A Statistical-Mechanical Analysis of Coded CDMA with Regular LDPC Codes

Toshiyuki Tanaka[1]
Dept. of Electro. & Info. Eng.,
Tokyo Metropolitan Univ.
1-1 Minami-Osawa, Hachioji,
Tokyo, 192-0397 Japan
e-mail: tanaka@eei.metro-u.ac.jp

David Saad[1]
Neural Computing Res. Grp.,
Aston Univ.
Aston Triangle, Birmingham,
B4 7ET United Kingdom
e-mail: d.saad@aston.ac.uk

I. Introduction

Performance of LDPC-coded CDMA system is analyzed, using the replica method of statistical mechanics (which has been separately applied to the analysis of CDMA [1] and of LDPC codes [2]), in the infinite codelength and large system (i.e., infinite number of users) limits. Most existing studies of such systems consider the case of a finite number of users. Caire et al. [3] consider the same limits as the current study, but theirs is based on stripping and successive detection/decoding. Our analysis, on the other hand, gives the information-theoretic and decoding thresholds of the optimum joint detection/decoding scheme.

II. Analysis

We consider the serially concatenated system of a bank of per-user LDPC encoders and the standard fully-synchronous, randomly-spread, BPSK CDMA channel. We assume that the users are of equal rate, that the LDPC codes used are drawn from the same random ensemble of a regular (C, L)-LDPC code (C and L are the column and row weights of the parity-check matrix), that the power control is perfect, and that the channel noise is additive white Gaussian with variance σ_0^2. The codelength M and the number of users K are sent to infinity, while the information rate $R \equiv N/M$ and the system load $\beta \equiv K/G$ are both kept to be $O(1)$, where N and G are the information-block length and the spreading factor, respectively.

Analytical evaluation of the performance of the optimum joint detection/decoding scheme requires solving the saddle-point equations for $\{m, q, E, F, \pi(x), \hat{\pi}(\hat{x})\}$:

$$m = \int \tanh\Big(\sqrt{F}z + E + \sum_{c=1}^{C} \tanh^{-1} \hat{x}_c\Big)\, Dz\mathcal{D}_{\hat{\pi}}^{C}\hat{\boldsymbol{x}}$$

$$q = \int \tanh^2\Big(\sqrt{F}z + E + \sum_{c=1}^{C} \tanh^{-1} \hat{x}_c\Big)\, Dz\mathcal{D}_{\hat{\pi}}^{C}\hat{\boldsymbol{x}}$$

$$E = \frac{1}{\sigma^2 + \beta(1-q)}, \quad F = \frac{\sigma_0^2 + \beta(1-2m+q)}{[\sigma^2 + \beta(1-q)]^2}$$

$$\pi(x) = \int \delta\Big[x - \tanh\Big(\sqrt{F}z + E + \sum_{c=1}^{C-1} \tanh^{-1} \hat{x}_c\Big)\Big]\, Dz\mathcal{D}_{\hat{\pi}}^{C-1}\hat{\boldsymbol{x}}$$

$$\hat{\pi}(\hat{x}) = \int \delta\Big(\hat{x} - \prod_{i=1}^{L-1} x_i\Big)\mathcal{D}_{\pi}^{L-1}\boldsymbol{x}$$

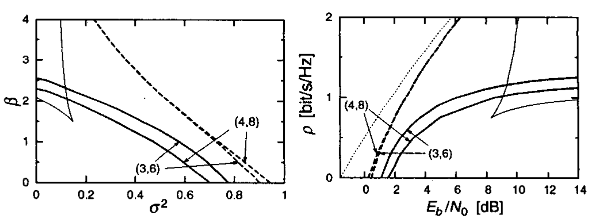
where $Dz \equiv (2\pi)^{-1/2}e^{-z^2/2}dz$ and $\mathcal{D}_{\hat{\pi}}^{C}\hat{\boldsymbol{x}} \equiv \prod_{c=1}^{C}[\hat{\pi}(\hat{x}_c)\, d\hat{x}_c]$ etc. The bit error rate P_b (which is the same for all users since they are statistically equivalent) is given by $P_b = \int_{-\infty}^{0} P(u)\, du$ where

$$P(u) = \int \delta\Big[u - \tanh\Big(\sqrt{F}z + E + \sum_{c=1}^{C} \tanh^{-1} \hat{x}_c\Big)\Big]\, Dz\hat{\mathcal{D}}^{C}\hat{\boldsymbol{x}}.$$

When the saddle-point equations have multiple solutions, the one minimizing the associated free energy (not shown) is information-theoretically relevant. The other solutions are also significant because they may determine the decoding threshold. Analytically found are the "error-free" solution ($m = q = 1$, $\pi(x) = \delta(x-1)$, $\hat{\pi}(\hat{x}) = \delta(\hat{x}-1)$), and, in the limit $L \to \infty$, the one corresponding to the uncoded system [1]. Any other solutions are obtained numerically.

III. Results

Figure 1 (a) shows the thresholds (decoding and information-theoretic), in terms of the critical load β, versus the noise level σ^2 for the joint detection/decoding scheme with (3,6)- and (4,8)-LDPC codes. The maximum achievable spectral efficiency $\rho = \beta f$ (f being the free energy or mutual information per symbol and user) versus signal-to-noise ratio E_b/N_0 is shown in Fig. 1 (b), which shows that, although the spectral efficiency of the joint detection/decoding scheme practically converges to a finite value, it diverges theoretically, approaching that of the AWGN single-user spectral efficiency (shown as dotted line), as $\beta \to \infty$, suggesting possible improvement in the decoding threshold by adopting irregular LDPC codes.



(a) β-σ^2 (b) ρ-E_b/N_0

Fig. 1: Decoding (thick) and information-theoretic (dashed) thresholds for (3,6)- and (4,8)-LDPC coded CDMA systems.

References

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