TANK GUNNERY PREDICTION SYSTEMS

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by

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This thesis is concerned with fire control prediction schemes for tanks employed in a defensive role against moving targets. The problem is considered in three parts: the determination of likely target movement patterns in an operational setting; the assessment and modelling of human operator response to those motions; and the utilisation of this response in optimal prediction schemes. In the first part the results from war games, tactical exercises and field trials are collated, and a method is devised for generating test target tracks for human operator study and prediction scheme evaluation. In the second part previous approaches to operator modelling are reviewed, laboratory experiments are described and a mathematical model of human response is developed. In the third part the general statistical properties of predictors are examined, a new class of predictive algorithm called the 'threshold' algorithm is devised, and this type of algorithm is then evaluated using the results of the previous two parts. The thesis ends with some consideration of further research requirements or possibilities, and of the steps needed to validate the results obtained so far.

CONTENTS

1. INTRODUCTION

Since its introduction in the 1914-18 war the tank has come to assume a central role in the conventional land battle. There has been a steady stream of technical improvements in the areas of mobility, protection and firepower, but all these changes have been more evolutionary than revolutionary. In particular the essentials of gun aiming and prediction have changed but little until recently, when the advent of the laser range finder (preceded by the optical range finder) and of compact and rugged digital computers (preceded by mechanical cam-based computers) has brought about a minor revolution (see, eg Ogorkiewicz, 1976).

The tank gun is still the preferred weapon for defence against other tanks. Its main advantage over the guided missile, which is its chief competitor in this role, lies in the fact that kinetic energy is more potent in the defeat of modern armour than is chemical energy. To reconcile very high missile velocity and mass with adequate in-flight guidance is a daunting technical challenge, and so the conventional anti-tank guided weapon relies on a shaped - charge warhead which acts by producing a penetrative jet of metal. The effectiveness of such a jet is much dissipated by modern spaced armour. Other disadvantages of the guided missile are the relative cost and complexity of the round, and (unless a so-called "fire-and-forget" missile is developed) the need for the firer to maintain line-of-sight to his target (and the involvement of the gunner) until impact, which in turn restricts the rate of fire. There are, of course, some offsetting advantages of the guided missile, especially its portability, its comparative accuracy at long range, and its ability to follow a manoeuvring target.

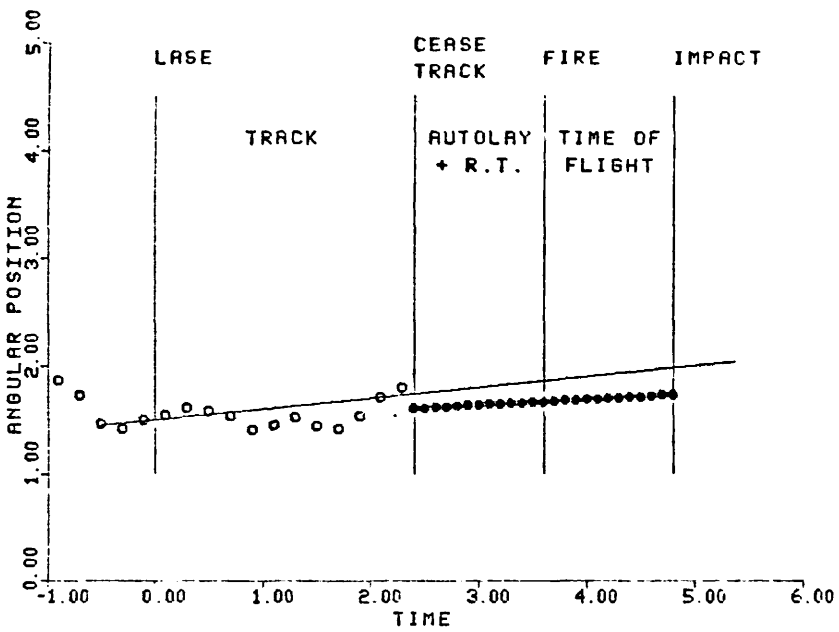
While the tank gun still reigns supreme in the context of defensive armoured warfare, the human gunner also retains his responsibility for aiming the gun and firing it. Given that a tank target is typically seen against a complex background, often with intermittent obscuration or partial masking, the human operator has many advantages over the automated aiming device. His resolving and discriminative powers are considerable. Although his effectiveness may be degraded to some extent by the stresses of the battlefield environment, compared to many complex automatic devices he is extremely reliable. But above all he is flexible and innovative, reacting intelligently to novel situations, including those which may be deliberately deployed against him in the form of enemy countermeasures. This having been said, it remains true that even in modern tank gunnery systems (at least with targets moving at a more or less steady rate) the gunner is the dominant source of system error or inaccuracy. It follows that our best chance of improving the system must lie in obtaining a thorough understanding of the properties of the gunner in his aiming role, and, armed with this knowledge, in devising intelligent schemes for utilising his input. This is the theme running through this thesis.

Apart from actual laying or tracking error, the main sources of inaccuracy introduced by the gunner are due to his inability to make direct estimates of target range or rates with precision (Harrison & Price, 1944; EASAMS, 1977). This in turn precludes accurate allowance for the fall of shot or lead-angle compensation for target motion. It is in these areas that the laser range finder and the digital computer have made their impact. The laser range finder is accurate to something like 5m (standard deviation) at ranges up to about 10km; and the computer can be used to store the gunner's tracking output over a period of time, and then, employing some smoothing algorithm, to produce a reasonably accurate estimate of target rates. Given estimates of other parameters which affect the likely flight of the shell (such as barrel wear, charge temperature and so forth, which will be spelled out in more detail in Chapter 6) the computer can then solve the ballistic and lead angle equations to lay the gun off with precision. Let us, then, run through the sequence of events in a normal engagement against a moving target, using as an example the Improved Fire Control System (IFCS) fitted to the Chieftain main battle tank (see Schreir, 1976).

Responsibility for target selection in an IFCS-like system rests with the tank commander. Having decided what target to engage and the type of ammunition to be used, he relays his decision to the gunner and passes tracking control to him. Looking through the right hand eyepiece of his sight the gunner lays his reference mark (accurately collimated with the gun barrel, as is the laser range finder) on the target and presses the "lase" button. An aiming ellipse is then injected electronically into his sight, centred on his reference mark and scaled in accordance with the laser-indicated range so that it should neatly enclose a broadside-on tank target. Provided that this visual evidence of range is credible the gunner continues tracking. When he decides that his tracking has been accurate for a sufficient length of time to give the computer a reasonable indication of target rate he may press his "autolay" button. No further tracking information is accepted after the start of "autolay". The engagement proceeds more or less automatically, and (except for firing) the gunner intervenes only in the case of evident malfunction. During "autolay" ballistic computations proceed; the aim-off required to allow for target motion is calculated; the turret continues to rotate at the set-in rate plus those needed to aim off the gun; and the gun is elevated appropriately. When the process is complete, and provided that the gunner has tracked accurately and the target has proceeded at a steady rate, the aiming ellipse should once more be centred on the target. If the ellipse is accurately centred the gunner may fire; if it is not he may, by pressing an appropriate button, assume control of the aiming ellipse once more, place it over the target, and then fire the gun.

A nominal engagement against a steady-rate target is illustrated schematicallyin Figure 1a. The computer stores the gunner's input during the "tracking" interval, and during the "autolay" period works out a prediction of target position at impact. The basis of prediction is a first order (unweighted) least squares fit to the sight position, as controlled by the gunner, over the "tracking" interval, yielding estimates of target position and velocity, and, were it not for the man's

a



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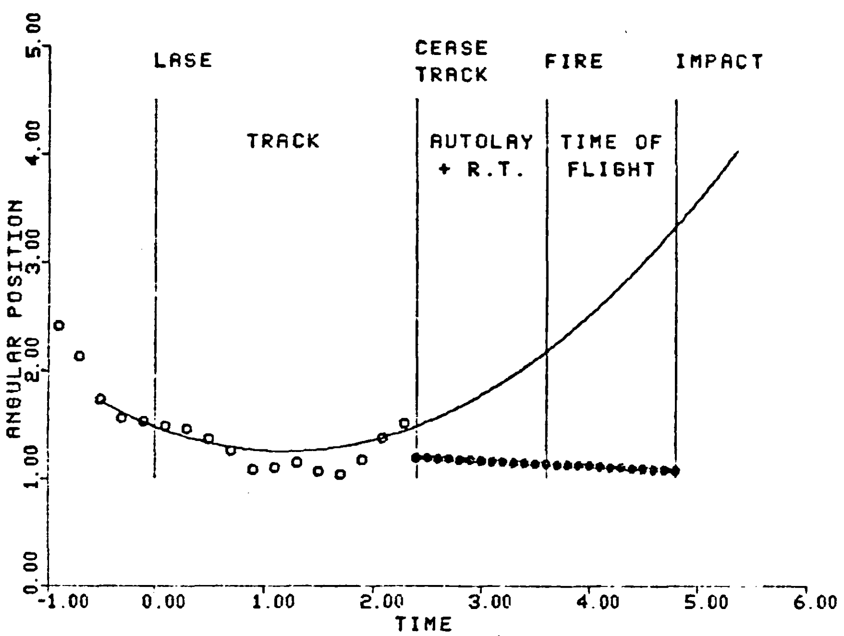

FIGURE 1. Schematic engagements for constant velocity target (a) and accelerating target (b). The fire control computer accepts sampled gunner tracking inputs (hollow circles) during the 'track' interval. Computations are carried out and the gun laid off in accord with the prediction equation (filled circles) during 'autolay', and a signal is then passed to the gunner who will fire with some reaction time delay. The time of flight of the shell then determines the nominal impact time.

tracking error and some slight system error, it would be exact. Figure 1b illustrates an engagement against an accelerating target. Clearly, the average target velocity over the "tracking" interval is not the same as that subsequent to that period, and so predictions made on this same basis are bound to be biased. When the "autolay" period is ended the gunner will face an unenviable task. He will see that the computer prediction is in error, but will find it difficult to judge whether this is due to initial quirks in his tracking or to variation in target rates. If he elects to switch back to "track" to take out his gross errors he runs the risk of the stored computer velocity estimate becoming even more out of date and inappropriate.

In essence, then, the problem of tank fire control system improvement is that of assessing system biases (introduced largely through target motion and systematic gunner response to that motion) and random variability (introduced largely through random gunner tracking response) and, then, by careful design of the predictive scheme used, of balancing and weighting these contributions to system error in such a way as to maximise overall effectiveness. These considerations were first set out by the author in a previous report (Speight, 1976a) where it was suggested that a reasonable programme of research to address the problem might contain the following strands:

a. Establish the characteristics of likely target motion and of exposure in operational conditions.

b. Describe in a statistical sense the human response to different target motions.

c. Via the construction of computer models utilising (a) and (b) above, devise and test different prediction algorithms and engagement routines.

d. Evaluate selected algorithms, in the first instance by using laboratory data, but then validating them under field conditions.

Accordingly, the first section (Chapter 2) of this thesis examines the available evidence on target motions and probable engagement geometry. In the second section (Chapters 3 to 5) previous research on human tracking characteristics is reviewed; laboratory experiments are described, a computer model of gunner response is derived; and possible field effects are considered. The third section of the thesis (Chapters 6 and 7) makes a start on the formulation and evaluation of predictive schemes which hold out some promise of improvement over the straightforward first order predictor. The programme of work does not extend to field evaluation, but some indication is given in Chapter 8 of ways in which research should be extended and validated.

2. TARGET MOTION CHARACTERISTICS AND ENGAGEMENT GEOMETRY

2.1 Introduction

The chief concern of this study is with fire control systems for tanks employed in a defensive role against attacking armour. The main emphasis, then, is given to the static tank firing against moving targets. It is assumed that next in priority is the static tank firing at stationary targets (particularly as some of the attacking force may engage from a short halt or from fixed fire positions). Firing on the move against moving targets is taken to be next in importance; and the engagement of static targets whilst on the move is accorded the lowest priority of all.

Any analysis of the effectiveness of present or potential fire control systems must start by establishing the relevant properties of the engagement situations which the systems must be designed to combat. Given the order of priorities outlined above, this implies that our first task must be to describe in some way the manner in which potential targets are likely to move on the battlefield. In order to translate target motions, specified in a geographical frame of reference, into the full prediction problem as seen by the defending tank, we also need details of what we shall call the 'engagement geometry' - the range to the target; the angle of the firer relative to the target's direction of travel; the height difference between them; and the angle of tilt of the firing platform itself. All of the foregoing will depend on tactics, conditioned by terrain, vehicle suspension and power/weight ratios; weapon capabilities; and so forth. Only the terrain (assuming a preoccupation with one theatre of operations) will remain more or less constant. All the other factors evolve with time. Accordingly, although operational researchers have for some years past been concerned with describing and quantifying important features of armoured warfare (see, eg, Gee, 1952) we would be wise to concentrate our attention on more recent studies, designed to predict and assess the characteristics of tank engagements for the decade or two ahead. Since many of these studies provide information on more than one aspect of engagement geometries, it will be as well to describe the more important ones at the outset rather than introducing them piece-meal as the results are collated.

This chapter starts with a very brief account of the games, exercises and field trials which have been used as the main sources of evidence about likely features of future tank engagements. The results of these studies are then combined with those of such other investigations as may be relevant to obtain our current best estimates of target motion characteristics and of engagement geometry. Conclusions are then drawn as to the most appropriate test situation features for assessing gunner response and for evaluating potential fire control systems. Finally, a description is given of the actual procedures used to generate such test situations.

2.2 Principal sources of evidence concerning tank engagement parameters

The data from low level tactical war games form one source of evidence concerning possible future engagement parameters. The Royal Armament Research and Development Establishment (RARDE) War Game 1 is a particular example played on conventional lines (Beresford, 1968a, 1968b). It consists of a central control room and two subsidiary rooms for the opposing Commanders, each room containing a relief model of the area in which the battle is to take place. The Commanders have complete and detailed information on the disposition of their own forces, and gather increasing information on the disposition of the enemy forces as games progress.

The relief models are accurately scaled in plan and elevation, the scaling being large enough to permit the placing of individual vehicles (normally to a resolution of 100m, but finer than this when, for instance, a tank is to take up a hull-down position behind a crest). The time resolution is typically 30 sec, but it may be increased to 1 sec when particularly detailed features of engagements are being examined. In playing any game special attention is paid to intervisibility aspects as governed by terrain, vegetation, buildings, smoke and the like.

Obviously, the further one moves from real battle and the more one abstracts the fine detail the greater is the chance of failing to represent accurately some significant effect of the true environment.

However, the war game does give one the chance to examine the interacting effects of terrain tactics, command decisions, enemy reaction and the characteristics of the weapon systems deployed. In particular war games are not restricted by peace time limitations concerning the use of terrain or terrain features; and, provided that reasonable estimates are available of likely system performance, they are not limited to presently available weapons. Accordingly, data from RARDE War Games have been given some weight in arriving at our best estimates of the likely features of tank engagements.

While war games can be used, among other things, to study the tactical movement and deployment of individual vehicles (as judged by a would-be Commander with complete oversight of his own forces) they are unlikely to reflect the influence of all the detailed terrain features (such as natural obstacles, varied going and the like) which may condition real vehicle movement on the ground. An assessment of such features requires physical inspection of the terrain in question. A complementary source of evidence is thus the TEWT(Tactical Exercise Without Troops) followed by eye witness examination. In Exercise PENANCE (Rowland, Dove & Thornton, 1976) a group of experienced Royal Armoured Corps officers were briefed on the probable characteristics of future NATO and Warsaw Pact main battle tanks, tactics and organisation. Four different map areas were selected, and the officers were asked to make detailed map deployments of combat teams in the setting of twelve possible tactical scenarios. For each nominal deployment map traces and data record sheets were produced, giving details of individual routes used by all tanks in the combat team; the section of each route during which a tank was assumed to be exposed to enemy fire, with an indication of the direction and range from which the fire was likely to come; the fire positions of all tanks; the chronological pattern of movement; and comments on likely speeds as influenced by tactical considerations. The map exercise was termed Exercise PENANCE I. While maps are normally used operationally to plan deployments and movements, the detailed execution of the latter depends on terrain features possibly not discernable on maps. Accordingly in Exercise PENANCE II a sample of the routes from PENANCE I was visited by teams in Landrovers. These teams (walking if the presence of fields or heavy going necessitated it) travelled the routes inspecting and recording details of obstacles, defiles, surface cover and other factors affecting probable speed.

While neither of the studies outlined so far were constrained to the use of present weapons and vehicles, both stopped some way short of real-time interaction of trained crews and officers with a supposed enemy, subject to all the constraints of the real environment. The nearest one can approach real battle conditions in a peace time setting is the full field exercise. Exercise CHINESE EYE III was such a field exercise, with elaborate provision for data acquisition, designed to yield information from realistic set piece situations against a Warsaw Pact type of attack. A tactical account is given by Rowland & Weeks (1976). Exercise CHINESE EYE III has been taken as the main source of evidence on all aspects of likely tank engagements except those pertaining to the fine (second-to-second) detail of target motions.

In Exercise CHINESE EYE III a BLUE combat team (consisting of two four tank troops, with anti-tank guided weapon and mechanised infantry support) fought twenty separate defensive actions against an attacking ORANGE tank battalion (consisting of three companies of ten tanks each, plus a headquarters group). The BLUE forces followed current 1(BR) Corps tactics and the ORANGE force, after considerable briefing, attempted to follow current Warsaw Pact doctrine. The effects of indirect fire weapons (such as artillery) were not reproduced, but the effects of direct fire weapons (tank guns and GW) were simulated with SIMFIRE equipments. These latter mimicked both the flash and smoke of firing, and the effects of a putative 'hit'. The twenty actions (including one practice engagement) covered a variety of set-piece scenarios, meteorological conditions and terrains. Visibilities varied from about 1km up to 10km, and terrains varied from the flat farmland of Scenario 1, through gradually more rolling country, to the valleys and wooded hills of Scenario 19.

Exercise CHINESE EYE III has yielded a host of descriptive statistics and insights on many aspects of tank engagements. Originally it was hoped that it might also yield information on the second-to-second motion of at least one attacking tank in realistic tactical settings (constrained of course by the mobility characteristics of the particular vehicle chosen). High resolution accelerometers were fitted to one ORANGE tank (Pymm & Yates,1976) but in the event the reduction of the recorded data from these accelerometers proved to be beset with difficulties. However, quasi-tactical information on target motion has been obtained from American sources. Realising the need for fine-grain estimates the US Army Materiel Systems Analysis Activity (AMSAA) mounted a series of free-play exercises with three different kinds of military vehicle. During these exercises drivers were instructed to move from one point to another, employing at their discretion evasive tactical manoeuvres compatible with the terrain and with vehicle mobility characteristics (Brown, 1977 and Myers, 1978). Drivers were not under the direction of their commanders and faced no simulated enemy. The trial was held in an instrumented and surveyed range area (with rather flat terrain) and the outputs of several tracking radars were combined to give estimates of target position at intervals of roughly 0.1 sec.

These, then, are the main sources of information about the likely features of future tank engagements. There have of course been other studies which have yielded useful data, and there have been many authors who have contributed to our present understanding of tank warfare.

2.3 Target movement

Central to the fire control prediction problem is the way that tanks may move operationally. The whole engagement sequence from lase to nominal impact takes something like 5 sec, and so we must consider variations in velocity in this sort of time frame. Surprisingly, there is little hard information on this score.

We can set the general scene in which attacking tanks may move by considering past studies. Exercise PENANCE (Rowland, Dove & Thornton, 1976) was directed towards establishing likely movement patterns for a proposed NATO main battle tank with advanced suspension characteristics. If we take this as a model for Warsaw Pact behaviour (although, as will be mentioned later, different tactical scenarios will doubtless yield very different sorts of movement patterns) we have a picture of tanks moving between fire positions, or fire positions and hides, with these 'bounds' averaging some 3.1 to 3.4km in length. The route will scarcely be in a straight line (as a ratio of movement distance to crows-flight distance of 1:1.3 shows) but roughly 50% could be on roads. Although it might be the aim of commanders to cover the majority of each bound at top speed (taken at some 55kph) they could well only be able to do so for roughly half the time. The remainder of the time might be spent in slowing or stopping for obstacles and into fire positions or in accelerating away from these. The median inter-obstacle distance could be of the order of 330 to 500m. At the time that Exercise PENANCE was conducted there was considerable interest in the possibility of employing a high speed weave to provide protection. The authors concluded that in rough figures 20% of the exposed route would consist of an avenue of approach too narrow for weaving; a further 30% would have sufficient width, but the tank would not be able to travel at full speed; leaving 50% suitable for the employment of this manoeuvre.

Exercise CHINESE EYE did yield some data on gross movement patterns. During the advance to contact the (crows-flight) movement rates of the attacking force averaged 3.4m/s with a standard deviation of 1.2m/s. In contact the corresponding figures were 3.2m/s and 1.4m/s (Rowland & Weeks, 1977; Rowland, Weeks & Attoe 1977). These surprisingly low figures were perhaps conditioned by the use of CHIEFTAIN tanks, which are not especially mobile. In any case, more important than actual velocity in our context is the second-to-second pattern of acceleration. As stated in the previous section, an attempt was made to obtain this sort of information by mounting very accurate accelerometers on one of the attacking tanks (Pymm & Yates, 1976). There were very considerable problems of data reduction, which seemed to be caused by the use of very small-scale ultraviolet traces as the recording medium. Reducing these traces by hand introduced considerable random error, which, compounded by inevitable recording amplifier biases and drifts, made it impossible to reconstruct target motions which would even approximately agree with those obtained from independent sources, such as aerial photography, or which would seem to be feasible on vehicle performance grounds. The Studies Branch of the Military Vehicles and Engineering Establishment devoted considerable effort to the analysis of these data, but all efforts were finally abandoned.

Intelligence sources have been consulted to obtain some rough guidelines on likely patterns of enemy movement. Roughly speaking, we may distinguish three rather different cases. Firstly, we have the formal set-piece massed attack. Here there is little room for evasive manoeuvre. Such manoeuvre as there is could be caused by obstacles, varied going, and the need to jockey to maintain relative positions within the attacking formation. Secondly, we could have a break-through scenario, where the main concern of the attacking force would be to move with maximum speed to exploit the tactical situation, and little attention should be paid to evasion. Lastly, we have the small scale or probing action (of which CHINESE EYE may be considered to be an example). The smaller the force the more will be the tendency to move with evasion and concealment in mind. In Exercise CHINESE EYE it was estimated that 6% of the attacking force's shots were fired from stationary positions, some 25% from a short halt, and the remainder on the move (Begley & Rowland, 1977). In this type of engagement, then, there will be a certain amount of deceleration and acceleration, if only to fire at the opposition.

Obviously, a wide range of movement patterns is possible. Neither is the picture necessarily a fixed one - presumably a tactic which results in inordinate losses will be a candidate for abandonment or revision. In view of the possible range of tactics it seems safest to evaluate present and proposed prediction schemes at the two extremes of possible manoeuvre. One extreme would consist of the stationary target and targets moving in strictly straight lines (the latter an ideal almost never approached in practice). The other would consist of targets employing the maximum degree of manoeuvre possible. There is still the problem of estimating what this maximum manoeuvre might entail with modern tank technology. As previously stated, hard data is difficult to come by, and so reliance has been placed on the US AMSAA tactical manoeuvre exercise.

Three vehicles took part in the AMSAA trial: an M60 tank; an XM800 Scout vehicle; and the Lockheed Twister high mobility vehicle (whose development has been discontinued, but which is wheeled and which has an articulated body pivoted about a central point). There are several problems attendant upon the use of these data:

(a) There was no simulated enemy present, and although we know in general terms that the drivers were asked to move from one point to another in evasive fashion, we have no information on the detailed instructions.

(b) The accelerations of the target will depend in part on the mobility characteristics of the vehicle concerned, and those actually deployed in the trial may not approximate future Warsaw Pact vehicles very closely. The power/weight ratio for the M60 and the T-72 Russian tank (estimated in the latter case, although the claimed top speed would indicate a much higher figure) are similar at 17bhp/ton. The next generation of tank will in all probability have a higher power/weight ratio, and advances in suspension are to be expected. These aspects affect the way in which a fighting vehicle may interact with the terrain. Large power/weight ratios permit high accelerations, but also mean that a tank is less impeded by heavy going or slowed by turning; and an advanced suspension could mean that the tank has less need to slow for obstacles.

(c) It has not been possible to secure any detailed information on the nature of the trial terrain. Inspection of the target position data reveals that the topography was very flat, and the fact that the trial took place in an instrumented range area suggests that there were very few natural or man-made obstacles.

(d) The form of the data themselves is not ideal:

(i) Position information at roughly 0.1 sec intervals was obtained in the horizontal plane by the use of a number of range radars. Radar outputs were passed through Kalman filters (whose properties are described by Kalman, 1960, and Kalman & Bucy, 1961), and the three 'best' (the three, presumably, which yielded the smallest error triangle) were combined to yield the final estimate. There is still considerable high frequency noise in this estimate; the substitution of new radars into the 'best' three produced occasional indicated position jumps of several metres (and the data for the Scout vehicle were not employed because of the frequency with which these jumps occurred); and the Kalman filtering to reduce the effects of high frequency noise (incorporating as it must some unknown model of target motion) must have introduced a certain amount of bias (particularly at points where the target changed manoeuvre and the target motion model was thus inappropriate).

(ii) Position information in the vertical plane was obtained not from radar information, but by reference to a map of this carefully-surveyed area. This means that the effects of minor terrain features were not included. The CHINESE EYE accelerometer records reveal appreciable accelerations in the vertical plane, and the US Army Human Engineering Laboratory 'HEL AST' field trial also indicated very considerable vertical accelerations (Eckles, Garry, Mullen & Aschenbrenner, 1973). HEL is colocated with AMSAA, and probably used the same test area.

For all their deficiencies, the AMSAA results are the highest-quality set of data available, and they have been utilised in this study. However, the conversion of raw motion data into the full prediction problem as seen by the defender requires estimates of other parameters and these aspects will be addressed in the sections which follow.

2.4 Target exposure times

Pressure of time is part and parcel of every operational engagement. In a duel it is, of course, the tank which fires the first shot which survives, other things being equal. Although a high first shot probability is a necessary condition for system effectiveness, this cannot be achieved at the expense of a long aiming, prediction and firing sequence. This latter point is reinforced by the fact that a moving tank target will be obscured intermittently, and so the longer the time taken by this sequence the higher will be the probability that the target is no longer exposed at the nominal instant of impact.

It is not our intention in this study to investigate in any detail the effect of altering the length of data-gathering interval in the engagement sequence, although a complete analysis of the fire control problem would require a careful consideration of the trade-offs between time, hit probability and probability of being hit. The matter of exposure times will not, therefore, be dwelt on at any length. This topic has been addressed by Finlay & Thornton (1972), who reviewed the results of a number of field trials, field exercises, war games and the prediction of terrain models to arrive at a general formula applicable to Northern Germany. They defined two kinds of exposure - 'unshortened' exposures, which commence with the attacker emerging from behind a concealing object and end with its disappearance behind another (or the same) concealing object; and 'shortened' exposures, which start in the same manner but end because the attacker overruns the defending position. If one regards obscuration as being due to the presence of concealing objects which are randomly placed in space with roughly constant mean inter-obstacle distances, then exposure distances correspond to the waiting times in renewal theory or the Poisson process model (see eg Parzen, 1962) and should be distributed in negative exponential fashion. In fact none of the studies examined by Finlay & Thornton yielded data that conformed too closely to this law, and there was immense variation from study to study. However, in the absence of any convincing theoretical alternative they adopted as a general formula for the probability distribution of 'unshortened' exposure lengths at long range:

$$p\,(r)\,d\,r = \frac{1}{588}\,\exp\left(-\frac{r}{588}\right)\,d\,r \qquad (1)$$

where r is the exposure length (in metres) and 588m is the mean inter-obstacle distance.

For 'shortened' exposures Finlay & Thornton accepted the form of probability distribution for distances to the first obstacle put forward by Hardison, Peterson & Benvenuto (1953);

$$g(r)\,d\,r = \frac{2\,r}{\bar{r}} \exp\left(-\frac{2\,r}{\bar{r}}\right) d\,r \qquad (2)$$

where r is again the exposure length, and $\bar{r}$ the mean distance to the first obstacle (given a value of 1095m by Finlay & Thornton). This is the gamma distribution with parameters $(2, \frac{2}{\bar{r}})$ which would, with a Poisson process, be the distribution of distances to the second event, assuming a mean inter-obstacle distance of $\frac{\bar{r}}{2}$. This form of distribution was first obtained as an empirical fit to World War II engagement range data by Peterson (1951). His application of the formula to first obstacle distances and the accompanying theoretical justification will not be dwelt on here, but will be touched on when engagement ranges are considered in section 2.5 below.

Finlay & Thornton's final formula for exposure distance (and hence, assuming a given vehicle velocity, of time) combined those for 'unshortened' and'shortened' exposure distances. The formula for 'shortened' exposures is as already shown. That for'unshortened' exposures must be modified to allow for the fact that the position of the first obstacle will curtail the length that an 'unshortened' exposure can have. The probability distribution of 'unshortened' exposures as a function of range is then given by:

$$p(r, R)\,d\,r = C\,p(r)\,d\,r \left\{ 1 - \int_{R}^{R-r} g(r)\,d\,r \right\} \qquad (3)$$

where C is a normalising constant, R is the range in metres, and the other symbols have the meanings already assigned to them.

That the negative exponential provides a reasonable description of exposure length data is suggested by Exercise PENANCE analysis (Rowland, Dove & Thornton, 1976), and by the CHINESE HORIZON exercises which followed CHINESE EYE (Stead and Rowland, 1977). In CHINESE HORIZON a selection of individual attack routes and defending positions from CHINESE EYE were examined in detail by observers on the ground, in both summer and winter. The mean exposure lengths were very scenario dependent, varying from 173 to 654m in summer and 247 to 1387m in winter, but all individual distributions approximated to the moving tank target will be obscured intermittently, and so the longer the time taken by this sequence the higher will be the probability that the target is no longer exposed at the nominal instant of impact.

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where r is the exposure length (in metres) and 588m is the mean inter-obstacle distance.

For 'shortened' exposures Finlay & Thornton accepted the form of probability distribution for distances to the first obstacle put forward by Hardison, Peterson & Benvenuto (1953);

$$g(r)\,dr = \frac{2r}{\bar{r}} \exp\left(-\frac{2r}{\bar{r}}\right) dr \qquad (2)$$

where r is again the exposure length, and $\bar{r}$ the mean distance to the first obstacle (given a value of 1095m by Finlay & Thornton). This is the gamma distribution with parameters $(2, \frac{2}{\bar{r}})$ which would, with a Poisson process, be the distribution of distances to the second event, assuming a mean inter-obstacle distance of $\frac{\bar{r}}{2}$. This form of distribution was first obtained as an empirical fit to World War II engagement range data by Peterson (1951). His application of the formula to first obstacle distances and the accompanying theoretical justification will not be dwelt on here, but will be touched on when engagement ranges are considered in section 2.5 below.

Finlay & Thornton's final formula for exposure distance (and hence, assuming a given vehicle velocity, of time) combined those for 'unshortened' and'shortened' exposure distances. The formula for 'shortened' exposures is as already shown. That for'unshortened' exposures must be modified to allow for the fact that the position of the first obstacle will curtail the length that an 'unshortened' exposure can have. The probability distribution of 'unshortened' exposures as a function of range is then given by:

$$p(r, R)\,dr = C\,p(r)\,dr \left\{ 1 - \int_{R}^{R-r} g(r)\,dr \right\} \qquad (3)$$

where C is a normalising constant, R is the range in metres, and the other symbols have the meanings already assigned to them.

That the negative exponential provides a reasonable description of exposure length data is suggested by Exercise PENANCE analysis (Rowland, Dove & Thornton, 1976), and by the CHINESE HORIZON exercises which followed CHINESE EYE (Stead and Rowland, 1977). In CHINESE HORIZON a selection of individual attack routes and defending positions from CHINESE EYE were examined in detail by observers on the ground, in both summer and winter. The mean exposure lengths were very scenario dependent, varying from 173 to 654m in summer and 247 to 1387m in winter, but all individual distributions approximated to the negative exponential shape. The data from these later studies thus confirm Finlay & Thornton's formula for exposure distances as a reasonable overall approximation.

2.5 Ranges of engagement

World War II casualty data have been examined by various authors (Benn & Shephard, 1951; Peterson, 1951; and Gee, 1952) to estimate the distribution of tank engagement ranges. In particular, Peterson (1951) examined the statistics of 556 British and US tank casualties in N W Europe, obtained an empirical fit for the gamma distribution discussed in the previous section, and attempted to give this distribution a theoretical justification. He did this by assuming that obstacles to intervisibility are distributed randomly in space, and are well described as a Poisson process. It was then assumed that the bulk of engagements occur when the attacker passes from behind the obscuration afforded by the nearest obstacle to the defender. The distribution of this distance to the nearest obstacle, assuming that the defender is co-located with another obstacle, is obtained by the argument that line of sight from the defender to the attacker must imply that an intermediate observer can plainly view both. Assuming a mean inter-obstacle range of $\bar{r}$ then the distance from defender to attacker is obtained by summing the two independent ranges from the observer to both attacker and defender with their attendant obstacles, and it then follows that this distance will be distributed as a gamma variate with parameters $(2, \frac{1}{\bar{r}})$. There are however some difficulties with this formulation:

(a) Terrain can vary from 'open' to 'close', and so, even if in one area the obstacles to visibility are distributed in Poissonian fashion, the aggregration of results from a number of areas would yield a compound Poisson, rather than a simple Poisson distribution. It is noteworthy that none of the results reviewed by Finlay & Thornton (1972) followed the negative exponential pattern very closely. In general the frequency of very short exposures was less than expected, although it was pointed out that this could be due to the methods used to quantify exposure lengths in the first place.

(b) Irrespective of the presence of an intermediate observer, the distance from one event to the next in a true Poisson process is distributed in negative exponential fashion (that is to say, as a gamma variate with parameters $(1, \frac{1}{r})$ see, eg, Parzen, 1962).

(c) An attacker will not move at random through a random terrain. If the attacker does not have precise knowledge of the whereabouts of the defender this may not alter the form of exposure distribution to any appreciable extent (simply moving the attacker instead into areas of greater average obstacle density). However that may be, the defender's position is presumably chosen very deliberately in non-random fashion, so that there are well-above average distances to the nearest inter-visibility obstacles in the direction where a threat is expected.

(d) Peterson (1951) put forward fairly convincing arguments that the majority of engagements in World War II took place at short range because of terrain screening. But it is by no means clear that with expected NATO and Warsaw Pact tactics the ranges of engagement will principally be determined by the range of the first obstacle to inter-visibility. Casualty statistics are in any case bound to be biased estimates of engagement ranges because of the reduced probability of hit at long range. Certainly, the evidence from post-war exercises (Mitchell, 1970) and just lately from CHINESE EYE (Begley & Rowland, 1977, Rowland & Begley, 1977) is for ranges to increase steadily from the World War II mean of 701 yards quoted by Peterson to something in excess of 1000m in 1976. War games show the same trend (Beresford, 1968a, 1968b; Platt, Devon & Edwards, 1976) and no doubt the increasing accuracy of gunnery systems over this period has made commanders more ready to engage at long range.

Because of these sorts of considerations NATO set up an international Working Party in 1964 to determine the maximum essential range for anti-tank weapons (including tank guns). Part of this study consisted of a field exercise to determine the range at which approaching tank targets first became visible (NATO Working Party, 1964). The tactical assumptions behind the trial are clearly implied - that attackers may be engaged at any range up to and including that at which they first become visible, which is not necessarily the same point as when there are no remaining obstacles between them and the defenders. Three separate training areas were used in the field exercise with rather different terrain characteristics, and the mean detection distances in each differed quite widely from one another. If the areas in combination are regarded as roughly representative of North German terrain (and they were chosen with this point in mind) then the distribution of ranges of first detection should set an upper limit to the actual engagement ranges which may be expected in practice.

Exercise CHINESE EYE has yielded some first estimates of engagement ranges, based on the 'dot' ranges ordered by commanders (which refer to graticule markings allowing for the fall of shot). These estimates are only approximate. The lowest 'dot' range corresponds to a distance of 1000m, and so there is no discrimination below this point. In addition, as has already been mentioned, visual range estimation is notoriously inaccurate with errors typically in the region of 20-25% (Harrison & Price, 1944). There is also a tendency to overestimate short ranges and to underestimate long ones (especially, one supposes, for targets which are selected as being worthy of engagement). The CHINESE EYE 'dot' ranges certainly seem to be distributed in rather a different fashion from those obtained in other ways.

Detailed accounts of some of the CHINESE EYE scenarios (4, 5, 12, 13, 14, 18 and 19) have now been provided by Hayes & Rowland (1978) and Rowland & Hayes (1978a, 1978b). The accounts include map traces of all tank movements, and tabulated time histories of these movements, which latter list map co-ordinates of stopping points (with a resolution of 100m) the instants of firing, and the identity of the tank fired upon. The present author has analysed 354 simulated firings from the detailed CHINESE EYE accounts, utilising the published co-ordinates (interpolating along map traces where necessary). The firing ranges, plotted in cumulative probability form, are shown in Figure 2. The Figure also shows the results

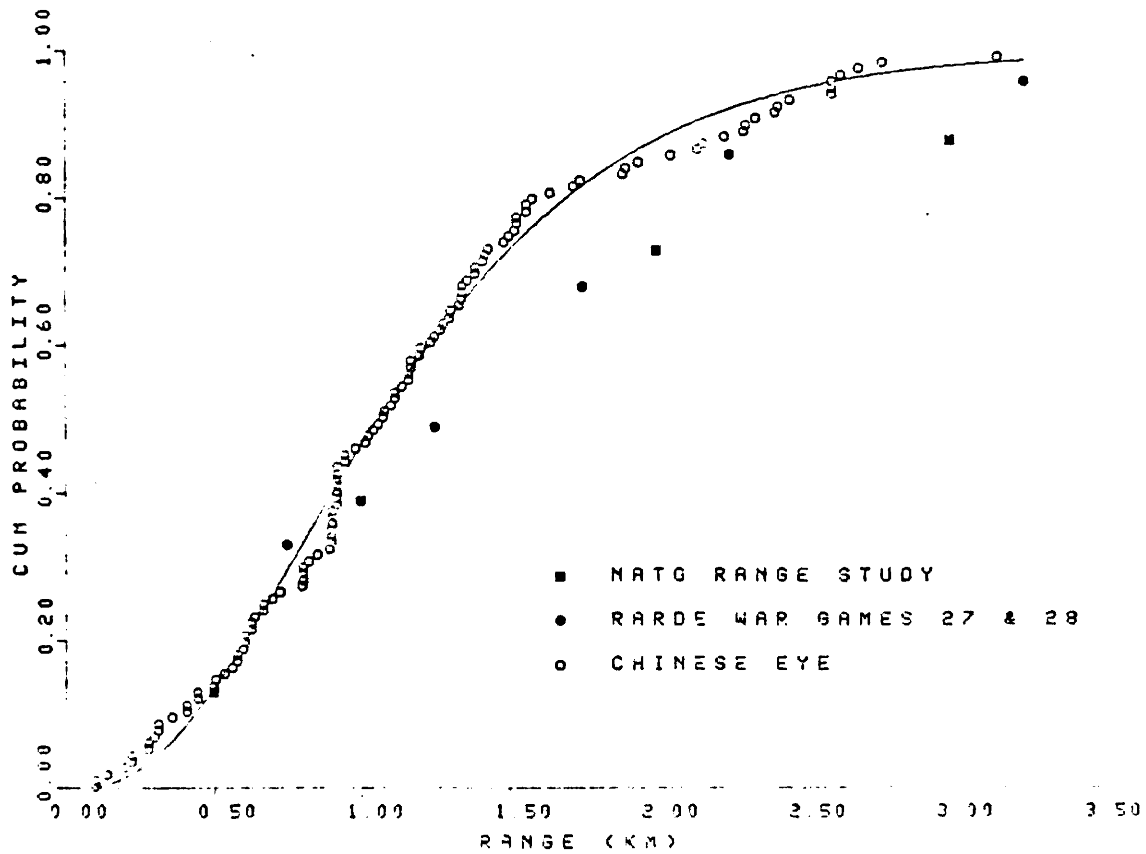


FIGURE 2. Range of engagement.

of the NATO range study (NATO Working Party, 1964) and the analysis of 146 firings from RARDE War Games 27 and 28 given by Platt, Devon and Edwards (1976).

The CHINESE EYE data plotted in Figure 2 are well described by a gamma distribution with parameters $(3, \frac{3}{\bar{r}})$:

$$p(r)\,dr = \frac{r^2}{2}\left\{\frac{3}{\bar{r}}\right\}^3 \exp\left(-\frac{3r}{\bar{r}}\right) dr \qquad (4)$$

where $\bar{r}$ is the mean engagement range (1188m in this analysis). This continuous curve has been depicted in the Figure. If we were indeed dealing with a true Poisson process the gamma distribution with these parameters would be the distance to the third event (obstacle) assuming a mean inter-obstacle distance of $\frac{\bar{r}}{3}$. It does seem, though, that arguments from Poisson processes rest on rather shaky ground, and the fitting of this particular curve is justified on empirical, rather than theoretical, grounds.

2.6 Angle of attack

Another item of information required to complete the engagement geometry picture is the aspect of the target at the instant of fire, and hence the direction of motion of the attacker with respect to the defender. The classical theoretical study of this topic was conducted by Whittaker in World War II (see Pennycuick, 1945a, 1945b). Whittaker considered a tank approaching a linear defence at constant velocity (with defending guns distributed uniformly at right angles to the line of approach). If the maximum range of the opposing guns is R and the length of the line of defence is 2C (that is to say, length C on each side of the line from the attacker normal to the defence) then these assumptions lead to a probability distribution of angles of attack.

$$p(\theta)\;d\theta = \begin{cases} \frac{R^2}{2K}\; d\theta & , 0 \geqslant \theta > k \\ \frac{R^2}{2K}\; \sin^2 k \; \text{cosec}^2\theta \; d\theta & , k \geqslant \theta \geqslant \pi/2 \end{cases} \qquad (5a)$$

where $\sin k = {}^{C}/_{R}$ and K is a constant of integration. For values of θ in the range $^{\pi}/_{2}$ to π it is simply assumed that the probability density will decline linearly (the defending guns being suppressed by this stage, and in any case not being optimally sited to attack) and so the distribution is completed by:

$$p\ (\theta)\ d\theta = \frac{R^2}{K}\ (\pi - \theta)\ \sin^2 k \qquad {}^{\pi}/_{2} > \theta \geqslant \pi \qquad (5b)$$

By symmetry we have a similar set of equations in the range 0 to $-\pi$, and K is evaluated by observing that

$$\int_{-\pi}^{\pi} p\ (\theta)\ d\theta = 1$$

from which it will be seen that

$$K = (k + \sin k \cos k + \frac{\pi}{4}\ \sin^2 k\ )\ R^2$$

It will be noted that R^2 thus disappears from the full equation, so that the maximum range of the guns is immaterial. However, the assumed ratio ${}^{C}/_{R}$ is still crucial, and this is commonly set to a value of ${}^{1}/_{2}$, so that $k = 30^0$.

Whittaker termed this probability distribution the 'directional probability variation', or 'dpv' for short, and 'Whittaker's dpv' has passed into the standard vocabulary of the tank warfare analyst. However, because of the obviously simplistic nature of the assumptions it has been subject to much re-examination and review (for a recent example see Wells, Wakelin, Kelly, Fredriksen & Pawley, 1976). Whittaker's dpv has been obtained by a gun density method (see Pennycuick, 1945a, 1945b) in which it is assumed that defending guns are distributed randomly and uniformly in space, but that guns further than some arbitary lateral distance from the axis of advance will not fire because they will be preoccupied or inappropriately sited. Clearly, though, the width of the firing strip (or the value of the constant k chosen for Whittaker's formulation) is both arbitary and yet crucial to the final form of the probability curve. As Beresford (1948a, 1948b) has shown in his War Game analyses, the dpv seems to become nearly circular in 'close' country, where engagements are practically all at short range (and where, one may suppose, defenders will engage in whatever sector a target may present itself). However Whittaker's dpv is derived, in real life it may be assumed that the attacker's heading will vary more or less about his mean axis of advance, and this too should affect the form of the observed dpv. Other factors which may affect the shape of the distribution are not too difficult to envisage.

An alternative formula has been preferred by American analysts. Peterson (1951) in his examination of the apparent direction of attack on 720 British and US tank casualties in World War II found that the percentage of hits on the front, sides and rear of the hull was consistent with a dpv of the form:

$$p\ (\theta)\ d\theta = \frac{1}{2\ \pi}\ (1 + \cos\theta)\ d\theta \qquad (6)$$

This he attempted to justify by the assumption that attacking tanks tended to be fired at very soon after emerging from an area of obscuration. If the further assumption is made that target motion (considering forward motion only) is completely random with respect of the contour of obscuration, then the probability density of the angle of emergence (α) will be:

$$p\ (\alpha)\ d\ \ = \cos\alpha\ d\alpha$$

where $\alpha = 0$ is taken to be the direction normal to the contour of obscuration (approximated by a straight line in the immediate vicinity of the point of emergence). Figure 3 illustrates this situation. Now at the instant that a tank emerges from obscuration at an angle α all azimuth aspects (θ) of the hull in the range $\alpha - \pi/2 < \theta < \alpha + \pi/2$ will be exposed to fire. If we now assume that defenders are distributed uniformly in azimuth with respect to the point of emergence, then we have for the dpv the cardioid equation:

$$p\ (\theta)\ d\theta = \frac{d\theta}{2\pi} \int_{-\pi/2}^{\pi/2\ -\ \theta} \cos\alpha\ d\alpha$$

$$= \frac{1}{2\pi}\ (1 + \cos\theta)\ d\theta$$

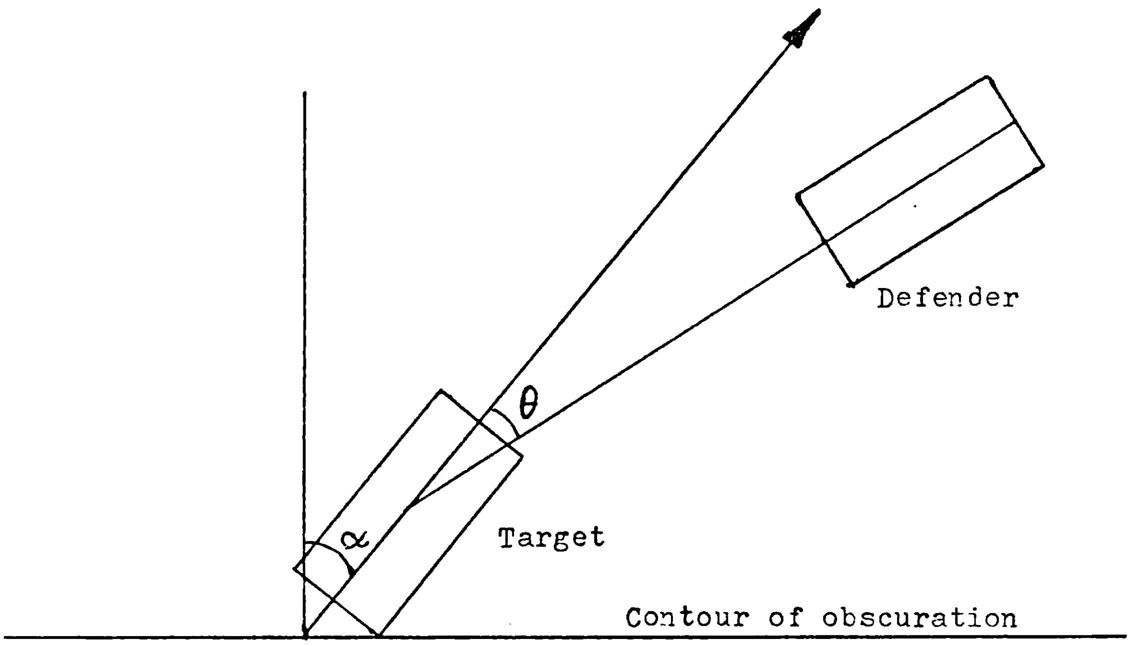


FIGURE 3. Target emergence from obscuration. The angle of emergence is α, and θ is the angle of attack.

