2}f_i + \lambda_{j1}d_i + \lambda_{j2}e_i = 0.0$$

Rearranging the above equation to calculate λ_{j2} gives:

$$\lambda_{j2} = (+\lambda_{j1}d_j - \lambda_{k1}d_j - \lambda_{k2}f_j + \lambda_{i1}d_i - \lambda_{i2}f_i$$

$$-\lambda_{j1}d_i)/(e_j + e_i) \qquad (8.36)$$

comparing the above equation with equation (2.17), it is possible to see that the above equation can be obtained from (2.17) by replacing each deflection by the relevant Lagrange multiplier. For example, λ_{j1} replaces y_j and λ_{k1} replaces y_k and so on.

8.5 THE DERIVATIVES OF THE LAGRANGIAN FUNCTION FOR A COLUMN JOINT

The specifications for a general configuration of a column joint j were shown in Figure 2.5 and repeated in Figure 8.3, in which two beams B_i and B_j and two columns C_m and C_j were connected to the joint. Their stiffness coefficients were defined in section 2.4.1. The derivatives $\partial F/\partial I_{cj}$, $\partial F/\partial x_j$ and $\partial F/\partial r_j$ are obtained here using the same notation.

a) For $\partial F/\partial I_{cj}$: The derivative of the Lagrangian function $F(X,\lambda)$ with respect to the second moment of area I_{cj} of column C_j is obtained from equation (8.26). In this equation I_{cj} appears in every element of matrix $\underline{S}_c$. The coefficients of all the other variables disappear when differentiating with respect to I_{cj}.

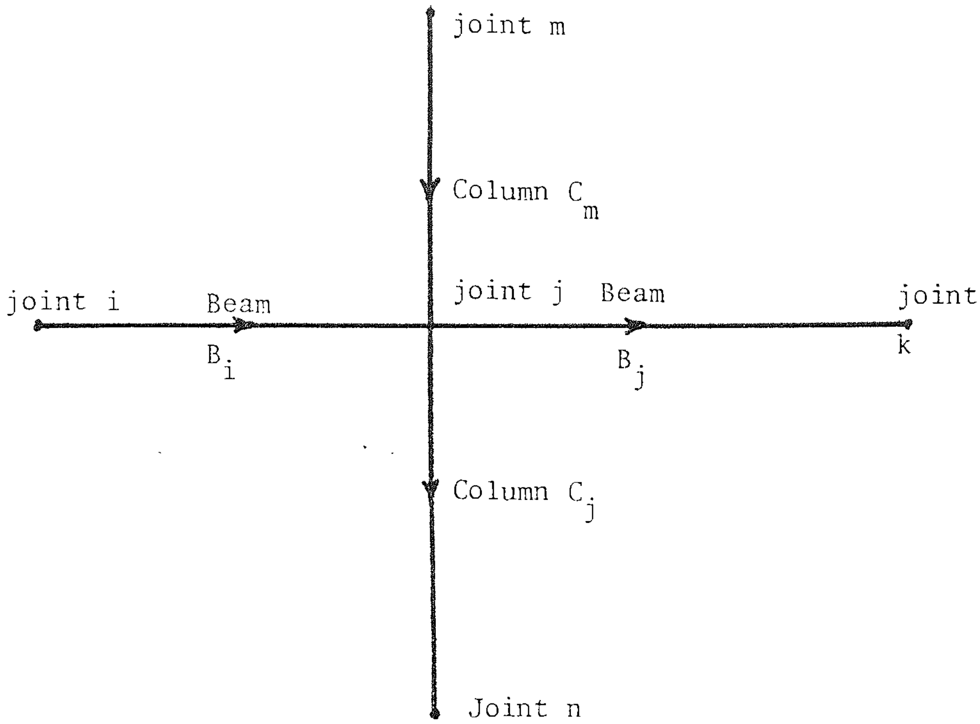


FIGURE 8.3 SPECIFICATIONS FOR A GENERAL CONFIGURATION OF A COLUMN JOINT j

Thus for column C_j with joint j at its first end and n at its second:

$$\begin{aligned} \partial F/\partial I_{Cj} &= L_{Cj} + \lambda_{j1}\, b_{Cj1}x_j - \lambda_{j1}d_{Cj1}r_j - \lambda_{j1}b_{Cj1}x_n \\ &- \lambda_{j1}d_{Cj1}r_n - \lambda_{j2}d_{Cj1}x_j + \lambda_{j2}e_{Cj1}r_j + \lambda_{j2}d_{Cj1}x_n + \\ &\lambda_{j2}f_{Cj1}r_n - \lambda_{n1}b_{Cj1}x_j + \lambda_{n1}d_{Cj1}r_j + \lambda_{n1}b_{Cj1}x_n \\ &+ \lambda_{n1}d_{Cj1}r_n - \lambda_{n2}d_{Cj1}x_j + \lambda_{n2}f_{Cj1}r_j + \lambda_{n2}d_{Cj1}x_n \\ &+ \lambda_{n2}e_{Cj1}r_n = 0.0 \end{aligned} \tag{8.37}$$

In the above equation the term L_{Cj} is obtained from differentiating the objective function $f(I)$ with respect to I_{Cj}. This gives the

Length L_{Cj} of column C_j. Equation (8.37) is rearranged to calculate λ_{j2}. Thus

$$\lambda_{j2} = [L_{Cj}+\lambda_{j1}(-b_{Cj1}x_j+d_{Cj1}r_j+b_{Cj1}x_n+d_{Cj1}r_n)$$

$$+ \lambda_{n1}(b_{Cj1}x_j - d_{Cj1}r_j - b_{Cj1}x_n - d_{Cj1}r_n) \tag{8.38}$$

$$+ \lambda_{n2}(d_{Cj1}x_j - f_{Cj1}r_j-d_{Cj1}x_n-e_{Cj1}r_n)]/$$

$$[-d_{Cj1}x_j+e_{Cj1}r_j + d_{Cj1}x_n+f_{Cj1}r_n]$$

b) $\underline{\partial F/\partial x_j}$: The derivative of the Lagrangian function $F(X,\lambda)$ with respect to the sway x_j, of joint j is obtained as follows:

i) The contributions of the column C_j to $\partial F/\partial x_j$ are calculated from the first column of the matrix S_c given in equations (8.26), since joint j is at the first end of column C_j and, n is at the second end, then in equation (8.26) j replaces T and n replaces D, thus:

$$\lambda_{j1}b_{Cj} - \lambda_{j2}d_{Cj} - \lambda_{n1}b_{Cj} - \lambda_{n2}d_{Cj} = 0.0 \tag{8.39}$$

ii) Joint j is at the second end of column C_m and replaces D in equation (8.26) while joint m which is at its first end replaces T. This means that the contributions of C_m to $\partial F/\partial x_j$ are given by the third column of the stiffness matrix, which gives:

$$-\lambda_{m1}b_{Cm} + \lambda_{m2}d_{Cm} + \lambda_{j1}b_{Cm} + \lambda_{j2}d_{Cm} = 0.0 \tag{8.40}$$

iii) There are no contributions from the beams B_i and B_j to $\partial F/\partial x_j$. This is because the axial extensions of these beams are ignored.

iv) The expression for $\partial F/\partial x_j$ is thus obtained by adding equations (8.39) and (8.40). Hence

$$\partial F/\partial x_j = \lambda_{j1}b_{Cj} - \lambda_{j2}d_{Cj} - \lambda_{n1}b_{Cj} - \lambda_{n2}d_{Cj} - \lambda_{m1}b_{Cm} + \lambda_{m2}d_{Cm} + \lambda_{j1}b_{Cm} + \lambda_{j2}d_{Cm} = 0.0 \tag{8.41}$$

From the above equation the value of λ_{j1} is calculated. This gives:

$$\lambda_{j1} = (+\lambda_{j2}d_{Cj} + \lambda_{n1}b_{Cj} + \lambda_{n2}d_{Cj} + \lambda_{m1}b_{Cm} - \lambda_{m2}d_{Cm} - \lambda_{j2}d_{Cm})/(b_{Cj} + b_{Cm}) \tag{8.42}$$

Once again, the above equation can be obtained from equation (2.20) by substituting, for each deflection, the relevant Lagrange multiplier and ignoring the load.

c) $\underline{\partial F/\partial r_j}$: The derivative of $F(X,\lambda)$ with respect to the rotation r_j, of joint j is obtained as follows:

i) The contributions of the beams B_i and B_j to $\partial F/\partial r_j$ are already derived and were given by equation (8.33) and (8.34).

ii) Column C_j contributes to $\partial F/\partial r_j$ by the coefficients of the second column of the metrix $\underline{S}_c$ in equation (8.26). Hence

$$-\lambda_{j1}d_{Cj} + \lambda_{j2}e_{Cj} + \lambda_{n1}d_{Cj} + \lambda_{n2}f_{Cj} = 0.0 \tag{8.43}$$

iii) Joint j is at the second end of column C_m. Hence the fourth column of matrix $\underline{S}_c$ in equations (8.26) gives the contributions of this member to $\partial F/\partial r_j$. Thus

$$-\lambda_{m1}d_{Cm} + \lambda_{m2}f_{Cm} + \lambda_{j1}d_{Cm} + \lambda_{j2}e_{Cm} = 0.0 \tag{8.44}$$

iv) Adding equations (8.33), (8.34), (8.43) and (8.44) together gives the final expression for $\partial F/\partial r_j$ as:

$$\partial F/\partial r_j = -\lambda_{j1}d_j + \lambda_{j2}e_j + \lambda_{k1}d_j + \lambda_{k2}f_j - \lambda_{i1}d_i$$

$$+ \lambda_{i2}f_i + \lambda_{j1}d_i + \lambda_{j2}e_i - \lambda_{j1}d_{Cj} + \lambda_{j2}e_{Cj} + \lambda_{n1}d_{Cj} \tag{8.45}$$

$$+ \lambda_{n2}f_{Cj} - \lambda_{m1}d_{Cm} + \lambda_{m2}f_{Cm} + \lambda_{j1}d_{Cm} + \lambda_{j2}e_{Cm} = 0.0$$

Notice that $d_j = -6EI_j/L_j^2$, $e_j = 4EI_j/L_j$ etc, thus equation (8.45) contains the second moment of area of the beam B_j and is used to calculate the value of I for this beam during one step of the iteration process. The use of equation (8.45) to calculate I_{Bj} is convenient because, apart from I_{Bj}, it contains λ(s) and I for the columns and these will be calculated at stages previous to the use of equation (8.45).

In equation (8.45) $d_j = (-6E/L_j^2)I_j$ which can be written as $d_j = I_j d_{j1}$ where $d_{j1} = -6E/L_j^2$. Similarly $e_j = I_j e_{j1}$ where $e_{j1} = 4E/L_j$ and $f_j = I_j f_{j1}$ where $f_{j1} = 2E/L_j$. Rearranging equation (8.45), we can calculate I_{Bj} as:

$$I_{Bj} = (\lambda_{i1}d_i - \lambda_{i2}f_i - \lambda_{ji}d_i - \lambda_{j2}e_i + \lambda_{j1}d_{Cj} - \lambda_{j2}e_{Cj}$$

$$-\lambda_{n1}d_{Cj} - \lambda_{n2}f_{Cj} + \lambda_{m1}d_{Cm} - \lambda_{m2}f_{Cm} - \lambda_{j1}d_{Cm} \tag{8.46}$$

$$- \lambda_{j2}e_{Cm})/(-\lambda_{j1}d_{j1} + \lambda_{j2}e_{j1} + \lambda_{k1}d_{j1} + \lambda_{k2}f_{j1})$$

Equations (8.32), (8.36), (8.38), (8.42) and (8.46) provide the right number of equations to calculate the Lagrange multipliers and the optimum values for the second moments of area for the beams. These are given for a general joint and they are repeated for the total number of joints in the frame. The iterative technique, used to solve the modified stiffness equation, is used here once again to obtain the values of λ and I_B.

