SIMULATION OF POWER STATION PROTECTION EQUIPMENT

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Thesis submitted to the

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

THE UNIVERSITY OF ASTON IN BIRMINGHAM

For the degree of

DOCTOR OF PHILOSOPHY

MAY 1984

Acknowledgments

I wish to thank those who have given help in this project, particularly the following:

My supervisor Dr R C Johnson.

My associate supervisor Dr A L Bowden, and

Mr J G Hadwick for their interesting discussions and useful contributions.

Finally, the financial support from the University of Aston in Birmingham, the Central Electricity Generating Board and the Committee of Vice Chancellors and Principals of the Universities of the United Kingdom is gratefully appreciated.

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Summary

Computer programs have been developed to enable the coordination of fuses and overcurrent relays for radial power systems under estimated fault current conditions. The grading curves for these protection devices can be produced on a graphics terminal and a hard copy can be obtained. Additional programs have also been developed which could be used to assess the validity of relay settings (obtained under the above conditions) when the transient effect is included.

Modelling of a current transformer is included because transformer saturation may occur if the fault current is high, and hence the secondary current is distorted. Experiments were carried out to confirm that distorted currents will affect the relay operating time, and it is shown that if the relay current contains only a small percentage of harmonic distortion, the relay operating time is increased.

System equations were arranged to enable the model to predict fault currents with a generator transformer incorporated in the system, and also to include the effect of circuit breaker opening, arcing resistance, and earthing resistance. A fictitious field winding was included to enable more accurate prediction of fault currents when the system is operating at both lagging and leading power factors prior to the occurrence of the fault.

Keywords: overcurrent protection, optimal relay settings, current transformer, synchronous generator modelling, circuit breaker opening

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For the degree of DOCTOR OF PHILOSOPHY

May 1984

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List of symbols

Ce	connection matrix
H	inertia constant (MW secs. on machine rating)
x_d	direct axis synchronous reactance (p.u. on machine rating)
x_d'	direct axis transient reactance (p.u. on machine rating)
x_d''	direct axis subtransient reactance (p.u. on machine rating)
x_q	quadrature axis synchronous reactance (p.u. on machine rating)
x_q'	quadrature axis transient reactance (p.u. on machine rating)
x_q''	quadrature axis subtransient reactance (p.u. on machine rating)
x_a	armature leakage reactance in (d,q) parameters (p.u.on machine rating)
x_{sa}	armature leakage reactance in (a,b,c) parameters (p.u. on machine rating)
x_o	zero sequence reactance (p.u. on machine rating)
T_d'	direct axis transient short circuit time constant (sec.)
T_d''	direct axis subtransient short circuit time constant (sec.)
T_q'	quadrature axis transient short circuit time constant (sec.)
T_q''	quadrature axis subtransient short circuit time constant (sec.)
T_L	loss torque due to friction and windage
T_e	electrical air gap torque (p.u. on machine rating)
T_m	mechanical torque

T_a	accelerating torque
p	$\frac{d}{d t} = w_o \frac{d}{d \bar{t}}$
$\bar{t}$	time in radians
r_a	armature resistance (p.u. on machine rating)
r_e	earthing resistance (p.u. on machine rating)
r_{fd}	d-axis field winding resistance (p.u. on machine rating)
r_{fq}	q-axis field winding resistance (p.u. on machine rating)
r_{kd}	d-axis damper winding resistance (p.u. on machine rating)
r_{kq}	q-axis damper winding resistance (p.u. on machine rating)
x_p	leakage reactance of the generator transformers delta connected windings.
x_s	leakage reactance of the generator transformer star connected windings.
r_p	resistance of the generator transformer delta connected windings
r_s	resistance of the generator transformer star connected windings
n	generator transformer tap ratio
r_T	equivalent resistance of the generator transformer as seen from the system side
x_T	equivalent reactance of the generator transformer as seen from the system side
r_L	equivalent resistance of the transmission system

x_L	equivalent reactance of the transmission system
e_a	armature voltage - phase a
i_a	armature current - phase a
v_a	infinite busbar voltage at phase a
i_o	zero sequence current in the generator
i_{fd}	field current in the direct axis
i_{kd}	direct axis damper winding current
i_{fq}	field current in the quadrature axis
i_{kq}	quadrature axis damper winding current
e_{fd}	field voltage in the direct axis
x_{afdo}	direct axis field/armature mutual reactance (unsaturated)
x_{fdl}	field winding leakage reactance
x_{kdl}	damper winding leakage reactance
S_{1s}	switching matrix for faults on LV side of the transformer.
S_{2s}	switching matrix for faults on HV side of the transformer.
M_{1s}^{-1}	connection matrix for faults on LV side of the transformer.
M_{2s}^{-1}	connection matrix for faults on HV side of the transformer.
$(M_{is}^{-1})^t$	the transpose of M_i^{-1} where i can be 1 or 2.
k_j	constant voltage across the arc used to simulate arcing resistance, where j will be a, b, and c depending on phase opening.

w_o	nominal system angular frequency, radian/second
λ_a	flux linkages - phase a
λ_{fd}	flux linkages - direct axis field winding
λ_{kd}	flux linkages - direct axis damper winding
λ_{fq}	flux linkages - quadrature axis field winding
λ_{kq}	flux linkages - quadrature axis damper winding
θ	the angle of the direct axis with respect to phase a
s	the angle between the direct axis and the synchronously rotating reference of the infinite busbar.

Matrix definition

The inductance matrix L is a 7x7 matrix and consists of X and Y.

$$
X = \begin{bmatrix}
-x_{aa} & -x_{ab} & -x_{ac} & x_{afd}\cos\theta \\
-x_{ab} & -x_{bb} & -x_{bc} & x_{afd}\cos(\theta - \frac{2}{3}\pi) \\
-x_{ac} & -x_{bc} & -x_{cc} & x_{afd}\cos(\theta + \frac{2}{3}\pi) \\
-x_{afd}\cos\theta & -x_{afd}\cos(\theta - \frac{2}{3}\pi) & -x_{afd}\cos(\theta + \frac{2}{3}\pi) & x_{fdl} + x_{afd} \\
-x_{afd}\cos\theta & -x_{afd}\cos(\theta - \frac{2}{3}\pi) & -x_{afd}\cos(\theta + \frac{2}{3}\pi) & x_{afd} \\
x_{afq}\sin\theta & x_{afq}\sin(\theta - \frac{2}{3}\pi) & x_{afq}\sin(\theta + \frac{2}{3}\pi) & \emptyset \\
x_{afq}\sin\theta & x_{afq}\sin(\theta - \frac{2}{3}\pi) & x_{afq}\sin(\theta + \frac{2}{3}\pi) & \emptyset
\end{bmatrix}
$$

$$
Y = \begin{bmatrix}
x_{afd}\cos\theta & -x_{afq}\sin\theta & -x_{afq}\sin\theta \\
x_{afd}\cos(\theta - \frac{2}{3}\pi) & -x_{afq}\sin(\theta - \frac{2}{3}\pi) & -x_{afq}\sin(\theta - \frac{2}{3}\pi) \\
x_{afd}\cos(\theta - \frac{2}{3}\pi) & -x_{afq}\sin(\theta + \frac{2}{3}\pi) & -x_{afq}\sin(\theta + \frac{2}{3}\pi) \\
x_{afd} & \emptyset & \emptyset \\
x_{kdl} + x_{afd} & \emptyset & \emptyset \\
\emptyset & x_{fql} + x_{afq} & x_{afq} \\
\emptyset & x_{afq} & x_{kql} + x_{afq}
\end{bmatrix}
$$

$x_{aa} = x_{aao} + x_{aa2}\cos 2\theta$, $\quad x_{ab} = -x_{abo} + x_{aa2}\cos(2\theta - \frac{2}{3}\pi)$

$x_{bb} = x_{aao} + x_{aa2}\cos(2\theta + \frac{2}{3}\pi)$, $x_{bc} = -x_{abo} + x_{aa2}\cos 2\theta$

$$x_{cc} = x_{aao} + x_{aa2}\cos(2\theta - \frac{2}{3}\pi), \quad x_{ca} = -x_{abo} + x_{aa2}\cos(2\theta + \frac{2}{3}\pi)$$

$$x_{aao} = \frac{x_d + x_q + x_o}{3}, \quad x_{abo} = \frac{x_d + x_q - 2x_o}{6}, \quad x_{aa2} = \frac{x_d - x_q}{3}$$

$$r_s = \begin{bmatrix} r_a + r_e & r_e & r_e \\ r_e & r_a + r_e & r_e \\ r_e & r_e & r_a + r_e \end{bmatrix}$$

$$r_r = \begin{bmatrix} -r_{fd} & \emptyset & \emptyset & \emptyset \\ \emptyset & -r_{kd} & \emptyset & \emptyset \\ \emptyset & \emptyset & -r_{fq} & \emptyset \\ \emptyset & \emptyset & \emptyset & -r_{kq} \end{bmatrix}$$

$$R = \begin{bmatrix} r_s & \emptyset \\ \emptyset & r_r \end{bmatrix}$$

$$C = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 & \emptyset \\ \emptyset & 1 & -1 \\ -1 & \emptyset & 1 \end{bmatrix} \qquad C^t = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & \emptyset & -1 \\ -1 & 1 & \emptyset \\ \emptyset & -1 & 1 \end{bmatrix}$$

$$R_T = \begin{bmatrix} r_T & \emptyset & \emptyset \\ \emptyset & r_T & \emptyset \\ \emptyset & \emptyset & r_T \end{bmatrix} \qquad L_T = \begin{bmatrix} l_T & \emptyset & \emptyset \\ \emptyset & l_T & \emptyset \\ \emptyset & \emptyset & l_T \end{bmatrix}$$

$$R_L = \begin{bmatrix} r_L & \emptyset & \emptyset \\ \emptyset & r_L & \emptyset \\ \emptyset & \emptyset & r_L \end{bmatrix} \qquad L_L = \begin{bmatrix} l_L & \emptyset & \emptyset \\ \emptyset & l_L & \emptyset \\ \emptyset & \emptyset & l_L \end{bmatrix}$$

$$\bar{i}_s = (i_a, i_b, i_c)^t$$

$$\bar{e}_s = (e_a, e_b, e_c)^t$$

$$\bar{v} = (v_a, v_b, v_c)^t$$

$$\bar{e}_T = (e_{Ta}, e_{Tb}, e_{Tc})^t$$

$$\bar{i}_T = (i_{Ta}, i_{Tb}, i_{Tc})^t$$

$$\bar{i}_r = (i_{fd}, i_{kd}, i_{fq}, i_{kq})^t$$

$$\bar{e}_r = (e_{fd}, \emptyset, \emptyset, \emptyset)^t$$

$$\bar{e}_L = (e_{La}, e_{Lb}, e_{Lc})^t$$

$$\bar{i}_L = (i_{La}, i_{Lb}, i_{Lc})^t$$

$$\bar{\lambda}_s = (\lambda_a, \lambda_b, \lambda_c)^t$$

$$\bar{\lambda}_r = (\lambda_{fd}, \lambda_{kd}, \lambda_{fd}, \lambda_{kq})^t$$

$$\bar{\lambda} = (\lambda_a, \lambda_b, \lambda_c, \lambda_{fd}, \lambda_{kd}, \lambda_{fq}, \lambda_{kq})^t$$

$\overline{e}_1 = (e_{a1}, e_{b1}, e_{c1})^t$

$\overline{i}_1 = (i_{a1}, i_{b1}, i_{c1})^t$

$\overline{k} = (k_a, k_b, k_c)^t$

$\overline{K} = (k_a, k_b, k_c, \emptyset, \emptyset, \emptyset, \emptyset)^t$

$\overline{E} = (e_a, e_b, e_c, e_{fd}, \emptyset, \emptyset, \emptyset)^t$

$\overline{I} = (i_a, i_b, i_c, i_{fd}, i_{kd}, i_{fq}, i_{kq})^t$

$\overline{V} = (v_a, v_b, v_c, e_{fd}, \emptyset, \emptyset, \emptyset)^t$

$p\overline{I} = (pi_a, pi_b, pi_c, pi_{fd}, pi_{kd}, pi_{fq}, pi_{kq})^t$

$p\overline{i}_s = (pi_a, pi_b, pi_c)^t$

$\overline{E}_T = (e_{Ta}, e_{Tb}, e_{Tc}, e_{fd}, \emptyset, \emptyset, \emptyset)^t$

$\overline{I}_T = (i_{Ta}, i_{Tb}, i_{Tc}, i_{fd}, i_{kd}, i_{fq}, i_{kq})^t$

$p\overline{I}_T = (pi_{Ta}, pi_{Tb}, pi_{Tc}, pi_{fd}, pi_{kd}, pi_{fq}, pi_{kq})^t$

$p\overline{i}_T = (pi_{Ta}, pi_{Tb}, pi_{Tc})^t$

U_4 is a 4 X 4 unit matrix, $\emptyset_4$ is a 4x4 zero matrix.

X_{1sr}, X_{2sr}, $C_T^{\,t}$, R'_{TL}, L'_{TL} are 7 X 7 matrices, X can be F or S.

$$X_{1st} = \begin{bmatrix} X_{1s} & \emptyset \\ \emptyset & U_4 \end{bmatrix}$$

$$C_T^t = \begin{bmatrix} C^t & \emptyset \\ \emptyset & U_4 \end{bmatrix}$$

$$X_{2st} = \begin{bmatrix} X_{2s} & \emptyset \\ \emptyset & U_4 \end{bmatrix} \quad L'_{TL} = \begin{bmatrix} F_{2s}L_L + L_T & \emptyset \\ \emptyset & \emptyset_4 \end{bmatrix}$$

$$R'_{TL} = \begin{bmatrix} F_{2s}R_L + R_T & \emptyset \\ \emptyset & \emptyset_4 \end{bmatrix}$$

$$Ce_T = \begin{bmatrix} Ce & \emptyset \\ \emptyset & U_4 \end{bmatrix}$$

$$M_{ist}^{-1} = \begin{bmatrix} M_{is}^{-1} & \emptyset \\ \emptyset & U_4 \end{bmatrix}$$

i can be 1 or 2.

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Chapter 1

Introduction

This chapter deals with the research background, showing why my research has been done and describing my research approaches.

1.1 Objectives of research

Discussions with the CEGB have pointed to the need for a computer program which will calculate the relay settings, plot the coordination curves and check the validity of the settings by using a transient model. Part of the research will develop this program and be concerned principally with auxiliary power supply systems in power stations and industrial power systems. In such systems the transient d.c. and the a.c. decrement components may be significant and their effect on relay performance should be considered. In addition the heat generated at junctions and the thermal ratings of cabling is dependent on the $(\text{transient current component})^2$ and should be included in design considerations. To meet the more complex operating conditions, transient studies and the inclusion of the characteristic of auxiliary devices, such as current transformers, must be included. My research restricts consideration to the case when the relay current contains a negligible harmonic component only. This can be justified because , in general, the waveforms of short-circuit current will approach a pure sine wave; and also the current transformer is assumed not to be saturated. The research, however, shows how the current transformer (CT) saturation can be included and a brief investigation indicates that harmonics affect the relay operations. Another main objective of the research is to study how the circuit breaker opening affects the translation to a current zero.

1.2 Research background

Inverse Definite Minimum Time Overcurrent Protection is widely used in the main and backup protection of power stations. The inverse definite minimum time (IDMT) relay is relatively cheap, very reliable, and robust.

IDMT overcurrent protection to a radial system has been applied for more than fifty years. The philosophy associated with protection system designs is that the circuit breaker which trips first is the one on the source side and nearest to the fault. In complex power systems where a fault may occur at several intermediate points between relays, it is necessary for just one relay to be selected and operated first. It is possible to obtain such discrimination by varying the current and time settings of each IDMT relay.

Coordination of protective devices by manual methods is a tedious process which is time consuming and does not allow for the numerous changes which occur between design data and data obtained during final commissioning. Non-manual methods such as computers can provide immediate feedback by producing graphical displays of all the protective device curves together with the boundaries on the coordination ranges for the devices being considered. Various protection devices, ratings, coordination margins, and coordination philosophies can be easily investigated. The usual method of plotting coordinated curves is to use a template curve.Since the selection of device parameters follows well-defined rules, it is possible to develop computer programs that will select the settings and trace the curves. As a result,a number of computer programs have been written for coordinating these devices.But they only provide numerical output, so manual procedures are still required to obtain the grading curves.More recently, computer graphic routines have been incorporated into a coordination program to provide a graphical display [107,108]. To use the procedures described in [108], a large computer and considerable computation time is required. In [107], a small computer and reduced computation time are achieved.One of the main reasons for the differences between the approaches of these authors is due to the way the relay characteristics are represented. The assumption usually made in the study and application of protective relay schemes is that the transient component of fault currents can be neglected. This assumption may not be true and the transient effect has to be taken into account. A method for finding the operating time of relays under transient conditions is described in [102], this method is particularly useful when the relay setting is unknown. It requires the slope of the relay characteristic to be computed.

A study of circuit breaker opening is required due to the fact that nuclear power stations use circuit breakers at the generator terminals. These circuit breakers have to interrupt highly inductive currents with X/R ratios of typically 100/1; and because of extreme offset due to large dc components, there are at least one or possibly two phases in which the current does not reach zero for many cycles. The circuit breakers are not designed to interrupt currents which do not pass through a current zero and hence the operating time of protective relays has to be delayed in some cases(e.g. Delay in relay operating occurs on the LV side of a generator transformer where the inductive current is high). This phenomenon prevents normal circuit breaker opening, with possible damage to the system. In practice, for a three-phase short circuit, at least one phase current will pass through zero within the first cycle and the appropriate circuit breaker will first interrupt the current in this phase. A reduction of the dc component is then achieved in the phase with the maximum dc offset and so the zero transition will occur much earlier and faster protection can be obtained [33,34]. Simulation of circuit breaker opening was described briefly in [92], but it did not include a transformer, so that the modified matrices introduced in that paper were easier to find. With a delta star transformer connected between the generator and the transmission line, the conditions will become more complicated. In [95], the authors simulated the open phases by means of infinite inductance.

1.3 Approaches in research

In [107] the representation of relay characteristics is by polynomial functions but this involves excessive computation time and only coordination of relays are considered. In my research they are represented by hyperbolic functions [106] which can minimize computation time. The coordination of fuses are also involved in the discrimination of protective devices. [107] was published after I had completed this part of my research.

The relay operating time under transient conditions is obtained by using a step-by-step method, since this method does not require the slope of the relay characteristic to be known.

The heat produced during the fault time can be calculated from the already computed short circuit current and the time of relay operation as the current squared multiplied by time (i^2t).

To study the delay in reaching a current zero, the rate of change of rotor current with respect to time cannot be ignored otherwise there will be no d.c. component in the calculated stator currents and hence the study would have no practical validity. The (d,q) method is usually adopted because it saves computing time as compared with the phase coordinate method. But in the (d,q) method, it is assumed that the effects due to the change of rotor currents is so small that they can be neglected; and computing time can be saved. If the change of rotor currents is not ignored, the (d,q) method does not provide any significant advantage when compared with the phase coordinate method.

It is thought that with the introduction of fault, connection and switching matrices, the transient analysis can be made more flexible. Arc resistances [35] may be taken into account by assuming that there is a constant voltage [117] across the arc during the period between the occurrence of the fault and the instant at which currents go through zero.

1.4 Work layout

This chapter has explained the reason for doing this research. In chapter 2, a typical auxiliary overcurrent protection system is examined. The criteria that are required for protection are described. Chapter 3 deals with the development of a coordination program and plotting program for fuses and relays. Optimisation of relay operating time under different conditions are considered. After the settings have been calculated under the steady state conditions, their validity can be assessed under transient conditions by carrying out a fault study. The radial system is basically composed of circuits as shown in figure 1.1. A detailed analysis on this system is carried out in chapter 4. The system is analysed under different fault conditions with the inclusion of circuit breaker opening using matrix methods. In the same chapter programs are also developed for calculating the relay operating time under transient conditions. Chapter 5 deals with current transformer modelling, where a simple model is used to check whether saturation exists or not. In chapter 6, experiments describe how the d.c. and distorted currents affect the operation of IDMT relays. Results obtained from the developed programs and experimental work are given in chapters 6&7. Finally, the conclusions are presented in chapter 8 together with the achievements of the project and suggestions for further work.

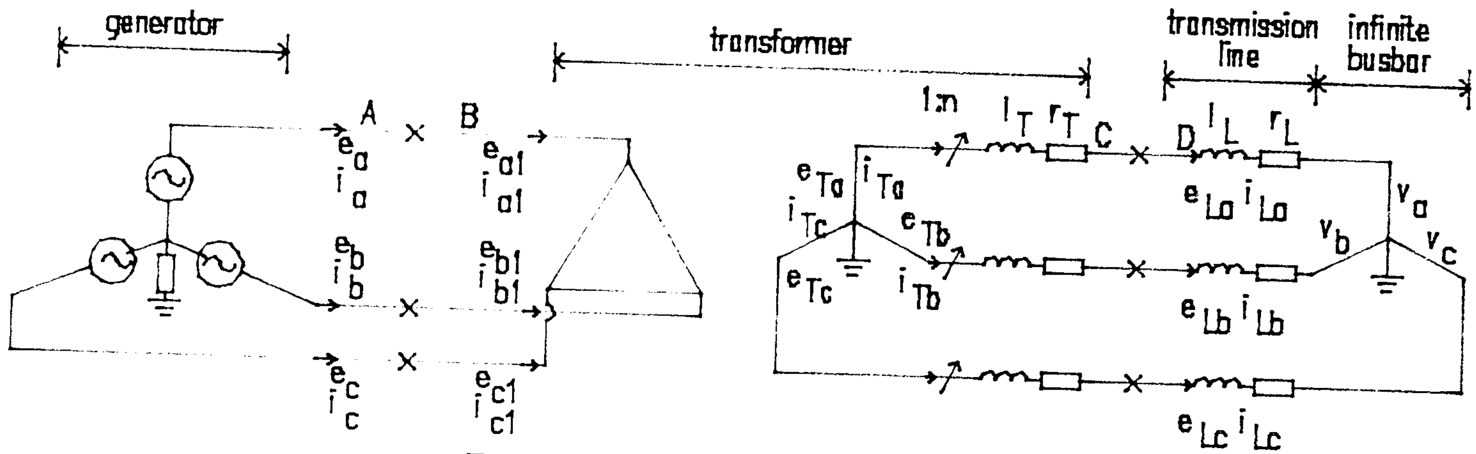


Figure 1.1 Circuit diagram for fault analysis

Chapter 2

Overcurrent protection

This chapter deals with three topics. The first one is an examination of both relays and an auxiliary overcurrent protective system. The second is a description of the discrimination criteria used for relays and fuses. The last one is a look at the manual method which is usually used for protective devices coordination.

2.1 Inverse time overcurrent relays

Inverse time overcurrent relays are avaliable with a wide variety of characteristics, providing a range that is suitable for all forms of inverse time graded protection of transformers, machines or feeders. They all have the following advantages:

(a) High torque,ensuring consistent timing even under adverse conditions.

(b) Very low overshoot.

(c) Simple construction.

2.1.1 Types of relays

The following three types are usually used in overcurrent protection. They are the inverse time, the very inverse time and the extremely inverse time relays.

(a) The inverse time relay gives selective phase and earth fault protection on time graded systems, ensuring that the minimum number of circuit breakers is tripped to clear a faulted section.

(b) The very inverse time relay provides greater time selectivity than the inverse time relay and is used where the fault current at any point does not vary too widely with system conditions.

(c) The extremely inverse time relay has an even steeper time-current characteristic than the very inverse time relay, permitting a graded setting in conjunction with fuses. Its characteristics most closely match the heating characteristics of the pro~~c~~tected apparatus.

2.1.2 Construction and operation

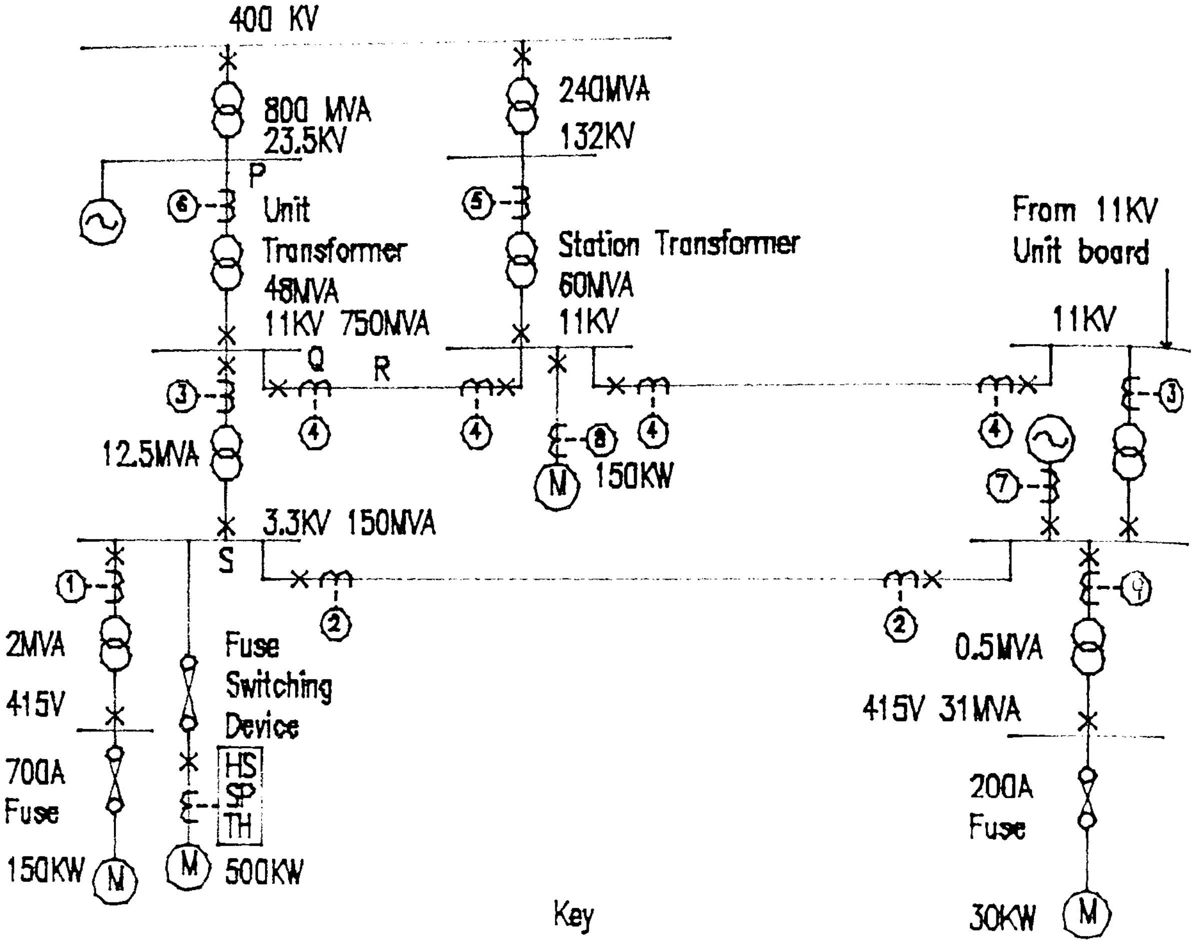
The relays mentioned above have an aluminium disc which is rotated by the torque exerted upon it by a C-shaped pole electromagnet. This electromagnet normally has copper shading rings. The relay disc is shaped so that as it rotates, the driving torque increases and offsets the changing restraining torque of the return spring. This feature, together with the high level of torque produced by the electromagnet, ensures good contact pressure even at currents near pickup.

Damping of the disc movement is by a removable permanent magnet of high retentivity steel. Sometimes, in addition to this, a damping flux is provided by a third pole on the electromagnet so that when the fault current is interrupted, the relay resets more rapidly.

A unique method of winding the operating coil ensures that the time/current characteristics are identical on each of the seven current taps. Selection of the required current setting is by means of a plug setting bridge which has a single insulated plug. When the plug is withdrawn from the bridge the maximun current tap is automatically connected, allowing the setting to be changed under load without risk of open circuiting the current transformers. Adjustment of the relay operating time is by movement of the disc backstop which is controlled by rotating a wheel at the base of the graduated time multiplier scale.

There are two general ways of obtaining a torque in induction type relays. One is by arranging the physical position of the flux-producing poles so that the flux of the pole reacts with that portion of the eddy currents induced by a second pole lying in the field of the first. The second method uses the shaded-pole principle. A short circuited turn around a part of the pole causes the flux under the turn to lag the flux in the rest of the pole. This results in a sweeping of the flux across the air gap under the pole, which induces eddy currents in the disc. The reaction of the induced currents with the pole flux produces the torque.

The torque of these relays is proportional to the sine product of the two magnetic fluxes cutting the disc. If both fluxes are produced by the same quantity of current, the torque is also proportional to the square of the current, i.e. $T = KI^2$, K decreases with increase of current owing to magnetic saturation and is used to give a definite minimum time feature to the characteristics of the overcurrent relays.

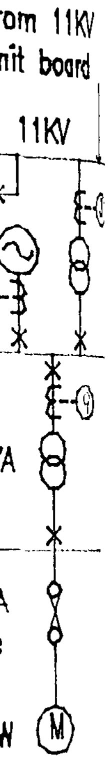


Key

HS : High-set SP : Single phasing
TH : Thermal × : Circuit breaker
M : Motor ~ : Generator
Current transformer

relays with the same label will have the same settings

Figure 2.1 Simplified power station overcurrent protection diagram



2.2 High set instantaneous overcurrent relay

The overcurrent relays can be provided with a high set instantaneous overcurrent unit. If there is no high set element then the grading is carried out at the maximum fault level. But if there is a high set element then the grading is carried out at the high set value. The important difference between them is that both the tripping time at high fault level and the time setting at all stations are reduced.

2.3 System description

A system which is typical of most CEGB power stations with voltage levels at 400 KV to 415 V is shown in figure 2.1. A description of the system and method of operation is given in [111]. This chapter is concerned with the grading between protective devices such as fuses and relays. The 415 V circuits are mainly protected by HRC fuses, the 3.3 KV circuits by fuse switching devices or circuit breakers while the 11 KV circuits only use circuit breaker control. In order to provide fast clearance of faults on the HV side of the transformer, all overcurrent relays have high set, overcurrent elements with low transient overreach. The elements must be set above the maximum throughput capability of the transformer in order to maintain discrimination with the relays on the LV side. All motors switched by the circuit breakers have instantaneous high set overcurrent elements to protect cables and connections to the motor. The elements are set above the motor starting current to allow healthy motor starting and give adequate short circuit protection to the supply cable and motor itself.

Fuses are used in the 415 V circuits or in circuits with lower voltage levels because they operate faster and also reduce effective fault level compared with circuit breakers of similar rating and breaking capacity. The most common fuse ratings are 25 and 35 MVA.

Circuit breakers are used at the higher voltage level because it takes less time to reclose a circuit breaker than to replace a fuse. In general, protective devices are easily graded with each other if they have similar operating characteristics. Since just the extremely inverse relay has a similar operating characteristic to that of a fuse, only it can provide the best means of coordination with the fuse. The relatively long operating time characteristic of the relay also makes it a suitable device for protecting circuits which are subjected to peak currents on switching, such as supplying power to pumps. In the 11KV circuits, standard inverse relays are used.

2.4 Factors influencing settings

The following factors will influence the coordination settings. They are the time multiplier setting and the pickup value.

2.4.1 Time grading

The time interval between the operation of two adjacent relays depends on the relay performance and the operating speed of the circuit breakers.

If we assume that each inverse time overcurrent relay complies with error class E 7.5 defined as normal British practice in [109], then the time interval t' required between relay and relay can be represented by the following relationship:

t' = Ø.25t + Ø.25 seconds, where t is the time for the first protective device to operate. The variable time takes into account the effects of temperature, frequency and departure from reference setting. Although the normal limits of error for an E 7.5 relay are 7.5%, a total effective error of 15% should be used. A further 1Ø% should be added for the overall current transformer error, hence the variable time value can be obtained. The fixed time value is made up of Ø.1s for the fault current interrupting time of the circuit breaker, Ø.Ø5s for the relay overshoot time and Ø.1s for the safety margin [11Ø].

As far as the time interval t' between the relay and the fuse is concerned, the following expression is used.

t' = Ø.4t + Ø.15 seconds, where t is the nominal operating time of a fuse.

The result is also based on the previous criteria, except that there is no circuit breaker time. Hence the fixed time value is Ø.15s. The variable time has been based on a 3Ø% error for the fuse and 1Ø% for the relay.

2.4.2 Current grading

Assuming a safety margin of 2Ø% [1Ø9,11Ø] to allow for relay errors and a further 1Ø% for variations in the system, at least a factor of 1.3 is required between the pickup values of relays. A factor of 2 is required between the normal ratings of fuses and a factor of 3 is required between the pickup value of a relay and the normal rating of a fuse.

2.4.3 Time multiplier setting (TMS)

The time setting must be high enough to prevent operation with starting and inrush currents but low enough to trip the breaker when the short circuit withstand rating of the protected cables or equipment is exceeded. The contact travel should not be made too small, in order to avoid the possibility of operation by mechanical shock.

2.4.4 Pickup setting

The pickup current setting should be high enough to permit the continuous maximum load current to flow but low enough to provide for their overload protection, and below the short circuit current level for a fault at the most remote downstream protective device under minimum fault conditions with the fewest number of generators and parallel lines in operation.

2.5 Discrimination criteria

The coordination of protective devices is determined by calculation from the lowest voltage level and related through intermediate voltage levels back to the power source. Time discrimination will be explained in terms of branches. A branch is defined as a circuit with protective elements, for example in figure 2.1, there is a single branch between the 23.5 KV and 11 KV busbars. The criteria described in this section may only be applied to the case where the operating time difference is a minimum at the maximum fault level.

The single branch representation is assumed to apply to the case where a busbar has only one infeed circuit, but one or many outfeed circuits. It requires discrimination between the infeed and outfeed circuits.

(1) If a fault occurs at F (see figure 2.2) then relay B has to operate but not relay A. Relay A operates only when there is a fault at G or if it is used for backup protection when relay B or its controlled circuit breaker fails to operate. These conditions also apply when relay B is replaced by a fuse.

(2) Consider the two protective devices, B and C, which are connected in parallel (see figure 2.3). B and C can be either a relay or a fuse. Three different cases can be considered.

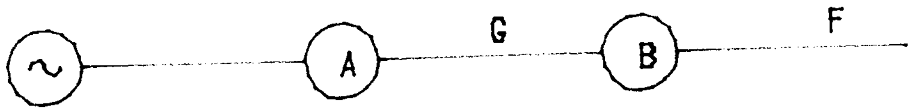
(a) If B and C are both fuses, then relay A will grade with the fuse having the largest normal current rating.

(b) If B and C are both relays , then relay A will be current graded with the one with the higher pickup value.

(c) If B is a relay and C is a fuse, then whichever of these has the highest operating time at the maximum fault level at which relay A must discriminate, will be chosen for coordination purposes.

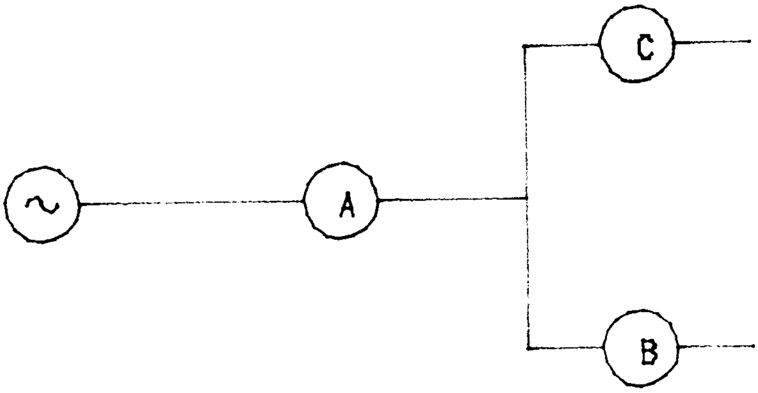
(3) Figure 2.4 represents part of the circuit shown in figure 2.1. To obtain discrimination between relays, three different conditions need consideration:

(a) During a starting process[111], there will be power flow from R to S; therefore relay 4 has to be discriminated from relay 3.



Typically A could be relay 1 and B could be a 200A fuse

Figure 2.2



Typically A could be relay 3, B could be a fuse switching device and C could be relay 1

Figure 2.3

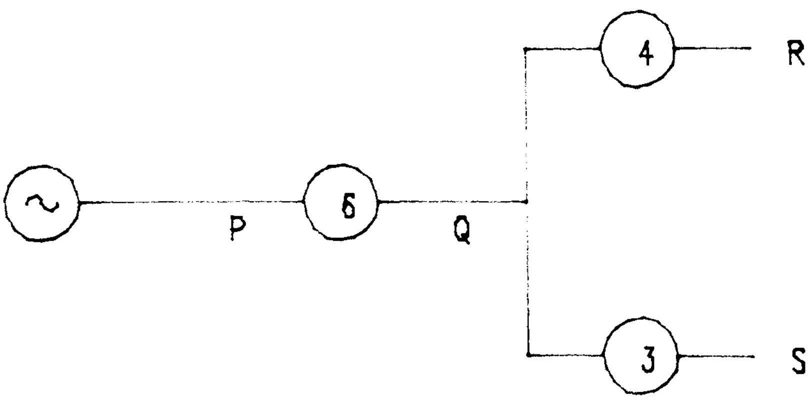


Figure 2.4

Simplified relays and fuses connection diagrams related to figure 2.1

(b) A current will flow from P to R if a fault occurs at R, therefore coordination is required between relay 6 and 4.

(c) During the normal operating condition, there is power flow from P to S, therefore relay 6 must discriminate from relay 3.

If the three cases are combined then relay 3 must discriminate from relay 4 which in turn discriminates from relay 6.

(4) No discrimination between relay 3 and relay 4 is required if there is no current flow between R and S.

The multiple branches representation is assumed to apply to the case where a busbar has more than one infeed circuit and one or more outfeed circuits. To obtain discrimination between infeed and outfeed circuits, it is required that the relay in the outfeed circuit which produces the longest operating time with any relay in the infeed circuits is chosen for grading with all the relays in the infeed circuits.

2.6 The template method

The template method has been used for a numbers of years [114] for graphically coordinating overcurrent relays. An overcurrent relay characteristic curve is a plot of contact closing time versus the magnitude of operating current. The setting of the time dial positions the fixed contact of the relay, and thereby sets the angle the induction disc requires to travel before the moving contact associated with it touches the fixed contact. If it is assumed that the time-current curves are related by ratios of time, then on a logarithmic time scale, a specific ratio of time values spans the same distance along the time axis regardless of where these values are. For example, the distance between Ø.1 and Ø.2 second ordinates is the same as the distance between 1.5 and 3 seconds. Therefore, with the logarithmic time scale, changes in the time setting multiplier shift the curves in a direction parallel to the time axis but do not otherwise change the curves.

The same argument can be applied to the current scale. Since the relay operation is independent of pickup, it is convenient to plot characteristic curves with the current axis dimensioned in multiples of pickup. On a logarithmic scale a given ratio of currents will span the same distance along the axis regardless of where the currents occur. A change in pickup current values will shift the time-current curves in a direction parallel to the current axis but does not change the curve.

It is this property which forms the basis for the use of templates to coordinate relays. An example is given in [114] of how to use this method for finding the relay settings.

2.7 Summary

The discrimination criteria for overcurrent protection and the manual method used to obtain the coordination settings and plot the grading curves have been discussed. This process is tedious and non-manual methods involving computers can achieve the coordination of protective devices faster and easier. In the next chapter, the development of software for these purposes is explained in detail.

Chapter 3

Coordination and plotting programs

This chapter deals with the development of programs that can be used for relay and fuse coordination and also with characteristics of these protective devices which can be plotted by using computers. In order to use a computer, the template curve has to be translated into an equation and, of course, the simpler the equation, the easier it is for programming. The decision of whether to use a mathematical representation for the device curves, or to store the device curves in the form of a table in memory or on disc, was based on the consideration of the accuracy and storage requirments of the methods used.

In the past, operating characteristics of overcurrent relays expressed in the form of either polynomial or hyperbolic equations have been used in computer programs for determining relay settings.

3.1 Representation of relay characteristics

3.1.1 The polynomial model

Albrecht, et al.[112] suggested that the operating characteristics of an overcurrent relay can be expressed as a polynomial in two variables, the time multiplier setting and multiples of pickup.

$$T = [\sum_{j=m}^{n} \sum_{i=o}^{p} a_{ji} (TMS)^j (I)^i]^k \text{ ------------ (1)}$$

where a_{ji}, k, m, n, o, and p are constants determined by each relay type.

Radke [113] expressed the logarithm of relay operating time as a logarithmic polynomial of I.

$$\log(T) = a_o + a_1 (\log I) + a_2 (\log I)^2 + \ldots\ldots \quad \text{---------}(2)$$

where I is the multiple of pickup. T is the relay operating time. a_o is a constant dependent upon the time multiplier setting, with I = 1. a_1, ... etc. are the curve characteristic coefficients.
Representation of a relay curve by (2) requires less storage than (1). The relay characteristics defined by these equations are not asymptotic to the minimum pickup current or to a minimum operating time when the fault current is high.

Sachdev, Singh and Fleming [104], examined the polynomial equations (3) to (7) for their suitability in representing characteristics of overcurrent relays.

$$\log(T) = a_o + \frac{a_1}{\log I} + \frac{a_2}{(\log I)^2} + \frac{a_3}{(\log I)^3} + \ldots \quad \text{-----}(3)$$

$$T = a_o + \frac{a_1}{I - 1} + \frac{a_2}{(I - 1)^2} + \frac{a_3}{(I - 1)^3} + \ldots\ldots \quad \text{----}(4)$$

$$T = a_o + \frac{a_1}{I - 1} + \frac{a_2 I}{(I - 1)^2} + \frac{a_3 I^2}{(I - 1)^3} + \ldots\ldots \quad \text{----}(5)$$

$$T = a_o + \frac{a_1}{I - 1} + \frac{a_2}{I^2 - 1} + \frac{a_3}{I^3 - 1} + \ldots\ldots \quad \text{-----}(6)$$

$$T = a_o + \frac{a_1}{I - 1} + \frac{a_2 I}{I^2 - 1} + \frac{a_3 I^2}{I^3 - 1} + \ldots \quad \text{----}(7)$$

The relay characteristics described by these equations are asymptotic to the pickup current and to a minimum operation time when the fault current is high.

The use of polynomial equations would require excessive complexity of computation, so the use of hyberbolic function is recommended whenever it is possible.

3.1.2 Hyberbolic equations

A.R.van C.Warrington [106] proposed a generalized hyperbolic equation for representing the relay contact closing time.

$$T = a_o + \frac{a_1 \text{ (TMS)}}{(I\ I_o)^n - I_o^m}$$

where I_o is the multiple of tap current at which pickup occurs, a_1 is a design constant of the relay, a_o is the minimum operating time of the relay.

3.2 Coordination between different types of protective devices

If relays are followed by their same types, then the operating time difference decreases as the MVA increases. But because the operating characteristics for the extremely inverse relay and the standard inverse relay when plotted on a graph have different slopes, the minimum time difference will usually occur within the range from the pickup value of the standard inverse relay to the maximum fault level or the high set value of the extremely inverse relay. For protection coordination, it is necessary to find out this minimum time difference, and a routine has been developed to search for the minimum time difference. It is based on the Golden Section routine which converges quickly, so that the number of computation steps required to obtain a specified accuracy is small. It was shown that there may not be just one minimum, and that the minimum can also occur at the maximum fault level of the extremely inverse relay. Therefore the routine has to be written in such a way that it can find out all the local minima. If no minimum occurs within the MVA range, then the time difference at the maximum fault level will be the minimum. If there is a minimum within the range, then all the minima will be compared with each other and the smallest of them will be chosen. The whole range of MVA is normalized and represented by one to make the Golden Section routine easier to write.

3.2.1 Minimun time difference searching routine

Figure 3.1a shows two relay characteristics where X1 is the pickup value for relay A and X2 is the maximum fault level for relay N. The MVA range (X1 to X2) is represented by one. The minimum time difference between these two relay characteristics is determined by a searching routine based on the Golden Section method. The Golden Section number is well established [126] to be Ø.618. Since the MVA range is represented by one, this number will be too large and lead to the possibility that some of the minima may be missed because the minima at the lower level may not be calculated. If the number is too small, a long computation time is required. To compromise, the number is chosen as Ø.Ø618. The whole MVA range is divided into sections as shown in figure 3.1b. In this figure, 1 and Ø represent X2 and X1 for the relays as shown in figu . 3.1a respectively. AØ to A1 is Ø.1 of

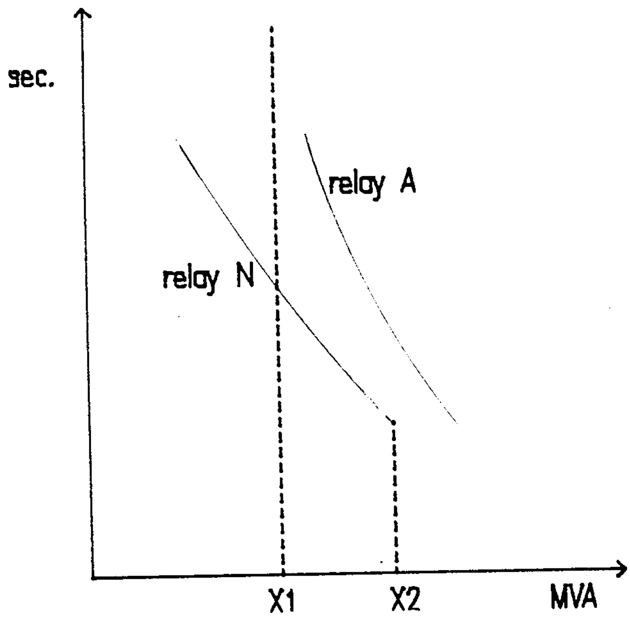


Figure 3.1a Relay coordination procedure

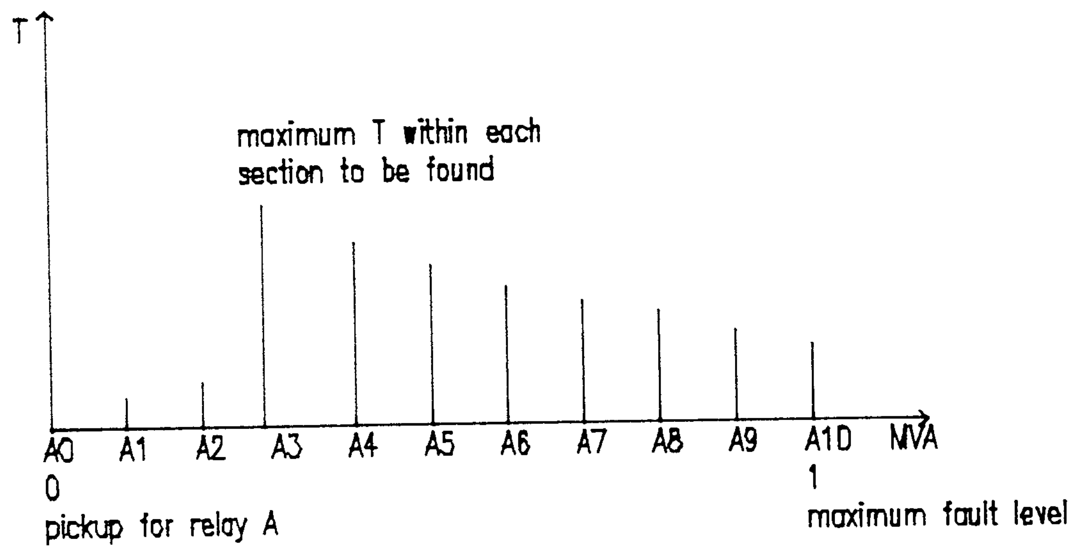


Figure 3.1b Searching for the minimum time difference

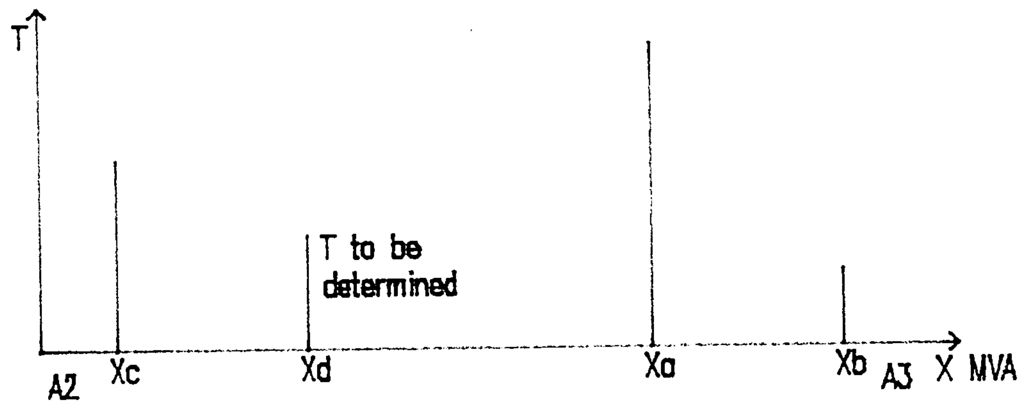


Figure 3.2 A typical example for the calculation of T

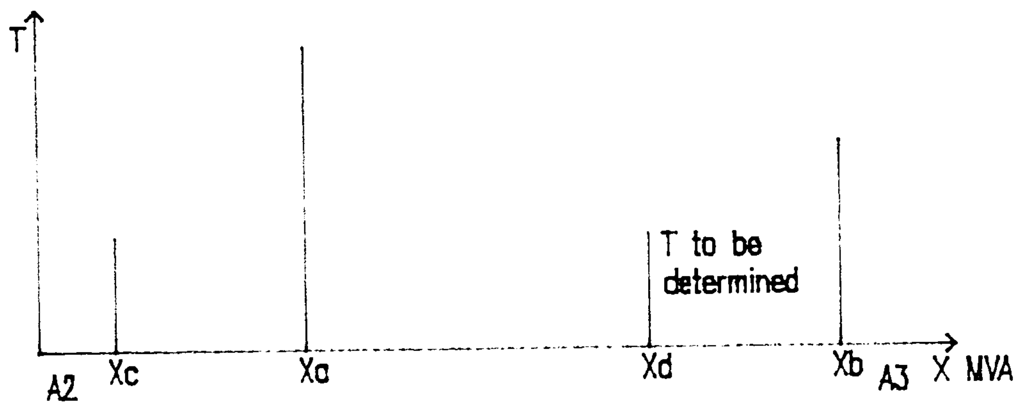


Figure 3.3 An alternative example for the calculation of T

the total range and A1 to A2 is also Ø.1 of the total range etc.. The smallest time difference within each section is compared. To simplify the algorithm, instead of finding the minimum time difference, the time difference is subtracted from a number, say 5Ø, and called T. So that now the maximum of T will be found. Let us call the fault level MVA which produces the corresponding T, X. Figures 3.2 and 3.3 show typical examples for the calculations of the maximum value of T. Whenever there is a T which is smaller than the previous one, it implies there is a local maximum. T will be arranged in an ascending order and Xa which corresponds to the present maximum T will be calculated. Two more X values are required, one greater than Xa, which we can call Xb; and one less than Xa, which we can call Xc. Xd is the value of X used to find the new value of T, where Xd = Xb + Xc - Xa. If the new T is greater than the previous T values, then this T is used as the maximum T and the procedure repeated. If T is not the biggest, then the previous maximum T is still used. The procedure is repeated until the required accuracy has been achieved or the number of iterations becomes excessive. After the minimum time difference has been found, it is necessary to check whether it is equal to the grading margin or not. If it is greater than the grading margin, the TMS of the standard inverse relay will be decreased by Ø.1. The TMS is checked and if it is within the range, the minimun time difference is found again. If the minimum time difference is less than the grading margin, the TMS is increased by Ø.1. For the case where the minimum time difference changes from greater than, to less than the grading margin or vice versa, the TMS will be increased or decreased by $\frac{0.1}{2^n}$ respectively. n is increased by one whenever such a change occurs.