Here again the assumptions on which this cardioid dpv is based may be queried on the grounds of over-simplification. It is by no means clear that the distance from the attacker to local obscuration is always small enough for the main approximation involved in the derivation to hold true. There must also be many major and dominating obscuring objects and terrain/features (such as hill crests) which attackers will take great care not to approach in random fashion, and in relation to which defenders will also site themselves in a non-uniform manner.

Turning to empirical data, the results of a small number of field trials were recently reviewed by Begley (1976) and Begley & Rowlands (1976). This information was in fact collated with a view to determining the likely azimuth distribution of targets as seen from the defender, although this will be equivalent to the angle of attack on the approaching target if the latter's heading is assumed to be the reciprocal of the defender's. The resolution of the raw data is very coarse, but within these limitations it was concluded that there was good agreement with Whittaker's dpv.

In the previous section it was mentioned that the author had made an analysis of 354 firings from the detailed CHINESE EYE accounts provided by Hayes & Rowland (1978) and Rowland & Hayes (1978a, 1978b). The angle of attack for these firings could be determined by locating the attacker and defender on the map traces, and, relying on the movement trace as giving a fair indication of the attacker's heading, by measuring the relevant angle with a protractor. These measurements were made to a resolution of 5°. The results are illustrated in Figure 4 together with data from Exercise PENANCE (Begley & Rowland, 1976 and Rowland, Dove & Thornton, 1976) and the cardioid and Whittaker dpv's (the latter with $k = 30°$). If the form of the Exercise PENANCE data favours the cardioid dpv for small angles the reverse could be said to be true for the CHINESE EYE data. There seems to be no pressing reason to depart from the Whittaker dpv, especially as it is roughly in line with the CHINESE EYE data over the angular interval where most of the firings occur. A more searching analysis would perhaps establish the degree to which the form of distribution varied as a function of range. This has

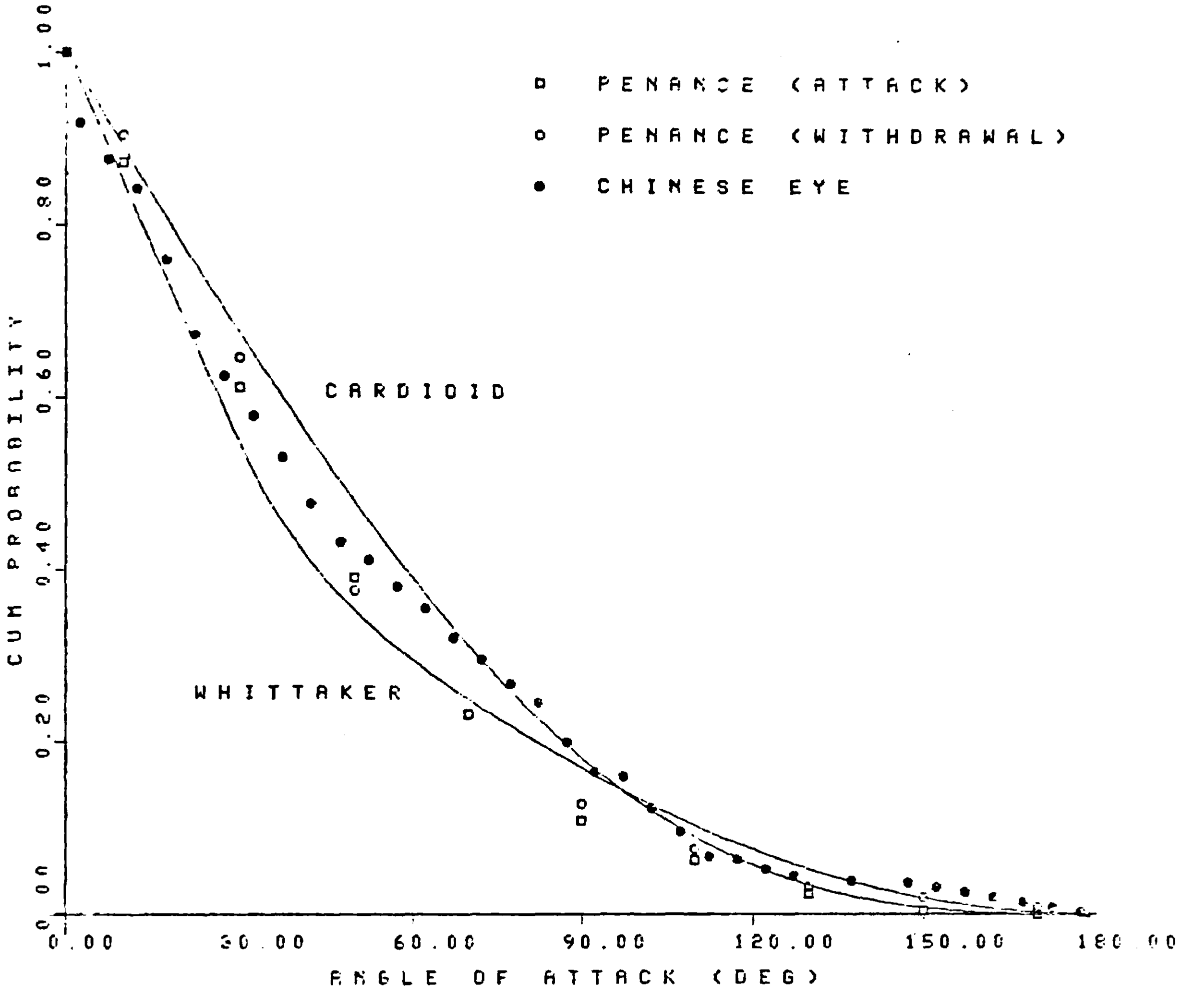


FIGURE 4. Angle of attack.

not been done on this occasion.

2.7 Height differences between defenders and attackers

Typically the defender has a height advantage over the attacker. Thus, when Stead & Rowland (1977) examined CHINESE EYE Scenarios 3, 5, 9, 11, 12 and 17 in detail they found that the mean height advantage of defenders over attackers varied from - 0.6m on the flat plain of Scenario 3 up to 70m for the valley of Scenario 11. In undulating or hilly country a high defensive position naturally yields the most extensive view, and by and large the maximum concealment is given to the attacking force by sticking to the valleys.

Unfortunately, by itself this height difference tells one little about the engagement geometry. What matters is the inclination of the current plane of motion of the attacking tank with respect to the defender. A large height advantage can entail negligible target rates in the vertical plane if the attacking tank is advancing up a slope towards the defender hull down on the crest; and conversely a height disavantage can be associated with appreciable elevation rates if the attacker is rolling down one side of a valley with the defender ensconced in a fire position on the other side. In the absence of a more detailed analysis of this aspect all one can say is that the defender must be above the plane of motion of the attacker or the intervening ground will screen him from view. Thus if one has motion data for a target manoeuvering on a (locally) near level plain, operation engagement geometries will need to be modelled by some distribution of positive height differences between the defending and attacking tanks.

In the author's analysis, already referred to, of detailed CHINESE EYE records, almost exactly one half of the defender's firings took place with the attacker at the higher altitude. For the 50% of engagements where the reverse was the case the distribution of height differences was approximately exponential with mean 9.88m. There appeared to be a negligible relationship between height difference and range. In the absence of more detailed information, engagement geometries have been modelled by assuming always non-negative height differences with distribution:

$$p\ (h)\ dh = \frac{1}{10}\ \exp\ (- \frac{1}{10} h)\ dh \qquad (7)$$

but with the restriction that the implied angle of depression of the defender's gun should in no case be greater than - 10°.

2.8 Angle of tilt of sight

Finally, the motion of the attacker as resolved into the sight axes of the defender will be affected by the angle of tilt of the sight. There is scant information on this point, although a detailed map analysis was made by Rowland, Dove & Thornton (1976) of 473 firing positions in Exercise PENANCE. Approximately 14% were on slopes with a gradient greater than 5%, which, on the assumption of a Gaussion distribution, would indicate a standard deviation of about 4° angular tilt. However, this does not necessarily imply that the direction of slope would always be sideways-on to the attacking tank. In any case a map analysis cannot really reveal local undulations which one would expect to be the dominant influence in determining tilt.

In the absence of any definitive study a standard deviation of 5° has gained wide acceptance among analysts as a working figure for sightline tilt. It has been used in many different fire control assessments (in some cases by international agreement) as a basis for estimating the contribution of gun cradle tilt to total system error. There does not seem to be any firm evidence for preferring any other value in this present study. Figure 5 illustrates most of the aspects of engagement geometry just discussed.

2.9 The generation of target courses for human operator modelling and for evaluation

Having reviewed the available evidence concerning the expected values of the different parameters which affect the engagement geometry, it remains to describe the procedures actually used to generate target courses for exercising the operator and for evaluating prediction schemes.

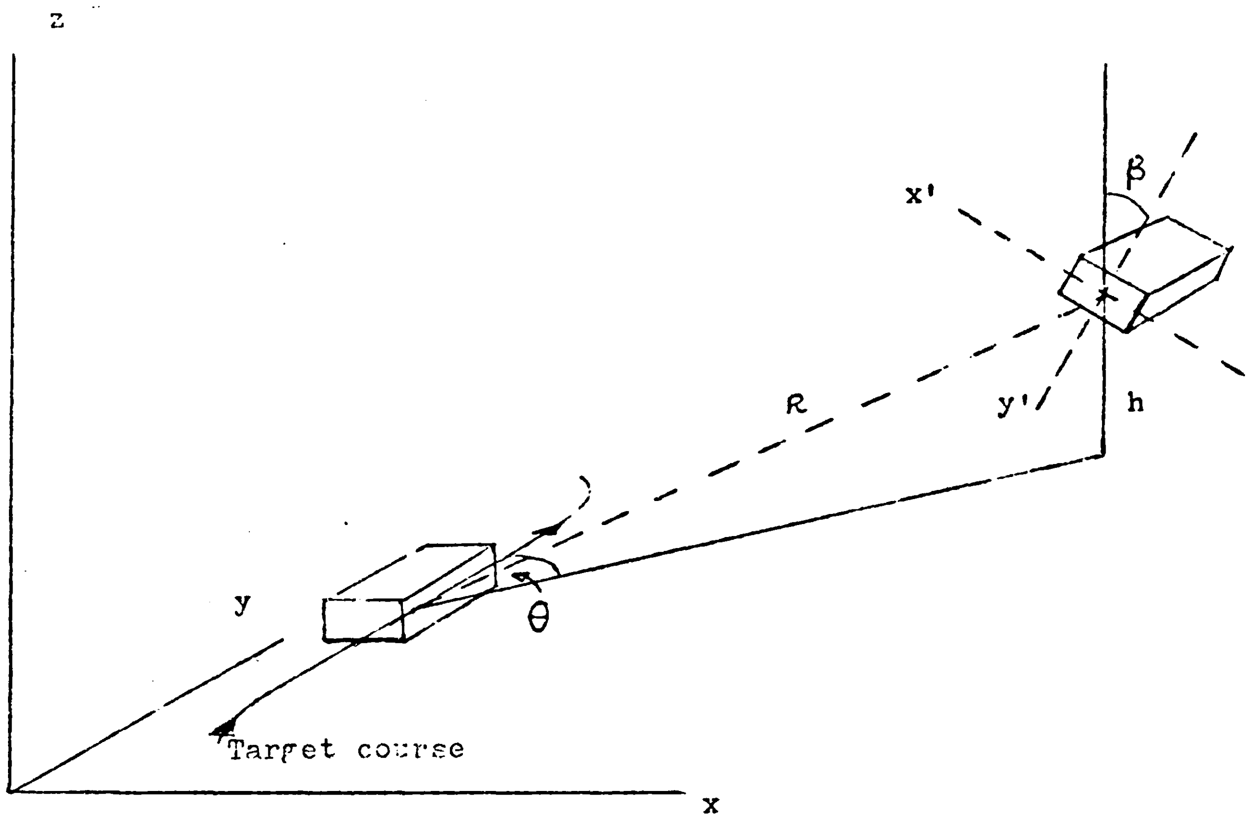


FIGURE 5. Engagement geometry. The geographic coordinate system is (x,y,z) and the sight axis system (x′,y′,z′). The target is at range R along the z′ axis. The angle of attack is θ, the angle of sight tilt is β, and the defender's height advantage is h.

It will be recalled that US AMSAA data were used as the basis for target motion inputs, and that these data consisted of target x, y and z co-ordinates at intervals of approximately 0.1 sec. A nominal engagement history (with movements in sightline axes) was determined as follows:

(a) Specify a nominal impact time (data record) or instant at which the defender's shell is deemed to pass through the plane of the target. By reference to the target's co-ordinates one second prior and one second subsequent to this time, determine the target heading.

(b) Specify an angle of attack, a slant range and a (positive) height difference between defender and attacker. This in turn determines the co-ordinates of the defender.

(c) Specify a firing delay from the ending of prediction and ballistic computations to the pressing of the fire button. (It is assumed that the final range determination will be made just before fire control computations cease and so the estimated target range will be that at the nominal impact time, less the time of flight of the shell and the firing delay).

(d) Translate the target position information for the 16 sec period prior to nominal impact from geographical co-ordinates to angular co-ordinates in sightline axes. (The mathematics of this translation are too straightforward to require spelling out in detail, but are described by Johnson, 1972). By the usual differencing techniques, calculate the apparent target angular velocities in each plane for each of the roughly 0.1 sec sampling intervals in the 16 sec period chosen. Plot these calculated velocities to provide a visual display.

(e) By visual examination select major points of inflection, thus determining 2 or more sub-intervals within each 16 sec period. Perform an unweighted least-squares linear fit to the velocity data within each subinterval. Re-plot the velocity information together with the lines just calculated, allowing the values of the fitted coefficients to determine new points of inflection.

(f) By visual examination, judge whether the equations just fitted represent a reasonable approximation to the velocity data. If they do not appear to do so select new points of inflection as at (e) and re-cycle this portion of the fitting routine.

Illustrative plots are given in Figure 6 showing the displayed velocities and the piece-wise linear fits for one target, taken at random from the many target courses fitted.

The absolute position of the target in angular terms from the defender is arbitrary. If we fix the nominal point of impact as our arbitrary origin, then it will be seen that the procedure which has just been described is a rough and ready method for fitting cubic splines with variable knots (expressing the angular position of the target as piece-wise cubic polynomials in time within two or more successive intervals in a 16 sec total period, the resulting function being smooth in angular position and velocity but with step changes in acceleration). It was originally proposed to fit cubic splines to the velocity data (and hence splines of degree 4 in position terms) by some more formal method (see, eg Schumaker, 1969; Esch & Eastman, 1969; and Mier Muth & Willsky, 1978). However, in view of the extent to which the radar-based position estimates are corrupted by noise (apparent when one views the velocity plots of Figure 6) this degree of sophistication hardly seemed to be justified. It is not claimed, therefore, that the target motion equations actually generated are very accurate representations of the particular target histories on which they are based. It must be born in mind, though, that such accurate representation is not required for the purposes of this study. Our objective, instead, is to generate a series of nominal target motions which will provide an adequate test for the human tracker and for prediction scheme evaluation.

The importance of the information, which has been surveyed in this chapter, concerning engagement ranges, angle of attack, height differences and angle of tilt is that, together with target motions, these parameters describe the envelope within which the prediction scheme must act. Taking the human operator first, our objective will be to fit some model which will describe the way in which he responds to target motions. This model will then be used to evaluate prediction schemes for a different, and wider, set of motions than those used for model fitting. The important

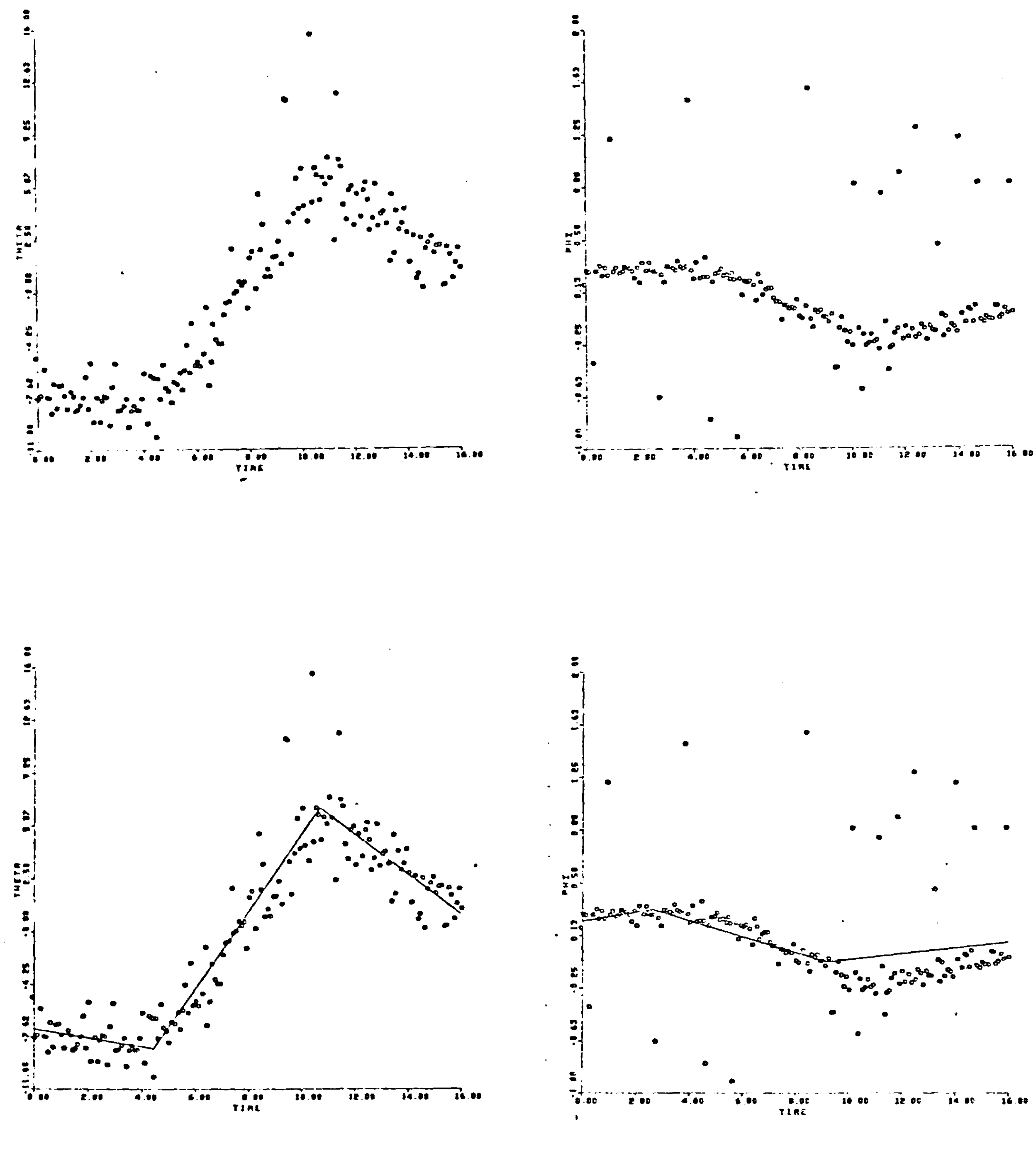


FIGURE 6. Computed instantaneous velocities (mrad/sec) in sightline axes, AMSAA position data. Theta is the horizontal dimension and phi the vertical. The first two (much reduced) plots show the raw data, and last two piecewise linear fits to these data.

thing, then, is truly to span the engagement envelope (and then, if possible, select points within it) so that the model is used to interpolate, rather than to extrapolate to a region not tested.

So far as the evaluation of prediction schemes is concerned, it is important to give the correct weight to all the points within the operational envelope. If this is not done any optimisation procedure to evolve a possible prediction scheme may capitalise on particular features of the atypical examples chosen; and any evaluation procedure may give an entirely false impression of the system's true operational utility. Having described the operational envelope, therefore, and the procedure used to generate target courses for test, the actual sampling methods employed in the operator modelling context and the evaluation context will be described in Chapters 4 and 7 respectively.

3 HUMAN OPERATOR MODELLING: A REVIEW OF PREVIOUS WORK

3.1. Introduction

There exists a voluminous literature on tracking and numerous experiments have been conducted in this field. Furthermore, models have been developed which describe the data on which they have been based reasonably well. Why, then, should we be hesitant about applying these results directly to any fresh tank gunnery application? The trouble lies in the fact that most of the experimental data is gathered in conditions which differ in important respects from the situation with which we are concerned, and the models lack generality: the values of their parameters depend to a great extent (and in a way which is not always predictable) on the precise experimental set-up used to generate them. Nevertheless, it is obvious that there are severe drawbacks to exclusive reliance on empirical results from live operators and real equipment for all speculative, exploratory or development work. There would seem to be some value in reviewing past approaches to modelling, noting their strengths and their weaknesses, and in trying to devise, if not a general model, a general approach to the description of human tracking data in this context.

The present chapter does not attempt any review of experimental results. Many such reviews can be found (and Poulton (1974), for instance, covers a great deal of tracking literature, attempting to devise a coherent frame-work and to give clear cut recommendations for design). Instead it provides a very brief overview of tracking models and their development. It picks out desirable properties, and then incorporates some of them in a suggested simple scheme for handling tracking data. The aim is not to construct a model which is any way embodies 'real' human processes or modes of operation as a tracker. Rather it is to devise as simple a scheme as possible to describe tracking data in the tank gunnery context, which will have sufficient accuracy and generality for system computations and preliminary evaluation.

This last point requires further emphasis. Mathematical models of the human operator, or some features of them, have attracted criticism in some quarters (see, eg Kelly, 1968, and Poulton, 1974). The main point made is that they present an excessively mechanistic vision of human response. The operator's output is tied in rigid fashion to the input (target motion in this case) modified to some extent by variability which is random rather than purposive in character. From most models one would never guess that the human being has a memory (and so can respond even if the target temporarily disappears), that he is affected markedly by the form of the display, that he takes note of all sorts of subtle features of the environment and builds up complex internal representations of his task, and so forth. A very recent review of modelling approaches concludes that 'existing human operator models are not sufficiently representative of known characteristics of human behavior (sic) to be useful for general performance measurement applications' (Knoop, 1978). The key word here must be 'applications'. It should be admitted at once that it would be a complex model indeed which could approach a complete description of human response in all its rich variety. But our limited field of application must be kept in mind. In a modern tank gunnery system the gunner's tracking output is fed directly into a digital computer at a regular sampling rate in the region of 0.1 sec. The computer at least reacts automatically, proceeding blindly to a computational stage, the output from which is fed on into the rest of the system. Our aim is simply to describe the input to the computer in a way that will reflect its main mathematical features. We shall perhaps be able to judge the extent to which we have succeeded in attaining this modest goal by the end of the next chapter.

In the typical visual tracking set-up the operator faces a display containing a target and an aiming mark or graticule. His task is to maintain coincidence between the target and the graticule in the face of target disturbance or motion. In the laboratory a distinction is usually made between compensatory and pursuit displays. In the former the graticule is fixed, and the operator's control output is subtracted from the target motion. With no operator supervision disturbance would drive the target away from the graticule, but the man attempts to compensate for this motion through the medium of his control. In a pursuit display both target and graticule are free to move. The operator's control output affects the graticule only, and so the man pursues the target with his aiming mark. The control itself may or may not be a simple device - there could be a straight forward relationship between the man's input and the control output, or it could be complicated by all sorts of lags and non-linearities.

A large proportion of the experiments on which past models have been based have used compensatory displays. This is because many real-life situations approximate to this mode for a variety of reasons (not least of which is that, if target motion is at all extensive and displays are to be kept to a reasonable size, it is difficult to obtain sufficient magnification with a pursuit display accurately to assess one's error). It is hard, however, to obtain any information about the nature of the target motion from a compensatory display, because one can only build up a picture of the disturbance indirectly by reference to one's remembered control actions and the displayed error. Such studies therefore tend to de-emphasise the human capability to predict and anticipate, and to emphasise instead the more mechanical aspects and limitations of human response. The tank gunner's set-up has both pursuit and compensatory features. Although the sight head or turret is free to rotate in space, the graticule or aiming mark is fixed in the centre of the gunner's eye-piece (and magnification is normally sufficient for him to be able to assess his aiming error with some accuracy). As he tracks he sees the scene outside move across his display in the opposite sense to that in which he moves his control. The gunner can, however, obtain some information on the nature of the target track, because he can note the motion of the target relative to the structured terrain background and foreground (and in some cases he can use changing target aspect as an additional cue). We should expect tank gunnery results to be more akin to those obtained from pursuit displays than compensatory displays.

Figure 7 displays a single error trace (the difference between target position and the gunner's control output) in one dimension only for a laboratory tracking run. The laboratory simulation (described in the following chapter) reproduced the main features of the tank gunner's tracking task(except for change of aspect). Errors were sampled at a

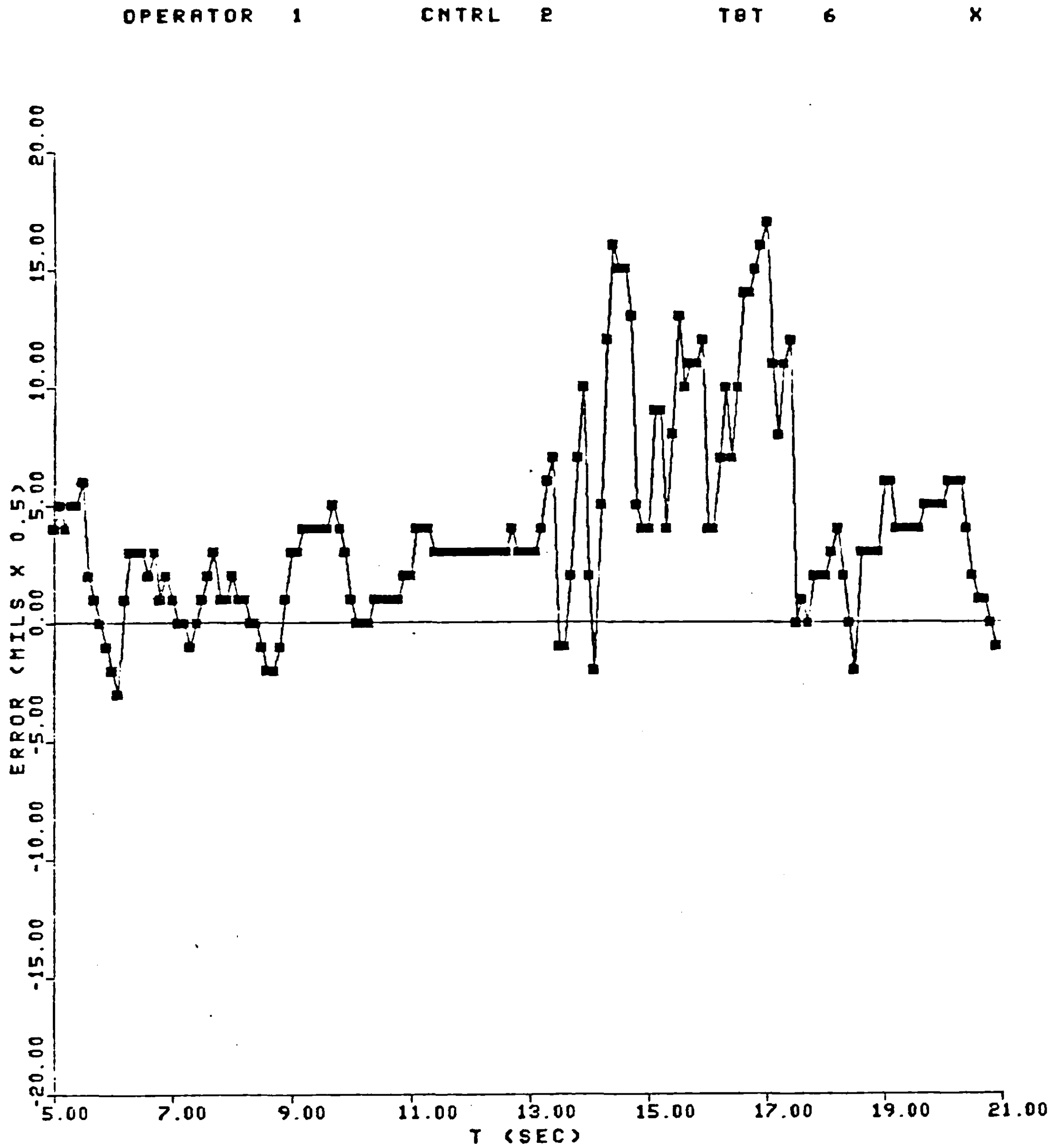


FIGURE 7. Individual tracking error trace.

rate of 10 per second. For the run in question the target started with a low angular velocity which remained constant until t = 13 sec. During the interval t = 13 sec to t = 17 sec the target maintained a constant positive acceleration, the velocity thereafter remaining steady at the t = 17 sec value. The most obvious feature of the error trace is the moment-to-moment variability, or high frequency 'jitter'. To a large extent this masks any systematic effect which may be present due to target motion, although it does seem that there is some positive displacement of the error trace during the period of positive acceleration. Clearly, then, any description of human tracking behaviour in this context must be a statistical one, and our task will be to describe how the mean and standard deviation of the error vary as a function of target motion, and the way in which error at one sampling instant may depend on that at a previous instant.

The pioneer in bringing quantitative methods to bear on the human tracker was Arnold Tustin (1944, 1947a, 1947b) during World War II. Tustin was studying the manual control of a power driven gun, in an anti-aircraft context. From an examination of tracking records Tustin concluded that there were regular features of operator response, and it was he who imported into this field the concept of the 'describing function' or 'transfer function' - a function transforming target motion input into human tracker output in a determinate manner. Furthermore he concluded that to a very reasonable approximation the describing function could be regarded as linear, with the useful result that the well developed theory of linear servomechanisms could be applied to manual control in the same way as it is applied to automatic following. However, when the regular, determinate portion of human response had been allowed for, there remained an inescapable stochastic element which could only be accounted for in statisitical terms. This portion, not correlated with the output, he termed the 'remnant'. This general scheme for describing tracking behaviour has dominated human operator modelling until this day, and in our review it will be convenient to retain this convention, considering first the regular features of operator response and then the random - appearing features.

3.2. Regular features of human tracking response

3.2.1 Quasi-linear operator (transfer function) models

As just mentioned, Tustin likened the determinate portion of the human tracking response to that of a linear servo. This analogy has gained wide acceptance, and since Tustin's studies took place experimentation and investigation within this general framework have proceeded apace. Early results have been summarised by Licklider (1960). The continued development of the linear transfer function is perhaps best exemplified in the work of McRuer and Krendel (McRuer & Krendel, 1957, 1959; McRuer, Graham, Krendel & Reisner, 1965; and McRuer, Graham & Krendel, 1967). Quite recently these same authors have reviewed the status of this kind of model in the aircraft pilot context (McRuer & Krendel, 1974).

The basis of the linear filter is the impulse response. Let us suppose, for the sake of argument, that the human operator is being provided with discrete inputs (target deviations) x_t, x_{t-1}, x_{t-2}, ... being observed at regular intervals t, t-1, t-2 ... . The outputs at these equispaced intervals of time are z_t, z_{t-1}, z_{t-2}, ... The linear filter relates the output to the input by a simple expression of the form:

$$z_t = h_o x_t + h_1 x_{t-1} + h_2 x_{t-2} + \ldots \qquad (8)$$

The weights h_o, h_1, h_2, ... (some of which could be zero) are called the impulse response function of the system. According to this formulation the output is regarded as the straight forward aggregation of a number of impulse response functions weighted by the input deviations x_t. Figure 8 depicts this graphically. At the top of the figure we have illustrated a particular impulse response function. On the next line is shown a particular input history, which differs from zero at times t = 1, t = 2, t = 3 and t = 4. The impulse responses to these four non-zero inputs are shown on the four lines below, and these responses are aggregated to produce the system output at the bottom of the Figure. We can of course, derive a continuous model, which the discrete model will closely approximate if the time intervals are made small enough.

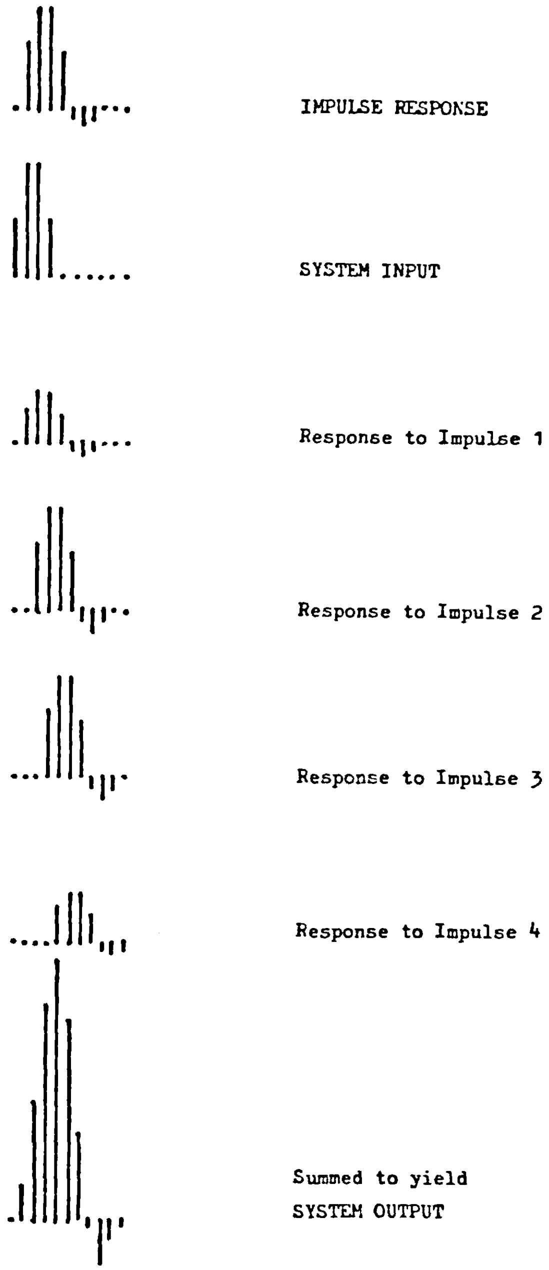


FIGURE 8. Impulse response.

The impulse response is then h(u), and the output and the input are related by the linear filtering operation:

$$z\ (t) = \int_{o}^{\infty} h(u)x(t-u)\ du \qquad (9)$$

z(t) is generated from x(t) as a continuously weighted aggregate, rather than as the discretely weighted aggregate in (8), and it is this continuous form which has been adopted by Tustin and his followers.

The impulse response function provides an easily understandable description of the way in which a linear filter operates. However, it is not a representation which is commonly encountered in the tracking literature. To avoid confusion it will be as well to pause and consider some of the other representations which have been used. The impulse response function is not always a very convenient tool for calculating the response of a system to a complex input. It generally requires the evaluation of complicated convolution integrals as indicated by (9) and this is often tedious. A simpler scheme, useful when the input can be regarded as a mixture of sinusoids of different phases and amplitudes (and virtually all target courses of practical interest can be represented in this way, see, eg Stuart, 1961)is to work with the <u>frequency response function</u>. This latter consists of two parts. The gain G(f) represents the amount by which the amplitude of a particular frequency f is multiplied, and the phase shift $\emptyset(f)$ represents the delay imposed on that frequency as it passes through the system. And so a sine wave input $x(t) = a \sin 2\pi ft$ will be transformed to an output $z(t) = a\, G(f) \sin (2\pi ft + \emptyset\ (f))$. In the same way as the linear filter can be regarded as responding via the linear aggregation of impulse responses, so it can be regarded as operating via the linear addition of sine waves of different amplitude and frequency, each sine wave being scaled and shifted appropriately as it passes through the system. The important point to note is that the two representations are entirely equivalent - the frequency response function is simply the Fourier transform of the impulse response function.

Another equivalent way in which to express the linear filter is to relate the deviations of the input and the output by a linear differential equation. The level, rate of change and higher derivatives of the output are proportional in some way to the level, rate of change and higher derivatives of the input, and so we have $(1 + u_1 D + \ldots + u_R D^R)\, z(t) = g(1 + w_1 D + \ldots + w_s D^s)\, x(t - \tau)$ where D is the differential operator d/dt, the u's and w's are parameters and τ is the 'dead time', or pure delay between input and output. R and s are assumed finite. This approach can be useful in model building. Direct evaluation of the impulse response function is often unsatisfactory (see, eg Jenkins & Watts, 1969, and Box & Jenkins, 1970) because it usually involves the estimation of very many parameters, and these in turn are functionally related and so unstable. Often a close approximation to the 'true' or observed transfer function can be obtained by using a differential equation representation with very few parameters.

Differential equations of this type may be solved by the use of the Laplace transform:

$$H(s) = \mathcal{L}[h(t)] = \int_0^{\infty} h(t)e^{-st}\, dt$$

The resulting transformed equations are algebraic and their solution is re-inverted to solve the original equations. The method is much used in control systems analysis. In most of the tracking literature, therefore, the <u>transfer function</u> H(s) is used in the same sense as the control systems engineer uses it - as the ratio of Laplace transforms of input and output (ie, as the Laplace transform of the impulse response). Just as a differential equation with a few parameters can often describe quite a complex impulse response tolerably well, so can a transfer function with a few terms provide a reasonable approximation. Written in this way, though, the terms do lend themselves to engineering analogies and, it must be said, jargon, so that the human being tends to be depicted as a device with inbuilt delays, lags, leads and the like, and he is talked of as adjusting his gains and so forth. It is certainly convenient, though, to have one's model in such a form that it can be assimilated immediately into the rest of control theory.

These, then, are the principal alternative ways of describing the linear filter. That the transfer function and the differential equation provide parsimonious means of representing the human impulse response function may be appreciated by considering the most common forms used to describe tracking data. The transfer function most commonly used is:

$$H(s) = \frac{Ke^{-\tau s}(1 + T_L s)}{(1 + T_N s)(1 + T_I s)}$$

The equivalent differential equation representation is:

$$\{1 + (T_N + T_I)D + T_N T_I D^2\} z(t) =$$

$$K(1 + T_L D) x(t - \tau)$$

It will be seen that the model contains only five parameters, and, as has been mentioned previously, there is a tendency to attach to these parameters a real meaning. The reaction time constant τ is held to be an inherent delay and the term incorporating it represents a dead time due to central processing. T_N is called the neuromuscular lag time constant, and the expression within brackets involving T_N is held to represent an exponential lag due to the time taken to actuate and move the operator's controlling limb. The remaining parameters (the gain K, the lead time constant T_L and the compensatory lag time constant T_I) together comprise the so-called 'series equalisation' - the means by which the operator compensates for and adjusts to the characteristics of the controlled element and the target course with which he is faced (assuming that he has some such objective as minimising rms error). This restricted model has been found to provide an adequate description of practically all the data on which it was based, although an extended 'precision' model with more paramters has been developed to provide a better fit at very low and very high frequencies.

The most serious limitation of these models for our application is that, while they describe the sets of data on which they are based reasonably well, the values of the parameters needed to obtain these good fits vary in a marked and unpredictable way on the particular controls used, the details of the tracking plant and to some extent on the characteristics of the target course. Furthermore, it is assumed that the course itself is a stationary, random-appearing process. These limitations are quite openly acknowledged by McRuer & Krendel (1974) who say that' ...

when all is said and done, a common thread through almost all situations we shall consider, and the only thread which has any pretensions to general applicability, is ... the ' crossover model''. The 'crossover model' is a simplification of that just put forward which suggests that, to a rough approximation, the operator and his control plus tracking plant, taken together, can be regarded as a simple delay (the gain and delay time now being the only parameters which must be estimated in each case). That the man should attempt to adjust his characteristics in accord with the properties of the control (or overall tracking plant) with which he is faced seems only natural. In the present study we are not directly concerned with a thorough understanding of human adjustment, and we should be content to model the man-plus-control as one entity. To do so should reduce the situation-dependent variation of model form (particularly if reasonable attention is given to control optimisation) although the unadorned 'crossover model' is perhaps a bit crude for our purposes. Some residual effect due to the tracking plant is probably inevitable, and with the present state of knowledge we must doubtless resign ourselves to not being able to allow in advance or in any detail for controller and tracking plant effects. To regard each combination as unique and to model the man/machine complex as a single entity probably represents a reasonable approach in the circumstances. However, we should not resign ourselves too readily to an approach which places severe restrictions on the type of target course we can deal with, and perhaps should consider some simple adaptive feature which might make our model less target course dependent.

3.2.2 Intermittent models

Tustin in his pioneering studies noted that there was some evidence that the human operator fashioned a succession of discrete reactions when he was tracking a smoothly-moving target, but felt nevertheless that a continuous model provided a reasonable approximation. Other workers active at this time (Craik, 1947; Bates, 1947; Hick, 1948; and Vince, 1948) put much more emphasis on the apparently intermittent nature of human response. In manual tracking the evidence is not absolutely clear cut (although fairly convincing evidence is assembled in the studies of Navas & Stark, 1968), but when eye movement tracking is considered (see Young & Stark, 1965 and Robinson, 1965) relative discontinuities of eye position and velocity are quite easily seen, presumably because the moment of inertia and friction of the eyeball is very small. An intermittent mode of operation fits in well with other evidence that the human being in the control of skilled reactions acts as a single channel device and that there is a 'psychological refactory period' after the initiation of one response before another can be fashioned (Hick, 1948; Welford, 1952; Davis, 1957; and Smith, 1967). These sorts of considerations have led a number of workers to put forward intermittent, or sampled-data human operator models. A number of these are summarised by Young & Stark (1965), but perhaps the best known is that due to Bekey (1962).

To a simple operator model (pure delay plus an exponential lag) Bekey added a periodic sampler with a first order hold. This latter addition records the target deviation at the start of a sampling interval and for the duration of the interval extrapolates it at a rate derived from the difference between this level and that previously sampled. In the rather simple situation which Bekey investigated his model gave every bit as good a fit to the results at low frequencies as a conventional continuous form, and in addition it accounted for a high frequency 'sampling peak' which the continuous model could not explain.

This sort of model has rather fallen out of favour, in part because an examination of much data by McRuer, Krendel and Graham (1964) failed to show any evidence of significant stationary sampling peaks in the frequency response plots which regular sampling should induce. However, this of itself is not a serious objection, as it is easy to show (Bekey, Biddle & Jacobson, 1967) that even modest variability in the sampling interval would obscure all such evidence. However, there is no doubt that compared to continuous models intermittent ones tend to be cumbersome and do not adapt themselves so readily to most applications, and so it would require more than a modest increase in goodness-of-fit to make them seem attractive. It is important to note, though, that there is no evidence that they are less accurate than continuous models - indeed there is a suggestion that they explain some of the fine detail rather better. If in a particular application it proved more convenient to work with a discrete model then we should not shrink from doing so.

3.2.3 Predictive and adaptive models

A major criticism of the models considered so far is not lack of fit to the data, but that they fail to provide any account of how the operator adapts to a new input or the response of the system to his own output (that is, the controlled element dynamics). It has been noted that the conventional model parameters must be estimated afresh for each new task situation. A feature of man is his adaptability and capacity to learn from the past, and so increasing attention has been paid more recently to these aspects of his response (see eg Baron & Kleinman, 1968, 1969; Kleinman, Baron & Levison, 1969; Weiranga, 1969; Paskin, 1970; and Kleinman & Baron, 1971a).

Perhaps the best-known model in this category is that of Kleinman (Kleinman, Baron & Levison, 1970; Baron, Kleinman & Levison, 1970; Kleinman & Baron, 1971). It has been developed with the pilot situation mainly in mind, with multiple inputs (from a variety of instruments) and with more than one dimension of control, and it especially attacks the problem of divided and distributed attention. We shall only attempt to describe it in the sketchiest detail. Figure 9a shows in schematic outline the conventional transfer function model, and Figure 9b shows how this is supplemented in the Kleinman development. Observation and motor noise are not strictly additional features: although the transfer function models do not explicitly detail these inputs they are included implicitly in the 'remnant' (and this aspect will be dealt with in the next section of this chapter). The novel features are the Kalman filter, the predictor and the optimal controller (one whose parameters can be adjusted according to the output of the first two components). The model starts by recognising that the human being has only imperfect information on the state of the world, and that he must estimate the true state of affairs (the position and rates of the target, the condition of his controller and so forth) on this basis. The Kalman filter is chosen because it can be shown to have optimal estimation properties (in the least squares sense, and provided that an accurate model of the real system exists) in the presence of input noise and time delays (Kalman, 1960; Kalman & Bucy, 1961). There is, of course, no guarantee that the human being 'truly' estimates in this optimal fashion.

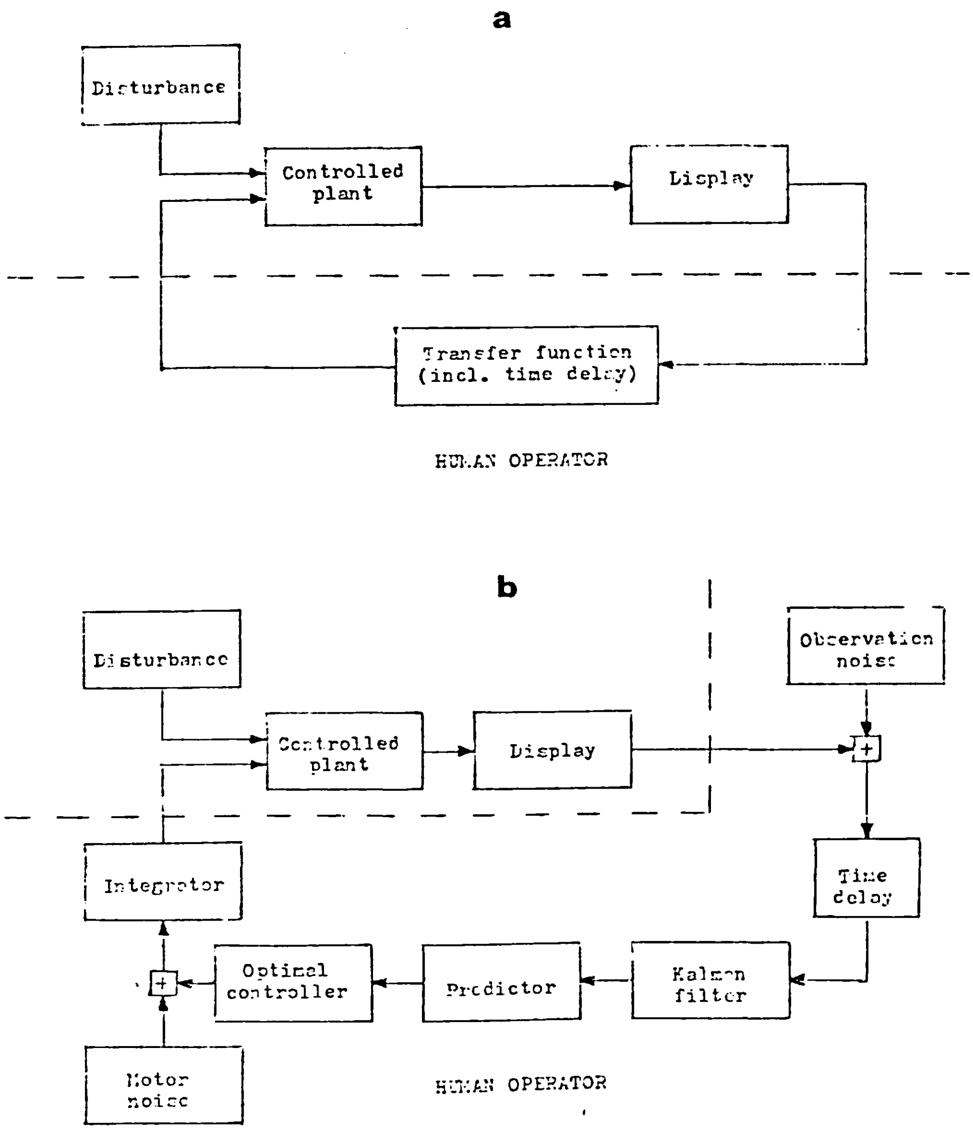


FIGURE 9. Standard transfer function model (a) and Kleinman model (b) of the human operator. (The portion representing the human operator is to the right and/or below the dotted line).