8.6 OUTLINE OF THE DESIGN PROCEDURE

The optimum design for a given frame is obtained by a two loop iteration. The first loop solves the modified stiffness equations (2.14), (2.17), (2.26), and (2.38) given in Chapter 2, section 2.4. The second iteration loop solves the derivative equations given above. The procedure is summarised in the flow-chart shown in Fig 8.4 and a step by step approach would be as follows:

STEP 1 - Select a set of lower bounds for the beam sections. This is decided by preventing the failure of each beam under vertical loads by the development of a beam mechanism. Under combined vertical and wind loads a beam section is not allowed to be smaller than its lower bound.

STEP 2 - Specify the horizontal deflection at each storey level. An equation for calculating this is given in section 2.9.

STEP 3 - Define the ratio of an internal to external column for the second moment of area as in equation (2.33).

STEP 4 - Select an initial set of beam sections using one of the methods given in section 2.5.

STEP 5 - Set the values of the Lagrange multipliers for each joint to zero.

STEP 6 - Solve the modified stiffness equations (2.14), (2.17), (2.26) and (2.38) by iteration to obtain the values of the unknown sectional property I of the columns and the unknown joint displacements. The method of solving the modified stiffness equations was explained in sections 2.6 and 2.7. Notice that the solution is carried out until convergence is fully achieved.

STEP 7 - Using these values of deflections and column sections, calculate the Lagrange multiplier for each beam joint from equations (8.32) and (8.36).

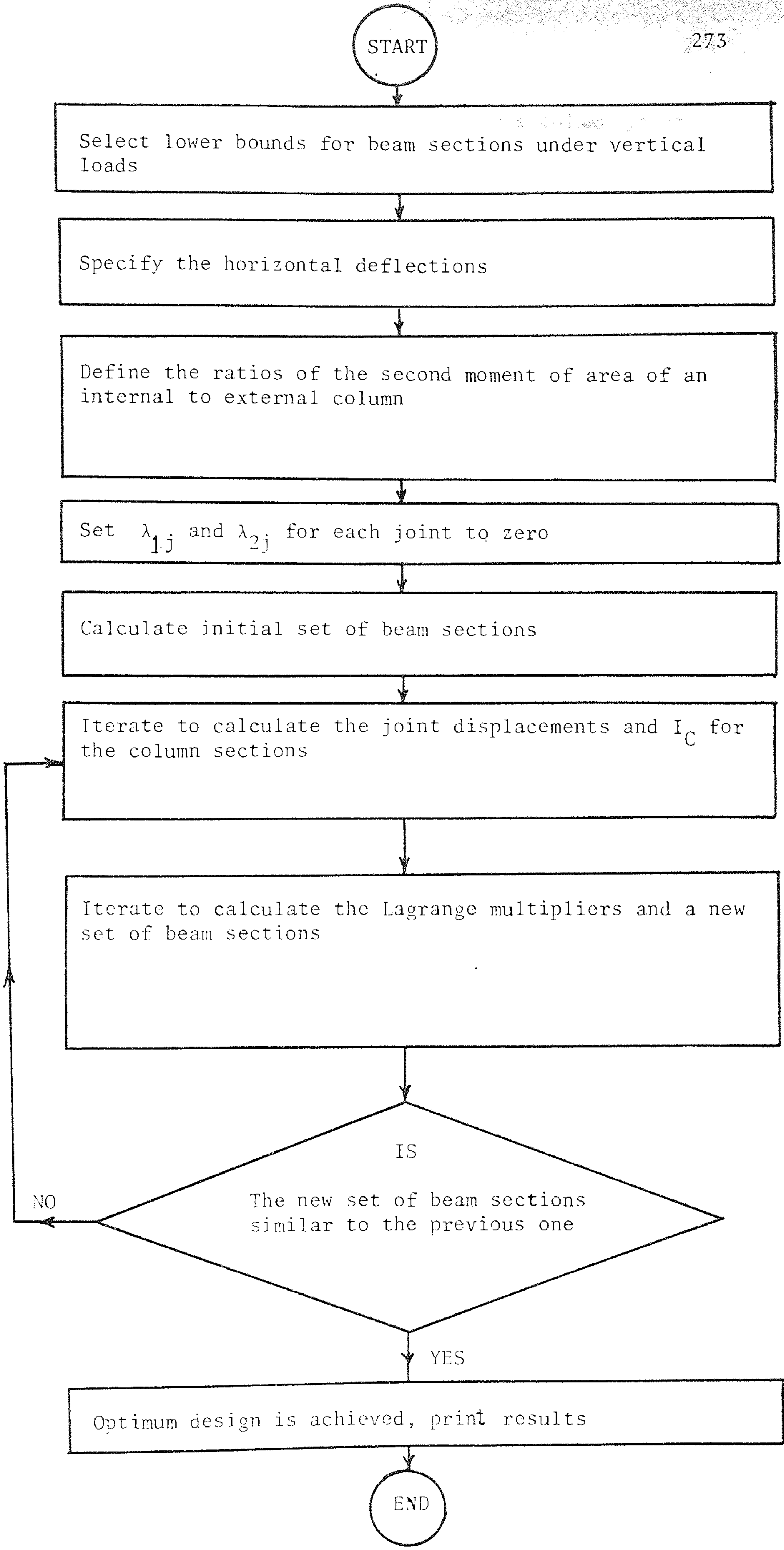


FIGURE 8.4 OPTIMUM DESIGN FLOW CHART

STEP 8 - Calculate the Lagrange multipliers for a column joint using equations (8.38) and (8.42).

STEP 9 - Calculate the second moment of area I_B from equation (8.46) for each beam associated with its corresponding column in step (8).

STEP 10 - Repeat steps (8) and (9) for all the column joints until all the derivative equations are used up.

STEP 11 - Repeat steps 7 to 10 until convergence is achieved. At this stage a new set of beam sections becomes available.

STEP 12 - With the new beam sections repeat steps 5 to 11 until no reduction in any beam section is possible. No beam section is allowed to reduce below its lower bound imposed by vertical loading conditions.

It is possible to solve the modified stiffness equations as well as the derivative equations in one iteration loop. In this case for any iteration cycle, the joint displacements are calculated first, followed by the calculation of the Lagrange multipliers and then I_B for the beam section. This process is repeated until convergence is achieved for the total number of the unknowns. However, the procedure of solving all the equations at one go was found to be inefficient, as it increases the number of unknowns to be calculated at one time.

8.7 THE COMPUTER PROGRAM

A computer program was written for the optimum design of rigidly jointed plane frames which makes use of the design procedure described in the previous section. The program consists of three segments. These are:

1 - The MASTER segment; this follows the optimum design flow-chart given in Figure 8.4.

2 - The subroutine ITERATE: as was given in Chapter 4.

3 - The subroutine OPTIMUM: This calculates the Lagrange multipliers and the optimum, values for the second moments of area of the beam sections, making use of the derivative equations. The flow-chart of this subroutine is similar to that of ITERATE. Except that for the subroutine OPTIMUM the contributions of a member to the derivative equations are calculated at each iteration cycle instead of the contributions to the modified stiffness equations.

The input data to this program is exactly the same as for the design of steel frame given in Appendix A of this thesis.

8.8 DESIGN EXAMPLE 1 - A FOUR STOREY SINGLE BAY FRAME

The four storey single bay frame which was shown in Figure 2.10a is designed here using the computer program described above. This frame was designed in section 2.11.1 using the linear extrapolation technique and the final reduced cost design was shown in Figure 2.18. The optimum design for this frame obtained by Saka (1975) using a programming type of optimisation was given in Figure 2.19.

Using the classical type of optimisation as described in this chapter the second moment of area for each member is shown in Figure 8.5. To calculate the weight of the frame the length L of each member is multiplied by the second moment of area I, and the sum ΣIL is calculated. It was found that ΣIL for the design with classic optimisation is 1.8940×10^{13} mm^5, while for the optimum design by Saka this was 1.8835×10^{13} mm^5 with a difference of 0.55%. This small difference may be due to the fact that Saka's objective function minimised the actual volume ΣAL and not ΣIL.

Five design cycles were needed to obtain the final optimum design, and the Lagrange multipliers for each joint at the final design are given in Table 8.1. The reduction in the sum ΣIL for each design cycle is represented in Figure 8.6. This shows that the biggest reduction occurs at the second design cycle, where the Lagrange multipler are first used. Table 8.2 shows the number of iterations per each design cycle for both the stiffness and the derivative iteration techniques.

The sum ΣIL for the reduced weight design which was shown in Figure 2.18 was 1.903×10^{13} mm^5 with an increase of 0.48% over the optimum. This difference is small and for this reason the economy by using the linear extrapolation technique may be sufficient for practical purposes.

The computer time used for obtaining the classical optimum design was 63 seconds. The time needed for the optimum design by Saka was 467 seconds which is more than seven times that of the present method.

8.9 DESIGN EXAMPLE 2 - A FOUR STOREY - TWO EQUAL BAY FRAME

The four storey two bay frame shown in Figure 8.7 was designed using the computer program described in section 8.7. The height of each storey and the wind load at each storey level is similar to that of the four storey single bay frame designed above. The sway deflections used here are shown beside the frame in Figure 8.7. These were obtained using the procedure described in section 2.9 with $\alpha = 1/300$.

Six design cycles were needed to obtain the optimum design shown in Figure 8.8, in which the second moment of area for each member is given. The initial I values for the beam sections were taken all equal to 111673 cm^4. The sum ΣIL for each design cycle is shown in Figure 8.9. This shows that the biggest reduction is obtained at the second design cycle, in which the Lagrange multipliers are first introduced. For the optimum design shown in Figure 8.5 all the beam sections are reduced to those needed by the beam mechanism type of collapse. No further reduction in the beam sections is allowed as this violates the strength requirements for vertical loading.

The same frame was designed by the method used in Chapter 2, in which the linear extrapolation technique was used to reduce the cost of the frame material. The initial I values for all beam sections were taken equal to 111673 cm^4, which are similar to those of the initial values for the optimum design. The reduced cost design obtained, is shown in Figure 8.10. The total ΣIL for this design was 2.0065×10^{13} mm^5 which is 5.68% more than the optimum of 1.8955×10^3 mm^5.