3.3 Coordination Program

3.3.1 Program description

The coordination program for the radial system determines relay settings based on fault current information, fuse rating, relay types and grading interval. The grading interval is calculated from circuit breaker opening times, relay errors including overshoot and current transformer errors. Current grading as well as time grading is required for coordination of protective devices. The relay settings are calculated sequentially from the lowest voltage level up to the highest voltage level. Since the characteristics of fuses, extremely and standard inverse relays have different slopes when they are plotted on log-log paper, the minimum time difference between them can occur at any level within the range from the pickup of the upstream relay to the maximum fault level or high set level of the downstream relay. So when different types of protective device are required to coordinate with each other, the fault level at which the minimum time difference occurs must be determined. There is a difference from the case where the protective devices are of the same type, because then the minimum time difference will always occur at the maximum fault level. The program can be applied to the following cases:

(1) A standard inverse relay followed by another standard inverse relay.

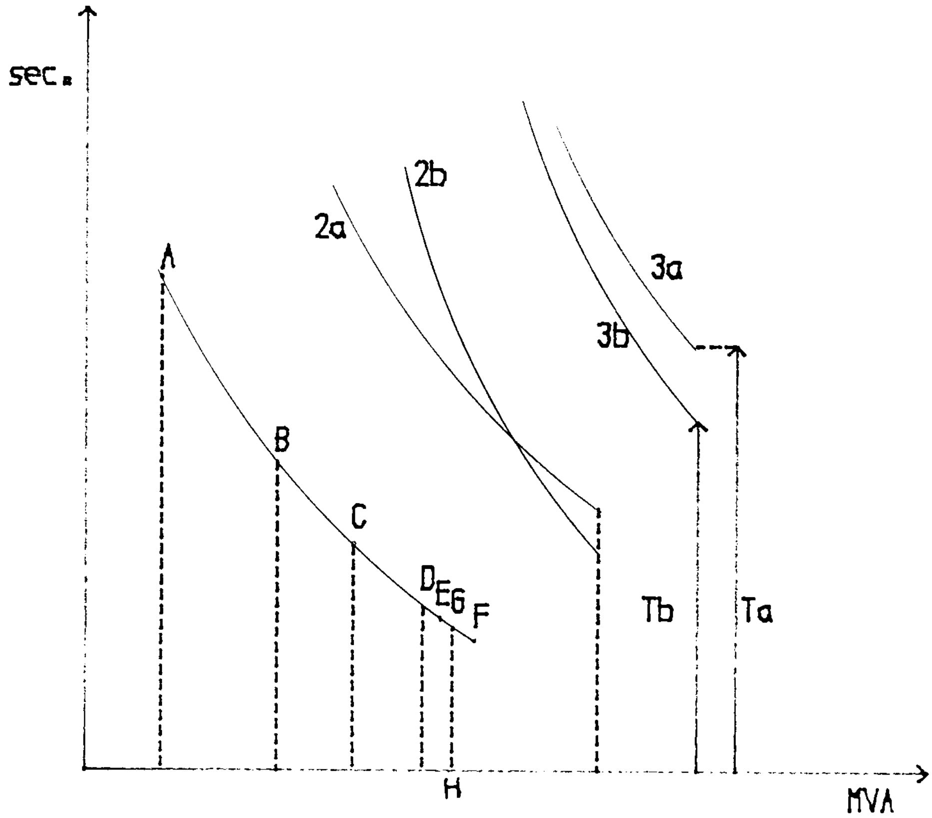


Figure 3.4 Effect of pickup on relay operating time

(2) An extremely inverse relay followed by another extremely inverse relay.

(3) An extremely inverse relay followed by a standard inverse relay.

The coordination program mainly consists of three subroutines. Two of them are used to find the TMS and pickup values for the relay nearer to the source when this relay is followed by a relay of the same type. One is for the standard inverse relay and the other for the extremely inverse relay. The third subroutine has the same purpose but is used for relays which are followed by a different type of relay.

If the calculated pickup value and TMS are outside the specified limit, a message will indicate whether there is difficulty in obtaining time grading or current grading, or both.

3.3.2 Program development

Consideration of figures 2.1 & 3.4 will assist in explaining how to obtain minimum relay operating time. For relay 3 to grade with relay 2, if relay 2 has the characteristic 2a, then relay 3 must have the characteristic 3a for proper grading; if relay 2 has the characteristic 2b, then relay 3 must have the characteristic 3b. It can be seen that relay 3 will operate faster (Ta > Tb) if it has the characteristic 3b, which means relay 2 should use a higher pickup value (2b). If a further relay is required to grade with relay 3 then it will also operate faster when relay 3 has characteristic 3b. This argument forms the basis for obtaining the shortest relay operating time.

The optimal operating time for the relays is obtained if all the pickup values for the relays are at their highest allowable values. If the coordination of relays cannot be achieved at these values, then they have to be changed by the operator for each stage. The process starts at the lowest voltage level. For example, the highest pickup value is assigned to the relay at the first stage. If coordination is obtained, then the highest pickup value for the relay at the second stage is assigned. If coordination is not achieved the relay tap position is decreased by one step, that is, if it is 200% in the previous case, then it will be reduced to 175%. This process is repeated until coordination is obtained. The relays at other stages are treated in the same way.

It is worth trying to coordinate the relays first before attempting the optimisation process. After the discrimination of relays is achieved, the optimum relay operating time can be obtained interactively as mentioned above.

The disadvantage of increasing the pickup value for each relay is that it may be difficult to obtain the correct current grading for other relays.

3.3.3 Algorithm

(a) If a fuse is in the first stage then its rating must be input to the program. If a relay is in the first stage then TMS, pickup value, maximum fault level and the device type are required.

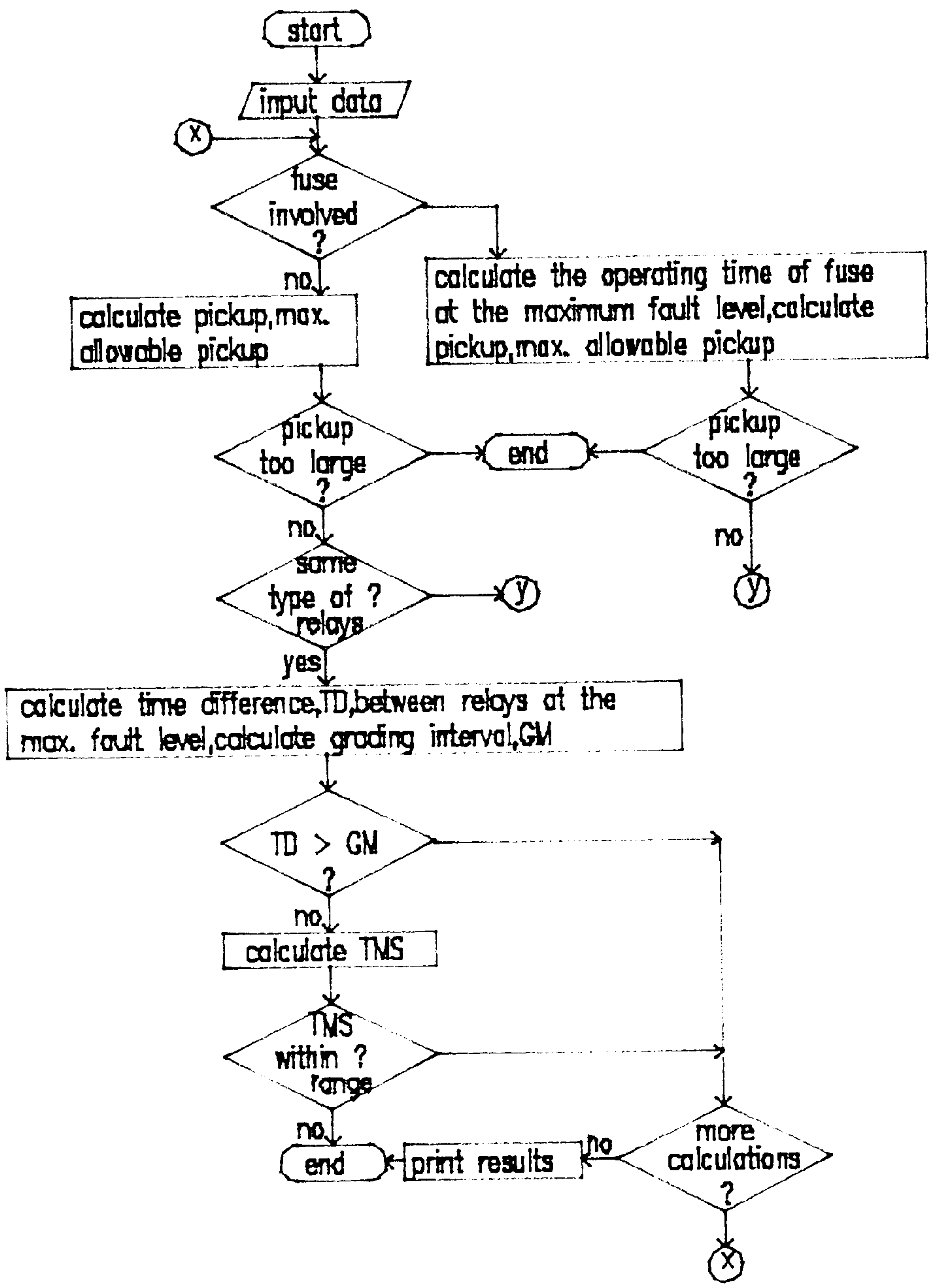


Figure 3.5a Flow diagram for coordination program

(b) Loading, maximum fault level, high set level, device type and maximum pickup level are then required for subsequent stages.

(c) The sum of the device types for the first two protective devices is calculated and used to select the appropriate subroutine for coordination.

(d) The pickup and TMS required for each relay is calculated, and if the time or current grading or both are outside the desired range, then a message is printed out.

(e) If changes are required then the program returns to (1), otherwise all the relay settings are printed.

3.3.4 Flow chart

A flow diagram of this program is shown in figures 3.5a & 3.5b and a listing of the program is shown in appendix J (** J1 **).

3.4 Curve plotting program

3.4.1 Program description

The plotting program produces a curve which represents the characteristic of the relay with the specified TMS and plug setting multiplier (PSM). The program consists of two parts; the first concerns the plotting of the log-log coordinates and the second consists of three subroutines, one for plotting the fuse characteristic, one for the standard inverse relay and one for the extremely inverse relay.

3.4.2 Program development

Only one curve for each relay type is required because the relay time-current curves, when plotted on a log-log scale are similar for all

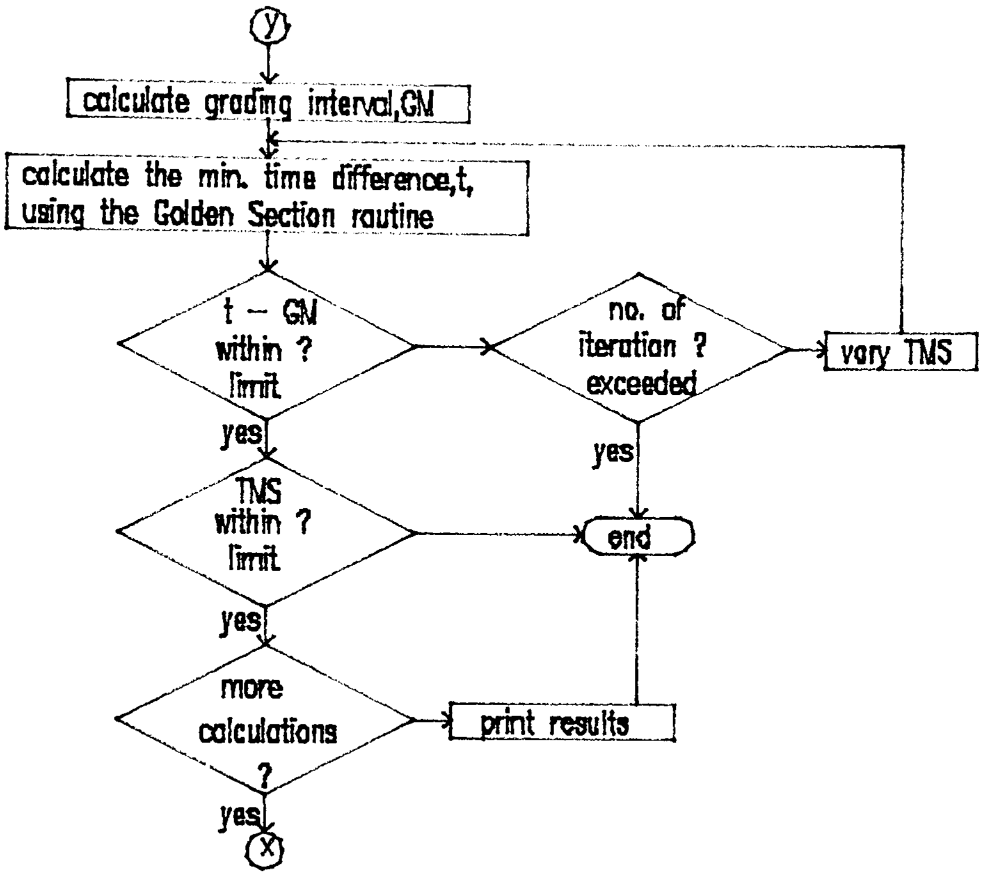


Figure 3.5b Continuation of flow diagram for coordination program

time settings. An adjustment of the relay tap setting shifts the curve horizontally; a change in time setting shifts the curve vertically. Thus a curve is shifted to the appropriate position by

$$x = \log_{10}(PSM) + \log_{10}(pu)$$

$$y = \log_{10}(t) + \log_{10}(TMS)$$

where pu is the pickup value of the relay, t is the operating time of the relay with the indicated PSM for TMS equal to one.

For small PSM, the relay characteristic has a large slope, but for large PSM, the slope is small. Because of this feature, it is found that the increment and the range of PSM can be increased as the characteristic is plotted. Referring to figure 3.4, plotting is divided into stages. Each stage will be assigned with a different increment in PSM. The number of stages and increments used in each stage will be obtained by trial and error. In practice, a set of increments has been found that enables satisfactory accuracy and speed of plotting. Plotting starts at A with a specified increment in PSM. When point B is reached, a new increment of PSM is used for the stage from B to C. This new increment will be bigger than the previous one. The process is repeated down to D. At D if the same increment of PSM is used, the calculated PSM becomes greater than the maximum PSM, specified by the maximum fault level or the high set value, and the curve DF will be plotted where F has a fault level higher than the maximum fault level. To overcome this problem 50% of the increment is subtracted from the PSM. If the new value of PSM is less than the maximum PSM then the point is plotted, for example, DE is plotted. The increment is updated and 50% of it is added to the new PSM. This

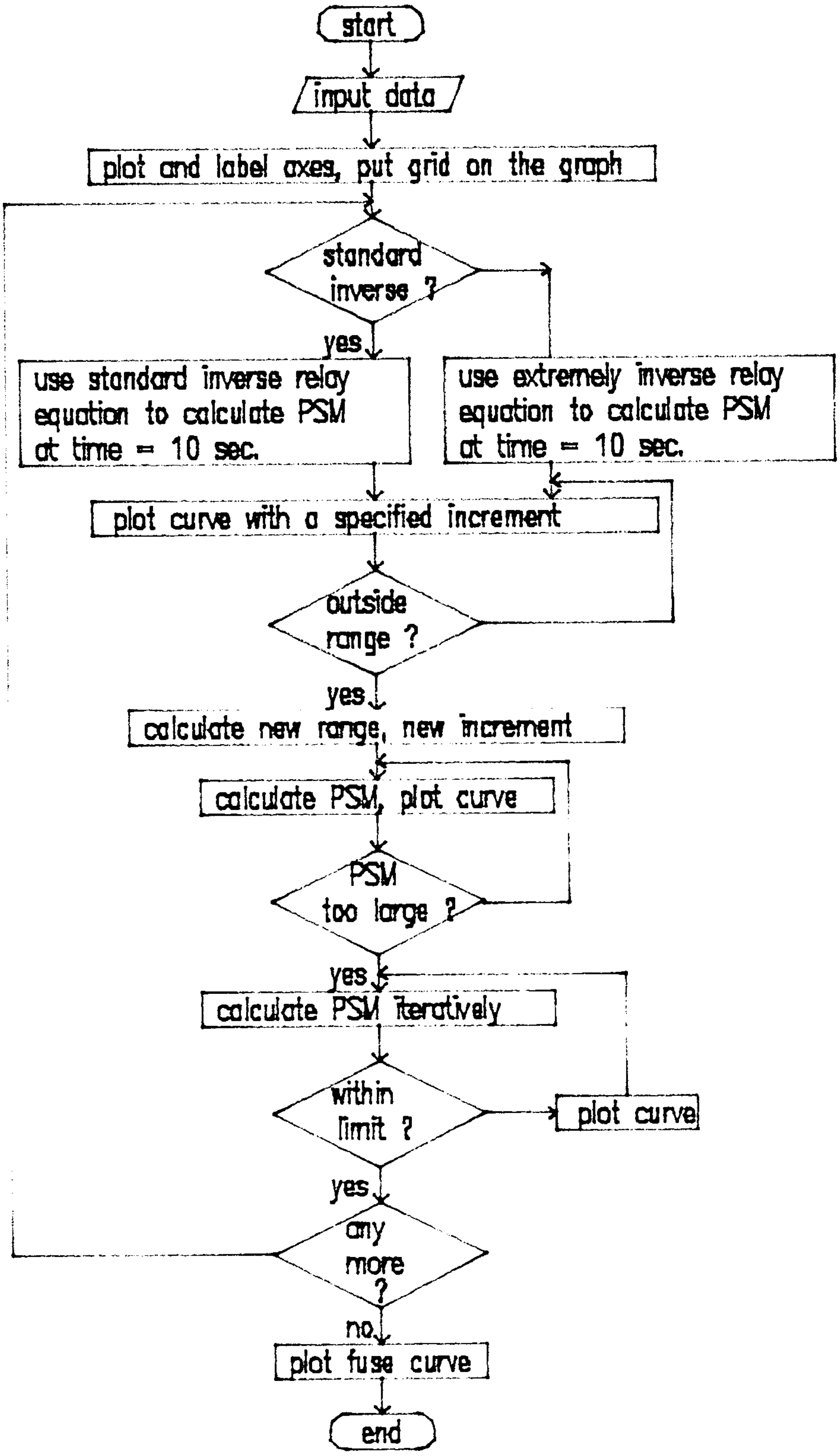


Figure 3.6 Flow diagram for plotting program

process is repeated until the specified difference between calculated and maximum PSM is reached, then GH is plotted.

The speed of plotting is determined by the increment of PSM and the range for which it applies; these are not fixed but vary with the values of TMS, pickup values and maximum fault level.
An additional program was developed to enable enlargement of a small area of the curve as required.

3.4.3 Algorithm

(a) The number of curves, device types, pickup, TMS, maximum fault level or high set level for each relay are input to the program.

(b) The log-log coordinates are plotted and the axes are labelled.

(c) The fuse characteristics represented by a polynomial function are plotted.

(d) The required subroutine is selected and the relay grading curve plotted for the relay type .

(e) If more grading curves are required, then the program returns to (d).

(f) If required, a hard copy of the grading curves can be obtained.

(g) Relay and fuse information can be printed out.

3.4.4 Flow chart

A flow diagram of this program is shown in figure 3.6 and a listing of the program is shown in appendix J (** J2 **).

3.5 Summary

Computer programs have been developed for calculating coordination settings and plotting of curves. The next chapter deals with the transient analysis and with the data required, such as the machine parameters, to assess the validity of relay settings under transient conditions.

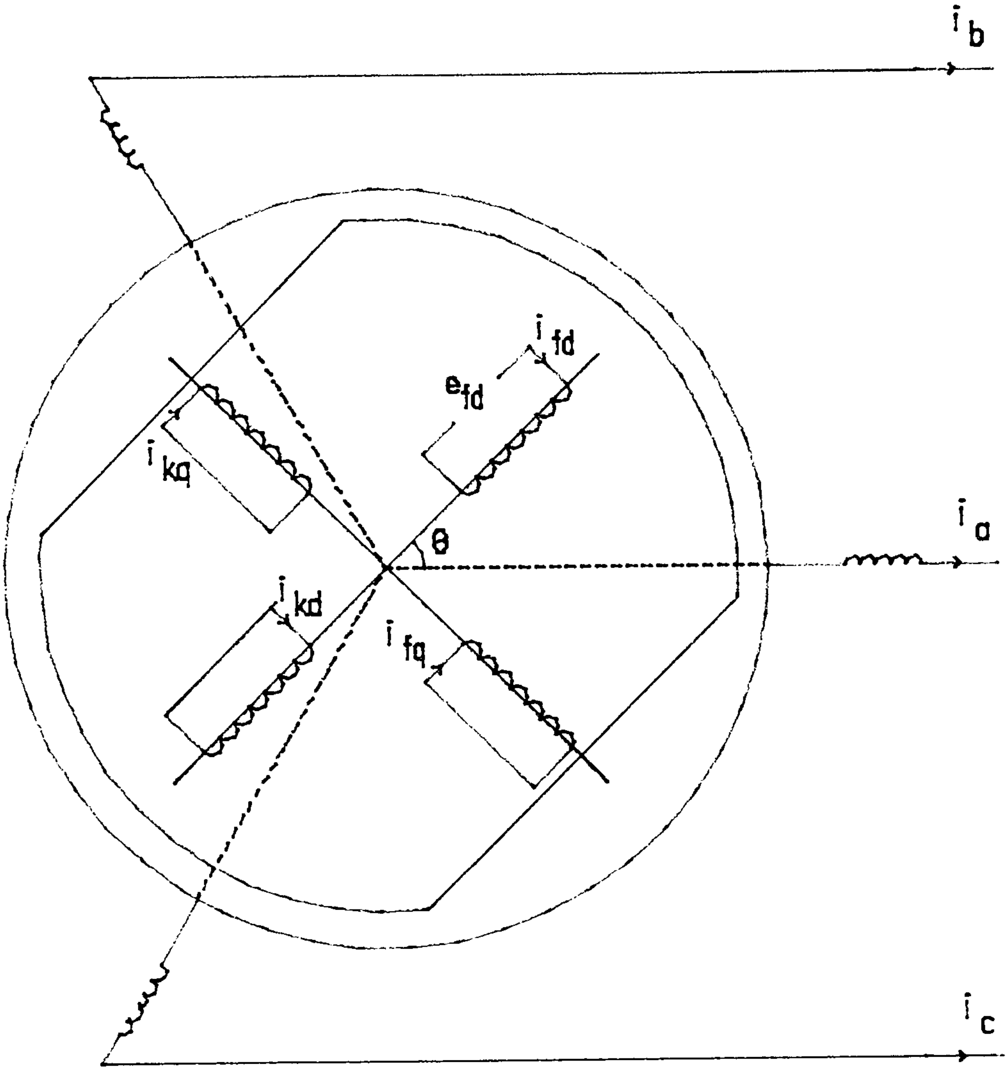


Figure 4.1 Schematic layout of the windings of the synchronous machine

Chapter 4

Transient analysis

This chapter develops programs which introduce the switching, fault, and connection matrices for transient analysis when faults occur on the system (figure 1.1) as mentioned in chapter 1. Programs are also developed to calculate the relay operating time under transient conditions. The chapter is divided into three main parts: the first part (4.7) deals with the faults which occur on the LV and HV side of the transformer where inclusion of circuit breaker opening is involved. In the second part (4.10) faults which occur on an open circuited generator are studied. Finally, in the third part (4.12), programs for calculating the operating time for relays are developed.

4.1 System modelling

The schematic layout of the windings of the synchronous machine is shown in figure 4.1. The behaviour of the system is specified by a set of three-phase variables arranged into first order differential equations which can be solved by numerical analysis using a digital computer. The system so formulated calculates the effects of circuit breaker opening occurring on either side of the generator transformer under fault or steady state conditions.

The generator is represented as a lumped parameter system, in which the space m.m.f. and the flux wave are assumed to be spaced sinusoidally around the generator air gap. There is one rotor damper winding along the direct axis and one along the quadrature axis. In addition to the field circuit on the direct axis, one fictitious field winding is added in the quadrature axis to simulate the system more accurately especially at the leading power factor operations. The transformer is approximated as an ideal delta star transformer in series with a reactance and a resistance for each phase winding. The transmission line is represented by a reactance and a resistance in each phase.

The following assumptions were made with respect to the generator, transformer and transmission system

(A) The generator and transformer

(a) The armature windings of the generator are sinusoidally distributed.

(b) The space m.m.f. and the flux density wave in the generator are assumed to be sinusoidal.

(c) The effect of armature slots on the rotor inductances is neglected.

(d) Hysteresis is both transformer and generator are negligible.

(e) Saturation in the generator transformer is neglected.

(B) Transmission system

It is assumed that the transmission system can be represented by a voltage source of constant frequency and amplitude. Impedance values are calculated from the fault MVA infeed to the HV busbar, and the estimated X/R ratio for the system.

4.2 Choice of approach

Due to the lack of sophisticated calculation methods in computer engineering in the past, the original untransformed equations of electrical machines expressed in phase variable form had to be solved by the (d,q) and (α,β) methods with approximations and assumptions to account for the difficulty of obtaining an exact solution.

The (d,q) method yields differential equations with constant coefficients. They are linear provided that speed is assumed to be constant, and this method usually ignores the derivatives in the stator voltage equations [3,45] for ease of computation. If these are not ignored then nothing is to be gained by using the (d,q) method. In addition, study of unsymmetrical faults necessitates further transformation of (d,q) equations. The (α,β) method results in differential equations with variable coefficients. The equations become nonlinear once the speed variation is taken into account and exact analysis is usually impossible. Moreover the (d,q) and (α,β) quantities cannot be used to study open circuit conditions and fault clearance by circuit breaker opening without going back to phase quantities.

The original untransformed equations of electrical machines have the greatest generality and allow all varieties of transients to be investigated. The possibility in principle of using untransformed equations for mathematical simulation was noted long ago [84,85]. Solutions of individual particular problems by means of these equations are also well known [86].

The phase coordinate method preserves the physical identity of the machine under consideration and also provides valuable insights to the machine performance. If different methods are adopted, the need for eventual reconversion to phase quantities is required. In recent years, where a modern digital computer is available in conjunction with suitable numerical methods, the solution in phase coordinate quantities appears to be the most natural one.

4.3 Methodology

Solution of the system equations for HV faults is not straightforward due to the need to invert the inductance matrix which is singular. To avoid this difficulty the variables in terms of flux linkages need to be expressed in terms of current variables.

Consider the following equation to be typical of the machine equations

to be solved $p\bar{\lambda} = w_o(\bar{E} + R\bar{I})$ -----(1).

where $\bar{E}$, $\bar{I}$, $\bar{\lambda}$ are the voltage, current, and flux linkage vectors respectively as defined in the matrix definition.

One method of solving equation (1) is to integrate the $p\bar{\lambda}$ equations to obtain a new value for $\bar{\lambda}$. If $\bar{\lambda} = L\bar{I}$ then $p\bar{\lambda} = w_o(\bar{E} + R\ (L)^{-1}\bar{\lambda})$ ---(2), where L is the machine inductance matrix.

There are two practical difficulties which can make equation (2) difficult to solve. Firstly the input vector $\bar{E}$ must be known. Secondly the inverse of $(L)^{-1}$ is not always easy to obtain. For machine faults

$(L)^{-1}$ is readily available but for HV faults L must be modified using connection matrices C_T and $C_T{}^t$ to enable the inclusion of the transformer.

The machine variables $(\overline{E},\overline{I})$ are related to the transformer and transmission line variables $(\overline{E}_T,\overline{I}_T)$ by a connection matrix C_T

$$E_T = C_T E \text{ and } I = C_T{}^t\, I_T$$

where $\overline{E}_T$, $\overline{I}_T$ are voltage, current vectors respectively; C_T, $C_T{}^t$ are the connection matrices and all are defined in the matrix definition.

Multiplying equation (1) by C_T and substituting for $\overline{E}$ and $\overline{I}$ gives

$$C_T p\overline{\lambda} = w_o(C_T R C_T{}^t \overline{I}_T + \overline{E}_T)$$

$$\text{Let } \overline{\lambda}' = C_T\overline{\lambda} = C_T L\overline{I} = C_T L C_T{}^t \overline{I}_T$$

$$= L'\overline{I}_T$$

and $R' = C_T R C_T{}^t$ so

$$p\overline{\lambda}' = w_o(\overline{E}_T + R'L'^{-1}\overline{\lambda}) \quad \text{-------(3)}$$

For HV faults it is not possible to determine $\overline{I}_T$ from equation (3) since C_T and $C_T^{\,t}$ are singular. Therefore equation (3) was rearranged in the following way using the substitution $p\bar{\lambda}' = pL'\bar{I}_T + L'p\bar{I}_T$.

Therefore $pL'\bar{I}_T + L'p\bar{I}_T = w_o(\bar{E}_T + R'\bar{I}_T)$,

So $L'p\bar{I}_T = w_o\bar{E}_T + (w_oR' - pL')\bar{I}_T$

This equation is used to solve for the armature and rotor currents for HV faults defined in the later sections. Although L' cannot be inverted, when $\bar{E}_T$ is related to the system voltage through the transmission system, the combination of L' and the transmission line inductance permits inversions of the inductance matrices.

4.4 Machine inductance matrix

$$L = \begin{bmatrix} Laa & Lab & Lac & Lafd & Lakd & Lafq & Lakq \\ Lba & Lbb & Lbc & Lbfd & Lbkd & Lbfq & Lbkq \\ Lca & Lcb & Lcc & Lcfd & Lckd & Lcfq & Lckq \\ Lfda & Lfdb & Lfdc & Lfdfd & Lfdkd & Lfdfq & Lfdkq \\ Lkda & Lkdb & Lkdc & Lkdfd & Lkdkd & Lkdfq & Lkdkq \\ Lfqa & Lfqb & Lfqc & Lfqfd & Lfqkd & Lfqfq & Lfqkq \\ Lkqa & Lkqb & Lkqc & Lkqfd & Lkqkd & Lkqfq & Lkqkq \end{bmatrix}$$

The armature self inductances Laa, Lbb, Lcc of a generator vary periodically with period π between a maximum when the armature winding is on a pole axis and a minimum when it is on an interpolar axis. Thus

$$Laa = A_0 + A_2 \cos 2\theta + A_4 \cos 4\theta + \ldots\ldots$$

Lbb & Lcc are given by the same expression with θ replaced by $\theta - \frac{2}{3}\pi$ and $\theta + \frac{2}{3}\pi$ respectively.

The armature mutual inductances Lbc, Lca, and Lab vary periodically with period π. Lbc is at a maximum when phase a is on the quadrature axis, hence

$$Lbc = -B_0 + B_2 \cos 2\theta + B_4 \cos 4\theta + \ldots\ldots$$

Lca & Lab are given by the same expression with θ replaced by $\theta - \frac{2}{3}\pi$ and $\theta + \frac{2}{3}\pi$ respectively.

The mutual inductances between an armature winding and a field or damper winding vary periodically with period 2π. Lafd is at a maximum when phase a is on the direct axis and zero when phase a is on the quadrature axis. Thus

$$Lafd = C_1 \cos \theta + C_3 \cos 3\theta + \ldots\ldots\ldots \quad \text{etc.}$$

$$Lakd = D_1 \cos \theta + D_3 \cos 3\theta + \ldots\ldots\ldots$$

$$Lafq = E_1 \sin \theta + E_3 \sin 3\theta + \ldots\ldots\ldots$$

$$Lakq = F_1 \sin \theta + F_3 \sin 3\theta + \ldots\ldots\ldots\ldots$$

The remaining inductances involving phases b and c are obtained by replacing θ by $\theta - \frac{2}{3}\pi$ and $\theta + \frac{2}{3}\pi$ in the appropriate expressions.

The self inductances Lfdfd, Lkdkd, Lfqfq, Lkqkq of the field and damper windings and the mutual inductances Lfdkd, Lfqkq are all constants.

The coefficient of coupling between the d and q axes is zero, and all pairs of windings with 90^o displacement have zero mutual inductance. Thus Lfdfq, Lfdkq, Lkdkq, Lkdfq are zero.

For many purposes the Fourier terms of order 3 and higher can be neglected, thus it was assumed that all the inductances vary sinusoidally, with an additional constant term in some cases.

4.5 Mathematical description of the model

It can be seen that the following sets of equations can be derived by referring to figure 1.1 and figure 4.1.

4.5.1 The generator voltage equations

4.5.1.1 Armature winding equations

The current, voltage and flux linkage for each phase on the armature will be related by the following equations.

$$e_a = \frac{1}{w_o} p \lambda_a - r_a i_a - 3r_e i_o - k_a$$

$$e_b = \frac{1}{w_o} p \lambda_b - r_a i_b - 3r_e i_o - k_b$$

$$e_c = \frac{1}{w_o} p \lambda_c - r_a i_c - 3r_e i_o - k_c$$

If no circuit breaker opening occurs, then k_a, k_b, k_c are zero.

and $3i_o = i_a + i_b + i_c$

4.5.1.2 Rotor equations

The current, voltage and flux linkage for each circuit on the rotor will be related by the following equations.

$$e_{fd} = \frac{1}{w_o} p \lambda_{fd} + r_{fd} i_{fd}$$

$$\emptyset = \frac{1}{w_o} p \lambda_{kd} + r_{kd} i_{kd}$$

$$\emptyset = \frac{1}{w_o} p \lambda_{fq} + r_{fq} i_{fq}$$

$$\emptyset = \frac{1}{w_o} p \lambda_{kq} + r_{kq} i_{kq}$$

4.5.2 The generator torque equation

The rotor swing can be obtained by considering the torque equations as follows:

The electrical air gap torque is expressed as

$$T_e = \frac{2}{3\sqrt{3}} \{\lambda_a(i_b - i_c) + \lambda_b(i_c - i_a) + \lambda_c(i_a - i_b)\}$$

The accelerating torque is equal to the mechanical torque minus the electrical air gap torque and the loss torque, so

$$T_a = T_m - T_e - T_L$$

The rotor angle will be related to the accelerating torque by

$$T_a = \frac{2H}{w_o} p^2\theta = \frac{2H}{w_o} p^2\varepsilon$$

$$p\theta = w_o + p\varepsilon$$

$$\theta = w_o t + \varepsilon - \frac{1}{6}\pi$$

4.5.3 The generator transformer voltage equations

$$e_{Ta} = \frac{e_{La}}{n} + (r_T + \frac{l_T p}{w_o}) \frac{i_{Ta}}{n^2} + k_a$$

$$e_{Tb} = \frac{e_{Lb}}{n} + (r_T + \frac{l_T p}{w_o}) \frac{i_{Tb}}{n^2} + k_b$$

$$e_{Tc} = \frac{e_{Lc}}{n} + (r_T + \frac{l_T p}{w_o}) \frac{i_{Tc}}{n^2} + k_c$$

4.5.4 The transmission line voltage equations

$$e_{La} = (r_L + \frac{l_L p}{w_o}) i_{La} + v_a$$

$$e_{Lb} = (r_L + \frac{l_L p}{w_o}) i_{Lb} + v_b$$

$$e_{Lc} = (r_L + \frac{l_L p}{w_o}) i_{Lc} + v_c$$

4.6 Normal steady state operating condition

Under normal steady state operating conditions, it can be seen that

$i_{La} = \frac{i_{Ta}}{n}$, $i_{Lb} = \frac{i_{Tb}}{n}$, $i_{Lc} = \frac{i_{Tc}}{n}$. So if

$Rtt = r_T + r_L$ and $Ltt = l_L + l_T$ then

$$ne_{Ta} = (Rtt + \frac{Lttp}{w_o}) \frac{i_{Ta}}{n} + v_a$$

$$ne_{Tb} = (Rtt + \frac{Lttp}{w_o}) \frac{i_{Tb}}{n} + v_b$$

$$ne_{Tc} = (Rtt + \frac{Lttp}{w_o}) \frac{i_{Tc}}{n} + v_c$$

The above equations can be written in matrix form as

$$\bar{e}_T = \frac{\bar{v}}{n} + \frac{1}{n^2} (Rtt + \frac{Lttp}{w_o}) \bar{i}_T$$

since $\bar{i}_s = C^t \bar{i}_T$, and $\bar{e}_T = C\bar{e}_s$ so

$$M\bar{e}_s = C^t \frac{\bar{v}}{n} + \frac{1}{n^2} (Rtt + \frac{Lttp}{w_o}) \bar{i}_s$$

where $M = C^tC$ is defined as the connection matrix.

These are the equations for a normally operating system and when a fault occurs on the LV side of the transformer, the equation is changed to the following general form $M\bar{e}_1 = C^t \frac{\bar{v}}{n} + \frac{1}{n^2} (Rtt + \frac{Lttp}{w_o})\bar{i}_1$

where $\bar{e}_1$ and $\bar{i}_1$ are defined in the matrix definition. To eliminate $\bar{e}_1$ and express the equation in terms of $\bar{e}_s$, the connection matrix M_{1s} is introduced such that

$$M_{1s}\bar{e}_s = C^t \frac{\bar{v}}{n} + \frac{1}{n^2}(r_{TL} + \frac{Lttp}{w_o})\bar{i}_1$$ where M_{1s} is a new connection matrix.

With the introduction of a fault matrix F_{1s} the equation can be expressed in terms of $\bar{i}_s$ rather than $\bar{i}_1$ such that

$$\bar{e}_s = F_{1s}M_{1s}^{-1}C^t \frac{\bar{v}}{n} + F_{1s}M_{1s}^{-1}\frac{1}{n^2}(r_{TL} + \frac{Lttp}{w_o})\bar{i}_s$$

This equation when combined with the machine equation can now be used to form a set of state equations which can be solved. M_{1s} and F_{1s} are discussed in detail in the next section.

4.7 LV and HV fault simulation

4.7.1 Fault & Connection matrices

4.7.1.1 LV faults

A fault matrix is used to take into account the voltages which become zero or equal to each other when a fault occurs. For example, if on the LV side of the transformer, a ground fault occurs on phase a, then $e_a = 0$. So

$\bar{e}_s = F_{1s}\,\bar{e}_s$, or

$$\bar{e}_s = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \bar{e}_s$$

Or if a line to line fault occurs between phase b and phase c, then

F_{1s} becomes

$$F_{1s} = \begin{bmatrix} 1 & \emptyset & \emptyset \\ -\emptyset.5 & \emptyset & \emptyset \\ -\emptyset.5 & \emptyset & \emptyset \end{bmatrix}$$

To include the rotor currents, 7x7 matrices are used, so F_{1s} is changed to F_{1sr} etc..

If faults occur the connection matrix has to be determined for different fault conditions to calculate the currents. The vector $\bar{e}_s$ can be used to define M_{1s} which enables the combination of system and machine equations to be defined for all fault conditions. To determine M_{1s}, an example is given below:

If phase a is earthed , then $e_a = e_{a1} = \emptyset$, $e_b = e_{b1}$, $e_c = e_{c1}$, $i_b = i_{b1}$, $i_c = i_{c1}$. With the inclusion of

$$F_{1s} = \begin{bmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & 1 & \emptyset \\ \emptyset & \emptyset & 1 \end{bmatrix},$$

therefore since the elements in the first row of $F_{1s}M_{1s}^{-1}$ must be zero; the elements in the first row of M_{1s}^{-1} are arbitrary. Also since

$M_{1s}\bar{e}_s = M\bar{e}_1$ then M_{1s} may be chosen such that it can be inverted i.e.

$$M_{1s} = \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

From [90], it was concluded that when the system equations and the machine equations are combined then, for LV faults and with the inclusion of rotor currents

$$(L - AL_{TL}\frac{1}{n^2})\frac{p\bar{I}}{w_o} = \frac{1}{n}AC_T{}^t\bar{V} + (R - \frac{pL}{w_o} + AR_{TL}\frac{1}{n^2})\,\bar{I} \text{ and}$$

$$A = F_{1sr}M_{1sr}{}^{-1}$$

where R_{TL} is the 7x7 resistance matrix for the transformer and the transmission line, with the first three diagonal elements equal to Rtt and the rest zero. L_{TL} is the 7x7 inductance matrix for the transformer and the transmission line, with the first three diagonal elements equal to Ltt and the rest are zero.

The rules used in calculating F_{1s} and M_{1s} can be stated as follows:

F_{1s} is a unit matrix where each column represents a phase subject to fault conditions, i.e., column 1 = phase a, column 2 = phase b, column 3 = phase c.

For earth faults replace corresponding column by $[0,0,0]^t$.

For phase to phase faults the corresponding columns are modified to $[-0.5,-0.5,1]^t$, $[1,-0.5,-0.5]^t$, $[-0.5,1,-0.5]^t$ for faults "a to b", "b to c" or "c to a" respectively, and the rest of the elements in the matrix are zero.

M_{1s} will be the unit matrix for phase to phase faults(not involving an earth fault), three phase to earth and normal operation.
For other earth faults the rows and columns of M corresponding to faulted phase or phases are replaced by zero's in the non-diagonal elements and a 3 in the diagonal elements.

4.7.1.2 HV faults

For HV faults

$$(L' - L'_{TL}\frac{1}{n^2})\frac{p\overline{I}_T}{w_o} = F_{2sr}\frac{\overline{V}}{n} + (R' - \frac{pL'}{w_o} + R'_{TL}\frac{1}{n^2})\overline{I}_T \text{ where}$$

$L' = C_T L C_T^t$, $R' = C_T R C_T^t$

F_{2sr} is the fault matrix for faults occurring on the HV side of the transformer with the rotor currents included.
R'_{TL} and L'_{TL} are the resistance and inductance matrices for the transformer and the transmission line respectively. They are defined on page (xvi).

F_{2s} is the fault matrix used to define faults at the HV side of the transformer and it is calculated in exactly the same way as F_{1s}. The matrix equations for HV and LV faults are similar, except that for faults on the HV side of the transformer a knowledge of M_{1s} is not required. The difference is due to different fault locations.

4.7.2 Switching matrix

To determine the effect of phase opening, a switching matrix, S, is introduced. It is a unit matrix when no opening of phases occurs, but changes according to the rule that when any phase is opened, the corresponding one in the matrix will be changed to a zero. For example, if phase a is opened, then the one in the first column will be changed to zero in order to set the current in phase a to zero.

4.8 Circuit breaker opening

In this thesis, the author has divided the circuit breaker opening cases into the general type and the particular type. In the general type, the matrices can be obtained by the rules as stated above, whereas in some cases the rules cannot be applied. For particular faults the vector matrix equations have to be redefined. The voltages which link with the system side and those which link with the machine side are equated to obtain the vector matrix equations. In some cases, difference in voltages have to be used to eliminate unknown variables which occur in the transmission system equations. Also when the summation of two of the phase currents are equal to zero, the fault, switching and connection matrices have to be redefined. For example, let a fault occurs between A and the LV circuit breaker. Assume a ground fault on phase b and that the circuit breaker in phase b is opened first, then in order to eliminate the voltage, e_{b1}, the difference of voltages in phase a and phase c has to be used. Also to account for the summation of currents in phase a and phase c to be zero as soon as phase b is cleared, a new set of fault, connection and switching matrices have to be defined. It should be noted that an extra connection matrix is introduced to allow for the fact that i_{b1} is zero but i_b is not necessarily zero. There are no particular rules for defining the matrices for faults of the particular type.

4.8.1 General type

Consider a phase to ground fault to occur between the circuit breaker and the primary side of the generator transformer. Presume the circuit breaker in phase a is opened first, so this case can be represented as in figure 4.2. It will be represented as `Fault @ B b-earth opening a'. It can be seen that the following conditions will be true.

Due to opening of phase a, $i_a = i_{a1} = \emptyset$

Since no zero phase sequence current can flow in a delta connected transformer, $i_{b1} + i_{c1} = \emptyset$

$i_c = i_{c1}$

$e_b = e_{b1} = \emptyset$

$e_c = e_{c1}$

Before the fault, the system equations will be

$$\frac{1}{3} (2e_{a1}-e_{b1}-e_{c1}) = \frac{v_a - v_c}{n\sqrt{3}} + (Rtt + \frac{Lttp}{w_o}) \frac{i_{a1}}{n^2} \qquad (4)$$

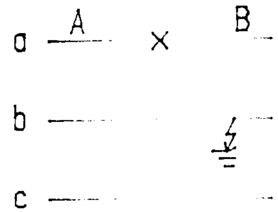


Figure 4.2 Fault at B, b to earth, phase a is opened

$$-\frac{1}{3}\ (-e_{a1}+2e_{1}-e_{c1}) = \frac{v_b - v_a}{n\sqrt{3}} + (Rtt + \frac{Lttp}{w_o})\ \frac{i_{b1}}{n^2} \qquad \text{------------(5)}$$

$$-\frac{1}{3}\ (-e_{a1}-e_{b1}+2e_{c1}) = \frac{v_c - v_b}{n\sqrt{3}} + (Rtt + \frac{Lttp}{w_o})\ \frac{i_{c1}}{n^2} \qquad \text{------------(6)}$$

For phase b-earth

$$F_{1s} = \begin{bmatrix} 1 & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & 1 \end{bmatrix} \qquad M_{1s}^{-1} = \begin{bmatrix} 2 & \emptyset & 1 \\ \emptyset & 1 & \emptyset \\ 1 & \emptyset & 2 \end{bmatrix} \text{ so}$$

$$e_a = \frac{2v_a - v_b - v_c}{n\sqrt{3}} + (Rtt + \frac{Lttp}{w_o})\ \frac{2i_a + i_c}{n^2}$$

$$e_b = \emptyset$$

$$e_c = \frac{v_a - 2v_b + v_c}{n\sqrt{3}} + (Rtt + \frac{Lttp}{w_o})\ \frac{i_a + 2i_c}{n^2}$$

To introduce the effect due to opening phase a, introduce the switching matrix, S_{1s}, which is equal to

$$S_{1s} = \begin{bmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & 1 & \emptyset \\ \emptyset & \emptyset & 1 \end{bmatrix}$$

$$\text{So } S_{1sr}\overline{E} = S_{1sr}AC_T^{\ t}\ \frac{\overline{V}}{n} + S_{1sr}A(\ \frac{L_{TL}p}{w_o} + R_{TL})S_{1sr}\frac{1}{n^2}\overline{I} \qquad \text{--------(7)}$$

That is, $e_b = \emptyset$

$$e_c = \frac{v_a - 2v_b + v_c}{n\sqrt{3}} + (r_{TL} + \frac{l_{TL}p}{w_o})\ \frac{2i_c}{n^2}$$

The machine equations will be

$$\overline{E} = (-R + \frac{pL}{w_o} + \frac{Lp}{w_o})\overline{I} - \overline{K}$$

where $\overline{K}$ is the vector for arcing resistance simulation.

$$\text{so} \quad S_{1sr}\overline{E} = S_{1sr}(-R + \frac{pL}{w_o} + \frac{Lp}{w_o})S_{1sr}\overline{I} - \overline{K} \qquad \text{-----}(8)$$

The elements for the $\overline{K}$ vector may be modified.

Equating (7) to (8)

$$S_{1sr}(-R+\frac{pL}{w_o} + \frac{Lp}{w_o})S_{1sr}\overline{I} - \overline{K} = S_{1sr}AC_T^{\ t}\frac{\overline{V}}{n} + S_{1sr}A(R_{TL} + \frac{L_{TL}p}{w_o})\frac{1}{n^2} S_{1sr}\overline{I}$$

or

$$S_{1sr}(L - \frac{A}{n}L_{TL})S_{1sr}\frac{p\overline{I}}{w_o} = S_{1sr}AC_T^{\ t}\frac{\overline{V}}{n} + S_{1sr}(R - \frac{pL}{w_o} + \frac{A}{n^2}R_{TL})S_{1sr}\overline{I} + \overline{K}$$

This equation can be solved by numerical methods to obtain the required currents.

For the HV side, the equation will be

$$S_{2sr}(L' - F_{2sr}L'_{TL})S_{2sr}\frac{p\overline{I}_T}{w_o} = S_{2sr}F_{2sr}\frac{\overline{V}}{n} +$$

$$S_{2sr}(R' - \frac{pL'}{w_o} + F_{2sr}R'_{TL}\frac{1}{n^2})S_{2sr}\overline{I}_T + \overline{K}$$

4.8.2 Particular type

For circuit breaker opening in cases where the general rules cannot be applied. The procedure used to define the matrices can be summarized as follows:

(1) Determine the conditions of the currents and voltages depending on the locations and type of faults, or circuit breaker opening.

(2) Arrange the system equations to satisfy the conditions in (1) and noting that the equations cannot involve any unknown variables.

(3) Arrange the machine equations with defined switching, connection and fault matrices such that the phase voltages can be eliminated when combined with the system equation to obtain the vector matrix equations for calculating the currents.

For illustration, the following examples are chosen to demonstrate how to define the switching, fault and connection matrices:

(a) Fault @ A b-earth opening b

This case is shown in figure 4.3; it can be seen that the following conditions will be true:

Since there is no zero phase sequence current $i_{a1} + i_{b1} + i_{c1} = 0$.