The predictor simply extrapolates forward to allow for the observation, processing and actuation delays which the operator recognises to be present. In the next step the operator is held to optimise some quadratic performance index or cost functional (including representations, say, of his mean square error; his 'effort' measured in terms of rate of control movement; the 'cost' of sampling different instruments; and so forth). This optimisation is effected according to the dictates of optimal control theory (see eg Obermeyer & Muckler, 1965; Elkind et al, 1967). Given the detailed assumptions of this model and a reasonably powerful computing facility it is possible to solve the various equations involved.

Clearly, the Kleinman model is at a much higher level of complexity than those considered so far. It has been applied to real data (see, for instance, Kleinman & Baron, 1971; Mattin, 1973; Mobley, 1974; Harvey & Dillow, 1974; Broussard & Stengel, 1976) but its routine use is not to be undertaken lightly. There are three interrelated features of this approach which militate against its employment in the present context. The first is just that of complexity. Although modern computers can ease the load of, for instance, solving a series of Ricatti equations (which model use necessitates), nevertheless when such time consuming operations are incorporated into extensive simulations and iterative optimisation schemes the computational load can still become excessive. The second point concerns the multiplicity of parameters which the model incorporates. As Phatak, Weinert & Segall (1975) have demonstrated, in its original form the model is not 'identifiable' (which is to say that its parameters are so numerous and interactive that their values cannot be jointly established in any unique fashion). The principal author and his associates have gone on to propose drastic simplifications of the model to remedy this defect (Phatak, Weinert, Segall & Day, 1976; Phatak & Kessler, 1975, 1977). Finally, the cost functional which the human operator is held to optimise itself raises problems. The form of the cost functional can be shown to have a marked influence on the other parameters in the model, and yet there are real doubts as to whether a universal cost functional can be identified or estimated (Anderson, 1974). Research on this topic has commenced (Jagacinski, Burke & Miller, 1976

Jagacinski & Miller, 1978) but in a slow-moving system some way removed from continuous tracking. The promise of providing a unified approach to the problem of human adaptability in the face of system variation has thus been met by substituting for it another at one remove - that of estimating the human cost criterion.

The difficulties in the Kleinman approach should not lead one to belittle its real successes. However, for present application some simplification is certainly required. To illustrate the direction one might travel, Wards' (1971) simple predictive scheme has shown some promise in a straight forward pursuit tracking application, and, at a more complex level, Pitkin's (1972) nonlinear model with adaptive features seems to provide a reasonable description of tracking results. It must be concluded, though, that for practical application there are real dangers in over-elaboration. If some attempt is to be made to incorporate adaptive features into our model it will be as well to keep these methods simple.

3.2.4 Modelling Implications

Relatively simple continuous filters (with the addition of a stochastic 'noise' element) provide reasonably accurate descriptions of most tracking data in straightforward applications. There is, though, no reason to prefer them to their discrete equivalents, except in terms of convenience of application. The main drawback to linear models is that their parameters change as situational elements change. To some extent this problem may be alleviated by considering the operator and his control (and the rest of the tracking plant) as a single entity, but nevertheless some variation due to target course or equipment features may remain. For this reason it would be as well to consider some elementary adaptive features in model development. Nevertheless, the overall aim must be relative simplicity, and over-elaboration is to be avoided.

3.3 Random-appearing characteristics of human response

3.3.1 Character and possible sources of 'remnant'

The random-appearing portion of the operator's response has not really received as detailed attention from research workers as has the determinate portion. In large part this is because with the vast majority of experimental set-ups and intended applications (in the main utilising relatively difficult control dynamics, compensatory displays and unpredictable inputs) it is the portion of the response modelled by the transfer function which dominates system error (see eg McRuer & Krendel, 1974, p65). With tank gunnery the control task is relatively easy, and in these conditions it is the so-called 'remnant' which dominates (see eg Poulton, 1974, p138). A good description of this component of the man's response is an essential ingredient of a useful model, and this description must cover both the expected amplitude of random response and its spectral content (what frequencies it contains, and hence how it may be expected to vary with time).

In practice the 'remnant' is estimated by subtraction. A form of model is chosen to describe a particular set of data, the parameters of the model are adjusted to maximise model fit, and that portion of the total variation which is not then accounted for is termed the 'remnant'. In practice, then, the remnant will always include an element due to lack of fit or model inadequacy. But over and above this there is always an element of variability in human response. McRuer, Graham, Krendel & Reisner (1965), Levison & Kleinman (1968), and Levison, Baron & Kleinman (1969a, 1969b) (using the standard linear transfer function as a reasonable descriptive framework) all considered the various ways in which this random variation may arise. Firstly, the operator will perceive the system state (target motion and deviation from the cursor) with error, and so there will be an element of 'observation noise'. Next there will be a time delay, and this also will be variable in practice. The operator's strategy (modelled by the so-called 'series equalisation') will not be absolutely fixed, and so included in his reaction will be adjustable gains, lags and the like. Finally, the operator will have an intended control motion, but he will not be able to execute his intended action accurately, and it will be corrupted by muscular tremor to some extent. Levison and his co-workers have demonstrated that in practice it will be almost impossible to separate out these different effects. To a first approximation, then, one might just as well lump all these sources of error together and treat them as injected 'observation noise'. They have gone on to examine regularities of the remnant noise treated in this way, and have built a theoretical construction around them.

Viewed as a component of the system output the error due to the remnant will be found to vary both in absolute value (variance) and in its frequency characteristics (spectral content) as a function of the task details. However, if one first normalises this error by dividing it by the total system error, and then treats it purely as 'observation noise' injected prior to the operator and the controlled element, a remarkable consistency taken over a variety of very different studies appears: the remnant seems to be constant, both as regards level and spectral content. This is a most useful empirical finding. What it says, in effect, is that at the point where the noise can conveniently be said to arise, and prior to the linear filters which approximate to the operator and to the controlled plant, the remnant may be regarded as random noise with a fixed (fairly flat and wide-band) spectrum. Furthermore, when regarded as a component of the system output subsequent to filtering, the remnant forms a fixed proportion (in power terms) of the total error. It has been pointed out that this last result is in accord with some of the earliest findings of experimental psychology (Weber, 1834 and Fechner, 1860) that when above the absolute threshold human errors are in rough proportion to the stimulus magnitude, and that they are more multiplicative than additive in nature, so that in the present context they tend to be in more-or-less constant ratio to the systematic errors which arise from the operator's response.

By and large the finding that, properly treated, the character of the remnant remains constant over a very wide range of conditions, has been confirmed by later evidence (see eg Gordon-Smith, 1969; Jex & Magdaleno, 1969; Jex, Allen & Magdaleno, 1971; and de Jonge & van Lunteren, 1972). The shape of the noise spectrum does depend to some small extent though, on whether the operator is faced with a position, rate or acceleration control. Also, the remnant is not just a proportional component of system error, because it does not simply disappear when target motion is completely absent (Sutton, 1957, and McDonnell & Jex, 1967). Levison (1970, 1971) later amplified his model to include an irreducible, or threshold component of noise. Whether it still has the same spectral composition when there is no system input has not been fully tested, but first indications are that even in these circumstances differences are not very great.

3.3.2 Modelling implications

From the properties of the remnant just reviewed it seems that the problems of modelling the random element of the human response could be more tractable than one might at first suppose. If a model can be found which reasonably predicts the determinate (or average) response then,because of the proportionality property, relatively straightforward modification or extension should predict the error variance. Furthermore, although it will vary according to the details of the controller and the tracking plant, the spectral composition of the error (and hence its autocorrelation properties) should remain constant over a very wide range of target motions.

3.4 Proposed modelling procedure

3.4.1 The question of closed-loop system identification

Up to now we have been discussing the operator as though he were part of an open-loop system, with his output (through the medium of the controlled plant) having no effect on his input. In practice, of course, there is a feed-back loop: the operator's output is subtracted from the input and then displayed to the operator as error (in real terms the gunner's sight moving in response to his joystick input; the deviation of the target from the centre of his graticule then being apparent to him; he then attempting to null this error; and so on). The general closed-loop arrangement - which could be, say, a manufacturing process - is shown in Figure 10, which in turn is loosely based on the Figures 1 of the two papers by Box & MacGregor (1974, 1976). The 'controller' is some sort of feedback scheme designed to minimise e_t - the deviation of the process output, z_t, from some target value, z_{opt}.

The usual system identification problem, see eg Akaike (1971), Aström & Eykhoff (1973) and Box & MacGregor (1974), is to discover the nature of the process dynamics so that the optimal feedback scheme (ie controller) can be designed. The use of open-loop methods in closed-loop situations will in general lead to a wrong identification. As Chatfield (1977) has put it in a recent review of time series analysis,

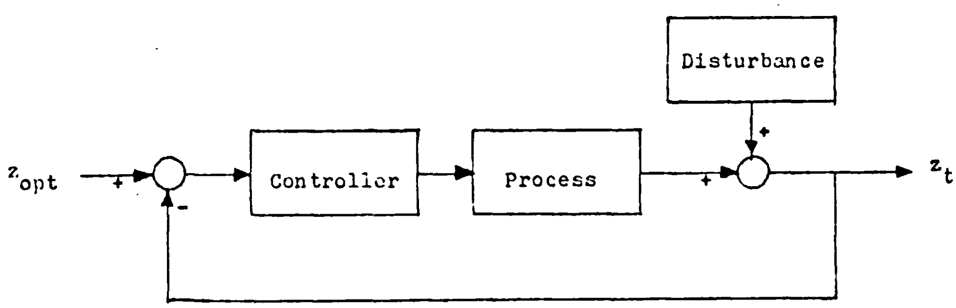


FIGURE 10. Schematic closed loop system.

from being thought of as an impossible task the identification of linear systems in the possible presence of feedback has progressed to become a popular research topic.

It should be noted, however, that in the present case we are not really involved in system identification in the true sense. We are instead merely concerned with describing the behaviour of the whole loop, and establishing regularities between target motion and consequent tracking error. Furthermore, a representation of the tracking situation which has the human operator simply reacting mechanically to the presented error (and this is a representation almost always put forward in diagrammatic form in texts on human tracking) is almost certainly misleading. Even with a compensatory tracking set up, the man has in principle available to him all the information necessary to make a perfect reconstruction of the target course. The operator can recover the target course details from the error history (although in practice he will naturally do this imperfectly) and use this for his prediction. It seems valid in our case, therefore, to regard the determinate portion of operator response as being open loop for descriptive purposes. The stochastic portion is clearly closed loop (and indeed feedback theories of motor regulation have been accepted for some time: see, eg Adams, 1971). To our (open-loop) transfer function model of determinate operator reaction we must add a model for the remnant which will be appropriate to a closed-loop system. The procedure put forward by Pandit and Wu (1977) is appropriate to either an open-loop or closed-loop set up, and it will form the basis of our approach here.

3.4.2 Transfer function modelling

The lessons we have drawn from our review of previous work have been set out in section 3.2.4, chief among these being the need for simplicity. In our application we shall be more than happy to settle for a discrete model, which will lend itself most conveniently to the regular sampling routine of the fire control computer. If elementary adaptive features are to be examined this is perhaps best done through the medium of the discrete impulse response. If this approach is adopted the simplest way to allow for some degree of variation of model properties with change of target motion characteristics (although such a scheme cannot cope with tracking plant variation) is to permit the addition of second - order terms in the model. The expanded model then becomes the quadratic form:

$$z_t = \underline{X}'\underline{HX} \qquad (10)$$

$$\underline{X}' = (1, x_t, x_{t-1}, \ldots, x_{t-m}, y_t, \ldots, y_{t-m})$$

$\underline{H}$ being an upper triangular matrix of weights or coefficients and x_t and y_t being target positions in the two planes. It is assumed that a finite (m+1) number of sampling instants are included in the model. In practice there will be a different weight matrix for the output in each plane. Also, one would expect to use absolute target positions as inputs in the plane normal to the output being predicted, it being assumed that interactions due to these position changes depend on the extent of motion, rather than on its sign. The extended model just outlined is, of course, linear in the coefficients and can be fitted by ordinary least squares.

Having accepted this expanded model as our starting point for the determinate portion of operator response, three problems of estimation immediately present themselves: correlated and non-homogeneous errors; multicollinearity of the data; and the multiplicity of parameters to be estimated. It is precisely these sorts of problems, resulting in unstable parameter estimates, which have led writers such as Jenkins & Watts (1969) and Box & Jenkins (1970) to advocate modelling approaches other than the evaluation of the impulse response function (and this has already been alluded to in section 3.2.1). However, not only is the impulse response function representation the simplest one in which to incorporate higher-order terms (which, nevertheless, exacerbate the problems just enumerated in turn) but comparative evaluations of this and other methods which have been used in the tracking field have shown that if anything it out-performs its competitors (Shirley, 1970, and Taylor, 1969). Even so, it will obviously be wise to proceed with caution.

Expressed in the usual statistical fashion our model will be

$$z_t = \sum b_i u_i + \varepsilon_t$$

where the b's represent the elements of the upper triangular portion of $\underline{H}$ arranged in vector form and the u's are the corresponding upper triangular elements of the matrix $\underline{X}'\underline{X}$. The final term is the random element of the operator's response. In matrix notation we have

$$\underline{z} = \underline{Ub} + \underline{\varepsilon} \tag{11}$$

where $\underline{z}$ is a vector of observed responses at all the sampling instants, $\underline{U}$ is the matrix of input variables (target positions and crossproduct terms), $\underline{b}$ is the vector of unknown coefficients, and $\underline{\varepsilon}$ is the vector of instantaneous 'remnant' errors.

The ordinary least squares (OLS) solution for $\underline{b}$ is

$$\underline{b} = (\underline{U}'\underline{U})^{-1}\underline{U}'\underline{z} \tag{12}$$

and this solution can be shown to be unbiased and of minimum variance if the usual assumption is made that the ε's are identically and independently distributed with mean zero and variance σ^2. That is to say:

$$\underline{E}(\underline{\varepsilon}) = \underline{0}\ , \qquad \underline{V}(\underline{\varepsilon}) = \underline{I}\ \sigma^2$$

Our review of past results has already shown that the assumption of uniform variance is unrealistic (the ε's being scaled in some way with systematic error) and that the covariance terms will be non zero (errors at adjacent sampling instants being positively correlated). If this is so the OLS solution will still be unbiased, but it will not have minimum variance. For the latter we need some form of weighted least squares (see, eg Draper & Smith, 1966)

If we assume instead that the error variance has the form $\underline{V}(\varepsilon)=\underline{V}\sigma^2$ where $\underline{V}$ is an arbitrary matrix, not necessarily diagonal in form, then it can be shown (Aitken, 1935) that it is possible to find a unique non-singular symmetric matrix $\underline{P}$ such that $\underline{P}'\underline{P} = \underline{V}$, and the minimum variance estimate of $\underline{b}$ is given by

$$\underline{b} = (\underline{U}'\underline{V}^{-1}\underline{U})^{-1}\underline{U}'\underline{V}^{-1}\underline{z} \tag{13}$$

In our case, of course, the matrix $\underline{V}$ (although sparse) would be extremely large. Not only is the use of this particular weighting scheme impractical on these grounds, but Obenchain (1975) has shown that it can produce curious (and pathalogical) patterns in the residuals, especially when the fitted model is incorrect. Obenchain advocated instead the weight matrix $\underline{T}$ where

$$\underline{T}^2 = \text{Diag}(\underline{V})$$

The elements of $\underline{T}^{-1}\underline{z}$ have uniform variance (but are not in general uncorrelated) and preserve certain optimality properties in the residuals (which are spelt out in detail in the original article). A straightforward weighting scheme therefore suggests itself. Let us suppose that the extended model is to be fitted to each experimental subject's results in turn, and that each subject undertakes n tracking runs with each target motion. Then at any elapsed time from the start of that target target motion the sample of n tracking records will have an instantaneous mean m_t and variance s_t^2:

$$m_t = \frac{1}{n} \sum_{i=1}^{n} z_{it} \tag{14}$$

$$s_t^2 = \frac{1}{n-1} \sum_{i=1}^{n} (z_{it} - m_t)^2 \tag{15}$$

Suitable weights would then be:

$$w_t^2 = \frac{1}{s_t^2} \tag{16}$$

and weighted estimates of $\underline{b}$ would be given by:

$$\underline{b} = (\underline{U}'\underline{w}\,\underline{U})^{-1}\underline{U}'\,\underline{w}\,\underline{z} \tag{17}$$

Having addressed the problem of correlated and non-homogeneous errors we must turn to the multicollinearity question - the possible high degree of dependence among the input variables. Strong linear relationships, and the near degeneracy of the matrix $(\underline{U}'\,\underline{w}\,\underline{U})$ which this implies, will result in unstable coefficient estimates with high variance. These estimates in consequence may be far from the true values. There are two ways to tackle this. The first is to recast our model in a form which will reduce the degree of dependence between the input variables. If one considers the (restricted) impulse response

$$z_t = h_0 x_t + h_1 x_{t-1} + h_2 x_{t-2} + \ldots$$

then it is obvious that in general the target positions at adjacent sampling points x_t and x_{t-1} will be highly correlated. If instead we write the model in equivalent form

$$e_t = x_t - z_t = h'_o + (h'_1\nabla + h'_2\nabla^2 + \dots)x_t \tag{18}$$

(where ∇ is the backward difference operator $\nabla x_t = x_t - x_{t-1}$) then the matrix of input variables will be much more nearly diagonal, even though it will not be possible to reduce the off-diagonal values completely to zero.

In recent years a great deal of attention has been given to the use of biased estimates where there are appreciable correlations among the input variables. The methods put forward include 'shrunken estimates' (Stein, 1960; Sclove, 1968); latent root regression (Hawkins, 1973; Webster, Gunst & Mason, 1974); principal components regression (Massy, 1965; Jeffers, 1967; Lott, 1973;Greenberg, 1975) which concept has been extended to the generalised inverse of fractional rank by Marquardt (1970); ridge regression (Hoerl & Kennard, 1970); and generalised ridge regression (Hoerl & Kennard, 1970; Hemmerle, 1975). Although the (appropriately weighted) least squares solution has minimum variance among all linear unbiased estimators, this last requirement carries with it the unfortunate penalty that this minimum variance can be extremely large as $(\underline{U}'\underline{W}\underline{U})$ moves from a near - diagonal matrix to an ill-conditioned one. The essence of the procedures just mentioned is that by permitting a small amount of bias the variance of the coefficient estimates is decreased, so that the coefficients may predict and extrapolate better in most cases than those produced by unbiased least squares. A review of the results and theory of biased estimation is beyodd the scope of this paper, but Marquardt & Snee (1975) have reviewed the use of generalised inverse and ridge regressions in practical situations and simulated experiments, and have shown that they do seem to possess the advantages claimed for them. Hocking, Speed & Lynn (1976) have made an analysis of many of the different biased estimators which have been put forward, and have shown that on theoretical grounds the generalised ridge procedure is superior to all others, although ordinary ridge regression is likely to be close behind.

Let us suppose that the usual least squares regression problem (ignoring possible weighting for convenience, and as previously given)

$$z = \underline{Ub} + \underline{\varepsilon} \qquad (11)$$

has been scaled so that $\underline{U'U}$ is in correlation matrix form. The ordinary least squares solution for $\underline{b}$ is

$$\underline{b} = (\underline{U'U})^{-1}\underline{U'y} \qquad (12)$$

The ordinary ridge estimates $\underline{b}^*$ are given by

$$\underline{b}^* = (\underline{U'U} + k\underline{I})^{-1}\underline{U'y} \qquad (19)$$

The criteria by which the value of the constant k is fixed are not straightforward (see the review by Hocking, 1976). Usually the choice of k is determined by a qualitative examination of the 'ridge trace' - a pictorial representation of the way that the individual components of $\underline{b}^*$ vary as a function of k. Fortunately, Marquardt & Snee (1975) report that the choice of k does not seem to be too critical in practice.

The generalised ridge procedure avoids any judgemental issues so long as an estimate of the residual standard deviation is available. This procedure reduces $\underline{U'U}$ to diagonal form by applying an orthogonal transformation $\underline{P}$. If $\underline{\Lambda}$ is a diagonal matrix whose elements are the latent roots of $\underline{U'U}$ we have

$$\underline{P}(\underline{U'U})\underline{P'} = \underline{\Lambda}$$

and writing $\underline{U}^* = \underline{UP'}$, $\quad \underline{a} = \underline{Pb}$

the usual least squares model given above can be rewritten as

$$\underline{y} = \underline{U}^*\underline{a} + \underline{\varepsilon} \qquad (20)$$

and the generalised ridge estimate, $\hat{\underline{a}}^*$ is

$$\hat{\underline{a}}^* = [(\underline{U}^*)'(\underline{U}^*) + \underline{K}]^{-1}(\underline{U}^*)'\underline{y} \qquad (21)$$

where $\underline{K}$ is a diagonal matrix with non-negative elements Given an estimate of the residual variance σ^2 the values of the k's which minimise

$$E(L_1^2) = E[(\underline{a}^* - \underline{a})' \ (\underline{a}^* - \underline{a})]$$

are given by the solutions to

$$k_i = \sigma^2/a_i^2$$

An iterative procedure for obtaining the solutions is given by Hoerl & Kennard (1970) and a closed form solution by Hemmerle (1975).

It seems sensible, then, to use both ridge and generalised ridge methods in this study. While the main motive for their employment resides in minimising the effects of multicollinearity, they have a role to play, too, in selecting a best subset of regressors from a large collection of possible input variables.

Turning now to this latter problem, then it is obvious that the extended impulse response human operator model which has been outlined is profligate in the extreme with the numbers of parameters which must be estimated. Even if the impulse response is based on some finite number, m, of sampling instants, the complete model would have (m+1)(2m+1) coefficients. It should certainly be possible to choose a limited subset of input variables, 'best' in the sense of producing a multiple correlation coefficient with the output nearly equivalent to that using the entire set, and better than that using any alternative subset of the same size. The usual subset selection procedure is the 'stepwise' approach outlined by Efroymson (1960) (see also Draper & Smith, 1966) in which variables are added one at a time in the order in which they contribute to the sum of squares accounted for by regression. It would not be appropriate to go into details here, but there are several unsatisfactory features about the stepwise procedure and it can miss the most important variables (see, eg, Mantel, 1970, and the discussion in Hocking, 1976). Two alternative approaches have been suggested: search all possible regressions for the 'best' subset of a specified size using routines such as those developed, eg by LaMotte & Hocking (1970), Furnival & Wilson (1974) and Hocking (1977); and the use of ridge or generalised ridge to aid in selection. The first approach is conceptually simple, although ingenuity is required in order to evolve an efficient search procedure so that search times stay within practicable bounds. In the present study it is proposed to make use of a routine based on Furnival & Wilson's (1974) 'leaps and bounds' algorithm. The employment of ridge procedures in the variable selection role is based on the premise that variables whose coefficients shrink markedly or change sign as the constant k is increased (ordinary ridge) or which have small coefficients (generalised ridge) contribute little to prediction in practice. Hocking (1976) has pointed out that there seems to be much promise in combining the two approaches.

Having reviewed the problems of linear least squares, and the methods used to attack them, it is now possible to outline the procedures actually to be used for data analysis in this study. Transfer function models will be fitted to each subject individually, based on the mean tracking errors for all targets. The full experimental routine, the targets to be used, etc, will be spelled out in the next chapter. The extended impulse response function will be cast in the form

$$e_t = \underline{X}'\underline{HX} \qquad (22)$$

where

$$\underline{X}' = (1, \nabla x_t, \nabla^2 x_t, \nabla^3 x_t, \nabla^4 x_t, \nabla y_t, \nabla^2 y_t, \nabla^3 y_t, \nabla^4 y_t)$$

the model being defined in terms of time increments of 0.2 sec. The following routine will be adopted:

(a) Calculate the weighted products and crossproducts for the solution of the extended model defined in equation (22). Rescale in correlation matrix form.

(b) Examine the ridge trace and the coefficients yielded by the generalised ridge procedure (with estimates of error variance for the latter based on degrees of freedom equivalent to from 1.5 to 4 independent samples a second).

(c) From a qualitative judgement based on (b) select no more than 16 variables to enter the Furnival & Wilson 'best subset' routine (the upper limit on numbers of variables being set by the limited core storage of the computer used).

(d) Collate the results from stage (c) for all the different subjects to propose a limited model adequate for the actual subject samples used.

It will be noted that variable selection is based on judgement rather than formal statistical test. Watson (1955), Watson & Hannan (1956) and Vinod (1976) have considered the effects of correlated errors on significance tests for regression coefficients. However the testing of significance is not a crucial issue for model building in our context. Since our model will be far from perfect it would be found in practice that strict statistical procedures would require the use of quite a large number of variables to describe the ensemble of results for one individual. Our chief problems, however, are those of maintaining parsimony (describing operator response in as simple a fashion possible, yet including features of practical - as opposed to statistical - significance) and of generalising to a wide range of subjects (and - a problem we shall briefly address later - of possible stress conditions).

3.4.3 'Remnant' modelling

Having decided on a means of representing the regular determinate portion of human response it is now necessary to develop a scheme for describing the random stochastically varying error which is superimposed. The natural extension of the linear transfer function to the randomly varying portion of the response is to model the latter as an Autoregressive Moving Average (ARMA) disturbance. An ARMA model of order p,q can be represented as:

$$\varepsilon_t = \phi_1\varepsilon_{t-1} + \phi_2\varepsilon_{t-2} + \dots + \phi_p\varepsilon_{t-p}$$

$$- \theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_q a_{t-q} \qquad (23)$$

where

ε_t is the stochastic part of the output

$\phi_1, \phi_2, \dots, \phi_p$ are the Autoregressive (AR) parameters

$\theta_1, \theta_2, \dots, \theta_q$ are the Moving Average (MA) parameters

The a_t's may be regarded as random shocks which enter the system at time t, and they are usually assumed to be normally distributed with mean zero and constant variance σ^2. The question of the distribution of the a's is, however, one which will be returned to later. Combinations of deterministic and stochastic response models have been used in fields other than human operator modelling, and the scheme proposed by Pandit, Goh & Wu (1976) for representing paper-making process data closely parallels that put forward here. Mention was made in section 3.4.1 of a method put forward by Pandit & Wu (1977) for fitting ARMA models in the presence of feedback. This method fits ARMA models of increasing complexity to operating data, judging the statistical significance of the contribution made by each extra term at each stage. However, while the principle of successive approximation is accepted, circumstances will force us to a more cautious and less formal approach than that adopted by Pandit & Wu.

Just as with the transfer function portion of the model, we shall hope to keep the noise portion to the lowest level of complexity possible. Autoregressive parameters can be fitted by linear least squares, but the inclusion of one or more moving average parameters forces one to a non-linear fitting procedure. The fitting of a nonlinear scheme to all the data represented by the individual tracking runs is an undertaking too horrendous to contemplate except in circumstances of direst need. Once again, it does not seem appropriate to appeal to formal statistical tests to decide what order of model to fit. Considerable attention has been paid to the development of such tests (see, eg Quenouille, 1947; Whittle, 1952; Box & Pierce, 1970; Pierce, 1972; Ljung & Box, 1978; and McClare, 1978) but they all depend on the assumption that the a's are uniformly distributed, and one of our major concerns will be to put forward a model which will account for and predict the changing variances of the shocks or impulses which enter the system. The development of the model for the stochastic 'remnant' therefore has two strands: firstly, to account for the dependencies between random errors at successive points in time; and, secondly, to account for the changing distributions of these errors.

The method suggested for fitting the ARMA parameters is to add the autoregressive variables one at a time (using linear least squares), judging the extent by which each new addition improves the goodness of fit by the extent to which it reduces the residual sum of squares. Only if the second AR parameter makes a practically significant reduction in this residual will the addition of an MA parameter (necessitating a nonlinear fitting routine) be contemplated. No obvious method of weighting in order to stabilise residual variances presents itself. (It should be said that there are other methods of model identification which have been put forward, and the cycle of tentative identification and successive improvement put forward by Box & Jenkins (1970) has gained wide acceptance. It could be used in our case if reasonably long intervals were found amongst our own data which could reasonably be described as near-stationary. But, to anticipate, a close inspection of our results has failed to reveal any such intervals.)

What scheme might account for the way that the variance of the added shocks appear to change as a function of target motion or of systematic error? As mentioned in section 3.3.1, one of the earliest findings of experimental psychology was that human errors in response to stimuli which are above the absolute threshold tend to be in rough proportion to stimulus magnitude (Weber, 1834, and Fechner, 1860). The Weber-Fechner law, as it has come to be known, has the form:

$$\sigma/S \simeq k \tag{24}$$

where σ is the standard deviation of human response and S is the stimulus magnitude. In practice it has been found that this constancy breaks down for very small values of the stimulus, and so the law is often re-written:

$$\frac{\sigma}{S + k_1} = k_2$$

and so

$$\sigma^2 = k_2^2(k_1 + S)^2$$

In truth it is doubtful whether this relationship has been established with sufficient accuracy for one to be dogmatic about its precise form, and it is probably adequate to regard the response variance as having two independent components: a constant absolute threshold term, and another which has a standard deviation proportional to stimulus magnitude, so

$$\sigma^2 = \sigma_o^2 + S^2\sigma_1^2 \tag{25}$$

Now in section 3.3.1 it was pointed out that Levison and his co-workers (Levison & Kleinman, 1968; Levison, Baron & Kleinman, 1969a, 1969b) in their treatment of the 'remnant' found that the stochastic component of the error scaled with the man's systematic error (although Levison, 1970, later added a further 'threshold' error term independent of systematic response). Furthermore, they proposed that although there may be many contributory causes for this error, in practice it would be impossible to disentangle them, and that they were best lumped together and treated as 'observation noise' injected prior to the motor and mechanical elements in the loop. Although the reasons for adopting this approach were pragmatic, recent findings of experimental psychology have underlined the primacy of visual feedback in regulating motor behaviour, rather than kinaesthetic feedback which was emphasised in the past (see, eg Laszlo, Shamoon & Sanson-Fisher, 1969; Laszlo & Baker, 1972; Jones, 1974; Stelmach & Kelso, 1975; Adams, Gopher & Lintern 1977; and Frank, Williams & Hayes, 1977). The most relevant results concerning the errors made by the human operator in judging visual subtenses in gunnery systems are those collected by early workers in the field (Bates, 1944; and Craik, 1948). These are entirely consistent with the formulation given above. Errors in reproducing pressures or movements may still be implicated to some extent. These have been studied by Jenkins (1947) with pressure controls and by Brown, Knauft & Rosenbaum (1948), Weiss (1954, 1955) and Bahrick, Bennett & Fitts (1955) with moving controls. Once again the results follow the general pattern which has been outlined. The collected results indicate that, while the values of the constants depend quite markedly on the particular situation being investigated, the general form of the relationship between human error and stimulus magnitude is applicable over a wide range of circumstances.

It is now possible to postulate a simple scheme to account for the variability of tracking errors as a function of target course characteristics, and hence of systematic error. If visual feedback is indeed the major factor influencing motor response, then the relevant stimulus should be the displayed error one reaction time prior to a corrective action. To a first approximation the extent of this corrective movement or pressure should also be proportional to the displayed error. The relationship governing the variance of the shocks entering the system at time t should thus be:

$$\sigma^2(a_t) = \sigma_o^2 + \sigma_1^2 e_{t-1}^2 \qquad (26)$$

where e_t is the total error at time t, and assuming that the effective reaction time approximates to one sampling period. (For the present, to avoid too much complication we shall ignore the probable contribution of corrective motion in one plane to random error in the other.) It will be as well to trace through some of the logical implications of this proposal. Strictly speaking we are saying that the shock entering the system at time t has two components, a_o and a_1, and these are random variables independently (and, we assume, approximately normally) distributed with zero mean and variances σ_o^2 and σ_1^2. a_t is a weighted sum of these two variables

$$a_t = a_o + a_1 e_{t-1} \tag{27}$$

where e_{t-1} is the total error at time t-1. The total error is itself made up of the systematic human response, which we shall now denote by $f(x_{t-1})$, and the stochastic element ε_{t-1}, and so

$$a_t = a_o + a_1\left(f(x_{t-1}) + \varepsilon_{t-1}\right) \tag{28}$$

and the expression for the variance will be

$$\sigma^2(a_t) = \sigma_o^2 + \sigma_1^2 f^2(x_{t-1}) + \sigma_1^2\sigma^2(\varepsilon_{t-1}) \tag{29}$$

Let us now assume for the sake of exposition that a first-order autoregressive process accounts for the dependencies between successive ε's (although we shall keep an open mind as to whether a more complex scheme may be demanded by the data)

$$\varepsilon_t = \phi\, \varepsilon_{t-1} + a_t$$

and so

$$\sigma^2(\varepsilon_t) = \phi^2\sigma^2(\varepsilon_{t-1}) + \sigma^2(a_t)$$

$$= \sigma_0^2 + \sigma_1^2\ f^2(x_{t-1}) + (\phi^2 + \sigma_1^2)\ \sigma^2(\varepsilon_{t-1})$$

From the limited development of the remnant variance scheme carried out this far it is easy to see that the system will be unstable unless σ_1^2 is small, and certainly stability boundaries will be crossed if its value exceeds unity. σ_1 corresponds to the traditional 'Weber fraction' (strictly speaking, the asymptotic value) which is generally well below unity. Bates' (1944) results indicate a value in the region of 0.04 for the judgement of angular subtense, although the

'Weber fraction' for movement and pressure reproduction seems to be a bit higher. In addition the pressure of time in continuous tracking, with responses successively strung together, could result in still higher Weber fractions.

The full procedure for identifying the noise model for each operator is now proposed as follows:

(a) At each sampling instant compute the determinate response $f(x_t)$. For each individual tracking record subtract this from the observed error to yield the 'remnant' errors:

$$\varepsilon_t = e_t - f(x_t)$$

(b) Develop a linear model to account for the dependencies in the ε's. Use an unweighted linear least squares analysis to fit successive AR parameters:

$$\varepsilon_t = \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} + \ldots + a_t$$

(c) Review the results of (b) to decide whether a low order AR model is adequate, or whether the addition of MA parameters is indicated. If an extended scheme seems to be demanded by the data then ARMA models of order (2,1), (3,1), (3,2), (4,2), ... should be fitted successively, using a nonlinear least squares routine.

(d) Having decided on a dependency scheme, develop a model to account for the nonstationary character of the random noise. Use unweighted linear least squares to fit

$$a_t^2 = (\varepsilon_t - \phi_1 \varepsilon_{t-1} - \phi_2 \varepsilon_{t-2} - \ldots)^2$$
$$= \sigma_o^2 + \sigma_{x1}^2 \varepsilon_{x\,(t-1)}^2 + \sigma_{y1}^2 \varepsilon_{y(t-1)}^2$$

(y referring to the plane normal to x). In practice this stage to be preceeded by an exploratory phase to determine such details as effective reaction time.

Our proposed modelling procedures are now complete. It remains to test them against experimental data. Before doing so it should be pointed out that the model fitting procedure in respect of the 'remnant' is far from ideal. The trouble lies in the fact that the residuals from fitting the determinate transfer function are not the 'true' ε's. They are, in effect, somthing like a collection of residuals which approach the average of the 'true' ε's (Kendall, 1975). The estimated values of the dependency coefficients ϕ (and Θ if required) will be inflated due to lack of fit of the transfer function portion of the model. The terms which account for the varying standard deviation of the 'shocks' as a function of systematic error will also be affected. The correlation between these (residual) standard deviations and the tracking error at the previous sampling instant will be less than the 'true' correlation. The result will be to underestimate the values of the σ_1 terms and overestimate the σ_o terms. Unfortunate though this may be, the use of more efficient methods (such as those of Box & Jenkins, 1970; Pierce, 1971; and Harvey & Phillips, 1979) is precluded by the nonstationary character of the noise process which we are striving to represent. This matter will be considered further when the results are examined in the next chapter.

4. HUMAN OPERATOR MODELLING: EXPERIMENTAL AND MODELLING RESULTS

4.1 Introduction

From our review of previous modelling work we have deduced that the prospects are good for developing a relatively simple descriptive scheme for the gunner's tracking response. However, even if we were to accept a model precisely in the form that other authors have devised, there would still be a need to estimate its parameters afresh for the sorts of target motion and kind of tracking plant representative of tank gunnery. We shall in any case wish to develop a model in a form convenient for our own application, and paying more attention than has been usual in the past to the random-appearing 'remnant' portion of human response. To do this we need a laboratory simulation which will reproduce the main features of the tank gunnery set up, and experimental results from a sample of operators in some way representative of tank gunners.

4.2 Experimental equipment

In essence the equipment consisted of a Sperry-Univac V76 computer, which generated target courses and controlled the sequencing and detailed timing of the experiment; a Hewlett-Packard CRT, on which the target was displayed; and two thumb joysticks (one pressure and one movement) through which media the operator attempted to track the target. The computer was linked to the joystick and display by suitable digital-analogue input-output channels. The pressure joystick was based on a Measurement System joystick (Model 465L/1), and the movement joystick on a BAC Swingfire Controller, both being mounted in a Swingfire control housing with a specially constructed handgrip. In Experiments 4 and 5 a further hand-grip was provided, with two thumb-operated buttons on it which the operator used to animate the display and (nominally) to 'lase'.

The target was an outline drawing of a Russian T72 tank, sideways on and scaled so that the dimensions corresponded to such a target seen at 2000m through a X10 magnifying sight. The aiming mark consisted of an inverted triangle, the bottom vertex of which the operator endeavoured to maintain in alignment with a designated point on the tank target. The point of aim chosen was the junction between the centre of the turret and the top of the tracks, so that it could be identified with very little ambiguity. The target and aiming mark were super-imposed on a schematic background consisting of a grid of points. The operator could move his aiming mark over the background, and the tank target moved relative to the background, the motion relationships being exactly the same as they would be with a genuine gunners's sight.

The joysticks were clamped as needed to wooden arms attached to an ordinary office chair. Operators tracked with their preferred hand, and the position of the joystick along the arm was adjusted to their convenience. The 'take control' and 'lase' buttons were similarly clamped to the arm of the chair on the non-preferred side. A straightforward first order control law was used, maximum deflection or pressure giving 200 mrad plus 500 mrad/sec at the eye (the ratio of these terms being selected from evidence presented by Frost, 1972 and Michael, 1977). The laboratory equipment was thus more representative of a high quality servo, with good frequency response in the higher range, than a system in which the gunner directly controls the turret in traverse and the gun itself in elevation. The effective resolution of the display was 0.5 mrad at the eye (0.05 mrad in simulated real space, allowing for display magnification). Except where stated, joystick output was linear with respect to deflection or pressure. Output was sampled every 0.05 sec, and the display was generated with the same timing.

The display measured 30cm wide by 26cm high and was arranged so that, when the operator placed his head against a browpad fixed to the front of the CRT, the eye-to-screen distance was 56cm (giving a horizontal angle of 30° at the eye). The browpad and screen were shielded so that when in the operating position everything except the display was excluded from view. The operator's station was placed in a small room remote from the computer and from the experimenter. Figure 11 shows an experimental subject ready to track, and Figure 12 is an oblique view of the display screen with the frontal mask removed.

Special purpose software was written to generate the target profile, aiming mark and background; to sample the joystick output and transform it to the control law indicated; to subtract the transformed joystick output from target position and background position, so that the display responded in appropriate manner to operator inputs; and to exercise detailed and overall control, timing and sequencing for the experiment. This software also computed summary statistics after each

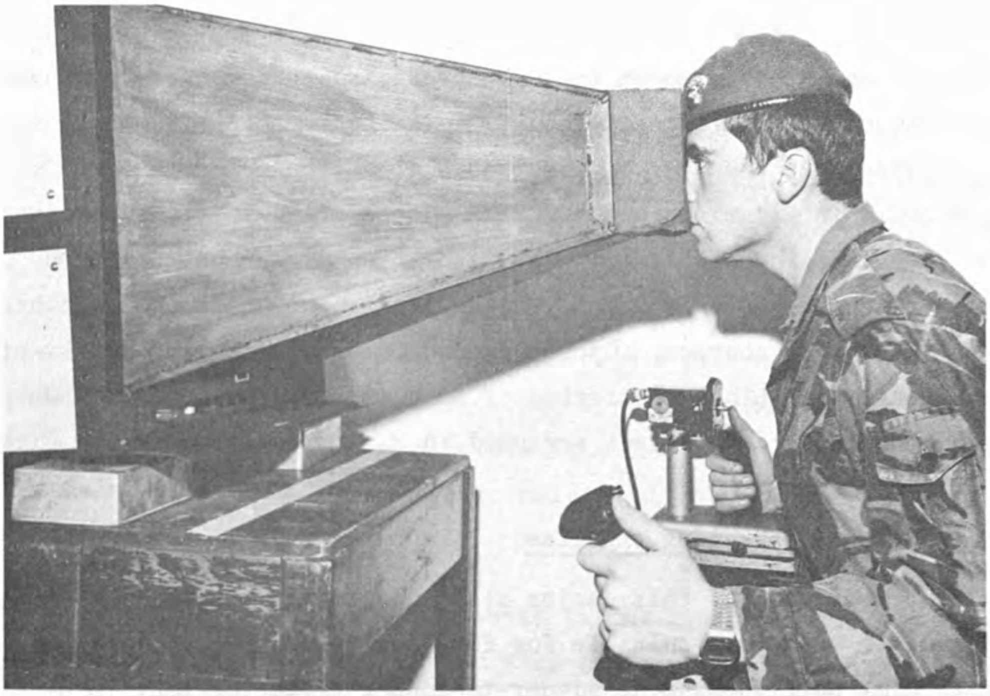


FIGURE 11. Experimental tracking station. Subject seated at controls.

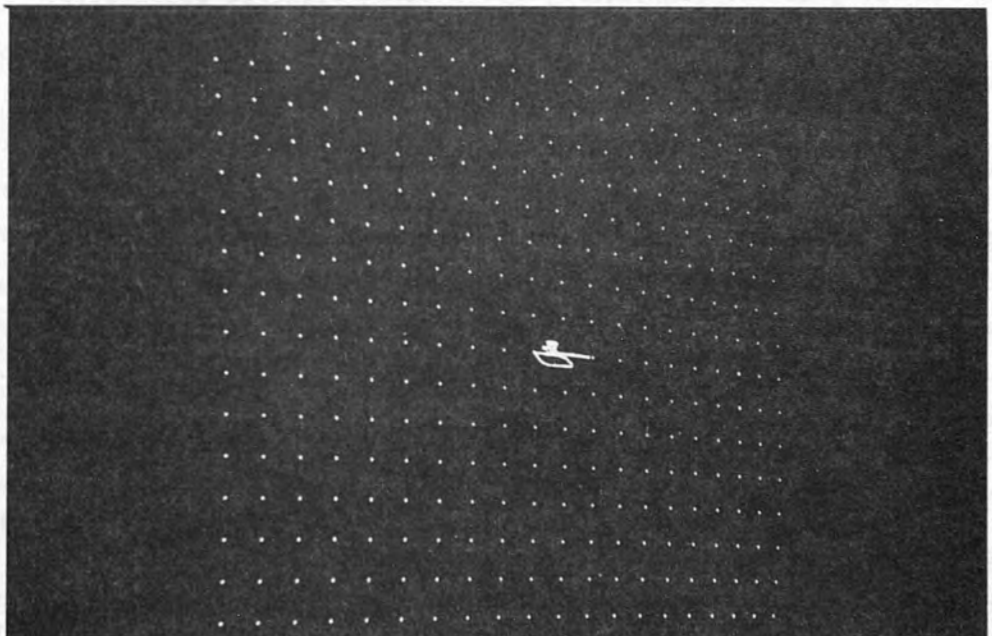


FIGURE 12. Experimental tracking station. Oblique view of display with mask removed.

tracking run and wrote them on the CRT screen, so that operators could be given immediate knowledge of results. In Experiments 1, 2 and 3 target motions were computed by a suitable algorithm prior to each tracking run. In Experiments 4 and 5 they were pre-computed according to the scheme outlined in section 2.9, stored on disc, and retrieved by the computer when required. Any given experiment utilised a set of 15 different target courses. Operators undertook a number of blocks of tracking runs, each block consisting of 30 runs, comprised of two sub-blocks of the 15 target courses arranged in random order.

4.3 Preliminary modelling exercise

The primary aim of this series of 3 experiments was to develop a tracking model in a form suitable for fire control system evaluation. Secondary aims were to examine gunner-to-gunner variation and, to a limited extent, to investigate the way in which the characteristics of the tracking plant itself might affect model parameters. The intention at this stage was to concentrate on the tracking portion of the gunner's task uncomplicated by the interacting effects of other activities in the engagement sequence (especially those which might be associated with 'lasing'). It was hoped that any such effects could be incorporated at a later stage in model development.

At the time that the experiments in this series were conducted there were no estimates available of target motion based on actual measurements of real vehicles manoeuvring in quasi-tactical fashion. Accordingly it was necessary to evolve synthetic target tracks, based on judgements of possible driving patterns as conditioned by vehicle capabilities and the tolerance of the crews. In most tracking studies in the past target motion has been represented by filtered white noise or by an admixture of a small number of sinusoids, yielding a function which is smooth in its derivatives. Due to the intervention of the driver this does not necessarily provide a good description of vehicle motion. While such motions must be smooth in position and velocity, there could well be step changes in acceleration and higher derivatives at the moment when the driver accelerates, brakes, changes gear or alters his heading. It was judged that, on average, a driver might introduce such step changes every 4 sec or so (and subsequent analysis suggests that this is very much in line with the AMSAA trials data). The target courses devised for the experiments in this series lasted for 21 sec, subdivided into successive intervals of 5, 4, 4, 4 and 4 sec. Within each interval target angular position in each axis was represented by a third order polynomial, but there could be a change in one or more of the polynomial coefficients from one interval to another. The values of the coefficients are given in Appendix B. Maximum target velocities and accelerations (at the eye) were 218 mrad/sec and 50 mrad/sec/sec respectively in x, and 40 mrad/sec and 10 mrad/sec/sec in y.

During this series of experiments target courses were presented with a fixed 9 sec interval between them (during which time the CRT screen was blank). During the first 2 sec of each run the computer, in effect, tracked the target with zero error, and so the ideal aiming point was displayed at the start of each run. The initial velocity of the target could also be judged by noting its motion relative to the background. After 2 sec control was passed to the subject, who then attempted to track the target through the medium of his joystick. Tracking errors were recorded during the interval 5 to 21 sec, and for each run a score was displayed based on the mean radial error over this 16 sec period. The 3 sec allowed from "take control" until the start of data recording was based on the average time taken to lase in the tank gunnery experiments conducted by Michael & Silverthorn (1978).

4.3.1 Experiment 1

The subjects for this experiment were 12 Infantrymen from the Trials Section of the Army Personnel Research Establishment (APRE). Prior to the experiment proper they had undertaken a minimum of 210 individual tracking runs.

The original aim of this experiment was to examine the effects of a stressor (isocapnic hypernoea induced by hyperventilation, which latter is known to be associated with fear) on tracking performance using two different types of control. The main results have been described by Speight, Withey, Labuc & Legge (1979) and it was shown that in the event the effect of the stressor was quite minor. The detailed tracking records from this experiment were therefore used for model development, and to throw light on individual differences and possible effects due to the type of control.

In this experiment all 12 subjects tracked one complete block of target runs with each control and under 3 different breathing regimes. The first regime was normal breathing; the second was hyperventilation with consequent reduction of partial pressure of carbon dioxide; and the third was hyperventilation but with added carbon dioxide to maintain this partial pressure constant. The joystick output for each type of control was linearly related to deflection or pressure. Each subject thus undertook 6 tracking runs for each of the 15 target courses: 2 under each of the 3 breathing regimes. The whole experiment was conducted to a balanced design (given in Speight et al, 1979) and great care was taken to minimise unwanted asymmetrical transfer of training when progressing from one type of control to another (Poulton & Freeman, 1966). The results from this and the following experiments will be exposed in the remainder of the chapter.

