The R&D Based Endogenous Growth Model: R&D workers, Skill Premium and Productivity Growth*

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Abstract

This paper attempts to explain the features of the recent U.S. economy by applying the transitional dynamics analysis of the R&D based endogenous growth model. It tries to explain the observed comovements of three variables: the relative wage rate of high skilled labour, the share of R&D workers and per capita output growth rate show a sharp decline in the

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beginning of 1970s followed by a gradual increase. The paper shows that an unexpected structural change, which induces a change in the allocation of high skilled workers, pushes the economy away from the steady state along the saddle path and the economy, thereafter, moves back gradually towards the steady state.

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1 Introduction

Contrary to the neo-classical growth models, various kinds of endogenous growth models provide the endogenized steady state growth rate by eliminating diminishing returns to the factors which can be accumulated. To eliminate diminishing returns to the factors, Lucus (1988) introduces human capital and Romer (1986) uses ideas of learning by doing and knowledge spillover. More recent endogenous growth models (Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992) and Jones (1995b)) focus on intentional knowledge creation and technological progress, i.e. the firms' R&D activities. The important difference between the R&D based endogenous growth models and the other types of endogenous growth models is that the R&D based models capture the fact that individuals and firms often earn monopoly profits in creating new knowledge.

Despite its promising features, the R&D based models focus rather on the steady state analysis mainly due to the complexity in analysing the transitional dynamics. Little work has been done to analyze the dynamic behaviour.[1] If we are interested in the behaviour of an economy over time, especially its trend rather than its business cycle, the transition analysis could be very important.

This paper attempts to explain the features of the recent U.S. economy by applying the transitional dynamics analysis of the R&D based endogenous growth model. It focuses on the effects of technological progress on the transitional path. In trying to explain the features of the recent U.S. economy, we

[1] Arnold (2000) provides a complete dynamic analysis of the Romer (1990) model.

particularly consider three variables: the relative wage rate of high skilled labour (i.e. the skill premium), the share of R&D workers in the total employment and the growth rate of per capita real GDP.

The U.S. skill premium dramatically fell during the 1970s and grew throughout the 1980s and the 1990s. The recent work on the skill premium pays a great attention to skill biased technological change, (e.g. Acemoglu (1998 and 2000), Kiley (1999) and Galor and Maov (2000)). For example, Acemoglu (1998) argues that a large exogenous rise in the number of U.S. college graduates in the 1970s (i.e. the rise in the relative supply of skill labour) first reduced the skill premium but induced the development of the skill biased technology which increased the skill premium in the subsequent period. There is, however, one interesting finding about the post-war U.S. economy, which has rarely been discussed in the literature. It is that the share of R&D workers in the total labour force and the number of R&D workers show the very similar patterns of dynamics to the skill premium. The share (and also the number) of R&D workers also fell sharply in the 1970s and increased in the 1980s and the 1990s. Considering this fact, this paper provides an alternative explanation of the recent pattern of the U.S. skill premium. It is argued that a change in the skill premium is caused by a change in the allocation of high skilled workers.

The model in this paper assumes that high skilled labour can work either in the final goods sector or in R&D, and low skilled labour can work only in the final goods sector. We take the supposition that there was an unexpected rise in the cost of producing new designs by R&D (which reflects the increased required amount of high skilled labour to produce new designs) in the beginning of the 1970s. This structural change leads to the decrease in the high skilled labour demand in R&D relative to the high skilled labour demand in the final goods sector. This implies the increase in the relative supply of high skilled labour in the final goods sector and then the reduction in the skill premium. The structural change pushes the economy away from the steady state. In the subsequent period, the share of high skilled labour employed in R&D starts increasing towards the steady state level and the relative supply of high skilled labour in the final goods sector starts falling. Thus, the skill premium gradually increases back towards the steady state level. The model also shows that the growth rate of output per labour first decreases due to the structural change and then increases back towards the steady state level.

This paper is organized as follows. Section 2 describes the R&D based endogenous growth model and analyzes the transitional dynamics towards the steady state. Section 3 shows some empirical findings about U.S. economy: the relative wage rate of high skilled labour (i.e. the skill premium), the share of R&D workers in the total employment and the growth rate of per capita GDP, and explains them by using the analysis of the transitional dynamics of the model. Section 4 concludes.

2 The Model

The model follows Romer (1990), Jones (1995b) and Barro and Sala-i-Martin (1995, Ch.6). We assume that there are two sectors in an economy: a final goods sector and an intermediate goods sector. The final goods sector produces goods by using labour and non-durable intermediate goods. The firm in the intermediate goods sector needs designs with forgone output to produce the non-durable intermediate goods. Research on a new design is undertaken by the firm in the intermediate goods sector. We also assume that labour is only input to the research and that firms and households are rational.

2.1 The Basic Setup

We consider an economy that produces homogenous final goods, Y. The production function for the final goods at time t is given by:

$$Y(t) = L_L(t)^{\alpha} L_{HY}(t)^{\beta} \int_0^{N(t)} X_j(t)^{1-\alpha-\beta}\, dj, \quad 0 < \alpha + \beta < 1. \tag{1}$$