Because $i_{b1} = 0$, then $i_{a1} + i_{c1} = 0$.

Also because $i_a = i_{a1}$ and $i_c = i_{c1}$, therefore $i_a + i_c = 0$

$e_a = e_{a1}$, $e_c = e_{c1}$, $e_b = 0$

Equations (4), (5), (6) become

$$\frac{1}{3}(2e_a - e_{b1} - e_c) = \frac{v_a - v_c}{n\sqrt{3}} + \left(Rtt + \frac{Lttp}{w_o}\right)\frac{i_a}{n^2} \quad \text{------} \quad (9)$$

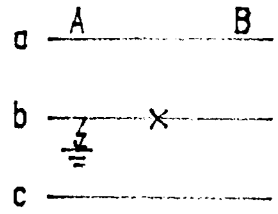


Figure 4.3 Fault at A, b to earth, phase b is opened

$$\frac{1}{3}(-e_a + 2e_{bl} - e_c) = \frac{v_b - v_a}{n\sqrt{3}} \text{ ---------- } (10)$$

$$\frac{1}{3}(-e_a - e_{bl} + 2e_c) = \frac{v_c - v_b}{n\sqrt{3}} + (Rtt + \frac{Lttp}{w_o})\frac{i_c}{n^2} \text{ ------ } (11)$$

(11) - (9)

$$-e_a + e_c = \frac{-v_a - v_b + 2v_c}{n\sqrt{3}} + (Rtt + \frac{Lttp}{w_o})\frac{i_c - i_a}{n^2}$$

An introduction of Ce, an extra connection matrix, is used to obtain proper modelling; e.g. $i_{bl} = 0$ but i_b may not be zero. If

$$M_{1sr}^{-1}\overline{E} = F_{1sr}M_{1sr}^{-1}C_T^{\ t}\frac{\overline{V}}{n} + F_{1sr}M_{1sr}^{-1}(R_{TL} + \frac{L_{TL}p}{w_o})(M_{1sr}^{-1})^{t}Ce_T\overline{I} \text{ --} (12)$$

To satisfy the above conditions

$$M_{1s}^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \qquad F_{1s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad Ce = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On the machine side

$$\overline{E} = (-R + \frac{pL}{w_o} + \frac{Lp}{w_o})\overline{I} - \overline{K}$$

$$\text{so } M_{1sr}^{-1}\overline{E} = M_{1sr}^{-1}(-R + \frac{pL}{w_o} + \frac{Lp}{w_o})(M_{1sr}^{-1})^{t}\overline{I} - \overline{K} \text{ --------} (13)$$

Since the currents i_a, i_b, i_c are not necessarily zero, so

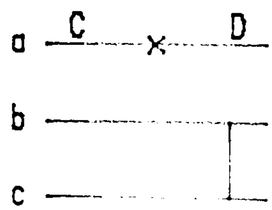


Figure 4.4 Fault at D, b to c, phase a is opened

$$S_{1s} = \begin{bmatrix} 1 & \emptyset & \emptyset \\ \emptyset & 1 & \emptyset \\ \emptyset & \emptyset & 1 \end{bmatrix}$$

Equating (12) & (13) and including the switching matrix

$$S_{1sr}(L - \frac{A}{n^2}L_{TL}(M_{1sr}^{-1})^t Ce_T)S_{1sr}\frac{p\overline{I}}{w_o} =$$

$$S_{1sr}AC_T^t \frac{\overline{V}}{n} + S_{1sr}(R - \frac{pL}{w_o} + \frac{A}{n^2}R_{TL}(M_{1sr}^{-1})^t Ce_T)S_{1sr}\overline{I} + \overline{K}$$

where $L = M_{1sr}^{-1}L(M_{1sr}^{-1})^t$, $R = M_{1sr}^{-1}R(M_{1sr}^{-1})^t$

(b) Fault @ D, b phase to c phase, opening a

This case is shown in figure 4.4; it can be seen that the following conditions will be true:

$i_{Ta} = \emptyset$, since there is no zero phase sequence current $i_{Tb} + i_{Tc} = \emptyset$

$e_{Lb} = e_{Lc}$, so

$$e_{Tb} - e_{Tc} = (r_T + \frac{l_T p}{w_o}) \frac{i_{Tb} - i_{Tc}}{n^2} + k_b - k_c$$

$$= (r_T + \frac{l_T p}{w_o}) \frac{2i_{Tb}}{n^2} + k_b - k_c$$

As far as the calculation of i_{Ta}, i_{Tb}, and i_{Tc} is concerned, the transmission line and infinite busbar are not involved, so L'_{TL}, R'_{TL} and busbar voltages are eliminated from the equations. All these conditions can be satisfied by setting

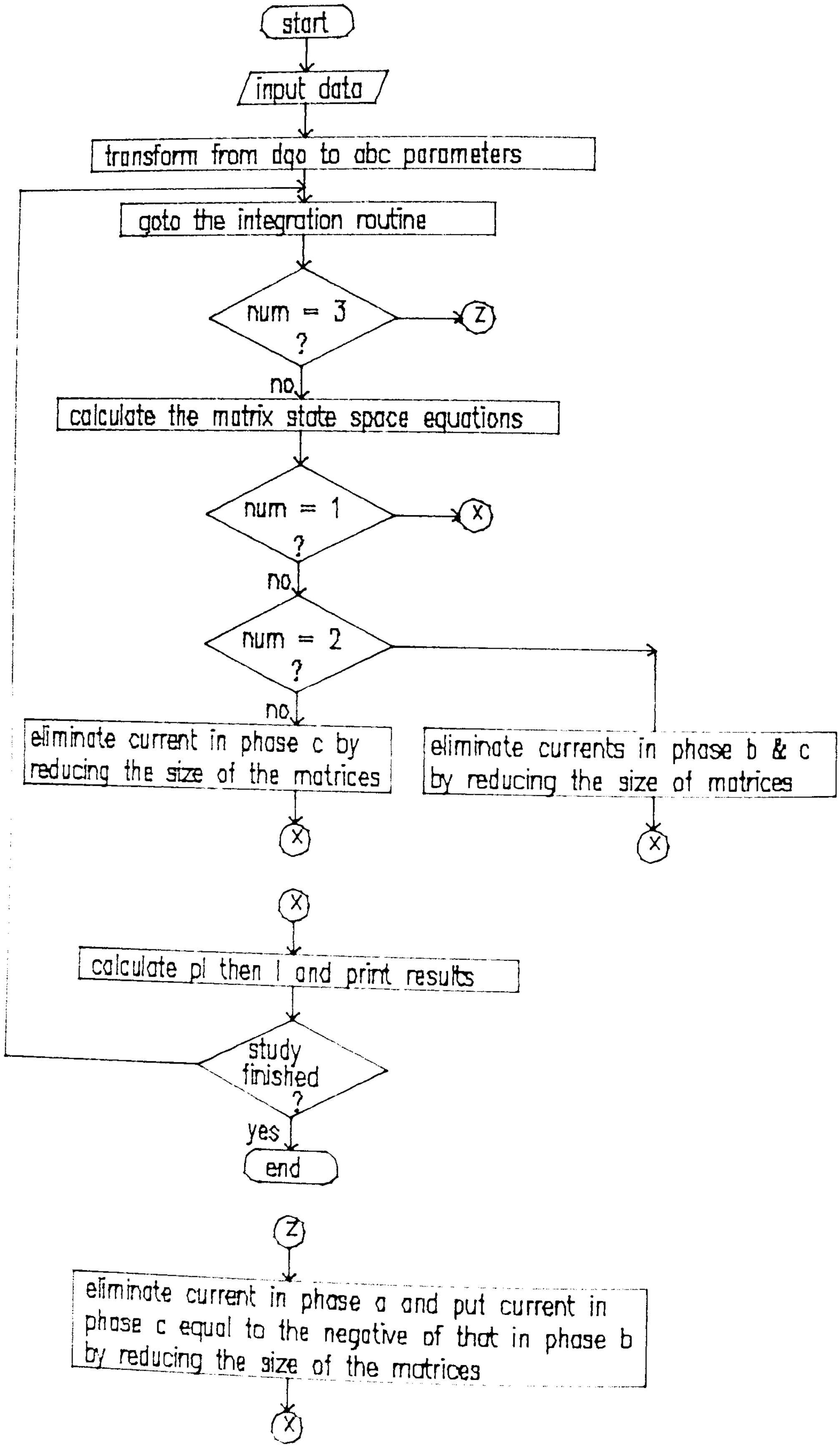


Figure 4.6 Flow chart for program calculating fault currents for a generator on open circuit

$$S_{2s} = \begin{bmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & 1 & \emptyset \\ \emptyset & \emptyset & \emptyset \end{bmatrix} (M_{2s}^{-1})^t = \begin{bmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & 1 & \emptyset \\ \emptyset & -1 & \emptyset \end{bmatrix} \quad F_{2s} = \begin{bmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \end{bmatrix} \quad Ce = \begin{bmatrix} 1 & \emptyset & \emptyset \\ \emptyset & 1 & \emptyset \\ \emptyset & \emptyset & 1 \end{bmatrix} \quad \text{for}$$

$$S_{2sr}\{L' - M_{2sr}^{-1}\frac{L'_{TL}}{n^2}(M_{2sr}^{-1})^t\}S_{2sr}Ce_T\frac{p\overline{I}_T}{w_o} = S_{2sr}F_{2sr}M_{2sr}^{-1}\frac{\overline{V}}{n} +$$

$$S_{2sr}\{R' - \frac{pL'}{w_o} + M_{2sr}^{-1}\frac{R'_{TL}}{n^2}(M_{2sr}^{-1})^t\}S_{2sr}Ce_T\overline{I}_T + \overline{K} \quad \text{---(14)}$$

where $L' = M_{2sr}^{-1}L'(M_{2sr}^{-1})^t$, $R' = M_{2sr}^{-1}R'(M_{2sr}^{-1})^t$

4.9 Software development

Programs have been developed which are used to analyse faults with circuit breaker opening. The flow chart for the program is shown in figure 4.5. A listing of the program is shown in appendix J(** J3 **).

4.10 Generator under open circuit conditions

It is possible to modify the computer programs to calculate generator fault currents for more elementary fault conditions i.e. when the generator is on open circuit before the application of a fault.

4.10.1 Line to line fault

For phase b connected to phase c, $i_a = pi_a = 0$ so

$$Lbbpi_b + Lbcpi_c + Lbfdpi_{fd} + Lbkdpi_{kd} + Lbfqpi_{fq} + Lbkqpi_{kq} =$$

$$w_oe_b + w_or_ai_b - pLbbi_b - pLbci_c - pLbfdi_{fd} - pLbkdi_{kd}$$

$$- pLbfqi_{fq} - pLbkqi_{kq} \text{ ----(15)}$$

$$Lcbpi_b + Lccpi_c + Lcfdpi_{fd} + Lckdpi_{kd} + Lcfqpi_{fq} + Lckqpi_{kq} =$$

$$w_oe_c + w_or_ai_c - pLcbi_b - pLcci_c - pLcfdi_{fd} - pLckdi_{kd}$$

$$- pLcfqi_{fq} - pLckqi_{kq} \text{ ---(16)}$$

$$Lfdbpi_b + Lfdcpi_c + Lfdfdpi_{fd} + Lfdkdpi_{kd} + Lfdfqpi_{fq} + Lfdkqpi_{kq} =$$

$$w_oe_{fd} - w_or_{fd}i_{fd} - pLfdbi_b - pLfdci_c - pLfdfdi_{fd} - pLfdkdi_{kd}$$

$$- pLfdfqi_{fq} - pLfdkqi_{kq} \text{ ------(17)}$$

$$Lkdbpi_b + Lkdcpi_c + Lkdfdpi_{fd} + Lkdkdpi_{kd} + Lkdfqpi_{fq} + Lkdkqp_{kq} =$$

$$- w_or_{kd}i_{kd} - pLkdbi_b - pLkdci_c - pLkdfdi_{fd} - pLkdkdi_{kd}$$

$$- pLkdfqi_{fq} - pLkdkqi_{kq} \text{ -----(18)}$$

$$Lfqbpi_b + Lfqcpi_c + Lfqfdpi_{fd} + Lfqkdpi_{kd} + Lfqfqpi_{fq} + Lfqkqpi_{kq} =$$

$$- w_or_{fq}i_{fq} - pLfqbi_b - pLfqci_c - pLfqfdi_{fd} - pLfqkdi_{kd}$$

$$- pLfqfqi_{fq} - pLfqkqi_{kq} \text{ --------(19)}$$

$Lkqbpi_b + Lkqcpi_c + Lkqfdpi_{fd} + Lkqkdpi_{kd} + Lkqfqpi_{fq} + Lkqkqpi_{kq} =$

$- w_o r_{kq} i_{kq} - pLkqbi_b - pLkqci_c - pLkqfdi_{fd} - pLkqkdi_{kd}$

$- pLkqfqi_{fq} - pLkqkqi_{kq}$ ----(2Ø)

and also $i_b + i_c = Ø$, and $e_b - e_c = Ø$ so (15)-(16), (17), (18), (19),

(2Ø) when written in matrix form will be $L_m p\bar{I}_m = w_o \bar{E}_m + (w_o R_m - pL_m)\bar{I}_m$

where

$$L_m = \begin{bmatrix} Lbb-Lbc-Lcb+Lcc & Lbfd-Lcfd & Lbkd-Lckd & Lbfq-Lcfq & Lbkq-Lckq \\ Lfdb-Lfdc & Lfdfd & Lfdkd & Lfdfq & Lfdkq \\ Lkdb-Lkdc & Lkdfd & Lkdkd & Lkdfq & Lkdkq \\ Lfqb-Lfqc & Lfqfd & Lfqkd & Lfqfq & Lfqkq \\ Lkqb-Lkqc & Lkqfd & Lkqkd & Lkqfq & Lkqkq \end{bmatrix}$$

$$R_m = \begin{bmatrix} 2r_a & Ø & Ø & Ø & Ø \\ Ø & -r_{fd} & Ø & Ø & Ø \\ Ø & Ø & -r_{kd} & Ø & Ø \\ Ø & Ø & Ø & -r_{fq} & Ø \\ Ø & Ø & Ø & Ø & -r_{kq} \end{bmatrix}$$

$\bar{E}_m = [\ e_b - e_c,\ e_{fd},\ Ø,\ Ø,\ Ø]^t$

$\bar{E}_{mo}$ defines the initial voltages such that

$\bar{E}_{mo} = [\ Ø,\ e_{fdo},\ Ø,\ Ø,\ Ø]^t$, e_{fdo} is the initial field voltage

$\overline{I}_m = [\ i_b,\ i_{fd},\ i_{kd},\ i_{fq},\ i_{kq}]^t$

$\overline{I}_{mo}$ defines the initial currents such that

$\overline{I}_{mo} = [\ 0,\ i_{fdo},\ 0,\ 0,\ 0\ ]^t$, i_{fdo} is the initial field current

Another way to simulate this case is to introduce fault and switching matrices. When under open circuit conditions, F_{os} and S_{os} are the fault and switching matrices respectively. To obtain the above conditions set

$$S_{os} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad M_{os}^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad (M_{os}^{-1})^t = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \quad F_{os} = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 0 & 0 \\ -0.5 & 0 & 0 \end{bmatrix}$$

where M_{os}^{-1} is used to account for the fact that $i_c = -i_b$

and $S_{osr} M_{osr}^{-1} L (M_{osr}^{-1})^t p\overline{I} =$

$$S_{osr} F_{osr} M_{osr}^{-1} \overline{E} + S_{osr} M_{osr}^{-1} (R - pL) (M_{osr}^{-1})^t \overline{I}$$

4.10.2 Three phase to earth

When a three phase to earth fault occurs,

$e_a = e_b = e_c = 0$. These conditions can be obtained by setting

$$S_{os} = \begin{bmatrix} 1 & \emptyset & \emptyset \\ \emptyset & 1 & \emptyset \\ \emptyset & \emptyset & 1 \end{bmatrix} \quad F_{os} = \begin{bmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \end{bmatrix}$$

For $S_{osr} \frac{L}{w_o} p\bar{I} = S_{osr} F_{osr} \bar{E} + S_{osr} (R - \frac{pL}{w_o}) \bar{I}$ ---------- (21)

where the initial $\bar{E}$ and $\bar{I}$ are $\bar{E}_o$ and $\bar{I}_o$ respectively.

$\bar{E}_o = [\ \emptyset, \emptyset, \emptyset, e_{fdo}, \emptyset, \emptyset, \emptyset\ ]^t$

$\bar{I}_o = [\ \emptyset, \emptyset, \emptyset, i_{fdo}, \emptyset, \emptyset, \emptyset\ ]^t$

4.1Ø.3 Single phase to earth

When an earth fault on phase a occurs,

$e_a = \emptyset$, $i_b = i_c = \emptyset$, so

$$S_{os} = \begin{bmatrix} 1 & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \end{bmatrix} \quad F_{os} = \begin{bmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & 1 & \emptyset \\ \emptyset & \emptyset & 1 \end{bmatrix}$$ for equation (21).

4.1Ø.4 Two phase to earth

When a line to line fault occurs on phases b and c,

$e_b = e_c = \emptyset$, and $i_a = \emptyset$, so

$$S_{os} = \begin{bmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & 1 & \emptyset \\ \emptyset & \emptyset & 1 \end{bmatrix} \quad F_{os} = \begin{bmatrix} 1 & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \end{bmatrix}$$ for equation (21).

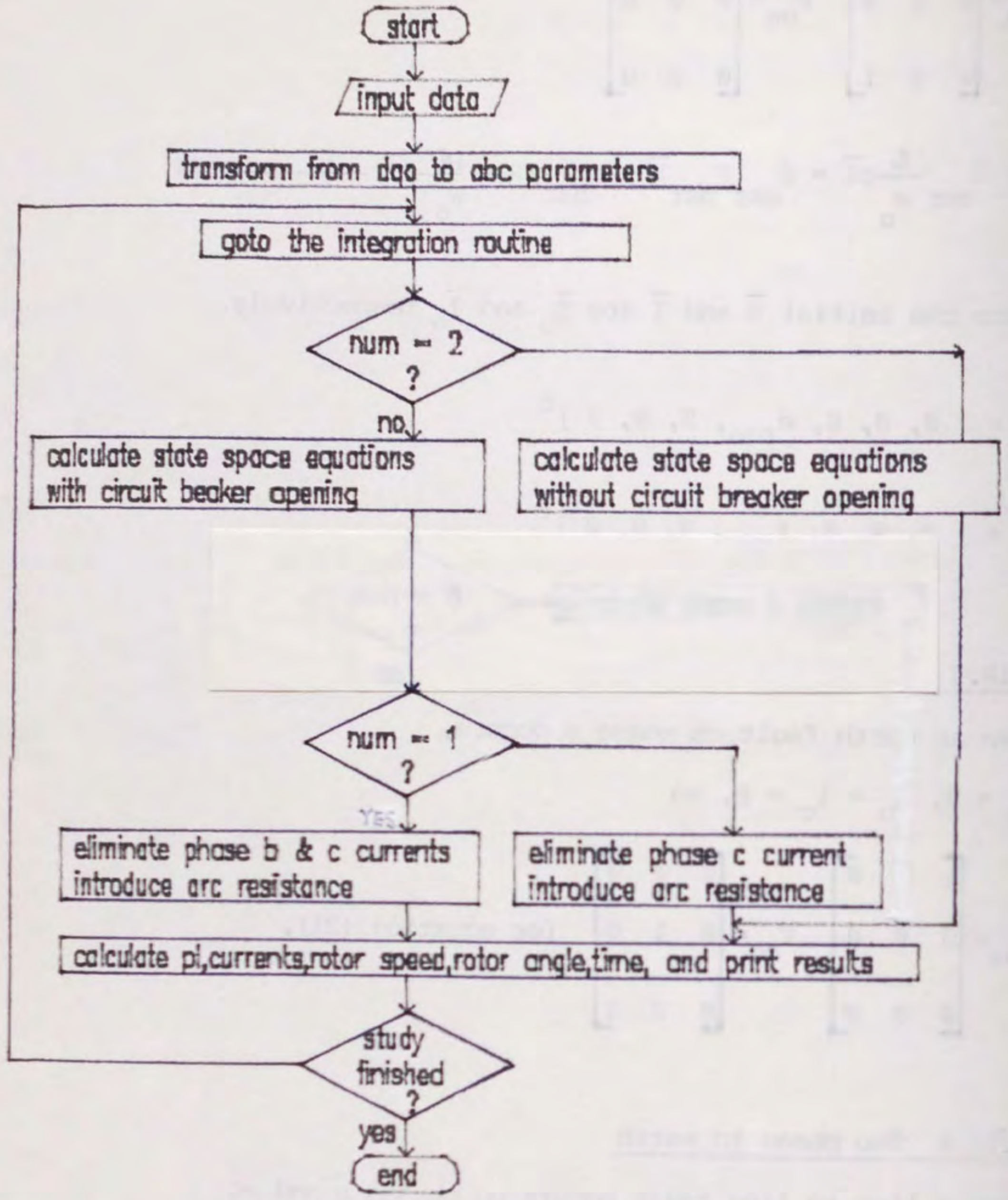


Figure 4.5 Flow chart for program calculating fault currents for faults occurring on LV side and with circuit breaker opening included

4.11 Software development

A program was developed to analyse fault currents for a generator on open circuit. A flow chart of the program is shown in figure 4.6, and a listing of the program is shown in appendix J (** J4 **).

4.12 Determination of relay operating time and i^2t under transient conditions

The secondary current of the current transformer is used to operate the relay. The current consists of d.c. and a.c. components, but only the a.c. component is used to determine the operating time of the relay.

4.12.1 Program development

(a) The upper and lower envelopes of the fault current were expressed as polynomials.

(b) The peaks of the symmetrical a.c. waveform were computed as half the sum of the upper and lower envelopes.

(c) The varying r.m.s. value of the fault current was determined as a function of time.

(d) The r.m.s. current was divided by the relay pickup current to give the PSM. If this was greater than one, then the operating time was calculated from the standard time-current characteristic as provided by the manufacturers. This time was divided into the time step to give the per unit travel of the relay. The travel was then held in a cumulative store.

(e) If the PSM is greater than one, procedure (d) is repeated until the summation is equal to or greater than one. That is, the relay

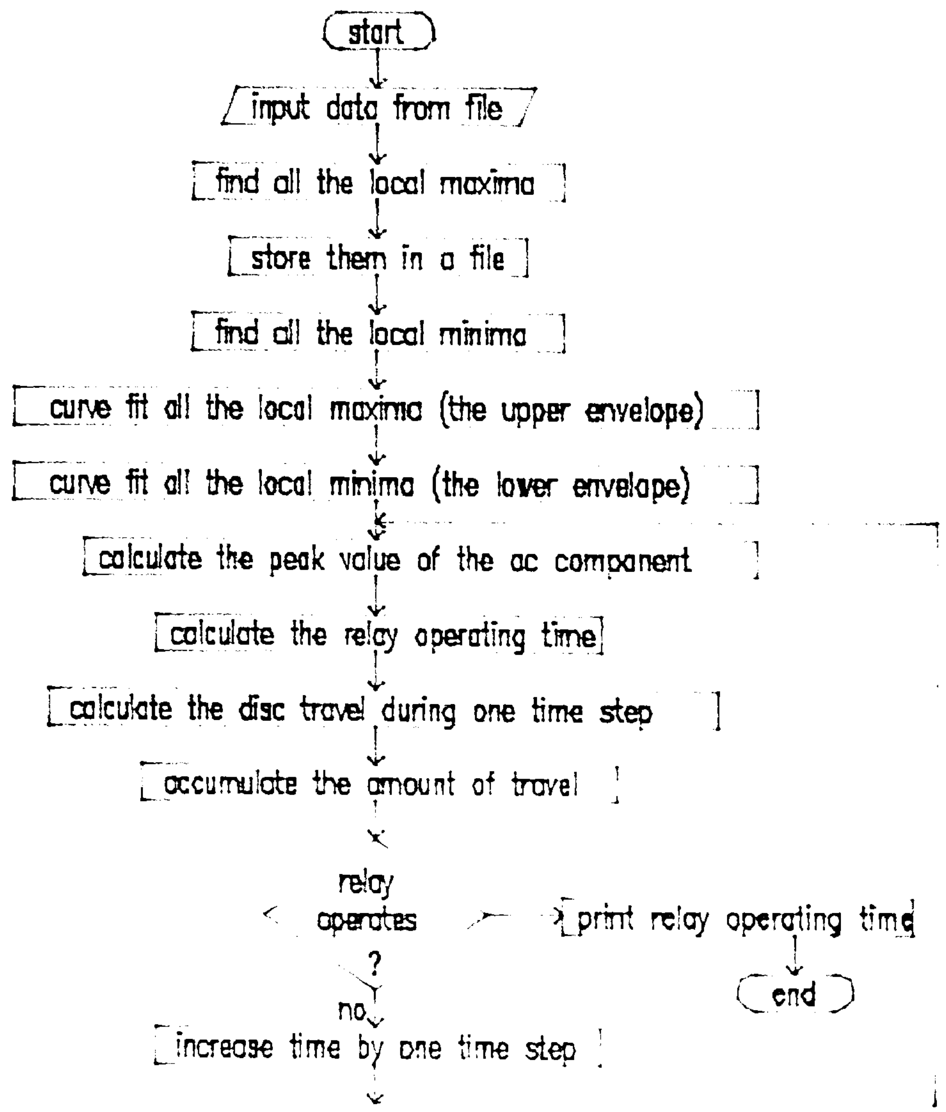


Figure 4.7 Flow chart for program calculating the relay operating time under transient conditions

operates. If the PSM is less than one, then the relay will not operate.

(f) The total relay operating time can be calculated and is equal to the time step multiplied by the number of steps needed to obtain unity in the cumulative store.

4.12.2 Program logic

The flow chart of the program is shown in figure 4.7 and a listing of the program is shown in appendix J (** J5 **).

4.12.3 i^2t calculation

A program has been developed to calculate the i^2t due to the fault current under transient conditions. A listing of the program is shown in appendix J (** J6 **).

4.13 Summary

Programs have been developed to analyse faults and circuit breaker opening. Since the secondary current of a current transformer is used to operate a relay, so the modelling of a current transformer is required. The next chapter deals with the current transformer modelling.

Chapter 5

Current transformer modelling

5.1 Introduction

Current transformers are four-terminal passive devices in which the output is a function of the input. In the simplest model of a current transformer, the relationship between the output current and the input current is a fixed number called the turns ratio. In an ideal transformer, the volt-ampere conservation applies, that is, input m.m.f. equals output m.m.f..

Coupling between the primary and secondary windings is determined by the flux patterns of the two windings. If the windings are suspended in air, the flux set up in the source winding extends to infinity in all directions with the result that only a small fraction of the available flux links the second winding. This arrangement is referred to as an air core transformer, and when applied to CT applications is sometimes called a `linear coupler'. Because only a small fraction of the available flux in the air-core transformer actually links the winding, such an arrangement is relatively inefficient. The coupling between windings can be radically improved by putting them on an iron core which acts to confine and maximize the flux linking the two windings. It is the iron core that produces the nonlinearity in practical current transformer circuits.

With the assumption that the leakage flux can be ignored, the current transformer can be described in terms of relatively few variables.

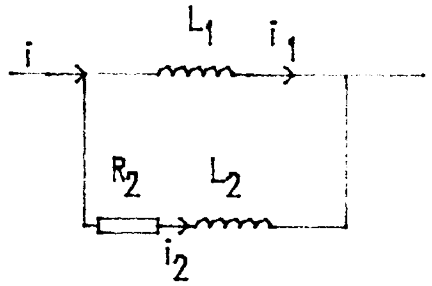


Figure 5.1 Equivalent circuit for a current transformer

Burden impedance $Z = R_2 + j\omega L_2$

They are the saturation flux, winding turns, resistance of the secondary winding, and residual flux.

5.2 Non-linear effects

There are three nonlinear effects introduced by iron cores:

(1) saturation

(2) eddy currents, and

(3) hysteresis.

Inaccurate reproduction of secondary current will result during unsymmetrical transients due to the large flux required to generate the voltage necessary to reproduce the transient in the burden of the transformer. The maximum value of transient flux is dependent on the magnitude of the current, the duration of the transient, the design of the transformer, and the secondary burden. Because of the increasing levels of short circuit currents and long time constants of present large power systems, the effect of CT saturation cannot be neglected.

5.2.1 Saturation

5.2.1.1 Reason for saturation

Inspection of figure 5.1 will show that the mutual impedance, $w\ L_1$, must be kept high with respect to the burden impedance, Z, if good current transformer performance is to be expected. The d.c. component of the primary current will divide between the branches of the circuit (figure 5.1). Since for all practical purposes, there is very little impedance to direct current in the magnetizing branch, the flow of direct current in this branch is retarded only by the time rate of change of the direct current, as it builds up in this circuit. The voltage which must be generated is determined by the d.c. component. If the d.c. time constant is long, this direct voltage must be maintained for a relatively long time, and it can only be achieved by a continuously increasing current through L_1 at a relatively high time rate of change. This will be in one direction only, without the advantage of positive and negative half-cycle characteristic of alternating current. This means that the knee of the saturation curve for the inductance L_1 may be reached within a few cycles, or within a fraction of a cycle in severe cases. When the iron core saturates, the effective value of the inductance, L_1, decreases to a fraction of the normal value. When this occurs, the ability of the mutual inductance to retard the flow of direct current through it collapses, so that practically all of the d.c. component goes through L_1. This condition will continue until the d.c. component approaches zero, after which the transformer can again perform with its normal a.c. characteristics.

5.2.1.2 Effect due to saturation

During the time that the magnetizing branch, L_1, remains severely saturated by direct current, it also offers much less impedance to the flow of alternating current, consequently, it is to be expected that the percentage of the a.c. component which is transmitted to the secondary will be materially less. That is, when a current transformer is failing to reproduce the d.c. component in the secondary, the apparent transformation ratio departs from its nominal value, even through its performance may be quite accurate for the same value of symmetrical current.

As far as relay operations are concerned, CT saturation can lead to the following relay misoperations.

(1) Detection of a fault where non exists

(2) Failure to detect a fault.

(3) Failure to detect a fault in an adequate time.

5.2.1.3 Reduction of saturation

The obvious solution of the saturation problem is to increase the core cross-section of the transformer, in order to allow a larger flux amplitude, but this method is expensive. A technique using the characteristics of asymmetrical fault currents, has been developed [121] in order to achieve dynamic control of the core flux, thus allowing a significant reduction of the required cross-section.

5.2.2 Eddy currents

Eddy currents cause the flux distributions in individual laminations to be nonuniform. Swift [65] has shown that the degree of nonuniformity is very small in laminations of the usual thickness over the range of transient and steady state conditions likely to be encountered in power systems. That is, the frequencies of interest are low enough so that eddy current effects are not large enough to materially influence the flux density profile. So eddy currents are neglected in the simulation model.

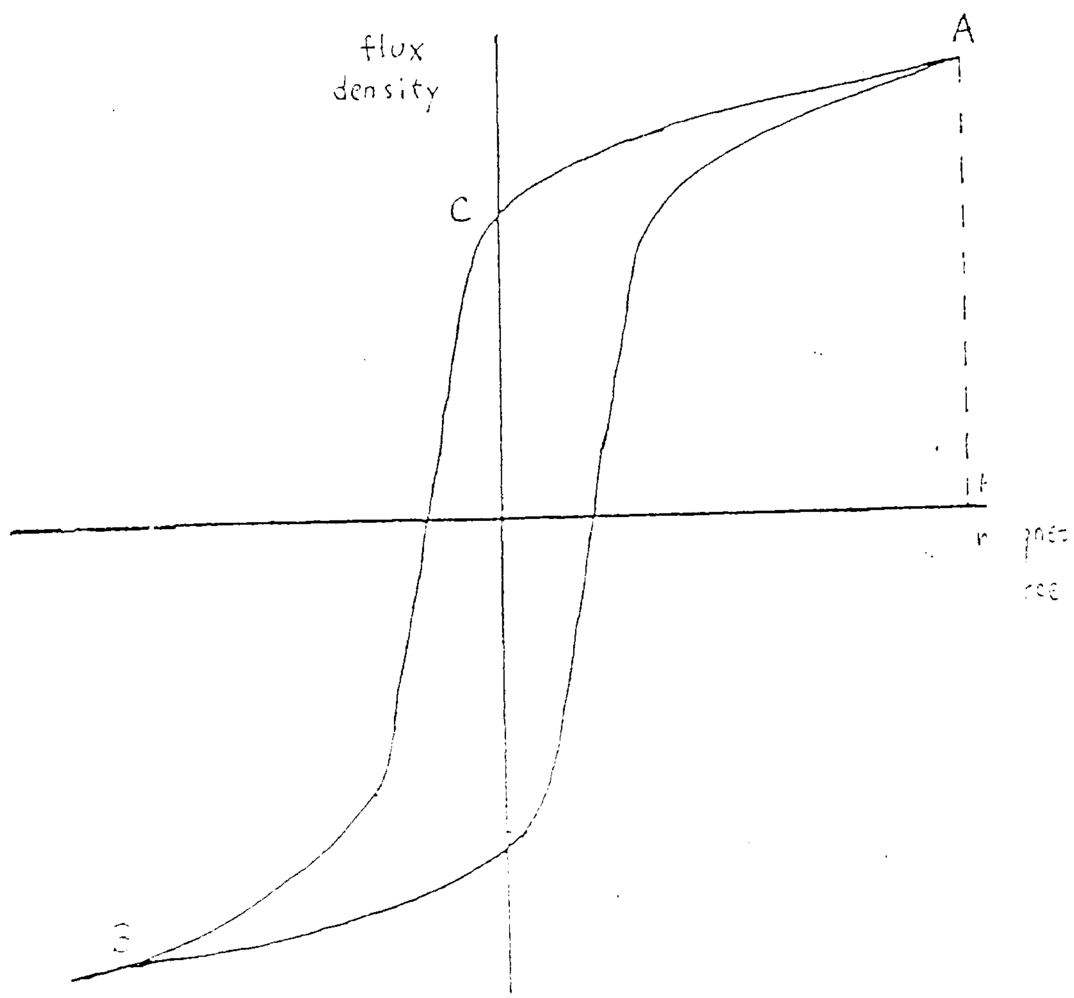


Figure 5.2 [illegible] loop of [illegible] material

5.2.3 Remanence

5.2.3.1 Reason for remanence

There are a number of conditions which may leave residual magnetism in the core of a current transformer connected to a power system. In general, the greatest residual fluxs imposed on a transformer will be under fault conditions. Such conditions can be summarised as follows:-

(a) Clearing of a fault under steady-state conditions

It can be assumed that the flux in a current transformer core is operating around the hysteresis loops shown in figure 5.2. When interruption of a fault occurs, the operating conditions of the iron will tend to be at one of two fixed points on the hysteresis loop, their position depending on the burden power factor. For example, with a purely resistive burden the flux will be at a maximum, that is , at A or B. For the working point to remain at A a direct magnetising force Ha must be available, and can only be provided by a direct current in the secondary circuit. To support this, the core flux must decrease to induce the necessary e.m.f. The operating point on the magnetisation characteristic will thus progress along the hysteresis loop until equilibrium is reached at point C, this will occur such that the rate of flux change at every point is sufficient to supply the magnetising force maintaining the flux at that point. The time constant of this decay will clearly be dependent on the losses in the transformer core and secondary circuit.

The protective current transformers are often designed to operate at very small flux levels under normal load conditions, this is to ensure that they function satisfactorily under fault conditions. Thus the steady state flux variations during a fault may not leave a significant level of residual magnetism.

(b) Clearing of faults during transient conditions

It is well known that the secondary and exciting currents consist of an exponentially decaying unidirectional component. In order to support a direct component of secondary current, it requires a continuous unidirectional flux change, and this leads to a high flux level during the transient period. The d.c. component of current will eventually decay to zero and the time constant will depend upon the losses in the transformer core and secondary circuit. This decay will be associated with a corresponding flux decay, however, owing to the nonlinear nature of the core material, magnetic equilibrium is obtained with the flux setting at different levels of residual magnetism.

5.2.3.2 Effect due to remanence

The presence of remanent flux in current transformer cores can significantly affect the performance of balanced forms of power system protective equipment. With balanced current relaying, Seeley [120] showed that with similar current transformers, equal burdens, equal primary current, and no residual flux, the relay would not operate. While with similar current transformers, equal burdens, and equal primary currents, but with residual flux in only one current transformer, the relay would operate. This false operation is due to the residual magnetism in the current transformer.

Due to remanence the core may be operating at a high peak flux density under normal system load conditions. Thus when a fault occurs, a current transformer with high residual flux may saturate in the first few cycles immediately following a system fault (with residual flux in the unfavourable direction).

5.2.3.3 Reduction of remanence

The most practical way to control residual flux is to put an air gap into the magnetic circuit, and it will be found that an extremely small gap causes a major reduction in possible residual flux [128]. The controlled residual flux current transformer can have an effective residual not greater than 10%. Very large air gaps in current transformers have the effect of increasing the reluctance to a high value such that saturation of the core is impossible even with fully offset fault currents. However this has the disadvantage that the transformer can only carry a very small burden.

The remanent flux in a core is reduced when an alternating current is passed through the windings of a transformer. The degree of reduction is dependent on the magnitude of the alternating flux produced. Large alternating flux variations are needed to effect complete removal of the residual flux in a core.

5.2.4 Hysteresis

One of the major consequences of hysteresis is that when a transformer is disconnected from the supply, residual core flux is usually established. In other words, the magnetic hysteresis determines the residual flux value in a current transformer. For transformers with closed cores, the highest expected remanent flux can exceed 0.5 ϕ_s, where ϕ_s is the saturated core flux. The average flux in the core of a transformer in service may be at the level of 0.25 ϕ_s. A transformer with an anti-remanent air gap in the core will have a remanent flux level of less than 0.1 ϕ_s. In normal circumstances the hysteresis loop is symmetrical, under short circuit conditions however the magnetic material has to deal with asymmetrical magnetising currents. There is a need to take into account the possible different magnetising loop characteristics created by a short circuit. Many papers [12,13,15,16,17,63,67,68] have considered the hysteresis effect; in this work, it was considered that neglect of hysteresis could be justified by the fact that the fault currents under consideration are many times the rated current of the transformer. Even the normal magnetising current is only a few percent of rated current, so the hysteresis effect will only produce a minor error at normal full load current.

5.3 Representation of saturation curves

For current transformer modelling, it is necessary to include the excitation characteristic. Papers [10,11] have reported on how to represent this characteristic in different ways. A brief summary of methods is given below:

5.3.1 Polynomial

It is possible to use a polynomial of the form

$$\emptyset = A_1 i_m + A_2 i_m^2 + A_3 i_m^3 + \ldots\ldots + A_n i_m^n$$

to represent the magnetisation curve, where $\emptyset$ is the flux and i_m is the magnetising current. But the use of a polynomial leads to computational errors because of the change in the polarity of the slope.

5.3.2 Arc-tangent function

Y.Miki et al. [59] reported the use of the following function for magnetising current simulation.

$$\emptyset = b_1 i_m + b_2 \tan^{-1}(b_3 i_m)$$

The use of this function has similar disadvantages to the method described in 5.3.1.

5.3.3 Exponential/hyperbolic functions

The use of exponential and hyperbolic functions do not provide a satisfactory method of representation over a wide range.

Trutt, Erdelyi and Hopkins [122] came to this conclusion after failing to find suitable polynomial, exponential, hyperbolic or logarithmic functions, and found that the only satisfactory way was by linear interpolation between a set of coordinate points.

5.4 Current transformer modelling

In [100], the author calculated the magnetising and secondary currents analytically using the equivalent circuit shown in figure 5.1 by knowing the equation of the fault current. Since a general equation

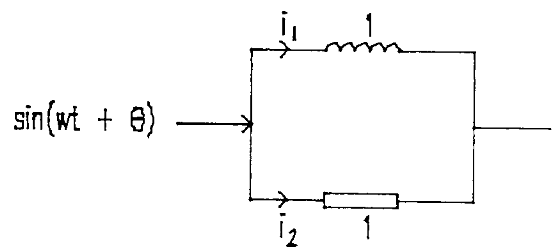


Figure 5.3 A sinusoidal current applied to a CT

i_1 is the real part of I_1

i_2 is the real part of I_2

$\sin(wt + \theta)$ is the real part of I

for the fault current is usually difficult to obtain, the magnetising and secondary current are calculated numerically. An equation for the magnetising current is derived in appendix A for this purpose.

5.4.1 Determination of initial conditions

To determine the initial current distribution for the current transformer, two methods can be used. They can be either analytical or graphical.

5.4.1.1 Analytical method

An example is used to illustrate this method. If an input is applied to a CT at $t = 0$ as shown in figure 5.3, with $w = 1$ assumed, then

$$i_1(s) = \frac{1}{s+1}\left(\frac{\cos\theta}{s^2+1} + \frac{s\sin\theta}{s^2+1}\right)$$

$$= \frac{(\cos\theta - \sin\theta)}{2(s+1)} + \frac{(\sin\theta - \cos\theta)s}{2(s^2+1)} + \frac{(\sin\theta + \cos\theta)}{2(s^2+1)}$$

from which,

$$i_1(t) = \frac{(\cos\theta - \sin\theta)}{2}e^{-t} + \frac{(\sin\theta - \cos\theta)}{2}\cos t + \frac{(\sin\theta + \cos\theta)}{2}\sin t$$

To find the steady state value, the transient term is neglected, so

$$i_1(t) = \frac{(\sin\theta - \cos\theta)}{2}\cos t + \frac{(\sin\theta + \cos\theta)}{2}\sin t$$

and this is the initial current for the magnetising branch of the CT before a fault develops.

5.4.1.2 Graphical method

If $\sin(wt + \theta) = 0.5$, $\theta = 0$ then $t = \frac{1}{6}\pi$

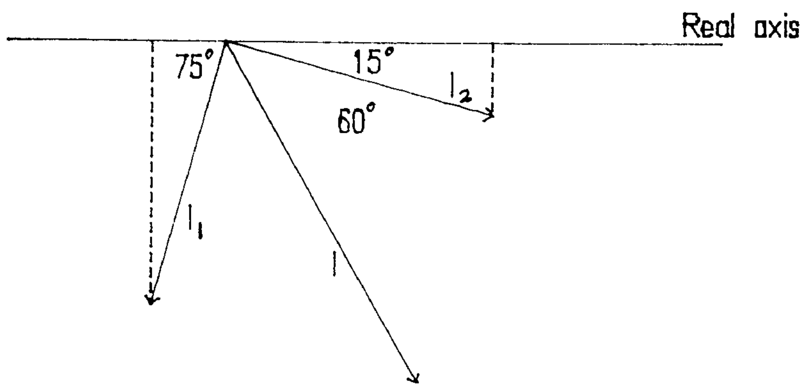


Figure 5.4 Phasor diagram to represent the current phasors in figure 5.3

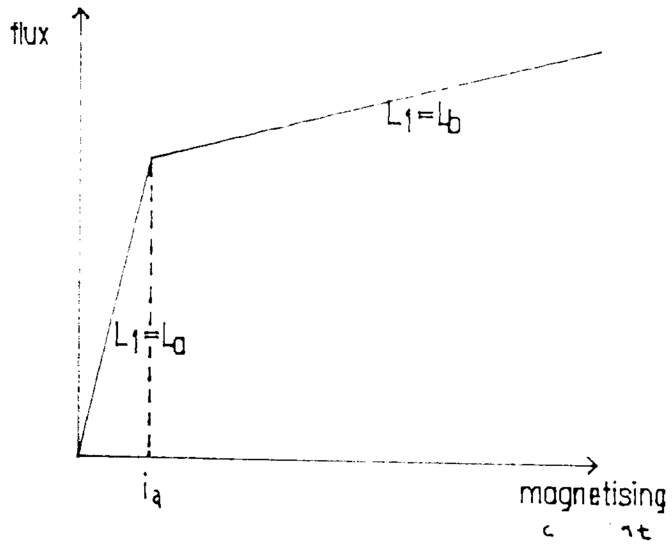


Figure 5.5 Approximate magnetisation characteristic of CT

$X_m = R = 1$

Since the amplitude of the input is one, therefore $|I_1| = |I_2| = \frac{1}{\sqrt{2}}$

From figure 5.4, the current in the magnetising branch,

$$i_1 = -\frac{1}{\sqrt{2}}\cos\frac{5}{12}\pi = -0.183$$

The current in the secondary circuit, $i_2 = \frac{1}{\sqrt{2}}\cos\frac{1}{12}\pi = 0.683$

These values can be checked with the analytical solution. Since

$t = \frac{1}{6}\pi$ and $\theta = 0$

$$i_1(\tfrac{1}{6}\pi) = -\frac{1}{2}\cos\frac{1}{6}\pi + \frac{1}{2}\sin\frac{1}{6}\pi = -0.183$$

So the two methods give the same result.

5.4.2 Inclusion of saturation

Saturation of the current transformer can be considered using a magnetisation curve (figure 5.5) represented approximately by two straight lines. This makes it possible to consider the magnetising inductance to be constant in a given region. The inductance is changed when the magnetising current is above or below a certain value. Higher accuracy can be obtained if additional straight lines are used to represent the magnetising characteristic. Whether CT saturation needs to be included will depend on many factors, such as the types of CT used.

5.4.3 Program

The flow diagram of the program is shown in figure 5.6, and the following data is required, e.g. the magnetising characteristic,

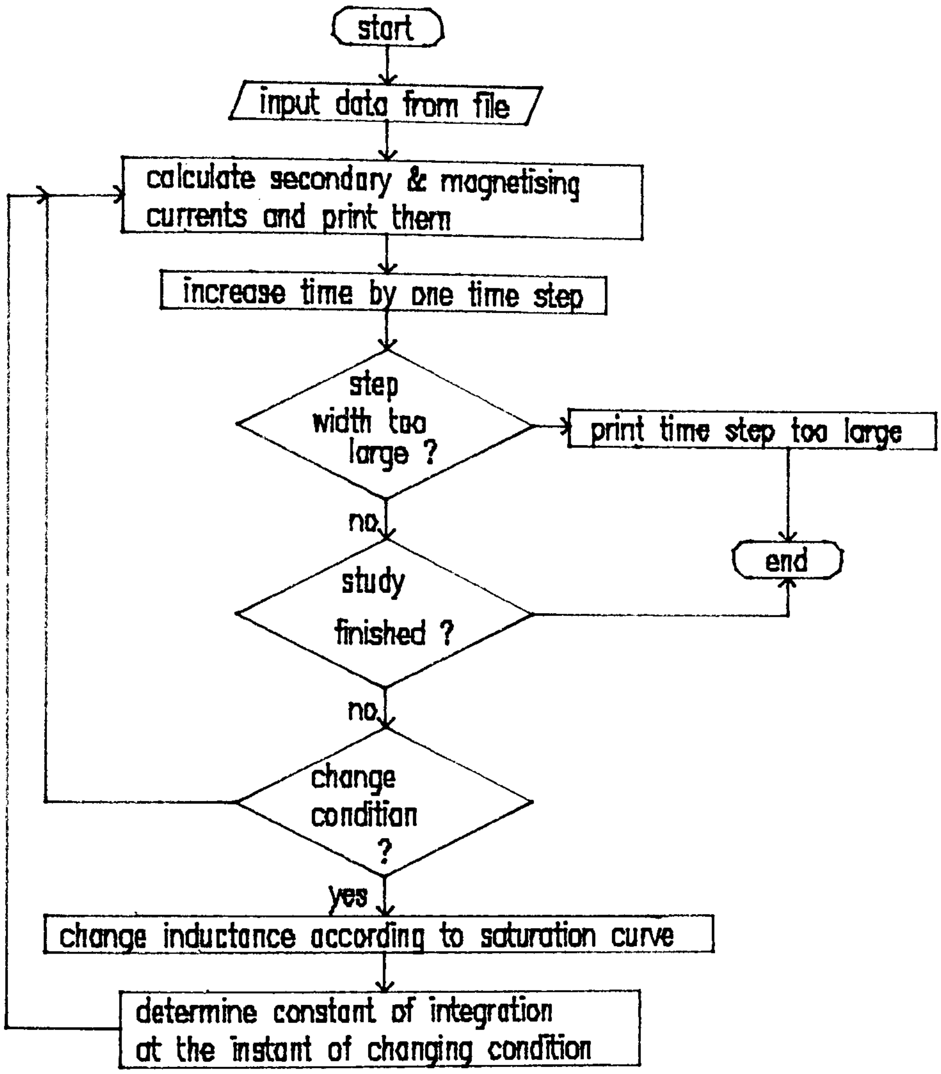


Figure 5.6 Flow diagram for current transformer modelling

the initial magnetising and burden currents, the burden inductance, L_2 and burden resistance, R_2.

The main features of the algorithm are described below:

(a) At $t = \emptyset$, for linear operation, then set $L_1 = L_a$, and calculate C from the transient equation for i_1(see appendix A).

(b) Calculate i_1 at various values of time and check for $|i_1| > i_a$, then $L_1 = L_b$ and C is re-calculated.

(c) Calculate i_1 and check for $|i_1| < i_a$, then $L_1 = L_a$, C is calculated and program returns to (c).

(d) Calculation stops when the time limit specified is reached.

A listing of the program is shown in appendix J (** J7 **).

5.5 Summary

A program has been developed for current transformer modelling. This program was used to check whether the fault current would be high enough to saturate the current transformer. If saturation occurs then the relay operating time cannot be determined by the program developed in chapter 4. Distorted current will affect relay operation, and the experiments carried out to determine these effects are described in chapter 6.