4.3.2 Experiment 2

The subjects for this experiment were 4 members of the APRE scientific staff. Once again, they had undertaken a minimum of 210 tracking runs during training.

Although useful for model development and preliminary investigation, Experiment 1 suffered from the fact that there were relatively few replicate tracking runs per target course, and, even though the different breathing regimes might not have resulted in marked differences of performance as measured by the overall miss distance statistic, they could well have introduced some variability in the fine detail of human response. It was felt advisable, therefore, to supplement the information from Experiment 1 with results based on a larger number of runs obtained under uniform conditions from a few subjects.

Once again, two kinds of control were used - movement and pressure thumb joysticks. However in this case the pressure joystick had a non-linear output. The voltage output v_o from the joystick (which was linearly related to the pressure placed upon it) was transformed according to the relationship

$$v_t = k\ (0.35\ v_o + 0.65\ v_o^3)$$

where v_t is the transformed voltage and k is a constant so arranged that maximum pressure yields the same voltage in both the transformed and untransformed case.

The reason for including a nonlinear joystick output is that this arrangement is frequently used in tank gunnery systems. The theory is that rapid acquisition of a target is facilitated by using the sensitive portion of the joystick response to make gross corrections, and at the same time accurate tracking after acquisition is aided by using the low gain portion of response to make small-scale corrections. (In practice informal experiments suggest that 'settling' on the target once the initial major correction has been made is difficult with a nonlinear law, and furthermore, since the gunner has no stable expectation of what his control response might be, the prospective advantage in steady state tracking is not in fact realised). In any event, it is obviously desirable to check whether such nonlinearity alters the form of model to an appreciable extent.

Subjects 2 to 4 each undertook 12 tracking runs per target course (6 tracking blocks) on each control. Subject 1 undertook double this number of tracking runs, his results being divided at random into 2 subsets in order to check on model stability within an individual. The controls were tackled in an unbalanced order, the movement control first, followed (after 2 tracking blocks for re-training) by the pressure control. The experiment was designed only to point up differences in model form, and certainly should not be taken as an unbiassed comparison of movement and pressure controls.

4.3.3 Model development (transfer function)

In section 3·4·2 it was decided that we should attempt to fit an extended impulse response function to the data in order to represent the regular features of gunner reaction. This would take the form

$$e_t = \underline{X}' \, \underline{H} \, \underline{X}$$

where

$$\underline{X}' = (1, \nabla x_t, \nabla^2 x_t, \nabla^3 x_t, \nabla^4 x_t, \nabla y_t, \nabla^2 y_t, \nabla^3 y_t, \nabla^4 y_t) \qquad (22)$$

$\underline{H}$ being an upper triangular matrix of coefficients and e_t being the expected value of the tracking error at time t. We would attempt to reduce the number of variables in the expression by using best subset, ridge and generalised ridge procedures.

The first casualties in the variable reduction programme for each operator were the vast bulk of the crossproduct and square terms in the prediction equation. In practice it was found that they added virtually nothing to the goodness of fit. Furthermore, although one or two individual terms from this subset improved the accuracy of fit by one or two percent for a few particular operators, there was no consistency in the identity of the crossproduct or squared terms which had this minor beneficial effect. The negligible role of squared terms is not so surprising, but the failure of movement in one plane to interact with response in the other is perhaps less to be expected. Taken over all operators the straightforward linear impulse response, based on four sampling instants spaced 0.2 sec apart, was a perfectly adequate description of the data:

$$e_{xt} = b_{xo} + (b_{x_1} \nabla + b_{x_2} \nabla^2 + b_{x_3} \nabla^3 + b_{x_4} \nabla^4) x_t \qquad (30)$$

$$e_{yt} = b_{yo} + (b_{y_1} \nabla + b_{y_2} \nabla^2 + b_{y_3} \nabla^3 + b_{y_4} \nabla^4) y_t \qquad (31)$$

Overall, the contributions of the b_2 terms were major, followed by those of the b_3 terms. Individual biases (b_o) were in general small, but one or two soldier subjects had very appreciable and consistent fixed errors. The contributions of the b_1 and b_4 terms were minor.

Applying the above 5 parameter transfer function model in turn to each subject (so that the coefficient values were allowed to differ for each) it was found that it accounted for a relatively high proportion of the observed variation in mean error (at least for experiments with humans). In very rough terms, for the APRE scientific staff the proportion of variance accounted for (with both movement and pressure controls) was nearly 80% in the horizontal plane and 55% in the vertical plane (vertical rates and accelerations averaging only about one fifth the horizontal ones). For the soldier subjects the corresponding figures were roughly 65% and 35%. There seemed to be two contributory factors to these lower figures for the military subjects: firstly, their means were based on 6 runs per target instead of 12 (and it will be remembered that for 4 of the 6 runs the subjects were hyperventilating, partial CO_2 pressures being allowed to drop on 2 of the hyperventilation runs); and, secondly, the random 'noise' element of their response seemed to be much larger than for the scientists (although the effects of hyperventilation *per se* cannot account for this).

While at first sight these results may seem to be wholly satisfactory some minor doubts appear when one considers the fine detail of operator response. The time now seems opportune to examine some visual records. Two target courses (Targets 6 and 15) have been selected from the set of 15 for illustrative purposes. Figures 13 and 14 show the patterns of acceleration for these two targets. Figure 15 shows the 5 parameter model detailed in equation (30) fitted to the average errors (over 12 runs) in the horizontal plane for these two targets for two records yielded by the APRE scientific staff. The unfilled circles represent the data and the continuous curve the model prediction. The curve follows the points reasonably well so far as general trends are concerned, but it does not seem to mimic what appears to be an underdamped, oscillatory response made by the subject. In part this periodic component seems to arise from the averaging of the stochastic noise element which is in turn random periodic in nature, and this can be shown by considering sections of target runs where there is virtually no target motion, and where the mean error is not significantly different from zero, but this quasi-periodic behaviour is

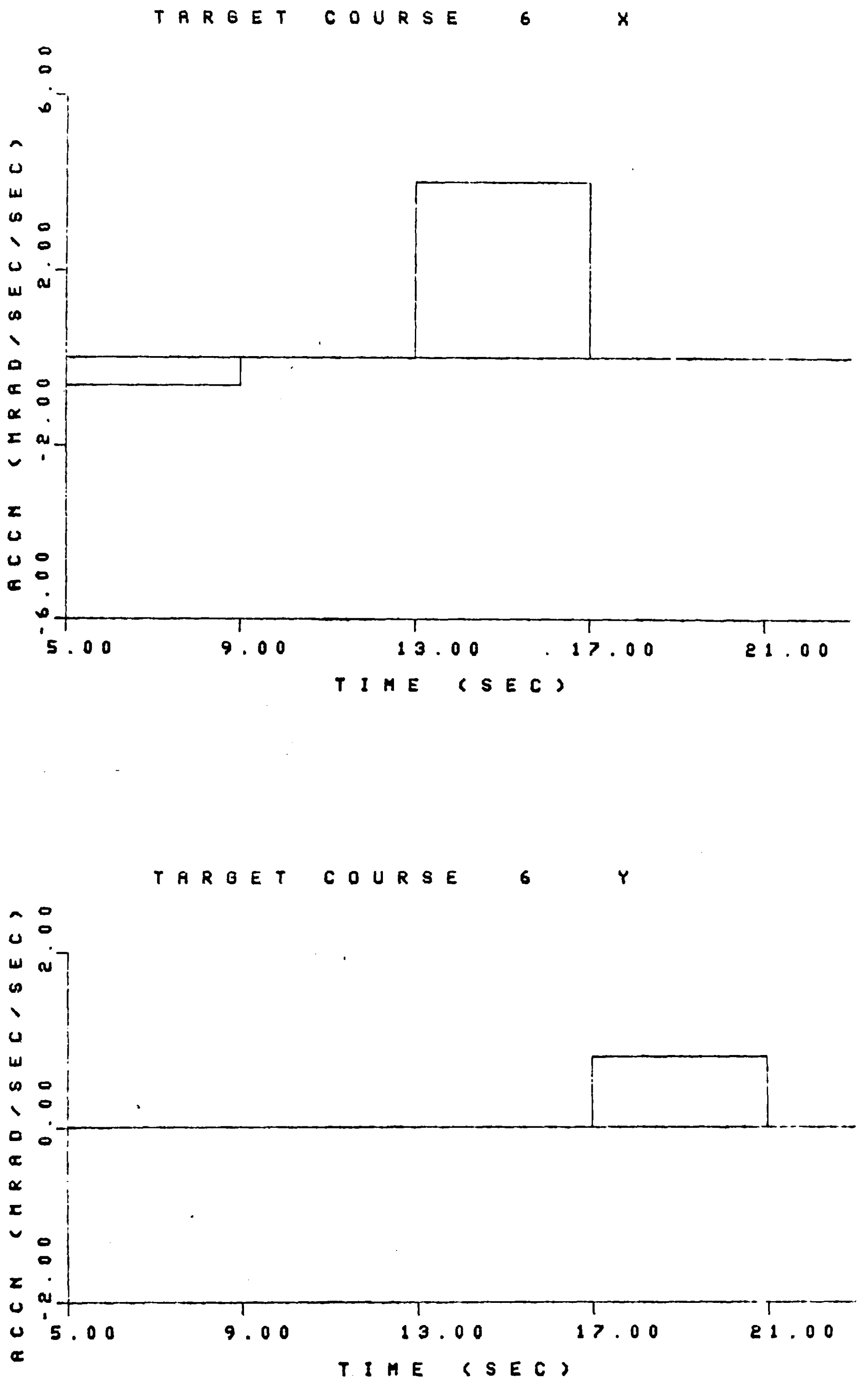


FIGURE 13. Acceleration time history, Target Course 6.

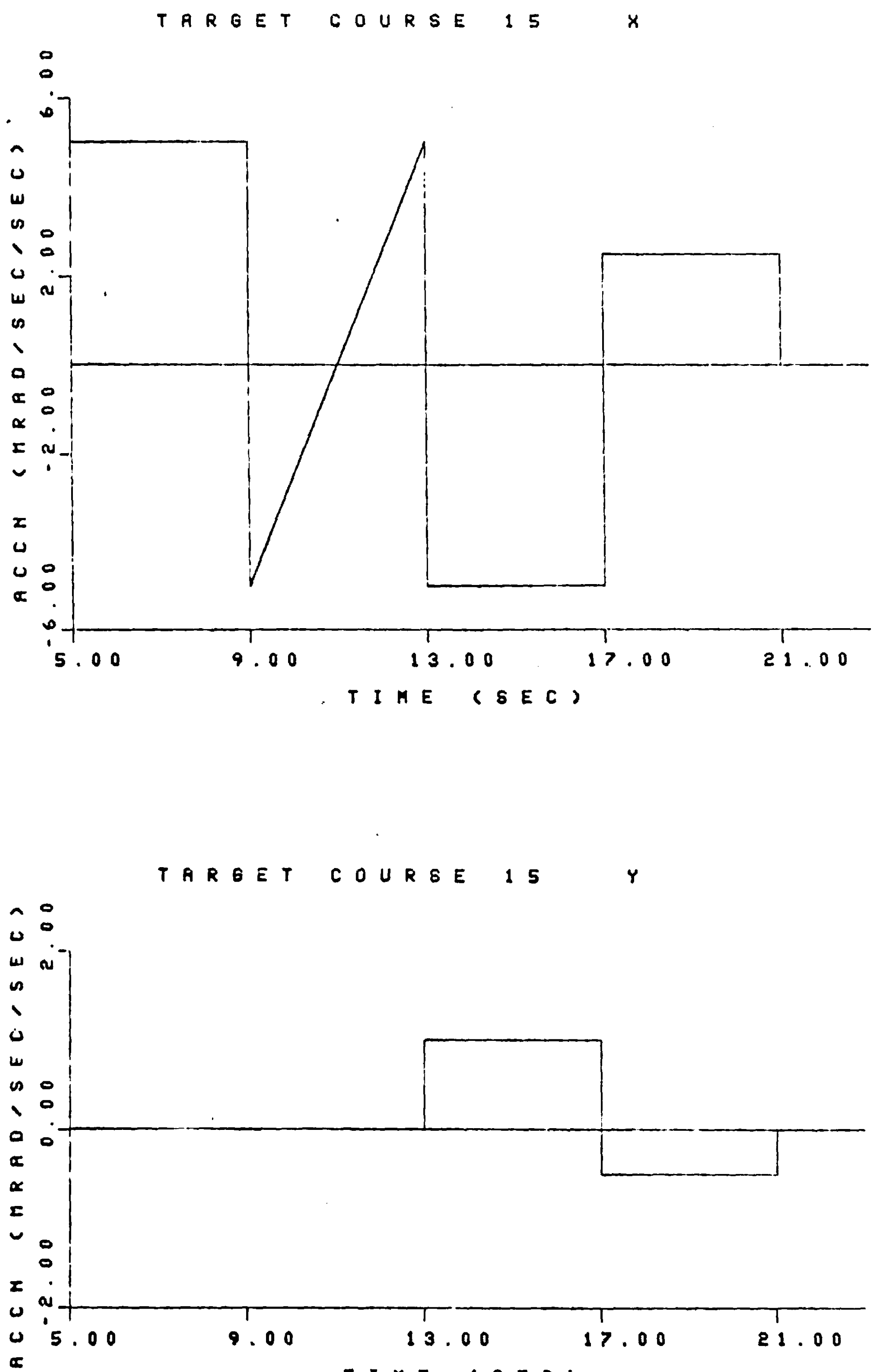


FIGURE 14. Acceleration time history, Target Course 15.

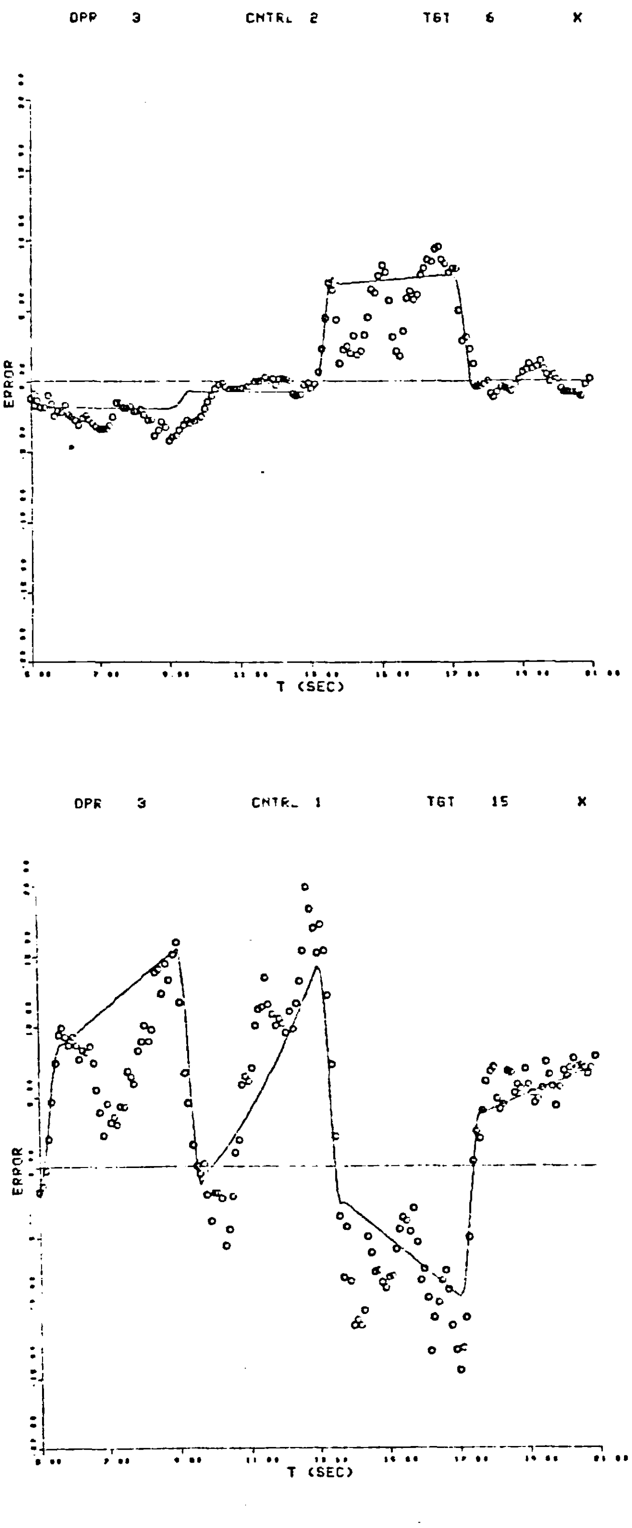


FIGURE 15. Fits to experimental data, 5 parameter impulse response model.

still evident. Nevertheless it would be most unwise to ignore this factor entirely at this stage: a real effect could interact with some prediction schemes yet to be devised in a non-trivial way. Accordingly, it was decided to augment our transfer function model in a way that might allow for a better representation of transient effects in the human operator.

We can augment the normal linear impulse response by allowing the level of the output to depend not only on current and past levels of the input but also on past levels of the output itself. The usual representation (see, eg, Box & Jenkins, 1970, pp 345ff) is of the general form

$$(1 + a_1\nabla + a_2\nabla^2 + \ldots + a_r\nabla^r)\, z_t = g(1 + c_1\nabla + c_2\nabla^2 + \ldots + c_s\nabla^s)x_t$$

which is, of course, the general linear difference equation (the discrete equivalent of the continuous differential equation introduced in Section 3.2.1). Writing this in equivalent form but using the backward shift operator $B = 1-\nabla$ (so that $Bx_t = x_{t-1}$) the model is

$$(1 - a_1'B^2 - a_2'B - \ldots - a_r'B^r)\, z_t = (c_0' - c_1'B - c_2'B - \ldots - c_s'B^s)x_t$$

or

$$\underline{a}'(B)\, z_t = \underline{c}'(B)x_t$$

Now referring to the original linear transfer function representation

$$\begin{aligned} z_t &= h_0x_t + h_1x_{t-1} + \ldots \\ &= (h_0 + h_1B + \ldots)x_t \\ &= \underline{h}(B)x_t \end{aligned}$$

we see that the transfer function for the expanded model is the ratio of two polynomials in B

$$\underline{h}(B) = \underline{a}'^{-1}(B)\underline{c}'(B)$$

An approximately equivalent representation of a limited parameter augmented model in impulse response terms would thus require a very large number of coefficients.

From the linear modelling exercise it seemed that it would be wise to include input terms at least up to $\nabla^3 x_t$. The minor contribution made by the $\nabla^4 x_t$ term could perhaps be more than compensated by adding e_{xt} terms. The fitting of a nonlinear model to very large sets of data is a tedious and time-consuming business, and so very little experimentation was conducted in order to determine how many e_{xt} terms to add. Instead it was decided virtually at the outset to fit a 7-parameter model:

$$e_{xt} = b_{xo}+(b_{x1}\nabla+b_{x2}\nabla^2+b_{x3}\nabla^3)x_t + (b_{x4}B+b_{x5}B^2+b_{x6}B^3)e_{xt} \quad (32)$$

$$e_{yt} = b_{yo}+(b_{y1}\nabla+b_{y2}\nabla^2+b_{y3}\nabla^3)y_t + (b_{y4}B+b_{y5}B^2+b_{y6}B^3)e_{yt} \quad (33)$$

This quaint mixture of backward shift and backward difference operators was chosen more for computational convenience than for aesthetic appeal.

With such extensive sets of data as those with which we are dealing here the computational load of fitting a nonlinear model is at least two orders of magnitude greater than that of fitting a linear one. Accordingly, considerable pains were taken to evolve an efficient nonlinear fitting routine. Inspection of equations (32) and (33) will reveal that the b's interact in a complex fashion, and a routine which requires the computation of partial derivatives is not a practicable proposition. An attempt was made to use Powell's (1964) general nonlinear function minimisation routine. However, not only did it require an excessively large number of sums-of-squares evaluations (running into several hundreds) but it converged at a result quite clearly remote from the true minimum. Powell's (1965) nonlinear least squares algorithm (as further developed and supplied by the AERE Harwell Applied Mathematics Group) required core storage of a size only found in large main frame computers. Correspondence with the National Physical Laboratory on the possible use of Gill-Murray routines (Gill & Murray, 1972) led to the conclusion that they too would require vast storage arrays. The method finally used was a development of the 'Dud' algorithm recently described by Ralston & Jennrich (1978). In evaluations of several standard test problems 'Dud' had compared very favourably with even the best derivative-based algorithms. As described, however, the method was not without its problems in practical use (which arose in part because of the highly nonlinear nature of the model fitted, which meant that stability boundaries could be crossed quite close to the real optimum; but also because as formulated the routine could collapse search prematurely into less than the full number of dimensions). A proper description of the way that the routine was developed will not be given here, but the problems were overcome by special provision to short-cut computational difficulties if search headed into a region of instability; by modifying the rules by which trial parameter vectors were accepted or discarded; and by incorporating ideas due to Marquardt (1963) in his well-known derivative-requiring algorithm. Some further details of the developed version of the 'Dud' routine are given in Appendix A where the computer programmes developed for this thesis are very briefly described.

The developed 'Dud' routine was employed to fit the model equations (32) and (33) to each subject in turn, minimising the sum of the squared weighted residuals. In section 3.4.2 it was stated that the intention was to base the weights on instantaneous sample variances:

$$w_t^2 = \frac{1}{s_t^2} \qquad (16)$$

In practice it was found that, due to the finite resolution of the simulation, instantaneous sample variances could very occasionally be zero, and so smoothed variances were used instead:

$$w_t^2 = \frac{1}{s_{t-1}^2 + s_t^2 + s_{t+1}^2} \qquad (34)$$

where <u>for this purpose only</u> successive time instants were spaced 0.1 sec apart. The transfer function itself was based on a spacing of 0.2 sec. Only half the recorded data were utilised, starting with the 5.1 sec point in each target course and proceding at a spacing of 0.2 sec to 20.9 sec.

The augmented transfer function model did seem to reproduce some of the underdamped oscillatory behaviour evident in the averaged operator records, and model fit (as judged by the weighted residual sum of squares) was improved. Figures 16 to 18 illustrate all the model fits to Target 6 (x plane only, there being virtually no output in the y plane) and Target 15 for the APRE scientific staff. Clearly, the

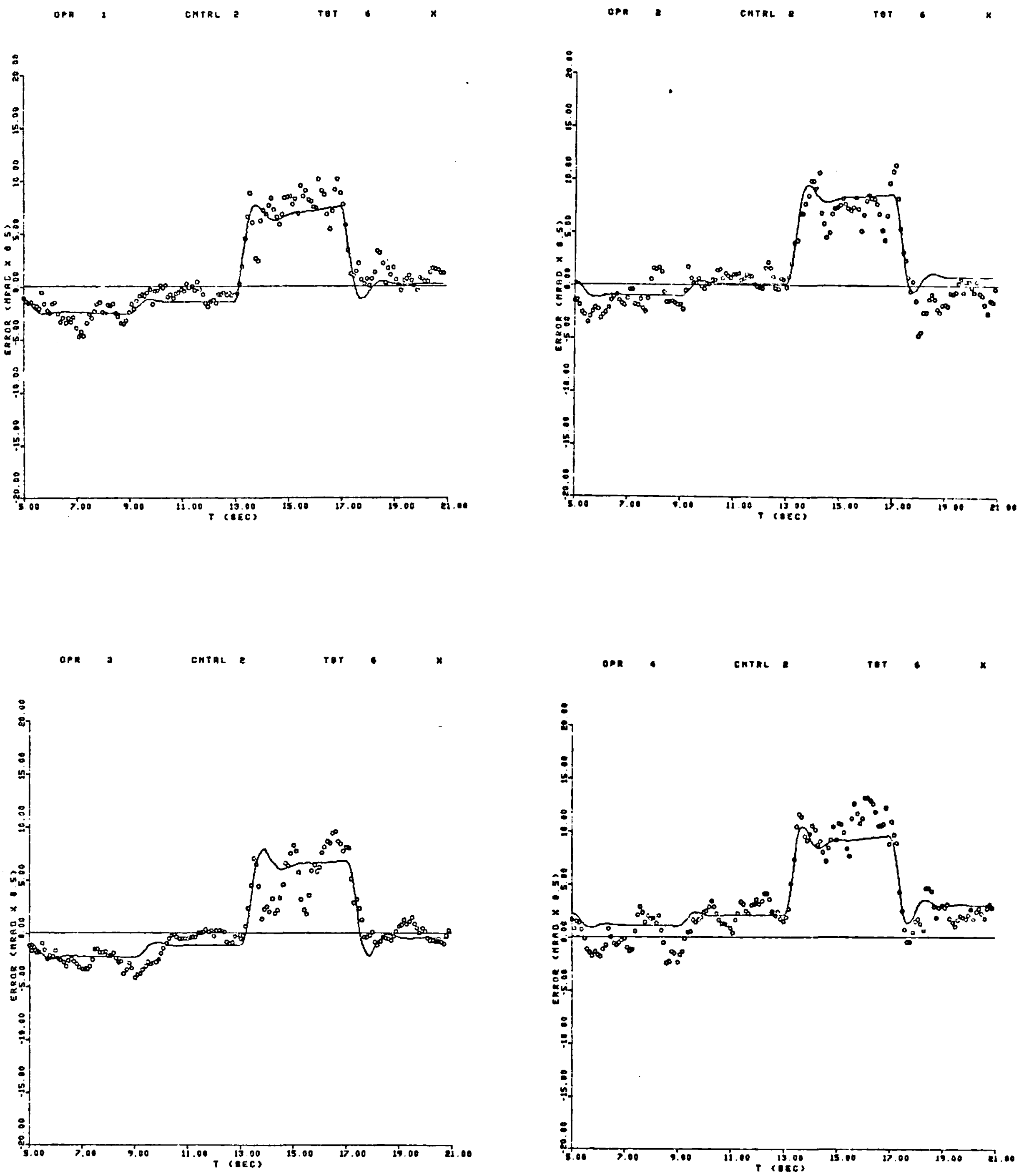


FIGURE 16. Fits to experimental data, difference equation model. APRE scientific staff, Target Course 6, x plane.

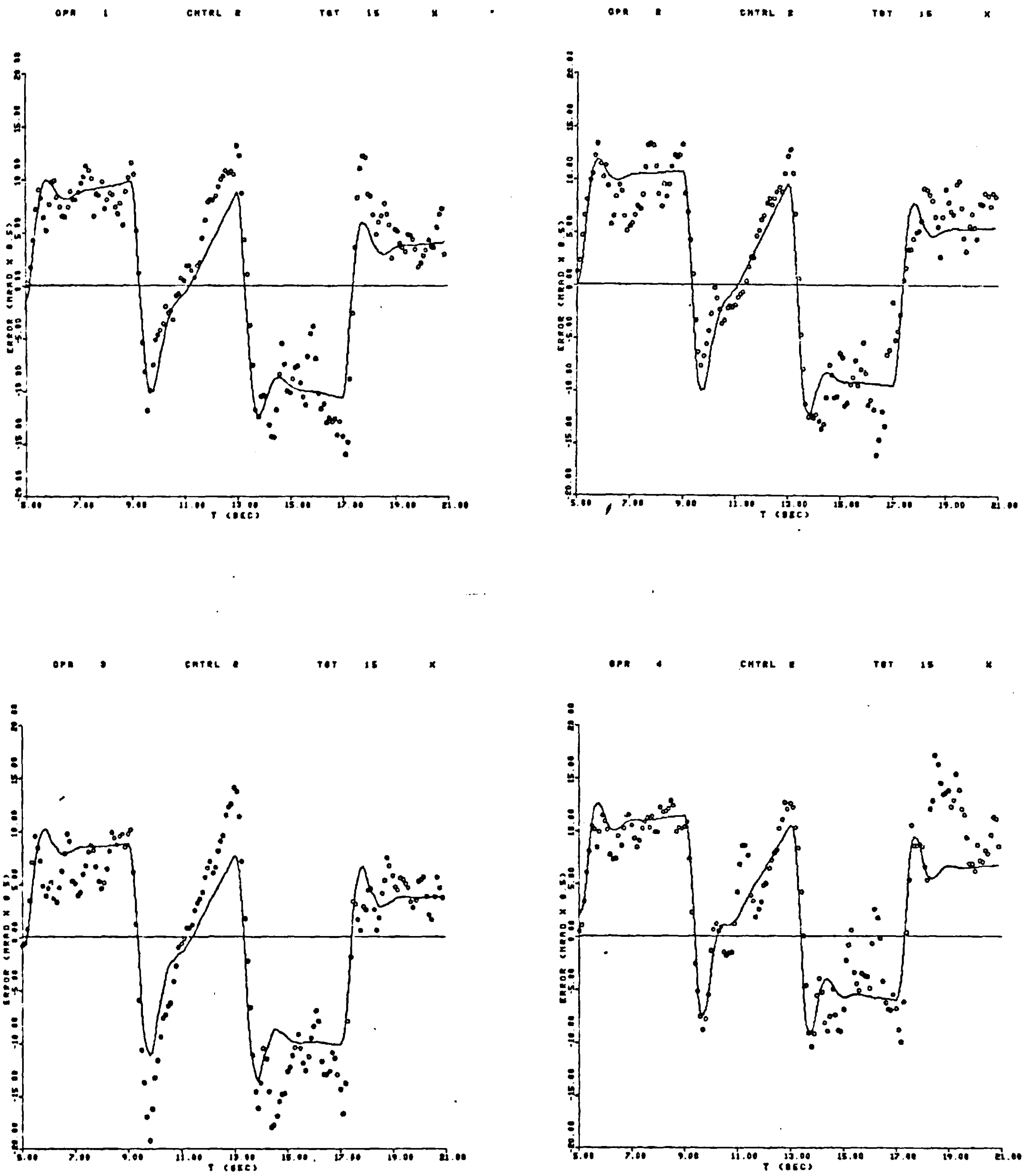


FIGURE 17. Fits to experimental data, difference equation model. APRE scientific staff, Target Course 15, x plane.

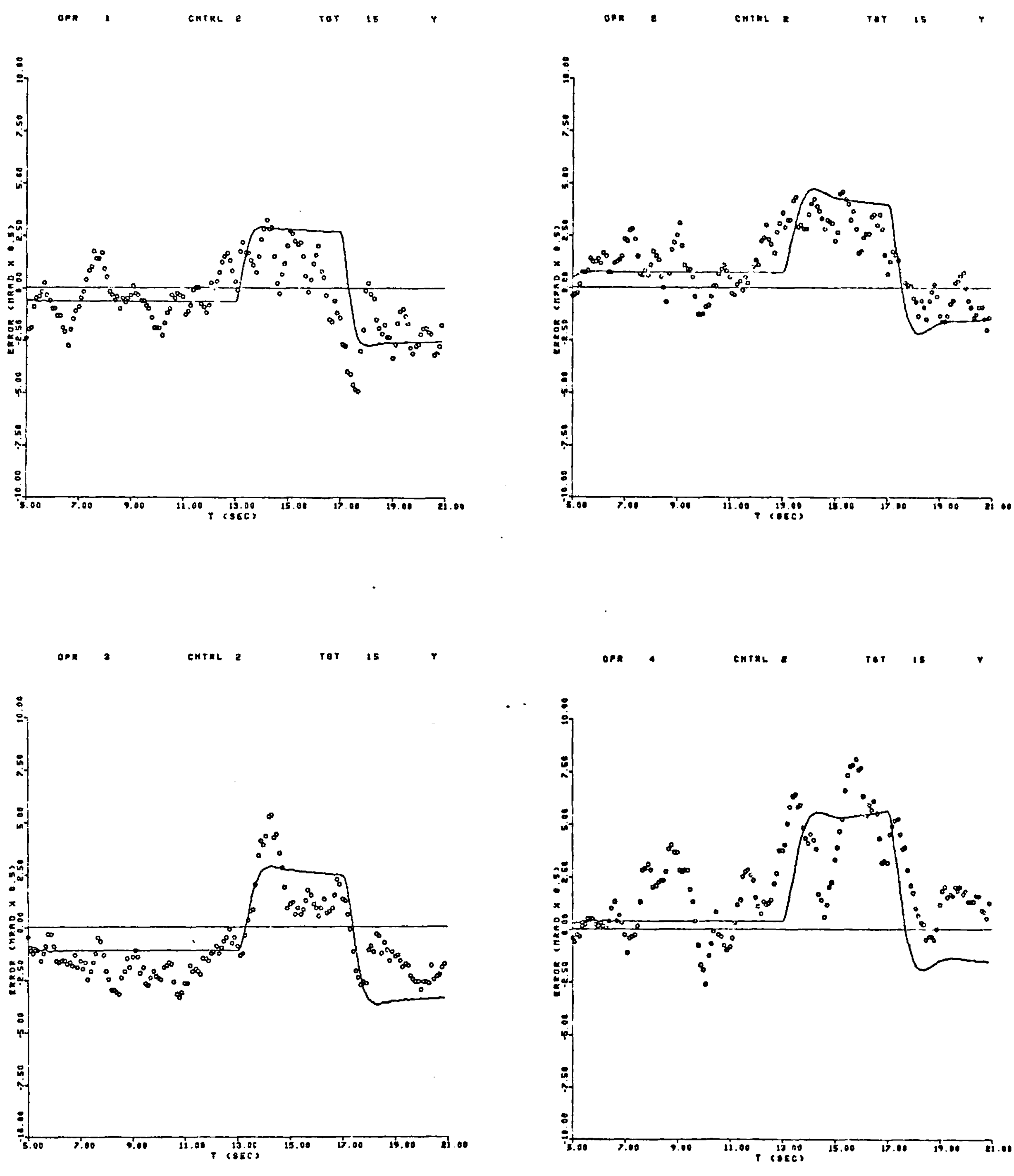


FIGURE 18. Fits to experimental data, difference equation model. APRE scientific staff, Target Course 15, y plane.

model does not reflect the quasi-periodic nature of the data fully, but it is highly unlikely that the incorporation of extra terms would be worthwhile. The unexplained (relatively) high frequency variation is to a large extent attributable to averaged (relatively) high frequency noise of comparatively large amplitude. As already mentioned, the records (particularly those from the less accurate trackers of the soldier sample) show this quasi-periodic response even in the absence of appreciable target motion input.

The model coefficients fitted to individual subjects are given in Appendix C for the sake of completeness. However, because of the highly interactive nature of the coefficients they are not terribly informative - two sets of constants with apparently very different numerical values can give very similar-seeming patterns of error response. The values of the last three parameters - b_4, b_5 and b_6 - should especially be viewed with suspicion: they are, in effect, concerned with mimicking the shape of quite short-lived transients, which occupy a small proportion of the total record and which are themselves corrupted by noise. A preliminary evaluation of these results will be deferred to section 4.3.5.

4.3.4 Model development (remnant)

In section 3.4.3 it was proposed that the 'remnant' should be modelled as an ARMA (autogressive moving average) process

$$\varepsilon_{xt} = a_{xt} + \sum_{i=1}^{p} \phi_{xi}\varepsilon_{x(t-i)} - \sum_{j=1}^{q} \theta_{xj} a_{x(t-j)} \qquad (35)$$

where

ε_{xt} is the stochastic element of the tracking error at time t,

a_{xt} represents an independent 'shock' entering the system at time t

$\phi_{x1}, \phi_{x2}, \ldots$ are the autoregressive (AR) parameters, and

$\theta_{x1}, \theta_{x2}, \ldots$ are the moving average (MA) parameters.

(and similarly for ε_{yt}). The issues to be resolved were the order (p,q) of the ARMA process, and the form of relationship governing the distribution of the a's.

The remnant data from the APRE scientific staff using the pressure joystick with nonlinear output were excluded from analysis. On a qualitative level the nonlinearity did seem to affect the form of distribution of the a's (so that errors in general seemed to be more pronounced where the slope of the output curve was steeper). However, time precluded a proper examination of this effect, and it was felt that it should properly be made the subject of a study in its own right.

So far as the order of the noise process is concerned, it was found that a (1,0) ARMA model was quite sufficient to account for the observed dependency for the APRE scientific staff (movement joystick). Increasing the order to (2,0) decreased the residual sum of squares on average by only 1% in x and 3% in y. For the Infantrymen the effect of the second AR parameter was not so minimal. For the 'worst' 6 subjects (as defined in the next section) and with the movement joystick residual sums of squares were decreased by something in excess of 9%. In the event the effect of adding MA parameters was not investigated, except indirectly by showing that the addition of further AR terms up to order 7 had a negligible effect. The choice of the most appropriate dependency scheme is deferred until section 4.3.5.

So far as the distribution of the a's is concerned, the formulation originally suggested seemed to be satisfactory:

$$\sigma^2(a_{xt}) = \sigma^2_{xo} + \sigma^2_{x1}\, e^2_{x(t-1)} + \sigma^2_{x2}\, e^2_{y(t-1)} \qquad (36)$$

where

$\sigma^2(a_{xt})$ is the variance of the independent 'shock' entering the system at time t,

σ^2_{xo} is a 'threshold' error term,

e_{xt} is the total tracking error in the x plane at time t, and

e_{yt} is the total tracking error in the y plane at time t

(and similarly for $\sigma^2(a_{yt})$). Methods for augmenting or adjusting this simple scheme were examined by testing whether there was any merit in extending the dependency from (t-1) to (t-2) or by adding lower order

($|e_x|$, $|e_y|$), crossproduct ($|e_x e_y|$), or higher order terms (e_x^4, e_y^4, $e_x^2 e_y^2$). They all had negligible effect on the residual sums of squares.

4.3.5 Evaluation of preliminary modelling results

For purposes of analysis the Infantrymen sample was divided into two. The soldiers were ranked in order of their average miss distance taken over both controls, sample A being those with the highest ('worst') scores and sample B being those with the lowest ('best') scores. In what follows the APRE scientific staff have been designated as sample C. To give one some sort of idea of the individual variation involved, the score ratio between the 'best' and 'worst' individuals in sample C was about 1:1½. The 'best' individual in sample B was about on a par with the 'worst' in sample 'C', and the score ratio between the former and the 'worst' in sample A was in excess of 1:3.

Fitted model constants for individual subjects are given in Appendix C. Average parameter values are shown in Tables 1 and 2, the former listing the results obtained with the movement joystick and the latter those obtained with the pressure joystick (with nonlinear output for sample C). The last line in each table lists the percentage reduction in residual sums of squares when the dependency scheme was increased from first order to second order.

As shown the model parameters relate target position measured in milliradians at the eye to tracking error similarly measured. A x10 sight is assumed. If input and output are required in milliradians in simulated real space the only change needed is to divide the σ_o^2 constant(s) by 100.

TABLE 1 Average fitted parameters, preliminary modelling exercise. Movement joystick

Plane / Sample	x A	x B	x C	y A	y B	y C
Transfer Function						
b_1	-0.602	-0.429	-0.112	0.820	0.100	-0.088
b_2	0.004	0.011	0.016	-0.057	-0.043	-0.014
b_3	1.951	1.840	2.163	3.553	1.803	2.798
b_4	-1.122	-0.949	-1.221	-2.071	-1.018	-1.242
b_5	0.609	0.674	0.129	0.436	0.867	0.189
b_6	0.249	-0.144	0.216	0.357	-0.126	0.499
b_7	-0.319	-0.148	-0.310	-0.255	-0.136	-0.269
Remnant (1st order)						
ϕ	0.727	0.575	0.621	0.774	0.657	0.650
σ_0^2	9.684	5.961	2.193	1.791	1.545	0.581
σ_1^2	0.092	0.119	0.171	0.022	0.031	0.022
σ_2^2	0.163	0.138	0.160	0.094	0.112	0.128
Remnant (2nd order)						
ϕ_1	0.949	0.711	0.682	1.023	0.816	0.757
ϕ_2	-0.300	-0.218	-0.097	-0.313	-0.231	-0.166
Residual SS reduction (%)	9.4	6.2	1.3	9.6	5.4	3.0

TABLE 2 Average fitted parameters, preliminary modelling exercise. Pressure joystick

Plane / Sample	x A	x B	x C	y A	y B	y C
Transfer Function						
b_1	-1.063	-0.945	-0.111	0.482	-0.130	-0.246
b_2	0.014	0.030	0.048	-0.024	-0.028	0.034
b_3	1.972	1.788	1.475	3.018	2.557	2.287
b_4	-1.443	-1.085	-0.544	-0.741	-1.866	-1.028
b_5	0.735	0.556	0.333	0.327	0.352	0.328
b_6	-0.279	-0.125	0.137	0.431	0.108	0.204
b_7	0.003	-0.177	-0.296	-0.240	0.053	-0.432
Remnant (1st order)						
ϕ	0.716	0.557		0.684	0.478	
σ_0^2	5.802	5.503		4.901	3.276	
σ_1^2	0.106	0.101		0.039	0.036	
σ_2^2	0.112	0.125		0.106	0.109	
Remnant (2nd order						
ϕ_1	0.886	0.674		0.787	0.529	
ϕ_2	-0.232	-0.204		-0.151	-0.085	
Residual SS reduction (%)	5.5	4.9		3.0	2.0	

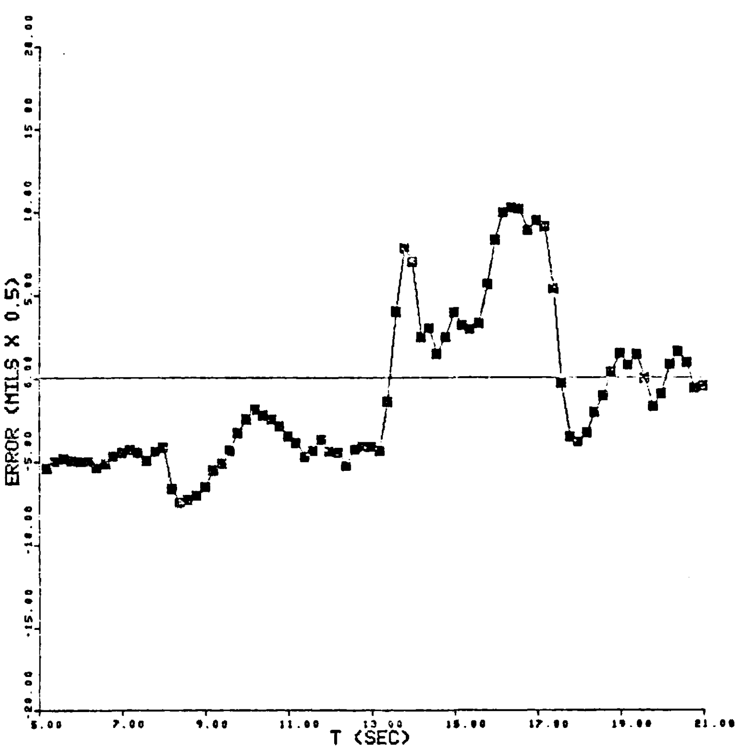
It has already been mentioned (section 4.3.3) that inspection of the values of the parameters in the fitted transfer function is not terribly informative. The model is nonlinear (due to the b_5, b_6 and b_7 terms) and so the parameter values interact in a fashion which is not easy to visualise. To obtain a better appreciation of the effects due to subjects and controls it is perhaps best to display the averaged tracking errors for the three different subject samples using the two different controls. This has been done in Figures 19 to 23. It has been concluded from these plots that the determinate portion of human operator response is similar for gunners with different tracking ability and at least for the two controls used here. There are differences in amplitude or gain, but differences in the shape of response seem to be second-order effects.

Operator and control type differences seem to show up most clearly in the 'remnant'. Taking the dependency characteristics first, the 'worst' trackers have the largest ϕ coefficients on average. Furthermore, while the 'best' trackers are quite well described by a first order dependency scheme, this description becomes progressively poorer as one moves to the other end of the ability scale.

Still on the question of the 'remnant', but turning now to the distribution of the entering shocks, a fairly clear pattern of results emerges. The coefficients relating the variance of these shocks to the errors at the previous sampling instant seem to be very similar from one subject sample to another, and from one control to another. However, the 'threshold' variance contribution (σ_0^2) is noticeably greater for the 'worst' trackers. The character of the 'remnant' for the two controls is quite similar in the x plane, but in the y plane it is quite different: with the pressure joystick serial dependency is less, but the contribution of the 'threshold' noise term is much greater.

The eventual application of the developed human tracking model is in fire control system simulation. Perhaps the best means of evaluating its adequacy is to compare the results of the model when used in a simulation mode with the experimental results on which it was based. Assuming a first order dependency scheme and a Monte Carlo simulation, then tracking errors within a run would be generated recursively in x (and similarly in y) by

MEAN CONTROL 1 TGT 6 X



MEAN CONTROL 2 TGT 6 X

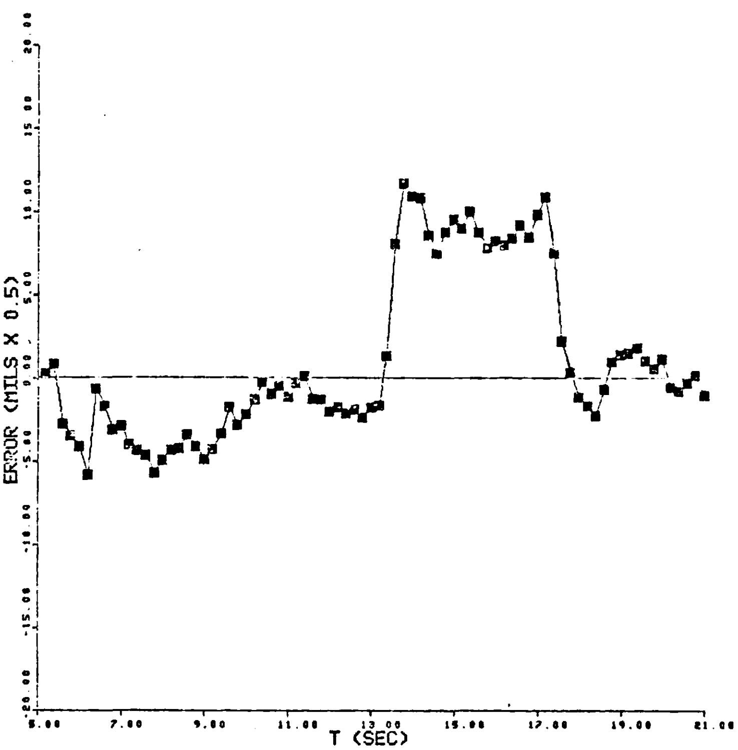


FIGURE 19. Comparison of pressure and movement joysticks. All Infantrymen pooled. Target Course 6, x plane. (The pressure joystick is Control 1 and the movement joystick is Control 2).