8.10 DESIGN EXAMPLE 3 - FOUR STOREY TWO UNEQUAL - BAY FRAME

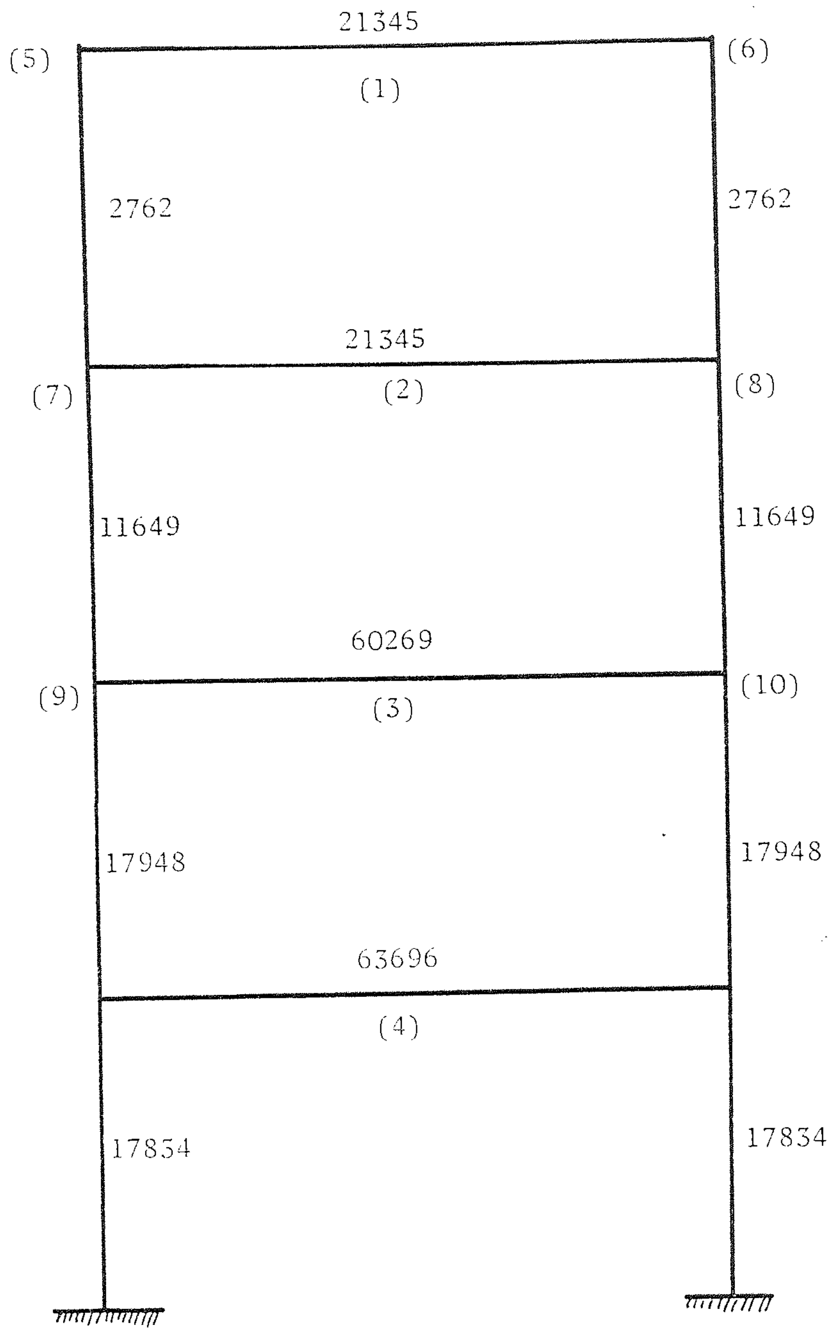
The four storey two unequal bay frame shown in Figure 8.11 was first designed by the optimum design computer program described in section 8.7, and then by the method used in Chapter 2. The reason for this is to provide a comparison of the two designs. The height of each storey and the deflection profile used here was the same as those used for the two equal-bay frames dealt with above. Both designs (the optimum and of Chapter 2) start by assuming the same initial set of beam sections, and a high I value of 111673 cm^4.

The optimum design for the frame is shown in Figure 8.12 for which $\Sigma IL = 1.53012 \times 10^{13}$ mm^5. Five design cycles were needed to reduce ΣIL from its initial value of 7.38128×10^{13} mm^5 to its optimum value. Some of the beam sections are reduced to those needed by the beam mechanism type of collapse, and marked by asterisks in

Figure 8.12. The reduction in ΣIL per design cycle is shown in Figure 8.13.

The design by the method proposed in Chapter 2 is shown in Figure 8.14, for which $\Sigma IL = 1.63879 \times 10^{13}$ mm^5 with an increase of 7.10%. However, it should be noticed that in this design the beam section at each storey is continuous over the two spans, which may reduce the erection cost of the frame.

Second moment of area in cm^4



$\Sigma IL = 1.8940 \times 10^{13}\ mm^5$

FIGURE 8.5 THE 4 STOREY SINGLE BAY FRAME - OPTIMUM DESIGN USING CLASSICAL OPTIMISATION

joint number	λ_1 $x10^{10}$	λ_2 $x10^7$	joint number	λ_1 $x10^{10}$	λ_2 $x10^7$
1	7.713	2.157	7	2.929	0.885
2	1.399	0.735	8	1.798	2.345
3	4.438	1.798	9	7.373	5.534
4	10.717	3.217	10	0.994	1.602
5	0.148	0.741	11	9.353	11.121
6	16.671	8.188	12	2.068	1.710

TABLE 8.1 THE 4 STOREY-SINGLE BAY FRAME LAGRANGE MULTIPLIERS FOR EACH JOINT IN THE FRAME

	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5
Number of iterations for modified stiffness equations	12	9	9	10	10
Number of iterations for derviatives equations	28	16	11	14	14