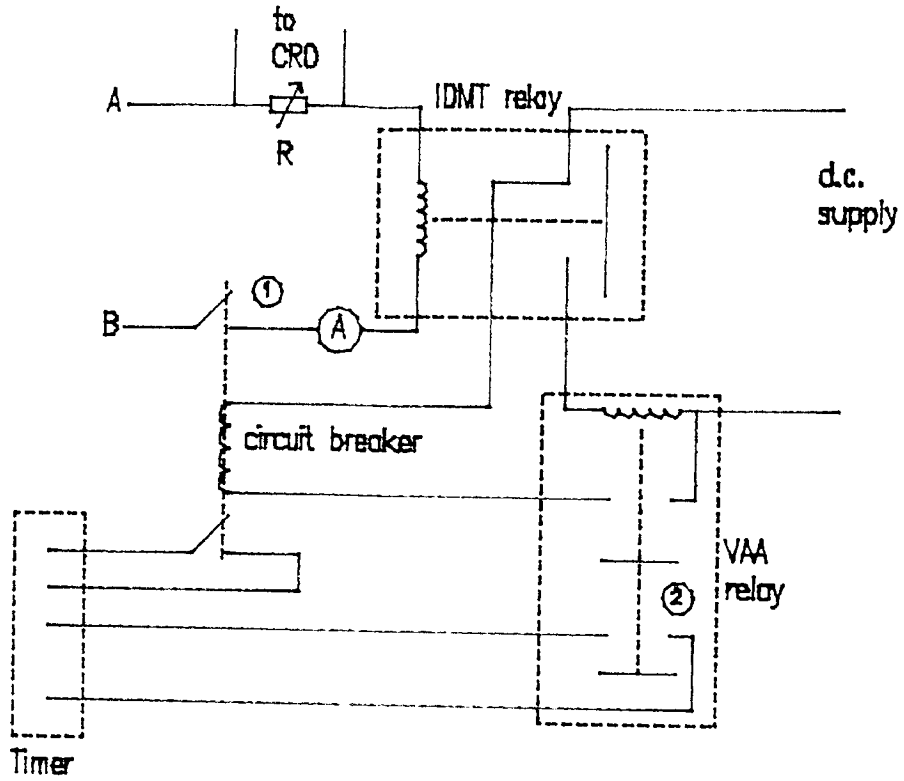


Figure 6.1 Circuit diagram for testing relay with different operating currents

Chapter 6

Experimental work and results

This chapter deals with the experiments which were carried out to study how d.c. and distorted a.c. currents affect relay operation. The experimental results are also presented.

6.1 Experimental conditions

By reference to figure 6.1, all the relay contacts are normally open when the relay coils are not energized.

When (1) is closed manually, the timer starts to count.

When (2) is closed, the timer stops.

Once the timer is started by closing (1), the IDMT relay will commence operation. When the contact of the IDMT relay is closed there will be a d.c. current through the d.c. trip coil of the circuit breaker and the VAA relay will also close and the timer stops counting. To provide an undistorted sine wave of current for the relay, it was necessary to ensure that the ratio of the circuit impedance to relay impedance be high and that the impedance of the circuit was not a function of current. That is, if the input to terminals AB is a sinusoidal voltage, and R is chosen to have a high value, compared with the impedance of the relay, then the current through the relay will be close to a sinusoid. As R is reduced then the impedance of the relay becomes increasingly significant. The inductance of the relay as in all iron cored magnetic circuits, is a function of current. The impedance to the flow of harmonic currents over the normal operating range of the relay is not directly proportional to the order of the

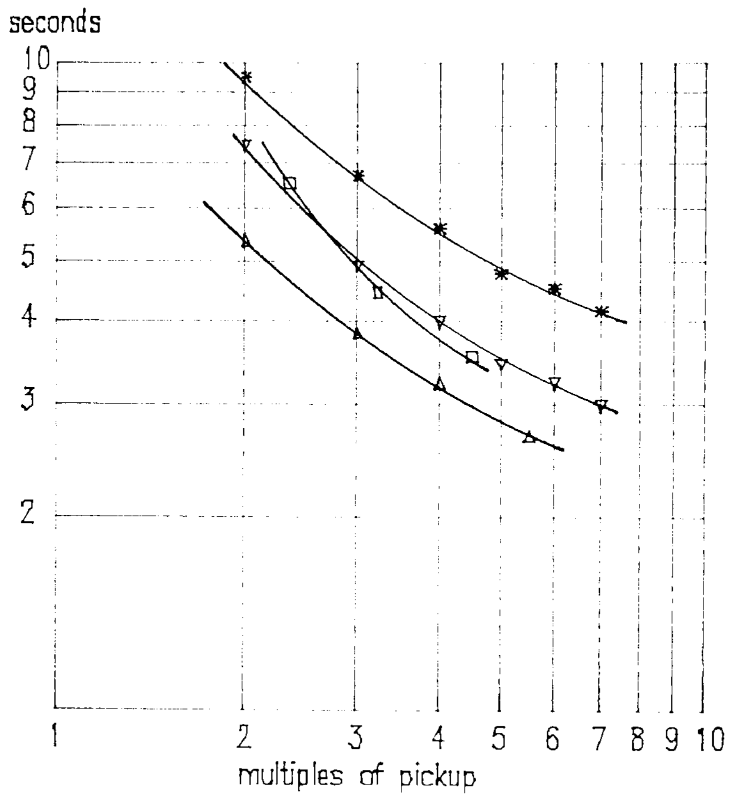


Figure 6.2 Relay operating time with distorted and pure third and fifth harmonic currents

* distorted current
▿ 50 Hz current
▵ 150 Hz current
□ 250 Hz current

TMS = 1.0

harmonic, due to saturation in the magnetic circuit. So if a sinusoidal voltage is input to the terminals AB, the current will be distorted; R was used to control the amount of harmonic distortion.

6.2 Relay operation with different current waveforms

6.2.1 With d.c. current

The pickup current of the relay was set at 1A. A d.c. source was applied to the terminals AB and R was varied so that the maximum current was limited to 5A. The relay did not operate.

6.2.2 Exponential current

A capacitor was charged and connected to the terminals AB. The pickup current setting was the same as described above. R was set such that the current started at about 5A and decayed to about 2A in approximately 10 seconds. This test was conducted to ensure that the relay did not operate with a varying d.c. current.

6.2.3 50 Hz sinusoidal current

R was set such that there was only a small harmonic component in the current waveform. The harmonic component was measured by using a wave analyser. The pickup current of the relay was set to 1A. The voltage level was varied such that a current up to 8A could be obtained. The operating time required to close the relay contacts was plotted against multiples of pickup, which is equal to the relay current divided by the pickup current. The result is shown in figure 6.2.

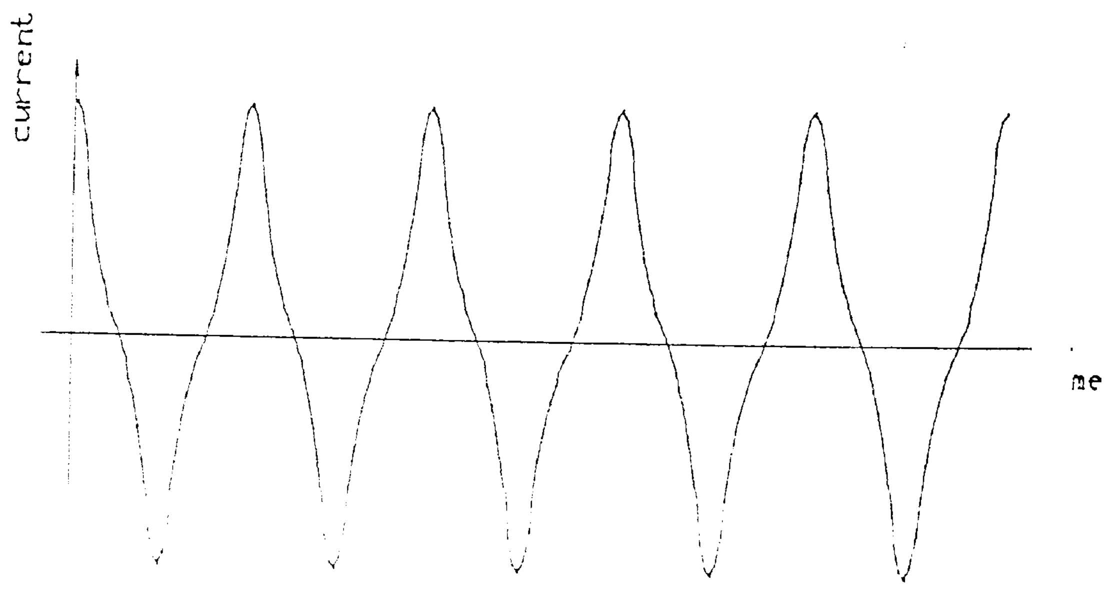


Figure 6.3 The waveform of a distorted relay current

6.2.4 Distorted current

The same procedure as described in 6.2.3 was used, except that R was set to a small value. The oscilloscope shows that the current waveform is distorted, and a trace was reproduced in figure 6.3. Using the wave analyser, the third harmonic component was estimated to be about 25% of the fundamental. The relay operating time was plotted against multiples of pickup current, and the result is shown in figure 6.2. The figure shows that distorted current delays relay operation. The delay is equal to the time difference between the two curves (✱ and ∇) as shown in figure 6.2. It was reported in [98] that different levels of distortion will produce different delays in relay operation.

6.2.5 Relays tests with 3rd and 5th harmonic currents

The same procedure used in 6.2.4 was adopted, except that R was set to a high value to prevent current distortion. The relay current was supplied from a generator with a normal rating of 5A. The frequency of the relay current was set to 150 Hz and checked with an oscilloscope. With the relay current set at 250 Hz, the same experiment was repeated again. The relay operating time was plotted against multiples of pickup and the results are shown in figure 6.2. The figure shows that a sinusoidal 150 Hz current will produce faster relay operation, and if the frequency is increased to 250 Hz, there will be a reduction in relay pickup sensitivity. At the higher frequency, relay operation is erratic for a PSM below 2.3.

6.3 Discussion

If the r.m.s. current is used to determine the relay operating time, the result will produce an erroneously high estimate of operating torque because the waveform distortion and its effect on torque will not be correctly accounted for. From [96,97], it was shown that the torque acting on the relay disc due to a distorted current is equal to the resultant torque produced by the fundamental and individual harmonics of the distorted current.

Torque in the relay is produced by the reaction of the flux from one pole and the current induced in the disc by the flux from the other pole. Experiment has shown that a distorted current will delay relay operation. One reason may be that the phase angle between pole flux and disc current is significantly different when compared with a sinusoidal current. The magnitude of the fundamentals of the two variables may be nearly the same, but the torque produced by the distorted wave will be much less. Another reason may be because the distorted current has a pronounced third harmonic peak. It will produce a greater m.m.f. and so produces a flat-topped flux waveform because of magnetic saturation, with a consequent reduction in torque. The torque curve in [96,97] shows that the reverse torque is much larger with a distorted current.

For a distorted current, usually the third harmonic is more important than the other harmonics. From test it was shown that pure third harmonic will produce faster relay operation. So it is not difficult to see that if harmonic distortion is increased the operating time will continue to fall [98].

6.4 Summary

In this chapter, the effect due to d.c. and a.c. distorted current on relay operations were studied. It is concluded that distorted current will affect relay operation. If no correction factor or modification to the relay characteristic is introduced, then the relay characteristic as provided by manufactures should not be used to calculate the relay operating time when the relay is operated with a distorted current. The next chapter deals with results from computer simulation. Part of the chapter deals with coordination protection, and it is assumed that the relay current is not distorted. Results on fault studies and circuit breaker opening are presented.

Chapter 7

Results on protection and fault analysis

Results obtained by simulation are presented here, and are divided into the following sections. Section 1 covering protection simluation and section 2 covering the simulation of faults and circuit breaker opening.

7.1 Results on protection coordination

7.1.1 Relay pickup and operating time

The relays (2,3,4,5) shown in figure 2.1 were graded by using the coordination program. The relay characteristics, plotted by the curve plotting program, are shown in figure 7.1. By increasing the pickup value of relay 3, other relay settings are obtained, and results are replotted in figure 7.2. It shows that the higher the pickup value, the faster the relays will operate at high fault levels.

7.1.2 Position of minimum time difference

For different types of relay to grade with each other, the minimum difference in operating time does not necessarily occur at the maximum fault level. Consider a standard inverse relay which is required to grade with an extremely inverse relay. If the TMS and pickup for the extremely inverse relay are Ø.25 and 15 MVA respectively and the maximum fault level is 8Ø MVA, then one setting for the standard inverse relay will be 27 MVA and Ø.1322 for the pickup and TMS respectively to achieve the grading margin of Ø.4 second at the 8Ø MVA level. The current grading is also satisfied. If the time difference

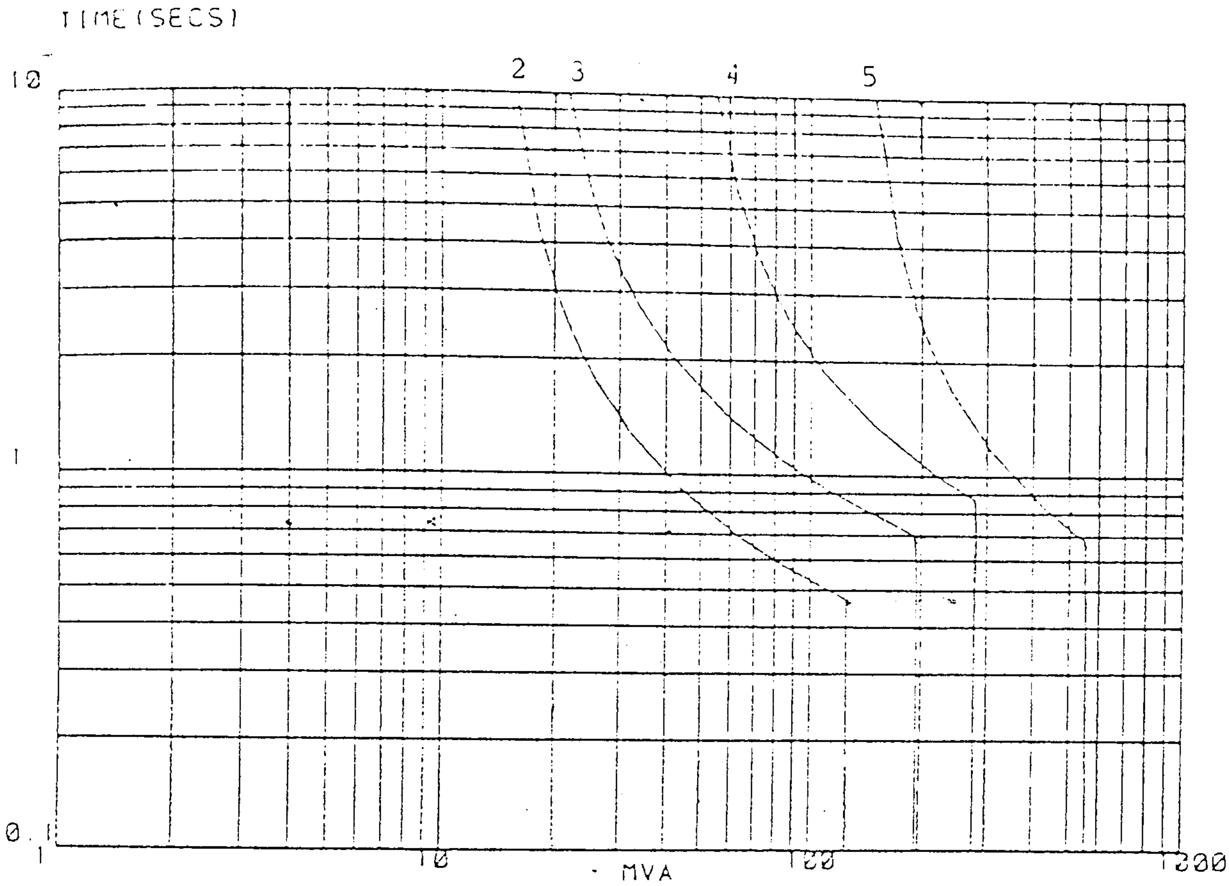


Figure 7.1 Relay curves with the data as shown below

Relay	Type	TMS	Pickup	Maximum fault level
2	IDMT(3/10)	0.15	14.28	125
3	IDMT(3/10)	0.24	18.57	195
4	IDMT(3/10)	0.225	47.57	280
5	IDMT(3/10)	0.138	137	550

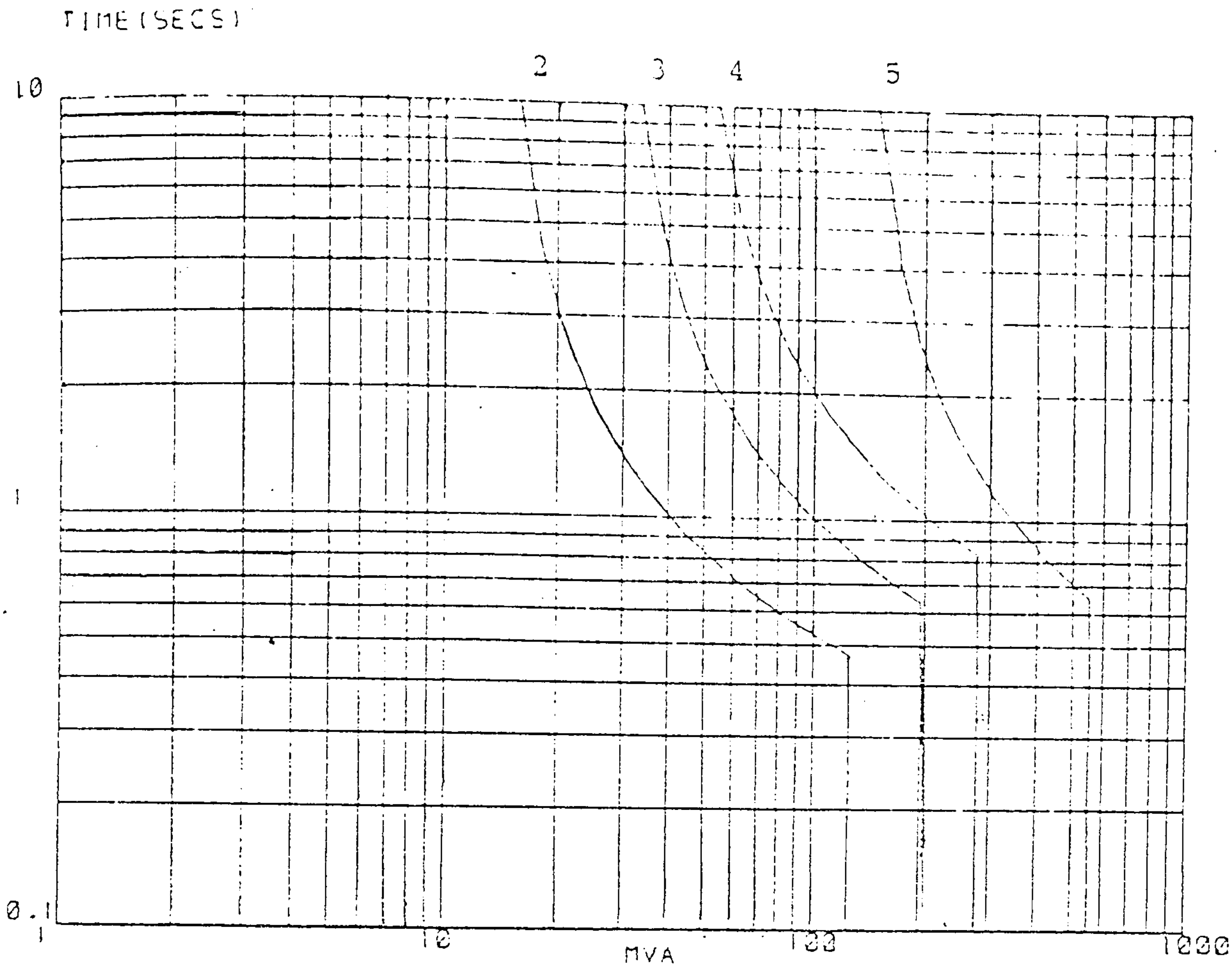


Figure 7.2 Relay curves with the data as shown below

Relay	Type	TMS	Pickup	Maximum fault level
2	IDMT(3/10)	0.15	14.28	125
3	IDMT(3/10)	0.174	30	195
4	IDMT(3/10)	0.214	47.57	280
5	IDMT(3/10)	0.133	137	550

Base MVA = 776 MVA

Machine parameters

$H = 3.16$	$r_a = 0.0021$	$x_d' = 0.32$	$x_d =$ .27
$x_d'' = 0.26$	$x_q = 2.15$	$x_q' = 0.3$	x_q''].26
$x_a = 0.17$	$x_o = 0.1933$	$T_d' = 0.95$	T_d''].02
$T_q' = 0.82$	$T_q'' = 0.02$	$T_L = 0.0$	$r_e =$ 0019

If no fictitious field winding is included, then $x_q' = 2.15$

Saturation characteristics

Flux linkage	0.6	0.7	0.8	0.9	1.0	1.1	1.2
Excitation	0.6	0.7	0.832	0.963	1.16	1.419	1.824

Generator transformer impedance (if transformer tap ratio = 1)

$r_T = 0.0039$ $x_T = 0.1551$

Transmission line impedance

$r_L = 0.0009$ $x_L = 0.031$

Initial currents

$i_{Ta} = 0.378$ $i_{Tb} = 0.613$ $i_{Tc} = -0.991$ $i_{fd} =$ 088

$i_{kd} = i_{kq} = i_{fq} = 0.0$

Infinite busbar voltage = 0.988, mechanical torque = 0.852

$\theta = 0.3514$, transformer tap ratio = 1.083

Table 1 Machine parameters and the initial conditions

between the two characteristics are studied, it can be established that the minimum time difference will occur at about 42.75 MVA, and the difference is only about 0.28 second.

7.1.3 Practical solution

In practice, usually a lower pickup value is chosen for relay coordination rather than a higher value as shown from the computer results. There are two reasons for this. The first is that it will be easier for current grading to be achieved; and the second reason is that if a higher pickup value is chosen then the relay operating time will be larger at the low fault level, so the relay cannot act as an effective backup device for other relays. As far as the grading of different types of relays is concerned, usually the difference between the pickup values are large. So the minimum time difference occurs at the maximum fault level and the problem as shown from the computed result will not happen.

7.1.4 Transient effect on relay operating time and i^2t

Let us refer to figure 1.1 to compare the relay operating time with and without transient conditions. Let a three-phase to earth fault occur on the HV side of the transformer. The initial conditions and data for the parameters are shown in table 1, except that the armature currents are replaced by the generator transformer secondary currents such that $i_{Ta} = 0.378$, $i_{Tb} = 0.613$, and $i_{Tc} = -0.991$. Assuming no CT saturation, after knowing the primary fault current the relay operating time due to the CT secondary current can be obtained by the programs developed in chapter 4. The a.c. amplitude of this current was obtained and the operating time for the relay under the transient conditions computed. The envelope of the a.c. peak is divided into small steps for the calculation of relay operating time if the pickup current is at a peak value. The i^2t can be calculated with the programs developed in chapter 4 if the transient effect is considered. In practice, if no transient effect is taken into account then the relay operating time is calculated as follows:

Usually the transient state of the machine is used for calculating the relay operating time. If the reactance is much bigger than the resistance then the relay operating current $Iop = \frac{Eop}{X}$ where Eop is the internal machine voltage and X is the total reactance from the machine to the fault location such that $X = x_d' + x_T$ when the tap ratio is one.

For this particular machine Eop = 1.08 p.u. and X = 0.1552 + 0.32 = 0.4752, so Iop = 2.27 p.u.. The relay operating time = $\frac{0.14 \times 0.1}{(2.27)^{0.02} - 1}$ = 0.847 sec. if TMS = 0.1 and pickup = 1.0 p.u.

If no transient effect is considered then the $i^2t = (2.27)^2 \times 0.847$ = 4.365.

The comparsion is summarized as below:

	relay operating time (second)
with transient effect taken into account	0.925
No transient effect is considered	0.847
	(i^2t)
with transient effect taken into account	2.544

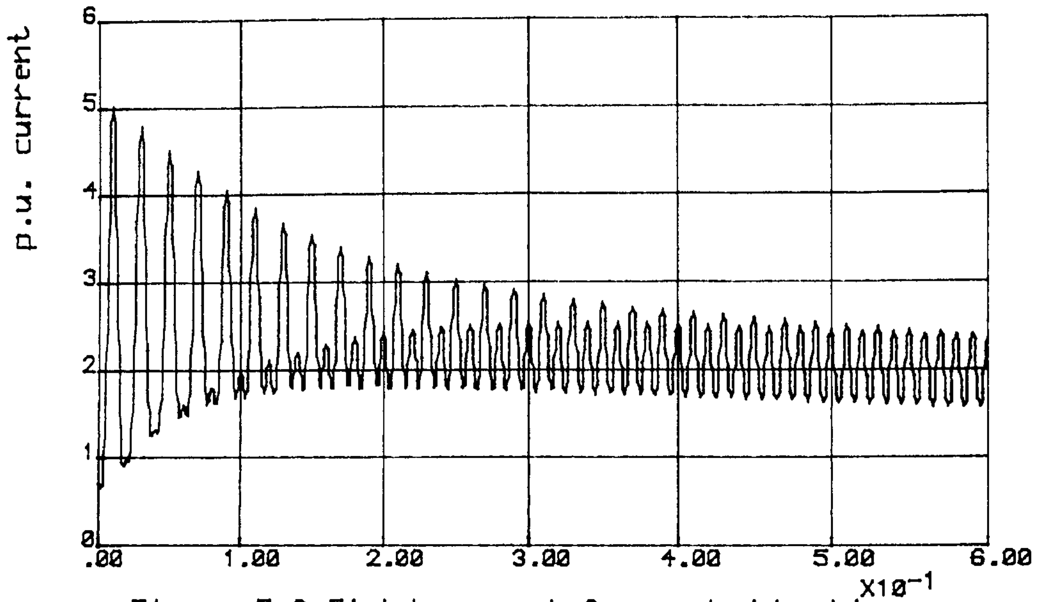


Figure 7.3 Field current for a double line (phases b&c) to earth fault

The generator is on open circuit
Initial currents are zero except $i_d = 0.71428$
$\theta = -0.5\pi$

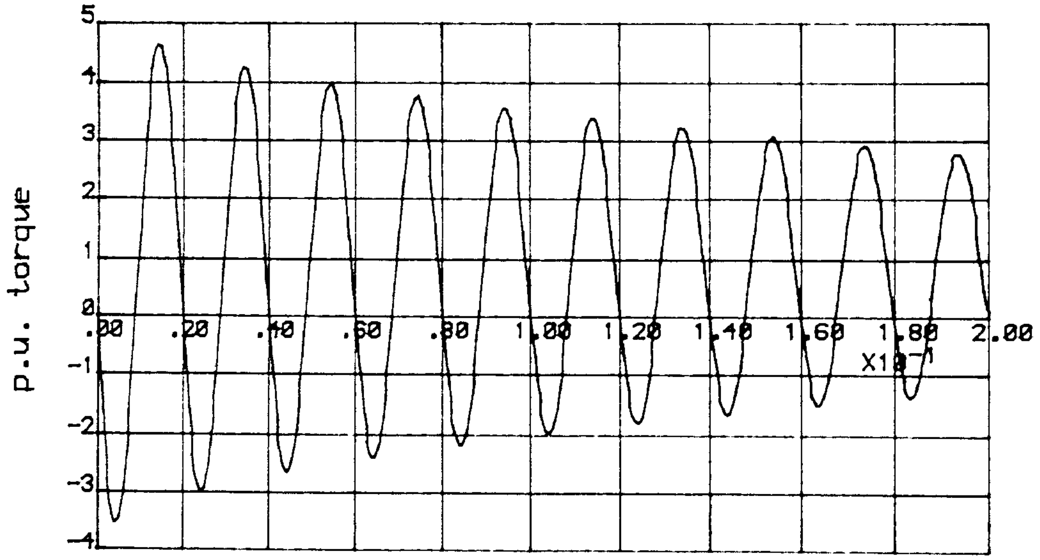


Figure 7.4 Accelerating torque when a three phase to earth fault occurs on a loaded generator

Required data are given in table 1

No transient effect is considered 4.365

7.2 Results on fault analysis

Many computer results were produced, but only the more important are shown in this chapter.

7.2.1 Open circuit condition

Figure 7.3 shows the field current when a double line to ground fault occurs on phases b and c with the generator on open circuit. The second harmonic component is evident. When negative sequence currents appear in the armature of a machine running at synchronous speed, the armature m.m.f. due to these currents produces a flux which rotates at synchronous speed in a direction opposite to the field produced by positive sequence currents. The negative sequence flux field is thus travelling at twice synchronous speed with respect to the rotor, and as it sweeps past the field poles, a second harmonic current is induced in the field winding.

7.2.2 Loading conditions

Figure 7.4 shows the accelerating torque for a generator when a three-phase to earth fault occurs on its terminals. The generator is on full load when the fault occurs. The initial conditions are given in table 1. The generator output becomes zero in the pure reactance circuits. As the electrical power is decreased by the short circuit and the mechanical power remains constant, there is an accelerating torque on the generator and consequently the generator speeds up. Figure 7.5 & 7.6 show the effect of varying the inertia constant on

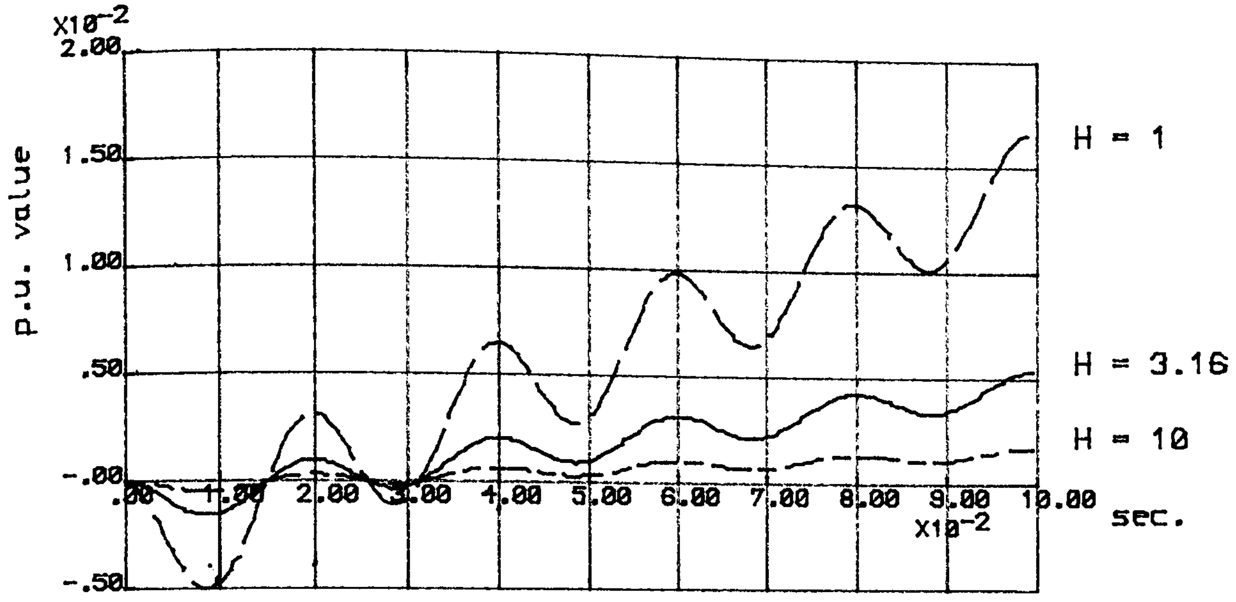


Figure 7.5 Effect of variation in inertia constant on rotor angular velocity

Data are given in table 1 except H is varied as shown above

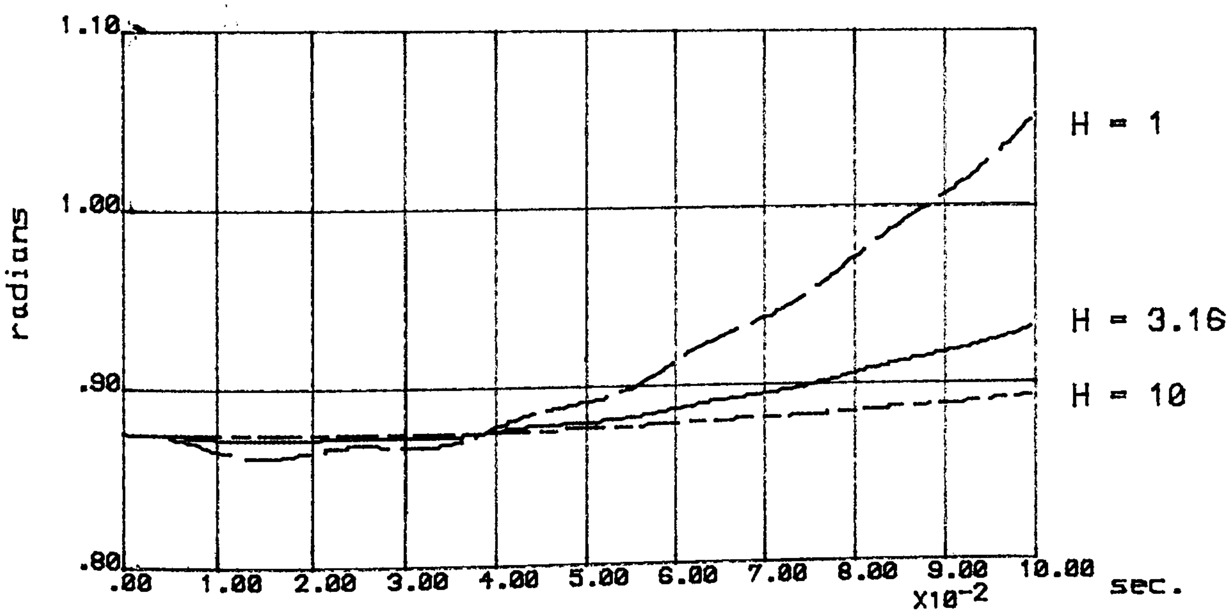


Figure 7.6 Effect of variation in inertia constant on rotor angle

the rotor speed and rotor swing. The smaller the inertia constant, the higher the variation in rotor speed and the greater the back swing.

7.2.2.1 Effect due to saturation

During most fault and short circuit conditions, the rotor winding is highly inductive and the rotor flux linkages before the short circuit are produced by $\emptyset + \emptyset_1$ where $\emptyset$ is the air gap flux and $\emptyset_1$ is the leakage flux which traverses paths outside the active transfer or conversion regions (for example, the core of a transformer or the air gap of a rotating machine). Immediately following the application of the short circuit it may be assumed that the flux linking the rotor remains constant, this being brought about by an induced current in the rotor which balances the heavy demagnetizing effect set up by the short circuited armature. So $\emptyset + \emptyset_1$ remains constant, but owing to the increased m.m.f involved, the leakage flux will increase considerably. Therefore there is a transfer of flux from the main air gap to the leakage paths. Much of the leakage occurs in the iron paths of the machine and hence must be affected by saturation. To calculate the saturation effect on leakage reactance rigorously is very time consuming. One must recognize that the calculation of leakage reactance of a machine is a three dimensional electromagnetic field problem complicated by the nonlinearity of iron and the fact that the flux density is different in different sections of the magnetic path and varies with operating conditions. In practice [11Ø] a simple engineering solution is possible by introduction of a correction factor of approximately Ø.9 to account for the reduction in reactance.

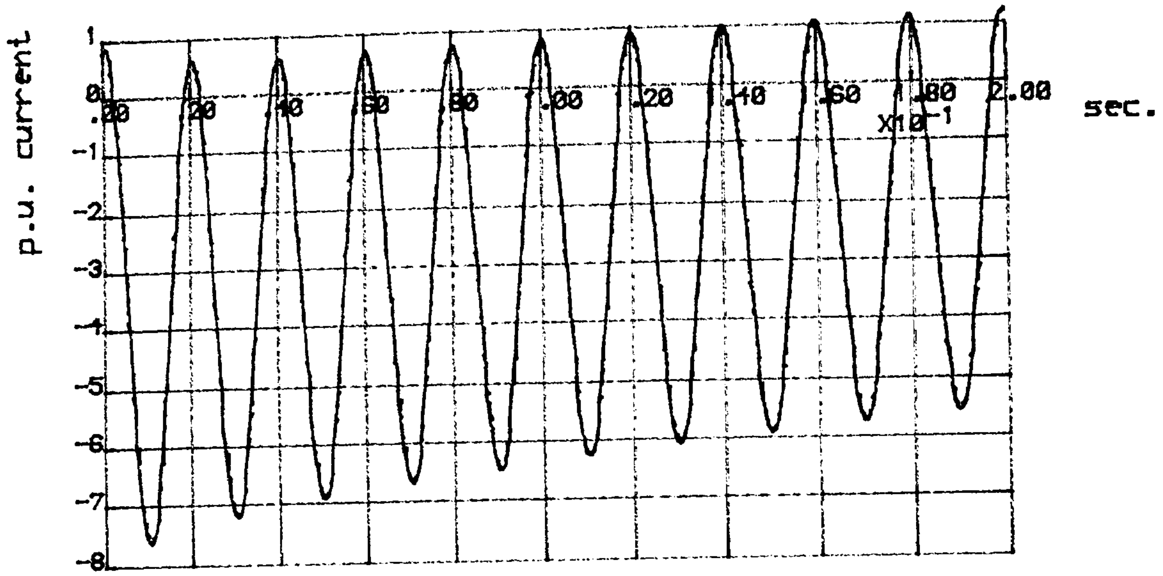


Figure 7.7 Effect of variation of mutual reactance due to saturation on armature current

Data are given in table 1.With saturation the following changes are made:
8 = 0.2946, saturation factor = 1.291
i_{fd} = 2.282

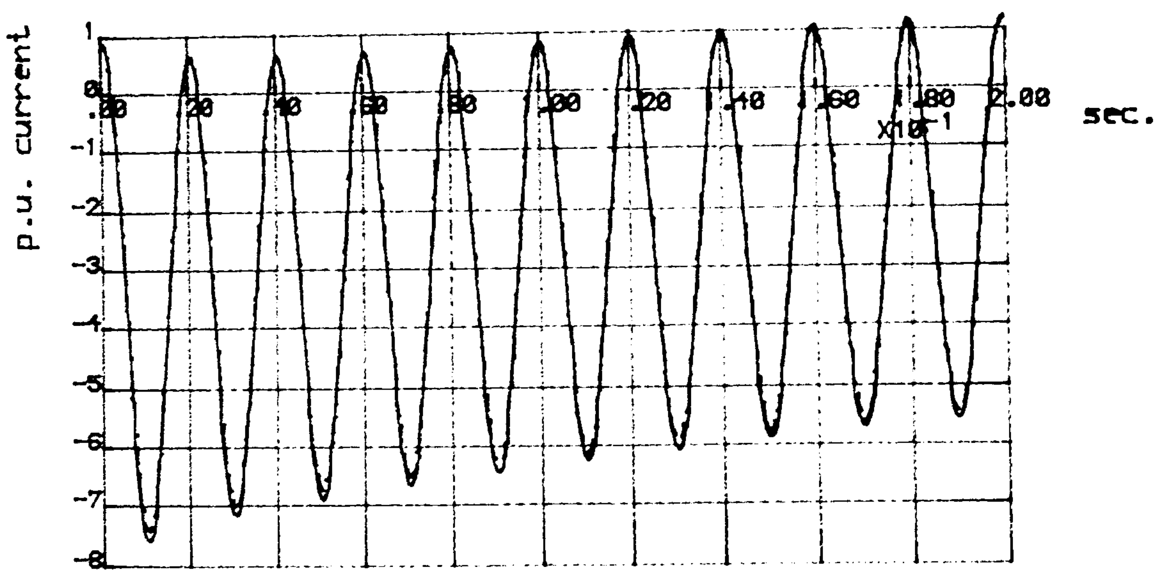


Figure 7.8 Effect of variation in armature leakage reactance on armature current

Data are given in table 1.When the leakage reactance is increased x_a = 0.221
i_{fd} = 2.123 8 = 0.3568

As the short circuit current continues, the main flux is reduced for the following reasons.

(1) The effects, which try to maintain constant flux linkages, due to damper windings and field circuits die away.

(2) Demagnetising action of the armature short circuit current.

A fall in the generated voltage results, such that the main air gap flux will fall below a saturation level.

The amplitude of the transient currents are mainly determined by leakage and not by mutual reactance. Figure 7.7 shows that saturation of mutual reactance does not affect the armature winding transient fault current. Figure 7.8 shows that by varying the leakage reactance, the armature winding current is varied. A variation of 30% in the armature winding leakage reactance only produces a variation of 4% in the armature winding current.

7.2.2.2 Effect due to the iron rotor

(a) Lagging power factor operation

Figure 7.9 shows the current in phase a when a three phase to earth short circuit occurs at the generator terminals (with a fictitious field winding on the quadrature axis). The fault current passes through a current zero within the first cycle.

With the same fault conditions as just described above but without a fictitious field winding on the quadrature axis, figure 7.10 shows that the current zero is delayed by about 80 msec.. This result shows the importance of including a fictitious winding.

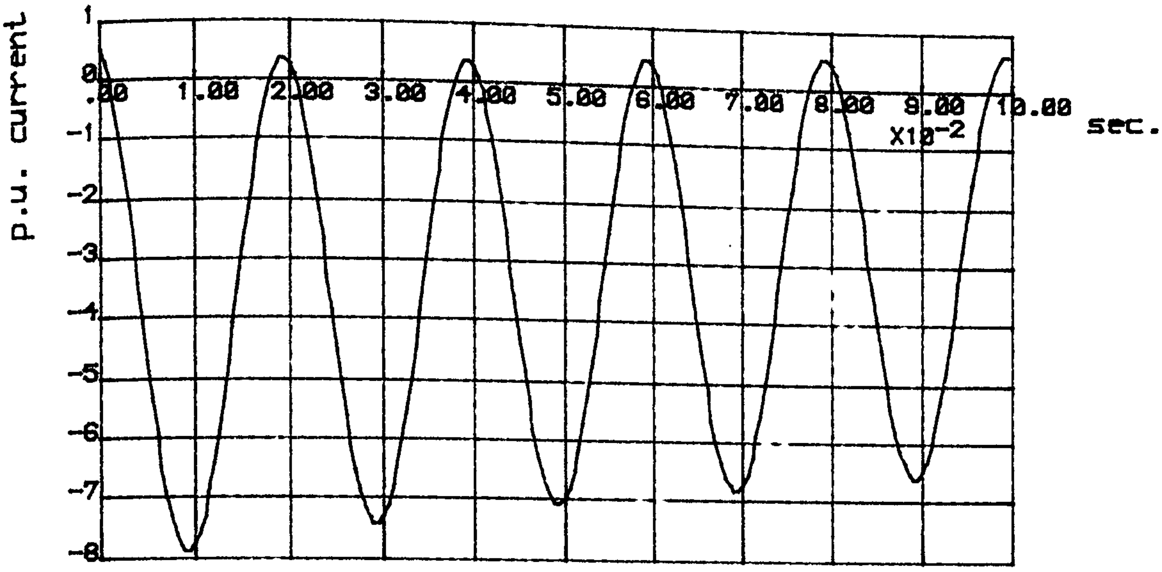


Figure 7.9 Current in phase a when a three phase to earth fault occurs at the generator terminals

Lagging power factor operation
With a fictitious field winding included
Initial conditions are given in table 1 except
$i_a = 0.5 \quad i_b = 0.5 \quad i_c = -1.0 \quad \theta = 0.739$

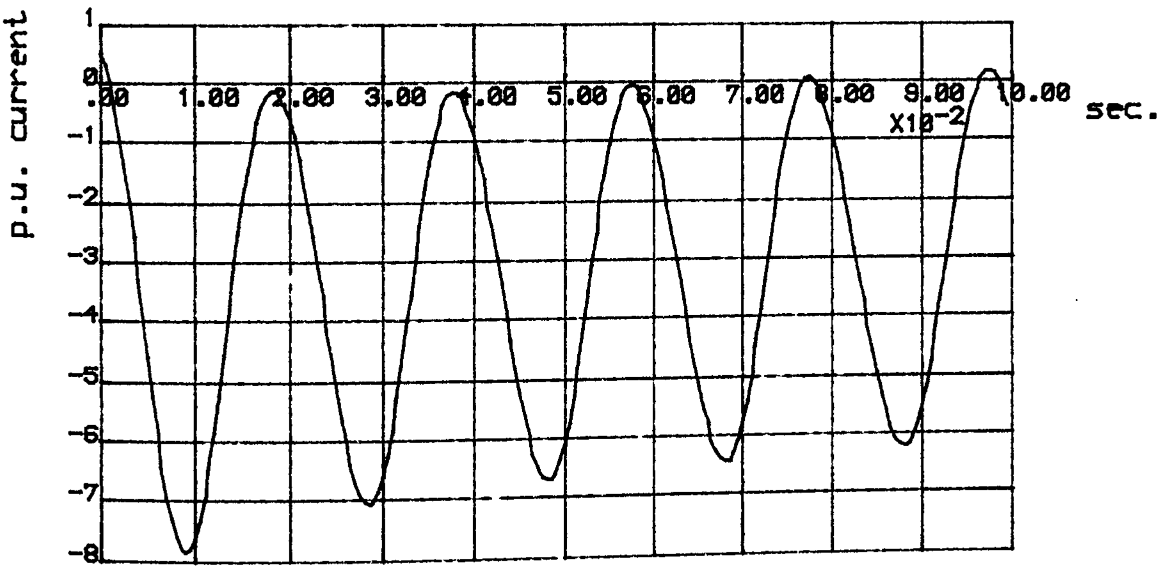


Figure 7.10 Same description as in figure 7.9

Except no fictitious field winding is included

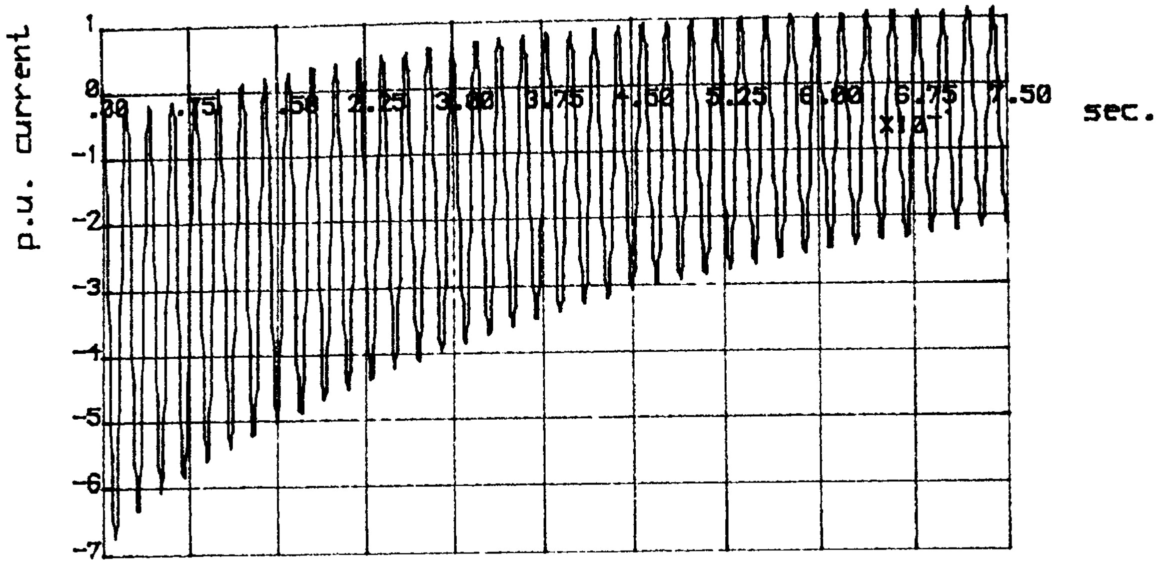


Figure 7.11 Current in phase a when a three phase to earth fault occurs at the generator terminals

Leading power factor operation
With a fictitious field winding included
Initial conditions are given in table 1 except
θ = 1.135 busbar voltage = 1.016 i_{fd} = 1.379
Transformer tap ratio = 0.935
i_a = -0.075 i_b = 0.853 i_c = -0.778

Figure 7.12 Same description as in figure 7.11

Except current in phase b is shown here

(b) Leading power factor operation

Figures 7.11, 7.12, 7.13 show that when the generator is operated at leading power factor before a three-phase to earth fault occurs at its terminals, the armature currents will be delayed in some phases before passing through a current zero. Current in phase a is delayed by about 100 msec..

With the same fault conditions as just described above but without a fictitious winding in the quadrature axis, figures 7.14, 7.15, 7.16 show that the currents in phase a and phase b are delayed by about 500 msec. and 400 msec. respectively before a current zero is reached.