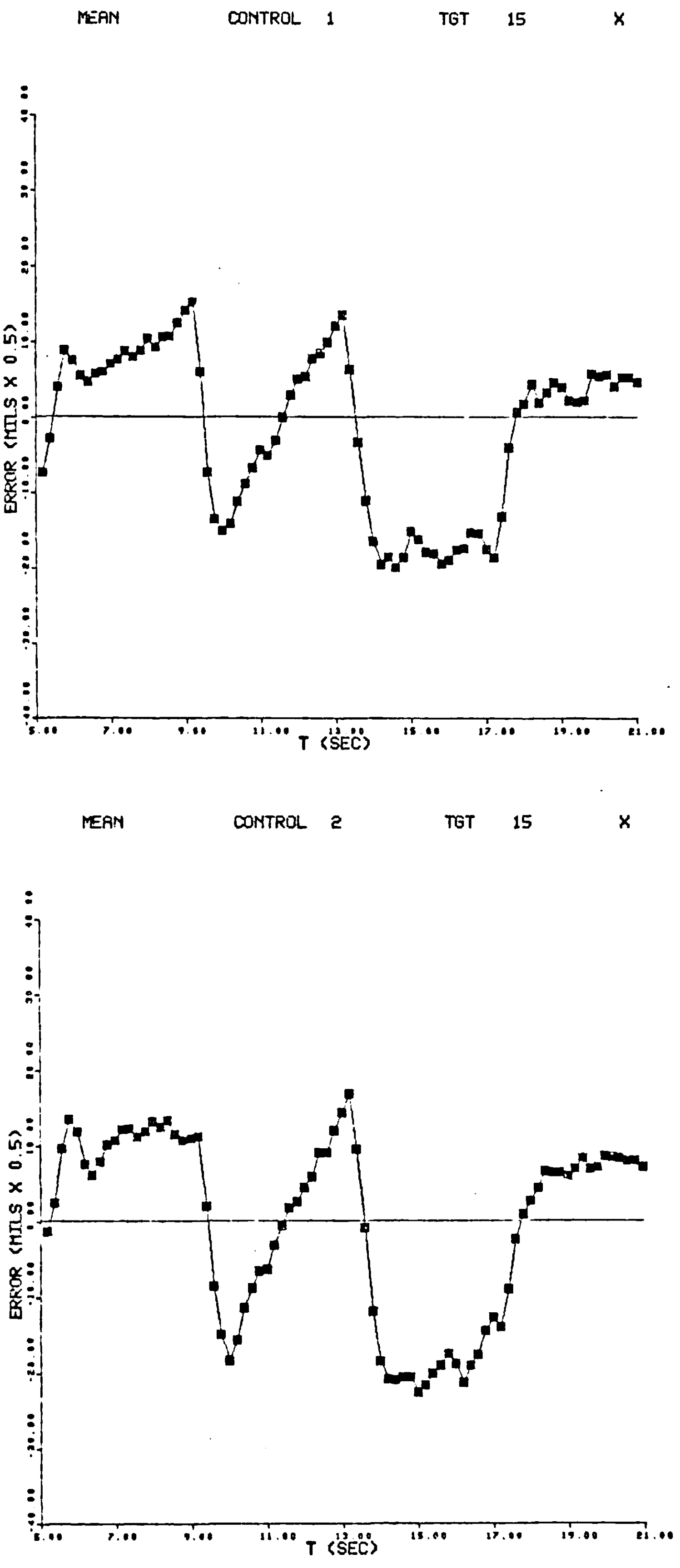
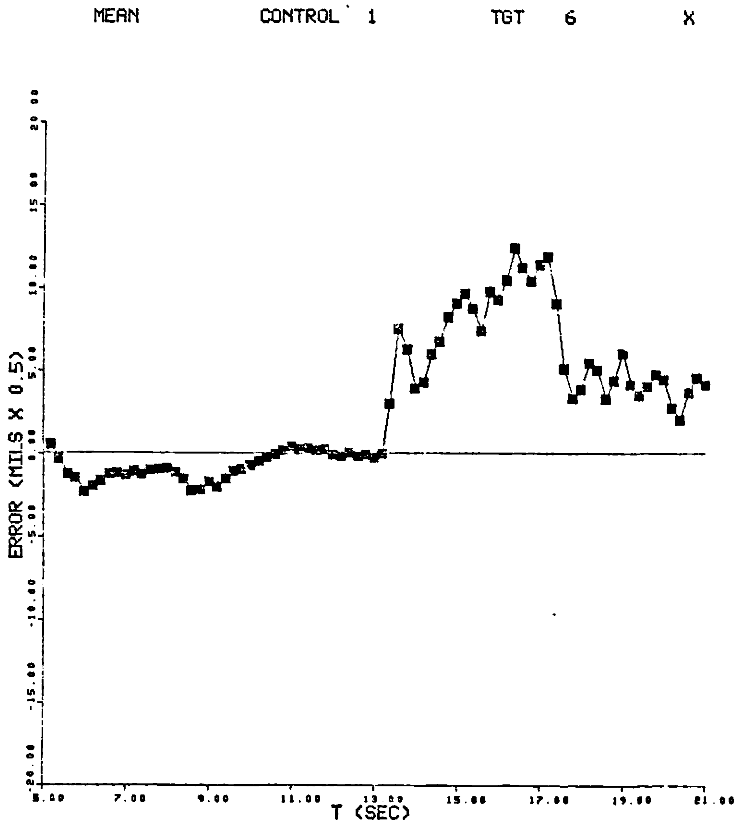


FIGURE 20. Comparison of pressure and movement joysticks. All Infantrymen pooled. Target Course 15, x plane. (The pressure joystick is Control 1 and the movement joystick is Control 2).



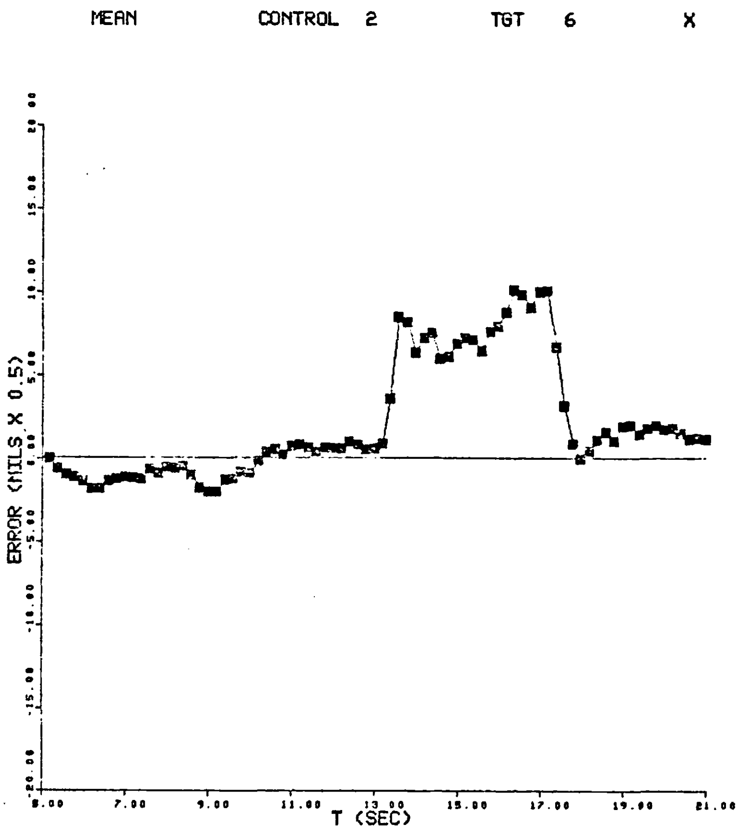


FIGURE 21. Comparison of pressure and movement joysticks. All APRE scientific staff pooled. Target Course 6, x plane. (The pressure joystick – with nonlinear output – is Control 1 and the movement joystick is Control 2).

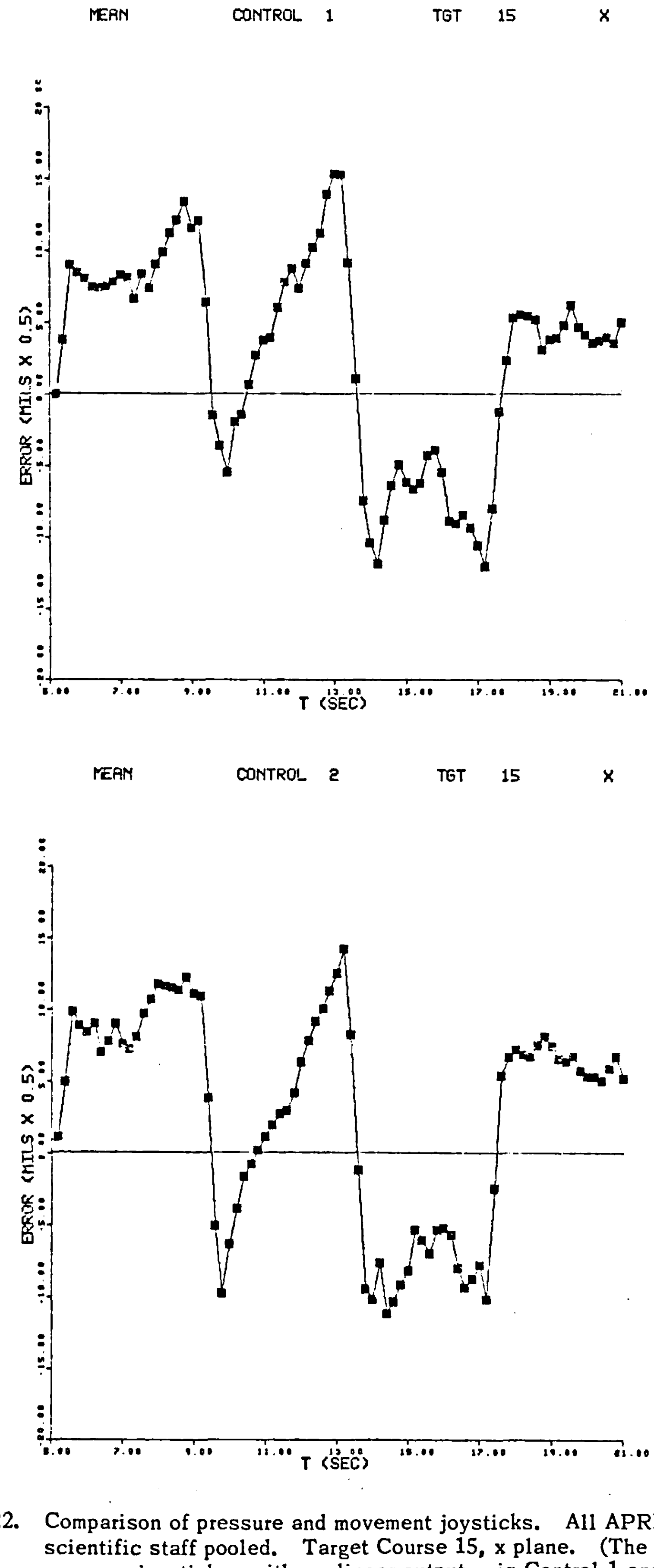


FIGURE 22. Comparison of pressure and movement joysticks. All APRE scientific staff pooled. Target Course 15, x plane. (The pressure joystick – with nonlinear output – is Control 1 and the movement joystick is Control 2).

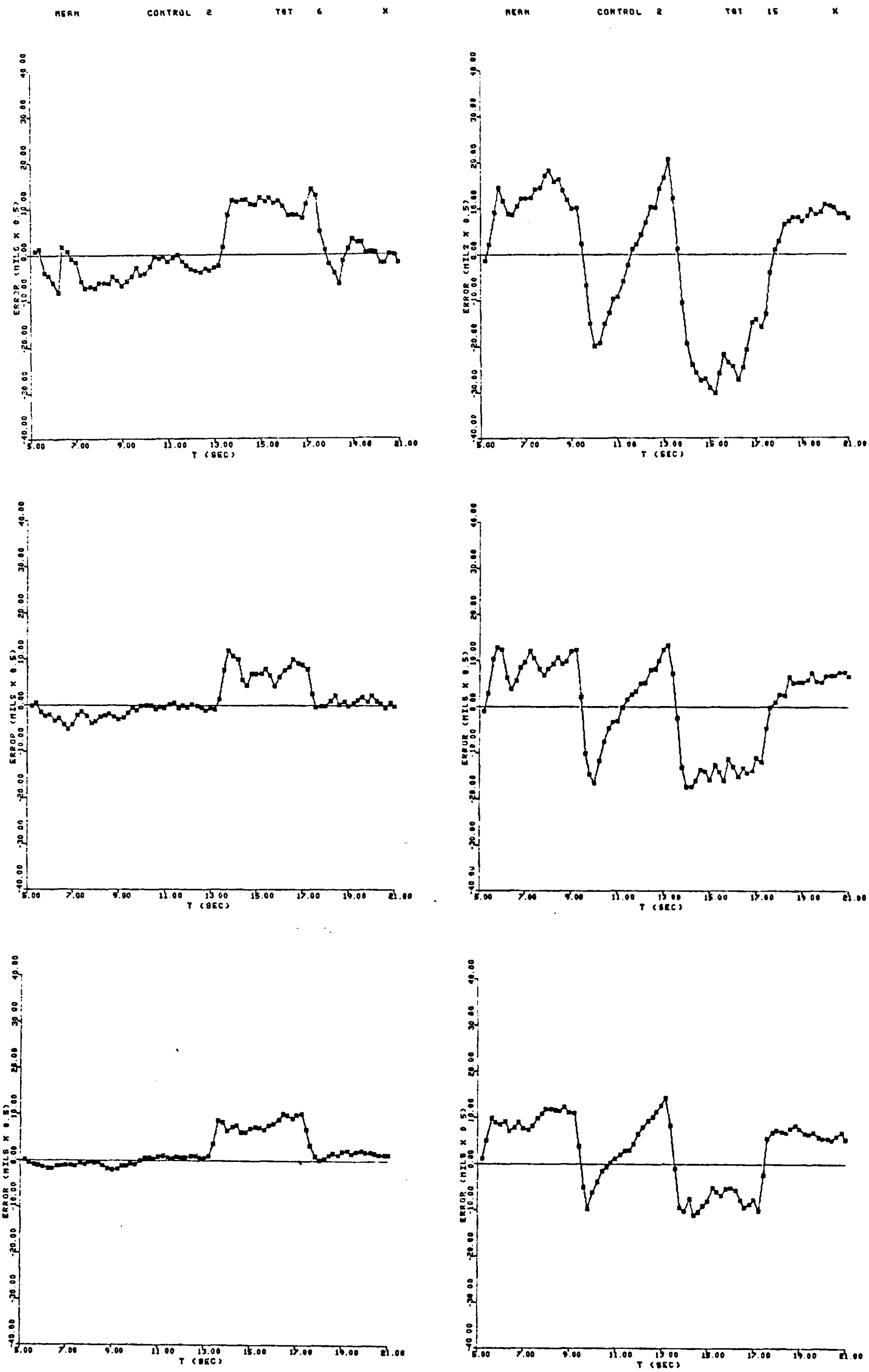


FIGURE 23. Comparison between subject samples. In descending order the plots show the averaged results for Samples A, B and C (the 6 'worst' Infantrymen, the 6 'best', and the APRE scientific staff)

the scheme:

$$e_{xt} = f_x(x_t) + \varepsilon_{xt} \tag{37}$$

$$\varepsilon_{xt} = a_{xt} + \phi_x \varepsilon_{x(t-1)} \tag{38}$$

$$a_{xt} = S_{xt}\left\{\sigma^2_{xo} + \sigma^2_{x1} e^2_{x(t-1)} + \sigma^2_{y1} e^2_{y(t-1)}\right\}^{\frac{1}{2}} \tag{39}$$

where

S_{xt} is a random sample from the population N(0,1),

e_{xt} is the total tracking error at time t,

$f_x(x_t)$ is the determinate ('transfer function') portion of that error,

ε_{xt} is the stochastic ('remnant') portion,

a_{xt} is the independent 'shock' entering the system at time t, and

ϕ_x, σ_{xo}, σ_{x1} and σ_{x2} are the model noise parameters previously discussed.

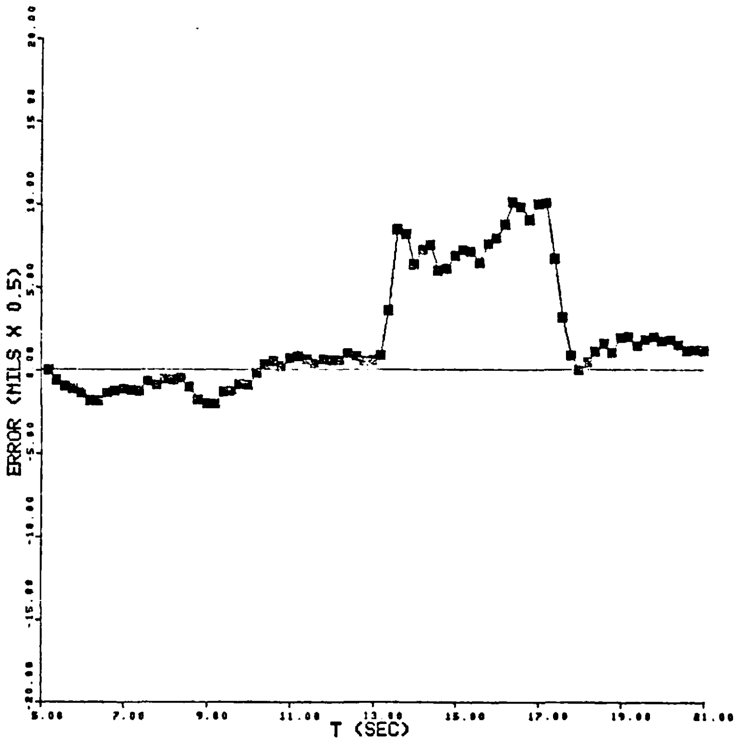
Such a recursive scheme requires 7 starting values: assuming a start at t=3 then values for x_o, x_1, x_2, $f_x(x_o)$, $f_x(x_1)$, $f_x(x_2)$ and ε_{x2} are needed. Provided that 'reasonable' values are chosen for these variables, and that we are only interested in tracking errors an appreciable time after the start (such as the 3 sec allowed in our experiments) then the effect of the particular initial values chosen should be minimal.

The model was used in Monte Carlo fashion to simulate 12 tracking runs against each target for each operator, employing the individually fitted model parameters and the first order dependency scheme. (Starting values of zero were used for all quantities other than ε_{xo} and ε_{yo}. ε_{xo} and ε_{yo} were random samples from N(0,7.5) and N(0,2.5) respectively). The simulation runs thus mimicked the live experimental procedure. Figures 24 and 25 compare some of the experimental and the simulation means and standard deviations, averaged over the APRE scientific staff.

It will be seen that the model and the simulation are in very close agreement so far as means are concerned. As for standard deviations, the simulation results are at about the right overall level, and they

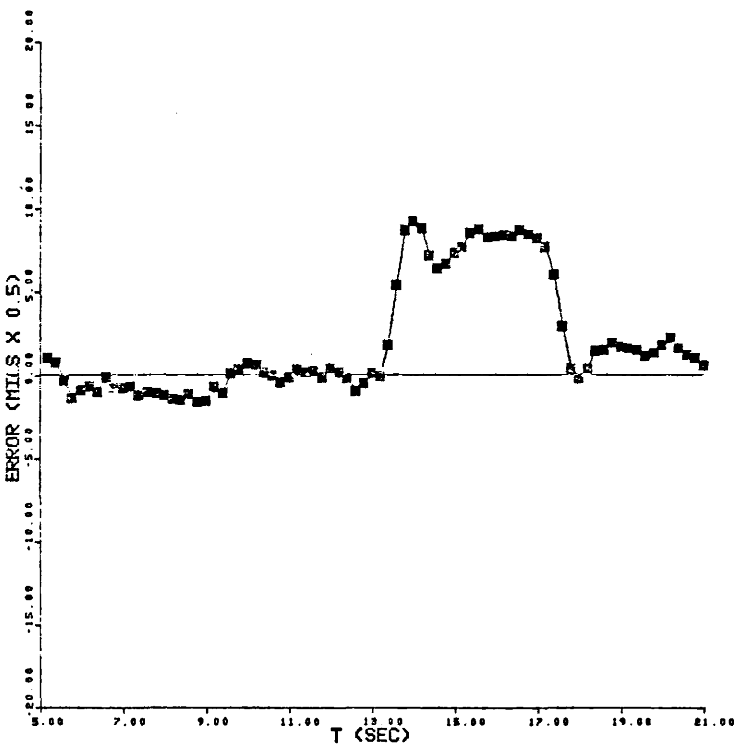
EXPERIMENTAL RESULTS

MEAN CONTROL 2 TGT 6 X



SIMULATION

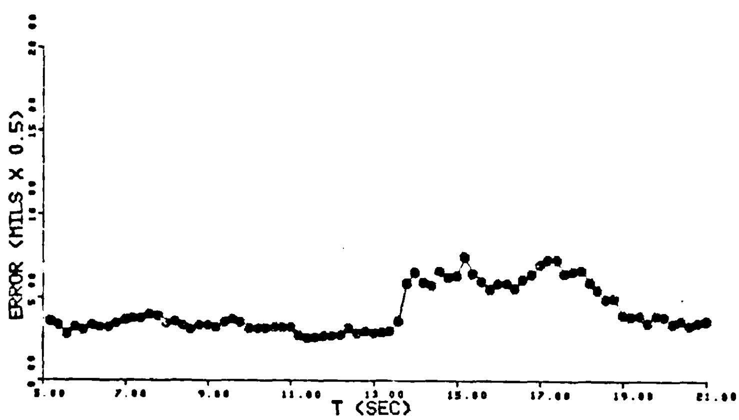
MEAN CONTROL 2 TGT 6 X



24a

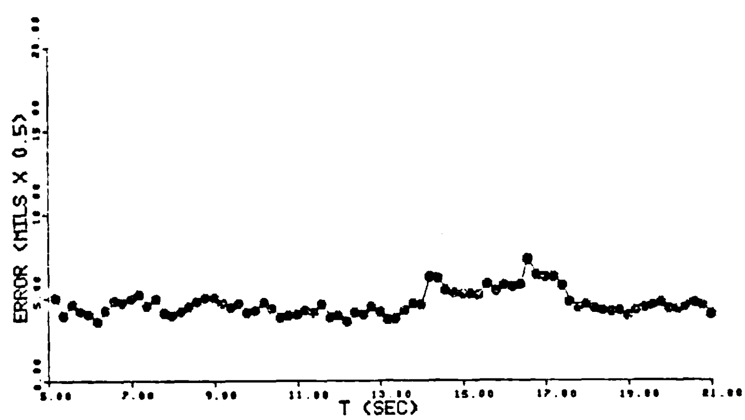
EXPERIMENTAL RESULTS

STD DEV CONTROL 2 TGT 6 X



SIMULATION

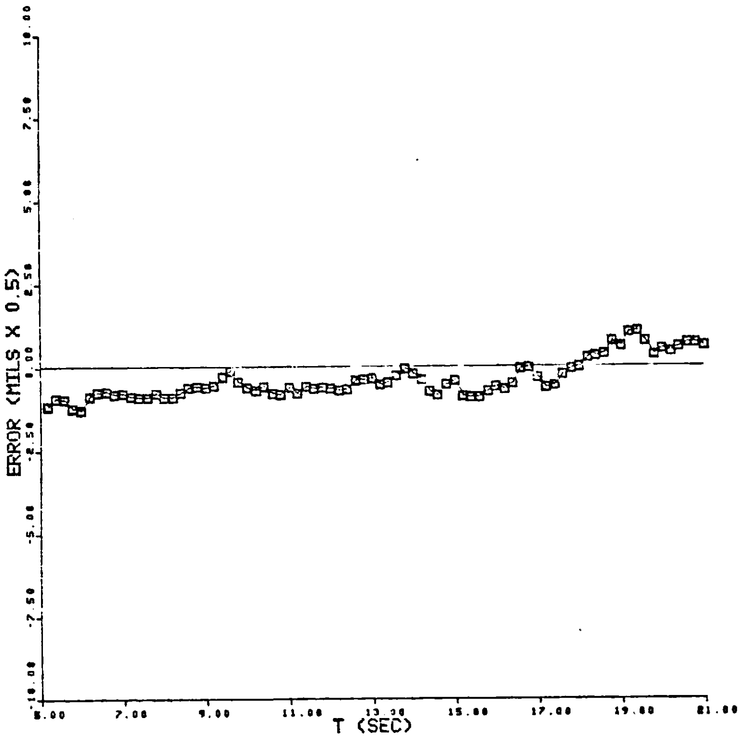
STD DEV CONTROL 2 TGT 6 X



24b

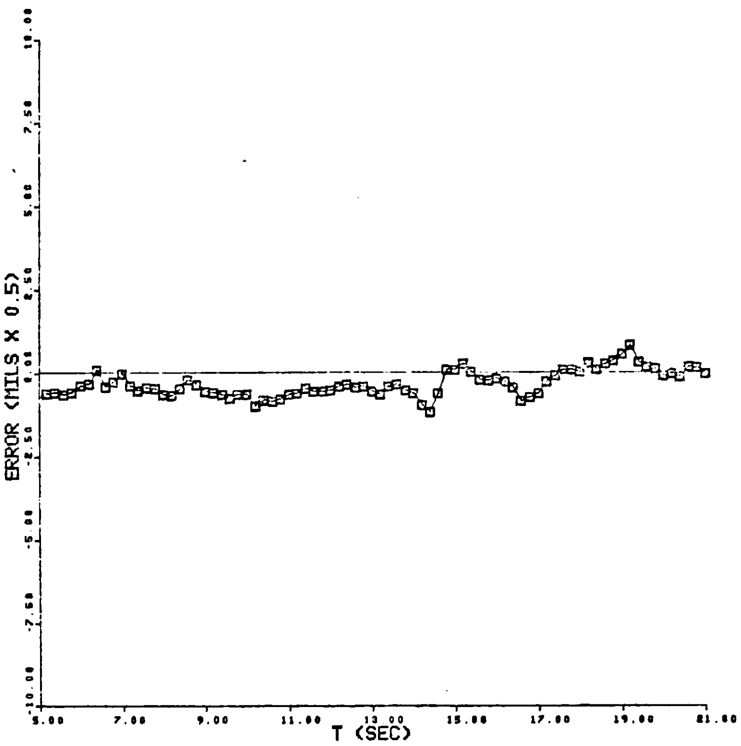
EXPERIMENTAL RESULTS

MEAN CONTROL 2 TGT 6 Y

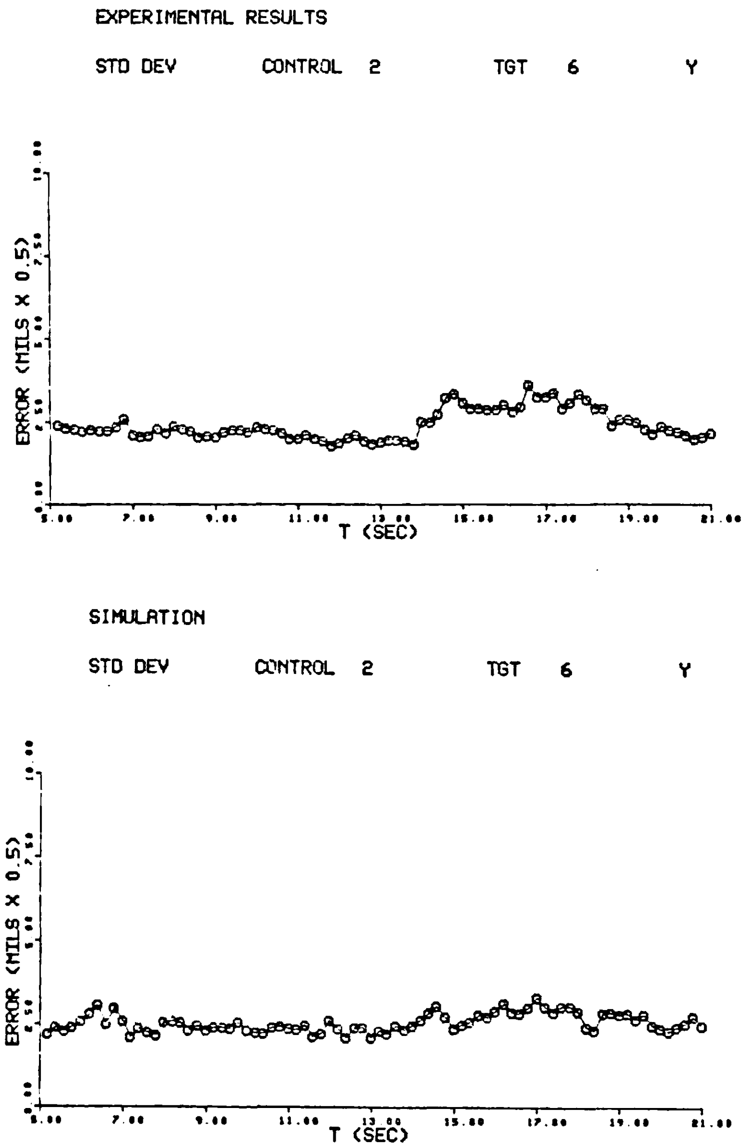


SIMULATION

MEAN CONTROL 2 TGT 6 Y



24c

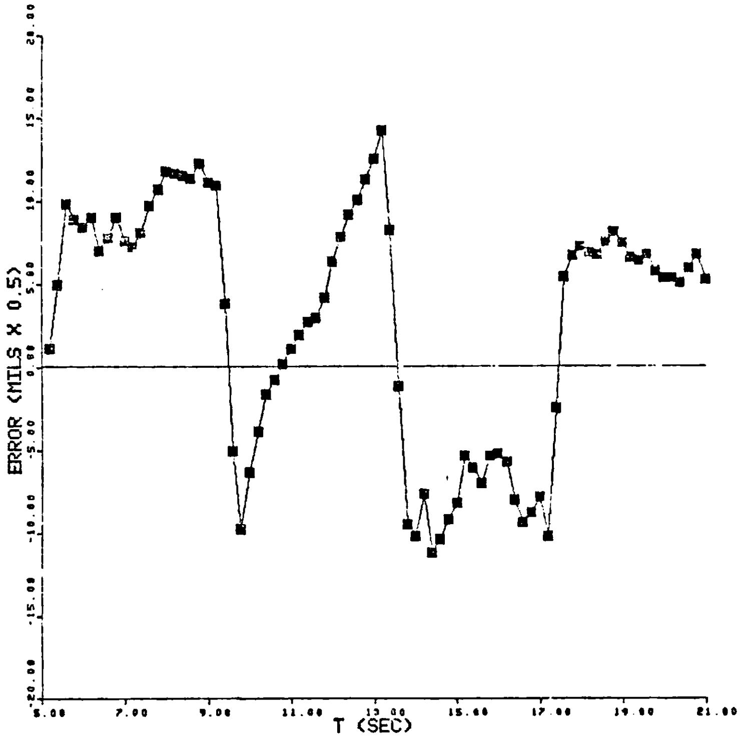


24d

FIGURE 24. Comparison between experimental results and Monte Carlo simularion. APRE scientific staff. Target Course 6.

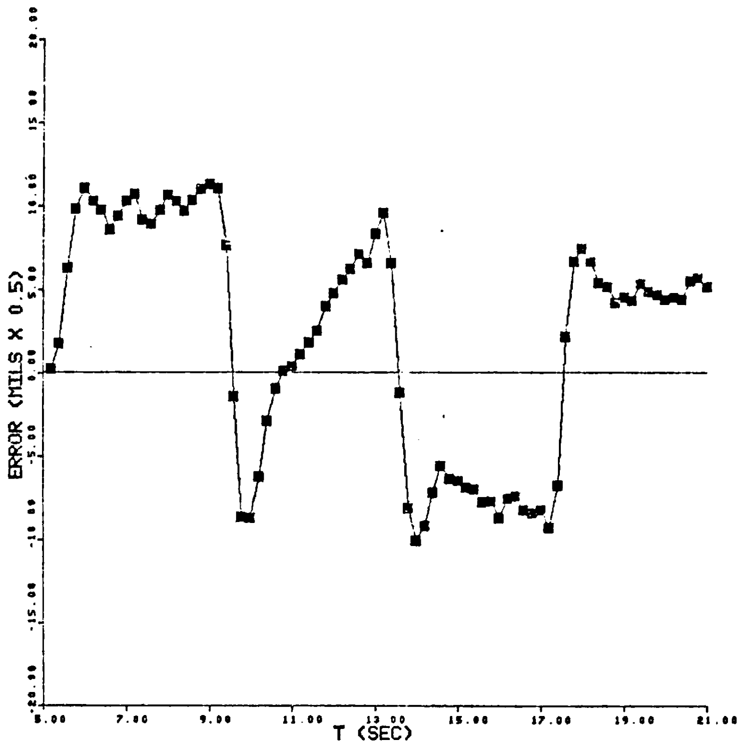
EXPERIMENTAL RESULTS

MEAN CONTROL 2 TGT 15 X



SIMULATION

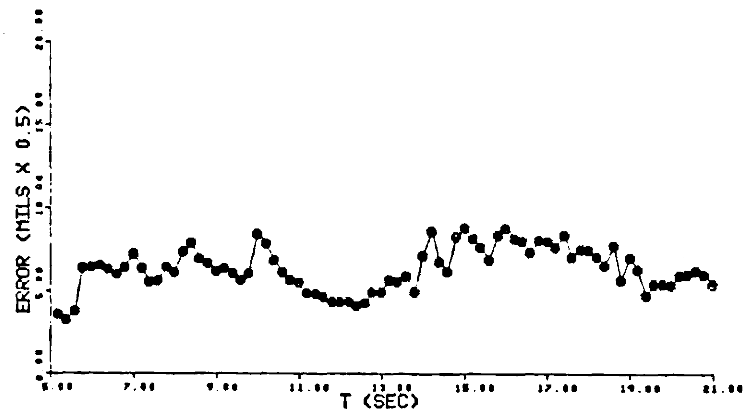
MEAN CONTROL 2 TGT 15 X



25a

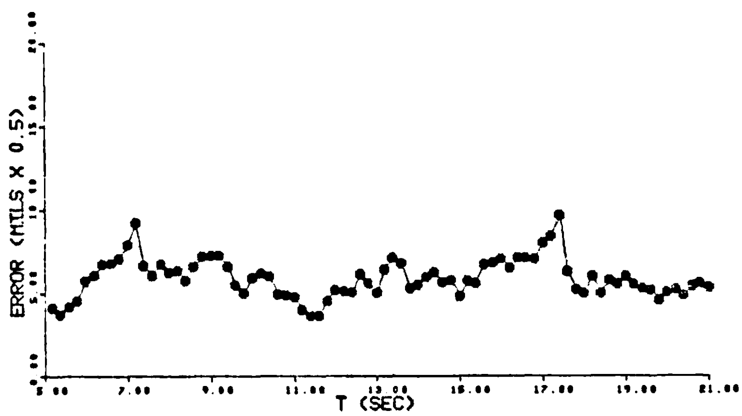
EXPERIMENTAL RESULTS

STD DEV CONTROL 2 TGT 15 X



SIMULATION

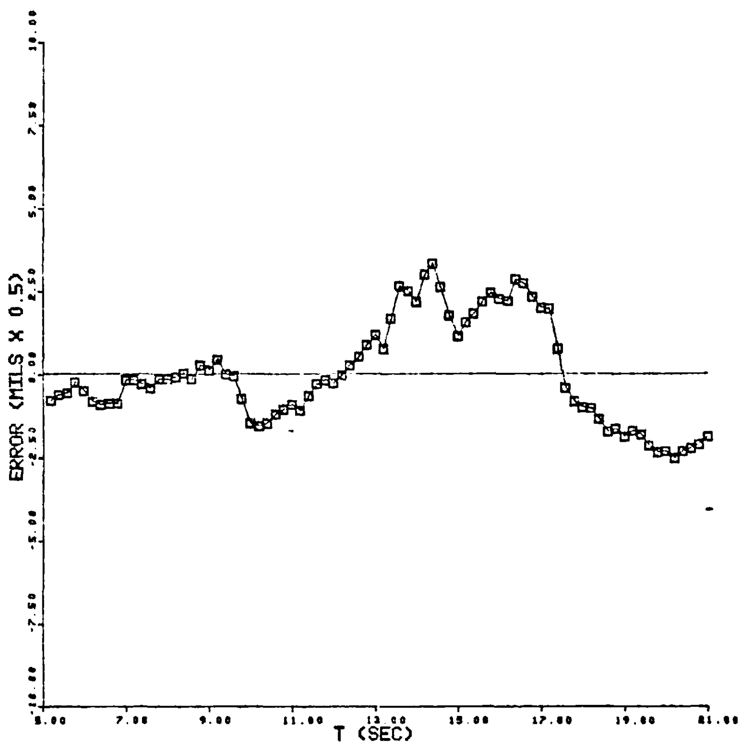
STD DEV CONTROL 2 TGT 15 X



25b

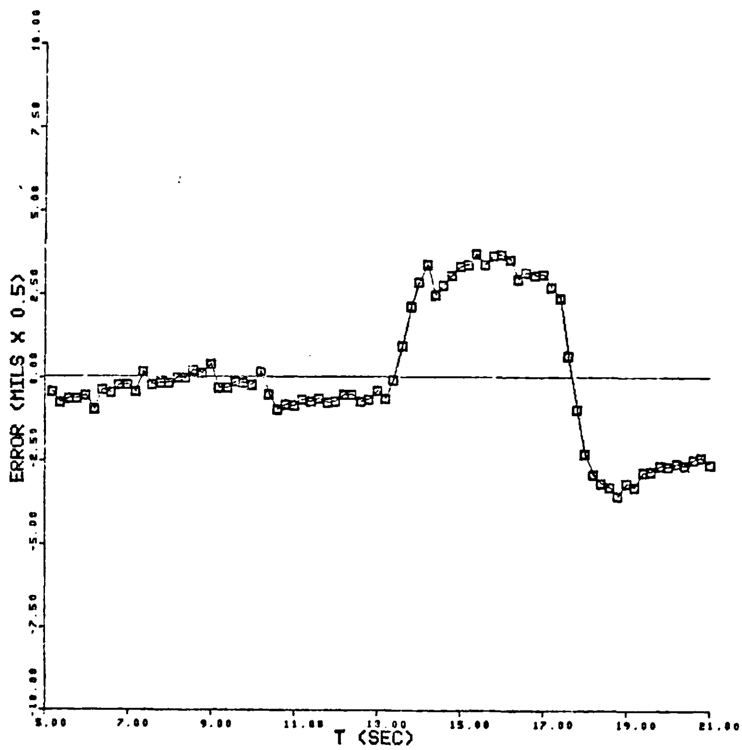
EXPERIMENTAL RESULTS

MEAN CONTROL 2 TGT 15 Y

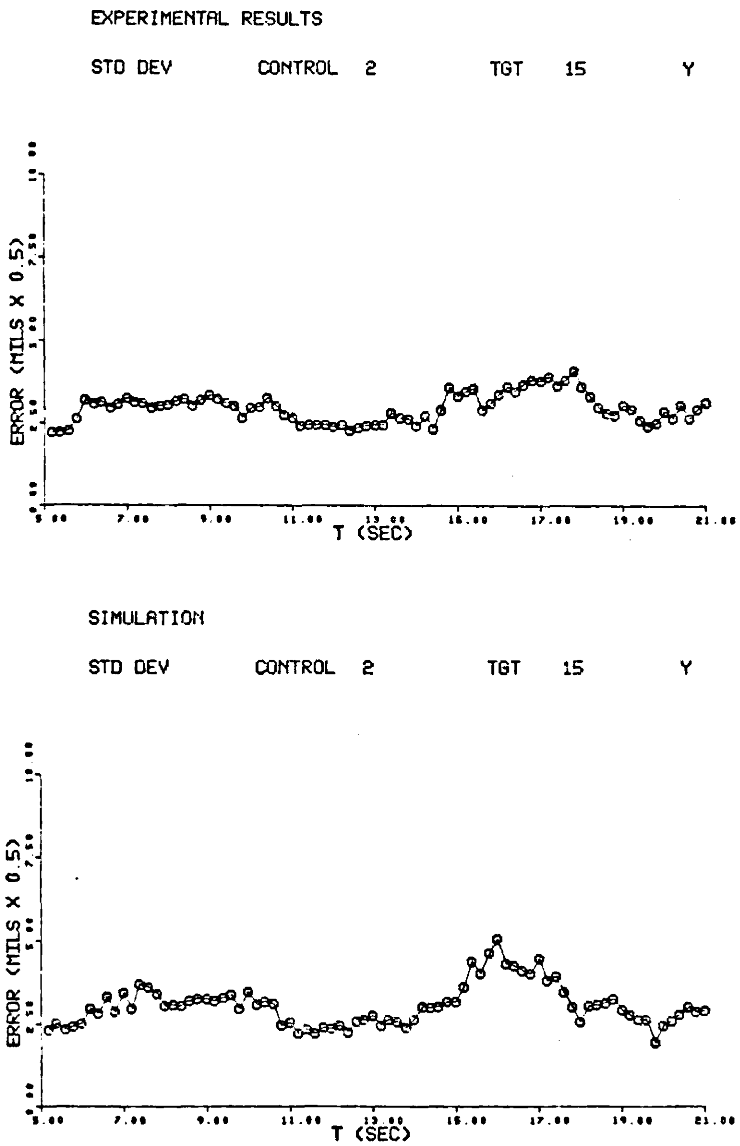


SIMULATION

MEAN CONTROL 2 TGT 15 Y



25c



25d

FIGURE 25. Comparison between experimental results and Monte Carlo simulation. APRE scientific staff. Target Course 15.

do show some tendency to increase where the target starts to manoeuvre. However, the model does appear to underestimate the extent of this relationship. There are probably two reasons for this. Firstly, the model introduces all its variability via the x and y dimensions. No attempt is made to represent variability in the time dimension by including variable time delays. To do so would undoubtedly introduce a sharper error rise where a target starts a new manoeuvre. More importantly, it was mentioned in section 3.4.3 that the model fitting regime (necessitated by the nonstationary character of the noise process) would result in overestimates of the dependency between the tracking responses at successive time instants, and in underestimates of the (indirect) relationships between the variance of the entering 'shocks' and target manoeuvres. It would be possible to adjust the 'remnant' parameters by a nonlinear least squares procedure (such as the developed Dud) to increase the goodness of fit between the model and the obtained sample standard deviations, although this has not been attempted here.

Finally, a comparison has just been provided between experiment and model using a first order dependency scheme. A similar comparison was carried out with a second order scheme, but this extra degree of complication had a negligible effect on the way that standard deviations varied with time. In practical use the degree of dependency assumed in the model would affect the covariance structure of the tracking noise. The effect of assumed order on total fire control system error would almost certainly be marginal. From present evidence the statistical properties of total system error would be more affected by our failure to reflect fully the 'true' nonstationary character of the noise. It is concluded that expanding the 'remnant' model from first to second order would not be worthwhile.

4.3.6 Experiment 3

The 'transfer function' portion of the model which has just been developed and fitted to experimental data is in line with current and past practice, although it has been produced in discrete rather than continuous form. However, the 'remnant' portion of the model treads rather new ground, especially that part of it which attempts to account for its nonstationary character. It would seem to be desirable, then, to look for further corroborative evidence.

If the 'remnant' model is accurate then it may be used to predict the after effects of a momentary disturbance entering the system (such as that which might occur when the gunner presses the 'lase' button). If the effect of such a disturbance is to produce a momentary increase in the variance of the tracking error, then the model would predict that this variance would not return immediately to base level. The variance dependency scheme introduced would imply (provided that no other disturbance entered the system) that the variance would decay exponentially towards the 'threshold' level as its asymptote. Furthermore, the model would predict a fairly minor, but nevertheless noticeable, transfer of disturbance from one plane to another. However, this induced variance would also decay with time.

The simplest way to introduce a disturbance and then to examine its subsequent decay is to use a stationary target, but with a step change of target position at a given instant. The operator response to step inputs is qualitatively different from those made while tracking continuously moving targets (Phatak & Weir, 1968) and the same transfer function will not be applicable in both cases. Laycock (1978) has shown that the initial reaction to a large step input is a rapid aimed, but not continuously guided response, thereafter followed by movements of a more continuously controlled nature. However, this ballistic movement phase will be very short lived, and we may assume that subsequent tracking responses will approximate to those associated with the static portions of more realistic target courses. It is of interest, then, to examine the history of the error variance following a step input, to determine whether it does indeed decay towards some asymptote. Not only would such evidence lend some support to the variance dependency ideas which have been put forward, but it would reinforce the concept of an irreduceable 'threshold' variance, present even with a zero input.

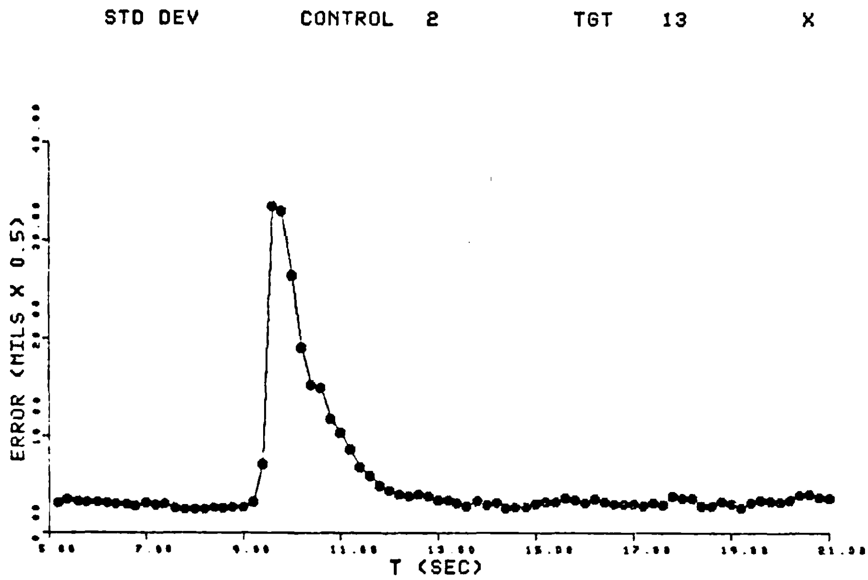
The procedure for Experiment 3 was precisely the same as that which was adopted for the movement joystick portion of Experiment 2. Subject 3 from that experiment was replaced with a new member of the APRE scientific staff (Subject 5) who tackled 180 training runs before embarking on the experiment proper. Once again Subject 1 undertook 12 blocks of tracking runs and the others 6 blocks (12 runs for each of the 15 target courses). 5 target courses from the set of 15 used in Experiments 1 and 2 were replaced by static targets with step changes in position. In 2 courses out of these 5 the change was in one plane only (one in the x plane and the other in the y plane) and in the remaining 3 the target moved in each plane.

Figure 26 shows the averaged standard deviations for 2 of the step input courses (all the other time histories having similar features). The variance decay does seem to follow the expected pattern. It was also established that disturbance in one plane did indeed feed across to the plane in which no target movement occurred, and that it, too, decayed in similar fashion.

The tracking model previously described was fitted to the data yielded by the 10 targets which were unchanged from the previous experiments. The averaged values of the fitted coefficients are shown in Table 3.

TABLE 3 Average fitted parameters, step tracking exercise (10 target subset). Movement joystick.

Plane	x	y
Transfer Function		
b_1	0.264	-0.286
b_2	0.028	-0.014
b_3	2.148	2.733
b_4	-1.045	-1.703
b_5	0.501	0.303
b_6	-0.050	0.139
b_7	-0.247	-0.016
Remnant (1st order)		
ϕ	0.602	0.725
σ_0^2	2.614	0.555
σ_1^2	0.106	0.010
σ_2^2	0.134	0.091
Remnant (2nd order)		
ϕ_1	0.680	0.904
ϕ_2	-0.132	-0.248
Residual SS reduction (%)	2.2	6.8



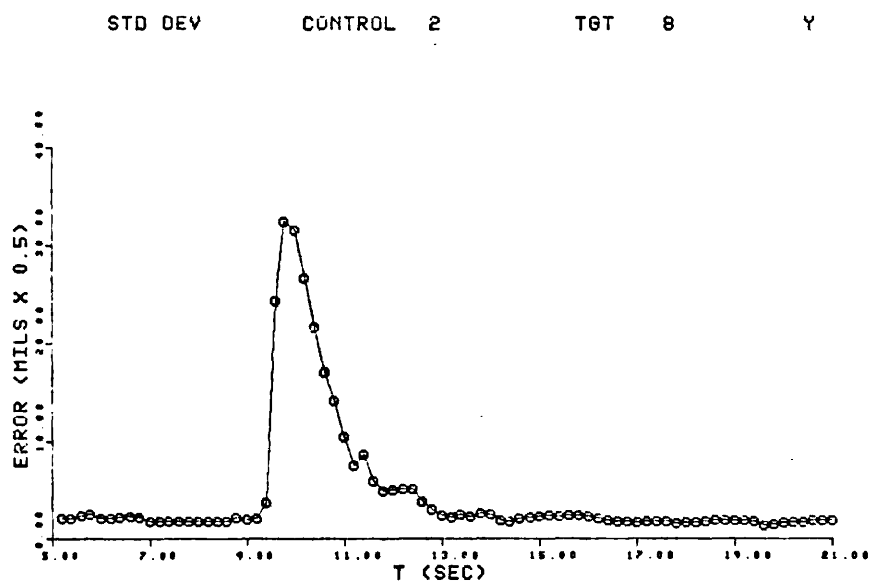


FIGURE 26. Reaction to step inputs. Averaged instantaneous standard deviations. APRE scientific staff.