All firms in the final goods sector access to this production function. The firm in the final goods sector employs low skilled labour L_L and high skilled labour L_{HY}, with nondurable intermediate goods. Equation 1 implies that unskilled labour and skilled labour are both essential to production of final goods. The assumption taken here is that the firm in the final goods sector needs skilled labour who can perform complicated tasks which are necessary for production of the final goods. As we will discuss later, the share of skilled labour in the total labour force is assumed to take such a value that the equilibrium level of skill premium - high skilled wage over low skilled wage - is greater than one. Thus, no high skilled labour work as low skilled labour. Low skilled labour who can only perform simple tasks is therefore also essential to production of the final goods. X_j is the jth type of nondurable intermediated goods and N is the number of available types of nondurable intermediate goods. We treat X_j as non-durable goods rather than durable goods because we focus on the effects of technological progress on the economy's transitional path rather than focus on the effects of capital accumulation. Normalizing the price for the final goods to unity, the firm's profit at time t is given by:

$$Y(t) - \int_0^{N(t)} p_j(t)\, X_j(t) dj - w_L(t) L_L(t) - w_{HY}(t) L_{HY}(t)\,,$$

where p_j is the price of nondurable intermediate j, w_L is the wage rate for low skilled labour and w_{HY} is the wage rate for high skilled labour. Since the final goods market is competitive, the firm takes p_j, w_L and w_{HY} as given. Assuming that there is no adjustment cost, we obtain the usual equations between factor prices and marginal products at all points of time as follows (the time argument is dropped):

$$p_j = L_L^{\alpha}\, L_{HY}^{\beta}\, (1-\alpha-\beta)\, X_j^{-\alpha-\beta}\,, \tag{2}$$

$$w_L = \alpha\, L_L^{\alpha-1}\, L_{HY}^{\beta} \int_0^N X_j^{1-\alpha-\beta}\, dj\,, \tag{3}$$

$$w_{HY} = \beta\, L_L^{\alpha}\, L_{HY}^{\beta-1} \int_0^N X_j^{1-\alpha-\beta}\, dj\,. \tag{4}$$

In the intermediate goods sector, once the firm invents a new design it can retain a perpetual monopoly over the production of the new type of intermediated good. R&D to invent new designs is undertaken within the firm and the cost of invention is one-time cost. Production of one unit of intermediate goods incurs η units of forgone final output (η is an exogenously determined positive and constant parameter). Thus, the flow of the monopolist's operational profit at a point of time is given by:

$$\pi_j = p_j \, X_j \, - \eta \, X_j \, .$$

The present value of return from the operation is, then, given by:

$$V_j = \int_t^{\infty} \pi_j(v) \, e^{-\int_t^v r(\omega) \, d\omega} \, dv = \int_t^{\infty} \left(p_j(v) \, X_j(v) \, - \eta \, X_j(v)\right) e^{-\int_t^v r(\omega) \, d\omega} \, dv \, , \tag{5}$$

where r is the interest rate. Since the monopolist faces the demand curve given by equation (2) at every period, it is faced with the following problem:

$$\max_{X_j} \int_t^{\infty} \left(p_j(v) \, X_j(v) \, - \eta \, X_j(v)\right) e^{-\int_t^v r(\omega) \, d\omega} \, dv$$
$$s.t. \; p_j = L_L^{\alpha} \, L_{HY}^{\beta} \, (1 - \alpha - \beta) \, X_j^{-\alpha-\beta} \, .$$

Solving this problem yields:

$$X_j = \overline{X} = \left(\frac{L_L^{\alpha} \, L_{HY}^{\beta} \, (1 - \alpha - \beta)^2}{\eta} \right)^{\frac{1}{\alpha+\beta}} \, . \tag{6}$$

and

$$p_j = \overline{p} = \frac{\eta}{1-\alpha-\beta}\,. \tag{7}$$

Thus, each monopolist in the intermediate sector produces the same amount of intermediate goods and charges the same price at every period. This also implies that the present value of the monopoly operational profit is the same for each firm in the intermediate sector: $V_j = \overline{V} = \int_t^\infty \overline{\pi}(v)\, e^{-\int_t^v r(\omega)\, d\omega}\, dv$ where $\overline{\pi}(t) = \overline{p}\,\overline{X}(t) \;-\; \eta\overline{X}(t)$.

In order for the intermediate firm to produce goods, it needs a design which is produced through R&D activities. R&D requires a certain amount of high skilled labour. We assume that the intermediate firm needs $\frac{\eta}{N^{\phi}}$ units of high skilled labour to innovate one unit of new design. This implies: (i) the existing stock of designs spills over (designs are non-rival goods) and (ii) the higher the level of existing stock of designs is, the lower the level of required high skilled labour for the innovation ("shoulders of giants" effect). The cost of innovation is, then, given by:

$$Z(t) = \frac{\eta}{N(t)^{\phi}}\, w_{HN}(t)\,, \tag{8}$$

where w_{HN} is the wage rate for high skilled labour engaged in R&D. We assume that there is free entry into R&D. Any firm can pay Z to secure the present value of monopoly operational profit. In equilibrium, therefore, $\overline{V} = Z$ must be satisfied. Thus, the free entry condition implies:

$$\int_t^\infty \overline{\pi}(v)\, e^{-\int_t^v r(\omega)\, d\omega}\, dv = Z(t)\,. \tag{9}$$

Differentiating both side of equation (9) yields:

$$r(t) = \frac{\overline{\pi}(t)}{Z(t)} + \frac{\dot{Z}(t)}{Z(t)} \,. \tag{10}$$

Since $\frac{\eta}{N^{\phi}}$ units of skilled labour are required to innovate one unit of new design, the amount of high skilled labour devoted to R&D is given by:

$$L_{HN}(t) = \dot{N}(t)\, \frac{\eta}{N(t)^{\,\phi}} \,. \tag{11}$$

In equilibrium, the high skilled labour employed in the final goods sector should receive the same wage rate as the high skilled labour employed in R&D. Thus, $w_{HY}(t) = w_{HN}(t)$ must hold. We denote this common wage rate for the high skilled labour as w_H. From equations (3) and (4), the skill premium - high skilled wage W_H, relative to low skilled wage W_L - is given by:

$$\frac{w_H}{w_L}(t) = \frac{\beta}{\alpha}\, \frac{L_L(t)}{L_{HY}(t)} \,.$$

Denoting s and u as the share of high skilled labour in the population and the fraction of R&D workers in the high skilled labour population, respectively, the skill premium can be rewritten as:

$$\frac{w_H}{w_L}(t) = \frac{\beta}{\alpha} \frac{1-s}{s} \frac{1}{1-u(t)}, \tag{12}$$

where we assume s is constant over time and exogenously given. We assume that s takes the value which satisfies $s < \frac{\beta}{\alpha+\beta}$. Therefore, $\frac{w_H}{w_L} > 1$ holds for any value of u between 0 and 1.[2] This implies that no high skilled labour

[2] We assume $0 < u(0) < 1$.

wishes to work as low skilled labour. Above all, $L_{HY}(t) = (1 - u(t))\, s\, L(t)$, $L_{HN}(t) = u(t)\, s\, L(t)$, and $L_L(t) = (1 - s)\, L(t)$ where L is the total labour force which grows at a constant rate n.