TABLE 8.2 THE 4 STOREY-SINGLE BAY FRAME NUMBER OF ITERATIONS PER DESIGN CYCLE

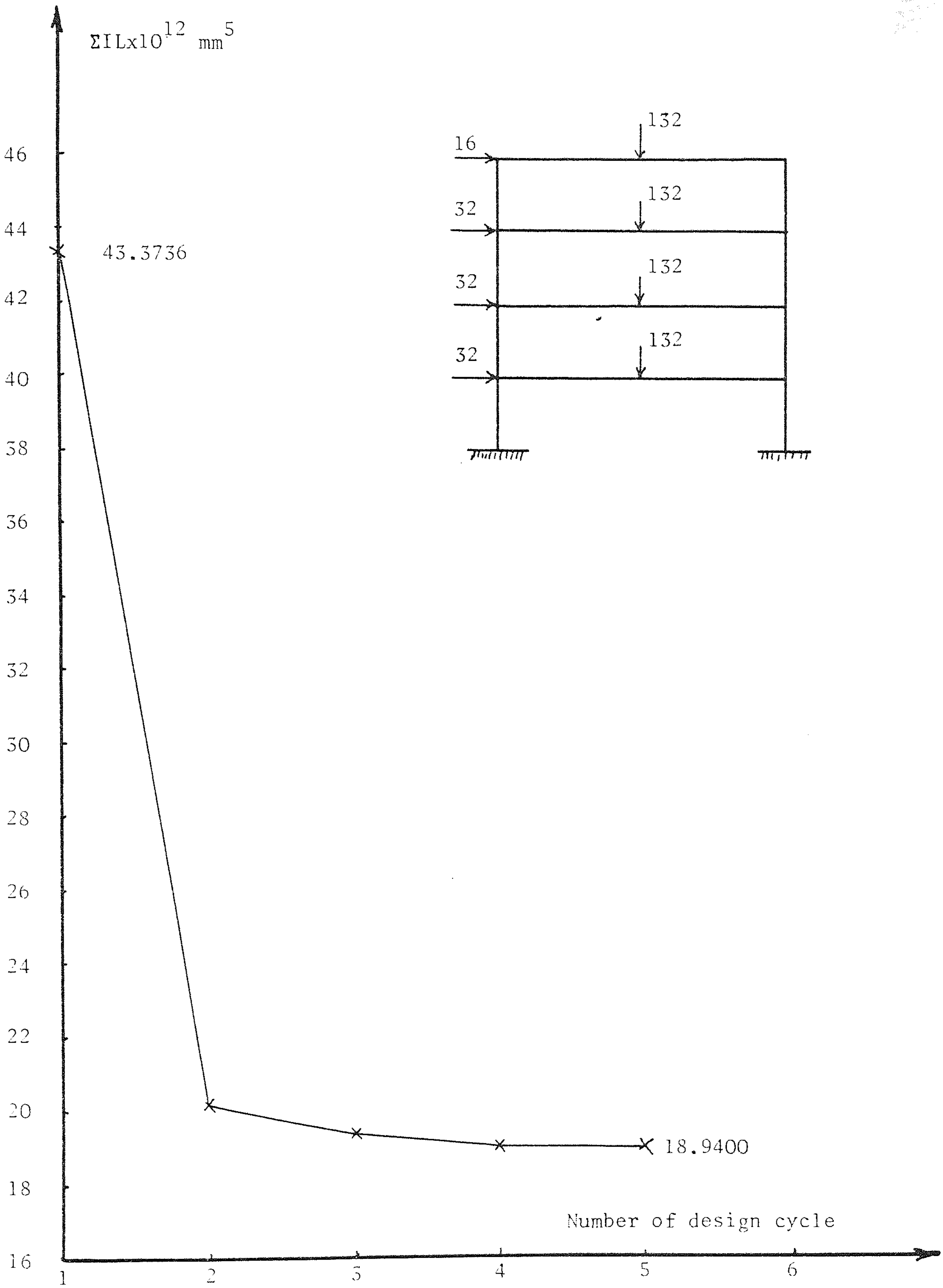


FIGURE 8.6 THE 4 STOREY SINGLE BAY FRAME ΣIL IN EACH DESIGN CYCLE

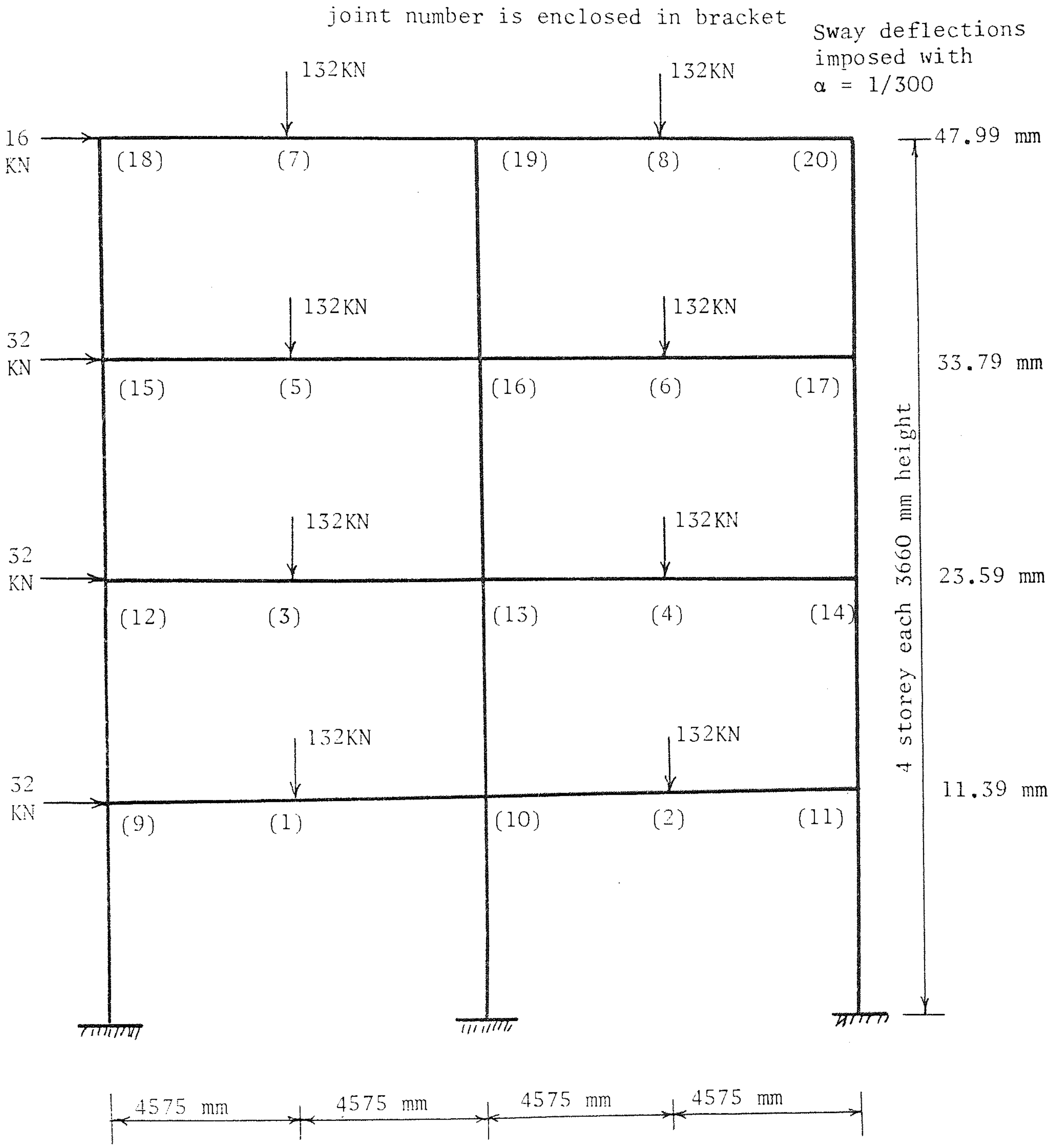
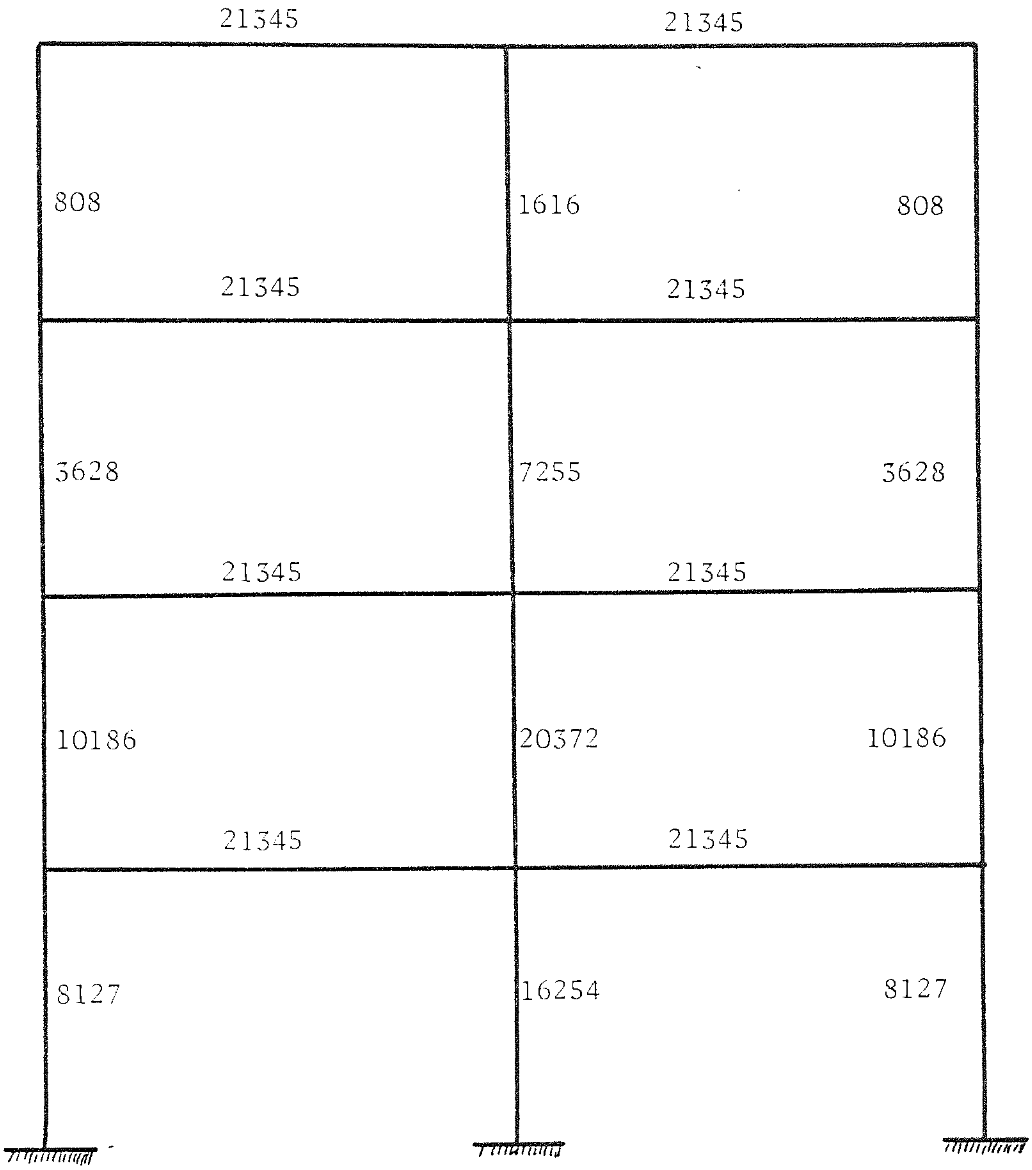


FIGURE 8.7 A 4 STOREY - 2 EQUAL BAY FRAME DIMENSIONS AND LOADS

Second moments of area in cm^4



$\Sigma IL = 1.8955 \times 10^{13}\ mm^4$

FIGURE 8.8 OPTIMUM DESIGN OF THE FRAME USING CLASSICAL OPTIMISATION

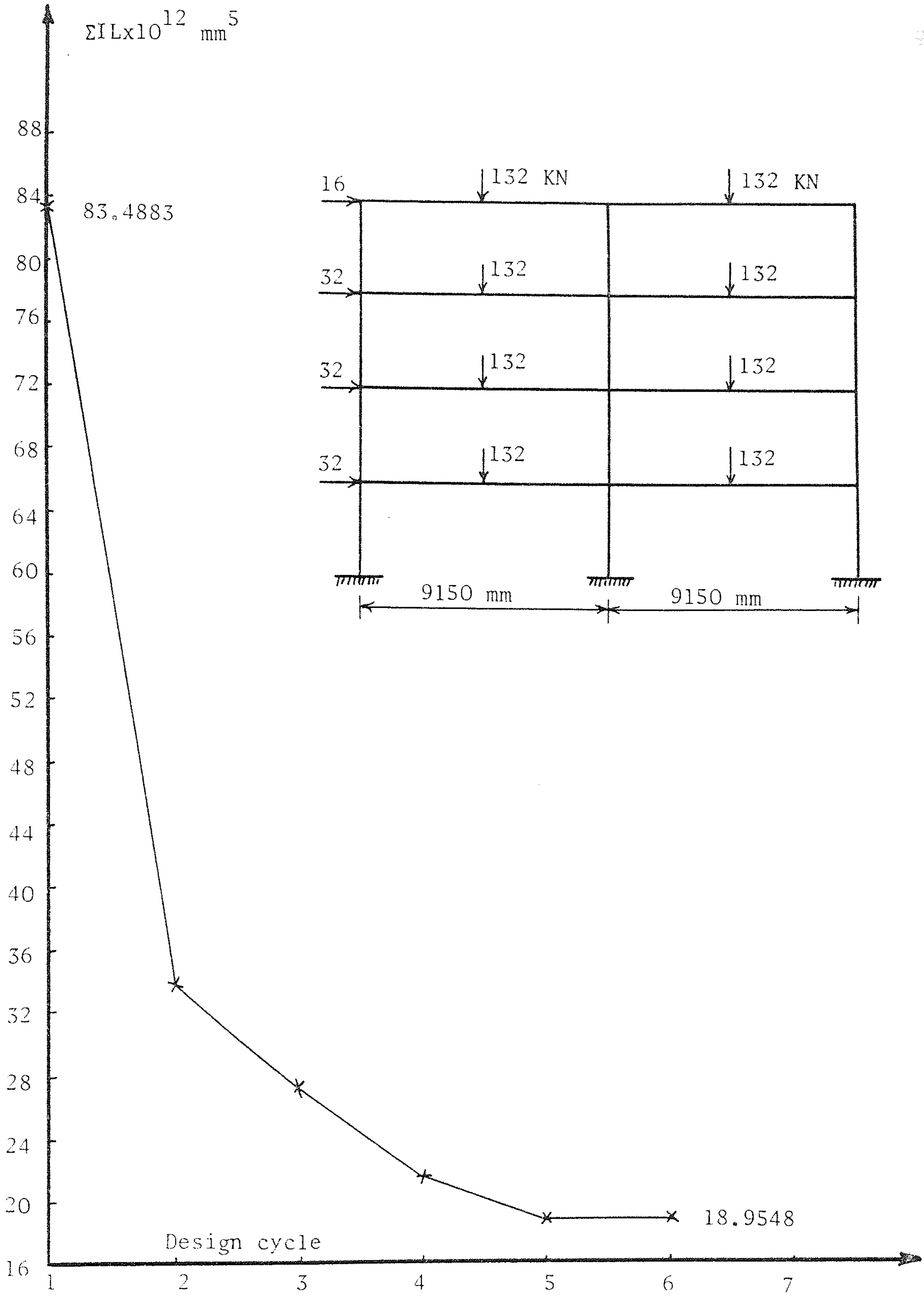
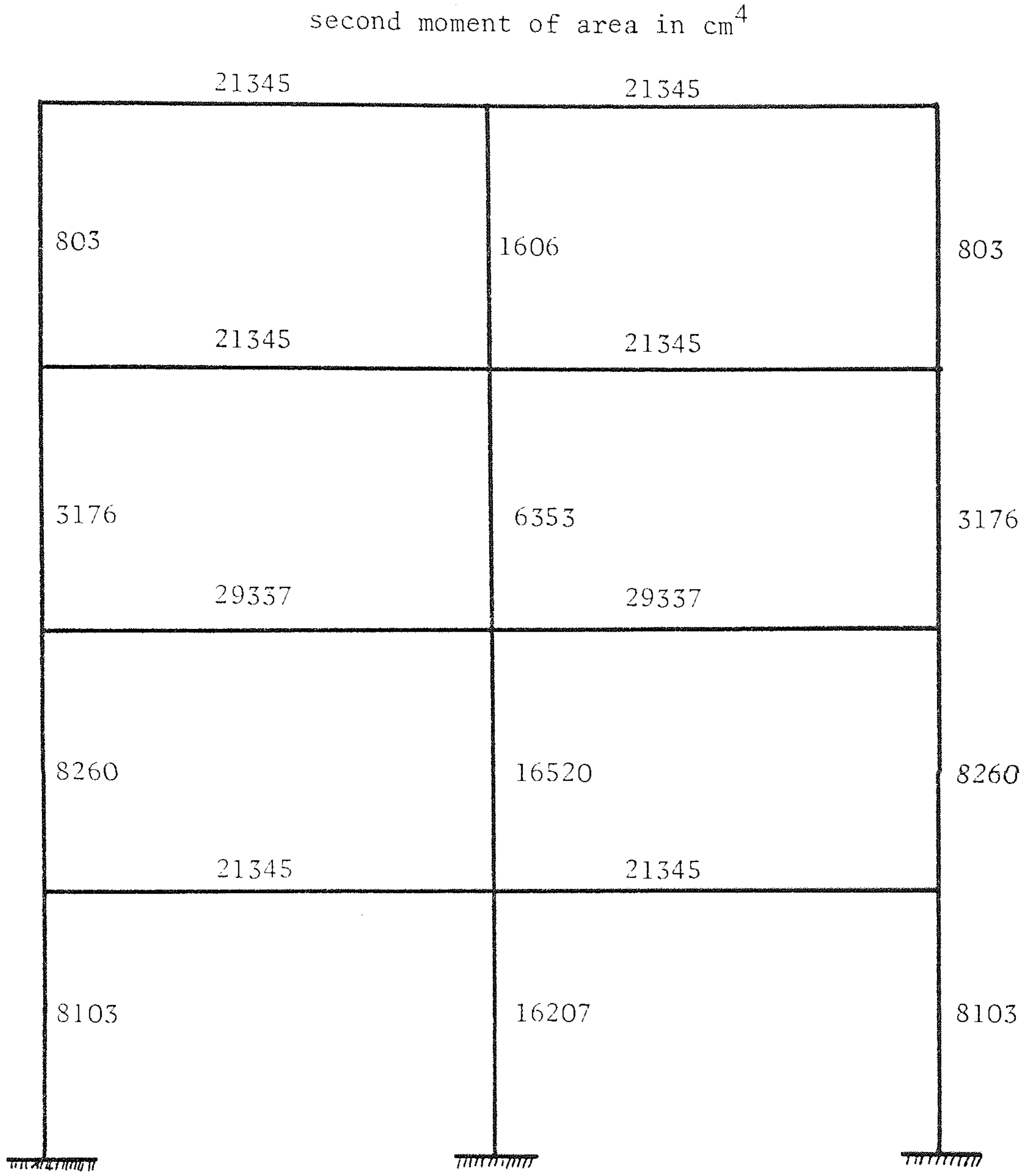