4.4 Confirmatory modelling exercise

From the preliminary modelling exercise it was concluded that the proposed operator model did provide a reasonable description of tracking behaviour. There remained three points which it was necessary to address. Firstly, the preliminary exercise was based on synthetic target motions, and so it was essential to check whether the model adequately described gunner reaction to target motions more closely in line with those which might be encountered operationally. Secondly, the preliminary exercise concentrated on tracking alone, and two essential features of a live engagement - the action of 'lasing' and the pressure of time - were absent. Obviously, any effects of these two features had to be incorporated into the model. And, thirdly, none of the subject samples used in the first three experiments could be said to be representative of the tank gunner population. (The Infantrymen were the veterans of very many human factors trials and experiments, and their approach to the tracking - plus-hyperventilation experiment was almost certainly different from that which might be expected from a less experienced service sample).

For the experiments of this exercise two synthetic target courses were retained from the original set of 15. (These were Targets 6 and 15, which have been singled out for demonstration purposes, and they were chosen to facilitate comparison from one experiment to another). The other 13 courses were generated from the AMSAA data according to the procedure described in section 2.9.

It was stated in 2.9 that, in generating target courses for use in a human operator modelling exercise, the most important consideration would be to forestall the danger of using the model to extrapolate beyond the range of conditions on which it was based. Now it is evident from the work carried out so far that the major determinants of operator model output are the level of target acceleration (which directly affects the size of the tracking error) and step changes of acceleration (which initiate error transients). The practical implication is that one should select some targets within the operational envelope (in terms of range, height difference between target and defender, etc) but which lie at the extremes of the two characteristics just mentioned. A judgemental procedure was used to do this. By visual inspection of the geographical time plots of the AMSAA targets it was possible to pick out sections of the track where the target scarcely deviated from a steady course, or alternatively where there were successive tight turns (perhaps coupled with changing forward velocities). Nominal defensive positions could then be defined (within a range bracket of 500 to 2000m, a height difference band of 0 to 70m, and with a maximum sight tilt of 10°) which appeared to result in a graded series of tracking exercises. Fifty nominal engagement geometries were generated in this way, and were then rated subjectively as 'very hard', 'hard', 'moderate' and 'easy' on the basis of the acceleration histories yielded by the polynomial fitting procedure given in section 2.9. 4 target courses were selected from the second category and 3 from each of the others to make up the new set of 13. Particulars of these target courses are given in Appendix B.

The details of the simulation were altered in order to incorporate the action of 'lasing' and to introduce a sense of urgency into the procedings. The subject initiated each tracking run himself by pressing his 'take control' button (the left hand button on the handle grasped with his non-preferred hand). The fact that the simulator was ready for him to do so was indicated by shining a static display on the CRT screen, with the aiming mark laid on the target in the zero error position. Immediately he pressed his 'take control' button the target was displaced randomly in both x and y, the displacements being sampled from approximate normal distributions with zero mean and standard deviations of 40 and 20 mrad (at the eye) in x and y respectively. The subject's task was to place his aiming mark over the target as rapidly and accurately as possible, to 'lase' (by pressing his right hand control button) and thereafter to track with minimal error for a further 16 sec. The three factors of lasing speed, lasing accuracy and subsequent tracking accuracy could, of course, be traded off against each other. Subjects were encouraged to concentrate on all three aspects, not allowing anyone to suffer unduly, by computing a total score at the end of each run.

$$s = mc(t_1 + 1)$$

where

s is the total score,

m is the mean miss distance for the 16 sec

t_1 is the elapsed time (sec) from 'take control' to 'lase' and

c is a multiplier whose value depended on accuracy at 'lase' (c = 1 if absolute errors were less than 14 and 7 milliradians, at the eye, in x and y respectively; and c=2 if errors lay outside this window).

The multiplier (t_1 + 1), rather than t_1, was used in order to prevent subjects achieving a very low score simply by 'lasing' almost immediately, irrespective of the effect that this might have on lasing accuracy or tracking accuracy.

Standard target motions were ensured subsequent to lase by holding the target position data in differenced form. The first difference stored in the target position array was added repeatedly until the 'lase' button was pressed, and thereafter the other differences in the array were added sequentially at each sampling instant. The effect was thus for the target to maintain a constant velocity until the 'lase' button was activated, this action then initiating the remaining 16 sec-worth of target manoeuvre.

4.4.1 Experiment 4

Subjects 1, 2, 3 and 5 from the APRE scientific staff sample participated in this experiment. Each undertook 2 blocks of 30 tracking runs in order to familiarise themselves with the new procedure. The experiment proper consisted of 6 blocks of 30 tracking runs for each subject (although once again Subject 1 undertook double this number) using the movement control.

Although considered in the main as a pilot experiment for the one which followed, its aim was also to serve as a basis for comparison between the two sets of conditions and two sets of target courses, with effects due to subject sample differences minimised. Results are reviewed in section 4.4.3.

4.4.2 Experiment 5

The subjects were 2 L/Sgts and 10 Guardsmen from the Grenadier Guards. At first sight it might have seemed preferable to obtain a sample of Royal Armoured Corps gunners. The difficulty here is that subjects in such a sample would almost certainly have had gun laying experience on equipment very different from that represented in the experimental set up. Previously learned responses of this type are extremely resistant to re-training, and might seriously have affected the results obtained. Guardsmen were chosen as representative of 'teeth arm' soldiers, with roughly comparable selection standards to RAC gunners, but without the complications due to previous training.

The subjects completed a minimum of 180 runs as training prior to the experiment proper. This latter consisted of 6 blocks of 30 tracking runs per subject, using the movement joystick.

4.4.3 Evaluation of confirmatory modelling results

In evaluating the results of the confirmatory modelling exercise the 'transfer function' and 'remnant' models were fitted to each subject's data by the procedures which have already been described. Once again, the soldier subjects were subdivided into two groups of 6 based on their average tracking scores taken over the whole experiment. The APRE scientific staff sample has in this instance been designated as sample D, the 6 'best' Guardsmen as sample E and the 6 'worst' as sample F. The suspicion that the Infantrymen who participated in Experiment 1 were an unusual sample is given some weight by the fact that individual variation in this exercise was much less than in the last. The range of average tracking scores is from 5.38 mrad (at the eye) for the 'worst' subject in any of these 3 samples to 3.36 mrad for the 'best'. The means for samples D, E and F are 3.80, 3.81 and 5.02 mrad respectively.

The average values of the fitted model parameters are given in Table 4. It will be seen that the pattern of results is rather similar to those obtained previously. However, a detailed consideration will be deferred until the effects of 'lasing' have been examined and incorporated into the model.

TABLE 4 Average fitted parameters, confirmatory modelling exercise. Movement joystick.

Plane / Sample	x D	x E	x F	y D	y E	y F
Transfer Function						
b_1	-0.276	0.093	0.208	0.059	0.096	0.283
b_2	0.034	0.015	0.020	-0.003	-0.024	-0.018
b_3	1.373	1.694	2.126	1.972	2.028	2.157
b_4	-0.612	-0.647	-1.242	-1.344	-2.630	-2.821
b_5	0.537	0.518	0.315	0.472	0.483	0.581
b_6	-0.199	-0.070	0.255	0.325	0.358	0.345
b_7	-0.231	-0.219	-0.319	-0.185	-0.220	-0.251
Remnant (1st order)						
ϕ	0.557	0.578	0.668	0.630	0.663	0.733
σ_0^2	5.405	5.690	5.848	0.676	0.977	1.211
σ_1^2	0.171	0.159	0.176	0.008	0.012	0.010
σ_2^2	0.274	0.244	0.194	0.240	0.158	0.147
Remnant (2nd order)						
ϕ_1	0.648	0.707	0.843	0.740	0.791	0.906
ϕ_2	-0.163	-0.212	-0.254	-0.170	-0.191	-0.233
Residual SS reduction (%)	3.0	4.8	7.3	3.0	4.0	6.0

During the early part of a live engagement the gunner has to track in the normal way, but he also has to monitor his accuracy continuously, assess whether it is sufficiently good for him to expect a correct laser range indication (rather than one which will merely reflect some foreground or background terrain feature) and at some time he must decide to 'lase' and then act in accord with his decision. Recent research has shown that all these activities up to the pressing of the 'lase' button itself are likely to have virtually no effect on the primary tracking task (Trumbo, Noble & Swink, 1967; Noble, Trumbo & Fowler, 1967; Kerr, 1975; Wickens, 1976; and McLeod, 1977). Whether the response itself will interfere with tracking depends on the response modality: a verbal response will produce practically no interference, but one made via the medium of any other motor channel will result in significant tracking disturbance. It is as though motor output processing capacity is strictly limited, so that controlling effort must be switched from the tracking task for a short interval prior and subsequent to making the required competing motor response. There is, then, good reason to expect a tracking disturbance injected, in effect, at the instant of 'lase'. The simplest scheme to represent this disturbance would be to assume that an abnormal 'shock' enters the tracking system at this time, and that this then decays in the same manner as all the other shocks which enter at each sampling instant. The variance of the 'lasing shock' could be estimated empirically.

TABLE 5 Lase statistics. Shown are the average error variances (in mrad at the eye) 0.1 sec after lase (σ_1^2) and the dependency coefficients based on the error data at the 0.1 and 0.3 sec points after lase (ϕ_1).

Plane / Sample	x			y		
	D	E	F	D	E	F
σ_1^2	17.460	28.058	48.384	3.941	6.153	12.070
ϕ_1	0.607	0.599	0.620	0.628	0.633	0.711

Table 5 assembles some relevant results. It shows the tracking error variance for each subject sample 0.1 sec after 'lase', averaged over all target courses and all subjects within the sample. The data were examined to determine whether the size of this variance depended on target velocities prior to lase or accelerations thereafter. No significant relationships could be found. Table 5 also shows the average regression coefficient relating tracking error at 0.3 sec after 'lase' to that at 0.1 sec. The values are sufficiently close to those of the ϕ coefficients in Table 4 for one to accept the simple scheme proposed at the end of the last paragraph as a reasonable means of incorporating 'lase' disturbances into our tracking model.

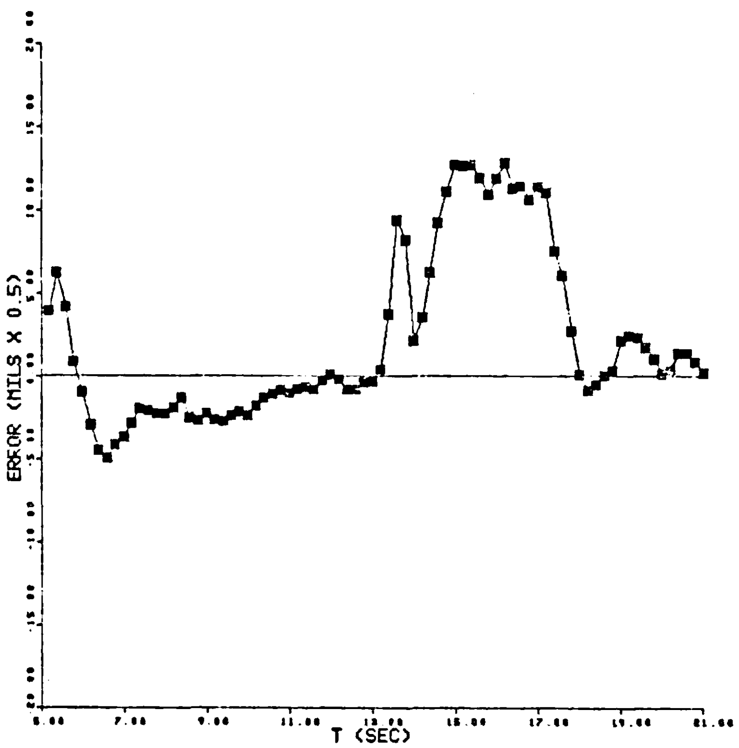
The Monte Carlo tracking simulation routine put forward in section 4.3.5 now needs but slight modification in order to incorporate 'lase' effects. Tracking errors are generated recursively according to the algorithm embodied in Equations (37) to (39), but the procedure used for starting up the simulation is slightly different. In what follows it is assumed that only the tracking history subsequent to 'lase' is of practical interest. The determinate 'transfer function' is computed from a moment 2 or 3 sec prior to 'lase', with arbitrary but 'reasonable' starting values, in the manner which was described in section 4.3.5. However, the stochastic 'remnant' contribution to error is calculated only from the instant of 'lase' onward, the starting values of ε_{xt} and ε_{yt} being sampled from normal distributions with the empirically determined lasing variances.

This Monte Carlo simulation scheme was used to mimic the results of Experiments 4 and 5 in exactly the same way as the unmodified scheme was used in section 4.3.5 to mimic Experiments 1 and 2. Figures 27 to 29, illustrate just some of the results with the 6 'worst' Guardsmen (sample F).

Turning first to the experimental results obtained for Target 6 (Figure 27) it will be seen from the standard deviation plots that there does indeed seem to be a disturbance at lase. This disturbance appears to decay exponentially until target manoeuvre acts to inflate error variance once again. However, a comparison between the experimental and the simulation results shows that agreement between the two is not so close

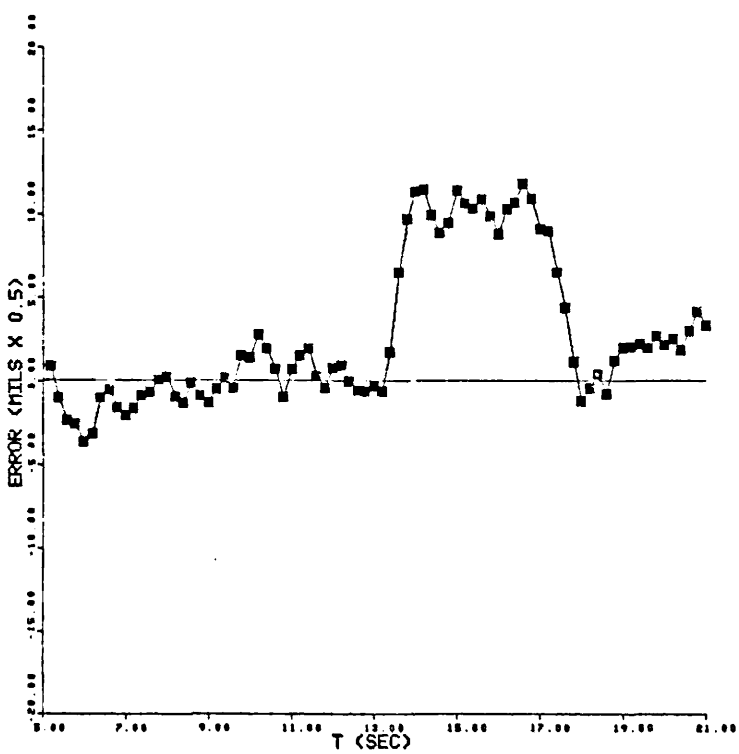
EXPERIMENTAL RESULTS

MEAN CONTROL 2 TGT 6 X



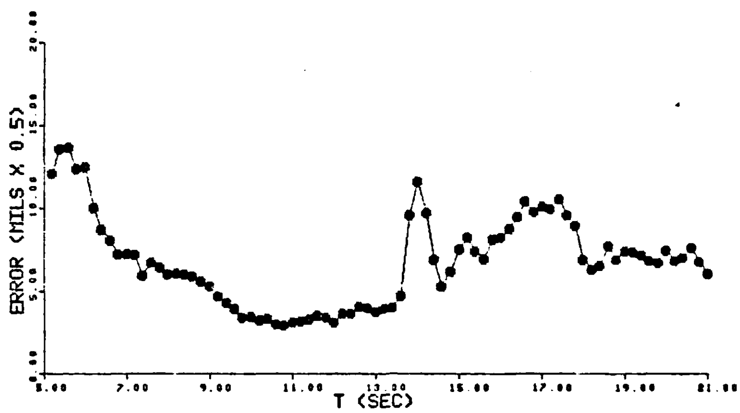
SIMULATION

MEAN CONTROL 2 TGT 6 X



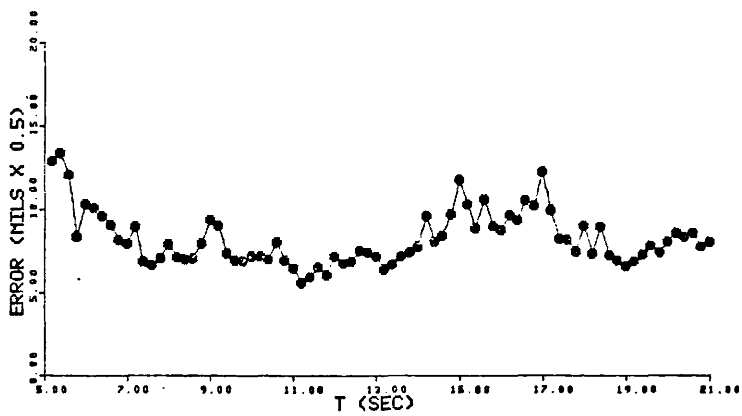
EXPERIMENTAL RESULTS

STD DEV CONTROL 2 TGT 6 X



SIMULATION

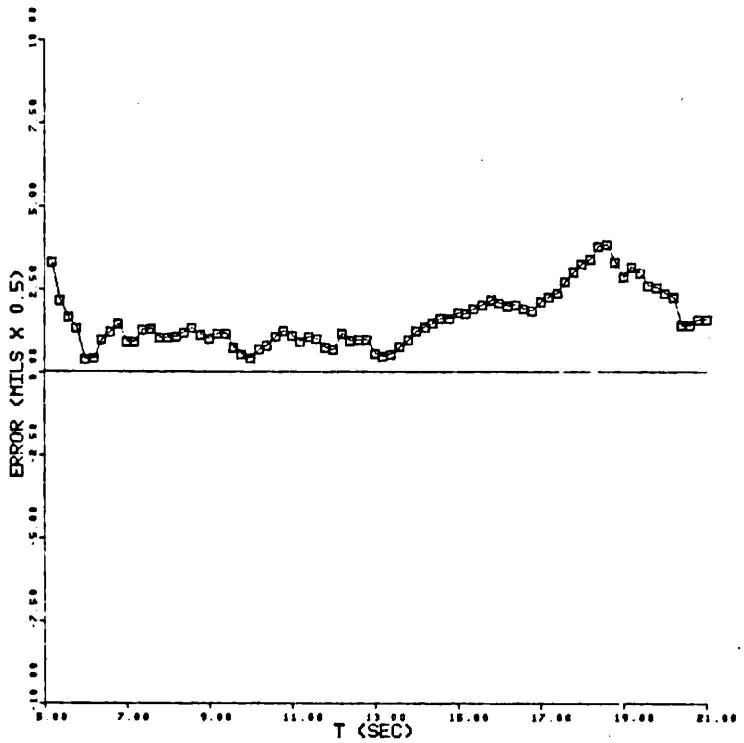
STD DEV CONTROL 2 TGT 6 X



27b

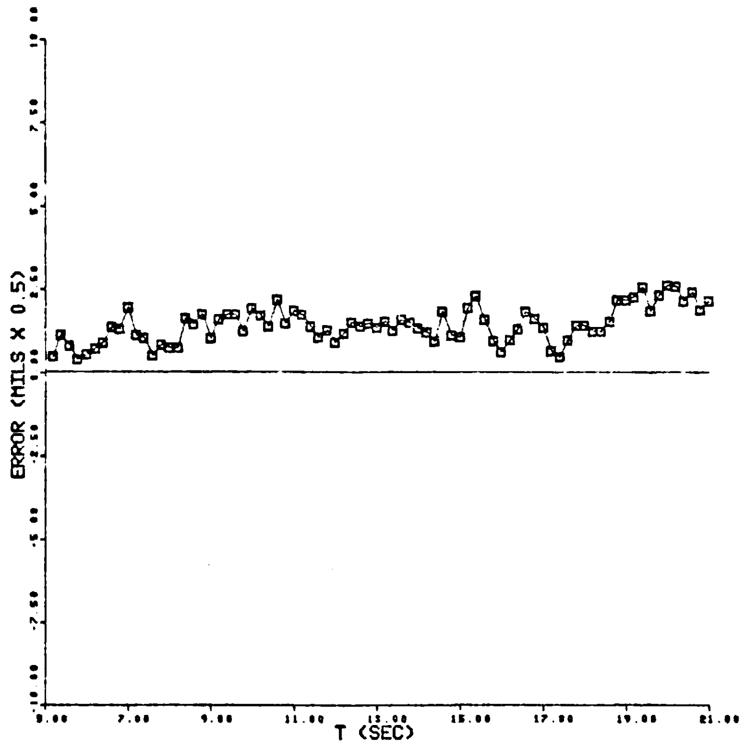
EXPERIMENTAL RESULTS

MEAN CONTROL 2 TGT 6 Y

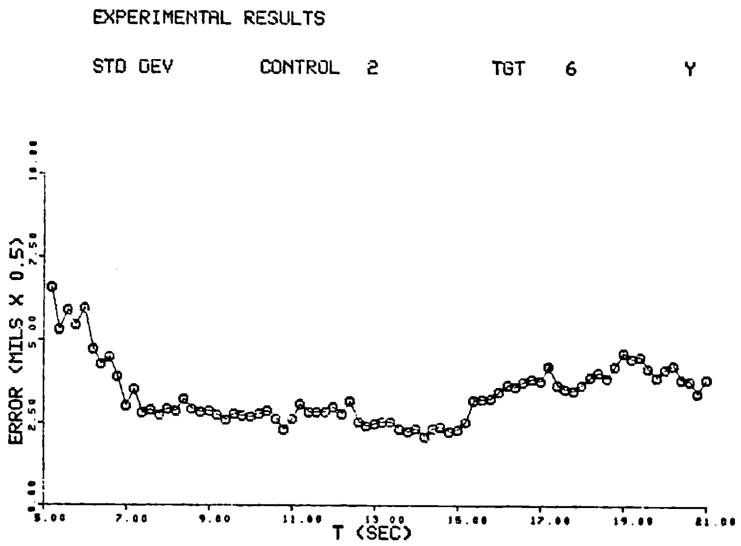


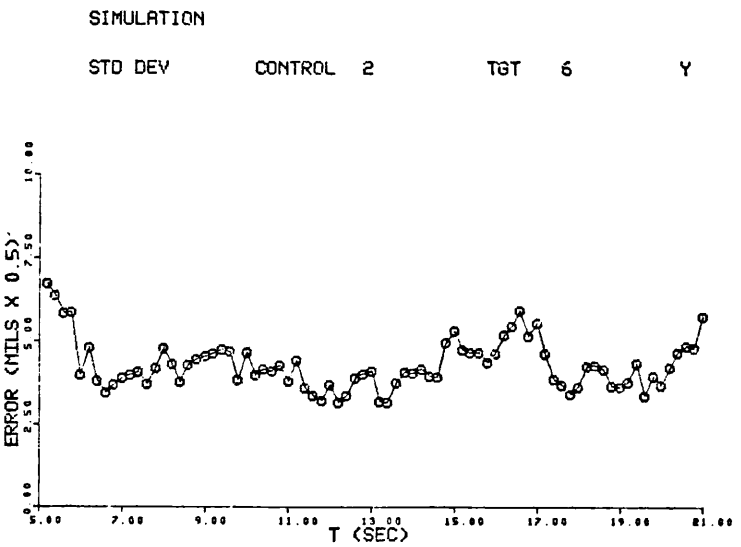
SIMULATION

MEAN CONTROL 2 TGT 6 Y



27c



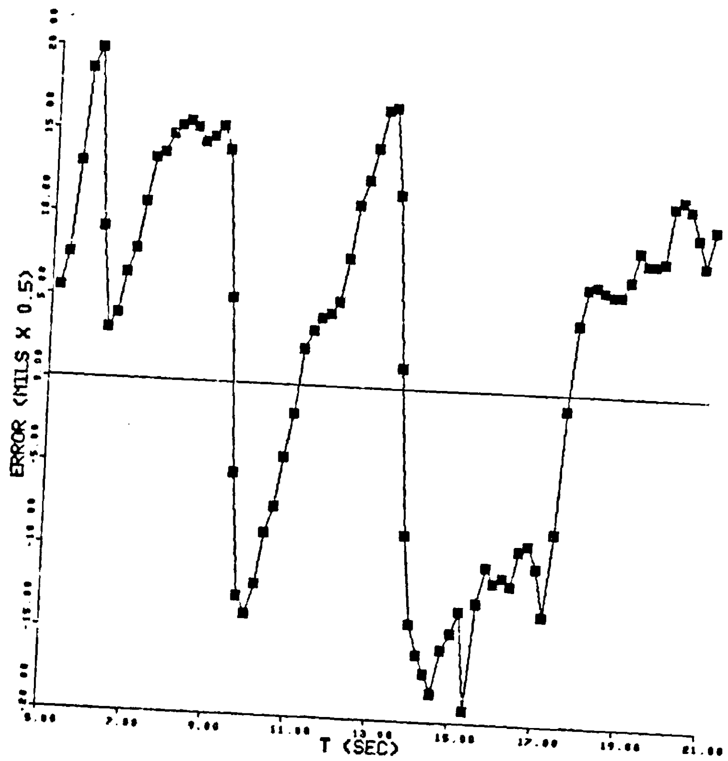


27d

FIGURE 27. Comparison between experimental results and Monte Carlo simulation. Sample F (6 'worst' Guardsmen). Target Course 6.

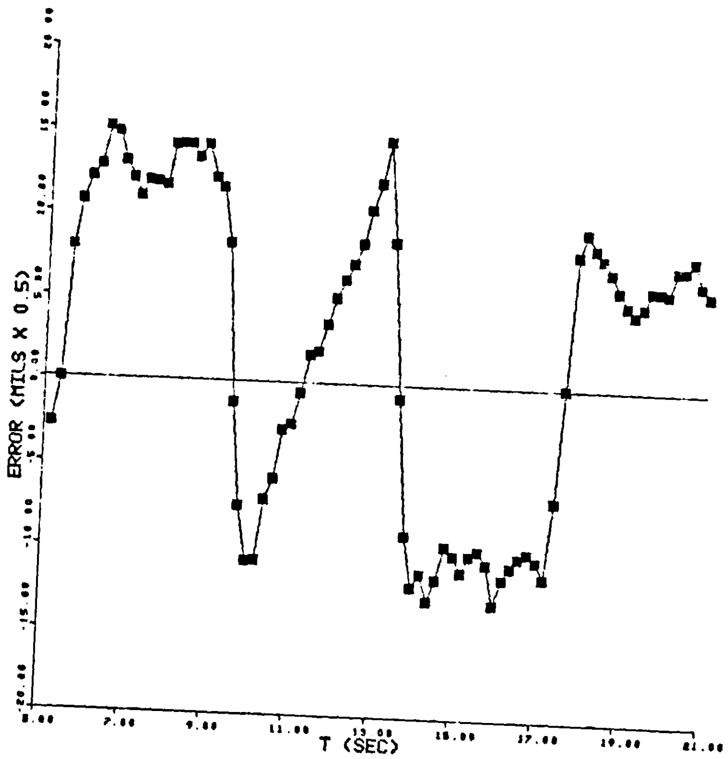
EXPERIMENTAL RESULTS

MEAN CONTROL 2 TGT 15 X



SIMULATION

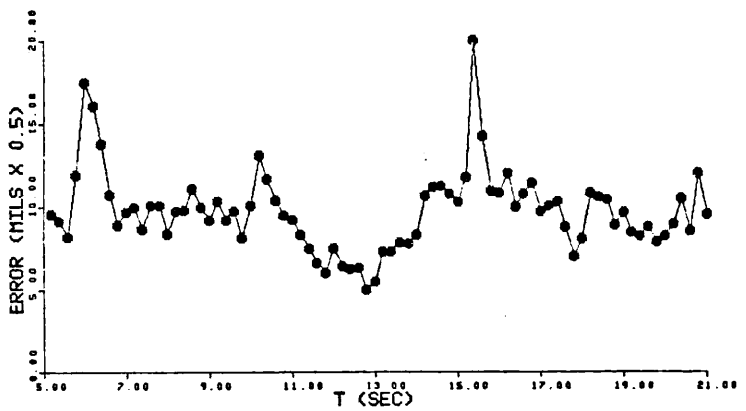
MEAN CONTROL 2 TGT 15 X



28a

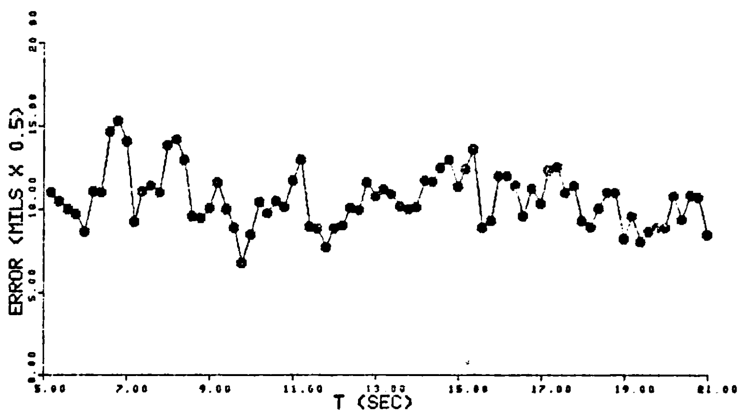
EXPERIMENTAL RESULTS

STD DEV CONTROL 2 TGT 15 X



SIMULATION

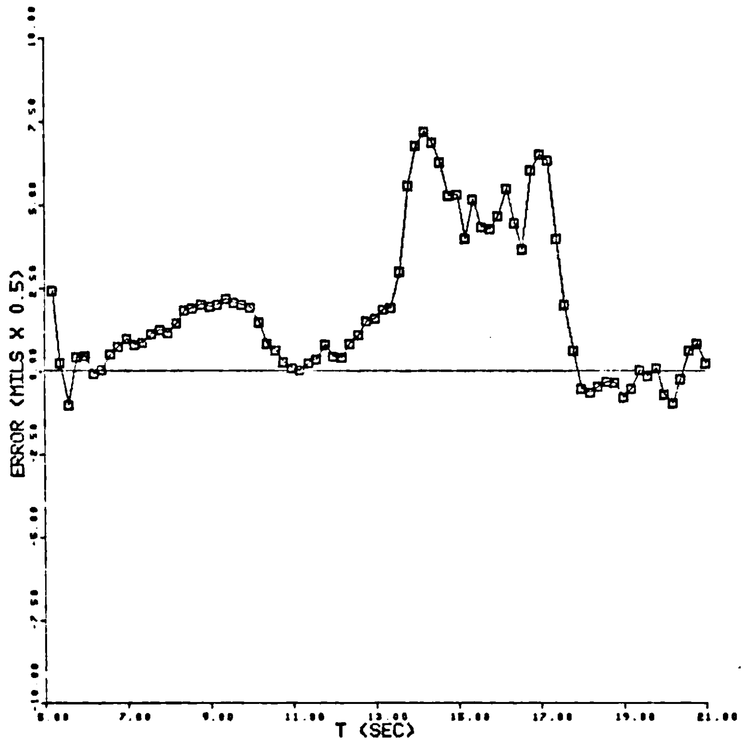
STD DEV CONTROL 2 TGT 15 X



28b

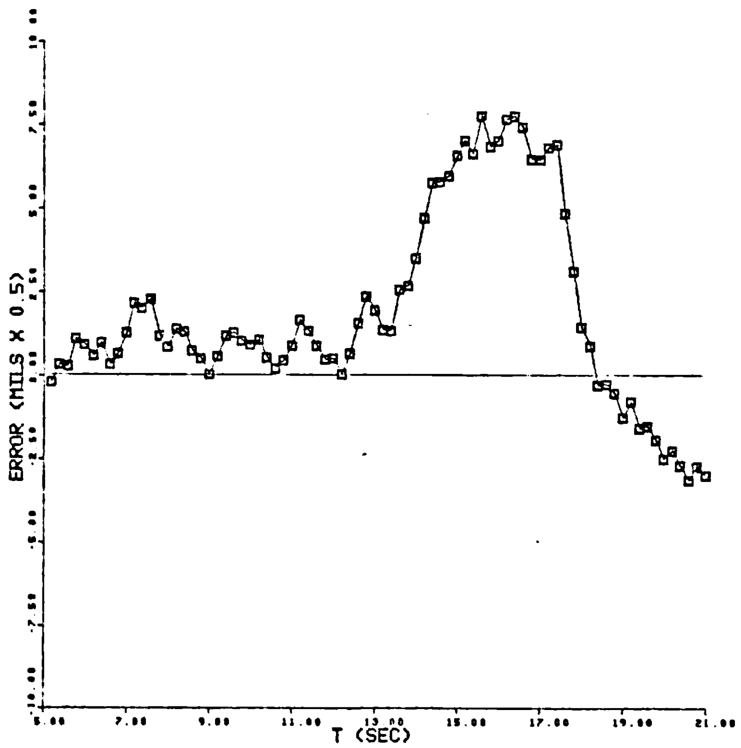
EXPERIMENTAL RESULTS

MEAN CONTROL 2 TGT 15 Y

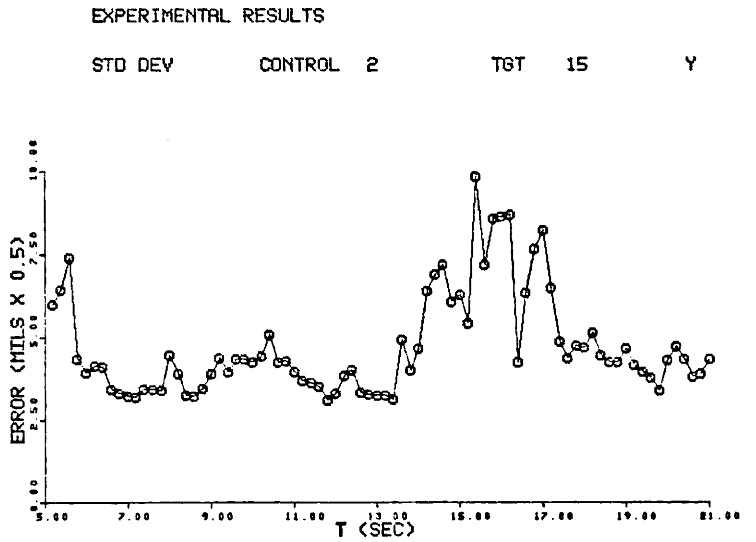


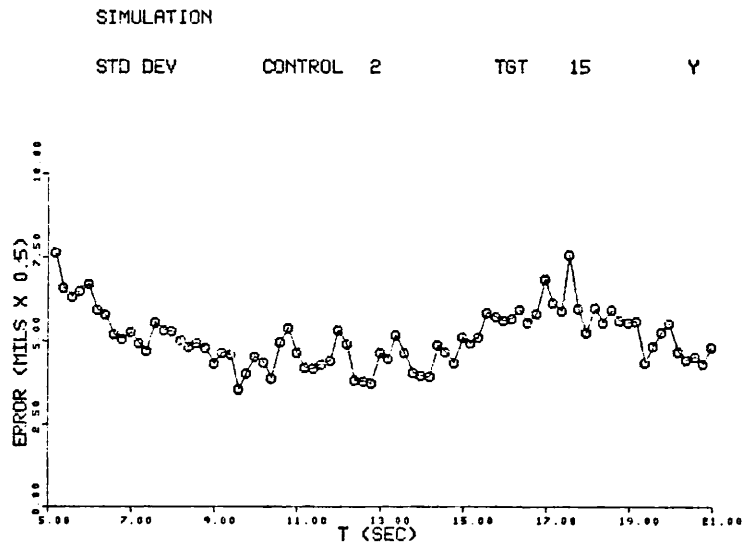
SIMULATION

MEAN CONTROL 2 TGT 15 Y



28c



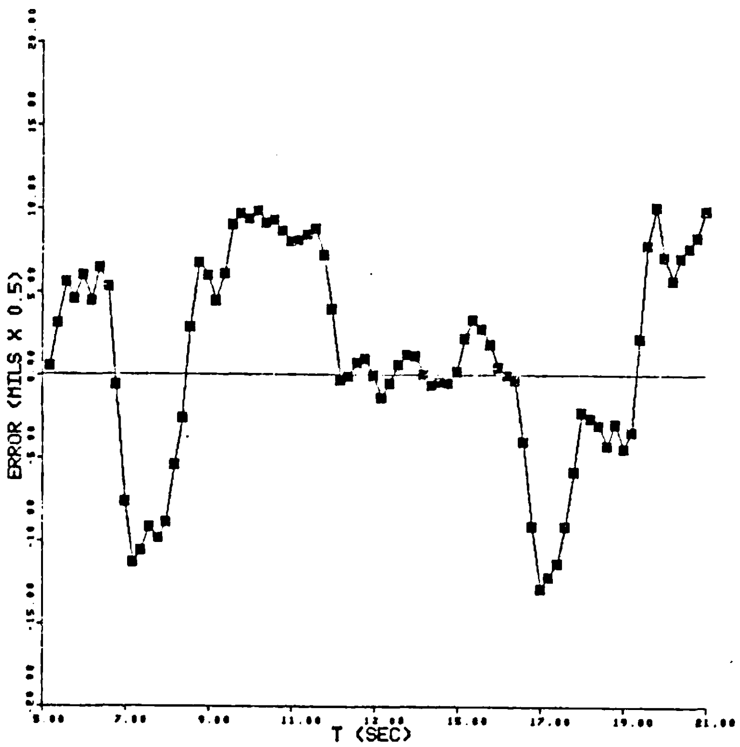


28d

FIGURE 28. Comparison between experimental results and Monte Carlo simulation. Sample F (6 'worst' Guardsmen). Target Course 15.

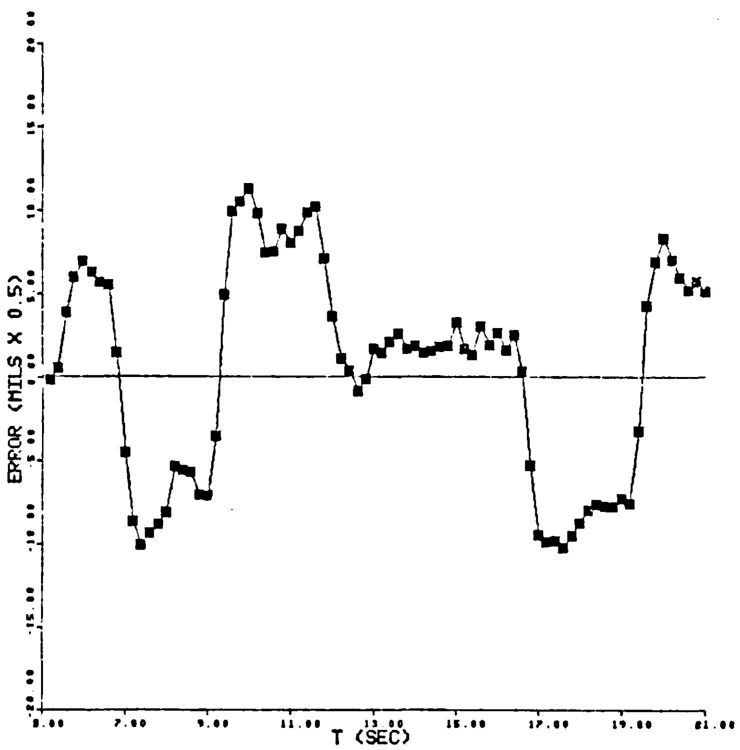
EXPERIMENTAL RESULTS

MEAN CONTROL 2 TGT 2 X



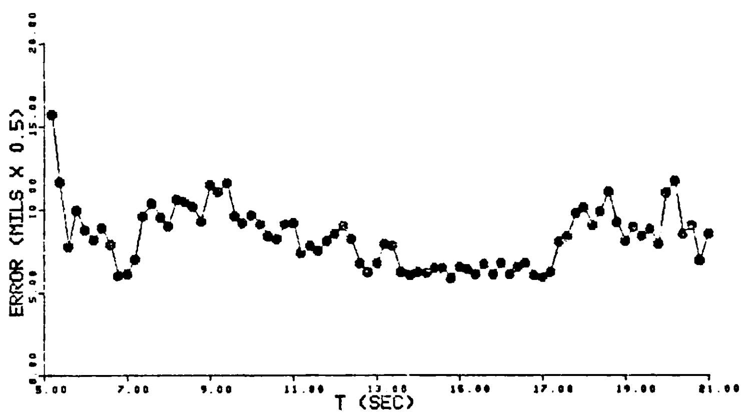
SIMULATION

MEAN CONTROL 2 TGT 2 X



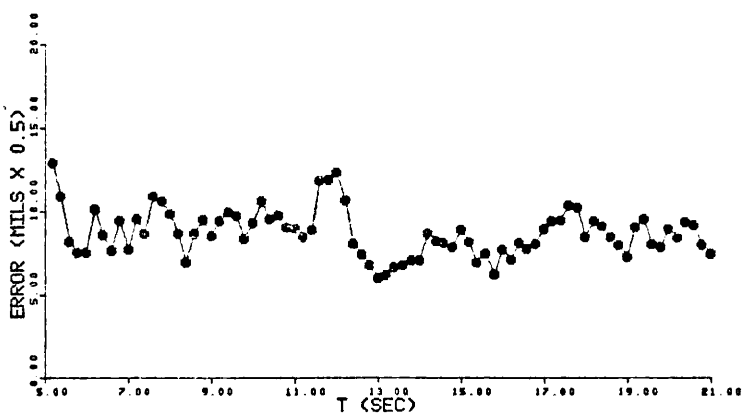
EXPERIMENTAL RESULTS

STD DEV   CONTROL 2   TGT 2   X



SIMULATION

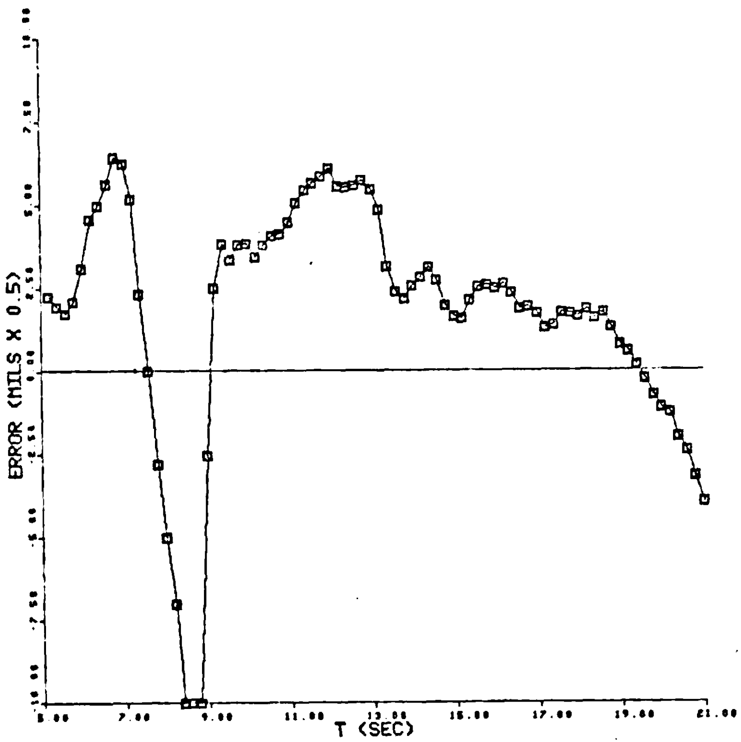
STD DEV   CONTROL 2   TGT 2   X



29b

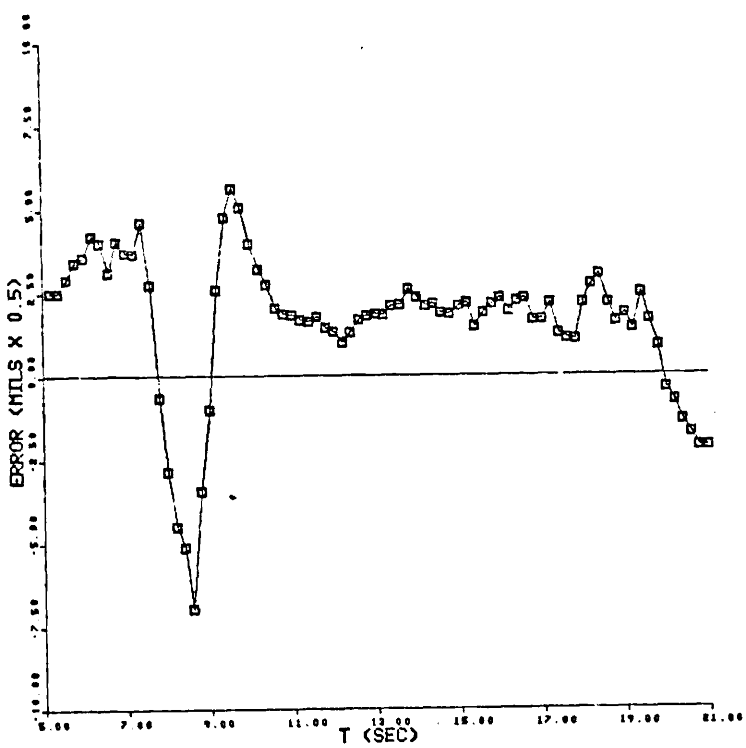
EXPERIMENTAL RESULTS

MEAN CONTROL 2 TGT 2 Y



SIMULATION

MEAN CONTROL 2 TGT 2 Y



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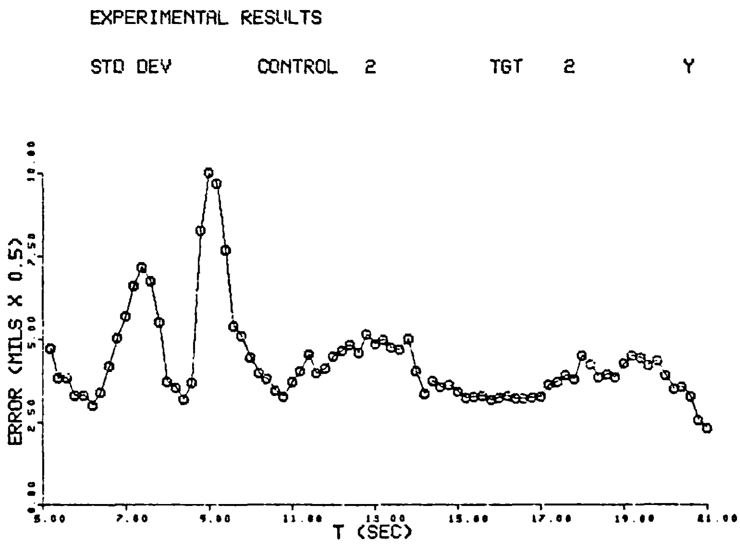

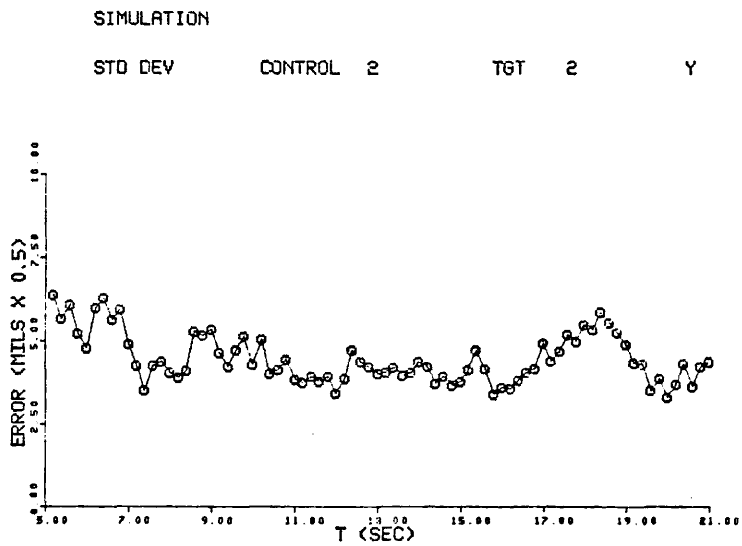

29d

FIGURE 29. Comparison between experimental results and Monte Carlo simulation. Sample F (6 'worst' Guardsmen). Target Course 2.

as it was in the preliminary modelling exercise. Concentrating again on the standard deviation plots, the experimental records show a very marked variance decay after lase, and a very sharp rise when systematic error increases at about the 13 sec point, but the strength of this dependence is scarcely mirrored in the simulation record. There would seem to be two main reasons why this should be so. The first reason has already been touched on in sections 3.4.3 and 4.3.5: the 'remnant' model is biased because it is fitted to the distorted residuals from the 'transfer function' fitting procedure, rather than to the 'true' residuals. The second reason has to do with a tracking phenomenon which has been noted for the first time under the conditions of this confirmatory modelling exercise.