Finally, we consider the household's utility maximization problem. Since we have assumed that s is fixed over time, its composition of low skilled and high skilled adults is constant. By normalizing the number of adults at time 0 to unity, each household wishes to maximize overall utility U as given by:

$$U = \int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta}\, e^{(n-\rho)\,t}\, dt\,, \tag{13}$$

where $c = \frac{C}{L}$, C is the total consumption, ρ is the rate of time preference, and $\rho > 0$.[3] We assume $\theta > 1$. The flow budget constraint for the household is given by:

$$\dot{a}(t) = \; (1-s)\, w_L(t) + s\, w_H(t) \; + r(t)\, a(t) - c(t) - n\, a(t)\,, \tag{14}$$

where $a = \frac{A}{L}$ and A is the total financial assets. The present value Hamiltonian is, then, given by:

$$H = \frac{c(t)^{1-\theta} - 1}{1-\theta}\, e^{(n-\rho)\,t} + \lambda \left[\, (1-s)\, w_L(t) + s\, w_H(t) \; + r(t)\, a(t) - c(t) - n\, a(t) \right]. \tag{15}$$

The first order conditions are reduced to the familiar Euler equation which is given by:

[3] We assume that $n - \rho < 0$ so that U is bounded if c is constant over time.

$$\frac{\dot{c}(t)}{c(t)} = \frac{r(t) - \rho}{\theta} \,. \tag{16}$$

The transversality condition is:

$$\lim_{t \to \infty} \left(\lambda(t)\, a(t)\right) = 0 \,. \tag{17}$$

Since the aggregate of the households' financial assets equals the market value of the firms, we can write the assets per person as:[4]

$$a(t) = \frac{Z(t)\, N(t)}{L(t)} \,.$$

Thus, the transversality condition (17) can be rewritten as:

$$\lim_{t \to \infty} \left(\lambda(t)\, \frac{Z(t)\, N(t)}{L(t)} \right) = 0 \,. \tag{18}$$

2.2 Dynamic Equilibrium

This section analyses the dynamic equilibrium by using phase diagrams. We show that if the steady state exists the saddle path is stable. Note that saddle path stability means that there is a unique and monotonic path converging towards the steady state. It is also shown that the stable saddle-path is the only possible dynamic equilibrium.

[4] Since the firm in the final goods sector earns zero profit, the market value of the firms in this economy equals the number of monopolists in the intermediate sector N, multiplied by the cost of innovation $Z(t)$.

In order to analyse the dynamics of the economy, we need to derive equations which explain the motions of three variables: c, u and N. One differential equation comes from Euler equation (16) together with the expression for the rate of return given by equation (10). Substituting equation (10) into equation (16) yields:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}\left(\frac{\overline{\pi}(t)}{Z(t)} + \frac{\dot{Z}(t)}{Z(t)} - \rho\right). \tag{19}$$

Since $\overline{\pi}(t) = \overline{p}\,\overline{X}(t) - \eta\overline{X}(t)$, from equations (6) and (7) the monopolist's operational profit can be rewritten as:

$$\overline{\pi}(t) = \eta^{\frac{-(1-\alpha-\beta)}{\alpha+\beta}}\,(\alpha+\beta)\,(1-\alpha-\beta)^{\frac{2-\alpha-\beta}{\alpha+\beta}}\,(1-s)^{\frac{\alpha}{\alpha+\beta}}\,s^{\frac{\beta}{\alpha+\beta}}\,(1-u(t))^{\frac{\beta}{\alpha+\beta}}\,L(t)\,. \tag{20}$$

Substituting equation (4) into equation (8) yields (in equilibrium $w_{HY} = w_{HN}$) :

$$Z(t) = \frac{\eta}{N(t)^{\phi}}\beta\,L_L(t)^{\alpha}\,L_{HY}(t)^{\beta-1}\int_0^{N(t)} X_j^{1-\alpha-\beta}\,dj\,.$$

By using equation (6), this expression can be rewritten as:

$$Z(t) = \beta\eta^{\frac{2(\alpha+\beta)-1}{\alpha+\beta}}\,(1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}}\,(1-s)^{\frac{\alpha}{\alpha+\beta}}\,s^{\frac{-\alpha}{\alpha+\beta}}\,(1-u(t))^{\frac{-\alpha}{\alpha+\beta}}\,N(t)^{1-\phi}\,. \tag{21}$$

Taking logs and differentiating with respect to time on both sides of equation (21) yields:

$$\frac{\dot{Z}(t)}{Z(t)} = (1-\phi)\,\frac{\dot{N}(t)}{N(t)} - \frac{\alpha}{\alpha+\beta}\,\frac{\dot{\overline{(1-u(t))}}}{(1-u(t))}\,. \tag{22}$$

Substituting equations (20), (21), and (22) into equation (19) gives:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}\left(\frac{(1-\alpha-\beta)\,(\alpha+\beta)}{\beta}G_N(t)T(t) + (1-\phi)G_N(t) - \frac{\alpha}{\alpha+\beta}G_T(t) - \rho\right), \tag{23}$$

where $G_N(t) = \frac{\dot{N}(t)}{N(t)}$, $G_T(t) = \frac{\dot{\overline{(1-u(t))}}}{(1-u(t))}$, and $T(t) = \frac{1-u(t)}{u(t)}$. Equation (23) is one of the differential equations which we use to analyse the dynamics of the economy.