From the results obtained for (a) and (b), it shows that an unrealistic decay in the a.c. component of the fault current occurs if a fictitious field winding is not included in the quadrature axis. Comparsion of figure 7.9 and figure 7.10 will show a delay of 4 cycles. The time taken for some phases to pass through a current zero is much longer when the generator is operated at a leading power factor. In practice, the a.c. component of the fault current does not decay as rapidly as when no fictitious field winding is included. So the quadrature axis presents subtransient and transient effects to the fault currents, and therefore the value of x_q' is an important factor in obtaining correct fault current prediction. For modelling a round rotor machine, a fictitious field winding should be included in the quadrature axis to simulate the solid iron section. This solid iron acts as an equivalent winding during both transient and subtransient periods, thus providing multiple paths for circulating eddy currents. The changing reactance can be represented as the sum of two

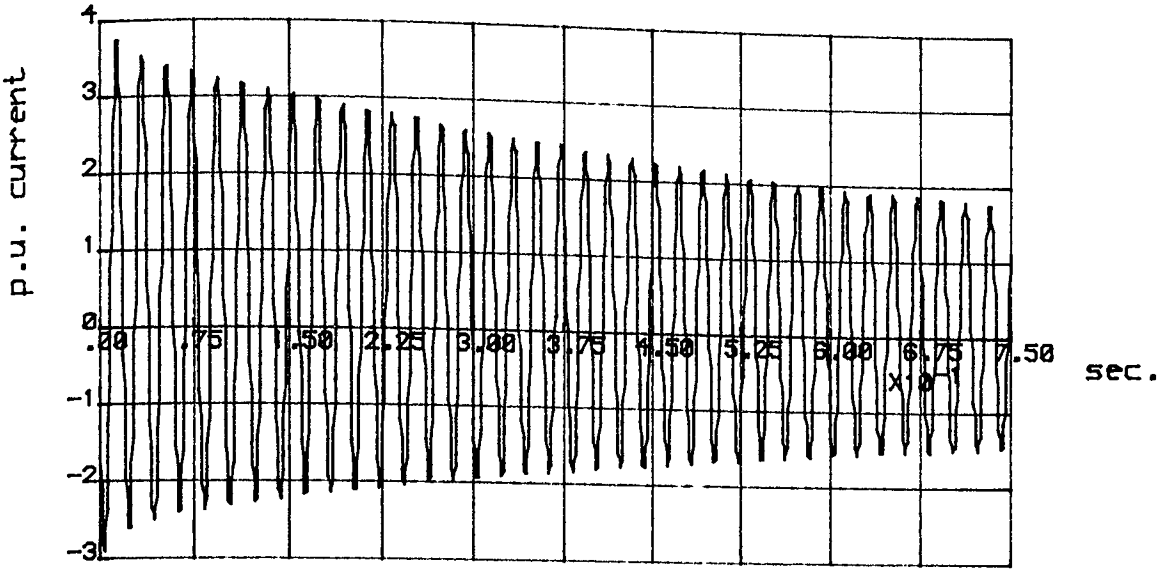


Figure 7.13 Current in phase c when a three phase to earth fault occurs at the generator terminals

Leading power factor operation
With a fictitious field winding included
Initial conditions are given in table 1 except
$i_a = -0.075$ $i_b = 0.853$ $i_c = -0.778$
$\theta = 1.135$ busbar voltage = 1.016 $i_{fd} = 1.379$
Transformer tap ratio = 0.935

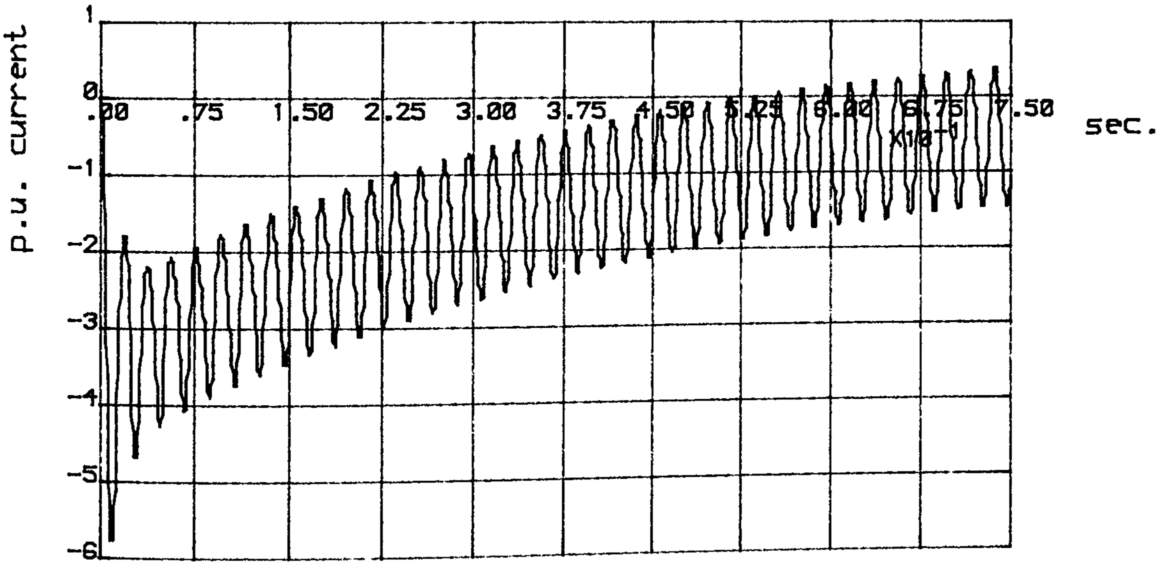


Figure 7.14 Same description as in figure 7.13

Except no fictitious field winding is included and current in phase a is shown here

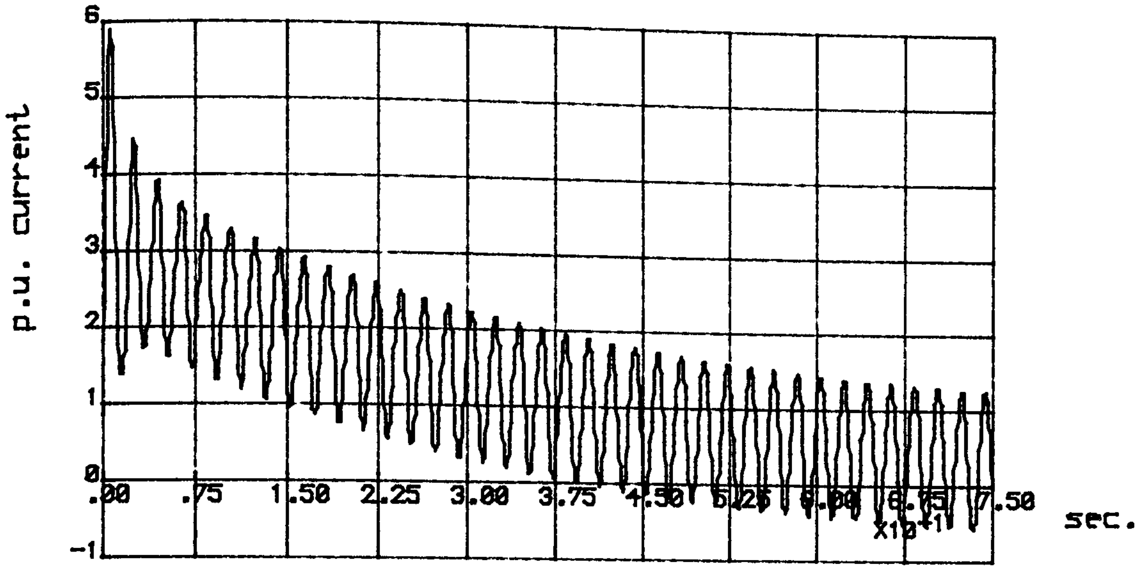


Figure 7.15 Current in phase b when a three phase to earth fault occurs at the generator terminals

Leading power factor operation
No fictitious field winding is included
Initial conditions are given in table 1 except
i_a = −0.075 i_b = 0.853 i_c = −0.778
θ = 1.135 busbar voltage = 1.016 i_{fd} = 1.379
Transformer tap ratio = 0.935

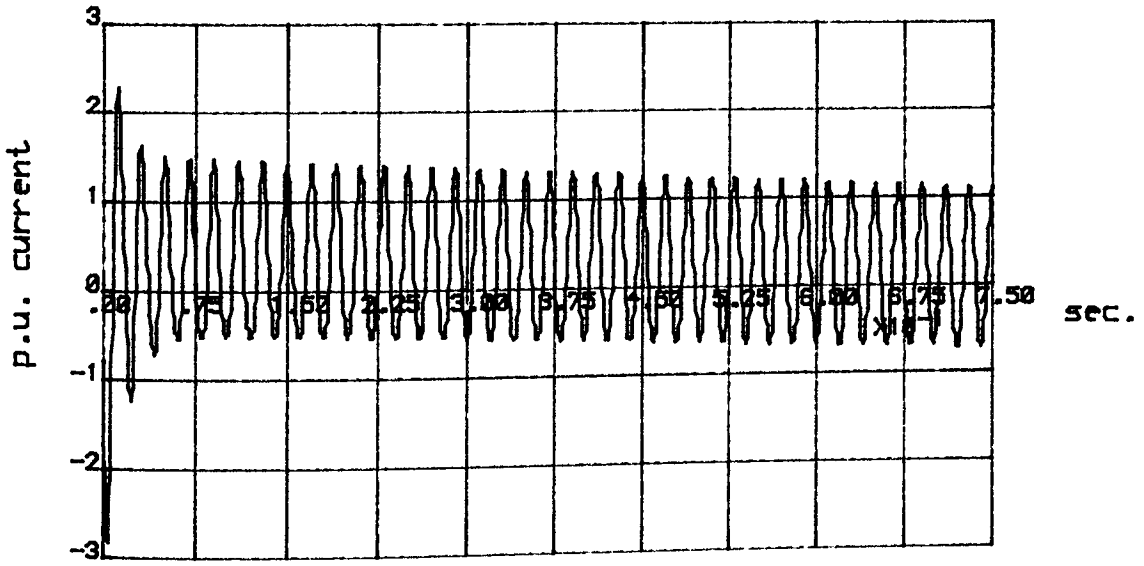


Figure 7.16 Same description as in figure 7.15

except current in phase c is shown here

exponentials, one of which decays rapidly and the other slowly. The slower transient is defined by x_q' and the faster transient by x_q''.

7.2.2.3 Effect due to circuit breaker opening

Consider a three-phase short circuit to occur at the generator terminals when the machine is running on no load, the a.c. component and the d.c. component are equal at t=0. The a.c. component decays initially with the time constant T_d'' and the d.c. component decays exponentially in accordance with the time constant $T_{dc} = \frac{X_d'' + X_t}{(R_a + R_t)w}$

where

X_d'' = Subtransient reactance of the generator

R_a = Resistance of the generator armature winding

X_t = Leakage reactance of the transformer

R_t = Winding resistance of the transformer

w = angular frequency

The a.c. subtransient time constant is smaller than the d.c. time constant so that after initiation of a fault the subtransient a.c. component decays more rapidly than the d.c. component. A current cannot pass through zero if its d.c. component is larger than the amplitude of the a.c. component. The result is that the lower envelope of the current curve rises above zero, and hence the time taken for the current to pass through a current zero is increased. After a given time has elapsed the first zero occurs, and the a.c. current decays

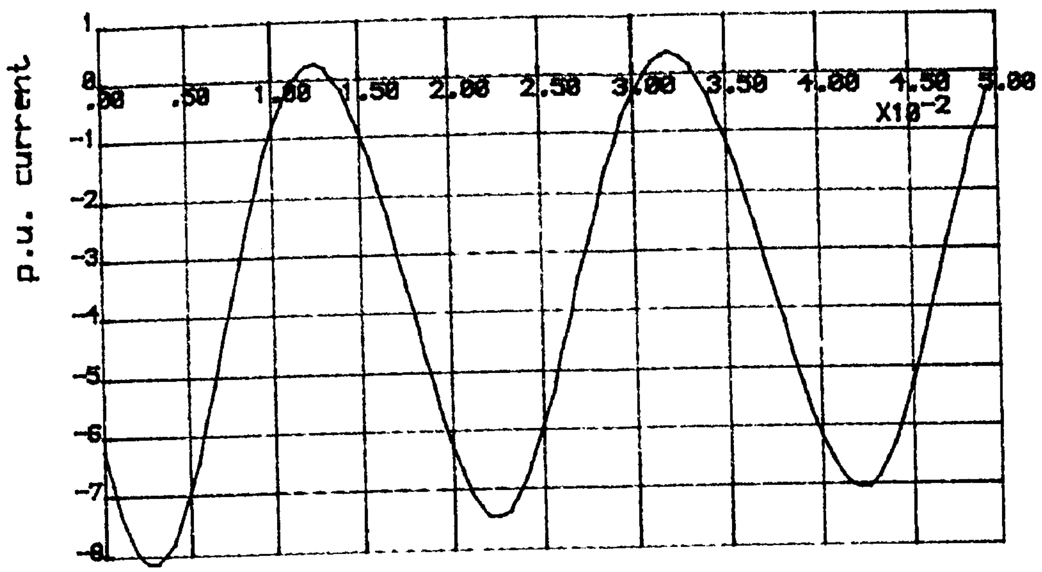


Figure 7.17 Current in phase a when a three phase to earth fault occurs at the generator terminals

Lagging power factor operation
No fictitious field winding is included
Initial conditions are given in table 1 except
$i_a = 0.5$ $i_b = 0.5$ $i_c = -1.0$ $\theta = 0.739$
Only shows the current after phase c is opened

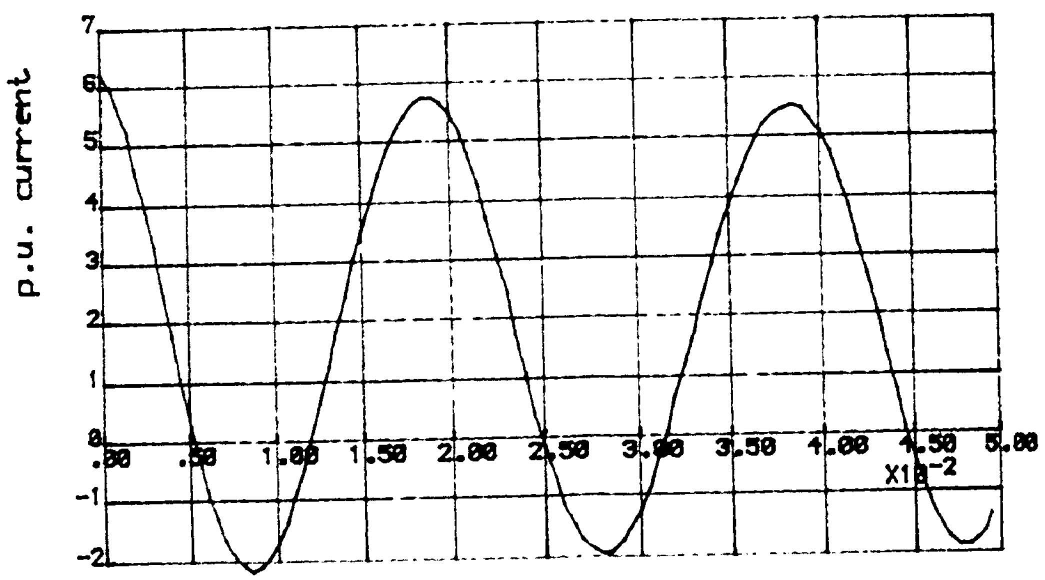


Figure 7.18 Same description as in figure 7.17

Except this figure shows the current in phase b

with the time constant T_d', where $T_d' > T_{dc}$, and so the a.c. current is greater than the d.c. current.

Operation with a wide load angle or underexcitation causes the first zero transition to be delayed. The increase in time to reach a current zero is a result of the large quadrature axis component with subtransient decay for wide load angle and underexcitation produces a diminished initial value of the alternating current.

(a) Lagging power factor operation

Figures 7.17 & 7.18 show that under the same fault conditions as described in figure 7.10 but with the circuit breaker in phase c opened, then the time taken for currents in phases a and b to pass through a current zero is shortened. Currents in phase a and phase b reach their first current zero in 11.5 and 5.5 msec. respectively. Figures 7.19 & 7.20 show that the time to reach current zeros for phases a and b is the same if the earthing resistance is very large.

(b) Leading power factor operation

Figures 7.21 and 7.22 show that under the same fault conditions as described in figure 7.14, but with the circuit breaker in phase c opened, then the currents in phases a and b reach a current zero faster by about 150 and 50 msec. respectively.

As far as circuit breaker opening is concerned, opening one phase has the effect of decreasing the d.c. component in the other phases, thus

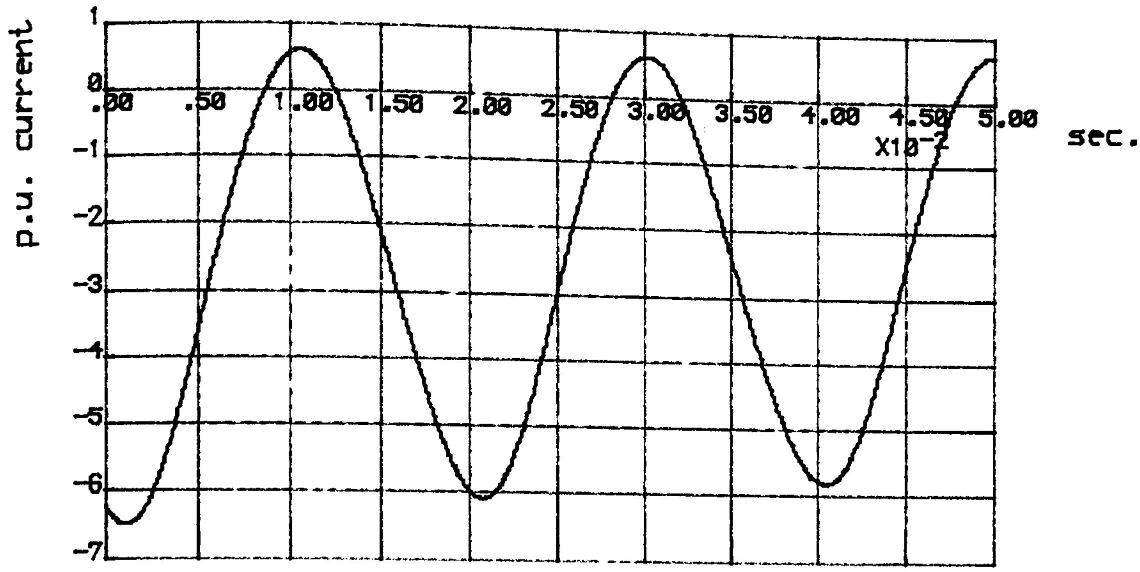


Figure 7.19 Current in phase a when a three phase to earth fault occurs at the generator terminals

Lagging power factor operation

No fictitious field winding is included

Initial conditions are given in table 1 except

$i_a = 0.5$ $i_b = 0.5$ $i_c = -1.0$ $\theta = 0.739$

Only shows the current after phase c is opened

Earthing resistance = 19 p.u.

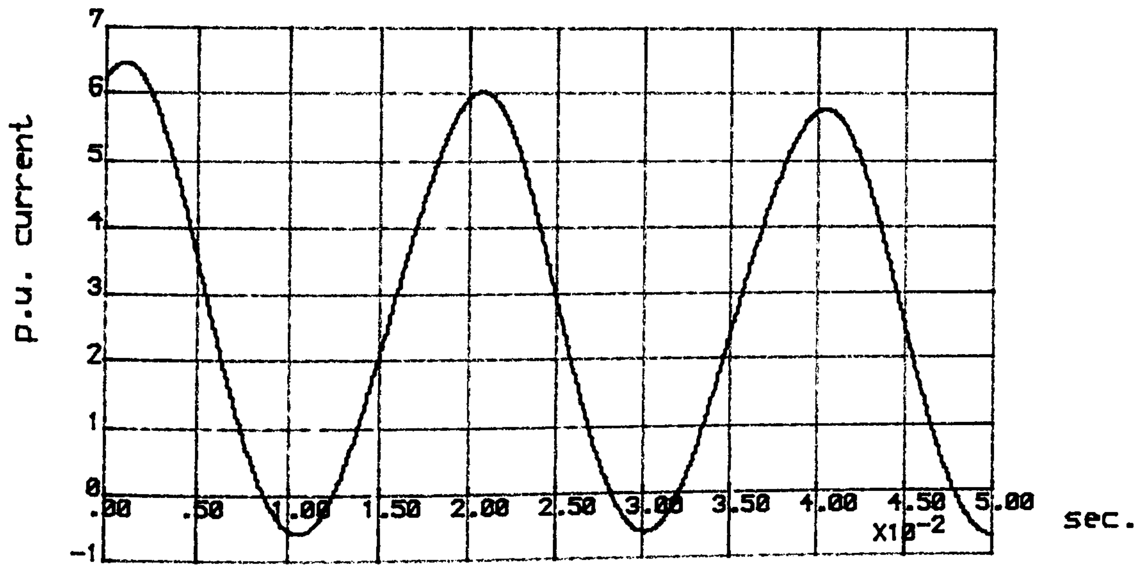


Figure 7.20 Same description as in figure 7.19

Except this figure shows the current in phase b

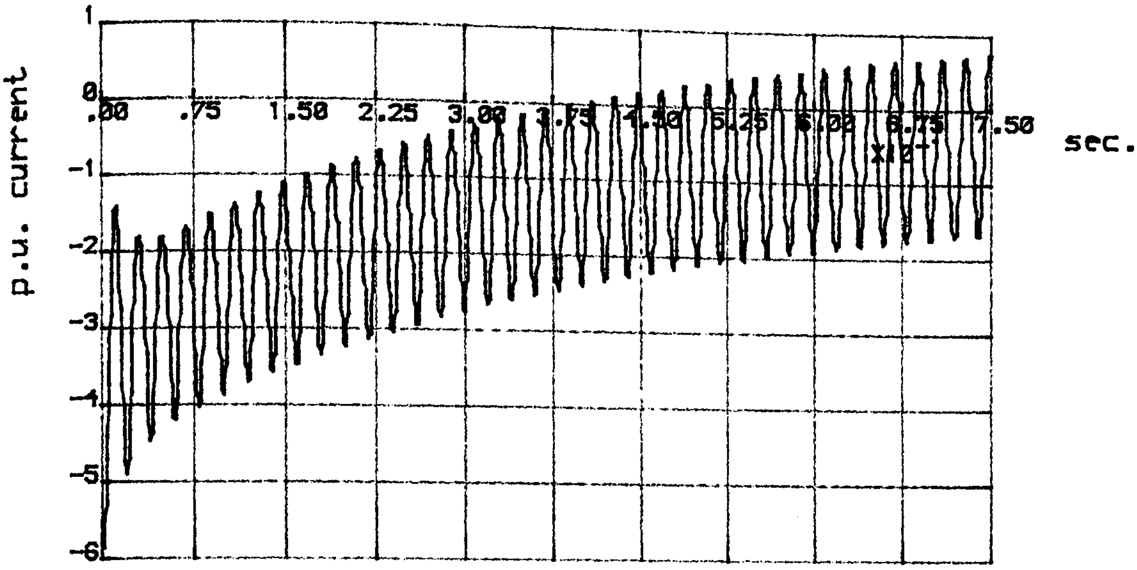


Figure 7.21 Current in phase a when a three phase to earth fault occurs at the generator terminals

Leading power factor operation
No fictitious field winding is included
Initial conditions are given in table 1 except
$i_a = -0.075 \quad i_b = 0.853 \quad i_c = -0.778$
$\theta = 1.135$ busbar voltage $= 1.016$ $i_{fd} = 1.379$
Transformer tap ratio = 0.935
Only shows the current after phase c is opened

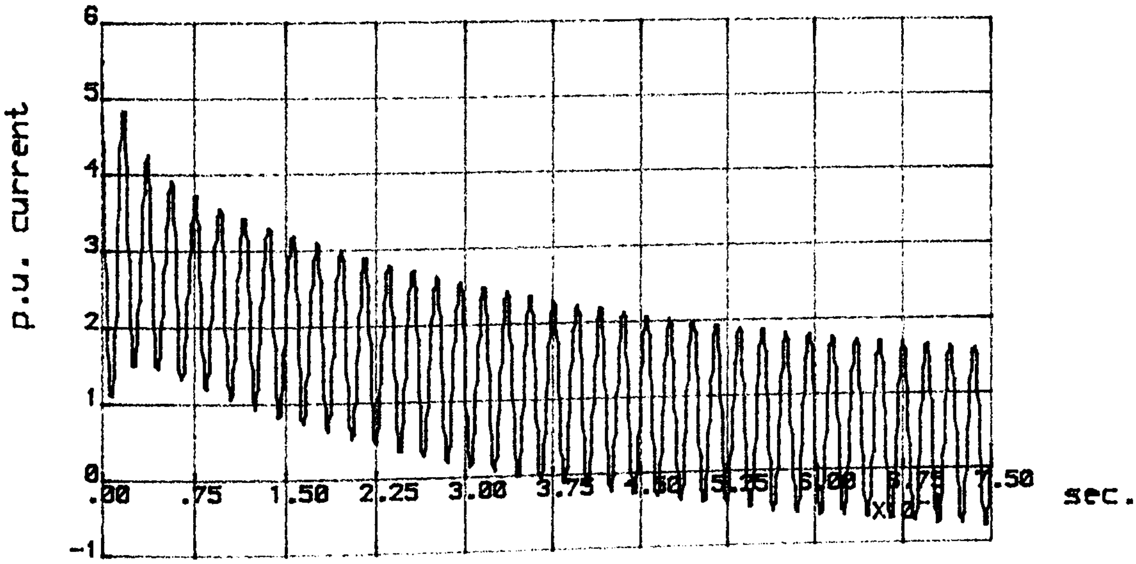


Figure 7.22 Same description as in figure 7.21

Except current in phase b is shown here

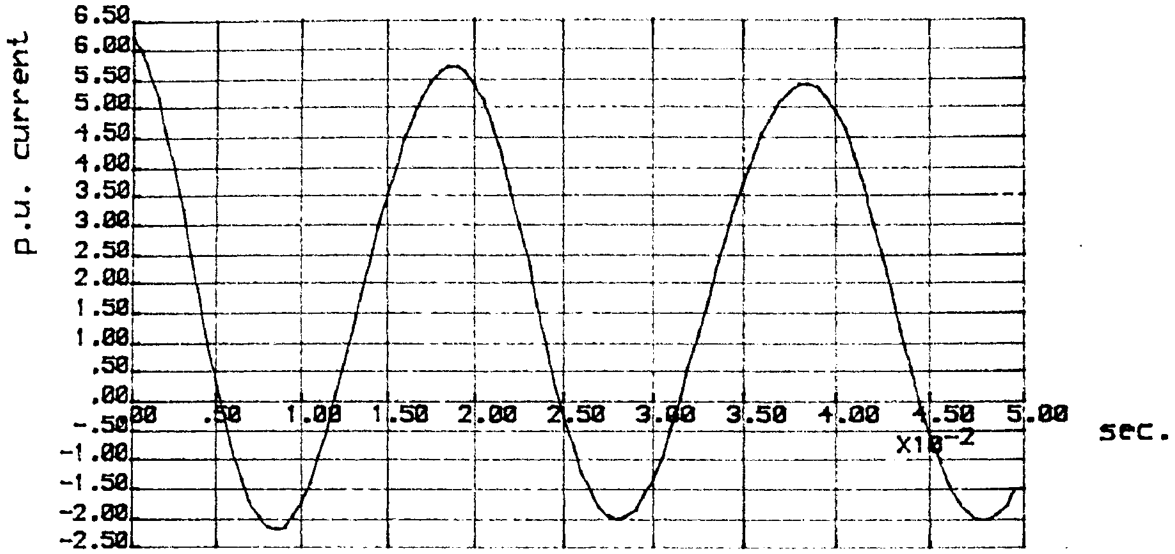


Figure 7.23 Fault current in phase b when arcing resistance is included

Lagging power factor operation
No fictitious field winding is included
Initial conditions are given in table 1 except
$i_a = 0.5$ $i_b = 0.5$ $i_c = -1.0$ $\theta = 0.739$
Only shows the current after phase c is opened
Arcing voltage = 0.0085 p.u.

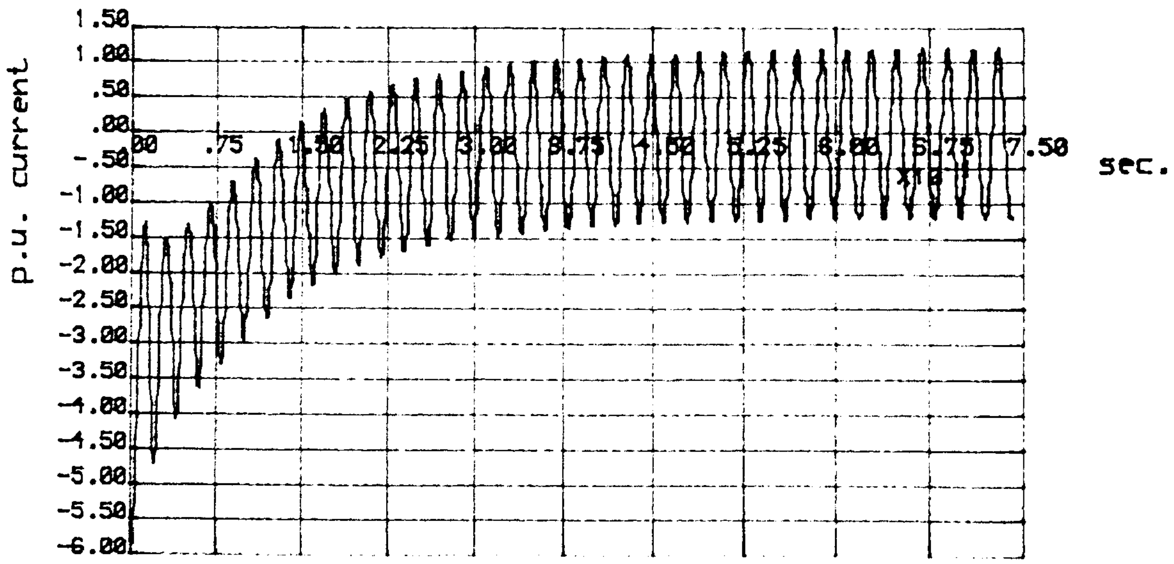


Figure 7.24 Same description as in figure 7.21

Except arcing resistance is included
Arcing voltage = 0.0085 p.u.

reducing the time for the fault current to pass through a current zero.

7.2.2.4 Effect due to arcing resistance

The arcing resistance has the effect of accelerating the decay of d.c. component.

(a) Lagging power factor operation

Figure 7.23 shows that the inclusion of arcing resistance does not have any noticeable effect on the time taken to reach a current zero when compared with figure 7.18. It shows that the arcing resistance is not important when the generator is operated at a lagging power factor.

(b) Leading power factor operation

Figures 7.24 and 7.25 show that under the same fault condition as described in figure 7.21, but with the inclusion of arcing resistance, both the currents in phase a and phase b reach a current zero much faster.

Arcing resistance has the effect of accelerating the fault current through a current zero. The inclusion of arcing resistance is considered to be important when the generator is operating at leading power factor because the time taken to reach a current zero is usually much greater when no arcing resistance is included.

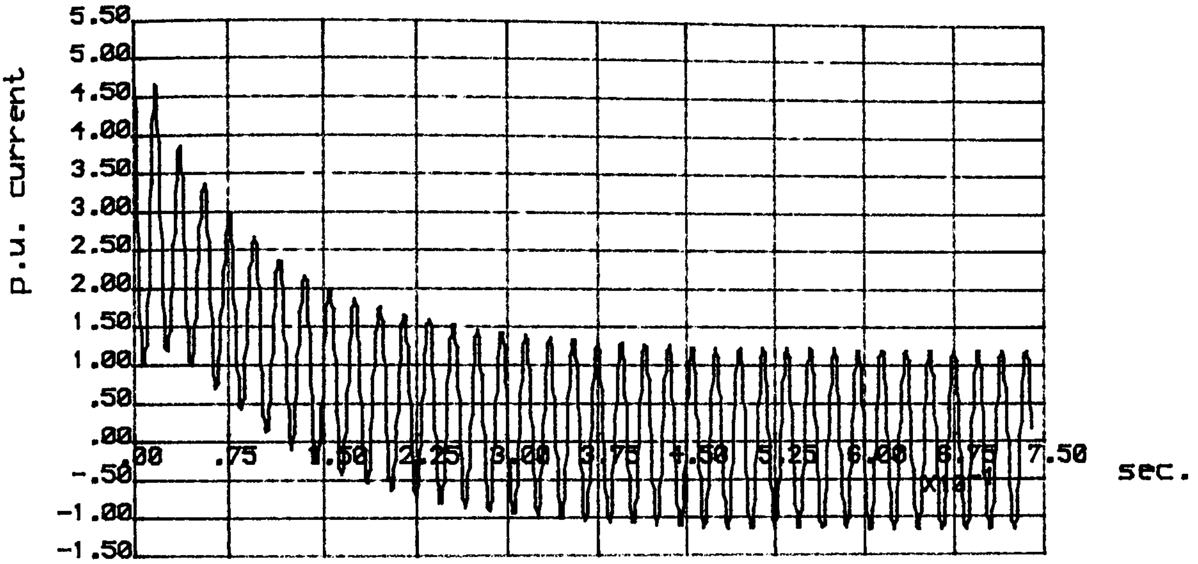


Figure 7.25 Current in phase a when a three phase to earth fault occurs at the generator terminals

Leading power factor operation
No fictitious field winding is included
Initial conditions are given in table 1 except
$i_a = -0.075 \quad i_b = 0.853 \quad i_c = -0.778$
$\theta = 1.135$ busbar voltage $= 1.016$ $i_{fd} = 1.379$
Transformer tap ratio = 0.935
Only shows the current after phase c is opened
Arcing resistance is included
Arcing voltage = 0.0085 p.u.

7.3 Summary

The effect due to circuit breaker opening, arcing and earthing resistance for both lagging and leading power factor operation are studied when faults occur. The results obtained in this chapter enable more detailed and accurate protection studies to be undertaken. With the required data, such as the machine parameters, it is possible to use the developed programs to assess the validity of the relay settings with the transient effect taken into account. The next chapter summarises conclusions and proposes areas of further work.

Chapter 8

Conclusions

8.1 Achievements of the project

The following original contributions to the subject of power system fault analysis and protection have been described in this thesis.

(1) Protection coordination and plotting programs for a radial system have been developed. These programs can provide immediate feedback by producing a graphical display of all the protective devices together with the boundaries on the coordination ranges for the devices being considered. Various protection devices, ratings, coordination margins, and coordination philosophies can be easily investigated. The optimal settings for relays can be determined interactively.

(2) Excessive computation time is required for a detailed analysis of a complete power station. Only part of a power station system (figure 1.1) was selected for analysis in detail using the phase coordinate method. The study included faults occuring on both the high voltage and low voltage sides of the generator transformer. Circuit breaker opening on both sides of the transformer were also involved to enable study of delays in current zeros. The effect of circuit breaker opening on protection operation was satisfactorily investigated, and found to produce faster relay operation. To unify the approach, switching, fault and connection matrices were introduced. Generalized matrix equations were developed which incorporated the required

switching and fault matrices, and enabled the computer algorithm to be written in a flexible manner.

(3) As far as auxiliary power supply systems in power stations and industrial power systems are concerned, the transient d.c. and a.c. decrement components may be significant and their effect on relay performance should be considered. In addition the heat generated at junctions and the thermal ratings of cabling is dependent on the $(\text{transient current component})^2$ and should be included in design considerations. Programs have been developed for these purposes.

8.2 Suggestions for further work

The following areas can be expanded and improved.

8.2.1 Modelling review

The armature winding, excitation winding and damper winding of a synchronous machine are magnetically coupled not only in the main field but also, owing to the geometric relationships, in the leakage field as well. According to mathematical analysis these three circuits must be magnetically coupled not through the main reactance $x_d - x_l$, but through the reactance $x_d - x_c$, where x_c is generally not equal to the leakage reactance x_l. With the assumption $x_c = x_l$ therefore the traditional circuit cannot represent the exciter and damper circuits correctly. The assumption used in the equivalent circuit that the magnetic coupling exists only in the main field in accordance with the main reactance $x_{ad} = x_d - x_l$ is not correct [38], and the model should be modified to enable rotor currents to be simulated accurately.

8.2.2 Generator saturation

An accurate transient and steady state model must allow for saturation in both the d-axis and q-axis. One approach is to determine the complete magnetic field distribution inside the machine with a step by step solution. This method will be time consuming. The other approach which is much simpler and faster, but less accurate, is to assume that only certain inductances in the equivalent circuits saturate. Following this latter approach, it is usual to assume that the armature and rotor winding leakage inductances have constant values. This implies that only mutual flux paths are subject to saturation.

To allow for saturation during a transient in which the machine flux is falling, a reasonably good result is obtained by using an appropriately selected constant value of the magnetising impedance. The value can be based on the slope of the saturation curve. The variation of mutual reactance can be expressed by using the open circuit saturation curve to define a saturation factor, sf, as a function of the total m.m.f.. The generator open-circuit saturation curve is presented such that terminal voltage in p.u. is plotted against field current in p.u., where 1 p.u. field current corresponds to 1 p.u. terminal voltage on the air gap line.

To calculate the mutual reactance at any operating condition, the procedure will be as follows:

$$\lambda = \sqrt{x^2_{afd}(i_{fd} + i_{kd} - 1.5i_d)^2 + x^2_{afq}(i_{fq} + i_{kq} - 1.5i_q)^2}$$

Then the open circuit saturation curve could be used to calculate the corresponding value of field current i_{fd}, such that

$$sf = \frac{i_{fd}}{\lambda}\text{, and}\quad x_{afd} = \frac{x_{afdo}}{sf}\text{,}\quad x_{afq} = \frac{x_{afqo}}{sf}$$

8.2.3 Governor and voltage regulator

If a transient is initiated by a fault, the armature reaction tends to decrease the flux linkage. This is particularly true for the generators electrically close to the location of the fault. The voltage regulator tends to force the excitation system to boost the flux level. Thus while the fault is on, the effect of the armature reaction and the action of the voltage regulator tend to counter act each other. Due to the relatively long effective time constant of the main field winding, these effects result in almost constant flux linkage during the first swing of 1 sec. or less. When the period of analysis extends beyond one second it is important to include the effects of the exciter and governor system. The exciter control system provides the proper field voltage to maintain a desired system voltage. An important characteristic of an exciter control system is its ability to respond rapidly to voltage deviations during both normal and emergency system operation. For sustained faults the speed and field control system should be incorporated in the model for simulation.

8.2.4 Resetting of relays

To include the resetting of relays in the program is straight forward. When the fault current in the protected circuit is interrupted or falls below the pickup value, the relays need to be reset. The induction relays reset gradually under the effect of a restraining spring. That is, the counter which stores the travel will be reduced gradually. The resetting rate can be determined by a subroutine.

8.2.5 Inclusion of induction motors

In isolated power systems, it is possible that many induction motors are supplied by one large generator. It is desirable to study the behaviour of these machines when a disturbance occurs. Initially the study can be simplified as in figure 1.1, except that the infinite busbar is replaced by induction motors. The behaviour of this system can be analysed when faults or circuit breaker opening occurs within the system.

8.2.6 Relay operating time calculation

The method used to calculate the relay operating time requires that the relay current in not distorted, i.e. the magnitude of harmonics should be small. To include the effect of harmonic distortion, the relay operating curves provided by the manufacturers need to be modified such that the same approach can be applied. One way is to obtain relay characteristics for relay currents with appropriate harmonic distortion, or alternatively to introduce correction factors.

8.2.7 Assessing the validity of relay settings

From chapter 7, the result shows that the operating time of a relay is different if the transient effect is taken into account. In other words, the relay settings as calculated by the coordination program may require recalculation in order to obtain proper coordination between relays during operation under both transient and steady state (without transient) conditions. In chapter 4, the transient fault currents can be calculated by the transient analysis program if the required data, such as the machine parameters, are available. Also in the same chapter, programs have been developed to calculate the relay operating time with transient conditions taken into consideration. It is therefore possible to use these programs to assess the validity of relay settings.

Appendix A

Current distribution in a current transformer

From the equivalent circuit shown in figure 5.1. The following relationships between the variables can be defined.

$$L_1 p i_1 = R_2 i_2 + L_2 p i_2 \quad , \; i_2 = i - i_1$$

$$\text{so } p i_1 + \frac{R_2}{L_1 + L_2} i_1 = \frac{1}{L_1 + L_2}(L_2 p i + R_2 i) \text{ ------(A.1)}$$

That is,

$$i_1 = \frac{1}{R_2 T_1} e^{-t/T_1} \int_{ts}^{t} (R_2 i + L_2 \frac{di}{d\tau}) e^{\tau/T_1} d\tau + C e^{-t/T_1} \text{ ---------(A.2)}$$

where $1/T_1 = \frac{R_2}{L_1 + L_2}$. Since i is represented by discrete points, and if pi is calculated then a cumulative error may result. To avoid this problem equation (A.2) will be expressed only in terms of i. That is,

$$i_1(t) = \frac{1}{R_2 T_1} e^{-t/T_1} \int_{ts}^{t} R_2 i e^{\tau/T_1} d\tau + \frac{L_2}{R_2 T_1} e^{-t/T_1} \Big[i e^{\tau/T_1} \Big]_{ts}^{t} -$$

$$\int_{ts}^{t} \frac{i}{T_1} e^{\tau/T_1} d\tau + C e^{-t/T_1}$$

Appendix B

Per-unit system

A recipirocal per-unit system [124,125] was used. Base armature winding voltage and current, were taken as the peak values per phase. The base current of a d-axis rotor circuit is that required to produce the same fundamental spacial component of air-gap flux as 1.0 p.u. direct axis current in the armature winding. The fundamental component of air-gap flux produced by 1.0 p.u. direct axis current is 1.5 times that produced by that current in a single phase.

Therefore $I_{fdo} = \frac{3}{2}\frac{Ns}{Nfd} I_{so}$,

where I_{so} is the base armature winding current, and I_{fdo} is the base field current.

Ns is the effective number of turns on the field winding.

Nfd is the effective number of turns on one phase of the armature winding.

The term "effective" refers to the ability of the windings to produce the fundamental spacial component of air-gap flux. Similarly, for any other direct axis rotor circuit, $Ixdo = \frac{3\ Ns}{2Nxd}$ Iso and for the quadrature axis, $Ixqo = \frac{3\ Ns}{2Nxq}$ Iso, where x can be f or k.

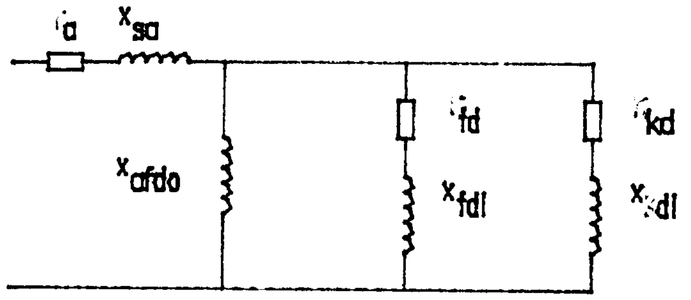


Figure C1 Equivalent circuit for the generator in the direct axis under subtransient condition

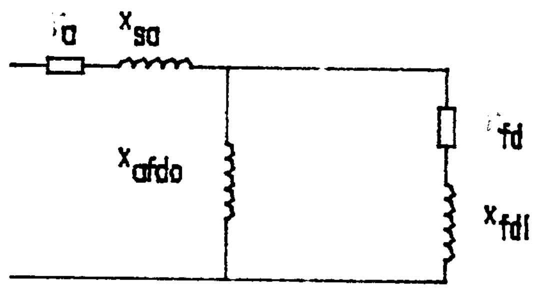


Figure C2 Equivalent circuit for the generator in the direct axis under transient condition

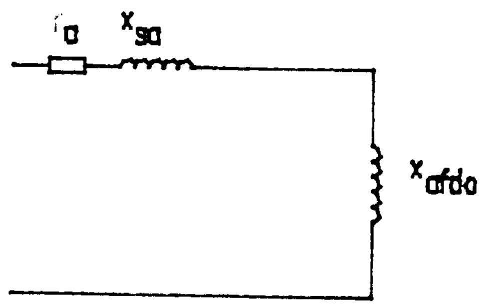


Figure C3 Equivalent circuit for the generator in the direct axis under steady state condition

Appendix C

Relationship between (a,b,c) and (d,q) parameters

Since the base value of current in the two-axis coils is one and a half times that in the phase coils, the p.u. rotor currents expressed in phase variables is 1.5 times the p.u. rotor currents expressed in (d,q) variables, so

$$x_{sd} = \frac{2}{3} x_d, \quad x_{sq} = \frac{2}{3} x_q$$

$$x_{sd}' = \frac{2}{3} x_d', \quad x_{sq}' = \frac{2}{3} x_q'$$

$$x_{sd}'' = \frac{2}{3} x_d'', \quad x_{sq}'' = \frac{2}{3} x_q''$$

$$x_{sa} = \frac{2}{3} x_a$$

x_{sa} and x_a is the armature leakage reactance defined in the (a,b,c) and (d,q) reference frames respectively. Other parameters are defined similarly. The (a,b,c) parameters can be obtained by considering the equivalent circuits of the turbo-generator. The equivalent circuit for the q-axis is the same as that of the d-axis except that d is replaced by q.

Figure C1 shows the equivalent circuit for the synchronous turbogenertor in the direct axis under subtransient conditions.

When the subtransient components have decayed, the equivalent transient circuit is shown in figure C2.

When the transient component has decayed, the equivalent circuit is shown in figure C3.

If the resistances of the windings are neglected

$$x_{sa} = x_{sd} - x_{afdo}$$

$$x_{fdl} = x_{fdo} - x_{afdo}$$

$$x_{sd}' = x_{sa} + \frac{x_{afdo}x_{fdl}}{x_{afdo} + x_{fdl}}$$

$$= \frac{(x_{sd} - x_{afdo})(x_{afdo} + x_{fdo} - x_{afdo}) + x_{afdo}(x_{fdo} - x_{afdo})}{x_{afdo} + x_{fdo} - x_{afdo}}$$

$$x_{sd}'\ x_{fdo} = x_{sd}x_{fdo} - x^2_{afdo}$$

$$x_{sd}' = \frac{x_{sd}x_{fdo} - x^2_{afdo}}{x_{fdo}}$$

$$x_{fdo}x_{sd}' = x_{sd}x_{fdo} - x^2_{afdo}$$

$$x_{fdo}(x_{sd}' - x_{sd}) = -x^2_{afdo}$$

$$x_{fdo} = \frac{x^2_{afdo}}{x_{sd} - x_{sd}'}$$

so $x_{fdl} = \dfrac{x^2_{afdo}}{x_{sd} - x_{sd}'} - x_{afdo}$

$$x_{sd}'' = x_{sa} + \frac{x_{afdo}x_{fdl}x_{kdl}}{x_{afdo}x_{fdl} + x_{afdo}x_{kdl} + x_{fdl}x_{kdl}}$$

$$= x_{sa} + \frac{x_{afdo}(x_{fdo} - x_{afdo})x_{kdl}}{x_{afdo}(x_{fdo} - x_{afdo}) + x_{afdo}x_{kdl} + (x_{fdo} - x_{afdo})x_{kdl}}$$

That is $x_{sd}''[x_{afdo}x_{fdo} - x^2_{afdo} + x_{fdo}x_{kdl}] =$

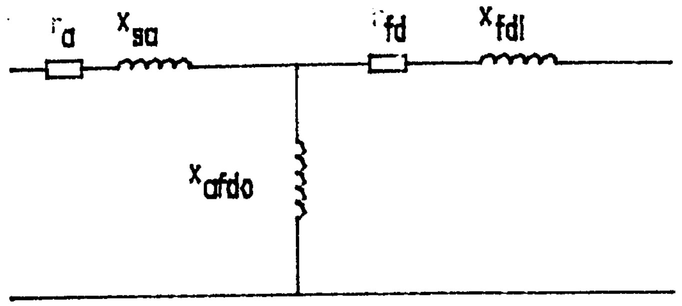


Figure C4 Equivalent circuit used to determine the transient short circuit time constant

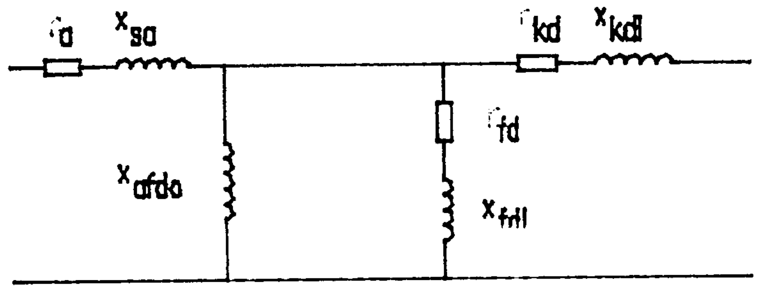


Figure C5 Equivalent circuit used to determine the subtransient short circuit time constant

$x_{sa}[x_{afdo}x_{fdo} - x^2_{afdo} + x_{fdo}x_{kdl}] + x_{afdo}x_{fdo}x_{kdl} - x^2_{afdo}x_{kdl}$

So $x_{sd}''[\frac{x_{afdo}x^2_{afdo}}{x_{sd} - x_{sd}'} - x^2_{afdo} + \frac{x^2_{afdo}x_{kdl}}{x_{sd} - x_{sd}'}] = x_{sa}[\frac{x_{afdo}x^2_{afdo}}{x_{sd} - x_{sd}'} -$

$x^2_{afdo} + \frac{x^2_{afdo}x_{kdl}}{x_{sd} - x_{sd}'}] + \frac{x_{afdo}x^2_{afdo}x_{kdl}}{x_{sd} - x_{sd}'} - x^2_{afdo}x_{kdl}$ That is

$x_{kdl}[x_{sd}''\, x^2_{afdo} - x_{sa}x^2_{afdo} - x^3_{afdo} + x^2_{afdo}(x_{afdo} + x_{sa}) - x^2_{afdo}\, x_{sd}']$

$= x_{sd}''[-x^3_{afdo} + x^2_{afdo}(x_{sd} - x_{sd}')] + x_{sa}x^3_{afdo} - x^2_{afdo}x_{sa}(x_{sd} - x_{sd}')$

So $x_{kdl}[x_{sd}'' - x_{sd}'] = x_{sd}''[-x_{afdo} - x_{sd}'] + x_{sa}x_{afdo} - x_{sa}x_{sd} + x_{sa}x_{sd}'$

That is $(x_{sd}'' - x_{sd}')x_{kdl} = -x_{sd}''x_{afdo} + x_{sd}x_{sd}'' - x_{sd}''x_{sd}' - x^2_{sa} +$

$x_{sa}x_{sd}'$

So $(x_{sd}'' - x_{sd}')x_{kdl} = -x_{sd}''\, x_{afdo} + x_{sd}x_{sd}'' - x_{sd}''\, x_{sd}' -$

$x^2_{sa} + x_{sa}x_{sd}'$

Therefore $x_{kdl} = \frac{(x_{sd}' - x_{sa})^2}{x_{sd}' - x_{sd}''} + x_{sd} - x_{sd}' - x_{afdo}$

Because $(x_{sd}' - x_{sa})^2 + (x_{sd} - x_{sd}')(x_{sd}' - x_{sd}'') - x_{afdo}(x_{sd}' - x_{sd}'')$

$= x_{sd}'^2 - 2x_{sd}'\, x_{sa} + x_{sa}^2 + x_{sd}x_{sd}' - x_{sd}x_{sd}'' - x_{sd}'^2 + x_{sd}'s_{sd}'' -$

$x_{afdo}x_{sd}' + x_{afdo}x_{sd}''$

$= x_{afdo}x_{sd}'' - x_{sd}x_{sd}'' + x_{sd}'x_{sd}'' - x_{sa}x_{sd}' + x_{sa}^2$

The subtransient and transient short circuit time constants can be found by the following equivalent circuits (figure C4 & C5) with the armature input terminals open-circuited.

$$T_{do}' = \frac{x_{afdo} + x_{fdl}}{w\, r_{fd}} = \frac{x_{fdo}}{w\, r_{fd}}$$

Because $T_d' = \dfrac{x_{sd}'}{x_{sd}} T_{do}'$, therefore

$$\frac{x_{sd} T_d'}{x_{sd}'} = \frac{x_{fdo}}{w\, r_{fd}} \quad ,\text{so}$$

$$r_{fd} = \frac{x_{fdo} x_{sd}'}{w\, T_d'}$$

$$T_{do}'' = \frac{1}{w\, r_{kd}} [x_{kdl} + \frac{x_{afdo} x_{fdl}}{x_{afdo} + x_{fdl}}] = \frac{1}{w\, r_{kd}} [x_{kdl} + \frac{x_{afdo} x_{fdl}}{x_{fdo}}]$$

$$T_d'' = \frac{x_{sd}''}{x_{sd}'} T_{do}'' \quad , \text{ so}$$

$$\frac{x_d' T_{sd}''}{x_{sd}''} = \frac{1}{w\, r_{kd}} [x_{kdl} + \frac{x_{afdo} x_{fdl}}{x_{fdo}}] \quad \text{i.e.}$$

$$r_{kd} = \frac{1}{w\, T_d''} [x_{kdl} + \frac{x_{afdo} x_{fdl}}{x_{fdo}}] \frac{x_{sd}''}{x_{sd}'}$$

Appendix D

Runge Kutta and Predictor Corrector methods

Given the first order differential equation

$$\frac{dy}{dx} = f(x,y)$$

where x is the independent variable and y is the dependent variable.