FIGURE 8.9 A 4 STOREY - 2 EQUAL BAY FRAME ΣIL PER DESIGN CYCLE



$$\Sigma IL = 2.0065 \times 10^{13}$$

FIGURE 8.10 A 4 STOREY 2 EQUAL BAY FRAME - THE REDUCED COST DESIGN OBTAINED BY THE METHOD OF CHAPTER 2

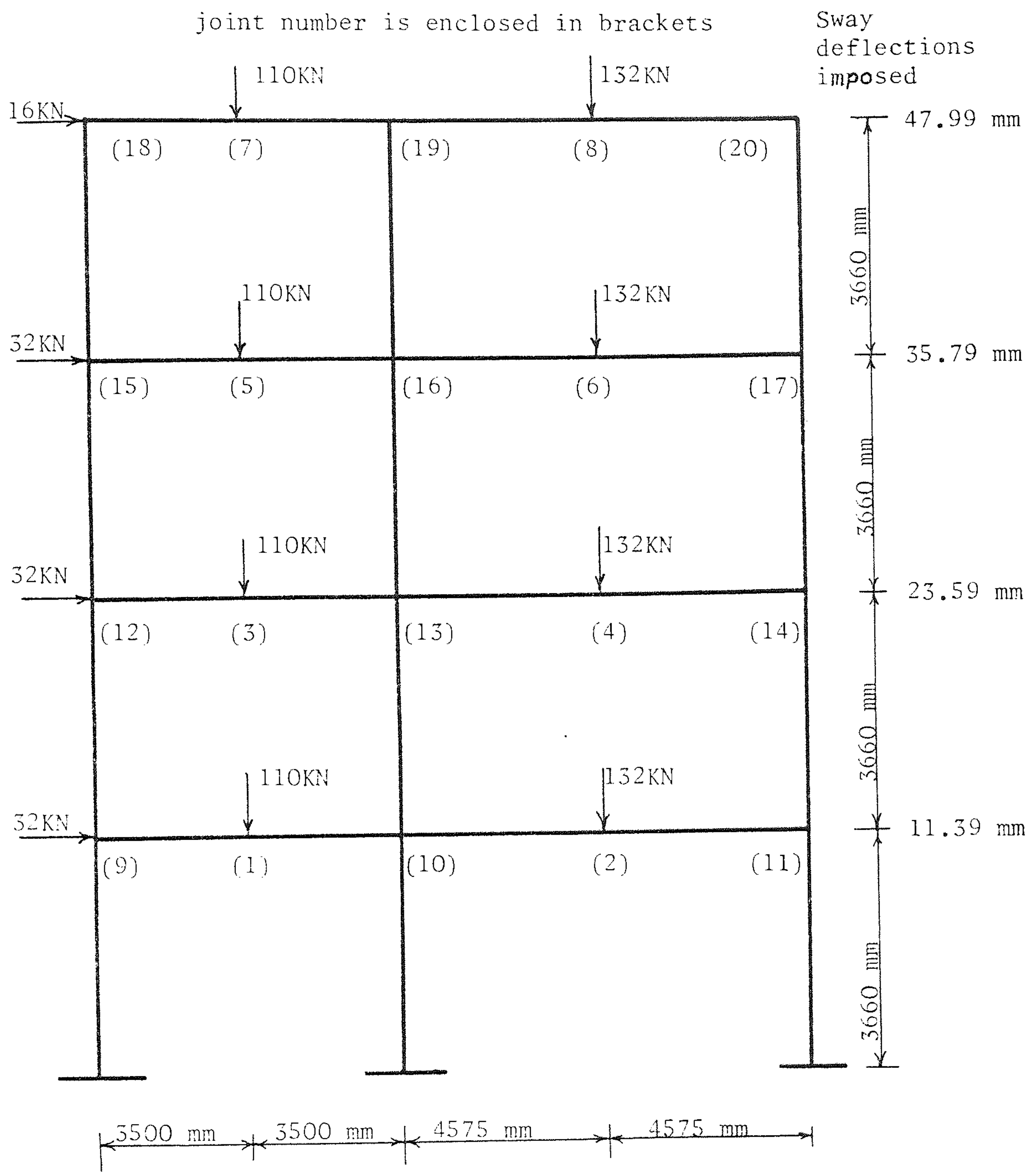


FIGURE 8.11 A 4 STOREY - 2 UNEQUAL BAY FRAME DIMENSIONS AND LOADS

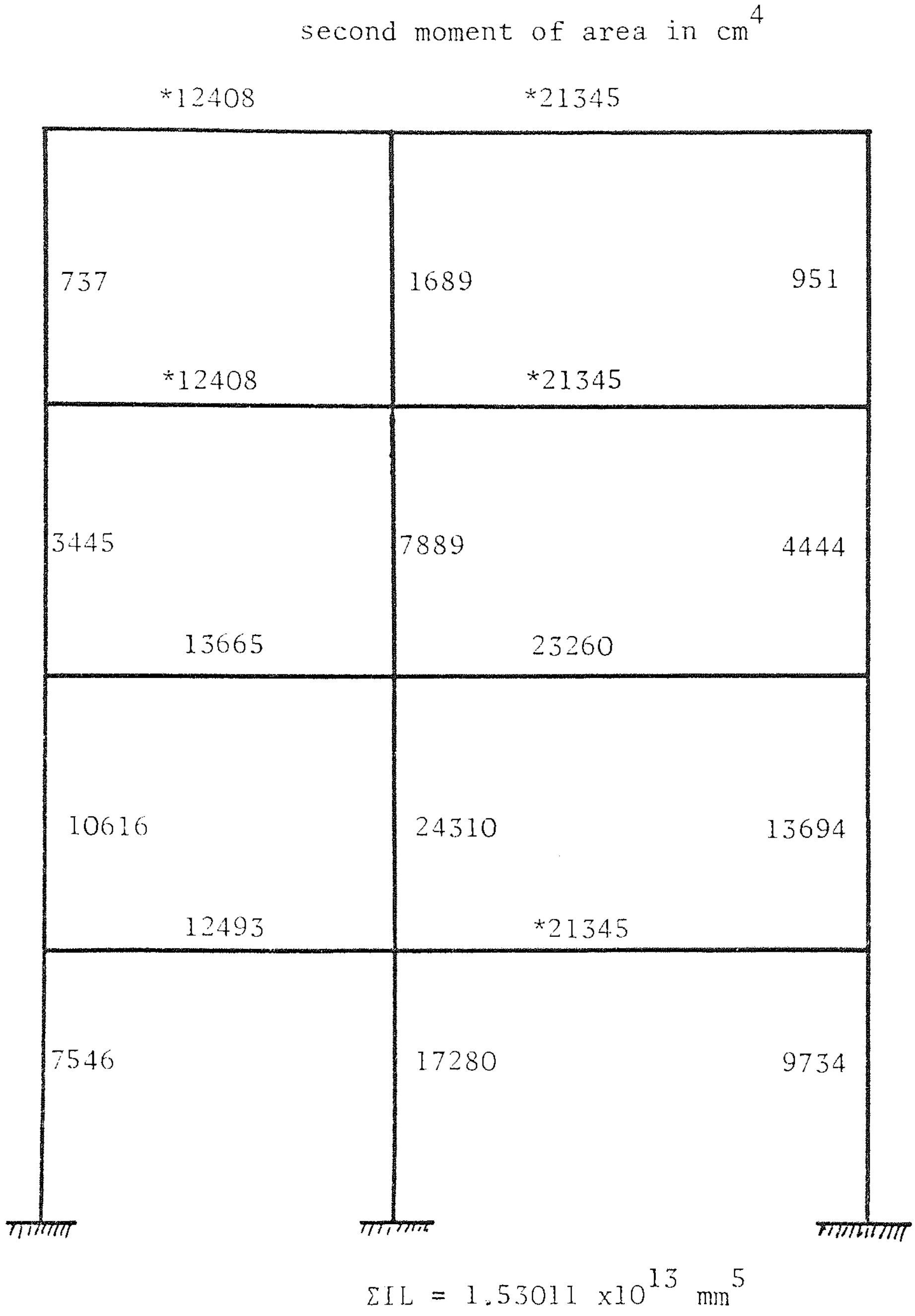


FIGURE 8.12 THE 4 STOREY 2 UNEQUAL BAY FRAME OPTIMUM DESIGN

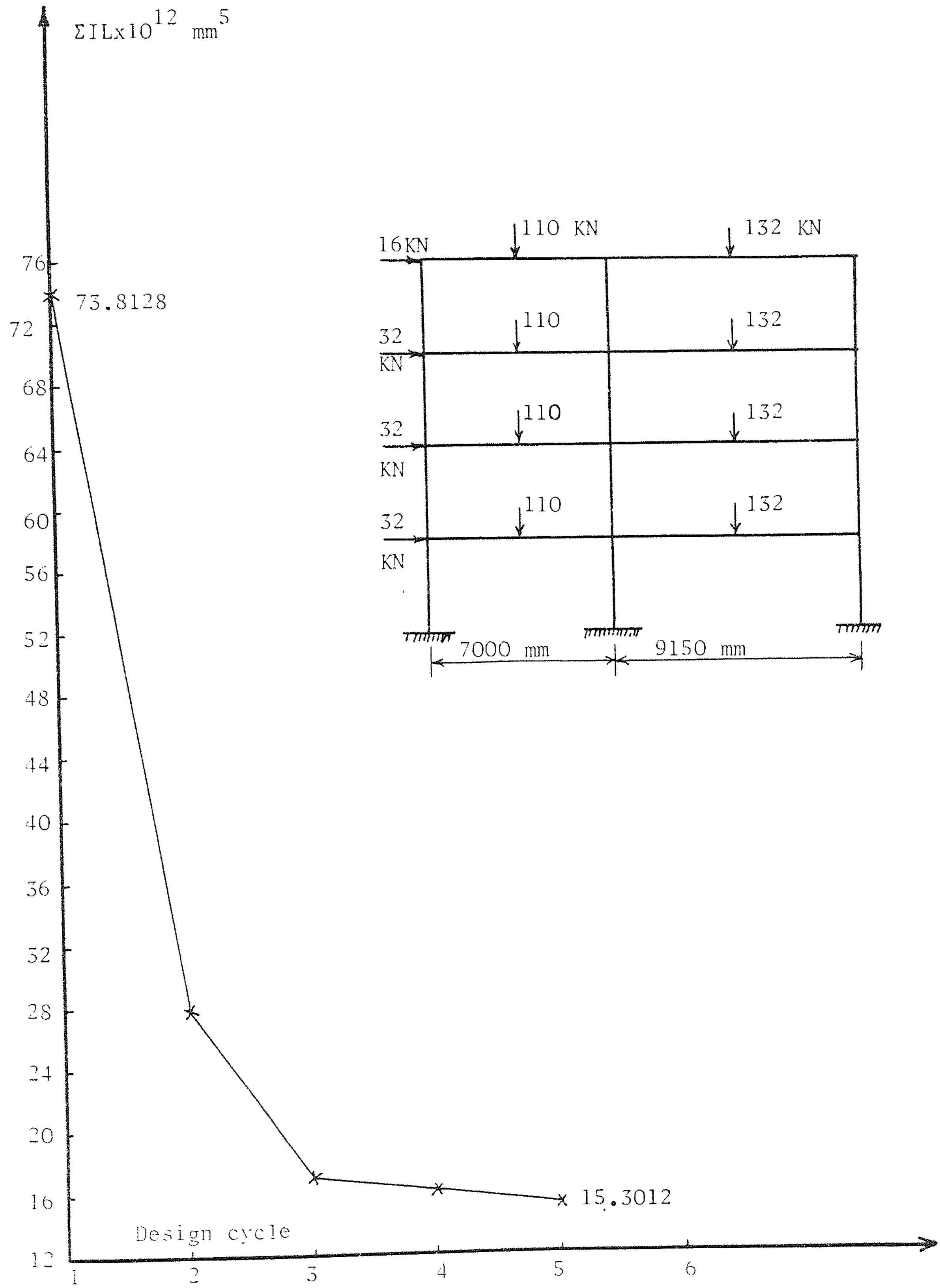
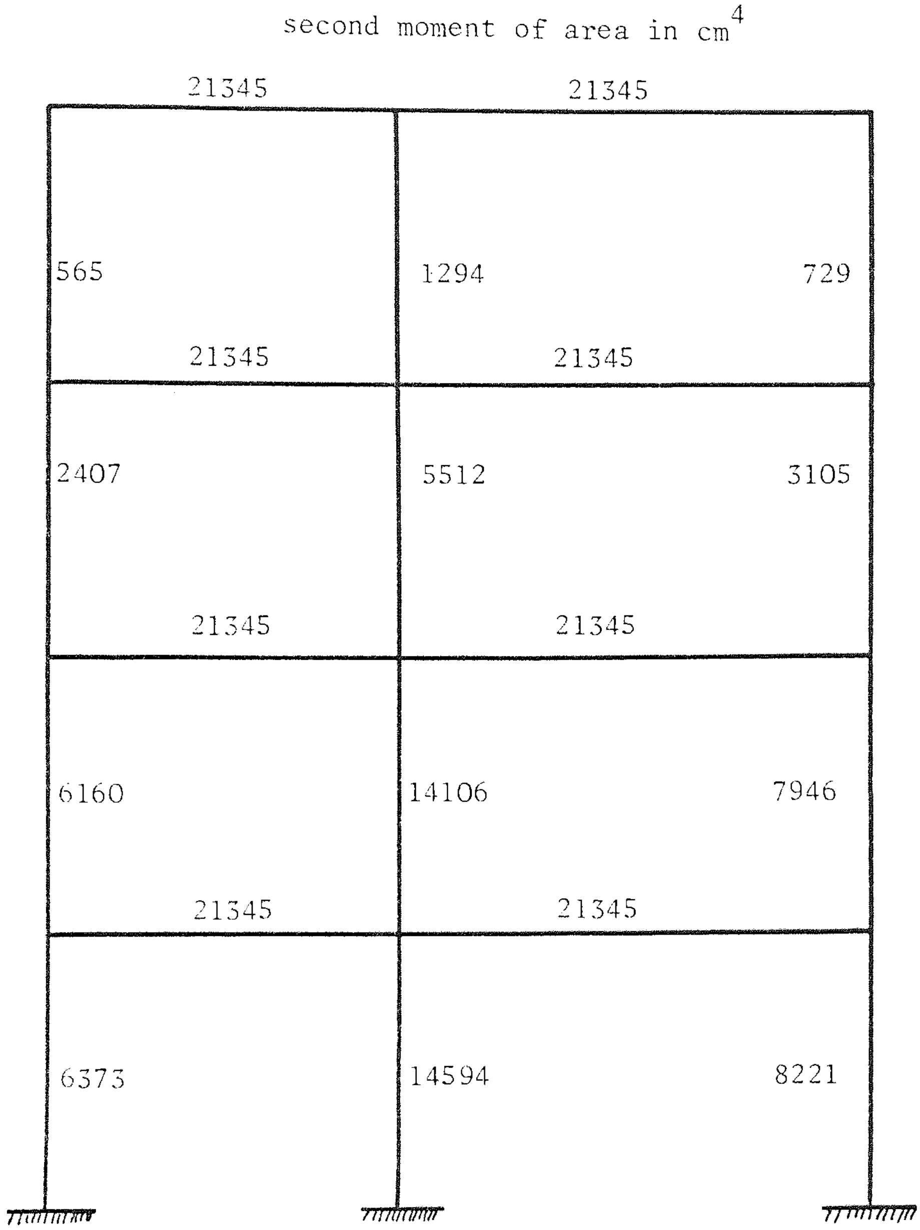


FIGURE 8.13 THE 4 STOREY 2 UNEQUAL BAY FRAME ΣIL PER DESIGN CYCLE



$\Sigma IL = 1.63879 \times 10^{13}$

FIGURE 8.14 THE 4 STOREY-2 UNEQUAL BAY FRAME - REDUCED COST OBTAINED USING THE METHOD OF CHAPTER 2

CHAPTER NINE

SUGGESTIONS FOR FURTHER WORK

The direct design method for deflection limitations presented in Chapter 2 to 5 is successful in designing multistorey steel frames in which limitations are imposed on their sway. The frames designed by this method proved to satisfy the strength requirements of the proposed new code. In fact, the design examples of Chapter 5 show that the frames designed to deflection requirement remain elastic under factorised combined loading, when a non-linear elastic plastic analysis is carried out.

It was found that many existing design methods for sway frames which satisfy strength requirements first prove to produce sections which do not reduce the storey sway to acceptable limits. This indicate that it is more realistic and advantageous to initiate the design activity by satisfying the sway requirements first.

Formulation of the design problem using the modified stiffness equation enables the structure to be considered in its entirety, without considering each storey individually as in the existing design methods (Moy, 1974), (Anderson and Islam, 1979). Hence it was possible to obtain an exact design without the need for simplifying assumptions, such as the insertion of points of contraflexure at midpoints of the members. The introduction of the axial load effect into the design method was made possible using stability functions.

The iteration technique used for solving the non-linear modified stiffness equations was found to be effective. Hitherto this technique has been used only to solve sets of linear equations, but it is shown that it may successfully be applied to the solution of non-linear equations. In all the design examples solved by this technique convergence was always obtained. However, experience showed that it is difficult to achieve convergence if the iteration process is started with a set of infeasible beam sections.

In this proposed design procedure, the list of universal beam and column sections included only those with the most economical second moment of area/cost ratios. However, if it is required to restrict the depth of a member, it may be necessary to select a section that is not among this 'economical' list. In this case it is necessary to alter the list accordingly. UB and UC are selected for the beams and the columns respectively, but the use of UB sections for some column members may sometimes be found economical.

The computer program is written for the design of fixed base frames. Although most of the multistorey frames in practice are designed as such, the program can be modified to include pinned base frames. Such modifications can be easily done by changing the modified stiffness equations to allow for a hinge rotation at the base of the frame.

The computer time needed for the direct design of the frame to deflection limitations is relatively small. The time consumed to obtain the final design for a frame, including the time for checking the strength requirements, is only twice that needed for a single direct analysis of solving the stiffness equations $\underline{L} = \underline{K}\ \underline{X}$ directly. However, for larger frames i.e. those with more than 24 storeys, the design time does increase and this is expected in such frames.

The computer storage requirements of the design method proved to be so small that it is in fact possible to use the method to design many smaller frames on programable pocket calculators. In fact, many of the frames designed in this thesis were obtained by desk-top computers using the basic language. This is an important advantage of the method and is achieved because the modified stiffness equations of only one joint is operated upon at any given stage of the iteration.

The proposed method was also applied successfully to design reinforced concrete sway frames. It was shown, in Chapter 6, that the limitations on sway deflections govern the design of some of the beams and columns in a frame. The examples show that for taller frames, the sway deflections of a frame become more and more demanding.