The second differential equation, which shows the dynamics of N, is derived from equation (11). From equation (11), the growth rate of $N(t)$ is given by:

$$G_N(t) = \frac{1}{\eta}\,u(t)\,s\,L(t)\,N(t)^{\phi-1}\,. \tag{24}$$

Thus, the growth rate of G_N is shown by:

$$\frac{\dot{G}_N(t)}{G_N(t)} = -G_T(t)\,T(t) - (1-\phi)\,G_N(t) + n\,. \tag{25}$$

Finally, we derive the equation which describes the motion of u. From equations (1) and (6), output per labour, $y(t)$, is given by:

$$y(t) = \eta^{\frac{-(1-\alpha-\beta)}{\alpha+\beta}}\,(1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}}\,(1-s)^{\frac{\alpha}{\alpha+\beta}}\,s^{\frac{\beta}{\alpha+\beta}}\,(1-u(t))^{\frac{\beta}{\alpha+\beta}}\,N(t)\,. \tag{26}$$

Taking logs and differentiating it with respect to time on both sides of equation (26) and rearranging it yield:

$$G_T(t) = \frac{\alpha+\beta}{\beta} \left(\frac{\dot{y}(t)}{y(t)} - G_N(t) \right) . \tag{27}$$

Since $C(t) = Y(t) - \eta\, N(t)\bar{X}(t)$, consumption per labour is given by:

$$c(t) = y(t) - \frac{\eta\, N(t)\, \bar{X}(t)}{L(t)} . \tag{28}$$

By using equation (6),

$$\begin{aligned} \frac{N(t)\, \bar{X}(t)}{L(t)} &= \eta^{\frac{-1}{\alpha+\beta}} (1-\alpha-\beta)^{\frac{2}{\alpha+\beta}} (1-s)^{\frac{\beta}{\alpha+\beta}} s^{\frac{\beta}{\alpha+\beta}} (1-u(t))^{\frac{\beta}{\alpha+\beta}} N(t) \\ &= y(t) \left(\eta^{-1} (1-\alpha-\beta)^{\frac{2(\alpha+\beta)}{\alpha+\beta}} \right) . \end{aligned} \tag{29}$$

Substituting equation (29) into equation (28) yields:

$$c(t) = y(t) \left[1 - (1-\alpha-\beta)^2 \right] . \tag{30}$$

This leads to: $\frac{\dot{c}(t)}{c(t)} = \frac{\dot{y}(t)}{y(t)}$. Thus, equation (27) is rewritten as:

$$G_T(t) = \frac{\alpha+\beta}{\beta} \left(\frac{\dot{c}(t)}{c(t)} - G_N(t) \right) . \tag{31}$$

Above all, equations (23), (25), and (31) together describe the dynamics of the economy. The three equations can be reduced to the following two equations:

$$\frac{\dot{\overline{(1-u(t))}}}{(1-u(t))} = \frac{(\alpha+\beta)}{(\beta\,\theta+\alpha)} \left(\frac{(1-\alpha-\beta)\,(\alpha+\beta)}{\beta} G_N(t)T(t) + (1-\phi-\theta)G_N(t) - \rho \right) \tag{32}$$

and

$$\frac{\dot{G}_N(t)}{G_N(t)} = \frac{-(1-\alpha-\beta)\,(\alpha+\beta)^2}{(\beta\,\theta+\alpha)\,\beta}\,G_N(t)\,T(t)^2$$
$$-\frac{(\alpha+\beta)\,(1-\phi-\theta)}{(\beta\,\theta+\alpha)}\,G_N(t)\,T(t) + \frac{(\alpha+\beta)}{(\beta\,\theta+\alpha)}\rho\,T(t) + n - (1-\phi)\,G_N(t)\,. \tag{33}$$

Thus, we can analyse the dynamics of the economy by using the phase diagram in the $(T,\, G_N)$ space: since $T(t) = \frac{1-u(t)}{u(t)}$, T increases (decreases) when $(1-\dot{u}(t))$ is positive (negative). The $(1-\dot{u}(t)) = 0$ and $\dot{G}_N(t) = 0$ loci are given by:[5]

$$G_N(t) = \frac{\beta\,\rho}{(1-\alpha-\beta)\,(\alpha+\beta)\,T(t) + (1-\phi-\theta)\,\beta} \tag{34}$$

and

$$G_N(t)$$
$$= \frac{(\alpha+\beta)\,\beta\,\rho\,T(t) + (\beta\,\theta+\alpha)\,\beta\,n}{(1-\alpha-\beta)\,(\alpha+\beta)^2\,T(t)^2 + (1-\phi-\theta)\,(\alpha+\beta)\,\beta\,T(t) + (\beta\,\theta+\alpha)\,(1-\phi)\,\beta}\,. \tag{35}$$

Equations (34) and (35) represent the $(1-\dot{u}(t)) = 0$ locus and the $\dot{G}_N(t) = 0$ locus, respectively. Solving equations (34) and (35) gives the steady state values for T and G_N:[6]

$$T^* = \frac{\beta\,\left(\rho\,(1-\phi) + n(\theta+\phi-1)\right)}{n\,(1-\alpha-\beta)\,(\alpha+\beta)} \tag{36}$$

and

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[5] $(1-\dot{u}(t)) = 0$ is also satisfied if $u = 1$, that is along the vertical axis in Figure 1 and $\dot{G}_N = 0$ is also satisfied if $G_N = 0$, that is along the horizontal axis in Figure 1.