Runge Kutta Method

$$p = h\, f(x_i, y_i)$$

$$q = h\, f(x_i + \frac{h}{2}, y_i + \frac{p}{2})$$

$$r = h\, f(x_i + \frac{h}{2}, y_i + \frac{q}{2})$$

$$s = h\, f(x_i + h, y_i + r)$$

h is the step length.

$$y_{i+1} = y_i + \frac{1}{6}(p + 2q + 2r + s)$$

The numerical method which is used to solve a set of differential equations by using Runge Kutta method is

$$p_k = h\, f_k(x_i, y_{1,i}, y_{2,i}, \ldots\ldots, y_{n,i}) \quad \text{----(D.1)}$$

$$q_k = h\, f_k(x_i + \frac{h}{2}, y_{1,i} + \frac{p_1}{2}, y_{2,i} + \frac{p_2}{2}, \ldots\ldots, y_{n,i} + \frac{p_n}{2}) \quad \text{--(D.2)}$$

$$r_k = h\, f_k(x_i + \frac{h}{2}, y_{1,i} + \frac{q_1}{2}, y_{2,i} + \frac{q_2}{2}, \ldots\ldots, y_{n,i} + \frac{q_n}{2}) \quad \text{---(D.3)}$$

$$s_k = h\, f_k(x_i + h, y_{1,i} + r_1, y_{2,i} + r_2, \ldots\ldots, y_{n,i} + r_n) \quad \text{---(D.4)}$$

$$y_{k,i+1} = y_{k,i} + \frac{1}{6}(p_k + 2q_k + 2r_k + s_k)$$

where $k = 1, 2, \ldots, n$

The set of equations above will give the values for each y_k at x_{i+1} when the solution at x_i is known.

The first step is to find p_k for $k = 1, 2, \ldots, n$. These are then used in the computation of q_k, $k = 1, 2, \ldots, n$. Each q_k is used to compute r_k, and then all values for r_k must be known to find s_k. Equations (D.1) to (D.4) must be solved in the order listed for each k before moving on to the next step.

Predictor Corrector method

Predictor formula

$$p = \frac{h}{24}[\,55\, f(x_i, y_i) - 59\, f(x_{i-h}, y_{i-h}) + 37\, f(x_{i-2h}, y_{i-2h}) - 9\, f(x_{i-3h}, y_{i-3h})] + y_i$$

$x_{i-h} = x_i - h$ etc.

Corrector formula

$$y_{i+h} = y_i + \frac{h}{24}[\,9\, f(x_{i+h}, p) + 19\, f(x_i, y_i) - 5\, f(x_{i-h}, y_{i-h}) + f(x_{i-2h}, y_{i-2h})]$$

When a predictor corrector method is used to solve a set of differential equations:

For the predictor formula

$$p_k = y_{k,i} + \frac{h}{24}[\,55\, f_{k,i} - 59\, f_{k,i-h} + 37\, f_{k,i-2h} - 9\, f_{k,i-3h}] \quad \text{------(D.5)}$$

For the corrector formula

$$y_{k,i+h} = y_{k,i} + \frac{h}{24} [\, 9\, f_{k,i+h} + 19\, f_{k,i} - 5\, f_{k,i-h} + f_{k,i-2h}] \text{ ------ (D.6)}$$

The solutions for y_k at $x = x_i + h$ can be found if the solutions for y_k at x_i, $x_i - h$, $x_i - 2h$ and $x_i - 3h$ are known.

The first step is to find p_k, the predicted value for y_k at $x = x_i+h$ for $k = 1, 2, \ldots\ldots, n$ from equation (D.5). The values for p_k are then used to find the values of $f_{k,\, i+h}$, $k = 1, 2, \ldots\ldots, n$ at $x = x_i + h$. $f_{k,i+h} = f_k(x_i + h, p_1, p_2, \ldots\ldots, p_n)$. The values of $f_{k,i+h}$ are used by equation (D.6) for computing the values of $y_{k,i+h}$.

The theory for the above derivations can be found in [127].

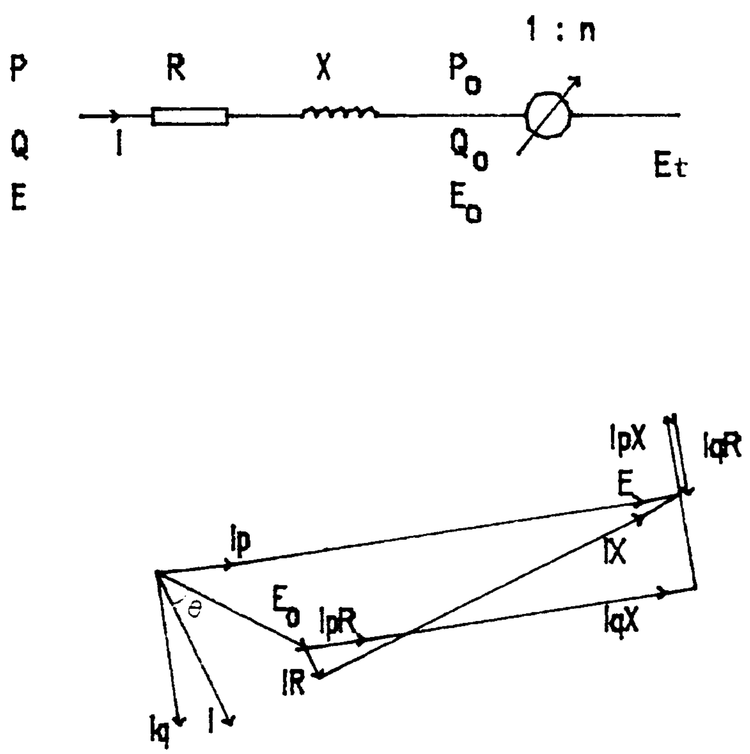


Figure E1 Phasor diagram for the transmission of power through a series impedance

Determination of initial conditions for the generator

From figure E1

$P = I_p E, \; Q = I_q E,$

$P_o = E_o I \cos \theta, \; Q_o = E_o I \sin \theta$

$P = P_o + I^2 R, \; Q = Q_o + I^2 X$

$I^2 = I_p^{\,2} + I_q^{\,2} = \dfrac{P^2 + Q^2}{E^2}$

therefore $P_o = P - \dfrac{P^2 + Q^2}{E^2} R$

$Q_o = Q - \dfrac{P^2 + Q^2}{E^2} X$

$E_o^{\,2} = [E - (I_p R + I_q X)]^2 + (I_p X - I_q R)^2$

$\quad = [E - (\frac{P}{E} R + \frac{Q}{E} X)]^2 + (\frac{P}{E} X - \frac{Q}{E} R)^2$

$\quad = (E - A)^2 + B^2$

where $A = \dfrac{PR}{E} + \dfrac{QX}{E}$, $B = \dfrac{QR}{E} - \dfrac{PX}{E}$, $C = E - A$

$\dfrac{Et}{E_o} = \dfrac{n}{1}$

therefore $n = \dfrac{Et}{\sqrt{C^2 + B^2}}$

From appendix F

$v_d = -r_a i_d - w\lambda_q + p\lambda_d$

$v_q = -r_a i_q + w\lambda_d + p\lambda_q$

During steady state $p\lambda_d = p\lambda_q = 0$

$\lambda_q = -L_q i_q$

$\lambda_d = -L_d i_d + L_{df} i_f$

where L_d is the self inductance of the d-axis coil, L_q is the self inductance of the q-axis coil, L_{df} is the mutual inductance on the direct axis. By Park's transformation

$e_a = v_d \cos\theta - v_q \sin\theta$

where $\theta = w_o t + \delta - \frac{1}{6}\pi$

A 30^o phase shift exists between the primary side and secondary side of the transformer, so the angle between the d-axis and D-axis is $\delta - 30^o$. The angle between the a-axis and the D-axis is $w_o t$. At t = 0, the angle is zero, therefore $\theta = \delta - 30^o = \theta o$. That is, the D-axis coincides with the a-axis of the 3-phase system. $r = r_a$,

$e_a = (-ri_d + wL_q i_q)\cos\theta - (-ri_q - wL_d i_d + wL_{df} i_f)\sin\theta$

$= (-ri_d + wL_q i_q)\cos\theta - (-ri_q - wL_d i_d + wL_{df} i_f)\cos(\theta - 0.5\pi)$

At steady state the angular speed is constant, $w = w_o$, and wL may be denoted as reactances, or $wL_q = x_q$, $wL_d = x_d$, $wL_{df} i_f = E_q$. The following phasor notation is used:

$\bar{A} = A e^{j\alpha}$, A is the peak value. The superior bar indicates a total phasor quantity in magnitude and angle (a complex number). This

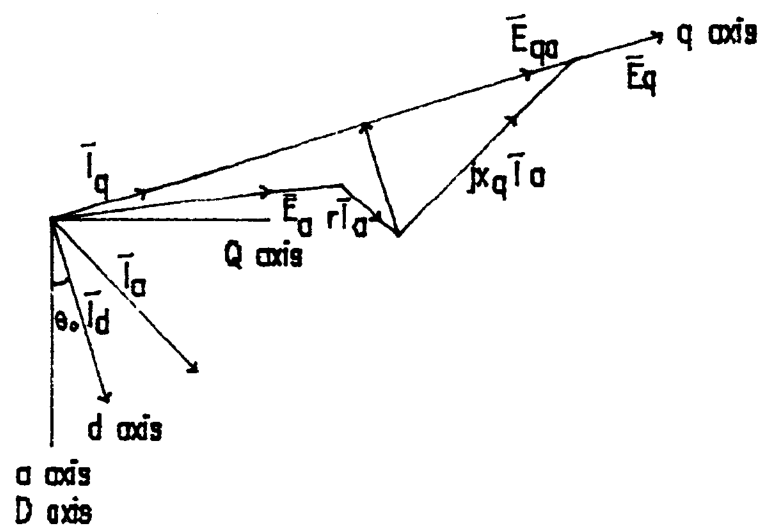


Figure E2 Phasor diagram

complex number is related to the corresponding time domain quantity a(t) by the relation $a(t) = Re\,(\bar{A}\, e^{jwt}) = A\cos(wt + \alpha)$, so the peak phasor voltage $\bar{E}_a = -r(I_d \underline{/\theta o} - I_q \underline{/\theta o - 0.5\pi}) + x_q I_q \underline{/\theta o}$

$+\, x_d I_d \underline{/\theta o - 0.5\pi} - E_q \underline{/\theta o - 0.5\pi}$.

By using the relation $j = \underline{/0.5\pi}$, $-j = \underline{/-0.5\pi}$

$\bar{E}_a = -r(I_d \underline{/\theta o} + jI_q \underline{/\theta o}) - jx_q jI_q \underline{/\theta o} - jx_d I_d \underline{/\theta o} + jE_q \underline{/\theta o}$

Let $\bar{I}_a = \bar{I}_d + \bar{I}_q = I_d \underline{/\theta o} + jI_q \underline{/\theta o}$, therefore $\bar{I}_a \underline{/-\theta o} = I_d + jI_q$. So

$I_d \underline{/\theta o} = \bar{I}_d$, $jI_q \underline{/\theta o} = \bar{I}_q$, $\bar{E}_q = jE_q \underline{/\theta o}$

$\bar{E}_a = -r(\bar{I}_d + \bar{I}_q) - jx_q \bar{I}_q - jx_d \bar{I}_d + jE_q \underline{/\theta o}$

$= -r\bar{I}_a - jx_q(\bar{I}_d + \bar{I}_q) + jx_q \bar{I}_d - jx_d \bar{I}_d + \bar{E}_q$

$= -r\bar{I}_a - jx_q \bar{I}_a + j(x_q - x_d)\, I_d \underline{/\theta o} + jE_q \underline{/\theta o}$

$= -(r + jx_q)\bar{I}_a + j[\,E_q - (x_d - x_q) I_d\,] \underline{/\theta o}$ ------ (E.1)

Let $E_{qd} = E_q - (x_d - x_q) I_d$, and $\bar{E}_{qa} = jE_{qd} \underline{/\theta o} = E_{qd} \underline{/\theta'}$,where

$\theta' = 0.5\pi + \theta o$, so $\bar{E}_{qa} = \bar{E}_a + (r + jx_q)\bar{I}_a$

$j[E_q - (x_d - x_q) I_d] \underline{/\theta o} = \bar{E}_q - j(x_d - x_q)\bar{I}_d = \bar{E}_{qa}$

The phasor diagram representing equation (E.1) is given in figure E2

The generator transformer has a nominal tap ratio n which can be adjusted to obtain the specified voltage at the infinite busbar from known values of generated power at the machine terminals. The equations are:

$$A = \frac{P(r_p + r_s) + Q(x_p + x_s)}{E}$$

$$B = \frac{Q(r_p + r_s) - P(x_p + x_s)}{E}$$

$$C = E - A$$

$$n = \frac{Et}{\sqrt{C^2 + B^2}}$$

$$P_T = P - \frac{(P^2 + Q^2)(r_p + r_s)}{E^2}$$

$$Q_T = Q - \frac{(P^2 + Q^2)(x_p + x_s)}{E^2}$$

$$P_i = P_T - \frac{(P_T^2 + Q_T^2)\, r_L}{Et^2}$$

$$Q_i = Q_T - \frac{(P_T^2 + Q_T^2)\, x_L}{Et^2}$$

$$\theta_i = \tan^{-1}\left(\frac{Q_i}{P_i}\right)$$

Calculation of E_i (infinite machine voltage)

$$E_i = \sqrt{\frac{[Et^2 - (P_T r_L + Q_T x_L)]^2 + (Q_T r_L - P_T x_L)^2}{Et^2}}$$

$$I_{Ti} = \frac{\sqrt{P_i^{\,2} + Q_i^{\,2}}}{E_i}$$

$$V_a = E_i \exp [\, j(t + \tfrac{1}{2}\pi)]$$

$$V_b = E_i \exp [\, j(t + \tfrac{1}{2}\pi - \tfrac{2}{3}\pi)]$$

$$V_c = E_i \exp [\, j(t + \tfrac{1}{2}\pi + \tfrac{2}{3}\pi)]$$

$$I_{La} = I_{Ti} \exp [j(t + \tfrac{1}{2}\pi - \theta_i\,)]$$

$$I_{Lb} = I_{Ti} \exp [\, j(t + \tfrac{1}{2}\pi - \tfrac{2}{3}\pi - \theta_i)]$$

$$I_{Lc} = I_{Ti} \exp [\, j(t + \tfrac{1}{2}\pi + \tfrac{2}{3}\pi - \theta_i)]$$

$$I_T = \left[\frac{I_{La}}{n}, \frac{I_{Lb}}{n}, \frac{I_{Lc}}{n}\right]^t$$

$$I_T = [\, I_{Ta} \,, I_{Tb} \,, I_{Tc} \,]^t$$

$$I_s = C^t \, I_T$$

where $I_s = [\, I_a \,, I_b \,, I_c \,]^t$

$$E = C^t \frac{1}{n} V + \frac{1}{n^2} [\, (r_L + r_T) + j(x_L + x_T)]\, I_s$$

where $V = [\, V_a \,, V_b \,, V_c \,]^t$, $r_L = (r_p + r_s) n^2$

$E = [\, E_a \,, E_b \,, E_c \,]^t$, $x_L = (x_p + x_s) n^2$

e_a = Real part of E_a

The time taken to reach a voltage zero ($e_a = 0$) can be calculated from the following,

$$t = \frac{1}{2}\pi - \cos^{-1}\left[\frac{\text{real } E_a}{|E_a|} \right],$$

Similarly to calculate the time taken to reach a current zero.

$$t = \frac{1}{2}\pi - \cos^{-1}\left[\frac{\text{real } I_a}{|I_a|} \right]$$

Calculation of θ.

From equation (E.1)

$$E_{qa} = E_a + [r_a + j(x_a + \frac{x_{aqo}}{sf}] I_a \quad \text{----(E.2)}$$

Arbitrary values of sf and I_{fd} were set to start the iterative procedure.

$$Eqd = |E_{qa}| \quad ,\theta' = \arg (E_{qa})$$

$$\theta' = \theta + \frac{1}{2}\pi = \delta - \frac{1}{6}\pi + \frac{1}{2}\pi$$

$$E_q = E_{qd} + \frac{I_d (x_{ado} - x_{aqo})}{sf}$$

where $I_a \exp(-j\theta) = I_d + jI_q$

If the previous value of I_{fd} was I_{fdold} then the new value is calculated as $I_{fd} = \frac{3}{2}\frac{E_q sf}{x_{ado}}$

$$\lambda = \frac{1}{sf}\sqrt{x_{ado}^2 (\frac{2}{3} I_{fd} - I_d)^2 + x_{aqo}^2 I_q^2}$$

A value of I_{fd} is calculated from the saturation characteristics of the machine to correspond to this value of λ, such that

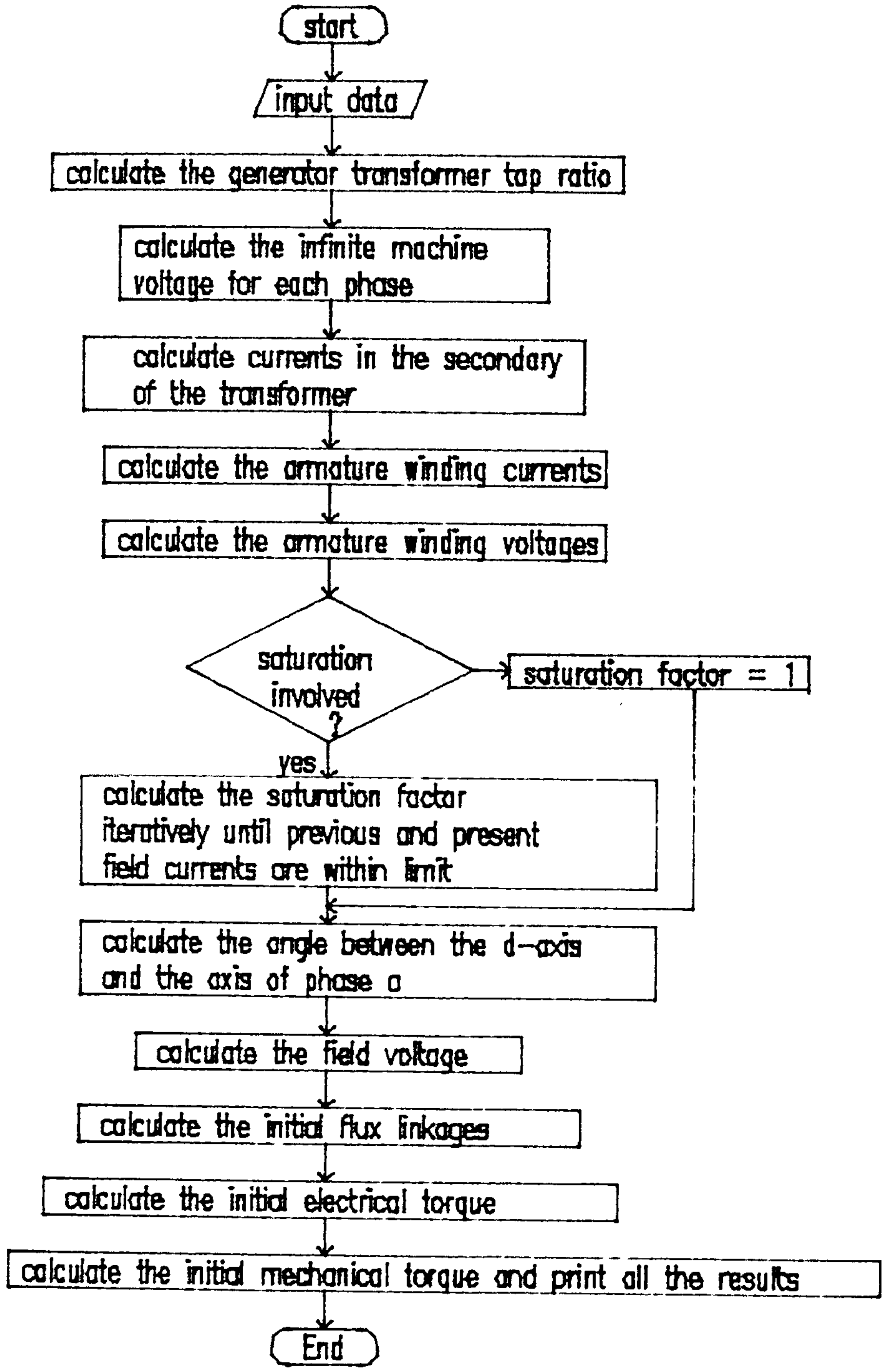


Figure E3 Flow chart for the program calculating the initial conditions

$$sf_{new} = \frac{I_{fd}}{\lambda} \qquad \text{(E.3)}$$

If $I_{fd} \neq I_{fdold}$, then I_{fdold} is replaced by the new I_{fd} and E_{qa} is recalculated from equation (E.2) using sf_{new}. Calculation was repeated down to equation (E.3). This calculation was repeated until two successive values of I_{fd} were equal. The last values of I_{fd} and sf are taken as the initial values for I_{fd} and sf.

The generator parameters that depend on saturation are shown below.

$$x_{ad} = \frac{x_{ado}}{sf}, \quad x_{aq} = \frac{x_{aqo}}{sf}$$

$$x_{afd} = \frac{2}{3} x_{ad}, \quad x_{afq} = \frac{2}{3} x_{aq}$$

The initial values of the quadrature field and damper currents are

$$I_{kd} = I_{kq} = I_{fq} = 0$$

The L matrix was calculated and the initial flux linkages $\lambda_a, \lambda_b, \lambda_c$, determined from the relationship $\lambda = LI$.

The electrical air gap torque obtained from the torque equation is,

$$T_e = \frac{2}{3\sqrt{3}} [\lambda_a(i_b - i_c) + \lambda_b(i_c - i_a) + \lambda_c(i_a - i_b)]$$

Since $T_a = 0$, therefore $T_m = T_e + T_L$

Lastly the field voltage E_{fd} was calculated from $E_{fd} = r_{fd} I_{fd}$.

A program has been developed for calculating the initial conditions and is described in the flow chart in figure E3. A listing is shown is appendix J (** J8 **).

Appendix F

Derivation of electrical torque, T_e

Notation used:

v, i positive for motoring

P power entering terminals electrically

T_e positive when torque is being removed from the shaft electrically

T_m mechanical driving torque is positive when the generator is driven

w, v_m positive speed means generating action

i_a, i_b, i_c were normalised to a base i_{ao}, the peak current per phase in the armature. i_d, i_q, i_o were normalised to a base $\frac{3}{2}\, i_{ao}$.

For the (a,b,c) reference frame, the base product is $v_{ao}\, i_{ao}$. For the (d,q) reference frame, the base product is $\frac{3}{2}\, v_{ao} i_{ao}$. The fundamental base of power is defined as the total rated apparent power, which is also the 2-axis voltage current base product so that $P_o = \frac{3}{2}\, v_{ao} i_{ao}$. Consider now the expression for instantaneous ordinary 3-phase output power of a synchronous generator. $P = v_a i_a + v_b i_b + v_c i_c$. Rewriting this equation, prior to normalising to bases P_o, v_{ao}, and i_{ao} yields

$$\frac{P}{\frac{3}{2} v_{ao} i_{ao}} = \frac{2}{3} \left(\frac{v_a i_a}{v_{ao} i_{ao}} + \frac{v_b i_b}{v_{ao} i_{ao}} + \frac{v_c i_c}{v_{ao} i_{ao}}\right)$$ hence the equation

for instantaneous per unit 3 phase power is

$\bar{P} = \frac{2}{3} (\bar{v}_a \bar{i}_a + \bar{v}_b \bar{i}_b + \bar{v}_c \bar{i}_c)$ the bar signifies p.u. values. With Park's transformation :

$$\begin{bmatrix} \bar{v}_a \\ \bar{v}_b \\ \bar{v}_c \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta - \frac{2}{3}\pi) & -\sin(\theta - \frac{2}{3}\pi) & 1 \\ \cos(\theta + \frac{2}{3}\pi) & -\sin(\theta + \frac{2}{3}\pi) & 1 \end{bmatrix} \begin{bmatrix} \bar{v}_d \\ \bar{v}_q \\ \bar{v}_o \end{bmatrix}$$

$\bar{P} = \bar{v}_d \bar{i}_d + \bar{v}_q \bar{i}_q + 2\bar{v}_o \bar{i}_o$, so the instantaneous ordinary 2-axis power is $P = v_d i_d + v_q i_q + 2v_o i_o$.

For simplicity, assume balanced but not necessarily steady state conditions, thus $v_o = i_o = \emptyset$. $P = v_d i_d + v_q i_q$ -----(F.1).

Expressing the axis voltage in terms of axis currents and flux linkages: $v_a = -r_a i_a + p\lambda_a$

$= -r_a(i_d \cos\theta - i_q \sin\theta + i_o) + p(\lambda_d \cos\theta - \lambda_q \sin\theta + \lambda_o)$

$= -r_a(i_d \cos\theta - i_q \sin\theta + i_o) + \cos\theta\, p\lambda_d - \sin\theta\, p\lambda_q + p\lambda_o - w\lambda_d \sin\theta$

$- w\lambda_q \cos\theta$

$= (-r_a i_d + p\lambda_d - w\lambda_q)\cos\theta - (-r_a i_q + p\lambda_q + w\lambda_d)\sin\theta + (-r_a i_o + p\lambda_o)$

$= v_d \cos\theta - v_q \sin\theta + v_o$, so

$v_d = -r_a i_d + p\lambda_d - w\lambda_q$

$$v_q = -r_a i_q + p\lambda_q + w\lambda_d$$

$$v_o = -r_a i_o + p\lambda_o$$

substituting into (F.1)

$$P = -r_a(i_d^2 + i_q^2) + (i_d p\lambda_d + i_q p\lambda_q) + (\lambda_d i_q - \lambda_q i_d)w$$

The last term is identifiable as the power transferred across the air gaps accounting for the electrical torque. With the notation used, the ordinary 2-axis electrical power associated with the motional terms may be written as $T_e v_m = (\lambda_d i_q - \lambda_q i_d)\ w$ -------(F.2) where $w = \frac{d\theta}{dt}$, θ is the electrical angle in radians, v_m is the mechanical angular velocity. Define w_o as base of w, $\frac{1}{w_o}$ as base of time. The per unit differential operator $\bar{p} = \frac{d}{d\bar{t}} = \frac{1}{w_o}\frac{d}{dt} = \frac{p}{w_o}$, $\lambda_o = \frac{v_{ao}}{w_o}$ as the flux linkage base. λ_o is the peak flux linkage which, varying at rated frequency, induces rated voltage; in other words, it is the peak flux linkage occurring in a coil which has rated voltage induced in it on open circuit. Defining the base mechanical angular velocity v_{mo} as that corresponding to base electrical velocity w_o. Thus $v_{mo} = \frac{w_o}{m}$ where m is the pole pairs. So $\bar{v}_m = \frac{v_m}{v_{mo}} = \frac{m}{w_o} p\theta_m = \frac{p\theta}{w_o} = \bar{p}\theta$, θ_m is the mechanical angle. The base torque corresponds to base power at base mechanical speed, that is,

$$T_o = \frac{3\, v_{ao}\, i_{ao}}{2\, v_{mo}}$$. Normalising equation (F.2)

$$\frac{3\, v_{mo}\, T_e}{2\, v_{ao}\, i_{ao}} \frac{v_m}{v_{mo}} = \frac{\lambda_d w_o}{v_{ao}} \frac{2\, i_q}{3\, i_{ao}} \frac{p\theta}{w_o} - \frac{\lambda_q w_o}{v_{ao}} \frac{2\, i_d}{3\, i_{ao}} \frac{p\theta}{w_o}$$

since $\frac{v_m}{v_{mo}} = \frac{p\theta}{w_o}$,therefore $\bar{T}_e = \bar{\lambda}_d\, \bar{i}_q - \bar{\lambda}_q\, \bar{i}_d$ ------(F.3)

On using the Park's transformation, such as

$$\begin{bmatrix} \bar{i}_d \\ \bar{i}_q \\ \bar{i}_o \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - \frac{2}{3}\pi) & \cos(\theta + \frac{2}{3}\pi) \\ -\sin\theta & -\sin(\theta - \frac{2}{3}\pi) & -\sin(\theta + \frac{2}{3}\pi) \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} \bar{i}_a \\ \bar{i}_b \\ \bar{i}_c \end{bmatrix}$$

$$\bar{T}_e = \frac{2}{3\sqrt{3}} [\, \bar{\lambda}_a\, (\bar{i}_b - \bar{i}_c) + \bar{\lambda}_b(\bar{i}_c - \bar{i}_a) + \bar{\lambda}_c(\bar{i}_a - \bar{i}_b)\,]$$

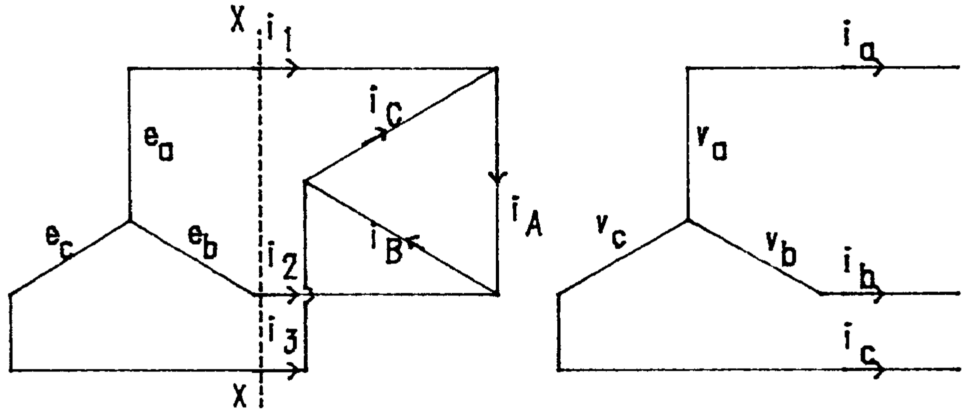


Figure G1 Circuit diagram for obtaining the voltages and currents between the star and delta sides of a transformer

Voltages and currents in a delta/star transformer

To prove that
$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \text{------} \quad (G.1)$$

Consider figure G1 and assume a transformer turns ratio of unity. If the same base of voltage is used for the system , then when the input line voltage is 1 p.u., the output line voltage at the star connected side will be $\sqrt{3}$ p.u.. In order to make sure that when the input line voltage is 1 p.u., the output line voltage will also be 1 p.u., the base voltage at the left hand side of XX has to be $\sqrt{3}$ times smaller than that at the right hand side of XX. In this case, when the input line voltage is 1 p.u., v_a will be $\frac{1}{\sqrt{3}}$ p.u., so the line voltage at the output will be 1 p.u.. Mathematically, $e_a - e_b = v_a$ for the same base. But if different base values are used (as above), then

$\frac{e_a - e_b}{\sqrt{3}} = v_a$, Similarly for the other phases, so the result as stated in (G.1) follows.

To prove that
$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} \text{------} \quad (G.2)$$

Consider the same figure again. Using the same current base value for the system, then when the input line current is 1 p.u., $i_A = i_a = \frac{1}{\sqrt{3}}$.

In order to make sure that when the input line current is 1 p.u., the output line current will also be 1 p.u., the base current for the transformer currents has to be $\frac{1}{\sqrt{3}}$ times the base current for the input currents. Mathematically, for the same base value, $i_1 = i_A - i_C$, but if different base values are used (as explained), the value in p.u. for i_A, i_B, i_C will be $\sqrt{3}$ times larger, so there is a requirement to divide by $\sqrt{3}$ in order to maintain the current relationship, therefore $i_1 = \frac{i_A - i_C}{\sqrt{3}}$, $i_2 = \frac{i_B - i_A}{\sqrt{3}}$,

$i_3 = \frac{i_B - i_A}{\sqrt{3}}$, so (G.2) follows.

Appendix H

Voltage equation and swing equation

The voltage equation in p.u. form is

$v = -ri + \bar{p}\lambda$ where $\bar{p}$ is the differential operator

so $v = -ri + \frac{1}{w_o} p\lambda$

The swing equation governs the motion of the machine rotor, relating the inertia torque to the resultant of the mechanical and electrical torques; i.e. $Jp^2\theta_m = T_a$, where J is the moment of inertia of all rotating masses attached to the shaft. θ_m is the mechanical angle of the shaft with respect to a fixed reference. T_a is the accelerating torque acting on the shaft.

For a generator, the driving torque T_m is mechanical and the retarding or load torque T_e is electrical. Thus $T_a = T_m - T_e$ Nm. The subscript R is used to mean 'rated' for all quantities.

t time, second.

n_R rated shaft speed, rev./min.

w_o rated angular frequency of the revolving magnetic field, ele.rad./sec.

f_R base frequency, cycles/sec.

S_{B3} rated three phase VA rating

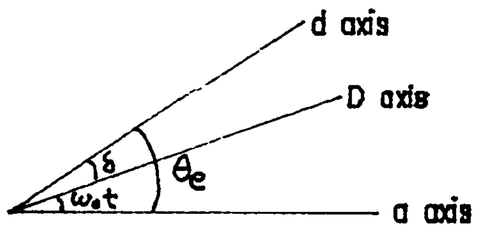


Figure H1 Angles between the d, D and a axes

The electrical angle θ_e is equal to the product of the mechanical angle θ_m and the number of pairs of poles $\frac{m}{2}$, that is,

$$\theta_e = \frac{m}{2}\theta_m \text{ and } f_R = \frac{mn_R}{2 \times 60} \text{ , so } \theta_e = \frac{60\, f_R}{n_R}\theta_m \quad \text{------(H.1)}$$

The electrical angular position δ, in radians, of the rotor with respect to a synchronously rotating reference axis is (as shown in figure H1)

$$\delta = \theta_e - w_o t$$

Then the angular velocity with respect to the reference axis is

$p\delta = p\theta_e - w_o$ and the angular acceleration is

$$p^2\delta = p^2\theta_e$$

Taking the second derivative of equation (H.1) and substituting

$$p^2\delta = \frac{60\, f_R}{n_R} p^2\theta_m \text{ , therefore}$$

$$\frac{Jn_R}{60\, f_R} p^2\delta = T_a \text{ , with}$$

$$w = p\theta_e \text{ , } pw = p^2\theta_e = p^2\delta\text{, so}$$

$$\frac{Jn_R}{60\, f_R} pw = T_a \quad \text{-------------(H.2)}$$

If this equation is normalised by dividing both sides by a rated torque,

$$T_B = \frac{60\, S_{B3}}{2\pi\, n_R},$$

also with the relation $w_o = 2\pi f_R$ (H.2) becomes

$$\frac{J\pi^2 n_R^2}{900\, w_o\, S_{B3}}\, pw = \frac{T_a}{T_B} \qquad \text{-------(H.3)}$$

Define $H = \dfrac{J\pi^2 n_R^2}{1800}$, then

Equation (H.3) becomes

$$\frac{2H}{w_o}\, pw = \overline{T}_a \qquad \text{---------(H.4)}$$

where H is in second, w and w_o are in electrical radians/second, and

the bar signifies p.u. values.

To normalise equation (H.4) completely, it is written as

$$2Hw_o\overline{p}\,\overline{w} = \overline{T}_a$$

Appendix I

Definition of fault, switching and connection matrices for faults of the particular type

Although the matrices have to be determined for each case, the generalised vector matrix equations can still be written as follows:
For LV circuit breaker opening, the equation will be

$$S_{1sr}(L - \frac{A}{n^2} L_{TL}Ce_T) S_{1sr} \frac{p\overline{I}}{w_o} =$$

$$S_{1sr}AC_T{}^t \frac{\overline{V}}{n} + S_{1sr}(R - \frac{pL}{w_o} + \frac{A}{n^2}R_{TL}Ce_T) S_{1sr}\overline{I} + \overline{K}$$

And for HV circuit breaker opening, equation (14) as defined in chapter 4 will be used.

Except where specified, otherwise, F_{2sr}, Ce_T and $M_{2sr}{}^{-1}$ will be unit matrices. The matrices for each case are listed below.

$i_a = i_b = i_c = \emptyset$ for

1 Fault @ B b-earth opening a & b & c

2 Fault @ B b-c opening a & b & c

3 Fault @ B b-earth opening a & c

$$S_{1s} = \begin{bmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & 1 & \emptyset \\ \emptyset & \emptyset & \emptyset \end{bmatrix} \quad F_{1s} = \begin{bmatrix} 1 & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & 1 \end{bmatrix} \quad M_{1s}^{-1} = \begin{bmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \end{bmatrix}$$

4 Fault @ B b–c–earth opening a & b

$$F_{ls} = M_{ls}^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad S_{ls} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5 Fault @ B b–c opening a

$$M_{ls}^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \qquad F_{ls} = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 0 & 0 \\ -0.5 & 0 & 0 \end{bmatrix} \qquad S_{ls} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$L = M_{lsr}^{-1} L \, (M_{lsr}^{-1})^{t}, \quad R = M_{lsr}^{-1} R (M_{lsr}^{-1})^{t}$$

6 Fault @ B b–c opening b

$$S_{lsr} M_{lsr}^{-1} \bar{E} = S_{lsr} M_{lsr}^{-1} \left(-R + \frac{pL}{w_o} + \frac{Lp}{w_o}\right) Ce_T \bar{I} - \bar{K}$$

$$= S_{lsr} \left[C_T^{t} \frac{1}{n} \bar{V} + \left(R_{TL} + \frac{L_{TL} p}{w_o}\right) Ce_T \bar{I}\right] \quad \text{where}$$

$$S_{ls} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad C^{t} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \qquad Ce = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \quad M_{ls}^{-1} = \frac{2}{3} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

7 Fault @ A b–earth opening a & b & c

$$F_{ls} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad M_{ls}^{-1} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \quad S_{ls} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad Ce = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$L = L, \; R = R$

8 Fault @ A b–c–earth opening b

$$F_{ls} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad M_{ls}^{-1} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad S_{ls} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad Ce = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$L = L, \; R = R$

9 Fault @ A b–c–earth opening c

$$F_{1s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad M^{-1}_{1s} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \quad S_{1s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Ce = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

L = L, R = R

10 Fault @ A b-c-earth opening a & b & c

$$F_{1s} = M^{-1}_{1s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad S_{1s} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

11 Fault @ A a-b-c-earth opening a or b or c or a & b & c

The switching matrix is a unit matrix and the fault and connection matrices are the same as those when no circuit breaker opening occurs.

12 Fault @ A b-c opening a

Same as case 5

13 Fault @ A b-c opening b

$$F_{1s} = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 0 & 0 \\ -0.5 & 0 & 0 \end{bmatrix} \quad M^{-1}_{1s} = -\frac{2}{3}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad S_{1s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Ce = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

L = L, R = R

14 Fault @ A b-c opening c

$$F_{1s} = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 0 & 0 \\ -0.5 & 0 & 0 \end{bmatrix} \quad M^{-1}_{1s} = -\frac{2}{3}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad S_{1s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Ce = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

L = L, R = R

15 Fault @ A b-c opening a & b & c

Same as case 5

16 Fault @ C a-earth opening a

$$S_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad F_{2s} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

17 Fault @ C a-earth opening a & b & c

$$S_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad F_{2s} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

18 Fault @ C b-c-earth opening b

$$S_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad F_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

19 Fault @ C b-c-earth opening a & b

$$S_{2s} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad F_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

20 Fault @ C b-c-earth opening c

$$S_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad F_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

21 Fault @ C b-c-earth opening b & c

$$S_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad F_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

22 Fault @ C b-c-earth opening a & c

$$S_{2s} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad F_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

23 Fault @ C b-c-earth opening a & b & c

$$S_{2s} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad F_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

24 Fault @ C 3 phase to earth opening any phase

The switching matrix is a unit matrix and the fault and connection matrices are the same as those when no circuit breaker opening occurs.

$i_{Ta} = i_{Tb} = i_{Tc} = 0$ for

25 Fault @ D b–c opening a & b & c

26 Fault @ D b–c opening c

$$S_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad Ce = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad F_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and $S_{2sr}Ce_T$ is replaced by Ce_T in equation (14) of chapter 4.

27 Fault @ D b–c opening b

$$S_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad Ce = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \quad F_{2s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and $S_{2sr}Ce_T$ is replaced by Ce_T in equation (14) of chapter 4.

28 Fault @ C b–c opening a

Same as fault @ D b–c opening a (page 52)

29 Fault @ C b–c opening a & b & c

Same as fault @ D b–c opening a (page 52)

30 Phase a is opened as LV side at steady state

$$M_{1s}^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad S_{1s} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(M_{1s}^{-1})^{t} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad F_{1s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$L = M_{1sr}^{-1} L (M_{1sr}^{-1})^{t}, \quad R = M_{1sr}^{-1} R (M_{1sr}^{-1})^{t}$$

31 Phase a is opened at HV side at steady state

$$S_{2s} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad M_{2s}^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$L' = M_{2sr}^{-1} L' (M_{2sr}^{-1})^{t}, \quad R' = M_{2sr}^{-1} R' (M_{2sr}^{-1})^{t}$$

Appendix J

Computer programs

Except where specified, all the programs were developed on a Harris 800 computer system in Fortran 66.

The following program listings are typical of the software development required for simulation.