When designing reinforced concrete frames for deflection limitation, the second moment of area of a cracked rectangular beam was considered to be constant along the length of the member. The proposed method is therefore only approximate in this respect as the sagging part of the beam acts as a T-beam. However, it is difficult to select a unique second moment of area which represents the member accurately throughout its length.

In Chapter 7, the proposed method was extended to design pin-jointed space frames also to satisfy deflection limitation first. For this category of structures, it was found that with high yield steel the design is also governed by deflection limitations rather than strength constraints. In fact, the author carried out a comparison of prices and found that the large space frame in example 5 of Chapter 7, is marginally cheaper when manufactured out of high yield steel than mild steel. In this practical type frame, it was found that only after one cycle of design the weight of the frame was reduced to a minimum. Further redistribution of the material throughout the frame and in accordance to strength requirements did not prove to be profitable. In fact, this realistic example showed that the strength requirements of the code played no significant part in selecting the member areas. These latter requirements were merely used in the second cycle to alter member sizes (i.e. the values of α), but in the resulting minimum weight design the deflection requirements proved to dictate the member sizes.

In many small hyperstatic space frames it was found that the final design is always statically determinant. This may cause misleading conclusions as the structures designed were small and grouping members together prevents such an outcome.

In the present computer program, for the design of pin-jointed space frames, the deflection was specified at one joint (e.g. at mid span) and the corresponding modified stiffness equations were used to calculate the member areas in the first group. Those of other groups were calculated by proportion. It is therefore suggested that the program is modified to specify the deflections in more than one joint. In that case, proportioning the members using stress requirements may become unnecessary. The problem arises as to what deflections should be used for these other joints and further research is needed in this field.

In Chapter 2 to 5 a linear extraploation technique was used to change the beam sections of rigidly jointed steel frames. Although this was found to be satisfactory from a practical point of view, the final design is dependent on the initial beam sections. It was however, found that these initial sections are difficult to be specified satisfactorily by a simple method. Further more, a given assumed set of initial sections can only be improved by the extrapolation method without ever obtaining the optimum design. This is inspite of the fact that the examples solved gave near optimum designs. However, the Lagrange multiplier method, applied in Chapter 8, overcame this problem and gives the optimum set of beam sections without the need of starting the procedure from any particular set of sections.

In this method of optimisation, the iteration technique was used successfully to solve the derivative equations. The aim was to reduce the sum, (for all the frame members), of the product of the second moment of area and length. It is therefore proposed to use the method of Lagrange multiplier to optimise the actual weight of the structure. Such an effort may improve the accuracy of the method but can also lead to mathematical complications.

APPENDIX A

DATA PREPARATION

THE DESIGN OF STEEL FRAMES

The data for the frame shown in Figure A1 are given below as an example.

a - Preliminary data

One card containing:

(1) The highest joint number. For the frame shown in Figure A1, this is equal to 6.

(2) Total number of joints in the beams (this is equal to 2 for the frame in Figure A1).

(3) The yield stress for the steel used in KN/mm^2 (0.250)

(4) The load factor for vertical loading (1.75)

(5) Modulus of elasticity in KN/mm^2 (207)

(6) Tolerance (0.001)

(7) $1/\alpha$ for Anderson and Islam's preliminary design (400)

(8) $1/\alpha$ allowed in the code (300)

(9) The load factor for combined loading (1.29)

b - Beam - joints Data (1 card per joint)

The specification of a beam-joint was given in Section 2.3. The data for beam-joints 1 and 2 of the frame shown in Figure A1 would appear as follows:

joint (1):	1	4575.	4	3	-132.0	0.0
joint (2):	2	4575	6	5	-132.0	0.0

The data refers to the following:

(1) The joint number j e.g. (1) for first joint

(2) The length of beam B_j in mm (4575.)

(3) The joint number following joint j (which is 4 for beam 1).