[6] We assume the transversality condition (18) is satisfied at the steady state. This implies that $n - \rho(1-\phi) - n(\theta+\phi-1) > 0$ is hold.

$$G_N^* = \frac{n}{1-\phi} \ . \tag{37}$$

In the steady state, y, c and N grow at the rate given by equation (37). T^* and G_N^* are both positive on the basis of our assumptions about parameters. Figure 1 shows the possible phase diagrams.

Figure 1 *here*

In Figure 1, the curves denoted by 1 show the $\dot{(1-u(t))} = 0$ locus and the curves denoted by 2 show the $\dot{G}_N(t) = 0$ locus. The $\dot{(1-u(t))} = 0$ and $\dot{G}_N(t) = 0$ loci intersect only once and both converge toward 0 as $T \rightarrow \infty$. The thick curves with arrows show the saddle path. There are possibly three kinds of phase diagrams: (a), (b) and (c). It depends upon the values of parameters. The important fact is that there exists a stable saddle-path towards the steady state in each phase diagram. Starting from a low level of $u(t)$ on the saddle path which corresponds to a high level of $T(t) = \frac{1-u(t)}{u(t)}$, both $u(t)$ and $G_N(t)$ monotonically increase towards their steady state levels. When the economy is not initially on the saddle path, it can take two kinds of dynamic paths. One is the path which eventually hit the vertical axis and the other is the path which asymptotically reach at the point where $u = 0$ and $G_N = 0$. The first violates Euler equation given by equation (16) and the latter violates the labour constraint. [7] Thus, the stable saddle-path towards the steady state is the only possible dynamic equilibrium in this model.

[7] see Appendix A for the proof.

Next, we consider the determination of the economy's starting point. Let $t = 0$ be the beginning of the planning period. $N(0)$ is historically given and $G_N(0)$ and $u(0)$ (i.e. $T(0)$) are not predetermined. From equation (24),

$$G_N(0) = \frac{1}{\eta} u(0) s N(0)^{\phi-1}, \tag{38}$$

where $L(0)$ is normalized to unity. With given $N(0)$, any pair of $G_N(0)$ and $u(0)$ which satisfies equation (38) describes the possible starting point of the economy. [8] By using equation (38), we can draw in the (T, G_N) space a locus which gives the possible starting points of the economy for a given value of $N(0)$. We call this locus the $N(0)$ locus.

Figure 2 *here*

A phase diagram in Figure 2 shows the $\dot{(1-u(t))} = 0$ and $\dot{G}_N(t) = 0$ loci with the $N(0)$ locus.[9] The $\dot{(1-u(t))} = 0$ locus and the $\dot{G}_N(t) = 0$ locus are denoted as 1-1 and 2-2, respectively. The solid curve and the dashed curve shows the saddle path and the $N(0)$ locus, respectively. Appendix B shows that there is at least a range of $N(0)$ which guarantees that the $N(0)$ locus intersects with a saddle path only once. We assume that $N(0)$ takes such a value in order to avoid the indeterminacy of the initial starting point of the economy. In Figure 2, the $N(0)$ locus intersects with the saddle path at point A with given $N(0)$.

[8] Equations (4.26) and (4.30) show that $c(0)$ is determined by $u(0)$ with given $N(0)$.

[9] Here, we use the phase diagram shown as (a) in Figure (1). The choice of the type of phase diagram is not important since all of the three phase diagrams in Figure (1) present the similar dynamics of the economy.

Since the economy is in equilibrium only on the saddle path, point A shows the economy's starting point. Note that $N(0)$ is historically determined so that N cannot change discontinuously in the event of any unexpected structural change, that is sudden decreases or increases in exogenous parameter values. All other variables would adjust discontinuously in the event of any unexpected structural change.

3 Skill Premium and Productivity Growth

There is one interesting finding about the post-war U.S. economy. After a sharp decrease in the beginning of the 1970's the U.S. skill premium has been increasing over time, and the share of R&D workers in the total labour force also fell sharply in the beginning of the 1970's and since then has been increasing back. The U.S. growth rate of GDP per worker also slowed dramatically in the 1970's and it partially recovered in the 1980's. Figure 3 shows the skill premium in the period between 1949 and 1996 and Figure 4 shows the share of R&D workers in the total labour force in the period between 1950 and 1992. Figure 3 is taken from Acemoglu (2000) and Figure 4 is from Jones (1995a). We can find a close relationship between the skill premium and the share of R&D workers. Table 1 shows the growth rate of GDP per worker. It seems that the growth rate of GDP per worker is also closely linked to the skill premium and the share of R&D workers.

Figure 3, Figure 4, and Table 1 here

In this section, we try to explain these findings by applying the transitional dynamics analysis presented in the previous section. The assumption taken here is that there is a sudden structural change which comes from an unexpected permanent increase in the cost of producing intermediate goods and new designs. In the model from the previous section, the production of one unit of intermediate goods incurs η units of forgone output, and the intermediate firm is required $\frac{\eta}{N(t)^{\phi}}$ units of high skilled labour to innovate one unit of new design. Thus, it is assumed that the structural change takes the form of an unexpected rise in η.[10]

We now analyse the effect of the unexpected rise in η by using the phase diagram shown in Figure 5.