Turning once more to the experimental record for Target 6, but this time concentrating on mean error, it will be noted that the response pattern is roughly similar to that yielded by previous experiments (Figures 19, 21 and 24). There is, however, one difference. Immediately after the sharp increase of error at t=13 sec the mean error trace returns to zero, before rising again to its plateau level. This temporary return to zero is associated with a peaking of the error variance. Inspection of other error traces shows that with violent manoeuvres this pattern is frequently repeated, often in more exaggerated form. Note the standard deviation in the y plane for Target 2. Under the conditions of this exercise a pronounced step change of acceleration is often followed by a short-lived, but very variable, response (which on occasion markedly over-compensates for the new motion). Response rapidly settles down thereafter to a much less variable pattern, approximating to a fixed lag. On the standard deviation plots these episodes show as pronounced but short-lived SD peaks. Our tracking model (and indeed all others known to the author) quite fails to account for these variable 'surges'. As the variance peaks are unpredictable the model fitting procedures sweep them into the residual error, where they show as an inflated 'threshold' variance term (σ_0). (Compare the σ_0 terms for the APRE scientific staff using the movement joystick obtained in Experiments 2 and 3, with those obtained in Experiment 4 as shown in Tables 1, 3 and 4. It will be recalled that these experiments all used subsets of 4 subjects from the same 5 member set).

The inflated 'threshold' error contributions estimated from the data yielded by the confirmatory exercise quite mask the rise, and subsequent decay, of error due to 'lasing' and target manoeuvre.

4.5 Review of human operator model features

The discrete time model we have developed regards human tracking error as being comprised of two components: a determinate 'transfer function', and a stochastically varying 'remnant':

$$e_{xt} = f_x(x_t) + \varepsilon_{xt} \qquad (40)$$

$$e_{yt} = f_y(y_t) + \varepsilon_{yt} \qquad (41)$$

where

e_{xt}, e_{yt} are total tracking errors in the x and y planes respectively at time t,

$f_x(x_t), f_y(y_t)$ are the 'transfer function' components

ε_{xt}, ε_{yt} are the 'remnant' contributions,

x_t, y_t being target positions in the two planes at time t.

The 'transfer function' is represented by a difference equation. With a time interval of 0.2 sec this has the general form:

$$f_x(x_t) = b_{x_1} + (b_{x_2}\nabla + b_{x_3}\nabla^2 + b_{x_4}\nabla^3)x_t + (b_{x_5}B + b_{x_6}B^2 + b_{x_7}B^3)f_x(x_t) \qquad (42)$$

(and similarly for $f_y(y_t)$). The values of the fitted parameters tend to be variable, in large part because of the interacting effects of the last three terms. However, the shape of the function yielded by the fitted parameters is very stable. 'Bad' trackers have similar 'transfer functions' to 'good' trackers, but with a higher gain or amplitude.

The 'remnant' contribution to tracking error has been modelled as a first order autoregressive process:

$$\varepsilon_{xt} = \phi_x \varepsilon_{x(t-1)} + a_{xt} \qquad (43)$$

$$\varepsilon_{yt} = \phi_y \varepsilon_{y(t-1)} + a_{yt} \qquad (44)$$

where

$\varepsilon_{xt}, \varepsilon_{yt}$ are the 'remnant' contributions to error in the x and y planes at time t,

ϕ_x, ϕ_y are autoregressive parameters, and

a_{xt}, a_{yt} are independent normally distributed 'shocks' entering the system at time t.

There is evidence to suggest that a second order autoregressive process. would provide a slightly more accurate representation of the stochastic element of tracking error, especially for 'bad' trackers. However, the first order scheme is much more straightforward in practical use, and the slight gain in accuracy consequent upon using a second order scheme is quite overshadowed by inaccuracies due to the limitations of the model fitting process, and by our failure to account for brief variable error 'surges' following a change of target acceleration.

Although independent, the a's cannot be regarded as having a fixed variance. The variance of the entering 'shocks' has been held to be dependent on the tracking errors at the previous sampling instant, according to the scheme:

$$\sigma^2(a_{xt}) = \sigma^2_{xo} + \sigma^2_{x1}\, e^2_{x(t-i)} + \sigma^2_{x2}\, e^2_{y(t-1)} \tag{45}$$

$$\sigma^2(a_{yt}) = \sigma^2_{yo} + \sigma^2_{y1}\, e^2_{x(t-1)} + \sigma^2_{y2}\, e^2_{y(t-1)} \tag{46}$$

where

$\sigma^2(a_{xt}), \sigma^2(a_{yt})$ are the variances of the 'shocks' entering the system at time t in the x and y planes,

$\sigma^2_{xo}\ \sigma^2_{yo}$ are the 'threshold' error terms, and

$\sigma^2_{x1}, \sigma^2_{x2}, \sigma^2_{y1}, \sigma^2_{y2}$ are the variance dependency parameters.

The following expressions for the variances and covariances of the tracking errors are implied:

$$\sigma^2(e_{xt}) = \sigma^2(a_{xt}) = \sigma^2_{xo} + (\phi^2_x + \sigma^2_{x1})\sigma^2(e_{x(t-1)}) + \sigma^2_{x2}\,\sigma^2(e_{y(t-1)}) \tag{47}$$

$$\mathrm{cov}(e_{xt}, e_{x(t-j)}) = \phi^j_x \sigma^2(e_{x(t-j)}) \tag{48}$$

Clearly the model just put forward stops some way short of providing a perfect description of tracking results. Ways have been suggested for countering the bias in estimating the 'remnant' parameters due to limitations in the model-fitting process, and doubtless empirical schemes could be devised to reflect to some extent the error 'surge' phenomenon identified in the last section. However, even as it stands the model probably provides a more accurate representation of gunner behaviour than most of the rough-and-ready schemes used in the past. It is probably adequate for a preliminary assessment of different fire control prediction schemes.

5. HUMAN OPERATOR MODELLING: FIELD EFFECTS

5.1 Introduction

In chapter 4 experimental results were presented, and a gunner tracking model was developed which could have some utility for the synthesis and evaluation of potential fire control systems. However, all the data were gathered in the laboratory in conditions very different from those which obtain on the battlefield, and it is, after all, with battlefield applications that we are ultimately concerned. It seems appropriate, therefore, to pause and consider the way that field effects may modify the results obtained so far.

The aim of this chapter is to review field effects under three main headings: target display characteristics; mechanical stresses; and environmental stresses. A comprehensive review would be an immense undertaking, quite beyond the scope of this thesis. The intention, instead, is to consider these factors very briefly, and to put forward tentative suggestions as to the way that model parameters may have to be adjusted to take account of battlefield effects. Most of these factors are amenable to research investigation, with one important exception - the human reaction of fear.

5.2 Target display characteristics

Up to now we have concentrated on the angular motion characteristics of the target. The modelling results have been obtained on the basis of gunners tracking, if not point targets, at least quite easily identifiable locations on those targets. Even in such constant and clear cut conditions individual operators tended to have characteristic (but relatively small) biases. However, an operational target will have an extended outline which may alter as it moves. Details may not be readily discernable (especially if a TV display of limited resolution is employed), the outline itself may not be distinct, and indeed the vehicle may be camouflaged. In addition, even if we assume that an engagement is always re-started if the target is totally obscured at any time, it is highly likely that a (varying) portion of the hull will be masked from view (Eckles, et al, 1973; Rowland & Stead, 1977). Even in good visibility and with clearly defined targets the data from field investigations indicate that tracking accuracy will be much degraded, lack of visual clarity having a further deleterious effect (Eckles, et al, 1973; Garry, 1974 quoted in Armour Team, 1976).

The extent to which tracking performance is impaired when gunners face live targets must depend in part on training, the aiming strategy which is taught, and details such as graticule design. Even so, it is likely that there will be appreciable bias, which may shift more-or-less slowly as the target changes its aspect. Under these conditions the characteristics of the remnant must change, if only because the gunner has no clearly defined spot to null his error to, and hence his assessment of error for visual feed back purposes must be more a matter of judgement than of observation. One would expect, then, that the autocorrelation coefficient in the remnant error term (ϕ) would increase, and that the variance of the entering shocks (at least as reflected in the σ_o term) would also increase. There could, however, be fairly regular biases dependent on range and aspect, most easily represented by some scheme which made the b_o term in the determinate portion of the model a function of these two parameters. The remaining b terms should, however, be little affected.

The evidence from the few carefully instrumented live trials which have been conducted (Eckles et al, 1973; Garry, 1974) suggests that target display features are major factors determining tracking response, and to date there are no good quantitative estimates of their effects. Clearly, our chances of obtaining such estimates would be much enhanced if we had in the laboratory a tracking tool capable of the same accuracy as that used in the research programme just described, but which would alter the target image in a controlled and predetermined fashion against a background more representative of operational terrain. Accordingly, the author put this requirement to the Cranfield Institute of Technology. The scheme which has been jointly evolved is to represent the target image in software terms, by specifying some 50 spatial coordinates of target surface and manipulating these by a collection of algorithms to yield the solid silhouette appropriate to a specified viewing angle and range. The background, on the other hand, is represented in hardware terms, by a coordinate table which responds in appropriate manner to joystick inputs and on which is superimposed a pictorial representation of a tactical scene. The apparent (possibly changing) tank image can then be moved in an appropriate manner across the background, and superimposed on it by electronic mixing techniques. The basic system had been proved, and the research simulator itself was on the verge of completion and delivery as this chapter was written.

5.3 Mechanical effects

The most obvious mechanical stresses imposed on the gunner are the shocks and vibration of his own vehicle if he is required to fire on the move. Even if his own vehicle is stationary the turret itself will move as the engagement proceeds (and there will not necessarily be a very direct relationship between joystick outputs and those motions) That shocks and vibration may affect tracking accuracy is not in the least surprising, and Collins (1973) and more recently Drennan, Curtin & Weaver (1977) have provided reviews attempting to link degradation to details such as the spectral characteristics of the vibration input, different features of the control, and so forth. However, these, and most other, authors have concentrated on some overall statistic of tracking performance, such as rms tracking error. It is only relatively recently that analytical tools have been brought to bear in attempts to discover how transfer functions and remnant characteristics have been affected by vibration (Allen, Jex & Magdaleno 1972, 1973; Jex & Allen, 1975; Levison & Houck, 1975; Levison, 1976; Lewis, 1976; Lewis & Griffin, 1976, 1978a 1978b).

Perhaps the most surprising finding of the more recent vibration studies mentioned above is that in most cases (depending on the control characteristics, limb support and so on) direct vibration feed through accounts for only a tiny proportion of the tracking error variance. The vibration input is much attenuated by the human frame itself, and, although there is a very noticeable effect due to vibration, that effect seems to be produced by a mechanism which is indirect. Lewis & Griffin (1976) argue that the mode of action of vibration is to corrupt the kinaesthetic feedback information (so that those sensations which inform the operator of the whereabouts of his controlling limb, or of the pressure that he is exerting, are masked by the sensations induced by vibration). As previously stated in section 3.4.3, more recent evidence suggests that visual feedback is in fact the dominant influence in governing motor behaviour (see especially Adams, Gopher & Lintern, 1977, and Jones 1974) However the effect of vibration is to degrade visual acuity also, and Shoenberger (1975) has produced evidence which suggests that it is indeed visual, rather than mechanical, interference which is mainly involved. The effects of vibration on visual acuity have been summarised and reviewed by Griffin & Lewis (1978). If we regard the transfer function portion of our model as being principally concerned with human ability to predict target motion, and the remnant as reflecting the inability to act in accord with those predictions without inducing some random error, then the reduction of visual (or kinaesthetic) feed back might be expected to increase the remnant leaving the transfer function virtually unchanged. This is precisely the effect observed by the research workers listed in the last paragraph.

In conclusion, it seems likely that vibration and turret motion will leave the nature of the transfer function in our model virtually unaffected. The size of the remnant will increase, but only a small part of this increase will be due to the mechanical feed through of input vibrations. As with degradation due to target display features, the main effect should be to increase the variance of the 'shocks' entering the system at each sampling instant.

5.4 Effects due to environmental factors

Vibration may have a non-specific effect, by increasing arousal, as well as a specific effect due to motor or visual interference. There are, however, hosts of other factors which may be operative on the battlefield and which could also have non-specific effects, which could be degrading or facilitating. Heat (Mackworth, 1945, 1961; Blockley & Lyman, 1951; Teichner & Wehrkamp, 1954; Pepler, 1958, 1959, 1960; Swisher & Maher, 1972); cold (Russell, 1957); noise (Harris & Shoenberger, 1970); and many other stresses have been found to affect tracking performance. Recently there has been an interest in applying tracking modelling techniques to describe some of these effects.

Replogle, Holden & Day, 1971; Phatak, 1973; Mehra & Tyler, 1972; Phatak, Mehra & Day, 1975; Repperger, Smiles, Neff & Summers, 1978). However, the effects of all these different stresses are subtle and complex, and it is known that their joint effects are not always predictable from the results of stresses applied one at a time (Wilkinson, 1969). Rather than attempt to unravel all these intricacies let us just appeal to the evidence which has been gathered, with difficulty, from war time records by Walker & DeSocio (1964). Whatever the combination of actual stresses (and it is felt that fear has the single most important effect) the evidence indicates that in battle tracking performance is much degraded when compared with laboratory or training range results. In aircraft crews, if severity of combat is measured by the per-sortie probability of becoming a casualty, then there is a linear relationship between rms error and severity. Furthermore, the more 'difficult' the tracking system (as measured by absolute errors in the nonstressed condition) the higher the percentage degradation. For extreme levels of combat,errors may be degraded by a factor of 2 if the operator is faced with a straightforward velocity control, and by a factor of 10 if the tracking system is 'difficult' (acceleration control with lags). Even live firing (as opposed to live tracking runs where there is no intention to fire) has been demonstrated to have an effect (Ford, Speight, Henschke & Readett, 1972).

For the kinds of degradation considered in this section it has been difficult to obtain any estimates of the manner in which the transfer function and remnant may be separately affected. Curtin (1971) carried out a limited investigation of the firing and non-firing tracking records mentioned in Ford et al (1972). To a first approximation the spectral contents of these records were not altered in the firing condition. but the overall gain was increased. This would imply that both transfer function and remnant were multiplied by a simple scaling factor. Similarly, the limited studies of other environmental stresses which were mentioned in the previous paragraph suggest that both the transfer function and the remnant are affected. In the absence of more reliable information it could be that the effects of those environmental factors, which we shall lump together and call 'battlefield stress', are best represented by utilising the modelling results from 'poor' or untrained gunners.

5.5 Tentative conclusions

The mechanical stresses of shock and vibration, the ambiguities induced by real-life target displays, and the environmental factors associated with the battlefield all have an undeniable effect on tracking performance. Degradation due to the first two categories is probably best modelled by altering the characteristics of the remnant. The precise way that remnant parameters should be modified can be established by research. 'Battlefield stress' is not amenable to research, however. From present knowledge the best means of incorporating this factor would be to concentrate on the modelling results obtained from 'poor' or relatively untrained experimental subjects.

6. PREDICTION ALGORITHMS: BASIC PRINCIPLES AND THEIR APPLICATION TO SIMPLE SCHEMATIC ENGAGEMENTS

6.1 Bias and random error in prediction

We have now indicated the manner in which operational targets may move, and have developed a model to represent the way in which the gunner may respond to those motions. It now remains to examine ways in which future target position may be predicted from present gunner tracking information, and then, by establishing the bias and random error properties of these predictive estimates, to compute the likely chances of a hit against specified targets.

Modern writers tend to favour the state space approach when considering the topic of estimation and prediction. The problem is regarded as one of obtaining an optimal estimate of a target's current state vector (say, its position, velocity and acceleration in cartesion or polar coordinates) and then, by multiplying by a suitable transformation matrix, of predicting what the state vector will be at some time ahead. This undoubtedly makes for compact development and exposition. However, in this thesis we prefer to regard the problem in the light of traditional regression theory. There are two reasons for this. Firstly, it is much easier in this way to obtain an intuitive understanding of the manner in which errors arise and combine in the overall system. And, secondly, it was pointed out in Chapter 1 that in essence the problem of optimising prediction consisted of obtaining estimates of likely random error and bias from treating tracking information in different ways, and then of balancing these two components of final system output in such a way as to maximise our chance of hit. Points at which these compromises can be made seem to be revealed more sharply when the problem is considered in regression terms.

Suppose, then, that we have a target course $x(t)$, where x is target angular position in sight axes in one plane and t is time referred to some convenient origin. We wish to gain knowledge of this function in one time period and use it as a basis for extrapolation forward into another period. Estimates of target position at sampling instants t_1, t_2,.... are given by the gunner's tracking output at these instants. Now we assume that the 'true' course x(t) can be approximated over a sufficiently small interval by a Taylor's series expansion

$$x(t) = b_o + b_1 t + b_2 t^2 + \ldots \tag{49}$$

and that we shall ignore higher order terms in this expansion. Estimates of the coefficients b_o, b_1, b_2, etc are obtained quite simply by differently weighting the observed instantaneous tracking errors. (Indeed, it is the very simplicity of the computational scheme, not requiring relatively slow iterative procedures, which leads in the first place to the polynomial approach). Let us therefore review the statistical properties of weighted sums of random variables.

Let Y_w and Y_z be linear combinations of variables $X_1, X_2, \ldots, X_i, \ldots, X_m$ with joint c.d.f. $F(x_1, x_2, \ldots, x_i, \ldots x_m)$ and means $a_1, a_2, \ldots, a_i, \ldots, a_m$ and variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_i^2, \ldots \sigma_m^2$

$$Y_w = \sum_{i=1}^{m} w_i X_i$$

$$Y_z = \sum_{i=1}^{m} z_i X_i$$

The w_i and z_i are constants (weights). Then the means, variances and intercorrelation of Y_w and Y_z are given by

$$E(Y_w) = \sum_{i=1}^{m} w_i a_i$$

$$E(Y_z) = \sum_{i=1}^{m} z_i a_i$$

$$\sigma_{Y_w}^2 = \sum_{i=1}^{m} \sum_{i=1}^{m} w_i w_j \sigma_i \sigma_j r_{ij}$$

$$\sigma^2_{Yz} = \sum_{i=1}^{m} \sum_{j=1}^{m} z_i \, z_j \, \sigma_i \, \sigma_j \, r_{ij}$$

$$r_{YwYz} = \left(\sum_{i=1}^{m} \sum_{j=1}^{m} w_i \, z_j \, \sigma_i \, \sigma_j \, r_{ij} \right) / \sigma_{Yw} \, \sigma_{Yz}$$

The results hold true, of course, whatever the form of the distributions of the X_i may be.

Estimates of the coefficients b_o, b_1, etc are usually obtained by ordinary least squares, and in this case the weighting scheme which yields each coefficient is easily deduced. Suppose that we intend to fit a second order polynomial. As already stated, we have sampled position information $x_1, x_2, \ldots, x_i, \ldots, x_m$ at the instants $t_1, t_2, \ldots, t_i, \ldots, t_m$. The x_i can be related to the 'true' target position x(t) by the expression

$$x_i = x\,(t_i) + e_{xi} \qquad (50)$$

e_{xi} being the instantaneous tracking error whose mean, variance and autocorrelation properties we have been at such pains to establish in the preceding three chapters. Suppose now that we do not attach the same importance to each of the sampling instants, and so we devise a scheme of 'primary' weights, $c_1, c_2, \ldots, c_1, \ldots c_m$ to reflect this importance. (The different kinds of filter introduced by, for example, Morrison (1969) are, in essence, different schemes for choosing these c_i so that the resultant estimator or predictor will have desirable statistical or computational properties). The b coefficients are then obtained by solving the so-called 'normal equations':

$$\begin{aligned}
b_o \Sigma c_i + b_1 \Sigma c_i t_i + b_2 \Sigma c_i t_i^{\,2} &= \Sigma c_i x_i \\
b_o \Sigma c_i t_i + b_1 \Sigma c_i t_i^{2} + b_2 \Sigma c_i t_i^{3} &= \Sigma c_i x_i t_i \qquad (51) \\
b_o \Sigma c_i t_i^{2} + b_1 \Sigma c_i t_i^{3} + b_2 \Sigma c_i t_i^{4} &= \Sigma c_i x_i t_i^{2}
\end{aligned}$$

If we refer all times to t_o, where

$$t_o = \Sigma c_i \, t_i \, / \Sigma c_i \tag{52}$$

and write

$$t_i' = t_i - t_o \tag{53}$$

and

$$s_j = c_i t_i'^{\,j} \tag{54}$$

we note that

$$s_1 = o$$

and that

$$\underline{S} = \begin{bmatrix} s_o & o & s_2 \\ o & s_2 & s_3 \\ s_2 & s_3 & s_4 \end{bmatrix}^{-1}$$

$$= \frac{1}{s_o s_2 s_4 - s_o s_3^{\,2} - s_2^{\,3}} \begin{bmatrix} s_2 s_4 - s_3^{\,2} & s_2 s_3 & - s_2^{\,2} \\ s_2 s_3 & s_o s_4 - s_2^{\,2} & - s_o s_3 \\ -s_2^{\,2} & - s_o s_3 & s_o s_2 \end{bmatrix}$$

$$= \frac{1}{G} \begin{bmatrix} A & B & C \\ B & D & E \\ C & E & F \end{bmatrix} \tag{55}$$

It is now easy to see what weights are attached to each of the sampled tracking input values in order to compute the b coefficients. If we use the first subscript to denote the order of the polynomial fitted and the second subscript to denote the relevant term we have:

$$b_{00} = b_{10} = \Sigma\, w_{00i} \quad x_i \quad = \Sigma \left(\frac{1}{s_o} c_i \right) x_i \tag{56}$$

$$b_{11} = \Sigma\, w_{11i} \quad x_i \quad = \Sigma \left(\frac{1}{s_2} c_i t_i' \right) x_i \tag{57}$$

$$b_{20} = \Sigma\, w_{20i} \quad x_i \quad = \Sigma \left\{ \frac{1}{G} (A + Bt_i' + Ct_i'^2) c_i \right\} x_i \tag{58}$$

$$b_{21} = \Sigma\, w_{21i} \quad x_i \quad = \Sigma \left\{ \frac{1}{G} (B + Dt_i' + Et_i'^2) c_i \right\} x_i \tag{59}$$

$$b_{22} = \Sigma\, w_{22i} \quad x_i \quad = \Sigma \left\{ \frac{1}{G} (C + Et_i' + Ft_i'^2) c_i \right\} x_i \tag{60}$$

We shall call these w terms the 'secondary weights'.

Knowing these secondary weights, and armed with the knowledge we have already obtained about the statistical properties of the human tracking error, we are now able to assess the statistical properties of the b coefficients. Furthermore, in order to predict target position at some time t_p' we use a prediction equation of the kind

$$\hat{x}_p = \sum_{j=o}^{m} b_{mj} \, t_p'^{\,j} \tag{61}$$

where m is the order of the polynomial which has been decided upon. The predicted position is thus just a further weighted sum of the random variables b_{mj}, and so a minor additional step is required to deduce its statistical properties.

Ignoring for the moment all contributions to system error other than those due to the human tracking input and the prediction computations, it will be seen that the random error arises from the remnant portion of our tracking model, transformed by the secondary weights and the prediction equation. There are, however, two sources of bias. One source is the regular transfer function portion of our tracking model (introducing leads and lags) undergoing the same transformation. The second source arises from the failure of the chosen polynomial to reproduce the 'true' target course exactly (introducing bias due to neglected higher order terms in the Taylor's series expansion). Schemes which may balance these two contributions in order to maximise hit probability will be considered in section 6.3.

6.2 Total system error

To achieve a kill a shell must hit (and penetrate) the target. In order to compute this chance of hit we must first determine the total system error, of which prediction error is but one part. If $\hat{x}_p$ is the predicted target position at time t_p (t_p being the predicted impact time of the shell) there will be various other errors which may cause the shell not to fly precisely down its predicted path.

There are firstly errors due to the ammunition itself. There are round-to-round and lot-to-lot variations in shape, size, propellant, etc, which, when combined with gun jump (defined below) will cause random variation in the fall of shot. There are then errors due to the weapon itself and its harmonisation with the rest of the system. The barrel will wear, affecting muzzle velocity, or it may droop or bend in different environmental conditions. Gun jump and throw off are due to the barrel shifting in its mounting and bending due to the stresses of gas expansion. Any inaccuracy in range finding will be reflected in an incorrect allowance for fall of shot. If the trunnion (or gun cradle) is tilted the gun will be pointed incorrectly, and then there are also the problems of accurately aligning the gun and the sight through what may be multiple links. The fire control computer itself will introduce rounding errors during computations, and the sight (and gun if firing on the move) may not be perfectly stabilised. Finally there may be errors due to environmental factors. Crosswind or headwind will introduce bias, and air density variations will affect the flight of the shell. There will be effects due to refraction and atmospheric turbulence disturbing the visual pathway. The temperature of the charge will also have a bearing on the muzzle velocity of the shell.

In the face of such a formidable list of factors which may affect system accuracy it is perhaps surprising that human tracking error should have a dominant effect. However, most of these other factors can be assessed by sensors, or by prior shooting-in of the gun with subsequent bias estimation, and some sort of allowance can be made when the ballistic computations are undertaken. What will remain will be residual errors and errors due to factors (such as atmospheric turbulence) which cannot be assessed. To a first approximation the sum of these several system errors may be regarded as normally distributed, with zero mean and a variance which may be range-dependent, but independent of the error due to prediction. If we denote the residual error in one plane due to the factors just discussed as x_r, the predicted position of the target at impact as $\hat{x}_p$ (without loss of generality taking the true target position at impact as zero) and the total system output as x_s, then the following assumptions are implied:

$$E(x_r) = 0 \tag{62}$$

$$E(x_s) = E(\hat{x}_p) \tag{63}$$

$$V(x_s) = V(\hat{x}_p)+V(x_r) \tag{64}$$

The means of assessing the expected value and variance of the prediction error have been outlined in section 6.1. The variance of the residual system errors is a function of system design and engineering and is generally provided by the appropriate research and development authorities.

6.3 'Threshold' prediction schemes

Having traversed the path leading to final estimates of total system bias and random error, we are now in a position to consider some ways in which these two factors may be balanced to affect chance of hit. Since bias is only encountered in the prediction scheme it must be within this part of the system that compromises are made. The first candidate for attention is the order of the polynomial which is selected as the basis for prediction, and the factors involved are best appreciated by considering this topic in a non rigorous and descriptive way.

It will be recalled that linear predictors are usually employed in fire control systems, so that target position at impact is estimated by an equation of the type

$$\hat{x}_p = b_o + b_1 t_p$$

As we have seen, b_o and b_1 are random variables making their contribution to the error of the composite $\hat{x}_p$ (with weights 1 and t_p). In the introductory remarks to this thesis in Chapter 1 it was pointed out (and illustrated in Figure 1) that the linear predictor was bound to be biased if the target accelerated during an engagement. The natural extension of the linear predictor is to introduce an extra term to account for the acceleration, so

$$\hat{x}_p = b_o + b_1 t_p + b_2 t_p^2$$

Almost certainly bias will be reduced, but the trouble here is that we have introduced another source of random variation: b_2 is a further random variable making its own contribution to the composite $\hat{x}_p$, and furthermore, since it is weighted by t_p^2, its effect will mount very rapidly as the length of the prediction interval is increased. (This argument is not exact, because it ignores the fact that the b_o and b_1 terms in the linear predictor are not identical to the b_o and b_1 terms in the quadratic predictor. For a more rigorous development see, eg Blackman, 1965, or Morrison, 1969). Clearly, then, we should not wish to add an acceleration term to our predictor unless we could be fairly certain that the advantage due to bias reduction outweighed the loss due to increased random error. This argument could be extended still further: if target rates are very low it might be preferable to use a zero order predictor rather than a first order one. Of course, before the engagement starts we do not know what the target velocity and acceleration may be, <u>and so any decision on the order of predictor to employ would have to be based on information collected as the engagement proceeds</u>.

All these considerations lead one to the proposition that one might set 'threshold' values for the incorporation of acceleration or velocity terms into one's predictor. The coefficients b_{20}, b_{21} and b_{22}, and b_{10} (=b_{oo}) and b_{11} would be computed according to the secondary weighting scheme set out in equations 56 to 60. A quadratic predictor would be employed if the absolute computed value of the b_{22} term exceeded a pre-set b_2 'threshold' (b_{c2} say), otherwise a linear or zero order predictor would be utilised - the former if the absolute computed value of the b_{11} term exceeded the b_{11} 'threshold' (b_{c1}), and the latter if it did not. In diagrammatic form the 'threshold predictor' algorithm would be:

$$
\begin{array}{ll}
 & |b_{22}| > b_{c2} \ ? \xrightarrow{\text{yes}} \hat{x}_p = b_{20} + b_{21}t_p + b_{22}\, t_p^{\,2} \\
 & \downarrow \\
\text{no} & |b_{11}| > b_{c1} \ ? \xrightarrow{\text{yes}} \hat{x}_p = b_{10} + b_{11}t_p \\
\text{no} & \downarrow \\
 & \hat{x}_p = b_{oo}
\end{array}
\qquad (65)
$$

If we elect to use one set of coefficients only if one number of that set lies above or below a certain cut-off point this will obviously have a direct effect on the mean and variance of the variate on which selection has taken place. It will also have an indirect effect on the intercorrelations and the means and variances of the other members of that set. This was a problem first considered by Pearson (1902) in the context of the natural selection of species. If we make the assumption that tracking error is approximately normally distributed and use the following notation

$$Z(a) = (2\pi)^{-\frac{1}{2}} \exp(-\tfrac{1}{2}a^2)$$

$$Q(a) = \int_a^{\infty} Z(a)\, da$$

we can allow for the effect of selection by methods outlined by Finney (1956, 1961) and Weiler (1959).

Let us consider the general case of selection being made on variable b_i with cut-off points $\pm\, b_c$, and with associated variables b_j and b_k. There will be three subpopulations: b_i' ($b_i > b_c$); b_i'' ($-b_c \geqslant b_i \geqslant b_c$) $_i$ and b_i''' ($b_i < -\, b_c$).

If we write

$$a_i = \frac{- b_c - E(b_i)}{\sigma_{b_i}} \tag{66}$$

$$a_2 = \frac{b_c - E(b_i)}{\sigma_{b_i}} \tag{67}$$

then the probability associated with $b_i{'}$, $b_i{''}$ and $b_i{'''}$ will be $Q(a_2)$, $Q(a_1) - Q(a_2)$ and $1 - Q(a_1)$ respectively. We now define the quantities.

$$u' = Z(a_2)/Q(a_2) \tag{68}$$

$$u'' = \left\{Z(a_1) - Z(a_2)\right\} / \left\{Q(a_1) - Q(a_2)\right\} \tag{69}$$

$$u''' = -Z(a_1)/ \left\{1 - Q(a_1)\right\} \tag{70}$$

$$v' = \left\{\frac{Z(a_2)}{Q(a_2)}\right\}^2 - \frac{a_2 Z(a_2)}{Q(a_2)} \tag{71}$$

$$v'' = \left\{\frac{Z(a_1) - Z(a_2)}{Q(a_1) - Q(a_2)}\right\}^2 - \frac{a_1 Z(a_1) - a_2 Z(a_2)}{Q(a_1) - Q(a_2)} \tag{72}$$

$$v''' = \left\{\frac{Z(a_1)}{1-Q(a_1)}\right\}^2 + \frac{a_1 Z(a_1)}{1-Q(a_1)} \tag{73}$$

Means, variances and intercorrelations in the subpopulation corresponding to the upper tail of the b_i distribution ($b_i{'}$) will be

$$E(b_i{'}) = E(b_i) + u'\sigma_{b_i} \tag{74}$$

$$E(b_j) = E(b_j) + u' r_{ij}\sigma_{b_j} \tag{75}$$

$$\sigma^2_{b'_i} = (1 - v')\, \sigma^2_{b_i} \tag{76}$$

$$\sigma^2_{b'_j} = (1 - r^2_{ij}\, v')\, \sigma^2_{b_j} \tag{77}$$

$$r_{ij}' = r_{ij} \left\{ \frac{1 - v'}{1 - r_{ij}^2 v'} \right\}^{\frac{1}{2}} \tag{78}$$

$$r_{jk}' = \frac{r_{jk} - r_{ij} r_{ik} v'}{(1 - r_{ij}^2 v')^{\frac{1}{2}} (1 - r_{ik}^2 v')^{\frac{1}{2}}} \tag{79}$$

The formulae for the b_i'' and b_i''' subpopulations are exactly analogous.

The methods above can be used to compute the statistical properties of the various coefficients in the threshold prediction scheme - first when selection takes place along the b_{22} dimension, and then iteratively when selection takes place along the b_{11} dimension. There will in general be some errror in the second computation, because if the correlation between b_{22} and b_{11} is non-zero b_{11}'' will not be normally distributed. The degree of non-normality could in fact be assessed by the methods of Tallis (1961) or Finney (1962b) and percentage points computed by the method of Cornish & Fisher (1937) and Fisher & Cornish (1960). However, we have chosen not to do this. Finney (1962a) has in fact used the method recursively as proposed here, and both his paper and the discussion of it suggest that the loss of accuracy will be minor.

To anticipate later queries , it should be remarked that the final computation of hit probabilities rests on a normality assumption for total system error. Clearly, some of the contributions to that error - those of the b coefficients after truncation - will be very far from normal. To assess these effects exact computations were carried out in a large number of selected cases and compared with the approximate methods described here. In no case were final computed hit probabilities affected in more than the third decimal place - an error quite obviously within the accuracy bounds of all the other model approximations involved in this thesis.

Having described a scheme which, it is hoped, might optimise the order of predictor used, there are two other areas where optimisation could perhaps be profitable. The first concerns the prediction interval one should employ when making the computations to lay off the gun. Intuitively one supposes that the value of t_p one should insert in one's prediction equation is the 'true' prediction time (the estimated time of flight of the shell plus the gunner's firing reaction time). There are, however, two reasons why one might prefer not to use the 'true' time. The first is simply a continuation of the theme of trading off bias against random error. As one reduces the values of t_p used in one's predictive equation the greater will be the bias (assuming, that is, that the b coefficient which it multiplies is unbiassed) but the smaller will be the variance of $\hat{x}_p$. The second reason arises out of the statistical properties of the b coefficients themselves after selection has taken place on them. To simplify matters let us concentrate on the first order predictor and the velocity dimension only (assuming for purposes of exposition that the b_{c2} threshold is set so high that the second order predictor is never employed). We now reject all those b_{11} coefficients below the threshold b_{c1}, and by this rejection ensure that the b_{11} subpopulation that we do use is itself biased. To see that this is so consider a constant rate target whose velocity is in fact exactly b_{c1}. Our threshold scheme will ensure that, if the algorithm in effect concludes that the target does have a 'significant' positive velocity, the 'true' velocity is never actually used in the computational scheme, but only velocities (b_{11} terms) greater than the 'true' velocity. This effect might perhaps be offset by using something other than the 'true' prediction interval in computing predicted target positions.

The second area one might wish to optimise is that of primary weights (the relative importance attached to each sampling instant during data capture) and once again there are two reasons for considering this possibility. The first arises from what we have already discovered about the disturbance at 'lase' - that an error is injected which seems to decay exponentially during subsequent tracking. The second arises from the observation that if the target is indeed manoeuvering in a way which does not exactly mirror our polynomial model, then we might suppose that we ought to attach a greater weight to the most recent target tracking information.

Our finally proposed threshold algorithm has 6 parameters K_1 to K_6, which are candidates for optimisation. Assuming sampling instants $t_1, t_2, \ldots, t_i, \ldots t_m$ then the system of primary weights is

$$c_i = \exp(K_1\tau_i)\ (1 + K_1K_2\tau_i) \tag{80}$$

where $\tau_i = t_i - t_m$. This scheme was proposed by Haddad (1971), the k_2 parameter increasing the system to a fourth order one.

The 'threshold' selection algorithm, as just described, is then employed, and the K_3 and K_4 parameters are identical to the threshold terms b_{c2} and b_{c1}. Final predictions are based on the formulae:

$$\hat{x}_p = \begin{cases} b_{20} + b_{21}\ (t + K_5) + b_{22}\ (t+K_6)^2, & b_{22} > K_3 \\ b_{10} + b_{11}\ (t + K_5) & , b_{22} < K_3,\ b_{11} > K_4 \\ b_{00} & , b_{22} < K_3,\ b_{11} < K_4 \end{cases} \tag{81}$$

The same primary weights, threshold values and prediction times are employed in both x and y planes (although the b coefficents will differ in each).

6.4 A computer-based system for prediction algorithm evaluation

The evaluation of predictive schemes has three strands: the generation of target motion information and other target data dependent on engagement geometry; the computation of the characteristics of human response to these inputs; and the assessment of total system effectiveness consequent upon a prediction algorithm operating in turn on these human responses. In this latter context we may wish to evaluate the prediction algorithm with a fixed set of parameters, or we may wish to optimise one or more parameters. There is a lot to be said for separating the three facets of evaluation into three separate quasi-inpendent layers, each layer using the output from the one below it. First of all, we may wish to generate different subsets of target engagements, either to demonstrate the effect of different target assumptions, or to cross-validate using independent samples from nominally the same population of engagement geometries. Secondly having generated an engagement subset we may wish to examine the effect of varying the assumed operator response (by simulating 'bad' operators, for example, or including the effects of vibration). Lastly, we might then wish to use these same gunner response characteristics with the prediction setup configured in different ways, especially if we have some iterative optimisation procedure in mind. The evaluation scheme we have developed is based on this three tier approach: one computer programme is used to generate engagement details and place these in a nominated file; another operates on the data in this file to produce expected gunner response information, placing this latter in the next file; and the final programme accepts information from this last file and operates on it in accord with prediction algorithm assumptions and parameters in order to compute hit probabilities.

Brief outlines of all these programmes are given in Appendix A, and they will not be elaborated on here. They are,of course, based on the methodology put forward in this and preceding chapters (with the exception of that used for generating the target courses used for expository purposes in section 6.5, but in this latter case the geometric principles involved are absolutely straightforward). There are, however, two areas which should be singled out for further attention: the philosophy and approach used for parameter optimisation; and the assumptions made about gunner reaction time, the flight time of the shell, the variance to be attributed to that part of the system error not including prediction error, and the physical dimensions of the target.

6.4.1 Optimisation of prediction algorithm parameters

The final evaluation programme basically computes hit probabilities for all those nominal engagements represented in its input file. When operating in its optimisation mode it seeks to maximise a weighted composite of those hit probabilities. The user divides the total engagements into one or more subsets and then specifies a weight for each subset. If there are N target engagements _in toto_, divided into m subsets, the i th subset containing n_i engagements $(\sum_{i=1}^{m} n_i = N)$ then the user specifies a different weight, w_i, for each subset. If for a given set of parameters we denote the hit probability for the j th engagement in the i the subset as P_{ij}, the statistic which is maximised is:

$$P = \sum_{i=1}^{m} \left(\frac{w_i}{n_i} \sum_{j=1}^{m_i} P_{ij}\right) \tag{82}$$

Powell's (1964) derivative - free minimization routine was at first used for optimisation. However, in this context, just as with human operator transfer function development, it was found that the routine was slow to converge and that often it converged to a point which was clearly remote from the true optimum. Once again, the author's own development of the Ralston & Jennrich (1978) 'Dud' algorithm was substituted and found to be satisfactory.

6.4.2 Assumptions on delay, time of flight, system error and target dimensions

Taking the time at which fire control computations cease, and a 'ready to fire' signal is passed to the gunner, as an arbitrary true origin, then the prediction time, t_p, has two components:

$$t_p = t_d + t_f \tag{83}$$

where

t_d is the gunner delay from receiving the 'ready' signal to actually firing, and

t_f is the time of flight of the shell.

A fixed value of 0.435 sec has been assumed for t_d. This value has been taken from experiments conducted by Stanberry (1975). The expression used for the time of flight of the shell is of the form:

$$t_f = k_1 R + k_2 R^2 \tag{84}$$

where

R is the target range in kilometres

For system errors, other than prediction error, in the x and y planes we have

$$V(x_r) = k_3 + k_4 R^2 \tag{85}$$

$$V(y_r) = k_5 + k_6 R^2 \tag{86}$$

R again being the target range in kilometres, and error being measured in milliradians. In the evaluations which follow values appropriate to a modern tank gun were chosen for the constants k_1 to k_6.

The assumed target in all computations of hit probability is based on the Russian T72 tank. The centre of rotation of the turret ring has been taken as the origin (and the gunner is always held to aim at this point). The turret itself is regarded as a cylinder of height 0.7m. and radius 1.3m. The hull is represented by a cube of depth 1.4m, width 2.8m and total length 5.9m, 3.1m of this length being forward of the the origin and 2.8m to the rear of the origin. In computing the area presented to the gunner due account is taken of the target's aspect in the horizontal plane, but no allowance is made for viewing angles in the vertical dimension which are not horizontal with respect to the tank's own axis.

6.5 'Threshold' and other predictors applied to simple schematic engagements

The principles we have discussed in this chapter are perhaps best appreciated by evaluating linear, quadratic and 'threshold' predictors against some simple target manoeuvre - perhaps one of maintaining a constant horizontal acceleration by turning with a fixed radius on a flat surface. Consider 3 basic engagement geometries. In each case the target maintains a straight line course until the laser range finder is activated, but thereafter turns with constant radius until the nominal instant of impact. The target range at 'lase' is 1000m, and the computer utilises 13 samples of tracking data, spaced at 0.2 sec intervals apart, the first data point used being 0.1 sec after lase. The target maintains a steady 10m/sec velocity along its momentary direction of travel throughout the engagement. The differing course features are:

Course Type A. The target approaches the tracker directly prior to 'lase' and thereafter turns to one side.

Course Type B. The target crosses in front of the tracker, reaching crossing point (so that the line from tracker to target is normal to the target path) at the instant of 'lase'. Thereafter it turns towards the tracker.

Course Type C. As for Type B, but the target turns away from the tracker.

For each type of target course 33 different turning radii were simulated. If we make use of the relationship between acceleration (g), radius of turn (r) and velocity, g = v/r, then targets 1 to 26 ranged from 0 to 0.5g in steps of 0.02g, and targets 27 to 33 from 0.55g to 0.85g in steps of 0.05g. (In practical terms high performance vehicles have been known to attain 0.7g for pre-programmed turns, and occasionally 1g with suitable ground conditions, but it seems unlikely that more than 0.5g would be encountered in operational conditions).

The hit probabilities for these 99 simulated engagements were evaluated for linear (first order) quadratic (second order), and 'threshold' predictors, with tracking model constants based on the results of the 6'best' and the 6 'worst' subjects from the final tracking experiment. The parameters for the 'threshold' predictor were optimised for the 6 'worst' subjects (sample F) and maintained at these values there-thereafter (and they are listed in Table 6). The weighting scheme used for optimisation gave heavy emphasis to the low - g targets. Since hit probabilities against these targets were in any case high, a straightforward uniform weighting for all targets would (and was actually shown to) produce dramatic improvements for high - g targets at the expense of some loss of effectiveness against the (presumably important) low - g targets. Accordingly the targets were divided into 9 different subsets. Within each type the first subset included targets 1 - 8, the second targets 9-19and the third targets 20-33. Within each target type these three subsets were then accorded relative weights of 16:4:1.

TABLE 6 'Threshold' predictor. Parameter values as optimised using 99 circular target courses, and tracking model constants yielded by 6 'worst' Guardsmen (sample F).

Parameter		Value
K_1	Primary weight constants	1.566
K_2		- 0.389
K_3	Time of flight adjustment, t^2	- 0.014
K_4	Time of flight adjustment, t	- 0.375
K_5	Threshold, quadratic term	0.803
K_6	Threshold, linear term	0.403

Figures 30 to 32 illustrate the results. They are based on the 6 'best' trackers (subject sample E). Those from the 6 'worst' trackers were so very similar that one could say that almost the only way to tell them apart would be to note that with the 6 'worst' trackers the overall level of hit probability is slightly lower.

The contrast between the linear and the quadratic predictor nicely illustrates the trade-off between bias and random error. The addition of an acceleration term does indeed give the system a higher performance against targets of moderate acceleration, but at the cost of serious degradation in the low-g area. At the highest acceleration levels even a second order equation fails to match closely the 'true' target course as transformed into sight axes. (It will be noted that, for the type A target, hit probabilities rise once more at the highest g levels. This is a freak result, the target in effect turning back into the path of the shell.)

On this showing the 'threshold' algorithm seems to hold out some promise of improved performance against targets with moderate levels of acceleration. It remains to be seen whether this promise is maintained with targets which may more closely approximate operational engagement geometries. Before doing so it will perhaps be as well to review other approaches to the manoeuvring target problem.

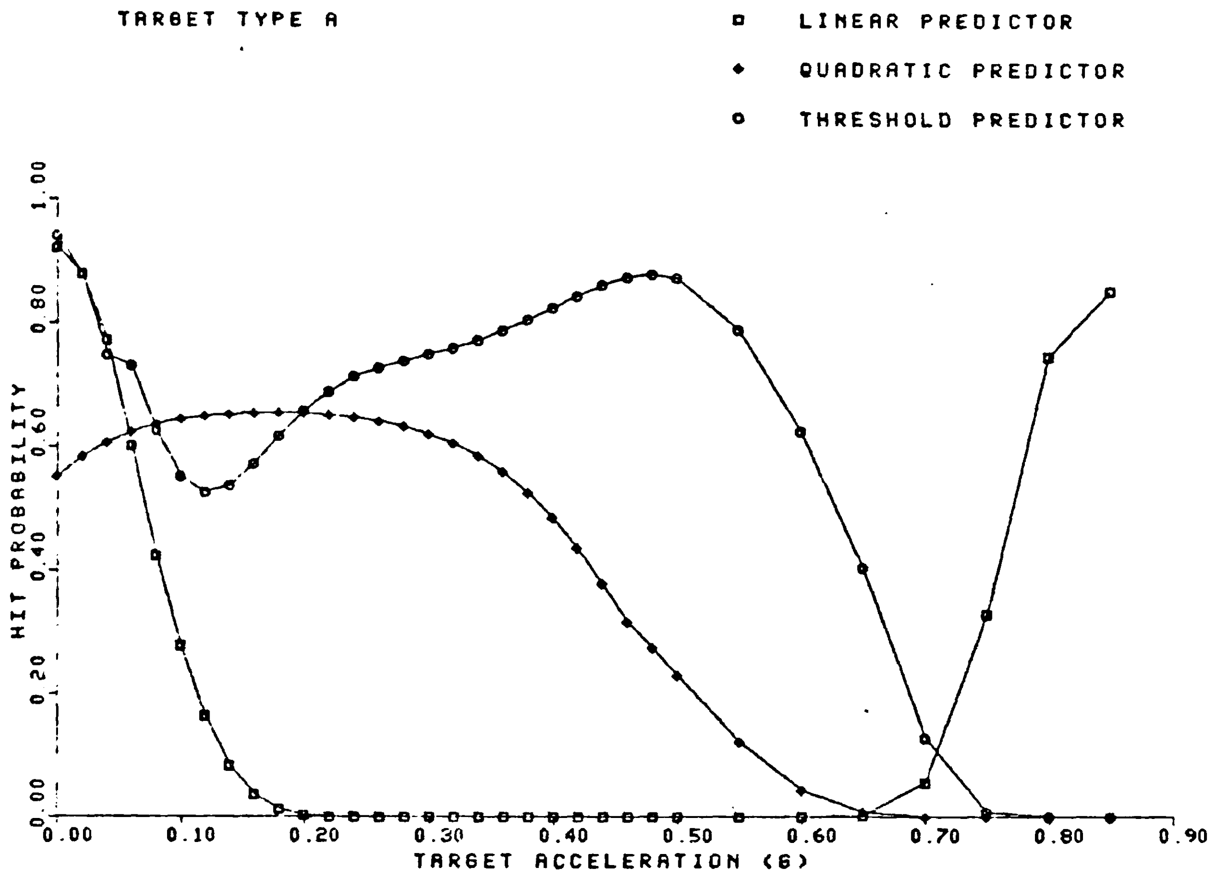

FIGURE 30. Comparison of predictor performance.
Type A targets.