** J1 **

```
C  Purpose
C  -------
C  This program is used to find the pickup value and time setting
C  multiplier for relays.
C  It requires the system under investigation to be a radial one.
C  Main parameters :
C  ---------------
C        A,B,C,D,E     Coefficients for polynomial of fuse
C        X             Set to be Ø.Ø1 sec. and the MVA/A corresponds
C                      to it is found and compared with Y2 which is
C                      the maximum fault level up to which the
C                      next stage required discrimination with the fuse
C        G1Ø,G11       The initial guessus in time(log value)
C                      which used to find the time corresponds
C                      to the MVA/A Y2
C        CULOAD        Current/MVA loading
C        CUTGRD        The factor required for the current grading
C        FTCMAX        Maximum fault level
C        FUSERT        Fuse rating
C        MRDMAR        Grading margin
C        HGSET         High-set
C        KEYNO         Key number
C                      1    for fuse
C                      2    for extremely inverse relay
C                      3    for standard inverse relay (3/1Ø)
C                      4    for standard inverse relay (1.3/1Ø)
C                      9    for stop
C        PICKUP        Pickup value
C        PRIREC        Primary relay current setting
C        PSMAX         Maximum allowable plug setting
C        TMS           Time multiplier setting
C
C
      DIMENSION CULOAD(1Ø),CUTGRD(1Ø),FTCMAX(1Ø),FUSERT(1Ø),
     1GRDMAR(1Ø),HGSET(1Ø),KEYNO(1Ø),PICKUP(1Ø),PLUGST(1Ø),
     2PRIREC(1Ø),PSMAX(1Ø),TM(2),TMS(1Ø)
C
C  Input data
C
      READ(1Ø,1)A,B,C,D,E,J2
1     FORMAT(5F11.7,I2)
      READ(1Ø,5)X,G1Ø,G11,Y2
5     FORMAT(4F12.5)
      X1Ø=G1Ø
      X11=G11
      DO 1Ø J3=1,J2
1Ø      READ(1Ø,15)CULOAD(J3),CUTGRD(J3),FTCMAX(J3),FUSERT(J3),
     1GRDMAR(J3),HGSET(J3),KEYNO(J3),PICKUP(J3),PRIREC(J3),
     2PSMAX(J3),TMS(J3)
15     FORMAT(F8.2,F4.1,F1Ø.2,F6.1,F4.2,F9.1,I2,1X,F8.2,F7.1,F5.2,F6.4)
      WRITE(11,2Ø)A,B,C,D,E,J2
```

```
20     FORMAT(1X,5F11.7,I2)
       WRITE(11,25)X,G10,G11,Y2
25     FORMAT(1X,4F12.5)
       DO 30 J3=1,J2
30     WRITE(11,35)CULOAD(J3),CUTGRD(J3),FTCMAX(J3),FUSERT(J3),
      1GRDMAR(J3),HGSET(J3),KEYNO(J3),PICKUP(J3),PRIREC(J3),
      2PSMAX(J3),TMS(J3)
35     FORMAT(1X,F8.2,F4.1,F10.2,F6.1,F4.2,F9.1,I2,F8.2,F7.1,F5.2,F6.4)
       MM=1
40     NN=MM+1
       L=KEYNO(MM)+KEYNO(NN)
       L55=KEYNO(NN)-KEYNO(MM)
C  L equal 3 means an extremely inverse followed by a fuse
C  L equal 4 may mean (3/10) followed by a fuse
C  L equal 5 may mean (1.3/10) followed by a fuse
C  L greater than 9 means the end
C
       IF ((L.EQ.3).OR.(L.EQ.4).OR.((L.EQ.5).AND.(L55.NE.1))) GOTO 45
       GOTO 50
45     CALL ROOT(A,B,C,D,E,X,X10,X11,X21,Y2)
50     IF (L.GT.9) GOTO 275
       N=0
       N1=1
       N2=1
       N3=1
       N4=0
       N5=0
       N6=1
       N7=0
       Y8=1.0
55     K=1
       L1=-1
       L2=999
       M4=1
       M7=1
       M15=1
       M16=0
C
C  To calculate the pickup value for the stage nearer to the source
C
       IF ((L.EQ.3).OR.((L.EQ.4).AND.(L55.NE.0))) GOTO 60
       IF ((L.EQ.5).AND.(L55.NE.1)) GOTO 60
       F=CUTGRD(NN)*PICKUP(MM)
       GOTO 65
C  A relay followed by a fuse
60     F=CUTGRD(NN)*FUSERT(MM)
       GOTO 65
C  Relay followed by relay
65     IF (F.GE.CULOAD(NN))GOTO 70
       F=CULOAD(NN)
70     F1=F/(PRIREC(NN)*0.25)
       M=F1
       F2=M
```

```
      IF (F1.LE.F2) GOTO 75
      M=M+1
75    F2=M
      PICKUP(NN)=F2*0.25*PRIREC(NN)
C
C  L equal 4 or 6 may mean relays followed by their same type
C
      IF ((L.EQ.4).AND.(L55.NE.0)) GOTO 175
      IF ((L.EQ.6).AND.(L55.NE.0)) GOTO 80
      IF (L.EQ.4.OR.L.EQ.6.OR.L.EQ.8) GOTO 290
      IF ((L.EQ.3).OR.((L.EQ.5).AND.(L55.NE.1))) GOTO 175
80    IF(HGSET(MM).GT.0.0) GOTO 85
      XI=FTCMAX(MM)/PICKUP(MM)
      GOTO 90
85    XI=HGSET(MM)/PICKUP(MM)
C  (3/10) followed by an extremely inverse relay
90    IF (L.EQ.5) GOTO 150
      IF (L.EQ.6) GOTO 110
      IF ((L.EQ.7).AND.(L55.EQ.1)) GOTO 130
C  (3/10) followed by (1.3/10)
      XI=ALOG10(XI)
      X2=1.167367+XI*(-2.66868+XI*(2.91788+XI*(-1.6677+0.36193*XI)))
      X2=EXP(X2*ALOG(10.0))*TMS(MM)
      PUMAX=PSMAX(NN)*PRIREC(NN)
      IF (PICKUP(NN).LE.PUMAX) GOTO 95
      GOTO 152
95    IF (HGSET(MM).GT.0.0) GOTO 100
      XI=FTCMAX(MM)/PICKUP(NN)
      GOTO 105
100   XI=HGSET(MM)/PICKUP(NN)
105   Y=0.02*ALOG10(XI)
      O=EXP(Y*ALOG(10.0))
      X1=0.14/(O-1)*TMS(NN)
      Z=X1-X2
      GOTO 200
C  (1.3/10) followed by an extremely inverse relay
110   XI1=XI-1
      Y=1.7*ALOG10(XI1)
      O=EXP(Y*ALOG(10.0))
      D1=19.0/O
      X2=(0.2+D1)*TMS(MM)
      PUMAX=PSMAX(NN)*PRIREC(NN)
      IF (PICKUP(NN).LE.PUMAX) GOTO 115
      GOTO 152
115   IF (HGSET(MM).GT.0.0) GOTO 120
      XI=FTCMAX(MM)/PICKUP(NN)
      GOTO 125
120   XI=HGSET(MM)/PICKUP(NN)
125   XI=ALOG10(XI)
      X1=1.167367+XI*(-2.66868+XI*(2.91788+XI*(-1.6677+0.36193*XI)))
      X1=EXP(X1*ALOG(10.0))*TMS(NN)
      Z=X1-X2
      GOTO 200
```

```
C  (1.3/1Ø) followed by (3/1Ø)
13Ø    Y=Ø.Ø2*ALOG1Ø(XI)
       O=EXP(Y*ALOG(1Ø.Ø))
       X2=Ø.14/(O-1)*TMS(MM)
       PUMAX=PSMAX(NN)*PRIREC(NN)
       IF (PICKUP(NN).LE.PUMAX) GOTO 135
       GOTO 152
135    IF (HGSET(MM).GT.Ø.Ø) GOTO 14Ø
       XI=FTCMAX(MM)/PICKUP(NN)
       GOTO 145
14Ø    XI=HGSET(MM)/PICKUP(NN)
145    XI=ALOG1Ø(XI)
       X1=1.167367+XI*(-2.66868+XI*(2.91788+XI*(-1.6677+Ø.36193*XI)))
       X1=EXP(X1*ALOG(1Ø.Ø))*TMS(NN)
       Z=X1-X2
       GOTO 2ØØ
C  (3/1Ø) followed by an extremely inverse relay
15Ø    XI1=XI-1
       Y=1.7*ALOG1Ø(XI1)
       O=EXP(Y*ALOG(1Ø.Ø))
       D1=19/O
       X2=(Ø.2+D1)*TMS(MM)
       PUMAX=PSMAX(NN)*PRIREC(NN)
       IF (PICKUP(NN).LE.PUMAX) GOTO 16Ø
152    WRITE(11,155)
155    FORMAT(1X,'CURRENT GRADING IS NOT OK')
       GOTO 447
16Ø    IF (HGSET(MM).GT.Ø.Ø) GOTO 165
       XI=FTCMAX(MM)/PICKUP(NN)
       GOTO 17Ø
165    XI=HGSET(MM)/PICKUP(NN)
17Ø    Y=Ø.Ø2*ALOG1Ø(XI)
       O=EXP(Y*ALOG(1Ø.Ø))
       X1=Ø.14/(O-1)*TMS(NN)
       Z=X1-X2
       GOTO 2ØØ
175     IF (HGSET(MM).NE.Ø.Ø) GOTO 18Ø
       XI=FTCMAX(MM)/PICKUP(NN)
       GOTO 185
18Ø    XI=HGSET(MM)/PICKUP(NN)
185    IF (L.EQ.3) GOTO 195
       IF (L.EQ.4) GOTO 19Ø
C  (1.3/1Ø) followed by a fuse
       XI=ALOG1Ø(XI)
       X2=1.167367+XI*(-2.66868+XI*(2.91788+XI*(-1.6677+Ø.36193*XI)))
       X2=EXP(X2*ALOG(1Ø.Ø))*TMS(NN)
       Y2=PICKUP(NN)*2.Ø
       IF (Y8.EQ.1.Ø) CALL ROOT(A,B,C,D,E,X,G1Ø,G11,X31,Y2)
       Y8=2.Ø
       Z=X2-X21
       GOTO 2ØØ
C  (3/1Ø) followed by a fuse
19Ø    Y=Ø.Ø2*ALOG1Ø(XI)
```

```
      O=EXP(Y*ALOG(10.0))
      X2=0.14/(O-1)*TMS(NN)
      Y2=PICKUP(NN)*2.0
      IF (Y8.EQ.1.0) CALL ROOT(A,B,C,D,E,X,G10,G11,X31,Y2)
      Y8=2.0
      Z=X2-X21
      GOTO 200
C  An extremely relay followed by a fuse
195   XI1=XI-1
      Y=1.7*ALOG10(XI1)
      O=EXP(Y*ALOG(10.))
      D1=19.0/O
      X2=(0.2+D1)*TMS(NN)
      Y2=PICKUP(NN)*2.0
      IF (Y8.EQ.1.0)CALL ROOT(A,B,C,D,E,X,G10,G11,X31,Y2)
      Y8=2.0
      Z=X2-X21
C  X2 ,X21 are the time at the maximum fault level
C  Z is the minimum time difference at the maximum fault level
200   IF ((L.EQ.3).OR.(L.EQ.4).OR.((L.EQ.5).AND.(L55.NE.1))) GOTO 205
      ACC=X2*0.25+0.25
      GRDMAR(NN)=ACC
      GOTO 210
205   ACC=X21*0.4+0.15
      GRDMAR(NN)=ACC
210   CALL GOLDEN(A,B,C,D,FTCMAX,HGSET,K,L,L1,L2,M4,M7,M15,M16,
     1MM,NN,PICKUP,TMS,X3,X21,X31,KEYNO)
C  Find the minimum time difference
      IF (X3.GT.Z) GOTO 220
      GOTO 225
220   X3=Z
C
C  To check whether the minimum time difference is within the
C  specified limit. If not they vary the time setting multiplier
C  according to the result of the logical and arithmetical statements
C
225   IF (ABS(X3-ACC)/ACC.LT.0.0005) GOTO 265
      IF (X3.GT.ACC.AND.N6.EQ.1) GOTO 235
      IF (X3.LT.ACC.AND.N1.EQ.1.AND.N2.EQ.1.AND.N3.EQ.1) GOTO 240
      IF (X3.GT.ACC.AND.N1.EQ.1.AND.N2.EQ.1.AND.N3.EQ.1) GOTO 245
      IF (X3.LT.ACC.AND.N1.EQ.1.AND.N2.EQ.2.AND.N3.EQ.1) GOTO 250
      IF (X3.GT.ACC.AND.N1.EQ.1.AND.N2.EQ.2.AND.N3.EQ.1) GOTO 245
      IF (X3.LT.ACC.AND.N1.EQ.1.AND.N2.EQ.2.AND.N3.EQ.2) GOTO 250
      IF (X3.GT.ACC.AND.N1.EQ.1.AND.N2.EQ.2.AND.N3.EQ.2) GOTO 245
235   N1=2
      N5=1
      IF (X3.GT.ACC.AND.N2.EQ.1.AND.N3.EQ.1) GOTO 240
      IF (X3.LT.ACC.AND.N2.EQ.1.AND.N3.EQ.1) GOTO 245
      IF (X3.GT.ACC.AND.N2.EQ.2.AND.N3.EQ.1) GOTO 250
      IF (X3.LT.ACC.AND.N2.EQ.2.AND.N3.EQ.1) GOTO 245
      IF (X3.GT.ACC.AND.N2.EQ.2.AND.N3.EQ.2) GOTO 250
      IF (X3.LT.ACC.AND.N2.EQ.2.AND.N3.EQ.2) GOTO 245
240   N6=2
```

```
      TMS(NN)=TMS(NN)+(-1)**N5*Ø.1
      GOTO 55
245   N2=2
      X2Ø=(2.Ø)**N
      TMS(NN)=TMS(NN)-(-1)**N5*Ø.Ø5/X2Ø
      N=N+1
      N7=N7+1
C
C  Test the number of iterations
C
      IF (N7.GT.5Ø) GOTO 255
      GOTO 55
25Ø   N3=2
      X2Ø=(2.Ø)**N
      TMS(NN)=TMS(NN)+(-1)**N5*Ø.Ø5/X2Ø
      N=N+1
      N4=N4+1
C
C  Test the number of iterations
C
      IF (N4.GT.5Ø) GOTO 255
      GOTO 55
255   WRITE(11,26Ø)
26Ø   FORMAT(1ØX,'NUMBER OF ITERATIONS EXCEEDED'/)
      GOTO 285
265   WRITE(11,27Ø)
27Ø   FORMAT(1ØX,'NORMAL COMPLETION'/)
      MM=MM+1
      GOTO 4Ø
275   WRITE(11,28Ø)
28Ø   FORMAT(1X,'EXECUTION COMPLETED NORMALLY'/)
285   GOTO 435
29Ø   PUMAX=PSMAX(NN)*PRIREC(NN)
      CALL OPTIME(FTCMAX,HGSET,L,MM,NN,PICKUP,TM,TMS)
      TMDFC=TM(2)-TM(1)
      GRDMAR(NN)=TM(1)*Ø.25+Ø.25
      IF (TMDFC.GE.GRDMAR(NN)) GOTO 295
      GOTO 3Ø5
295   WRITE(11,3ØØ)PICKUP(NN)
3ØØ   FORMAT(1X,'OPTIME TOO LONG, MIN.T.M.S. AND PICKUP USED',F17.7/)
      GOTO 43Ø
3Ø5   IF (PICKUP(NN).GT.PUMAX) GOTO 375
      CALL TSM1(FTCMAX,GRDMAR,HGSET,L,MM,NN,PICKUP,TM,TMS)
      IF (TMS(NN).GE.Ø.1.AND.TMS(NN).LE.1.Ø) GOTO 31Ø
      GOTO 32Ø
31Ø   WRITE(11,315)PICKUP(NN),TMS(NN)
315   FORMAT(1X,2F17.7/)
      GOTO 43Ø
32Ø   WRITE(11,325)
325   FORMAT(1X,'TIME GRADING IS NOT OK')
      GOTO 447
375   WRITE(11,38Ø)
38Ø   FORMAT(1X,'CURRENT GRADING IS NOT OK')
```

```
      GOTO 447
430   MM=MM+1
      GOTO 40
435   DO 440 J3=1,J2
440   WRITE(11,445)CULOAD(J3),CUTGRD(J3),FTCMAX(J3),FUSERT(J3),
     1GRDMAR(J3),HGSET(J3),KEYNO(J3),PICKUP(J3),PRIREC(J3),
     2PSMAX(J3),TMS(J3)
445   FORMAT(1X,F8.2,F4.1,F10.2,F6.1,F4.2,F9.1,I2,F8.2,
     1F7.1,F5.2,F6.4/)
447   STOP
      END
C
C PURPOSE
C -------
C Due to the difference in slope in fuse, extremely and standard
C inverse relays characteristics. The minimum time difference occurs
C within the range from the pickup of the relay (nearer to the
C source) to the maximum fault level or high-set level of the
C relay (further from the source) up to which discrimination
C is required. In case of fuse, maximum fault level is used,
C since no high-set level is defined for fuse.
C This routine is used to find the minimum time difference
C and the level at which it occured by using the Golden-section
C method.
C
      SUBROUTINE GOLDEN(A,B,C,D,FTCMAX,HGSET,K,L,L1,L2,M4,M7,
     1M15,M16,MM,NN,PICKUP,TMS,X3,X21,X31,KEYNO)
      DIMENSION FTCMAX(10),HGSET(10),PICKUP(10),T1(40),T2(4),T3(2),
     1TMS(10),X6(40),KEYNO(10)
C
C Test whether an extremely inverse relay followed by a fuse or a
C standard inverse relay followed by an extremely inverse relay.
C
      L55=KEYNO(NN)-KEYNO(MM)
      IF ((L.EQ.3).OR.(L.EQ.4).OR.((L.EQ.5).AND.(L55.NE.1))) GOTO 450
      GOTO 455
450   P=X21
      T1(1)=1.0
      H1=X31-X21
      GOTO 465
455   P=2.00*PICKUP(NN)
      T1(1)=0.0
      IF (HGSET(MM).GT.0.0) GOTO 460
      H1=FTCMAX(MM)-P
      GOTO 465
460   H1=HGSET(MM)-P
465   DO 2600 K=1,39
      IF ((L.EQ.3).OR.(L.EQ.4).OR.((L.EQ.5).AND.(L55.NE.1))) GOTO 490
      IF (T1(K).GT.1.0) GOTO 2670
      IF (M16.GT.39) GOTO 2650
      M16=M16+1
      H2=P+H1*T1(K)
      XI=H2/PICKUP(NN)
```

```
      IF ((L.EQ.7).AND.(L55.EQ.1)) GOTO 475
      IF (L.EQ.7) GOTO 48Ø
      IF (L.EQ.6) GOTO 485
47Ø   Y=Ø.Ø2*ALOG1Ø(XI)
      O=EXP(Y*ALOG(1Ø.Ø))
      X4=Ø.14/(O-1.Ø)
      X1=X4*TMS(NN)
      XI=H2/PICKUP(MM)
      XI1=XI-1
      Y=1.7*ALOG1Ø(XI1)
      O=EXP(Y*ALOG(1Ø.Ø))
      D1=19.Ø/O
      X5=Ø.2+D1
      X2=X5*TMS(MM)
      X3=X1-X2
      GOTO 13ØØ
C  (1.3/1Ø) followed by (3/1Ø)
475   XI=ALOG1Ø(XI)
      V=1.167367+XI*(-2.66868+XI*(2.917875+XI*(-1.6677+Ø.36193*XI)))
      X1=EXP(V*ALOG(1Ø.Ø))*TMS(NN)
      XI=H2/PICKUP(MM)
      Y=Ø.Ø2*ALOG1Ø(XI)
      O=EXP(Y*ALOG(1Ø.Ø))
      X4=Ø.14/(O-1.Ø)
      X2=X4*TMS(MM)
      X3=X1-X2
      GOTO 13ØØ
C  (3/1Ø) followed by (1.3/1Ø)
48Ø   Y=Ø.Ø2*ALOG1Ø(XI)
      O=EXP(Y*ALOG(1Ø.Ø))
      X4=Ø.14/(O-1.Ø)
      X1=X4*TMS(NN)
      XI=H2/PICKUP(MM)
      XI=ALOG1Ø(XI)
      X2=1.167367+XI*(-2.66868+XI*(2.91788+XI*(-1.6677+Ø.36193*XI)))
      X2=EXP(X2*ALOG(1Ø.Ø))*TMS(MM)
      X3=X1-X2
      GOTO 13ØØ
C  (1.3/1Ø) followed by an extremely inverse relay
485   XI=ALOG1Ø(XI)
      X1=1.167367+XI*(-2.66868+XI*(2.91788+XI*(-1.6677+Ø.36193*XI)))
      X1=EXP(X1*ALOG(1Ø.Ø))*TMS(NN)
      XI=H2/PICKUP(MM)
      XI1=XI-1
      Y=1.7*ALOG1Ø(XI1)
      O=EXP(Y*ALOG(1Ø.Ø))
      D1=19.Ø/O
      X5=Ø.2+D1
      X2=X5*TMS(MM)
      X3=X1-X2
      GOTO 13ØØ
C
C  Relay followed by a fuse
```

```
C
49Ø    IF (T1(K).LT.Ø.Ø) GOTO 267Ø
       IF (M16.GT.39) GOTO 265Ø
       M16=M16+1
       X1=P+H1*T1(K)
       XX=ALOG1Ø(X1)
       YY=A+XX*(B+XX*(C+D*XX))
       H2=EXP(YY*ALOG(1Ø.Ø))
       XI=H2/PICKUP(NN)
       IF (L.EQ.3) GOTO 5ØØ
       IF (L.EQ.4) GOTO 495
C  (1.3/1Ø) followed by a fuse
       XI=ALOG1Ø(XI)
       X2=1.167367+XI*(-2.66868+XI*(2.91788+XI*(-1.6677+Ø.36193)))
       X2=EXP(X2*ALOG(1Ø.Ø))*TMS(NN)
       X3=X2-X1
       GOTO 13ØØ
C  (3/1Ø) followed by a fuse
495    Y=Ø.Ø2*ALOG1Ø(XI)
       O=EXP(Y*ALOG(1Ø.Ø))
       X4=Ø.14/(O-1.Ø)
       X2=X4*TMS(NN)
       X3=X2-X1
       GOTO 13ØØ
C  An extremely inverse relay followed by a fuse
5ØØ    XI1=XI-1
       Y=1.7*ALOG1Ø(XI1)
       O=EXP(Y*ALOG(1Ø.Ø))
       D1=19/O
       X5=Ø.2+D1
       X2=X5*TMS(NN)
       X3=X2-X1
C
C  To find the minimum time difference by comparing the previous
C  value with the present value.
C
13ØØ   X6(K)=5Ø-X3
       M=K/2
       IF (K.EQ.1) GOTO 144Ø
       IF(K.GT.L1.AND.2*M.LT.K.AND.X6(K).GT.X6(K-1).AND.K.LT.L2)GOTO146Ø
       IF(K.GT.L1.AND.2*M.EQ.K.AND.X6(K).GT.X6(K-1).AND.K.LT.L2)GOTO149Ø
       IF((K.LT.L1.OR.X6(K).LE.X6(K-1)).AND.K.LT.L2.AND.M4.EQ.1)GOTO152Ø
       GOTO 192Ø
C
C  Form the Golden section number
C
144Ø   T1(K+1)=1-Ø.Ø618
       IF ((L.EQ.3).OR.((L.EQ.4).AND.(L55.NE.Ø))) GOTO 145Ø
       IF ((L.EQ.5).AND.(L55.NE.1)) GOTO 145Ø
       T1(K+1)=1-T1(K+1)
145Ø   GOTO 26ØØ
146Ø   X7=K+1
       X8=K-1
```

```
      T1(K+1)=1-(X7/2.Ø*Ø.Ø618+X8/2.Ø*Ø.Ø382)
      IF ((L.EQ.3).OR.((L.EQ.4).AND.(L55.NE.Ø))) GOTO 145Ø
      IF ((L.EQ.5).AND.(L55.NE.1)) GOTO 145Ø
      T1(K+1)=1-T1(K+1)
148Ø  GOTO 26ØØ
149Ø  X9=K
      T1(K+1)=1-(X9/2.Ø*(Ø.Ø382+Ø.Ø618))
      IF ((L.EQ.3).OR.((L.EQ.4).AND.(L55.NE.Ø))) GOTO 145Ø
      IF ((L.EQ.5).AND.(L55.NE.1)) GOTO 145Ø
      T1(K+1)=1-T1(K+1)
151Ø  GOTO 26ØØ
152Ø  L1=999
      L2=K
      M4=2
      IF (M15.EQ.2) GOTO 17ØØ
      IF (K.EQ.2) GOTO 27ØØ
      T1(K+1)=T1(K)-T1(K-1)+T1(K-2)
      GOTO 176Ø
17ØØ  T1(K+1)=T2(1)-T2(2)+T2(3)
      T2(4)=T1(K+1)
176Ø  IF (M15.EQ.2) GOTO 183Ø
C
C  Test the range of uncertainty and accuracy
C
      T4=T1(K+1)-T1(K-1)
      T5=(T1(K)-T1(K-1))/T1(K)
      IF (ABS(T4).LT.Ø.ØØØØ1.AND.T5.LT.Ø.Ø3) GOTO 267Ø
      IF (ABS(T4).LT.Ø.ØØØØ1) GOTO 267Ø
      GOTO 26ØØ
183Ø  T4=T2(4)-T2(2)
      T5=(T2(1)-T2(2))/T2(2)
      IF (ABS(T4).LT.Ø.ØØØØ1.AND.T5.LT.Ø.Ø3) GOTO 267Ø
      IF (ABS(T4).LT.Ø.ØØØØ1) GOTO 267Ø
      GOTO 26ØØ
192Ø  IF (M7.EQ.1) GOTO 194Ø
      GOTO 232Ø
194Ø  A2=X6(1)
      DO 2ØØØ M9=2,K
      IF (A2.LE.X6(M9)) GOTO 198Ø
      GOTO 2ØØØ
198Ø  A2=X6(M9)
      M11=M9
2ØØØ  CONTINUE
      B2=T1(M11)
      M6=K-1
      M5=2
      DO 215Ø M8=1,M6
      DO 213Ø M9=M5,K
      IF (T1(M9).LT.T1(M8)) GOTO 21ØØ
      GOTO 213Ø
21ØØ  U1=T1(M8)
      T1(M8)=T1(M9)
      T1(M9)=U1
```

```
2130  CONTINUE
      M5=M5+1
2150  CONTINUE
      DO 2230 M9=1,K
      IF (T1(M9).GT.B2) GOTO 2240
2230  CONTINUE
2240  M15=2
C
C  Arrange T2 in ascending order
C
      T2(1)=T1(M9)
      T2(2)=T1(M9-1)
      T2(3)=T1(M9-2)
      M7=2
      GOTO 1520
2320  IF (X6(K).GT.A2) GOTO 2340
      GOTO 2370
2340  B2=T1(K)
      A2=X6(K)
      GOTO 2380
2370  B2=T2(2)
2380  M5=2
      DO 2480 I=1,3
      DO 2460 J=M5,4
      IF (T2(J).LT.T2(I)) GOTO 2430
      GOTO 2460
2430  U1=T2(I)
      T2(I)=T2(J)
      T2(J)=U1
2460  CONTINUE
      M5=M5+1
2480  CONTINUE
C
C  Chose the required T2 which produces minimum time difference
C
      DO 2550 I=3,4
      IF (T2(I).GT.B2) GOTO 2560
2550  CONTINUE
C
C  Form the T2 array
C
2560  E1=T2(I)
      E2=T2(I-1)
      E3=T2(I-2)
      T2(1)=E1
      T2(2)=E2
      T2(3)=E3
      GOTO 1520
2600  CONTINUE
2650  WRITE(11,2651)
2651  FORMAT(10X,'NUMBER OF ITERATIONS EXCEEDED ****'/)
2670  GOTO 3000
2700  WRITE(11,2750)
```

```
2750  FORMAT(10X,'K=2   '/)
3000  RETURN
      END
C
C  Purpose
C  -------
C
C  This routine is used to find the operating time of the fuse
C  at the fault level Y2. If the operating time is less than
C  0.01 sec. Then the nominal operating time for the fuse is
C  assumed to be 0.01 SEC..
C
       SUBROUTINE ROOT(A,B,C,D,E,X,X10,X11,X41,Y2)
      IF (X.GT.0.0) GOTO 1
      Y=A+X*(B+X*(C+X*(D+E*X)))
      Y1=EXP(Y*ALOG(10.0))
      IF (Y2.GT.Y1) GOTO 3600
C  Calculate the function values at the initial guesses
1     FX10=-ALOG10(Y2)+A+X10*(B+X10*(C+X10*(D+E*X10)))
3100  FX11=-ALOG10(Y2)+A+X11*(B+X11*(C+X11*(D+E*X11)))
C  Increase X11 by 1 if FX10 and FX11 are of same sign
      IF (FX10.NE.SIGN(FX10,FX11)) GOTO 3200
      X11=X11+1.00
      GOTO 3100
C  Calculate and print the new estimate
3200  X12=X10-FX10*(X10-X11)/(FX10-FX11)
      FX12=-ALOG10(Y2)+A+X12*(B+X12*(C+X12*(D+E*X12)))
C  X12 replaces X10 or X11 depending on the sign of FX12
      IF (FX10.NE.SIGN(FX10,FX12)) GOTO 3300
      BETA=ABS(X12-X10)
      X10=X12
      FX10=FX12
      GOTO 3400
3300  BETA=ABS(X12-X11)
      X11=X12
C  Normalize the change if ABS(X12)  > 1.0
3400  IF (ABS(X12).GT.1.0) BETA=BETA/ABS(X12)
C  Repeat calculation until beta is small enough
      IF (BETA.GT.1.0E-05) GOTO 3200
      X41=X12
      X41=EXP(X41*ALOG(10.0))
      RETURN
3600  X41=0.01
      RETURN
      END
C
C Purpose
C -------

C This routine is used to calculate the operating time for relays
C
      SUBROUTINE OPTIME(FTCMAX,HGSET,L,MM,NN,PICKUP,TM,TMS)
      DIMENSION FTCMAX(10),HGSET(10),PICKUP(10),TM(2),TMS(10)
```

```
      I=1
      DO 372Ø J=MM,NN
      IF (HGSET(MM).GT.Ø.Ø) GOTO 361Ø
      XI=FTCMAX(MM)/PICKUP(J)
      GOTO 3612
361Ø  XI=HGSET(MM)/PICKUP(J)
3612  IF (L.EQ.4) GOTO 37ØØ
      IF(L.EQ.8) GOTO 37Ø1
C  (3/1Ø) followed by (3/1Ø)
      Y=Ø.Ø2*ALOG1Ø(XI)
      O=EXP(Y*ALOG(1Ø.Ø))
      T=Ø.14/(O-1.Ø)
      GOTO 371Ø
C
C  Extremely inverse relay followed bt its own type
C
37ØØ  XI1=XI-1.Ø
      Y=1.7*ALOG1Ø(XI1)
      O=EXP(Y*ALOG(1Ø.Ø))
      D1=19.Ø/O
      T=Ø.2+D1
      GOTO 371Ø
C  (1.3/1Ø) followed by (1.3/1Ø)
37Ø1  XI=ALOG1Ø(XI)
      T=1.167367+XI*(-2.66868+XI*(2.91788+XI*(-1.6677+Ø.36193*XI)))
      T=EXP(T*ALOG(1Ø.Ø))
371Ø  TM(I)=T*TMS(J)
      I=I+1
372Ø  CONTINUE
      RETURN
      END
C
C  Purpose
C  -------
C  This routine is used to find the time setting multiplier
C
      SUBROUTINE TSM1(FTCMAX,GRDMAR,HGSET,L,MM,NN,PICKUP,TM,TMS)
      DIMENSION FTCMAX(1Ø),GRDMAR(1Ø),HGSET(1Ø),PICKUP(1Ø),TM(2),TMS(1Ø)
C
C  Test whether high-set exists or not
C
      IF (HGSET(MM).GT.Ø.Ø) GOTO 38ØØ
      XI=FTCMAX(MM)/PICKUP(NN)
      GOTO 38Ø2
38ØØ  XI=HGSET(MM)/PICKUP(NN)
38Ø2  IF (L.EQ.4) GOTO 39ØØ
      IF (L.EQ.8) GOTO 39Ø2
C
C  Standard inverse relay followed bt its own type
C
      Y=Ø.Ø2*ALOG1Ø(XI)
      O=EXP(Y*ALOG(1Ø.Ø))
      T=Ø.14/(O-1.Ø)
```

```
      GOTO 4ØØØ
C
C  Extremely inverse relay followed by its own type
C
3900  XI1=XI-1.Ø
      Y=1.7*ALOG1Ø(XI1)
      O=EXP(Y*ALOG(1Ø.Ø))
      D1=19.Ø/O
      T=Ø.2+D1
      GOTO 4ØØØ
3902  XI=ALOG1Ø(XI)
      T=1.167367+XI*(-2.66868+XI*(2.91788+XI*(-1.6677+Ø.36193*XI)))
      T=EXP(T*ALOG(1Ø.Ø))
4000  TM(2)=TM(1)+GRDMAR(NN)
      TMS(NN)=TM(2)/T
      RETURN
      END
£VX
```