Figure 5 *here*

In Figure 5, the curve 1-1 shows the $(1-\dot{u}(t)) = 0$ locus and the curve 2-2 shows the $\dot{G}_N(t) = 0$ locus. The thick curve with arrows shows the saddle path. Note that $G_N = \frac{\dot{N}}{N}$ and $T = \frac{1-u}{u}$. Assume that the economy is initially at point A on the saddle-path at time $t = 0$ where $G_N = G_N^{'}$ and $T = T^{'}$. The curve 3-3 shows the $N(0)$ locus. With given $N(0)$ (note N cannot change discontinuously at the time of shock), the unexpected increase in η leads to a downward shift in the $N(0)$ locus according to equation (38). The shifted $N(0)$ locus is shown by the curve 3'-3'. Notice that since η does not enter in equations (34) and (35),

[10] Perron (1997) argures that there is a break in the post-war quarterly real GDP (or GDP) time series for U.S. and other G7 countries. He found out that the break occured around in 1970.

the $(1-\dot{u}(t))=0$ and $\dot{G}_N(t)=0$ loci do not shift. Since the saddle path is an unique equilibrium, the economy jumps from point A to point B. At point B, $G_N = G_N^{''}$ and $T = T^{''}$. Thus, T jumps up from T' to T'' and G_N jumps down from G_N' to G_N'' at the time of unexpected increase in η. The increase in T implies the decrease in u. Since the skill premium is given by equation (12): $\frac{w_H}{w_L}(t) = \frac{\beta}{\alpha}\frac{1-s}{s}\frac{1}{1-u(t)}$, the decrease in u leads to a fall in the skill premium. The intuitive explanation is as follows. The structural change represented by the unexpected increase in the cost of producing the intermediate good and the new design first causes a decrease in the high skilled labour demand in R&D relative to the high skilled labour demand in the final goods sector. Since labour is supplied inelastically, the number of high skilled labour employed in R&D decreases. This, in turn, implies an increase in the number of high skilled labour available in the final goods sector. That is, the relative supply of high skilled labour in the final goods sector, $\frac{\overline{L}_{HY}}{\overline{L}_L}$, increases. As a result, the skill premium falls: it reflects the downward movement along the relative demand curve.

The behavior of c at the time of the shock is revealed by equations (26) and (30). By substituting equation (26) into equation (30), consumption per labour at $t=0$ can be given by:

$$
\begin{aligned}
c(0) &= (1-(1-\alpha-\beta)^2)y(0) \\
&= \Omega\,\eta^{\frac{-(1-\alpha-\beta)}{\alpha+\beta}}\,(1-u(0))^{\frac{\beta}{\alpha+\beta}}\,N(0)\,,
\end{aligned}
$$

where $\Omega = (1-(1-\alpha-\beta)^2)(1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}}(1-s)^{\frac{\alpha}{\alpha+\beta}} s^{\frac{\beta}{\alpha+\beta}}$. Thus, c can jumps up or down when there is an unexpected increase in η. It depends on how large the effect of the change in η on the change in $u(0)$ is. The discontinuous change in c does not imply the violation of the Euler equation given by equation (16) since the sudden fall or rise in c is the optimal response to the new information.

Since the economy is on the stable saddle-path towards the steady state at point B, T start decreasing towards T^*. This implies that u and $\frac{w_H}{w_L}$ start increasing back towards their steady state levels. The link between u and $\frac{w_H}{w_L}$ can be explained intuitively in the followings. The share of high skilled labour employed in R&D rises towards the steady state level. This implies that the relative supply of high skilled labour in the final goods sector decreases over time. Therefore, the economy gradually move up along the relative demand curve (note that since technology, N, is not skill biased the relative demand curve does not shift over time). As a result, the skill premium gradually increases towards the steady state level.

We now consider the dynamics of per labour output growth followed by the unexpected increase in η. From equation (26), we can write the growth rate of output per labour as:

$$\frac{\dot{y}(t)}{y(t)} = \frac{\beta}{\alpha+\beta} \frac{(1-\dot{u}(t))}{(1-u(t))} + G_N(t)\,. \tag{39}$$

Substituting equation (32) into equation (39), we can write the growth rate of output per labour as: [11]

[11] Substituting $T = \frac{1-u}{u}$ and equation (24) into equation (32) yields:

$$\frac{\dot{y}(t)}{y(t)} = \frac{(1-\alpha-\beta)\,(\alpha+\beta)}{\beta\,\theta+\alpha}\,\Lambda\,\frac{1}{\eta} + \frac{(\alpha+\beta)^2-\beta\,\phi}{\beta\,\theta+\alpha}\,\Lambda\,\frac{1}{\eta}\,u(t) - \frac{\beta}{\beta\,\theta+\alpha}\,\rho\,, \quad (40)$$

where $\Lambda = s\,e^{n\,t}N(t)^{\phi-1}$. Assuming $(\alpha+\beta)^2-\beta\,\phi \geq 0$, the unexpected rise in η and the resulting fall in u reduce the growth rate of output per labour. In Figure 5, the growth rate of output per labour at point B is, thus, less than at point A. Since the economy goes back towards the steady state after the initial shock (i.e. u increases back), output per labour starts growing faster (note that Λ increases over time since the growth rate of Λ is positive at any point on the saddle path where $T > T^*$).

Above all, the unexpected increase in η first reduces the share of R&D high skilled labour in the high skilled (and also total) labour population, the skill premium and growth rate of output per labour, and then they gradually rise back. Thus, the transitional dynamics of the model can explains the findings about the post-war U.S. economy described in the beginning of this section.

4 Conclusion

This paper analyses the transitional dynamics of the R&D based endogenous growth model. It shows that if the steady state exists the saddle path is stable and the stable saddle-path towards the steady state is the only possible dynamic

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$$\frac{\dot{(1-u(t))}}{(1-u(t))} = \frac{(\alpha+\beta)}{(\beta\,\theta+\alpha)}\left(\frac{(1-\alpha-\beta)\,(\alpha+\beta)}{\beta}\,\frac{s\,e^{n\,t}\,(1-u(t))}{\eta\,N(t)^{1-\phi}} + \frac{s\,e^{n\,t}\,u(t)}{\eta\,N(t)^{1-\phi}}(1-\phi-\theta) - \rho\right).$$

Equation (40) can be obtained by substituting this expression into equation (39).

equilibrium. The analysis of the transitional dynamics is then used as a tool to explain the observed comovements of three variables in the U.S. over the past several decades: the relative wage rate of high skilled labour, the share of R&D workers in the high skilled (and also total) labour population and per capita output growth rate show a sharp decline in the beginning of 1970s followed by a gradual increase. It is argued that the structural change, which is represented by the unexpected rise in the cost of R&D and production of intermediate goods in the beginning of the 1970s, first pushed the economy away from the steady state along the saddle path, and the economy, thereafter, have been moving back towards the steady state. Although the suspected structural change is not empirically identified, the bottom line is that R&D activities seem to be an important factor in explaining the recent U.S. economy's trend.