(4) The joint number preceding B_i (which is 3 for joint 1)

(5) The vertical load acting at joint j in KN (-132.0)

(6) The externally applied moment on joint j in KN/mm (0.0)

c - Column-joints Data (2 cards per joint)

The specification of a column-joint was given in section 2.3. The data is punched on two cards. For example, the data for joint 3 of the frame of Figure A1 would appear as follows:

first card	3	1	0	0	5	10.17	3	4
second card	3	4575.	3660.		16.0	0.0	0.0	

The First Card contains the following information:

(1) The joint number j (e.g. 3)

(2) The joint number at the second end of beam B_j (which is 1 for joint 3)

(3) The joint number at the first end of beam B_i (which is 0 for joint 3, because B_i does not exist)

(4) The joint number at the second end of column C_j (which is 0 for joint 3, because C_j is fixed at its second end)

(5) The joint number at the first end of Column C_m (which is 5 for joint 3)

(6) The sway of the joint in mm (10.17)

(7) The number of the extreme left joint of the storey of j (3)

(8) The number of the extreme right joint of the storey of j (4)

The Second Card, which follows the first one, contains:

(1) The joint number (e.g. 3)

(2) The length of beam B_j in mm (4575.0)

(3) The length of column C_j in mm (3660.0)

(4) The external horizontal load applied at joint j in KN (16.0)

(5) The external vertical load applied at joint j in KN (0.0)

(6) The external applied moment on joint j in KN.mm (0.0)

Notice that a joint is given a number zero if it does not exist, or if it is fixed to a support. For example, for joint 3 the joint number at the first end of beam B_i is zero, because there is no beam B_i. Also the joint number at the second end of column C_j is zero, because this end is fixed to the ground. The rest of the data for joints 4, 5 and 6 of the frame shown in Figure A1 is given in Table A1.

joint 4	Card 1:	4	0	1	0	6	10.17	3	4
	Card 2:	4	0.0	3660.	16.0	0.0	0.0		
joint 5	Card 1:	5	2	0	3	0	20.33	5	6
	Card 2:	5	4575.	3660.	8.0	0.0	0.0		
joint 6	Card 1:	6	0	2	4	0	23.33	5	6
	Card 2:	6	0.0	3660.0	8.0	0.0	0.0		

TABLE A1 DATA FOR COLUMN-JOINTS 4, 5 AND 6 OF THE FRAME OF FIGURE A1

d - Frame Data (two cards)

The first card contains the number of storeys and the number of bays in the frame. The second card contains the length of each bay in mm starting from the left bay. For the frame shown in Figure A1 these two cards are: 2 and 1 on the first card and 9150 on the second.

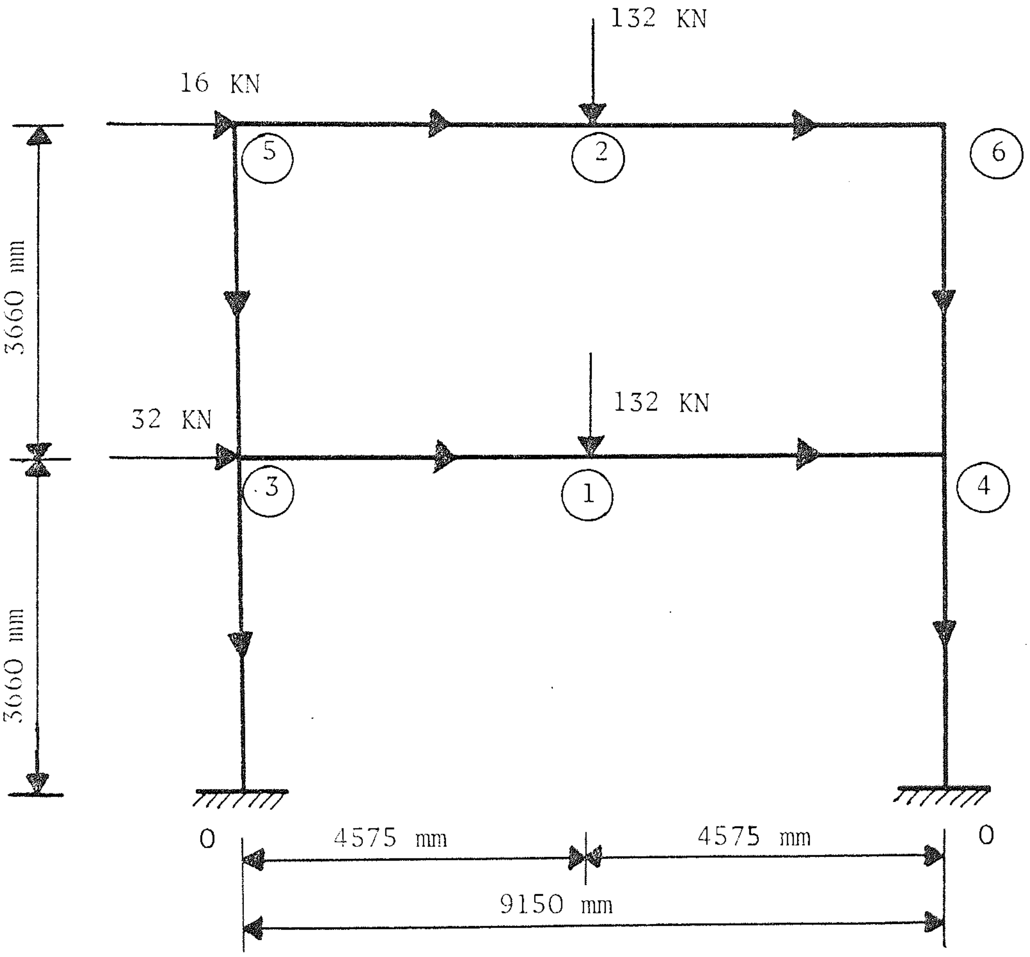


FIGURE A1 EXAMPLE ON DATA INPUT FOR THE STEEL DESIGN PROGRAM

APPENDIX B

DATA FOR THE CONCRETE PROGRAM

It is obvious that the steel universal column and beams are not used as data for concrete program. Instead some of the sectional properties of the concrete sections are used. These properties differ from one frame to another and should be given for each individual frame. The data consists of:

(i) Beam Sections Data: (One card per section) Each card contains the following information.

1) The joint number j;
2) The width of the section b;
3) The cover of the tensile steel d';
4) The cover of the compression steel G;
5) The flange depth h_f for a T-beam section;
6) The width of the flange B_f for a T-beam section.

(ii) Column Sections Data: (One card per section). Each card provides three numbers. These are:

1) The joint number.
2) The cover (of the tensile and compression steel).
3) The percentage of reinforcement ($100A_{sc}/bh$) assumed to resist the sway of the column. This percentage is taken equal to 1 at the top storey and increases gradually at the lower storeys.

The width of a column is not given as data because this is taken to be equal to the width of the beam.

APPENDIX C

DATA PREPARATION

THE DESIGN OF PIN-JOINTED SPACE FRAMES

The data for the frame shown in Figure C1 are given below as an example, (this frame was shown in Figure 7.3 and designed in section 7.7).

a - Preliminary Data

One card containing:

(1) Total number of members in the frame (this is equal to six for the frame in Figure C1).

(2) Total number of joints including supports. For the frame shown this is equal to six.

(3) Total number of supports (equal to four for the frame).

(4) Total number of joints at which deflection is specified. This is equal to one.

(5) Modulus of elasticity in KN/mm^2. (200)

(6) Design stress for members in tension in KN/mm^2. (0.250)

(7) Design stress for members in compression in KN/mm^2. (0.200)

(8) Tolerance. (0.001)

(9) Total number of groups. (For the frame in Figure C1 this is equal to six, as members were not grouped in this example).

b - Section Data (one card per group)

Each card contains the group number and the member areas for the group in mm^2 e.g. for the first group the card appears as follows:

1 100

If the ratios between the member areas in the groups are to be taken initially as equal to one, then the area of the members in

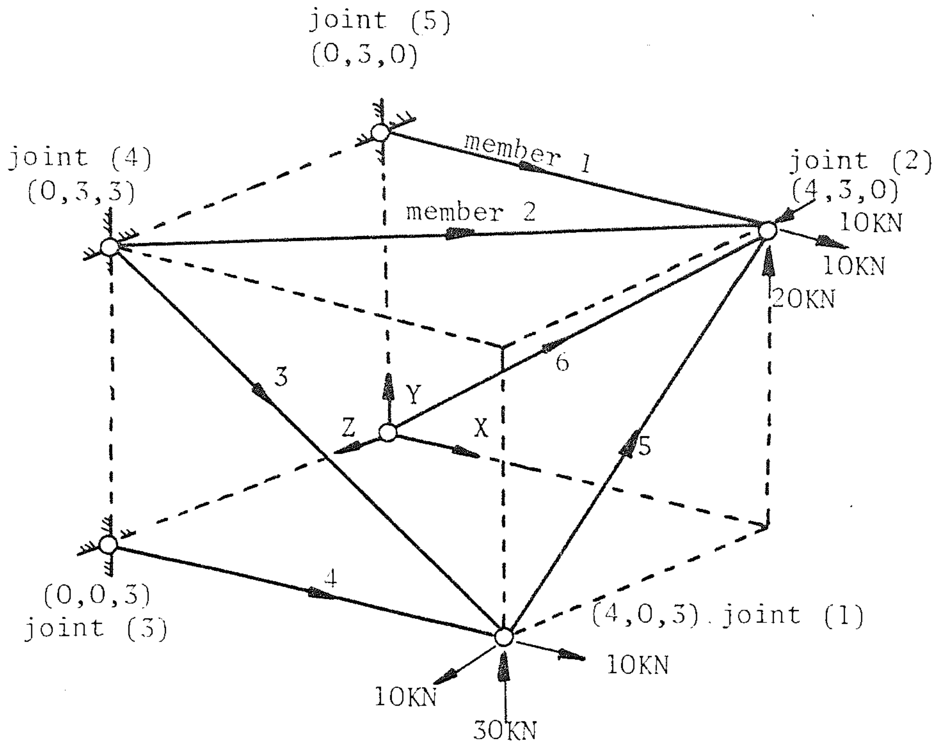


FIGURE C1 6 MEMBERS - ISOSTATIC SPACE FRAME

the remaining groups should be initially set to 100. Thus

2	100
3	100
4	100
5	100
6	100

c - Member Data (one card per member)

The data for member 1 of the frame of Fig. C1 would appear as follows:

1	5	2	1

the data refers to the following:

(1) the member number e.g. (1) for first member

(2) the joint number at the first end (which is 5 for member 1)

(3) the joint number at the second end (which is 2 for member 1)

(4) the number of the group in which the member belongs (e.g. 1).

The remaining data fro members 2, 3, 4, 5 and 6 of the frame, shown in Figure C1, are given in Table C1.

Member 2	2	4	2	2
Member 3	3	4	1	3
Member 4	4	3	1	4
Member 5	5	1	2	5
Member 6	6	6	2	6

TABLE C1 DATA FOR MEMBERS 2, 3, 4, 5, AND 6 OF THE FRAME OF FIGURE C1

d - Joint Data (one card per joint)

The data for joint 1 of the frame of Figure C1 would appear as follows:

1 4000 0 3000 10.0 30.0 10.0

the data refers to the following:

(1) the joint number (e.g. 1)

(2) the joint coordinate X in mm (4000)

(3) the joint coordinate Y in mm (0)

(4) the joint coordinate Z in mm (3000)

(5) the externally applied load to the joint in the X direction in in KN (10.0)

(6) The externally applied load to the joint in the Y direction in KN (30.0)

(7) the externally applied load to the joint in the Z direction in KN (10.0)

The rest of the data for joints 2, 3, 4, 5 and 6 of the frame, shown in Figure C1, are given in Table C2.

joint 2	4000	3000	0	10.0	20.0	10.0
joint 3	0	0	3000	0.0	0.0	0.0
joint 4	0	3000	3000	0.0	0.0	0.0
joint 5	0	3000	0	0.0	0.0	0.0
joint 6	0	0	0	0.0	0.0	0.0

TABLE C2 DATA FOR JOINTS 2, 3, 4, 5 AND 6 OF THE FRAME OF FIGURE C1

e - Restraint Data (one card per support)

Each card contains:

(1) The number of the joint which is restrained in some way

(2) Degree of freedom in X, Y and Z directions. (one if the joint is constrained in this direction and 0 if it is not).