** J2 **

```
100 INIT
105 REM This program is developed in Tektronix 4051.
107 REM This program is for plotting the log-log graph. After the
108 REM graph is plotted, another program is loaded into the memory
109 REM for plotting the relay and fuse characteristics.
110 DIM T5(10),S5(10),P5(10),L5(10),O1(10),V2(10),U2(10)
120 PRINT "INPUT THE TITLE"
130 INPUT F
140 PRINT "NUMBER OF RELAYS ?";
150 INPUT N
152 PRINT
155 PRINT "1-----(3/10)"
156 PRINT
157 PRINT "2-----EXTREMELY INVERSE"
158 PRINT
159 PRINT "3-----(1.3/10)"
160 PRINT
168 PRINT "RELAY TYPE","SETTING","PICKUP","LEVEL"
170 FOR I=1 TO N
180 INPUT T5(I),S5(I),P5(I),L5(I)
190 NEXT I
200 PRINT "NUMBER OF FUSES ?";
210 INPUT O2
213 PRINT " MVA=A+BT+CT^2"
215 PRINT "A","B","C"
220 FOR I=1 TO O2
230 INPUT O1(I),U2(I),V2(I)
240 NEXT I
250 PRINT "FILE NUMBER USED TO STORE THE DATA"
260 INPUT K
270 PRINT "NUMBER OF LOCATION IN THE H ARRAY"
280 INPUT M
290 FIND K
300 DELETE 100,300
310 DIM H(M)
320 FOR H1=1 TO M
330 INPUT @33:H(H1)
340 NEXT H1
350 PAGE
360 VIEWPORT 10,120,10,90
370 WINDOW H(1)+LGT(H(8)),H(2)+LGT(H(8)),H(3)+LGT(H(7)),H(4)+LGT(H(7))
380 AXIS H(5),H(6),H(1)+LGT(H(8)),H(3)+LGT(H(7))
390 IF LGT(H(8))=>0 THEN 420
400 R=H(2)
410 GO TO 430
420 R=H(2)+LGT(H(8))
430 FOR V=H(8) TO 10*H(8) STEP 9*V
440 W=V*10
450 FOR U=V TO W STEP V
460 A=LGT(U)
470 MOVE A,H(3)+LGT(H(7))
```

```
480 RMOVE Ø,R
490 RDRAW Ø,-2*R
500 RMOVE Ø,R
510 NEXT U
520 NEXT V
530 FOR U=1ØØ*H(8) TO 1ØØØ*H(8) STEP 1ØØ*H(8)
540 A=LGT(U)
550 MOVE A,H(3)+LGT(H(7))
560 RMOVE Ø,R
570 RDRAW Ø,-2*R
580 RMOVE Ø,R
590 NEXT U
600 FOR A=H(1)+LGT(H(8)) TO H(2)+LGT(H(8))
610 MOVE A+H(9),H(1Ø)+H(3)+LGT(H(7))
620 U=EXP(A*LOG(1Ø))
630 PRINT U
640 NEXT A
650 FOR V=Ø.1*H(7) TO H(7) STEP 9*V
660 W=V*1Ø
670 FOR U=V TO W STEP V
680 A=LGT(U)
690 MOVE H(1)+LGT(H(8)),A
700 RMOVE R,Ø
710 RDRAW -2*R,Ø
720 RMOVE R,Ø
730 NEXT U
740 NEXT V
750 FOR A=H(3)+LGT(H(7)) TO H(4)+LGT(H(7))
760 MOVE H(1)+H(11)+LGT(H(8)),A+H(12)
770 U=EXP(A*LOG(1Ø))
780 PRINT U
790 NEXT A
800 MOVE (H(1)+H(2))/2+LGT(H(8)),H(3)+H(13)+LGT(H(7))
810 PRINT "MVA"
820 MOVE H(1)+LGT(H(8)),H(14)+H(4)+LGT(H(7))+Ø.1
830 PRINT "TIME(SECS)"
840 MOVE (H(1)+H(2))/2+LGT(H(8))-Ø.5,H(14)+H(4)+LGT(H(7))+Ø.1
850 PRINT F
860 HOME
900 FOR I=1 TO O2
920 FOR T1=-1 TO 1 STEP Ø.1
930 T=-T1
940 R=O1(I)+T*(U2(I)+V2(I)*T)
950 DRAW R,T
960 NEXT T1
970 MOVE H(1),H(4)
980 NEXT I
990 HOME
995 DELETE 1ØØ,995
1000 REM
1010 INPUT O3
1020 FIND O3
1030 APPEND 1ØØØ,1Ø
```

```
1040 END

1000 REM This program is for plotting the relay and fuse characteristics.
1002 REM After finishing plotting, another program is loaded into the
1003 REM memory
1005 REM for outputting the information for the relays and fuses plotted.
1010 W5=0
1040 FOR I5=1 TO N
1140 Z=1
1240 S=S5(I5)
1340 S1=P5(I5)
1440 T3=L5(I5)
1540 REM   start the plotting from the maximum time
1640 MOVE LGT(S1),H(4)
1740 REM   CHECK THE TYPE OF RELAYS
1840 IF T5(I5)=1 THEN 2340
1850 IF T5(I5)=3 THEN 7350
1940 X4=LGT(19/(H(15)/S-0.2))/1.7
2040 REM calculate the PSM which corresponds to the maximum time H(15)
2140 X5=EXP(X4*LOG(10))+1
2240 GO TO 2640
2340 X4=LGT(0.14*S/H(15)+1)/0.02
2440 X5=EXP(X4*LOG(10))
2540 REM   compute the range of PSM and increment
2640 N3=H(16)
2740 N6=H(17)
2840 X6=X5+H(18)
2940 N7=1
3040 X7=T3/S1
3140 N9=1
3240 FOR I=X5 TO X6 STEP N3
3340 REM   test accuracy
3440 IF ABS(X7-I)<H(19) THEN 5940
3540 REM   is present PSM too large
3640 IF X7-I<0 THEN 5240
3740 GOSUB 8240
3840 NEXT I
3940 REM   calculate new range of PSM and new increment
4040 X5=X6+N3
4140 X6=X6+N6
4240 N6=N6+H(20)
4340 REM   is the increment too large
4440 IF N6>H(21) THEN 4940
4540 REM   increase previous increment by a specified number of times
4640 N3=N3*H(22)
4740 GO TO 3240
4840 REM   use fixed value for the increment
4940 N3=H(23)
5040 GO TO 3240
5140 REM   decrease PSM with 50% of the present increment
5240 I=I-N3/(2*N7)
5340 N7=N7+1
```

```
5440 REM    N9=2 if PSM is within the limit
5540 IF N9=2 THEN 6940
5640 IF ABS(X7-I)<H(19) THEN 5940
5740 IF X7-I<0 THEN 6240
5840 IF X7-I>0 THEN 6540
5940 N9=2
6140 GO TO 6540
6240 I=I-N3/(2*N7)
6340 N7=N7+1
6440 GO TO 5540
6540 GOSUB 8240
6640 I=I+N3/(2*N7)
6740 N7=N7+1
6840 GO TO 5540
6940 N7=N7-1
7040 I=I-N3/(2*N7)
7140 C=LGT(I)+LGT(S1)
7240 D=-1
7340 DRAW C,D
7350 I=0.301
7355 A=1.167367
7356 B=-2.66868
7357 C=2.917875
7358 D=-1.6677
7359 E=0.36193
7360 T=A+I*(B+I*(C+I*(D+E*I)))
7362 MOVE I+LGT(S1),T+LGT(S)
7365 I=I+0.05
7370 IF I>LGT(T3/S1) THEN 7385
7372 T=A+I*(B+I*(C+I*(D+E*I)))
7375 DRAW I+LGT(S1),T+LGT(S)
7380 GO TO 7365
7385 I=LGT(T3/S1)
7390 DRAW I+LGT(S1),-1
7440 NEXT I5
7540 HOME
7640 DELETE 100,7640
7840 INPUT R
7940 FIND R
8040 APPEND 7740,10
8140 END
8240 IF T5(I5)=1 THEN 8540
8340 T=19/EXP(1.7*LGT(I-1)*LOG(10))+0.2
8440 GO TO 8640
8540 T=0.14/(EXP(0.02*LGT(I)*LOG(10))-1)
8640 C=LGT(I)+LGT(S1)
8740 D=LGT(T)+LGT(S)
8840 IF Z=2 THEN 9440
8940 MOVE C-0.05,D
9040 W5=W5+1
9140 PRINT W5
9240 Z=2
9340 MOVE C,D
```

```
9440 DRAW C,D
9540 RETURN
9640 END
```

```
7740 REM This program is for outputting the information
7742 REM for the relays and fuses plotting.
7745 PAGE
7745 PRINT "RELAY","SETTING","PICKUP","LEVEL"
9594 FOR I=1 TO N
13594 IF T5(I)=1 THEN 25594
13596 IF T5(I)=3 THEN 25598
17594 PRINT I;" EXTREMELY",S5(I),P5(I),L5(I)
21594 GO TO 29594
25594 PRINT I;" STANDARD",S5(I),P5(I),L5(I)
25596 GO TO 29594
25598 PRINT I;"(1.3/10)",S5(I),P5(I),L5(I)
29594 NEXT I
30000 PRINT
30002 PRINT
30004 PRINT
30006 PRINT
30008 PRINT "  MVA=A+BT+CT^2 "
30010 PRINT
30020 PRINT
30030 PRINT "A","B","C"
30035 FOR I=1 TO O2
30040 PRINT O1(I),U2(I),V2(I)
30050 NEXT I
33594 END
```

```
C  This program is for finding the fault currents for the following cases
C  (1) A fault occuring at the terminal of the generator
C  (2) a-b-c-e @ B opening c including the arcing resistance
C  (3) a-b-c-e @ B opening b & c including the arcing resistance
C  If NUM = 1 then b & c are opened.
C  If NUM = 2 then fault on the terminal of the generator
C  MATINV is a subroutine for matrix inversion
C  MATM1, MATM2, MATM3, MATM4, MATM5 are the subroutines for matrix
C  multiplication
C  Array CU is used to store the initial currents
C  E is the infinite machine voltage
C  XA is the armature leakage reactance
C  XØ is the earthing reactance
C  XD is the direct axis synchronous reactance
C  XQ is the quadrature axis synchronous reactance
C  XD1 is the direct axis transient reactance
C  XQ1 is the quadrature axis transient reactance
C  XD11 is the direct axis subtransient reactance
C  XQ11 is the quadrature axis subtransient reactance
C  TD1 is the direct axis transient short circuit time constant (sec.)
C  TQ1 is the quadrature axis transient short circuit time constant
C  TD11 is the direct axis subtransient short circuit time constant.
C  TQ11 is the quadrature axis subtransient short circuit time constant.
C  H is the step width
C  ANI is the angle between the direct axis and phase a
C  W is the system angular frequency
C  V4 is the field voltage
C  R is the armature resistance
C  RØ is the earthing resistance
C  TIME is the initial time
C  N is the number of steps required for studying
C  NUM is a parameter used to select different types of study
C  RS1 is the saturation factor
C  SIC is the initial speed variation
C  SIC1 is the initial speed
C  CI is the inertia constant
C  TM is the mechanical torque
C  TL is the loss torque due to friction and winding.
C  TA is the accelerating torque.
C  ARV is the voltage used to simulate the arc resistance
      DOUBLE PRECISION CU(7),CUN(7),TN,T,H,PI(7),ANI,
     1E, T1ØØØ,CL(7,7),W,RC(3,3),XC(3,3), X,SIC1,SIC
     2,ZN(3,3),RA(7,7),F1(3,3), A(3,3),XA,XØ,XD,XQ,XD1,XQ1,
     3XD11,XQ11,TD1,TQ1,TD11,TQ11,B,XAAØ,XABØ,ZMI(3,3), TM
     4,XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,XKQL,XKD11,RS1,
     5XKQ11,RFD,RFQ,RKD,RKQ,CT4(3,3),ZN2(3,3),RC2(3,3),XC2(3,3),TA
     6,A1(3,3),PI1(7),PI2(7),PI3(7),PI4(7),V4,R,RØ,CT1(3,3)
     7,XADNØ,XAQNØ,AIC,TIME, S1(7,7),TL,TE,F(7),TE1,ARV,ZM(3,3)
C  Input data
      READ(13,1)CU(1),CU(2),CU(3),CU(4),CU(5),CU(6)
1     FORMAT(6F1Ø.5)
```

```
      READ(13,1)CU(7),E,XA,XØ,XD,XQ
      READ(13,1)XD1,XQ1,XD11,XQ11,TD1,TQ1
      READ(13,1)TD11,TQ11,B,T,H,ANI
      READ(13,2)W,V4,R,RØ,TIME,N,NUM
2     FORMAT(5F1Ø.5,2I4)
      READ(13,1)RS1,TA,TL,AIC,SIC,SIC1
      READ(13,51)CI,TM,ARV
51    FORMAT(3F1Ø.5)
C  F1 is the fault matrix
C  S1 is the switching matrix and CT1, ZM, ZMI are the connection
C  matrices
C  RC and XC are the resistance and reactance matrices for both
C  transformer
C  and transmission line
      READ(13,5)((F1(I,J),J=1,3),I=1,3)
      READ(13,5)((ZM(I,J),J=1,3),I=1,3)
      READ(13,5)((ZMI(I,J),J=1,3),I=1,3)
       DO 17 I=1,7
       DO 17 J=1,7
17     S1(I,J)=Ø.Ø
      READ(13,5)((S1(I,J),J=1,3),I=1,3)
      S1(4,4)=1.Ø
      S1(5,5)=1.Ø
      S1(6,6)=1.Ø
      S1(7,7)=1.Ø
      READ(13,5)((CT1(I,J),J=1,3),I=1,3)
      READ(13,5)((RC(I,J),J=1,3),I=1,3)
      READ(13,5)((XC(I,J),J=1,3),I=1,3)
C  ZN is matrix for 1/tap
      READ(13,5)((ZN(I,J),J=1,3),I=1,3)
5     FORMAT(3F1Ø.5)
      M=Ø
C  Transform from d-q to a-b-c
      XAAØ=((XD+XQ-2*XA)/RS1+2*XA+XØ)/3.Ø
      XABØ=((XD+XQ-2*XA)/RS1+2*XA-2.Ø*XØ)/6.Ø
      XAA2=(XD-XQ)/(3.Ø*RS1)
      XADNØ=B*(XD-XA)
      XADØ=XADNØ/RS1
      XAQNØ=B*(XQ-XA)
      XAQØ=XAQNØ/RS1
      XAFD=XADNØ*XADNØ/((XD-XD1)*B)
      XFDL=XAFD-XADNØ
      XFDL=1.Ø*XFDL
      XKDL=B*((XD1-XA)*(XD1-XA)/(XD1-XD11)+XD-XD1)-XADNØ
      XKDL=1.Ø*XKDL
      XAFQ=XAQNØ*XAQNØ/((XQ-XQ1)*B)
      XFQL=XAFQ-XAQNØ
      XFQL=1.Ø*XFQL
      XKQL=B*((XQ1-XA)*(XQ1-XA)/(XQ1-XQ11)+XQ-XQ1)-XAQNØ
      XKQL=1.Ø*XKQL
      XKD11=XKDL+XADNØ*XFDL/XAFD
      XKQ11=XKQL+XAQNØ*XFQL/XAFQ
      RFD=XAFD*XD1/(W*TD1*XD)
```

```
      RFQ=XAFQ*XQ1/(W*TQ1*XQ)
      RKD=XKD11*XD11/(W*TD11*XD1)
      RKQ=XKQ11*XQ11/(W*TQ11*XQ1)
      WRITE(2Ø,71)XAAØ,XABØ,XAA2,XADNØ,XADØ,XAQNØ
      WRITE(2Ø,71)XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL
      WRITE(2Ø,71)XKQL,XKD11,X Q11,RFD,RFQ,RKD
      WRITE(2Ø,72)RKQ
71    FORMAT(1X,6F1Ø.5)
72    FORMAT(1X,F1Ø.5)
C  Form R matrix
      DO 114 I=1,7
      DO 114 J=1,7
114   RA(I,J)=Ø.Ø
      RA(1,1)=R+RØ
      RA(1,2)=RØ
      RA(1,3)=RØ
      RA(2,1)=RØ
      RA(2,2)=R+RØ
      RA(2,3)=RØ
      RA(3,1)=RØ
      RA(3,2)=RØ
      RA(3,3)=R+RØ
      RA(4,4)=-RFD
      RA(5,5)=-RKD
      RA(6,6)=-RFQ
      RA(7,7)=-RKQ
      IF (NUM.EQ.2) GOTO 665
      GOTO 666
665   CALL MATINV(ZM,ZMI,3)
666   CALL MATM2(F1,ZMI,A)
C  Form F7C7tN
      CALL MATM2(CT1,ZN,CT4)
      CALL MATM2(A,CT4,CT1)
C  Form F7N2RC
      ZN2(1,1)=ZN(1,1)*ZN(1,1)
      ZN2(2,2)=ZN(2,2)*ZN(2,2)
      ZN2(3,3)=ZN(3,3)*ZN(3,3)
      DO 112 I=1,3
      DO 112 J=1,3
112   RC2(I,J)=Ø.Ø
      RC2(1,1)=RC(1,1)*ZN2(1,1)
      RC2(2,2)=RC(2,2)*ZN2(2,2)
      RC2(3,3)=RC(3,3)*ZN2(3,3)
      CALL MATM2(A,RC2,RC)
C  Form F7N2XC
      DO 113 I=1,3
      DO 113 J=1,3
113   XC2(I,J)=Ø.Ø
      XC2(1,1)=XC(1,1)*ZN2(1,1)
      XC2(2,2)=XC(2,2)*ZN2(2,2)
      XC2(3,3)=XC(3,3)*ZN2(3,3)
      CALL MATM2(A,XC2,A1)
      T1ØØØ=T/W
```

```
      TN=T
      I9=1
      DO 15 K=1,7
15    CUN(K)=CU(K)
      TN=T
      X=ANI+T+TIME
      WRITE(14,1Ø)T,CU(1),CU(2),CU(3),ANI
2Ø    CALL PDI(W,ANI,CUN,PI,TN,CL,CT1,RC,A1,RA,E,X,SIC1,
     1V4,XAAØ,XABØ,XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,
     2XKQL,XKD11,XKQ11,RFD,RFQ,RKD,RKQ,TIME,ZMI, S1,NUM,ARV)
      DO 3Ø K=1,7
      PI1(K)=PI(K)*H
3Ø    CUN(K)=CU(K)+PI1(K)*Ø.5
      TN=T+H*Ø.5
      CALL PDI(W,ANI,CUN,PI,TN,CL,CT1,RC,A1,RA,E,X,SIC1,
     1V4,XAAØ,XABØ,XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,
     2XKQL,XKD11,XKQ11,RFD,RFQ,RKD,RKQ,TIME,ZMI, S1,NUM,ARV)
      DO 5Ø K=1,7
      PI2(K)=PI(K)*H
5Ø    CUN(K)=CU(K)+PI2(K)*Ø.5
      CALL PDI(W,ANI,CUN,PI,TN,CL,CT1,RC,A1,RA,E,X,SIC1,
     1V4,XAAØ,XABØ,XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,
     2XKQL,XKD11,XKQ11,RFD,RFQ,RKD,RKQ,TIME,ZMI, S1,NUM,ARV)
      DO 7Ø K=1,7
      PI3(K)=PI(K)*H
7Ø    CUN(K)=CU(K)+PI3(K)
      TN=T+H
      CALL PDI(W,ANI,CUN,PI,TN,CL,CT1,RC,A1,RA,E,X,SIC1,
     1V4,XAAØ,XABØ,XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,
     2XKQL,XKD11,XKQ11,RFD,RFQ,RKD,RKQ,TIME,ZMI, S1,NUM,ARV)
      DO 9Ø K=1,7
      PI4(K)=H*PI(K)
      CU(K)=CU(K)+(PI1(K)+2.*PI2(K)+2.*PI3(K)+PI4(K))/6.
9Ø    CUN(K)=CU(K)
      T=T+H
      TN=T
      T1ØØØ=T/W
      X=ANI+T+TIME
C  Speed variation
      CALL MATM1(CL,CUN,F)
      TE1=F(1)*(CUN(2)-CUN(3))+F(2)*(CUN(3)-CUN(1))+F(3)*(CUN(1)-CUN(2))
      TE=TE1*Ø.3849
      TA=TM-TE-TL
      SIC1=TA*H/(314.16*4.Ø*CI)+SIC
      AIC=TA*H*H/(314.16*4.Ø*CI)+SIC*H+AIC
      SIC=SIC1
      ANI=AIC-Ø.5236
      SIC1=SIC1+1.Ø
      WRITE(14,1Ø)T1ØØØ,CU(1),CU(2),CU(3),ANI
1Ø    FORMAT(1X,F12.7,1X,F12.7,1X,F12.7,1X,F12.7,1X,F12.7)
       IF (I9.EQ.N) GOTO 1ØØ
       M=M+1
      I9=I9+1
```

```
      GOTO 2Ø
100   STOP
      END
C
C
C
      SUBROUTINE PDI(W,ANI,CUN,PI,TN,CL,CT1,RC,A1,RA,E,X,SIC1,
     1V4,XAAØ,XABØ,XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,
     2XKQL,XKD11,XKQ11,RFD,RFQ,RKD,RKQ,TIME,ZMI, S1,NUM,ARV)
      DOUBLE PRECISION ANI,CUN(7),PI(7),TN,CL(7,7),PL(7,7)
     1,V(7),RA(7,7), X,W,RC(3,3),A1(3,3),SIC1,
     2V4,XAAØ,XABØ,XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,E,CT1(3,3),
     3XKQL,XKD11,XKQ11,RFD,RFQ,RKD,RKQ,CI2(7),CI3(7),V1(7),TIME
     4,ZMI(3,3),S1(7,7),CL66(6,6),CL66I(6,6),CI4(6),
     5V2(7),RA1(7,7),PI6(6),CL2(7,7)
     6,CL55(5,5),CL55I(5,5),CI5(5),PI5(5),ARV,CLI(7,7)
      X=ANI+TN+TIME
C Form L matrix
      CL(1,1)=-(XAAØ+XAA2*DCOS(2.Ø*X))
      CL(1,2)=-(-XABØ+XAA2*DCOS(2.Ø*X-2.Ø944))
      CL(1,3)=-(-XABØ+XAA2*DCOS(2.Ø*X+2.Ø944))
      CL(1,4)=XADØ*DCOS(X)
      CL(1,5)=CL(1,4)
      CL(1,6)=-XAQØ*DSIN(X)
      CL(1,7)=CL(1,6)
      CL(2,1)=CL(1,2)
      CL(2,2)=-(XAAØ+XAA2*DCOS(2.Ø*X+2.Ø944))
      CL(2,3)=-(-XABØ+XAA2*DCOS(2.Ø*X))
      CL(2,4)=XADØ*DCOS(X-2.Ø944)
      CL(2,5)=CL(2,4)
      CL(2,6)=-XAQØ*DSIN(X-2.Ø944)
      CL(2,7)=CL(2,6)
      CL(3,1)=CL(1,3)
      CL(3,2)=CL(2,3)
      CL(3,3)=-(XAAØ+XAA2*DCOS(2.Ø*X-2.Ø944))
      CL(3,4)=XADØ*DCOS(X+2.Ø944)
      CL(3,5)=CL(3,4)
      CL(3,6)=-XAQØ*DSIN(X+2.Ø944)
      CL(3,7)=CL(3,6)
      CL(4,1)=-XADØ*DCOS(X)
      CL(4,2)=-XADØ*DCOS(X-2.Ø944)
      CL(4,3)=-XADØ*DCOS(X+2.Ø944)
      CL(4,4)=XFDL+XADØ
      CL(4,5)=XADØ
      CL(4,6)=Ø.Ø
      CL(4,7)=Ø.Ø
      CL(5,1)=-XADØ*DCOS(X)
      CL(5,2)=-XADØ*DCOS(X-2.Ø944)
      CL(5,3)=-XADØ*DCOS(X+2.Ø944)
      CL(5,4)=XADØ
      CL(5,5)=XKDL+XADØ
      CL(5,6)=Ø.Ø
      CL(5,7)=Ø.Ø
```

```
      CL(6,1)=XAQØ*DSIN(X)
      CL(6,2)=XAQØ*DSIN(X-2.Ø944)
      CL(6,3)=XAQØ*DSIN(X+2.Ø944)
      CL(6,4)=Ø.Ø
      CL(6,5)=Ø.Ø
      CL(6,6)=XFQL+XAQØ
      CL(6,7)=XAQØ
      CL(7,1)=XAQØ*DSIN(X)
      CL(7,2)=XAQØ*DSIN(X-2.Ø944)
      CL(7,3)=XAQØ*DSIN(X+2.Ø944)
      CL(7,4)=Ø.Ø
      CL(7,5)=Ø.Ø
      CL(7,6)=XAQØ
      CL(7,7)=XKQL+XAQØ
      DO 1Ø I=1,7
      DO 1Ø J=1,7
1Ø    CL2(I,J)=CL(I,J)
C  Form PL matrix
      PL(1,1)=SIC1*2.Ø*XAA2*DSIN(2.Ø*X)
      PL(1,2)=SIC1*2.Ø*XAA2*DSIN(2.Ø*X-2.Ø944)
      PL(1,3)=SIC1*2.Ø*XAA2*DSIN(2.Ø*X+2.Ø944)
      PL(1,4)=-XADØ*SIC1*DSIN(X)
      PL(1,5)=PL(1,4)
      PL(1,6)=-XAQØ*SIC1*DCOS(X)
      PL(1,7)=PL(1,6)
      PL(2,1)=PL(1,2)
      PL(2,2)=PL(1,3)
      PL(2,3)=PL(1,1)
      PL(2,4)=-XADØ*SIC1*DSIN(X-2.Ø944)
      PL(2,5)=PL(2,4)
      PL(2,6)=-XAQØ*SIC1*DCOS(X-2.Ø944)
      PL(2,7)=PL(2,6)
      PL(3,1)=PL(1,3)
      PL(3,2)=PL(2,3)
      PL(3,3)=PL(1,2)
      PL(3,4)=-XADØ*SIC1*DSIN(X+2.Ø944)
      PL(3,5)=PL(3,4)
      PL(3,6)=-XAQØ*SIC1*DCOS(X+2.Ø944)
      PL(3,7)=PL(3,6)
      PL(4,1)=XADØ*DSIN(X)*SIC1
      PL(4,2)=XADØ*DSIN(X-2.Ø944)*SIC1
      PL(4,3)=XADØ*DSIN(X+2.Ø944)*SIC1
      PL(5,1)=PL(4,1)
      PL(5,2)=PL(4,2)
      PL(5,3)=PL(4,3)
      PL(6,1)=XAQØ*DCOS(X)*SIC1
      PL(6,2)=XAQØ*DCOS(X-2.Ø944)*SIC1
      PL(6,3)=XAQØ*DCOS(X+2.Ø944)*SIC1
      PL(7,1)=PL(6,1)
      PL(7,2)=PL(6,2)
      PL(7,3)=PL(6,3)
      PL(4,4)=Ø.Ø
      PL(5,5)=Ø.Ø
```

```
      PL(6,6)=Ø.Ø
      PL(7,7)=Ø.Ø
      PL(4,5)=Ø.Ø
      PL(4,6)=Ø.Ø
      PL(4,7)=Ø.Ø
      PL(5,4)=Ø.Ø
      PL(5,6)=Ø.Ø
      PL(5,7)=Ø.Ø
      PL(6,4)=Ø.Ø
      PL(6,5)=Ø.Ø
      PL(6,7)=Ø.Ø
      PL(7,4)=Ø.Ø
      PL(7,5)=Ø.Ø
      PL(7,6)=Ø.Ø
C Form infinite busbar voltage
      V(1)=E*DCOS(TN+1.57Ø8+TIME)
      V(2)=E*DCOS(TN+1.57Ø8-2.Ø944+TIME)
      V(3)=E*DCOS(TN+1.57Ø8+2.Ø944+TIME)
      V(4)=V4
      V(5)=Ø.Ø
      V(6)=Ø.Ø
      V(7)=Ø.Ø
C Form R-PL
      DO 1 I=1,7
      DO 1 J=1,7
1     PL(I,J)=-PL(I,J)+RA(I,J)
C Form F7N2RC+R-PL
      DO 2 I=1,3
      DO 2 J=1,3
2     PL(I,J)=PL(I,J)+RC(I,J)
      IF (NUM.EQ.2) GOTO 89
C Form S1(F7N2Rc + R - PL)S1
      CALL MATM3(S1,PL,RA1)
      CALL MATM3(RA1,S1,PL)
C Form S1(F7N2Rc + R - PL)S1I
      CALL MATM1(PL,CUN,CI2)
C Form F7C7tNV
      DO 5 I=1,3
      V1(I)=Ø.Ø
      DO 5 K=1,3
5     V1(I)=V1(I)+CT1(I,K)*V(K)
      V1(4)=V4
      V1(5)=Ø.Ø
      V1(6)=Ø.Ø
      V1(7)=Ø.Ø
C Form S1F7C7tNV
      CALL MATM1(S1,V1,V2)
C Form S1F2C7tNV + S1(F7N2Rc + R - PL)S1I
      DO 3 I=1,7
3     CI3(I)=CI2(I)+V2(I)
      IF (NUM.EQ.1) GOTO 88
      IF (CUN(1).GT.Ø.AND.CUN(2).GT.Ø) GOTO 3Ø3
      IF (CUN(1).GT.Ø.AND.CUN(2).LT.Ø) GOTO 3Ø2
```

```
      IF (CUN(1).LT.Ø.AND.CUN(2).LT.Ø) GOTO 3Ø1
      CI4(1)=CI3(1)-ARV
      CI4(2)=CI3(2)+ARV
      GOTO 3Ø4
3Ø1   CI4(1)=CI3(1)-ARV
      CI4(2)=CI3(2)-ARV
      GOTO 3Ø4
3Ø2   CI4(1)=CI3(1)+ARV
      CI4(2)=CI3(2)-ARV
      GOTO 3Ø4
3Ø3   CI4(1)=CI3(1)+ARV
      CI4(2)=CI3(2)+ARV
3Ø4   CI4(3)=CI3(4)
      CI4(4)=CI3(5)
      CI4(5)=CI3(6)
      CI4(6)=CI3(7)
C  Form L-F7N2XC
      DO 4 I=1,3
      DO 4 J=1,3
4     CL2(I,J)=CL2(I,J)-A1(I,J)
C  Form S1(L - F7N2Xc)S1
      CALL MATM3(S1,CL2,RA1)
      CALL MATM3(RA1,S1,CL2)
      DO 78 I=3,6
      DO 78 J=3,6
78    CL66(I,J)=CL2(I+1,J+1)
      CL66(1,1)=CL2(1,1)
      CL66(1,2)=CL2(1,2)
      CL66(1,3)=CL2(1,4)
      CL66(1,4)=CL2(1,5)
      CL66(1,5)=CL2(1,6)
      CL66(1,6)=CL2(1,7)
      CL66(2,1)=CL2(2,1)
      CL66(3,1)=CL2(4,1)
      CL66(4,1)=CL2(5,1)
      CL66(5,1)=CL2(6,1)
      CL66(6,1)=CL2(7,1)
      CL66(2,2)=CL2(2,2)
      CL66(2,3)=CL2(2,4)
      CL66(2,4)=CL2(2,5)
      CL66(2,5)=CL2(2,6)
      CL66(2,6)=CL2(2,7)
      CL66(3,2)=CL2(4,2)
      CL66(4,2)=CL2(5,2)
      CL66(5,2)=CL2(6,2)
      CL66(6,2)=CL2(7,2)
      CALL MATINV(CL66,CL66I,6)
C  Form PI
      CALL MATM4(CL66I,CI4,PI6)
      PI(1)=PI6(1)
      PI(2)=PI6(2)
      PI(3)=Ø.Ø
      PI(4)=PI6(3)
```

```
      PI(5)=PI6(4)
      PI(6)=PI6(5)
      PI(7)=PI6(6)
      GOTO 99
88    IF (CUN(1).GT.Ø) GOTO 3Ø5
      CI5(1)=CI3(1)-ARV
      GOTO 3Ø6
3Ø5   CI5(1)=CI3(1)+ARV
3Ø6   CI5(2)=CI3(4)
      CI5(3)=CI3(5)
      CI5(4)=CI3(6)
      CI5(5)=CI3(7)
      DO 41 I=1,3
      DO 41 J=1,3
41    CL2(I,J)=CL2(I,J)-A1(I,J)
      CALL MATM3(S1,CL2,RA1)
      CALL MATM3(RA1,S1,CL2)
      DO 77 I=2,5
      DO 77 J=2,5
77    CL55(I,J)=CL2(I+2,J+2)
      CL55(1,1)=CL2(1,1)
      CL55(1,2)=CL2(1,4)
      CL55(1,3)=CL2(1,5)
      CL55(1,4)=CL2(1,6)
      CL55(1,5)=CL2(1,7)
      CL55(2,1)=CL2(4,1)
      CL55(3,1)=CL2(5,1)
      CL55(4,1)=CL2(6,1)
      CL55(5,1)=CL2(7,1)
      CALL MATINV(CL55,CL55I,5)
      CALL MATM5(CL55I,CI5,PI5)
      PI(1)=PI5(1)
      PI(2)=Ø.Ø
      PI(3)=Ø.Ø
      PI(4)=PI5(2)
      PI(5)=PI5(3)
      PI(6)=PI5(4)
      PI(7)=PI5(5)
      GOTO 99
89    DO 9Ø I=1,3
      V1(I)=Ø.Ø
      DO 9Ø K=1,3
9Ø    V1(I)=V1(I)+CT1(I,K)*V(K)
      V1(4)=V4
      CALL MATM1(PL,CUN,CI2)
      V1(5)=Ø.Ø
      V1(6)=Ø.Ø
      V1(7)=Ø.Ø
      DO 91 I=1,7
91    CI3(I)=CI2(I)+V1(I)
      DO 92 I=1,7
      DO 92 J=1,7
92    CL2(I,J)=CL(I,J)
```

```
      DO 93 I=1,3
      DO 93 J=1,3
93    CL2(I,J)=CL(I,J)-A1(I,J)
      CALL MATINV(CL2,CLI,7)
      CALL MATM1(CLI,CI3,PI)
99    RETURN
      END
```

```
C  This program is used to compute the transient currents for a
C  three-phase synchronous generator on open circuit by the phase
C  coordinate method.
C  Started by knowing the initial currents and voltages, the
C  derivative of currents is found and then the currents
C  at an time increment H can be found by the RUNGE-KUTTA method.
C  If NUM = 1 then three phase to earth
C  If NUM = 2 then phase a to earth
C  If NUM = 3 then phase b to phase c fault
C  otherwise phase a and phase b to earth
C  J5 is the number of steps studied
      DOUBLE PRECISION CU(7),CUN(7),TN,T,H,PI(7),ANI,
     1 T1ØØØ,CL(7,7),W,W1,PI1(7),PI2(7),PI3(7),PI4(7),
     2V4,XA,XØ,XD1,XQ1,ZMØ(7,7),ZMØT(7,7),
     3XD11,XQ11,TD1,TQ1,TD11,TQ11,B,RE,R2,XAAØ,XABØ,
     4XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,XKQL,XKD11,
     5XKQ11,RFD,RFQ,RKD,RKQ,FØ(7,7),SØ(7,7)
      READ(13,1)CU(1),CU(2),CU(3),CU(4),CU(5),CU(6)
1     FORMAT(6F1Ø.7)
      READ(13,1)CU(7),XA,XØ,XD,XQ,XD1
      READ(13,1)XQ1,XD11,XQ11,TD1,TQ1,TD11
      READ(13,1)TQ11,B,V4,RE,R2,T
      READ(13,2Ø)H,ANI,W,J5,NUM
2Ø    FORMAT(3F1Ø.5,2I4)
C  FO and SO are the fault and switching matrices respectively
      DO 16 I=1,7
      DO 16 J=1,7
16    FØ(I,J)=Ø.Ø
       READ(13,5)((FØ(I,J),J=1,3),I=1,3)
5      FORMAT(3F1Ø.5)
       FØ(4,4)=1.Ø
       FØ(5,5)=1.Ø
       FØ(6,6)=1.Ø
       FØ(7,7)=1.Ø
       DO 15 I=1,7
       DO 15 J=1,7
15     SØ(I,J)=Ø.Ø
       READ(13,5)((SØ(I,J),J=1,3),I=1,3)
       SØ(4,4)=1.Ø
       SØ(5,5)=1.Ø
       SØ(6,6)=1.Ø
       SØ(7,7)=1.Ø
      DO 17 I=1,7
      DO 17 J=1,7
C  ZMO and ZMOT are the connection matrices
17    ZMØ(I,J)=Ø.
      READ(13,5)((ZMØ(I,J),J=1,3),I=1,3)
      ZMØ(4,4)=1.Ø
      ZMØ(5,5)=1.Ø
      ZMØ(6,6)=1.Ø
      ZMØ(7,7)=1.Ø
```

```
      DO 18 I=1,7
      DO 18 J=1,7
18    ZMØT(I,J)=Ø.Ø
      READ(13,5)((ZMØT(I,J),J=1,3),I=1,3)
      ZMØT(4,4)=1.Ø
      ZMØT(5,5)=1.Ø
      ZMØT(6,6)=1.Ø
      ZMØT(7,7)=1.Ø
      WRITE(15,3)CU(1),CU(2),CU(3),CU(4),CU(5),CU(6)
      WRITE(15,3)CU(7),XA,XØ,XD,XQ,XD1
      WRITE(15,3)XQ1,XD11,XQ11,TD1,TQ1,TD11
      WRITE(15,3)TQ11,B,V4,RE,R2,T
      WRITE(15,3Ø)H,ANI,W,J5,NUM
3Ø    FORMAT(1X,3F12.7,2I4)
3     FORMAT(1X,6F12.7)
      XAAØ=(XD+XQ+XØ)/3.Ø
      XABØ=(XD+XQ-2.Ø*XØ)/6.Ø
      XAA2=(XD-XQ)/3.Ø
      XADØ=B*(XD-XA)
      XAQØ=B*(XQ-XA)
      XAFD=XADØ*XADØ/((XD-XD1)*B)
      XFDL=XAFD-XADØ
      XKDL=B*((XD1-XA)*(XD1-XA)/(XD1-XD11)+XD-XD1)-XADØ
      XAFQ=XAQØ*XAQØ/((XQ-XQ1)*B)
      XFQL=XAFQ-XAQØ
      XKQL=B*((XQ1-XA)*(XQ1-XA)/(XQ1-XQ11)+XQ-XQ1)-XAQØ
      XKD11=XKDL+XADØ*XFDL/XAFD
      XKQ11=XKQL+XAQØ*XFQL/XAFQ
      RFD=XAFD*XD1/(W*TD1*XD)
      RFQ=XAFQ*XQ1/(W*TQ1*XQ)
      RKD=XKD11*XD11/(W*TD11*XD1)
      RKQ=XKQ11*XQ11/(W*TQ11*XQ1)
      V4=RFD*CU(4)
      WRITE(15,3)XAAØ,XABØ,XAA2,XADØ,XAQØ,XAFD
      WRITE(15,3)XFDL,XKDL,XAFQ,XFQL,XKQL,XKD11
      WRITE(15,8)XKQ11,RFD,RFQ,RKD,RKQ,V4
8     FORMAT(1X,6F1Ø.5)
      T1ØØØ=T/W
      WRITE(16,1Ø)T1ØØØ,CU(1),CU(2),CU(3)
      WRITE(17,11)T1ØØØ,CU(4),CU(5),CU(6),CU(7)
11    FORMAT(1X,F13.8,2X,F13.8,2X,F13.8,2X,F13.8,2X,F13.8)
      DO 8Ø K=1,7
8Ø    CUN(K)=CU(K)
      TN=T
      I8=1
2     CALL PDI(W,ANI,CUN,PI,TN,CL,ZMØ,ZMØT,V4,
     1XAAØ,XABØ,XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,
     2XKQL,XKD11,XKQ11,RFD,RFQ,RKD,RKQ,R2,RE,FØ,SØ,NUM)
      DO 83 K=1,7
      PI1(K)=PI(K)*H
83    CUN(K)=CU(K)+PI1(K)*Ø.5
      TN=T+H*Ø.5
      IF (NUM.EQ.3) GOTO 9Ø
```

```
      GOTO 91
90    CUN(3)=-CUN(2)
91    CALL PDI(W,ANI,CUN,PI,TN,CL,ZMØ,ZMØT,V4,
     1XAAØ,XABØ,XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,
     2XKQL,XKD11,XKQ11,RFD,RFQ,RKD,RKQ,R2,RE,FØ,SØ,NUM)
      DO 85 K=1,7
      PI2(K)=PI(K)*H
85    CUN(K)=CU(K)+PI2(K)*Ø.5
      IF (NUM.EQ.3) GOTO 92
      GOTO 93
92    CUN(3)=-CUN(2)
93    CALL PDI(W,ANI,CUN,PI,TN,CL,ZMØ,ZMØT,V4,
     1XAAØ,XABØ,XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,
     2XKQL,XKD11,XKQ11,RFD,RFQ,RKD,RKQ,R2,RE,FØ,SØ,NUM)
      DO 7 K=1,7
      PI3(K)=PI(K)*H
7     CUN(K)=CU(K)+PI3(K)
      TN=T+H
      IF (NUM.EQ.3) GOTO 94
      GOTO 95
94    CUN(3)=-CUN(2)
95    CALL PDI(W,ANI,CUN,PI,TN,CL,ZMØ,ZMØT,V4,
     1XAAØ,XABØ,XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,
     2XKQL,XKD11,XKQ11,RFD,RFQ,RKD,RKQ,R2,RE,FØ,SØ,NUM)
      DO 9 K=1,7
      PI4(K)=H*PI(K)
      CU(K)=CU(K)+(PI1(K)+2.*PI2(K)+2.*PI3(K)+PI4(K))/6.
9     CUN(K)=CU(K)
      IF (NUM.EQ.3) GOTO 96
      GOTO 97
96    CUN(3)=-CUN(2)
      CU(3)=-CU(2)
97    T=T+H
      TN=T
      T1ØØØ=T/W
      WRITE(16,1Ø)T1ØØØ,CU(1),CU(2),CU(3)
1Ø    FORMAT(1X,F13.8,2X,F13.8,2X,F13.8,2X,F13.8)
      WRITE(17,11)T1ØØØ,CU(4),CU(5),CU(6),CU(7)
      IF (I8.EQ.J5) GOTO 2ØØ
      I8=I8+1
      GOTO 2
2ØØ   STOP
      END
C
C
C
      SUBROUTINE PDI(W,ANI,CUN,PI,TN,CL,ZMØ,ZMØT,V4,
     1XAAØ,XABØ,XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,
     2XKQL,XKD11,XKQ11,RFD,RFQ,RKD,RKQ,R2,RE,FØ,SØ,NUM)
      DOUBLE PRECISION ANI,CUN(7),PI(7),TN,CL(7,7),PL(7,7)
     1,V(7),R(7,7),CLINV(7,7),X,R1(7), W,V4,
     2XAAØ,XABØ,XAA2,XADØ,XAQØ,XAFD,XFDL,XKDL,XAFQ,XFQL,
     3XKQL,XKD11,XKQ11,RFD,RFQ,RKD,RKQ, R2,RE,CL2(7,7),
```

```
4 CI3(7),R3(7),PI6(6),PI5(5),CI4(6),FØ(7,7),SØ(7,7),
5CI5(5),CL55(5,5),CL66(6,6),CL55I(5,5),CL66I(6,6)
6,ZMØ(7,7),ZMØT(7,7)
 X=TN+ANI
 CL(1,1)=-1*(XAAØ+XAA2*DCOS(2.Ø*X))
 CL(1,2)=-1*(-XABØ+XAA2*DCOS(2.Ø*X-2.Ø944))
 CL(1,3)=-1*(-XABØ+XAA2*DCOS(2.Ø*X+2.Ø944))
 CL(1,4)=XADØ*DCOS(X)
 CL(1,5)=CL(1,4)
 CL(1,6)=-1*XAQØ*DSIN(X)
 CL(1,7)=CL(1,6)
 CL(2,1)=CL(1,2)
 CL(2,2)=-1*(XAAØ+XAA2*DCOS(2.Ø*X+2.Ø944))
 CL(2,3)=-1*(-XABØ+XAA2*DCOS(2.Ø*X))
 CL(2,4)=XADØ*DCOS(X-2.Ø944)
 CL(2,5)=CL(2,4)
 CL(2,6)=-1*XAQØ*DSIN(X-2.Ø944)
 CL(2,7)=CL(2,6)
 CL(3,1)=CL(1,3)
 CL(3,2)=CL(2,3)
 CL(3,3)=-1*(XAAØ+XAA2*DCOS(2.Ø*X-2.Ø944))
 CL(3,4)=XADØ*DCOS(X+2.Ø944)
 CL(3,5)=CL(3,4)
 CL(3,6)=-1*XAQØ*DSIN(X+2.Ø944)
 CL(3,7)=CL(3,6)
 CL(4,1)=-1*XADØ*DCOS(X)
 CL(4,2)=-1*XADØ*DCOS(X-2.Ø944)
 CL(4,3)=-1*XADØ*DCOS(X+2.Ø944)
 CL(4,4)=XFDL+XADØ
 CL(4,5)=XADØ
 CL(4,6)=Ø.Ø
 CL(4,7)=Ø.Ø
 CL(5,1)=-1*XADØ*DCOS(X)
 CL(5,2)=-1*XADØ*DCOS(X-2.Ø944)
 CL(5,3)=-1*XADØ*DCOS(X+2.Ø944)
 CL(5,4)=XADØ
 CL(5,5)=XKDL+XADØ
 CL(5,6)=Ø.Ø
 CL(5,7)=Ø.Ø
 CL(6,1)=XAQØ*DSIN(X)
 CL(6,2)=XAQØ*DSIN(X-2.Ø944)
 CL(6,3)=XAQØ*DSIN(X+2.Ø944)
 CL(6,4)=Ø.Ø
 CL(6,5)=Ø.Ø
 CL(6,6)=XFQL+XAQØ
 CL(6,7)=XAQØ
 CL(7,1)=XAQØ*DSIN(X)
 CL(7,2)=XAQØ*DSIN(X-2.Ø944)
 CL(7,3)=XAQØ*DSIN(X+2.Ø944)
 CL(7,4)=Ø.Ø
 CL(7,5)=Ø.Ø
 CL(7,6)=XAQØ
 CL(7,7)=XKQL+XAQØ
```

```
PL(1,1)=2.Ø*XAA2*DSIN(2.Ø*X)
PL(1,2)=2.Ø*XAA2*DSIN(2.Ø*X-2.Ø944)
PL(1,3)=2.Ø*XAA2*DSIN(2.Ø*X+2.Ø944)
PL(1,4)=-XADØ*DSIN(X)
PL(1,5)=PL(1,4)
PL(1,6)=-1*XAQØ*DCOS(X)
PL(1,7)=PL(1,6)
PL(2,1)=PL(1,2)
PL(2,2)=PL(1,3)
PL(2,3)=PL(1,1)
PL(2,4)=-XADØ*DSIN(X-2.Ø944)
PL(2,5)=PL(2,4)
PL(2,6)=-1*XAQØ*DCOS(X-2.Ø944)
PL(2,7)=PL(2,6)
PL(3,1)=PL(1,3)
PL(3,2)=PL(2,3)
PL(3,3)=PL(1,2)
PL(3,4)=-XADØ*DSIN(X+2.Ø944)
PL(3,5)=PL(3,4)
PL(3,6)=-1*XAQØ*DCOS(X+2.Ø944)
PL(3,7)=PL(3,6)
PL(4,1)=XADØ*DSIN(X)
PL(4,2)=XADØ*DSIN(X-2.Ø944)
PL(4,3)=XADØ*DSIN(X+2.Ø944)
PL(4,5)=Ø.
PL(4,6)=Ø.Ø
PL(4,7)=Ø.Ø
PL(5,1)=PL(4,1)
PL(5,2)=PL(4,2)
PL(5,3)=PL(4,3)
PL(5,4)=Ø.Ø
PL(5,6)=Ø.Ø
PL(5,7)=Ø.Ø
PL(6,1)=XAQØ*DCOS(X)
PL(6,2)=XAQØ*DCOS(X-2.Ø944)
PL(6,3)=XAQØ*DCOS(X+2.Ø944)
PL(6,4)=Ø.Ø
PL(6,5)=Ø.Ø
PL(6,7)=Ø.Ø
PL(7,1)=PL(6,1)
PL(7,2)=PL(6,2)
PL(7,3)=PL(6,3)
PL(7,4)=Ø.Ø
PL(7,5)=Ø.Ø
PL(7,6)=Ø.Ø
PL(4,4)=Ø.Ø
PL(5,5)=Ø.Ø
PL(6,6)=Ø.Ø
PL(7,7)=Ø.Ø
R(1,1)=R2+RE
R(1,2)=Ø.Ø
R(1,3)=Ø.Ø
R(1,4)=Ø.Ø
```

```
R(1,5)=Ø.Ø
R(1,6)=Ø.Ø
R(1,7)=Ø.Ø
R(2,1)=Ø.Ø
R(2,2)=R2+RE
R(2,3)=Ø.Ø
R(2,4)=Ø.Ø
R(2,5)=Ø.Ø
R(2,6)=Ø.Ø
R(2,7)=Ø.Ø
R(3,1)=Ø.Ø
R(3,2)=Ø.Ø
R(3,3)=R2+RE
R(3,4)=Ø.Ø
R(3,5)=Ø.Ø
R(3,6)=Ø.Ø
R(3,7)=Ø.Ø
R(4,1)=Ø.Ø
R(4,2)=Ø.Ø
R(4,3)=Ø.Ø
R(4,4)=-RFD
R(4,5)=Ø.Ø
R(4,6)=Ø.Ø
R(4,7)=Ø.Ø
R(5,1)=Ø.Ø
R(5,2)=Ø.Ø
R(5,3)=Ø.Ø
R(5,4)=Ø.Ø
R(5,5)=-RKD
R(5,6)=Ø.Ø
R(5,7)=Ø.Ø
R(6,1)=Ø.Ø
R(6,2)=Ø.Ø
R(6,3)=Ø.Ø
R(6,4)=Ø.Ø
R(6,5)=Ø.Ø
R(6,6)=-RFQ
R(6,7)=Ø.Ø
R(7,1)=Ø.Ø
R(7,2)=Ø.Ø
R(7,3)=Ø.Ø
R(7,4)=Ø.Ø
R(7,5)=Ø.Ø
R(7,6)=Ø.Ø
R(7,7)=-RKQ
V(1)=Ø.Ø
V(2)=Ø.Ø
V(3)=Ø.Ø
V(4)=V4
V(5)=Ø.Ø
V(6)=Ø.Ø
V(7)=Ø.Ø
DO 12 I=1,7
```

```
      DO 12 J=1,7
12    R(I,J)=R(I,J)-PL(I,J)
      IF (NUM.EQ.3) GOTO 89
      CALL MATM1(R,CUN,R1)
      CALL MATM1(FØ,V,R3)
      DO 9ØØ I=1,7
9ØØ   R3(I)=R1(I)+R3(I)
      CALL MATM1(SØ,R3,CI3)
      CALL MATM3(SØ,CL,CL2)
      IF (NUM.EQ.1) GOTO 88
      IF (NUM.EQ.2) GOTO 87
      CI4(1)=CI3(1)
      CI4(2)=CI3(2)
      CI4(3)=CI3(4)
      CI4(4)=CI3(5)
      CI4(5)=CI3(6)
      CI4(6)=CI3(7)
      DO 78 I=3,6
      DO 78 J=3,6
78    CL66(I,J)=CL2(I+1,J+1)
      CL66(1,1)=CL2(1,1)
      CL66(1,2)=CL2(1,2)
      CL66(1,3)=CL2(1,4)
      CL66(1,4)=CL2(1,5)
      CL66(1,5)=CL2(1,6)
      CL66(1,6)=CL2(1,7)
      CL66(2,1)=CL2(2,1)
      CL66(3,1)=CL2(4,1)
      CL66(4,1)=CL2(5,1)
      CL66(5,1)=CL2(6,1)
      CL66(6,1)=CL2(7,1)
      CL66(2,2)=CL2(2,2)
      CL66(2,3)=CL2(2,4)
      CL66(2,4)=CL2(2,5)
      CL66(2,5)=CL2(2,6)
      CL66(2,6)=CL2(2,7)
      CL66(3,2)=CL2(4,2)
      CL66(4,2)=CL2(5,2)
      CL66(5,2)=CL2(6,2)
      CL66(6,2)=CL2(7,2)
      CALL MATINV(CL66,CL66I,6)
      CALL MATM4(CL66I,CI4,PI6)
      PI(1)=PI6(1)
      PI(2)=PI6(2)
      PI(3)=Ø.Ø
      PI(4)=PI6(3)
      PI(5)=PI6(4)
      PI(6)=PI6(5)
      PI(7)=PI6(6)
      GOTO 99
87    CI5(1)=CI3(1)
      CI5(2)=CI3(4)
      CI5(3)=CI3(5)
```

```
      CI5(4)=CI3(6)
      CI5(5)=CI3(7)
      DO 77 I=2,5
      DO 77 J=2,5
77    CL55(I,J)=CL2(I+2,J+2)
      CL55(1,1)=CL2(1,1)
      CL55(1,2)=CL2(1,4)
      CL55(1,3)=CL2(1,5)
      CL55(1,4)=CL2(1,6)
      CL55(1,5)=CL2(1,7)
      CL55(2,1)=CL2(4,1)
      CL55(3,1)=CL2(5,1)
      CL55(4,1)=CL2(6,1)
      CL55(5,1)=CL2(7,1)
      CALL MATINV(CL55,CL55I,5)
      CALL MATM5(CL55I,CI5,PI5)
      PI(1)=PI5(1)
      PI(2)=Ø.Ø
      PI(3)=Ø.Ø
      PI(4)=PI5(2)
      PI(5)=PI5(3)
      PI(6)=PI5(4)
      PI(7)=PI5(5)
      GOTO 99
88    CALL MATINV(CL2,CLINV,7)
      DO 952 I=1,7
      PI(I)=Ø.Ø
      DO 952 K=1,7
952   PI(I)=PI(I)+CLINV(I,K)*CI3(K)
      GOTO 99
89    CALL MATM1(ZMØT,CUN,R1)
      CALL MATM1(R,R1,R3)
      CALL MATM1(ZMØ,R3,R1)
      CALL MATM1(SØ,R1,R3)
      CALL MATM1(ZMØ,V,R1)
      CALL MATM1(FØ,R1,V)
      CALL MATM1(SØ,V,R1)
      DO 9Ø I=1,7
9Ø    CI3(I)=R1(I)+R3(I)
      CALL MATM3(CL,ZMØT,CL2)
      CALL MATM3(ZMØ,CL2,CLINV)
      CALL MATM3(SØ,CLINV,CL2)
      CI5(1)=CI3(2)
      CI5(2)=CI3(4)
      CI5(3)=CI3(5)
      CI5(4)=CI3(6)
      CI5(5)=CI3(7)
      DO 91 I=4,7
      DO 91 J=4,7
91    CL55(I-2,J-2)=CL2(I,J)
      CL55(1,1)=CL2(2,2)
      CL55(1,2)=CL2(2,4)
      CL55(1,3)=CL2(2,5)
```

```
      CL55(1,4)=CL2(2,6)
      CL55(1,5)=CL2(2,7)
      CL55(2,1)=CL2(4,2)
      CL55(3,1)=CL2(5,2)
      CL55(4,1)=CL2(6,2)
      CL55(5,1)=CL2(7,2)
      CALL MATINV(CL55,CL55I,5)
      CALL MATM5(CL55I,CI5,PI5)
      PI(1)=Ø.Ø
      PI(2)=PI5(1)
      PI(3)=Ø.Ø
      PI(4)=PI5(2)
      PI(5)=PI5(3)
      PI(6)=PI5(4)
      PI(7)=PI5(5)
99    RETURN
      END
```

** J5 **

```
C   This program is used to find all the maxima and minima for a
C   current waveform.
C   Array A is used to store the time.
C   Array B is used to store the corresponding current.
C   These maxima and minima are used as data for finding the upper and
C   lower envelopes.
      DIMENSION A(2000),B(2000)
      DO 10 I=1,2000
      READ(13,1)A(I),B(I)
1       FORMAT(4X,2F12.7)
10    CONTINUE
C   To find all the max.
      IF (B(1).GT.B(2)) GOTO 300
      GOTO 200
300   WRITE(14,30)A(1),B(1)
200    DO 100 I=2,1999
      IF ((B(I+1).LT.B(I)).AND.(B(I-1).LT.B(I))) GOTO 20
      GOTO 100
20    WRITE(14,30)A(I),B(I)
30    FORMAT(1X,2F12.7/)
100   CONTINUE
C   To find all the min.
      IF (B(1).LT.B(2)) GOTO 400
      GOTO 500
400   WRITE(14,30)A(1),B(1)
500   DO 700 I=2,1999
      IF ((B(I+1).GT.B(I)).AND.(B(I-1).GT.B(I))) GOTO 600
      GOTO 700
600   WRITE(14,30)A(I),B(I)
700   CONTINUE
      STOP
      END
£VX
```

```
C   This program is used to calculate relay operating time
C   under transient conditions.
C   H is the step width, TMS is the time multiplier setting, PU is the
C   pickup
      READ(13,1)H,TMS,PU
1     FORMAT(3F10.5)
      I=1
      SUM=0.0
      X=H
C   Polynomial for the upper envelope
10    A1=0.41174379+7.8587433*X-15.686015*X*X
      A2=13.769419*X*X*X-4.6226993*X*X*X*X
      Y1=A1+A2
C   Polynomial for the lower envelope
      A1=-4.8949375+12.201269*X-23.084667*X*X
      A2=22.255607*X*X*X-8.3056757*X*X*X*X
```

```
      Y2=A1+A2
C    Calculate the fault current in p.u.
      FAULT=(Y1-Y2)/2.Ø
      PSM=FAULT/PU
      Y=Ø.Ø2*ALOG1Ø(PSM)
      O=EXP(Y*ALOG(1Ø.Ø))
C    Calculate the operating time
      T=Ø.14/(O-1)*TMS
C    Calculate the disc travel during step width
      TRAVEL=H/T
C    SUM is the accumulative store
      SUM=SUM+TRAVEL
C    Test whether the relay operates
      IF (SUM.GE.1.Ø) GOTO 2Ø
      I=I+1
      X=I*H
      GOTO 1Ø
2Ø    OPTIME=I*H
      WRITE(14,2)SUM,OPTIME,TMS,H,I
2     FORMAT(1X,4F12.7,2X,I5)
      STOP
      END
£VX
```

```
C  This program is used to find the i^2t.
C  Arrary A is used to store the current waveform.
C   Sum is the i^2t.
       DIMENSION A(1850)
       DO 10 I=1,1850
       READ(13,1)B,A(I),C,D
1      FORMAT(4X,4F12.7)
10     CONTINUE
       SUM=0.0
       DO 20 I=1,1849
       X=(A(I)*A(I))*0.0005
       SUM=SUM+X
20     CONTINUE
       WRITE(14,2)SUM
2      FORMAT(1X,F12.7)
       STOP
       END
£VX
```

** J7 **

```
10 REM This program is used for calculating the currents in the
11 REM magnetising and secondary branches of a current transformer when a
14 REM fault occurs.
20 REM There is no inductance in the secondary circuit.
22 REM Tl is the time constant for the secondary circuit.
24 REM R is used to account for the slope change in the saturation
25 REM curve
26 REM I4 is the magnetising current
28 REM C is the constant of integration and S is the step width.
30 REM Array D9 is used for storing a current waveform
32 DIM D9(1000)
40 INPUT Tl,S,R,C
41 AS 14="DATA1"
42 FOR M=1 TO 1000
44 INPUT #14,D9(M)
46 NEXT M
50 AS 15="DATA10"
60 I5=C
70 I2=0
80 N=1
82 M=2
90 Nl=N-1
92 Ml=M-1
100 I3=D9(M)*S*EXP(S*N/Tl)
110 Il=D9(Ml)*S*EXP(S*Nl/Tl)
120 I2=I2+(Il+I3)/2
130 I4=EXP(-N*S/Tl)*(I2/Tl+C)
140 PRINT #15, I4,S*N,D9(M)-I4
145 REM check whether the step width is too large
150 X=ABS(I5)
160 Y=ABS(I4)
170 IF (I5*I4)<0 THEN 190
180 GOTO 210
190 IF (X>1 AND Y>0) THEN 720
200 IF (X>0 AND Y>1) THEN 720
210 I5=I4
218 REM check whether saturation or not
220 IF I4<-1 OR I4>1 THEN 250
230 N=N+1
232 M=M+1
235 IF N>999 THEN 730
240 GO TO 90
250 Tl=Tl*R
255 REM saturation region
260 PRINT #15, "SATURATION"
262 REM calculate new constant C
270 C=I4/EXP(-S*N/Tl)
280 PRINT #15, C
290 N=N+1
292 M=M+1
300 I2=0
```

```
310 N1=N-1
312 M1=M-1
320 I3=D9(M)*S*EXP(S*N/T1)
330 I1=D9(M1)*S*EXP(S*N1/T1)
340 I2=(I1+I3)/2+I2
350 I4=EXP(-S*N/T1)/T1*I2+C*EXP(-N*S/T1)
360 PRINT #15,I4,S*N,D9(M)-I4
370 X=ABS(I5)
380 Y=ABS(I4)
390 IF (I4*I5)<0 THEN 410
400 GOTO 430
410 IF (X>1 AND Y>0) THEN 720
420 IF (X>0 AND Y>1) THEN 720
430 I5=I4
440 IF I4>-1 AND I4<1 THEN 470
450 N=N+1
452 M=M+1
455 IF N>999 THEN 730
460 GO TO 310
465 REM unsaturated region
470 PRINT #15, "NO SATURATION"
480 T1=T1/R
490 C=I4/EXP(-S*N/T1)
500 PRINT #15, T1,C,I4,N*S
510 N=N+1
512 M=M+1
520 I2=0
530 N1=N-1
532 M1=M-1
540 I3=D9(M)*S*EXP(S*N/T1)
550 I1=D9(M1)*S*EXP(S*N1/T1)
560 I2=I2+(I1+I3)/2
570 I4=EXP(-S*N/T1)/T1*I2+C*EXP(-N*S/T1)
580 PRINT #15, I4,S*N,D9(M)-I4
590 X=ABS(I5)
600 Y=ABS(I4)
610 IF (I5*I4)<0 THEN 630
620 GOTO 650
630 IF (X>1 AND Y>0) THEN 720
640 IF (X>0 AND Y>1) THEN 720
650 I5=I4
660 IF I4<-1 OR I4>1 THEN 680
670 GOTO 690
680 GOTO 250
690 N=N+1
692 M=M+1
700 IF N>999 THEN 730
710 GOTO 530
720 PRINT #15, "DECREASE THE STEP WIDTH"
730 END


10 REM This program is used to calculate the currents in the secondary
```

```
15 REM and magnetising branches for a current transformer when a fault
17 REM occurs.
20 REM There is no resistance in the secondary circuit
22 REM L1 and L2 are the inductance for the magnetising and secondary
24 REM branches respectively.
26 REM I4 is the magnetising current
35 DIM D9(1000)
40 INPUT L1,L2,S,R,C
42 AS 10="DATA1"
44 FOR M=1 TO 1000
46 INPUT #10,D9(M)
47 NEXT M
50 AS 15="DATA10"
60 I5=C
80 N=0
82 M=1
100 I4=L2*D9(M)/(L1+L2)+C
140 PRINT #15, I4,S*N,D9(M)-I4
145 REM check whether step width is too long
150 X=ABS(I5)
160 Y=ABS(I4)
170 IF (I5*I4)<0 THEN 190
180 GOTO 210
190 IF (X>1 AND Y>0) THEN 720
200 IF (X>0 AND Y>1) THEN 720
210 I5=I4
212 REM check saturation or not
220 IF I4<-1 OR I4>1 THEN 250
230 N=N+1
232 M=M+1
235 IF N>999 THEN 730
240 GO TO 100
245 REM saturated region
250 L1=L1*R
260 PRINT #15, "SATURATION"
265 REM find the constant C at the instant of changing condition
270 C=I4
280 PRINT #15, C
290 N=N+1
292 M=M+1
310 N1=N-1
312 M1=M-1
315 I3=D9(M1)
320 I1=D9(M)
324 I4=(I1-I3)*L2/(L2+L1)+C
360 PRINT #15, I4,S*N,D9(M)-I4
370 X=ABS(I5)
380 Y=ABS(I4)
390 IF (I4*I5)<0 THEN 410
400 GOTO 430
410 IF (X>1 AND Y>0) THEN 720
420 IF (X>0 AND Y>1) THEN 720
430 I5=I4
```

```
440 IF I4>-1 AND I4<1 THEN 470
450 N=N+1
452 M=M+1
455 IF N>999 THEN 730
460 GO TO 320
465 REM unsaturated region
470 PRINT #15, "NO SATURATION"
480 L1=L1/R
490 C=I4
500 PRINT #15, T1,C,I4,N*S
510 N=N+1
520 M=M+1
530 N1=N-1
532 M1=M-1
535 I3=D9(M1)
540 I1=D9(M)
546 I4=(I1-I3)*L2/(L1+L2)+C
580 PRINT #15, I4,S*N,D9(M)-I4
590 X=ABS(I5)
600 Y=ABS(I4)
610 IF (I5*I4)<0 THEN 630
620 GOTO 650
630 IF (X>1 AND Y>0) THEN 720
640 IF (X>0 AND Y>1) THEN 720
650 I5=I4
660 IF I4<-1 OR I4>1 THEN 680
670 GOTO 690
680 GOTO 250
690 N=N+1
692 M=M+1
700 IF N>999 THEN 730
710 GOTO 540
720 PRINT #15, "DECREASE THE STEP WIDTH"
730 END
```

```
C  This program is used to calculate the initial conditions for a
C  synchronous generator connected to a busbar through a transformer
C  and a transmission line
C  All parameters are in pu
C  P is the generated real power at the machine terminals
C  Q is the generated reactive power at the machine terminals
C  RT is the sum of the resistances of the generator
C  transformer delta and star connected windings.
C  XT is the sum of the reactances of the generator
C  transformer delta and star connected windings.
C  SRE is the equivalent resistance of the transmission system
C  SXE is the equivalent reactance of the transmission system
C  SE is the absolute value of the machine terminal voltage
C  ET is the absolute value of the HV voltage
C  RA is the armature resistance and XA is the armature leakage
C  reactance
C  RK is the saturation factor
C  XADO is the mutual reactance in the direct axis
C  CT connection matrix
C  W is the system angular frequency (radians/sec.)
C  XO is the zero phase sequence reactance
      DIMENSION CT(3,3),ZNI(3,3),ZN(3,3)
      COMPLEX V(3),ZIT(3),ZSIT(3),ZICT(3),E(3),EQA,AI
     1,PUT1,PUT2,PUT3,ANGN,U1,U2,XE(3)
      READ(13,1)P,RT,Q,XT,SE
1     FORMAT(5F8.4)
      READ(13,2)ET,SRE,SXE,RA,XA,RK,XAQØ,XADØ
2     FORMAT(8F8.4)
      READ (13,41)XD,XQ,XD1,TD1,TL,W,XØ
41    FORMAT(7F8.4)
      WRITE(14,42)XD,XQ,XD1,TD1,TL,W,XØ
42    FORMAT(1X,7F8.4)
      READ(13,3)((CT(I,J),J=1,3),I=1,3)
3     FORMAT(3F9.5)
      WRITE(14,1ØØ)P,RT,Q,XT,SE
1ØØ   FORMAT(1X,5F8.4)
      WRITE(14,1Ø1)ET,SRE,SXE,RA,XA,RK,XAQØ,XADØ
1Ø1   FORMAT(1X,9F8.4)
      WRITE(14,1Ø2)((CT(I,J),J=1,3),I=1,3)
1Ø2   FORMAT(1X,3F9.5)
      A=(P*(RST+RT)+Q*(XST+XT))/SE
      B=(Q*(RST+RT)-P*(XST+XT))/SE
      C=SE-A
C  Calculate the generator transformer tap position.
      TN=ET/SQRT(C*C+B*B)
      PT=P-((P*P+Q*Q)*(RST+RT))/(SE*SE)
      QT=Q-((P*P+Q*Q)*(XST+XT))/(SE*SE)
      PI=PT-((PT*PT+QT*QT)*SRE)/(ET*ET)
      WRITE(14,1Ø8)A,B,C,TN,PT,QT,PI
1Ø8   FORMAT(1X,7F1Ø.5)
      QI=QT-((PT*PT+QT*QT)*SXE)/(ET*ET)
```

```
      AN=ATAN(QI/PI)
      EI1=((ET*ET)-(PT*SRE+QT*SXE))
      EI2=EI1*EI1
      EI3=(QT*SRE-PT*SXE)*(QT*SRE-PT*SXE)
C  Calculate the AEI (infinite machine voltage)
      AEI=SQRT((EI2+EI3)/(ET*ET))
      WRITE(14,1Ø9)QI,AN,EI1,EI2,EI3,AEI
1Ø9   FORMAT(1X,6F1Ø.5)
      PUT1=1.57Ø8*(Ø.Ø,1.Ø)
      PUT2=(1.57Ø8-2.Ø944)*(Ø.Ø,1.Ø)
      PUT3=(1.57Ø8+2.Ø944)*(Ø.Ø,1.Ø)
      WRITE(14,1Ø9)PUT1,PUT2,PUT3
C  Calculate infinite machine voltage for each phase
      V(1)=AEI*CEXP(PUT1)
      V(2)=AEI*CEXP(PUT2)
      V(3)=AEI*CEXP(PUT3)
      WRITE(14,1Ø9)V(1),V(2),V(3)
      TIE=(SQRT(PI*PI+QI*QI))/AEI
      PUT1=(1.57Ø8-AN)*(Ø.Ø,1.Ø)
      PUT2=(1.57Ø8-2.Ø944-AN)*(Ø.Ø,1.Ø)
      PUT3=(1.57Ø8+2.Ø944-AN)*(Ø.Ø,1.Ø)
      WRITE(14,1Ø8)TIE,PUT1,PUT2,PUT3
      ZIT(1)=TIE*CEXP(PUT1)
      ZIT(2)=TIE*CEXP(PUT2)
      ZIT(3)=TIE*CEXP(PUT3)
      WRITE(14,1Ø9)ZIT(1),ZIT(2),ZIT(3)
C  Calculate the currents in the star winding of the generator
C  transformer
      ZSIT(1)=ZIT(1)*TN
      ZSIT(2)=ZIT(2)*TN
      ZSIT(3)=ZIT(3)*TN
C  Calculate armature currents
      CALL MATM1(CT,ZSIT,ZICT)
      DO 2ØØ I=1,3
      DO 2ØØ J=1,3
2ØØ   ZNI(I,J)=Ø.Ø
      ZNI(1,1)=1.Ø/TN
      ZNI(2,2)=1.Ø/TN
      ZNI(3,3)=1.Ø/TN
      CALL MATM2(CT,ZNI,ZN)
      CALL MATM1(ZN,V,E)
      U=(SRE+TN*TN*(RST+RT))/(TN*TN)
      U1=(SXE+TN*TN*(XST+XT))*(Ø.Ø,1.Ø)/(TN*TN)
      U2=U+U1
      XE(1)=U2*ZICT(1)
      XE(2)=U2*ZICT(2)
      XE(3)=U2*ZICT(3)
      WRITE(14,1Ø8)U,U1,XE(1),U2
C  Calculate armature voltages
      E(1)=E(1)+XE(1)
      E(2)=E(2)+XE(2)
      E(3)=E(3)+XE(3)
      WRITE(14,11)E(1),E(2),E(3)
```

```
11     FORMAT(1X,6F10.5/)
       WRITE(14,11)ZICT(1),ZICT(2),ZICT(3)
       EH=REAL(E(1))
       EV=AIMAG(E(1))
C  Calculate time to voltage zero
       TIME=(1.5708-ATAN(EV/EH))
       E(1)=E(1)*(COS(TIME),SIN(TIME))
       ZICT(1)=ZICT(1)*(COS(TIME),SIN(TIME))
       ZICT(2)=ZICT(2)*(COS(TIME),SIN(TIME))
       ZICT(3)=ZICT(3)*(COS(TIME),SIN(TIME))
       WRITE(14,150)E(1),ZICT(1),TIME
150    FORMAT(1X,5F10.5)
C  Calculate the angle between the d axis and the axis of phase a
       EQA=E(1)+(RA+(0.0,1.0)*(XA+XAQ0/RK))*ZICT(1)
       EQAH=REAL(EQA)
       EQAV=AIMAG(EQA)
       EQD=SQRT(EQAH*EQAH+EQAV*EQAV)
       IF ((EQAV.GT.0).AND.(EQAH.GT.0)) GOTO 10
       IF ((EQAV.GT.0).AND.(EQAH.LT.0)) GOTO 22
       IF ((EQAV.LT.0).AND.(EQAH.GT.0)) GOTO 30
       IF ((EQAV.LT.0).AND.(EQAH.LT.0)) GOTO 40
10     ANG=ATAN(EQAV/EQAH)
       GOTO 50
22     ANG=ATAN(EQAV/EQAH)+3.1416
       GOTO 50
30     ANG=ATAN(EQAV/EQAH)+6.2832
       GOTO 50
40     ANG=ATAN(EQAV/EQAH)+3.1416
50     ANGN=(ANG-1.5708)*(0.0,1.0)
       AI=ZICT(1)*CEXP(-ANGN)
       DI=REAL(AI)
       QI=AIMAG(AI)
       EQ=EQD+DI*(XAD0-XAQ0)/RK
       FDI=EQ*RK*1.5/XAD0
C  Calculate the initial flux linkages
       FS1=(0.666667*FDI-DI)*(0.666667*FDI-DI)*XAD0*XAD0/(RK*RK)
       FS2=QI*QI*XAQ0*XAQ0/(RK*RK)
       WRITE(14,107)FS1,FS2
107    FORMAT(1X,2F10.5)
       FS=SQRT(FS1+FS2)*100.0
       FDI1=FS
       RKN=FDI1/FS
       FDI1=FDI1/100.0
       WRITE(14,12)FDI1,FDI,FS,RK,RKN,ANGN
12     FORMAT(1X,7F10.5)
       WRITE(14,106)EQA,EQAH,ANG,AI,DI,EQ
106    FORMAT(1X,8F9.5)
C Calculate EFD, and the initial mechanical torque
       XAA0=((XD+XQ-2.0*XA)/RKN+2.0*XA+X0)/3.0
       XAB0=((XD+XQ-2.0*XA)/RKN+2.0*XA-2.0*X0)/6.0
       XAA2=(XD-XQ)/(3.0*RKN)
       XAD0=0.666667*XAD0
       XAQ0=0.666667*XAQ0
```

```
      XAFD=XADØ*XADØ/((XD-XD1)*Ø.666667)
      RFD=XAFD*XD1/(W*TD1*XD)
      X=ANG-1.57Ø8
      AAI=REAL(ZICT(1))
      BI=REAL(ZICT(2))
      CI=REAL(ZICT(3))
      CL11=-XAAØ-XAA2*COS(2.Ø*X)
      CL12=XABØ-XAA2*COS(2.Ø*X-2.Ø944)
      CL13=XABØ-XAA2*COS(2.Ø*X+2.Ø944)
      CL14=XADØ*COS(X)/RKN
      CL22=-XAAØ-XAA2*COS(2.Ø*X+2.Ø944)
      CL23=XABØ-XAA2*COS(2.Ø*X)
      CL24=XADØ*COS(X-2.Ø944)/RKN
      CL33=-XAAØ-XAA2*COS(2.Ø*X-2.Ø944)
      CL34=XADØ*COS(X+2.Ø944)/RKN
      FA=CL11*AAI+CL12*BI+CL13*CI+CL14*FDI
      FB=CL12*AAI+CL22*BI+CL23*CI+CL24*FDI
      FC=CL13*AAI+CL23*BI+CL33*CI+CL34*FDI
      EFD=RFD*FDI
      TE=Ø.3849*(FA*(BI-CI)+FB*(CI-AAI)+FC*(AAI-BI))
      TM=TE+TL
      WRITE(14,116)AAI,BI,CI,FA,FB,FC,TM,EFD,RFD
116   FORMAT(1X,9F9.5)
      STOP
      END
```

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