Appendix A

Assume that the economy is on the path which eventually hit the vertical axis in finite time in Figure 1. When it hit the vertical axis, $u = 1$. This implies $y = 0$ according to equation (26): all high skilled workers are employed in the R&D. Therefore, c must jump downward to 0 at this point. This violates Euler equation (16). Thus, the path can not be an equilibrium.

Next, we assume that the economy is on the path which asymptotically reach at the point where $u = 0$ and $G_N = 0$. If the economy is on this path, u and G_N will monotonically decrease after some point of time. By using equation (31), one can write the growth rate of $(1-u)$ as:

$$\frac{(1-\dot{u}(t))}{(1-u(t))} = \frac{(1-\alpha-\beta)(\alpha+\beta)^2}{(\beta\theta+\alpha)\beta} \frac{s\, e^{n\,t}(1-u(t))}{\eta\, N(t)^{1-\phi}} + \frac{(\alpha+\beta)(1-\phi-\theta)}{(\beta\theta+\alpha)} G_N(t) - \frac{(\alpha+\beta)}{(\beta\theta+\alpha)}\rho\,. \qquad (4A.1)$$

Thus, $\frac{(1-\dot{u}(t))}{(1-u(t))}$ will monotonically increase towards infinity after some point of time (i.e. $\lim_{t\to\infty} \frac{(1-\dot{u}(t))}{(1-u(t))} = \infty$ holds on this path). This violates the labour constraint ($0 \le 1-u \le 1$). Thus, the path can not be an equilibrium.

Above all, the saddle path in Figure 1 is an unique equilibrium in this model.

Appendix B

In order to show that there is at least a rage of $N(0)$ which guarantees that the $N(0)$ locus intersects with the saddle path only once, we first consider the $(1-\dot{u(t)})=0$ locus. Substituting equation (38) into equation (34) and solving for $u(0)$ yield:

$$u(0) = \frac{\beta\eta\rho - s\,(1-\alpha-\beta)(\alpha+\beta)N(0)^{\phi-1}}{s\,N(0)^{\phi-1}(\beta(1-\phi-\theta)-(1-\alpha-\beta)(\alpha+\beta))} \tag{B.1}$$

The denominator in equation B.1 is negative. We assume that $N(0)$ is sufficiently low to satisfy $u(0) > 0$ in equation B.1. Thus, the $N(0)$ locus intersects with the $(1-\dot{u(t)})=0$ locus only once at a point where $T(0) > 0$ and $G_N(0) > 0$ in Figure 2. To the left (right) of the intersection point, the $N(0)$ locus is below (above) the $(1-\dot{u(t)})=0$ locus.

Since the $(1-\dot{u(t)})=0$ locus also intersects with the saddle path only once at the steady state, there is a value of $N(0)$ which makes the $N(0)$ locus go through the steady state. We define this value as $N(0)^*$. Thus, the $N(0)$ locus and the saddle path have only one intersection point when $N(0) = N(0)^*$. This implies that at least when $N(0)$ is in the neighborhood of $N(0)^*$ the $N(0)$ locus intersects with the saddle path only once.

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	Growth rate of GDP per worker
1960-1970	2.2 % per year
1970-1980	0.4 % per year
1980-1990	1.5 % per year

Table 1: U.S. Productivity Growth Slowdown. Source: Jones (1998, Table 2.1)