For the frame shown in Figure C1 joints 3, 4, 5 and 6 are restrained in the X, Y and Z directions, thus the data for these joints are:

3	1	1	1
4	1	1	1
5	1	1	1
6	1	1	1

f - Deflection specification (only for the joints at which deflection is specified - one card per each such joint)

For the frame of Figure C1 there is only one card. This would appear as:

1	0.0	11.11	0.0	0	1	0

The above data refers to the following:

(1) The joint number at which deflection is specified.

(2) Deflections x, y and z for this joint (the value of the specified deflection in mm, or 0.0 if the deflection in the corresponding direction is not specified).

(3) Direction specification X, Y or Z (1 for the specified direction and 0 for the rest).

REFERENCES

AL-PASHA, S,'Optimum design of complete structures', thesis to be submitted to the University of Aston for the degree of PhD, 1982.

ALLEN, A H, 'Reinforced Concrete Design to CP110 - Simply Explained' Cement and Concrete Association, 1977.

ALLWOOD, B O, et al, 'Steel Designers Manual.' London : Crosby Lockwood, 1972 edn.

ALLWOOD, B O, et al, 'Steel Frames for Multi-Storey Buildings: Some Design Examples to Conform with the Requirements of BS449:1959. BCSA Publication No 16, 1961.

ANDERSON, D,'Investigations into the Design of Plane Structural Frames." PhD thesis, The Victoria University of Manchester, 1969.

ANDERSON, D and ISLAM, M A, 'Design of Multi-Storey Frames to Sway Deflection Limitations.' The Structural Engineer, Vol 57B, No 1, March 1979.

ASTILL, A W and MARTIN, L H, 'Elementary Structural Design in Concrete to CP110.' London : Edward Arnold, 1975.

BAKER, J F, 'The Steel Skeleton.' Vol 1, Cambridge University Press, 1954.

BAKER, J F, HORNE, M R and HEYMAN, J, 'The Steel Skeleton.' Vol 2., Cambridge University Press, 1956.

BEEBY, A W and TAYLOR, H P J, 'The Use of Simplified Methods in CP110 - Is Rigour Necessary?' The Structural Engineer, Vol 56A, No 8, August 1978.

BENNETT, E W, 'Structural Concrete Elements.' London : Chapman and Hall, 1975.

BOLTON, A, 'A Simple Understanding of Elastic Critical Load.' The Structural Engineer, Vol 54, June 1976.

BRITISH STANDARD CP114:1957 edn, 1967 edn, and 1969 edn. 'The Structural Use of Reinforced Concrete in Building.' British Standard Institution.

BRITISH STANDARD 449:Part 2:1969. 'Specification For the Use of Structural Steel in Building.' British Standard Institution.

BRITISH CONSTRUCTIONAL STEELWORK ASSOCIATION LTD, and the CONSTRUCTIONAL STEEL RESEARCH and DEVELOPMENT ORGANISATION. 'Handbook on Structural Steelwork.' 1971 edn.

BRITISH STANDARD CP110:1972. 'The Structural Use of Concrete.' British Standard Institution.

BRITISH STANDARD 4360:1972. 'Specification for Weldable Structural Steels.' British Standard Institution.

BRITISH STANDARD B/20, DOCUMENT 77/13908 DC. 1977 'Draft Standard Specification for the Structural Use of Steelwork in Building, Part 1:Simple Construction and Continuous Construction.' British Standard Institution.

BRITISH STEEL CORPORATION. 'Price List of Steel Sections' effective from 10 July 1977.

CELIK, T 'Elastic-Plastic Analysis of Complete Structures with Shear Walls and Frames.' PhD thesis, University of Aston, 1977

CLARK, C V, 'An Investigation of the Criteria for Deflection in Design.' PhD thesis, University of Leeds, 1974.

COATES, R C, COUTIE, M G and KONG, F K. 'Structural Analysis.' London : Nelson, 1972.

COHEN, J F; CUTTS, T F; FIELDER R; JONES, D E; RIBBANS, J and STUART, E. 'Numerical Analysis.' London:McGraw-Hill, 1973.

CROXTON, P C L. 'The Analysis of Complete Structures Consisting of Bare Frames, Shear Walls and Plate Components.' PhD thesis, University of Aston, 1974.

GALLAGHER, R H and ZIENKIEWICZ, O C. (Eds.) 'Optimum Structural Design; Theory and Applications.' John Wiley and Sons, 1973.

GRINTER, L E. 'Theory of Modern Steel Structures.' Vol II, Mac-Millan, 1937.

HADLEY, G. 'Non-linear and Dynamic Programming.' Addison-Wesley Publishing Company, 1970 edn.

HEYMAN, J. 'An Approach to the Design of Tall Steel Buildings.' Proc. Inst. Civ. Engrs., Vol 17, Dec 1960.

HOLMES, M and GANDHI, S N. 'Ultimate Load Design of Tall Steel Building Allowing for Instability.' Proc. Inst. Civ. Engrs., Vol 30, Jan 1965.

HOLMES, M and SINCLAIR-JONES, H W, 'Plastic Design of Multi-Storey Sway Frames.' Proc. Inst. Civ. Engrs., Vol 47, Sept 1970.

HORNE, M R. 'Plastic Design of Columns.' British Constructional Steelwork Association Publication, No 23, 1964.

HORNE, M R and MAJID, K I. 'Elastic-Plastic Design of Rigid Jointed Sway Frames by Computer.' 1st Report, Study of Analytical and Design Procedures for Elastic and Elastic-Plastic Structures, Department of Civil Engineering in the Faculty of Science, University of Manchester, March 1966.

HORNE, M R and MORRIS, L J. 'Optimum Design of Multi-Storey Rigid Frames.' Optimum Structural Design: Theory and Applications, (Eds.) Gallagher, R H and Zienkiewiez, O C, Wiley, 1973.

HORNE, M R. 'An Approximate Method for Calculating the Elastic Critical Loads of Multi-Storey Plane Frames.' The Structural Engineer, Vol 53, June 1975.

ISLAM, M A 'Optimum Design of Sway Frames.' PhD thesis, University of Warwick, 1978.

JENKINS, W M, 'Matrix and Digital Computer Methods in Structural Analysis.' London : McGraw-Hill 1969.

JENNINGS, A and MAJID, K I, 'An Elastic-Plastic Analysis for Framed Structures Loaded up to Collapse.' Structural Engineer, Vol 43, Dec, 1965.

JENNINGS, A, 'A Compact Storage Scheme for the Solution of Symmetric Linear Simultaneous Equation.' The Computer Journal, Vol 9, Nov. 1966.

JOINT COMMITTEE'S SECOND REPORT on 'Fully-Rigid Multi-Storey Welded Steel Frames.' The Institution of Structural Engineers and The Welding Institute, May 1971.

LENNOX, S C and CHADWICK, M. 'Mathematics for Engineers and Applied Scientists.' London : Heinemann, 1974 edn.

LIVESLEY, R K. 'The Application of an Electronic Digital Computer to Some Problems of Structural Analysis.' Structural Engineer, Vol 34, January 1956.

LIVESLEY, R K. 'The Application of Computers to Problems Involving Plasticity.' Symposium on the use of Electronic Computers in Structural Engineering, University of Southampton 1959.

LIVESLEY, R K. 'Matrix Methods of Structural Analysis.' Oxford: Pergamon Press, 1975 edn.

MACGINLEY, T J. 'Reinforced Concrete Design Theory and Examples.' London E and F N Spon Ltd, 1978.

MAJID, K I and ANDERSON, D. 'Elastic-Plastic Design of Sway Frames by Computer.' Proc. Inst. Civ. Engrs. Vol 41, December 1968.

MAJID, K I. 'Non-Linear Structures.' London : Butterworths, 1972.

MAJID, K I. 'Optimum Design of Structures.' London : Newnes-Butterworths, 1974.

MAJID, K I. 'Theory of Structures - with Matrix Notation.' London: Newnes-Butterworths, 1978.

MAJID, K I; STAJANOVSKI, P; and SAKA, M P. 'Minimum Cost Topological Design of Steel Sway Frames.' The Structural Engineer, Vol 58B, No 1, March 1980.

MAJID, K I. 'Matrix and Numerical Methods for Engineers.' to be published, 1980.

MERCHANT, W. 'The Failure Load of Rigid Jointed Frameworks as Influenced by Stability.' The Structural Engineer, Vol 32, July 1954.

MERCHANT, W. 'Critical Loads of Tall Building Frames.' The Structural Engineer, Vol 33 and 34, 1956.

MOY, F C. 'Control of Deflections in Unbraced Steel Frames.' Proc. Inst. Civ. Engrs. Vol 57, December 1974.

MOY, F C. 'Inelastic Sway Buckling of Multi-Storey Frames.' ASCE Journal of The Structural Division, Vol 102, No ST1, January 1976a.

MOY, F C. 'Multi-Storey Frame Design Using Storey Stiffness Concept.' Proc. ASCE, Vol 102, No ST6, June 1976b.

NEAL, B G and SYMONDS, P S. 'The Rapid Calculation of Plastic Collapse Loads for a Framed Structure.' Proc. Inst. Civ. Engrs. Vol 1, April 1952.

NEEDHAM, F H, 'The Economics of Steelwork Design.' The Structural Engineer, Vol 55, No 9, September 1977.

REYNOLDS, C E and STEEDMAN, J C. 'Reinforced Concrete Designer's Handbook.' A Viewpoint Publication by Cement and Concrete Association, 1974 edn.

ROWE, R E: GRANSTON, W B and BEST, B C. 'New Concept in the Design of Structural Concrete.' The Structural Engineer, Vol 43, No 12, December 1965.

SAKA, M P. 'Optimum Design of Structures.' PhD thesis, University of Aston, 1975.

SCHMIDT, L C; MORGAN, P R; O'MEAGHER, A J and COGAN, K. 'Ultimate Load Behaviour of a Full-Scale Space Truss.' Proc. Inst. Civ. Engrs. Vol 69, Part 2, March 1980.

SPIEGEL, M R, 'Mathematical Handbook of Formulas and Tables.' McGraw-Hill, 1968.

STEEL STRUCTURES RESEARCH COMMITTEE. 'Final Report' HMSO 1936.

STEVENS, L K, 'Direct Design by Limiting Deformations.' Proc. Inst. Civ. Engrs., Vol 16, July 1964.

STEVENS, L K, 'Control of Stability by Limiting Deformations.' Proc. Inst. Civ. Engrs. Vol 28, July 1964.

TOAKLEY, A R, 'Optimum Design Using Available Sections.' Proc. ASCE, Vol 94, No ST5, May 1968.

WILLIAMS, F W, 'Simple Design Procedures for Unbraced Multi-Storey Plane Frames.' Proc. Inst. Civ. Engrs., Vol 63, 1977.

WILLIAMS, F W, 'Consistent, Exact, Wind and Stability Calculations for Substitute Sway Frames with Cladding.' Proc. Inst. Civ. Engrs. Vol 67, Part 2, June 1979.

WOOD, R H, 'The Stability of Tall Building.' Proc. Inst. Civ. Engrs. Vol 11, September 1958.

WOOD, R H, 'Effective Length of Columns in Multi-Storey Buildings.' The Structural Engineer, Vol 52, July-September 1974.