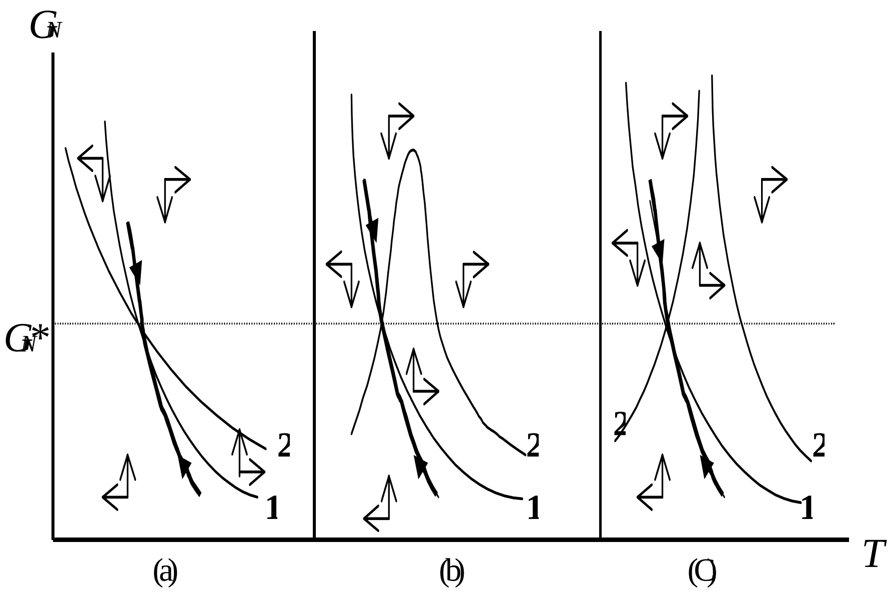

Figure 1: Phase diagrams

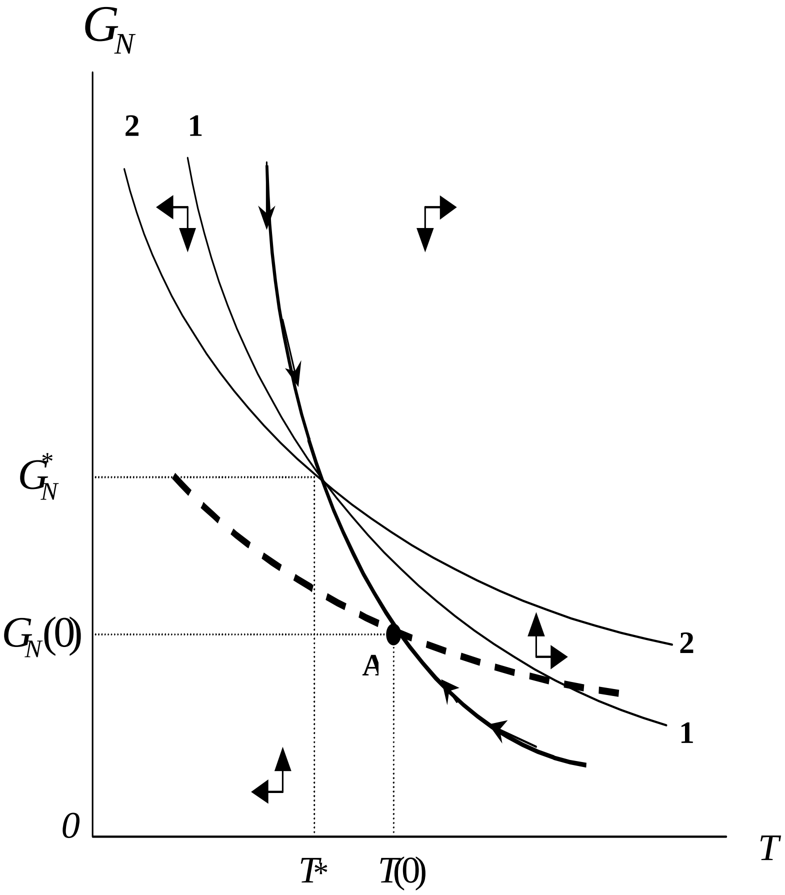

Figure 2: $N(0)$ locus

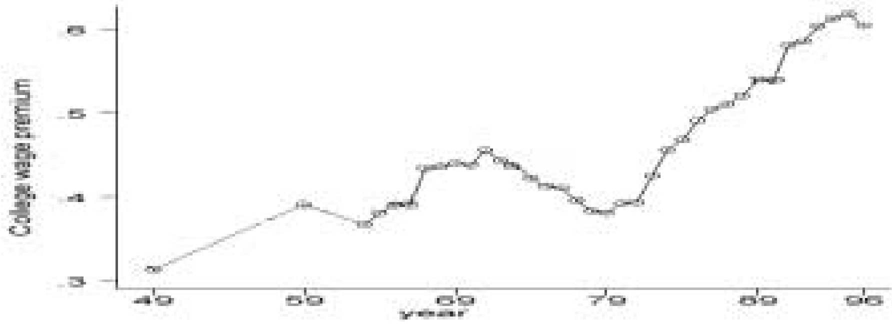

Figure 3: U.S. skill premium (log form)

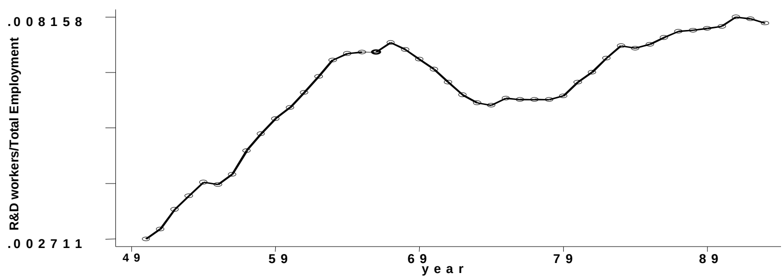

Figure 4: U.S. R&D worker (share in the total employment)

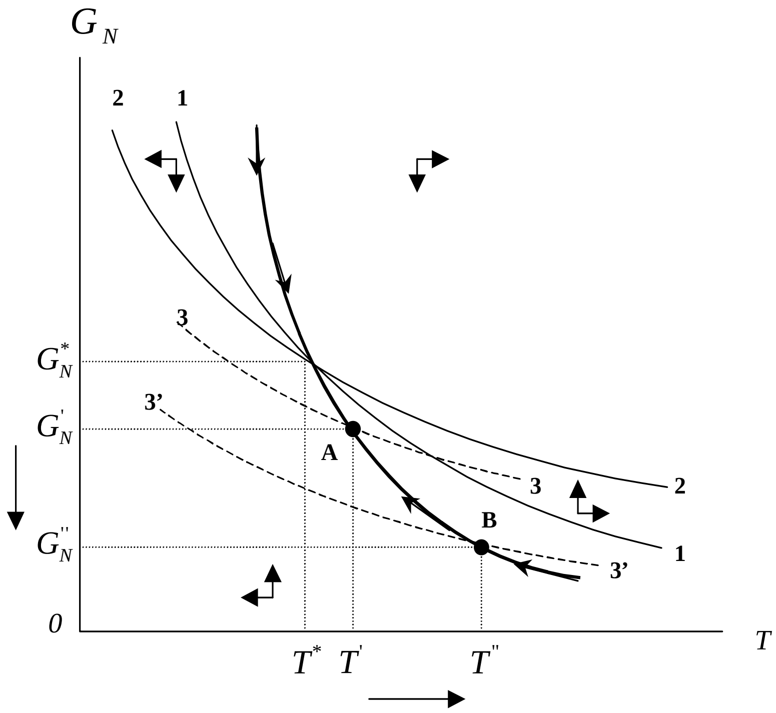

Figure 5: Dynamic response to the unexpected increase in η