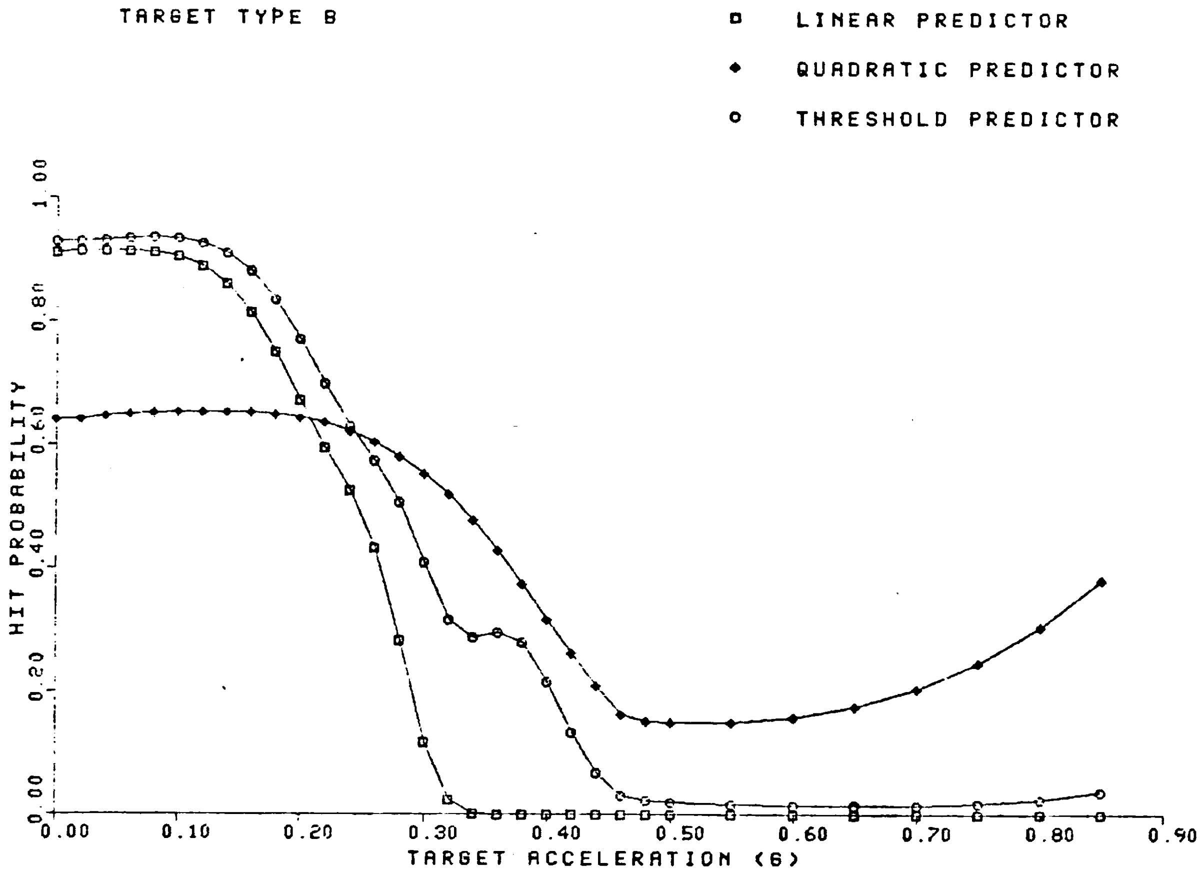

FIGURE 31. Comparison of predictor performance. Type B targets.

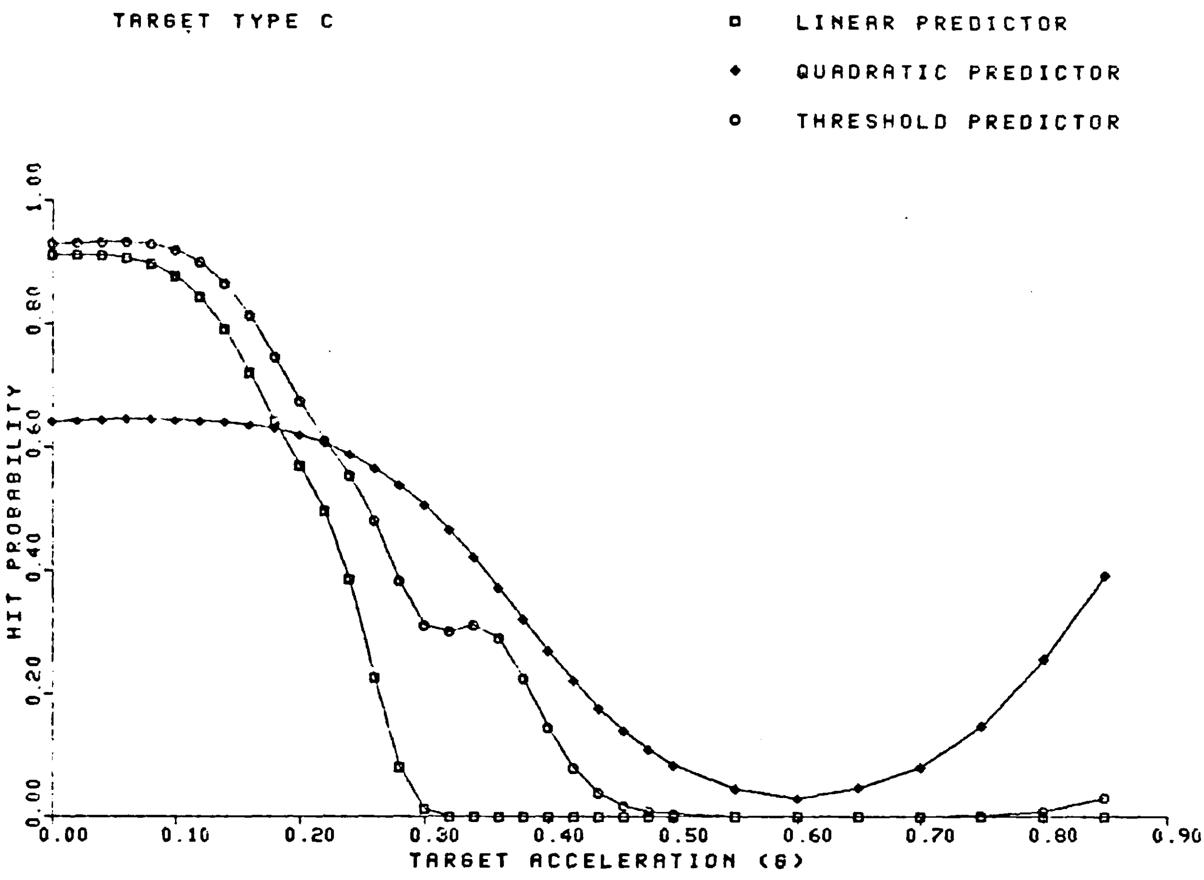

FIGURE 32. Comparison of predictor perfomance. Type C targets.

7. PREDICTION ALGORITHMS: THE 'THRESHOLD' ALGORITHM COMPARED WITH APPROACHES IN RELATED FIELDS, AND EVALUATED AGAINST QUASI-OPERATIONAL TARGETS.

7.1 A review of other approaches to the manoeuvring target problem

We have now examined the prediction problem in the tank gunnery context, developed a human tracking model, and have assessed traditional and modified prediction algorithms against three variants of a simple schematic tank engagement. But the problem of estimating the present and future position of targets which may be manoeuvring is met in many other contexts - air warfare, anti-submarine warfare, and manned spacecraft tracking for instance. Before passing to a more searching evaluation of the threshold scheme which has just been put forward it would perhaps be as well to pause and consider the approaches which have been developed in these other fields. As was mentioned in section 6.1, it is fashionable to regard the prediction problem primarily as one of estimation (that is to say, of establishing the current target state vector, and then, via a suitable model of target motion, projecting this forward). If this very brief review seems to concentrate on estimation, prediction is also covered by implication.

In the other fields of application which have been mentioned the tracking medium is commonly an electronic or mechanical device, such as radar or sonar, whose noise characteristics are well understood. Various kinds of tracking filter have been devised with desirable properties in steady state conditions, and given the known sensor characteristics it has been relatively easy to establish their properties for straight-line constant-velocity targets. In evaluating their performance against manoeuvring targets it is first of all necessary to develop some sort of model for assumed target motion (since to rely on real data collected by fallible sensors would presumably beg the question it was intended to settle). Examples of target course model development are given by Singer (1970), Singer & Behnke (1971), Merhav (1971), Merhav & Silbershatz (1973) and Morgan (1976). A common approach is to assume that the pilot (or captain) of the manoeuvring craft will have a range of possible accelerations open to him, and that he will select from this range according to some probability distribution, initiating a level of acceleration and maintaining it until the next change, the change points themselves being randomly distributed in time. This sort of model is not incompatible with the scheme used in this thesis to represent the AMSAA data (and although the writers mentioned specify their models in geographical coordinates, the translation to sightline axes in our case should have only a slight effect).

Having established a model of target motion, several authors (for example Bhagavan & Polge, 1974, and Singer & Behnke, 1971) assess the likely performance of different kinds of filter based on the model assumptions. Simple sorts of filter can be shown to perform tolerably well in some sets of circumstances, but by and large very considerable emphasis is placed on the now-ubiquitous, but relatively complex Kalman filter (Kalman, 1960; Kalman & Bucy, 1961) which certainly sets the standard by which other filtering schemes tend to be judged. Exceptions are provided by Spingarn & Weidemann (1972), Merhav (1971) and Merhav & Silbershatz (1973). The first two authors compare traditional regression techniques with the Kalman filter in the aircraft tracking context. They show that, provided that computations are carried out in Cartesian coordinates and an exponential weighting scheme is used to provide a fading memory feature, the regression approach compares very favourably with the line-of-sight coordinate Kalman filter (and indeed has a lower bias error bound). The last two authors, using a manoeuvre model of the kind described in the previous paragraph, show that optimal performance (in terms of hit probability in their fire control context) is provided by a finite time polynomial predictor. They then go on to establish the optimal length of the data - gathering interval and the weighting scheme within this interval as a function of the assumed mean time between manoeuvres, and the time lapse at the end of which prediction is required.

The Kalman filter has gained such popularity because with a linear estimation scheme it can be shown to have minimum variance properties (see, eg Morrison, 1969) and practical experience has confirmed its utility. A formal exposition of its structure and properties would be out of place here, but it relies for its good performance on an accurate model of target motion. (The Kalman filter also requires knowledge of the variance covariance properties of the measurement noise, and of the input noise if a random element of target motion is assumed, although, in certain conditions, there are methods to estimate these on line: see, eg, Mehra, 1960). The filter proceeds sequentially. An inital estimate of the target state vector is provided, and thereafter the filter updates this vector, attaching weights to each new data set in accord with the extent to which it is in line with filter predictions at that sampling instant. If the target motion equations or the measurement scheme are truly nonlinear then steps must be taken to reduce the effects of this nonlinearity and several authors have reported attacks on problems of this kind in particular applications (Pearson& Stear, 1974; Chang, Whiting & Athans, 1977; Fitts, 1977; Tenney, Herbert & Sandell, 1977).

While the Kalman filter gives excellent results for predictable target motions, it is less than satisfactory when the target starts to manoeuvre. This is because the filter builds up a very good prediction of the target state vector while the target is behaving in predictable fashion, and then when it fails to behave in accord with that prediction it gives the fresh data very low weight. If this danger is countered by incorporating a noise element into the target model then its good performance during fixed-manoeuvre portions of the track will be sacrified. Several authors have therefore turned their attention to adaptive schemes. In almost all cases the first step is to detect the onset of a manoeuvre by an examination of the filter residuals (the differences between the data and the filter predictions). Once bias has been detected a number of different schemes have been suggested. Demetry & Titus (1968) propose continuous storing of the most recent data, and then re-initialising the Kalman filter and re-working the filter predictions with this stored subset. Hampton & Cook (1973) and McAulay & Denlinger (1973) also propose re-initialising the filter, but they would in addition alter the filter properties, in the first instance by introducing an exponential weighting scheme and in the second by introducing input noise terms into the Kalman filter. Other authors have two or more filters running concurrently, using the residual information to select between, or weight, the alternatives. Thus Haddad (1969,1971) would select one of two filters with different exponential decay rates; Thorp (1973) would weight a filter assuming no input noise and one assuming Gaussian noise according to a likelihood ratio; and Ricker & Williams (1978) would similarly weight the state vectors derived from a bank of filters each assuming a different target noise characteristic. Finally, Moose (1975) and Gholson & Moose (1977) describe a recursive technique for estimating the pilot's demanded acceleration, and incorporating this estimate into an extended Kalman filter.

On reviewing these different applications some very obvious differences between the tank gunnery case and most others stand out. Firstly, the characteristics of radar or sonar noise are different from those introduced by human tracking variability, and the measurement noise assumed in all of the above papers is not at all similar to that implied by our tracking model. Secondly, the computational schemes envisaged in other settings are an order or two more complex than those which can be contemplated at present in the tank gunnery context. It is true that computer technology is advancing at a very fast rate indeed, so that present restraints due to the cramped conditions and hostile physical environment inside a main battle tank will not have the same force in the future, but there is still the need to justify increased complexity and cost by real increases in system effectiveness. The third major difference concerns the situational demands of the different applications. The time period of data gathering and prediction in tank gunnery is very short, and the autocorrelation properties of human tracking error are such that there are, effectively, just a few independent samples of target position during this period. There is scarcely time for the influence of the values used to initialise a Kalman filter to decay to a reasonable level, let alone to provide an opportunity to establish steady track characteristics and then detect deviations from them.

At first sight the differences just discussed would suggest that there are no implications for tank gunnery from manoeuvring target work in other contexts. There are, however, some general themes of relevance. Several authors have stressed the utility of time weighting, or of altering the length of the data gathering interval as a function of the assumed motion characteristics of the target. Another theme is the use of residuals to determine the kind of filter which one should employ. Schematically, the threshold scheme which we have espoused has a close affinity to this latter approach. We could in fact modify our algorithm to point up the similarity in concept: examine the residuals from a zero order predictor to determine whether to switch to a first order predictor, and then examine the residuals from the latter to determine whether a second order predictor should be used (although this way of approaching it would not be so efficient as the scheme actually put forward). None of the authors we have reviewed have suggested that the order of the target model should be changed, but have instead concentrated on the inclusion of target noise additions (and the state space approach discourages one from altering the number of states being estimated as one goes along). However, it can be seen from this review that the kind of filter employed need not be an 'either-or' decision - it is quite possible to devise a relative weighting scheme for different filters (perhaps based on coefficient estimates, perhaps based on residuals, or perhaps on both). There is clearly much scope for ingenuity in this area.

7.2 'Threshold' and other predictors applied to quasi-operational engagements

In section 6.5 'threshold' linear and quadratic predictors were evaluated against some simple circular manoeuvres. In this section the evaluation is extended to targets moving in quasi-operational fashion.

When the topic of target motion was reviewed in Chapter 2 it was concluded that a wide range of movement patterns was possible, depending on the tactical situation, and that a definitive description of these patterns did not exist. Consequently it was suggested (section 2.3) that proposed prediction schemes should be evaluated at two extremes of possible manoeuvre: one extreme would consist of stationary and straight line targets; and the other would consist of targets employing the maximum degree of operational manoeuvre possible (to be based on the AMSAA trials data). Accordingly, there were three classes of target included in the set of evaluation engagements: stationary targets (5); straight line, constant velocity targets (20); and manoeuvring targets (100), making a total of 125 targets in the set. The last class of target was divided into two: light vehicles (20) and heavy vehicles (80). This last subdivision was introduced because it may be expected that tanks will not only engage other main battle tanks: scout vehicles and the like will be a class of target of some operational importance.

For the stationary targets ranges were sampled from the gamma engagement range distribution given in equation (4) (section 2.5), and angles of attack were sampled from the Whittaker dpv (equations 5a and 5b, section 2.6). For the straight line targets a constant velocity of 10m/sec was assumed. Engagement range and angle of attack were sampled as above, and in addition the height differences were sampled from the exponential distribution given in equation (7) (section 2.7) and the angle of tilt of the sight from a Gaussian distribution with standard deviation 5° (section 2.8). The light vehicle subset of the manoeuvring targets was based on one of the Lockheed Twister tracks from the AMSAA trial (because, as reported in section 2.3, the data from the XM800 Scout vehicle proved unsuitable for this purpose) and the heavy vehicle subset was based on an M60 track from the same trial. In each case the engagement times were sampled uniformly from the total track time, less 16 sec at the start. The other engagement parameters were sampled in the same fashion as was used for the straight line targets, and the equations of motion were computed by the scheme described fully in section 2.9. (Occasionally this procedure included an apparent substantial step jump in target position within the nominal 16 sec engagement period. The engagement times were re-sampled in these instances).

For all the above targets, target position histories and other engagement parameters were computed using the first programme in the computer evaluation suite described in section 6.4; nominal human response statistics were calculated, based on the average model parameters for the 6 'best' Guardsmen (sample E) and the 6 'worst' (sample F) separately, using the second programme. Finally, overall hit probabilities were computed via the third programme in the evaluation suite, based on linear, quadratic and 'threshold' predictors. This last programme was used in the straightforward evaluation, rather than the optimisation, mode; and the 'threshold' predictor parameters were fixed at the values previously optimised using subject Sample F and the circular targets, as given in Table 6 (section 6.5). Once more, a set tracking time of 2.4 sec was assumed, followed by a 0.435 sec firing delay. The values of the weights accorded to the different target subsets have no bearing on the computations unless the evaluation programme is used in the optimisation mode.

The mean hit probabilities for each class of target are given in Table 7, the 'overall mean' arbitrarily assigning stationary, straight line, manoeuvring (light) and manoeuvring (heavy) targets weights of 0.1, 0.5, 0.08 and 0.32 respectively.

TABLE 7. Average hit probabilities, evaluation targets. (Tracking model constants were as yielded by the 6 'best' and 6 'worst' Guardsmen - Sample E and F)

Targets		Sample E			Sample F		
		T	L	Q	T	L	Q
Stationary	(5)	0.754	0.630	0.236	0.691	0.539	0.179
Straight line	(20)	0.807	0.826	0.558	0.768	0.780	0.488
Manoeuvring	(100)	0.450	0.402	0.332	0.445	0.390	0.298
Weighted mean		0.659	0.637	0.436	0.631	0.600	0.381

T - 'threshold' predictor

L - linear predictor

Q - quadratic predictor

7.3 Provisional evaluation of 'threshold' predictors

We can perhaps dismiss quite quickly the quadratic predictor from further detailed consideration. The results of this assessment confirm the impression gained during that described in section 6.5: that the straightforward, unadorned quadratic predictor is unlikely to have any practical utility in an operational setting.

It is perhaps encouraging to note that, on this evidence, the straightforward linear predictor does not fare too disastrously, even with manoeuvring targets. Compared to the linear predictor, the 'threshold' predictor, even in its undeveloped state, does appear to have a modest advantage. It seems to do better with the stationary targets (although, to be fair, those devising a practical fire control scheme might well prefer to institute a different drill procedure for stationary targets, asking the commander or gunner to identify the fact that the target was indeed stationary, and then bypassing the whole tracking-plus-prediction routine). Taken over all the manoeuvring targets the 'threshold' predictor does show some improvement over the linear predictor, but for the idealised straight line targets its performance is slightly worse.

Too much should not be read into the detailed figures at this stage. What we have done so far is to evaluate a crude 'threshold' predictor in a preliminary fashion. Perhaps enough has been done to suggest that the scheme might have some merit, particularly if growing computer power permits a greater degree of sophistication. It remains to sketch in very lightly the way that future developments might go, bearing in mind some of the lessons of the previous section. In large part these developments are conceptual and logical, but some consideration must be given to practical utilities and constraints.

The problem in marking out paths of possible future development is not that such paths are few or difficult to discern: it is more one of restricting one's imagination in a growing network of alternatives, picking out only those directions which seem to hold out promise of worthwhile gain.

Firstly, the 'threshold' logic here proposed does seem to be primitive. The 'all-or-nothing' transfer from one order of predictor to another (coupled with an implicit bias adjustment via the medium of assumed time of flight) could probably do with refinement. Intuitively one feels that a smoother transition from one order of predictor to another would be beneficial, but the mathematical properties of continuously weighted composites of this kind have not been examined in this thesis, so that intuition has not been backed with logic. It is possible to envisage zero, first and second order predictions (each of which may utilise a different set of primary weights) being computed on-line, and being weighted to produce a final prediction according to some scheme which could, perhaps, take account of the range of the target, the size of the residual error and the plane in which the prediction is being made. (Certainly, the a priori probabilities of different levels of target acceleration will not be the same in each plane). The length of the data-gathering interval has not been examined in this thesis. At this point one begins to become uncomfortably aware of the 'growing network' mentioned in the last paragraph.

Up to now the emphasis has been on the probability of hitting the target, and on ways that this might be improved. A second theme of development might be to concentrate on the probability of not hitting the target. The possibility has not been examined of using the residuals from the prediction process to estimate whether, due either to inaccurate tracking or markedly nonlinear target motion, the probability of hit is very low. Operationally there may be value in suppressing such firings (not only to preserve ammunition, but also to reduce the chances of attracting attention to one's position unnecessarily).

It is not too dificult, then, to think of lines of research which may lead in the end to improved effectiveness. However, the other side of the effectiveness coin is cost - equipment costs, development costs and the cost of the research itself. Devising a scheme which is mathematically feasible, and which can be shown to have desirable mathematical properties, does not guarantee that a practical embodiment of that scheme can be achieved at all, let alone achieved simply and cheaply. The practical embodiment of the 'threshold' predictor which has been evaluated in the previous section is not straightforward: the completion of the ballistic computations, the solution of the prediction equations, and laying the gun within tolerance to some demanded offset, all take time. Deciding the order of equation to be fitted cannot therefore be left to the last moment, and so one can see an immediate complication: perhaps a two stage procedure, deciding the order and computing a provisional offset, followed by a later update to the offset near the instant of fire. Even discounting computational and time complications the admission of a quadratic term to the prediction equation increases the physical complexity of the scheme: a zero order predictor implies a zero offset, a linear predictor a fixed nonzero offset, but the presence of a quadratic term implies an offset which is changing linearly with time. Once having accepted this latter principle there would seem to be much to be said for a sequential arrangement, in which a provisional prediction was made and the gun offset in accord with that prediction, predictions and offsets being updated after each computational cycle. (If the initial offset were zero it would be a simple matter to encompass an accelerated drill for the stationary target.) It would probably be easier to base a sequential scheme on continuous weighting rather than discrete thresholds, but whatever approach was adopted enough has been said to make it plain that there would undoubtedly be some development costs involved.

The decision whether or not to pursue a line of development is thus a sequential process. The first stage is the formation of a concept, followed by its embodiment in logical form, and the pursuance of that logic in order to obtain a provisional assessment of its potential. It is then the turn of the designer and developer to assess its practical feasibility, and the costs in time, money and complexity involved in the progression from logical and abstract formulation to embodiment in concrete form. The final balancing of costs and effectiveness is the task of the decision maker, aided by such operational assessment as he can muster. To venture too far into these last two stages would be to stray beyond the confines of this thesis. It is perhaps just worth making the observation that the development of a modern main battle tank, its armament and all its associated systems is an immense and costly undertaking. All this development must be complete, and all the hardware must be produced and proved, almost irrespective of the prediction scheme which will finally be utilised. Given that a digital computer will figure in the tank's equipment, the realisation of the predictor is in software rather than hardware terms. The principal aim of predictor improvement is to increase the probability of hit in operational conditions, and, while a high hit probability is not a sufficient condition for high overall system effectiveness, it is certainly a necessary condition. Hit probability is a factor which multiplies all others in the effectiveness equation. It follows that predictor enhancement is mainly an item on the research bill, rather than an addition to the vastly expensive hardware development costs, but any improvement which ensues from that research increases the effective return from the development spend in direct proportion. However, whatever the cost-effectiveness decision in a particular case, the necessary precursor to any advance is conceptual innovation. So long as there is a case for predicted - fire weapons there will probably be a case for predictor research.

8. TANK GUNNERY PREDICTION: THE RESEARCH PROGRAMME REVIEWED

8.1 Introduction

In Chapter 1 it was suggested that a programme of research on tank gunnery prediction might contain the following strands:

(a) Establish the characteristics of likely target motion and of exposure in operational conditions.

(b) Describe in a statistical sense the human response to different target motions.

(c) Via the construction of computer models utilising (a) and (b) above, devise and test different prediction algorithms and engagement routines.

(e) Evaluate selected algorithms, in the first instance by using laboratory data, but then validating them under field conditions.

We have now addressed the first three topics in turn, and to some extent we have evaluated the results as they have been obtained. It remains to look back critically, and from an overall point of view, attempting to identify the most important areas of deficiency and suggesting how they might be remedied. We might speculate, too, on how the results could be extended. Having done this we can look ahead to the last topic on the list, especially to the crucial task of validation.

8.2 Deficiencies

Quite obviously there are many deficiencies in the programme of work which has been completed. It is not the purpose of this review to list and describe them in all their detail. The aim instead is to identify those which may have the most serious and dominant effects.

There are two rather different approaches which we may adopt in dealing with these deficiencies. The most obvious way is to augment our present knowledge, and in the light of this extra information improve the description of the system which we are trying to represent in model form. In many ways this is the most satisfactory approach. But if extra information is impossible or costly to obtain we may have recourse to sensitivity analyses: we would speculate on the most probable differences which may occur in practice, try to incorporate them in model form, and then run our model with excursions along these supposedly critical dimensions. We would then note how sensitive were our conclusions to the assumptions we had made. The difficulty here is that we can seldom be certain that our model variations truly mirror the effect of the factor about which we have imperfect knowledge. However, in some of our present areas of ignorance (such as the effects of fear, or the battlefield environment) we may be forced to a sensitivity analysis approach.

8.2.1 Target motion characteristics and engagement geometries

It has been noted that there is some variation in the results of the different studies on which we have based our assumptions about engagement parameters (range, angle of attack, height difference and sight tilt). It follows that there must be some uncertainty about the values we have actually chosen. However, the very variability which has been observed leads one to the conclusion that just one or two extra investigations would add but little to our sense of certainty: the next reviewer would probably have but one more disparate set of results to reconcile with the rest. There is a case to be made, though, for the occasional updating study, because the expected values of engagement parameters are likely to change with time. There may also be a case for examining the relationship between firing range and angle of attack: Whittaker's formulation (Pennycuick, 1945a, 1945a,1945b) would entail a changing angle of attack distribution (dpv) with range, whereas Peterson's (1951) assumptions would imply independence. However, it seems likely that fairly major changes would have to be made to our assumptions about engagement parameter distributions in order to have a significant effect on the relative standing of different prediction algorithms.

Potentially a more serious gap in our understanding of tank gunnery prediction concerns our knowledge of what operational target motions may be. Until we are fairly certain on this point we cannot be sure what problems we should truly be asking our fire control computer to solve. While we have been fortunate to have access to the AMSAA trials data, these latter cannot be regarded as entirely satisfactory for our purposes. We have seen that the data are affected by measurement noise; we have reason to regard the information in the vertical plane with some suspicion; and there are difficulties in placing the results in an operational context (because, although we may believe that a fighting vehicle could approach these levels of manoeuvre, we do not know how often it might do so in a genuine tactical setting, nor what the typical level of manoeuvre might be). It seems, then, that there is a real requirement for some tactically based trial which could yield relevant target motion information. The intention of the CHINESE EYE exercise was entirely laudable, and even the accelerometer method of motion measurement used there could be appropriate if it were linked with a more suitable scheme for recording the data. Obviously, there are the usual sampling problems in exercises of this kind, and we may have difficulty in convincing ourselves that the results are 'typical'. It is possible, too, that vehicle motion characteristics will change more rapidly with technical advance and with evolving tactical concepts than will all the other engagement parameters put together. But at least if one has some snapshot of this shifting scene one has a base from which one can extrapolate, and this seems preferable to continued ignorance.

8.2.2 Human operator modelling

The most serious deficiency in the human operator modelling work is that it is based exclusively on laboratory studies. We have reason to suspect that field effects may be major, and so it hardly seems worthwhile at this stage to devote significant effort to detailed improvement of the model which has already been devised. (It is this consideration which has led us to leave the 'remnant' results as they stand).

In Chapter 5 field effects were discussed under three headings: target-associated effects, mechanical stresses, and environmental factors. It was suggested that means for investigating the first two factors were to hand, or would shortly be to hand, but that research into the effects of the battlefield environment is beset with methodological problems. This last topic has been addressed elsewhere (Speight, 1976b; see also the short review by Allnutt, Cox & Huddleston,1978) and so only the briefest consideration will be given to it here. Within the ethical constraints a free society sets itself the effects due to threat or fear cannot be simulated in a peacetime setting. However, it seems very probable that in a live engagement a gunner will be in a highly aroused state and will be very strongly motivated to succeed. Under these conditions performance of control tasks is typically degraded (Duffy, 1962; Martens & Landers,1970; Martens, 1971). A practical approach to this problem might therefore be to compare tracking records obtained in strongly arousing conditions (such as those of live firing, which are known to degrade performance: see Ford, Speight, Henschke & Readett, 1972) with those obtained under more optimally arousing conditions (such as the laboratory, with adequate and immediate score feed back to operators). The next step would be to characterise and extrapolate the observed differences in a series of sensitivity analyses. It was argued in Chapter 5 that the scant evidence so far collected suggests that the difference between 'over aroused' and 'optimally aroused' conditions is roughly akin to that between 'bad' and 'good' operators.

The first priority in human operator modelling must be to allow for field effects. Only when this is done should model refinement (or improved parameter assessment) be considered. But it must be born in mind that the model itself is simply a descriptive framework, and not a fundamental account of human behaviour. It does seem from the limited data collected so far that model parameters are not overly sensitive to the characteristics of the plant which the gunner controls, but any pronounced changes in this area (a radically different kind of joystick, markedly nonlinear output, or direct command of gun control equipment with poor high frequency response or with nonlinearities such as backlash) will require model reassessment.

8.2.3 Prediction algorithms

The work that has been carried out on this topic has been exploratory in nature, and as such there are bound to be deficiencies. A critique has been provided in section 7.3. A point not made there, but which is important nevertheless, is that any algorithm which is put forward for serious practical consideration must be shown to be robust with respect to the human operator characteristics assumed, and in addition should not be dependent on the particular sample of target courses employed. There is a requirement, therefore, for sensitivity analyses and cross validation studies.

8.3 Validation and extension of modelling work

The paragraphs above have identified deficiencies which should be remedied, and in section 7.3 some directions were indicated in which prediction algorithms could be extended. The 'threshold' algorithm concept is a novel one, which on the face of it does seem to have some potential. It has been suggested that one way in which this potential might be increased would be to soften the'all-or-nothing' feature of the fixed threshold, so that the zero-one weights employed here are replaced by a continuous weighting scheme. The suspicion that such a scheme may be advantageous is reinforced by examination of the detail of the hit probability graphs yielded by the schematic engagement analyses in section 6.5 (Figures 30 to 32). Taking the type A targets first (those initially approaching the defender but turning to one side at lase) there appears to be a disturbance in the hit probability plot for the 'threshold' predictor at an acceleration of about 0.07g. The disturbance seems to reflect the algorithm switching, in effect, from zero order to linear and then quadratic predictions at about this point. The other two target types (initially crossing, but then turning towards or away from the defender) yield smoother but more pronounced disturbances at about the 0.35g point, and this is presumably associated with the transition from linear to quadratic predictions. A smoother transition would in all probability be beneficial, especially if it was accompanied by a more sophisticated bias compensation than that provided by the present straightforward time of flight adjustment.

Another line of investigation which has been put forward is to examine whether useful information may be gleaned from the residuals yielded by the curve fitting routines embodied in our prediction algorithms. This suggestion is more speculative. What we are looking for here is abnormal or 'pathalogical' patterns which might indicate very low conditional hit probabilities.

There are, of course, many ways in which prediction algorithms and fire control schemes may be extended, and in exploring different alternatives the modelling approach which we have developed has an undoubted utility. But as Ackoff (1979) has put it very recently: ' The optimal solution of a model is <u>not</u> an optimal solution of a problem unless the model is a perfect representation of the problem, which it never is. Therefore, in testing a model and evaluating solutions derived from it, the model itself should never be used to determine the relevant comparative performance measures'. He goes on to say: 'All models are simplifications of reality. If this were not the case, their usefulness would be significantly reduced'. At some stage in the investigative process one or more fire control solutions may be deemed to have sufficient promise to warrant proper evaluation, and at this stage we must validate our model predictions against live data. It is suggested here that validation should be at least a two stage process, with a laboratory phase preceding final field testing.

The requirements for an entirely satisfactory field test are indeed stringent. The problems lie not so much in physical development of the whole tank gunnery and fire control system - certainly these are very considerable, but they must perforce be solved in pursuit of the overall aim of providing an effective defensive capability. The problems lie instead in the interacting areas of target behaviour specification, performance measurement and sample size. Establishing likely patterns of target manoeuvre has been identified as a major area of deficiency at present. If this deficiency were remedied, the possible variation of engagement geometries within the established operational envelope is such that only a very small subset of all possible geometries could be evaluated in a live trial. The ultimate test of any fire control scheme is, of course, to fire live rounds and determine whether the observed hit and kill probabilities agree with those predicted. However, even with a single specified target motion (presumably preprogrammed, with an unmanned target vehicle) the number of firings needed to provide an estimate of the hit probability within narrow confidence bands is very large. If target motions are not repeatable this will further reduce the precision of any comparison.

Already we can see that practical considerations lead one to suppose that there must be a lower level of test than the ultimate one of live firing. If we could measure tracking error and target position sufficiently accurately we could run the whole engagement sequence up to the moment of firing and (ignoring for the moment the anticipatory effect that live firing is known to have on the gunner) ascertain the workings of the prediction and laying off routines, and then could compare their outputs with measured target position at nominal impact. The measurement requirement is a demanding one. We have noted from this study that, in the vertical plane and with a well defined aiming point, the human operator can produce errors with an instantaneous standard deviation of less than one tenth of a milliradian in real space. Film techniques (splitting the optical path to the gunner's eyepeice) certainly cannot provide the required resolution (if only because the accuracy of the human film reader is of the same order as the gunner's performance in the first place: see, eg, Ford & Speight, 1970). Infra red beacons and television edge following devices can probably be developed to a pitch where measurement noise does not mask the true signal of tracking error, but some signal corruption is inevitable. If the problem of tracking error measurement can be solved, that of estimating the true target position can probably be solved also (because the bearing of the sight axis itself can be determined with some accuracy, and laser range indications would be sufficiently exact for the required computations). Nevertheless, the measurement and recording schemes we are discussing are complex, costly and require skilled setting up and supervision. The conclusion from all this is that even non firing live trials are expensive and difficult to run if their results are to be of value. If such trials are to be kept within tolerable cost bounds the data samples are likely to be sufficiently small for confidence intervals to be embarassingly large.

It may be possible to decouple the predictive portion of a live engagement from the actual gathering of tracking and target position data. If this is so, and assuming that the measurement problem has been solved in the first place, then the precision of any comparison between competing predictive schemes will be much improved. These schemes can be run with the same recorded tracking inputs, so that comparison is within, rather than across, engagements. The main problem is then to ensure that the engagenents themselves are not atypical, and that they are indeed representative of the ensemble of possible operational engagements.

With the introduction of data recording and off-line comparison of predictors we have moved some way from the ultimate live firing test. Laboratory evaluation is a further step in the same direction. (At this point it is perhaps worth noting that there is a growing prospect of extremely complex, realistic and accurate simulators being produced for training purposes.) The idea here is to face gunners with an extended sample of simulated engagements, record their tracking responses, and then use these as the inputs to mock-up embodiments of competing and practicable fire control schemes. To be sure, the problem of generating, measuring and recording representative target motions still has to be faced. Nevertheless, the problem of measurement is much eased by the fact that target position does not have to be rigidly linked to tracking response at the time when the recording is made. It is not just that different techniques can probably be used if simultaneous tracking records are not required: measurement errors themselves have a different effect if tracking data are generated from recorded motions. Instead of measurement errors distorting the true relationship between live tracking (and predictor) response and live target motion, one obtains, in effect, the true tracking response to possibly distorted recorded motion. The advantages of the laboratory set up, once proper samples of target motion have been secured, are considerable. It is flexible: extensive sets of movement data can be stored, and can be used in conjunction with varied sets of the other engagement parameters. It is accurate and repeatable. Data recording and storage is simple and rapid. And, compared to live trialling, it is cheap.

Ultimate and complete validation is an unattainable ideal. Validation is, after all, concerned with the confidence one has in a solution one proposes, and in practice there is a balance between an acceptable level of confidence and the price one is willing to pay for it. For a limited outlay it is suggested that an effective approach to validation might be to start with modelling, progress to laboratory evaluation, pass on to field trialling and end with live firing. To expand on these phases:

(a) Modelling. This is the initial exploratory stage, devising alternative approaches and testing them in simplified fashion. Sensitivity analyses should give one the assurance that the most promising solutions are not too dependent on the base case assumptions, and so may stand up to more searching test in the later stages of evaluation.

(b) Laboratory evaluation. The main aims here are twofold. The first is to reduce the effects of the approximations involved in human operator modelling. (Not only have we admitted that our model cannot reproduce all the effects apparent in the data, but the use of averaged coefficients from a nonlinear model is not an ideal procedure.) The second aim is to represent more fully the rest of the fire control system and assess real-time interactions. Modifications to the proposed system may be made (if necessary by recycling through the modelling phase). It has already been suggested that the amount of data yielded by the next two phases will be insufficient to build up a complete system description from these sources alone. It follows that our basic system understanding must be complete by the end of stage (b): subsequent stages should be planned to provide critical tests.

(c) Non-firing trials. The main aim is to confirm (to the extent permitted by limited data) the basic relationships established in (a) and (b), but incorporating such modifications as are dictated by unpredicted field effects.

(d) Firing trials. At best these can confirm one or two points within the envelope covered in the preceding phases, ensuring that eventual results are not markedly at variance with all that has gone before. A large part of the firing trial function will be to check the interaction of all the parts of the complete system. So far as gunner behaviour is concerned, the main emphasis should be to compare tracking performance (and related drills) in the firing and non firing modes.

If all has gone well in this programme one should feel by its completion that one has a reasonably accurate descriptive scheme, in line with live results to the degree of confidence one can afford, and taking due account (although perhaps in a rough and ready way) of live effects. We would then be in a position to extrapolate, with caution, to other areas where further assessments are required, and to speculate profitably on the effects that any system modification may have.

8.4. Summary and conclusions

In this thesis we have been concerned with fire control prediction schemes for tanks employed in a defensive role against moving targets. The problem has been considered in three parts: the determination of likely target movement patterns in an operational setting; the assessment and modelling of human operator response to those motions; and the utilisation of this response in optimal prediction schemes. In the first part the results from war games, tactical exercises and field trials were collated, and a method was devised for generating test target tracks for human operator study and prediction scheme evaluation. In the second part previous approaches to operator modelling were reviewed, laboratory experiments were described and a mathematical model of human response was developed. In the third part the general statistical properties of predictors were examined, a new class of predictive algorithm called the 'threshold' algorithm was devised and was evaluated using the results of the previous two parts.

It was concluded that the 'threshold' algorithm did hold out some promise of practical utility. It was suggested that further assessment of this and other algorithms is at present hampered by limited information on likely operational target manoeuvres, and by a lack of knowledge about the probable influence on the gunner of field effects (due to mechanical and environmental stresses, and associated with appearance of the live targets). Lines were indicated along which predictive schemes might be developed. Finally, some consideration was given to the practical problems of evaluation and validation, and a general approach to these topics was suggested.

APPENDIX A. COMPUTER PROGRAMS

A1. Introduction

A large number of computer programs were developed during the course of this thesis (including many brief programs, concerned with data handling, illustrative plots and the like, too numerous to mention). The principal programs are described below in just sufficient detail to outline their function and working methods. An exception is the nonlinear least squares routine, ADUD. This last-named has involved the development and extension of a method which has only recently appeared in the literature (Ralston & Jennrich, 1978) and so it has seemed appropriate to expand a little on the program logic and background.

All the programs are written for use on the Sperry-Univac V72 and V76 computers. Except where indicated the programming language is FORTRAN IV, although frequent use has been made of subroutines peculiar to the Sperry-Univac VORTEX system (especially for data plotting, disc file management and for handling the digital-analogue input-output channels).

A2. Engagement geometry: CHINESE EYE III analysis

The basic data for the CHINESE EYE III engagements (map coordinates of attacker and defender, height differences and angle of attack) were read into a disc file (CH3DTA) via a short data-handling program. Two further programs were used for analysis.

A2.1 Program LATTCK

Rank orders angles of attack and produces a cumulative probability plot. Plots the cardioid dpv (Equation 6) and Whittaker dpv with a specified value of k (Equation 5). Also computes the mean engagement range.

A2.2 Program ENGTR

Rank orders engagement ranges and produces a cumulative probability plot. Also draws the cumulative curve for a (scaled) chi-square distribution with specified mean (m) and degrees of freedom (k) - translatable into the gamma distribution by the relationship $m\chi^2(k) = G(\frac{1}{2m}, \frac{k}{2})$.

Subroutines called:

PCHISQ. Computes the probability that chi-square with k degrees of freedom exceeds a specified value x. Based on the algorithm of Hill & Pike (1967).

CDFN. Evaluates the integral of the standardised normal curve from minus infinity to x. Based on the algorithm of Hill (1973).

A3. Engagement geometry: AMSAA data analysis

For each of the AMSAA tracks the (edited) 0.1 sec interval target coordinates were placed on disc file AM315B, a separate disc being used for each track.

A3.1 Program TGEN2

Given a nominal impact time, the program describes the target course (as seen by a defender at a nominated position) in terms of cubic spline functions (that is to say, describing target angular position in each dimension by piecewise quadratic functions). The program yields the location of the knots and the values of the quadratic coefficients. The working procedure is described in section 2.9 of the thesis.

Subroutines called:

BOUNDS. Given visually judged points of inflection for the velocity plots produced by the main program, fits linear velocity equations (and hence quadratic equations in position terms) to the data within these bounds, and then computes the implied knots.

LINFIT. Called by BOUNDS. Least squares linear fit to specified data.

TFLT. Computes the time of flight of the shell for the indicated engagement range.

A4. Operator modelling: Simulation for tracking experiments

A4.1 Program FMTRK

The main program reads in constants governing the appearance of the display, and subject, control and experimental details. It sets up the conditions for two blocks of 15 target courses to be tackled (randomising the target order in each); produces a printout of operator summary statistics; and permits the experimenter to re-run particular target courses if there should be evidence of gross malfunction in the equipment (or markedly aberrant subject behaviour). Checks are incorporated that the right subject disc is loaded, that experimental conditions are not doubly specified, etc.

Subroutines called:

RUN. Sets an individual tracking run in motion, computes summary statistics, and stores tracking errors on disc files (PRESS if a pressure joystick is specified and MVT if a movement joystick is used). Target motion data is read from disc file DELTA.

TRACK. Called by RUN. This is the inner tracking block, refreshing the subject's visual display every 0.05 sec. When first called it computes an initial (random) target offset, and then paints a scene with stationary background grid, aiming triangle and (centrally placed) tank target. When an above-threshold output from the 'take control' button is sensed the target is immediately moved to its initial offset, and thereafter the joystick output is sampled every 0.05 sec and transformed in accord with the cubic output function (if specified - see section 4.3.2) and with the position-plus-velocity control law (see section 4.2). The target position in simulated real space is initially the offset just mentioned and thereafter it is computed by adding target position differences (the first in the array read from DELTA prior to activation of the 'lase' button, and, once activated, the array members in sequence). The background position in real space is, of course, fixed. The target and background deviations in sight axes are obtained by subtracting the transformed joystick output from the currently computed target position, and from the arbitrary starting position, respectively.

WTANK3, GRID2, TRIAN3. Assembly language subroutines to draw the tank target, grid background and aiming triangle respectively. (Programmed by Mr R O'Connor).

IRAND. Produces a random integer, uniformly distributed within a specified range.

IRPERM. Produces a random permutation of the integers from 1 to n (inclusive).

RAND. Produces a random real number, uniformly distributed in the interval (0,1). Based on the GFSR routine given in Lewis (1974).

A4.2 Program DELTGEN

For a given set of target motion equations (as yielded by TGEN2) and for a specified target identification number, computes target positions at 0.05 sec intervals over a 16 sec period, and stores these in differenced form in disc file DELTA, and in a fashion suitable for use by FMTRK. The equations actually employed are given in Appendix B.

A4.3 Program MTRK3

FMTRK was used in Experiments 4 and 5 to simulate the whole engagement including 'take control' and 'lasing' (see Chapter 4). MTRK3 was used for the tracking-only Experiments 1, 2 and 3. It has the same features as FMTRK, but uses modified RUN and TRACK subroutines (the latter setting individual target tracks in motion without reference to the output from any 'take control' button, and according to the routine outlined in section 4.3). Instead of reading (differenced) target position data from disc file DELTA, the following subroutine is used:

TGTMOT. Produces target positions at 0.05 sec intervals over a 21 sec period. The target motion equations are piecewise third order polynomials (with stored coefficients) as given in Appendix B.

A5. Operator modelling: analysis of tracking experiments

The analysis of the data from the tracking experiments involved a large number of small-scale programs, yielding information required by the major analysis routines. Thus ANLS3 produced instantaneous means and variances for each target course for each subject (it being the objective of transfer functions to predict these means; and the variances being used, among other things, in the weighting scheme described in section 4.3.3 - Equation 34). Other programs yielded correlation matrices needed as the input to ridge and best subset procedures, and so on. Rather than list all these minor programs only the major ones are outlined below.

A5.1 Program RRIDGE

Equation 19 (section 3.4.2) gives the formula by which the ridge estimates of the extended impulse response function are computed. This program yields the values of these coefficients for k=0(0.001) 0.01(0.01) 0.1(0.1)1.0.

A5.2 Program GRIDGE

Equation 21 (section 3.4.2) gives the formula by which generalised ridge estimates of the extended impulse response function are computed. As stated there, a unique (and optimal) diagonal matrix $\underline{K}$ can be obtained if the true error variance is known, although this is usually estimated from the residuals of the model-fitting procedure on the assumption that they are independently distributed. This latter assumption is not tenable in our case. This program yields the generalised ridge coefficients, estimating the error variance from the residuals, but assuming 1(0.5)4 effectively independent samples of tracking error per second. The method used is that due to Hemmerle (1975)

Subroutines used:

EIGEN. Produces the eigenvectors and eigenvalues of a real symmetric matrix, using the JK method outlined by Kaiser (1972).

A5.3 Program LANDB

Given an n by n correlation matrix, this program produces impulse response (or regression) coefficients based on a 'best m' subset of variables, using the 'leaps and bounds' routine of Furnival & Wilson (1974).

A5.4 Program ADUD

As explained in the introduction to this Appendix, the description of ADUD breaks the pattern established with the other programs. For the reasons stated there, instead of just a very brief statement of function and of working method some account is given of the Dud algorithm on which ADUD is based, and of the considerations and experiences leading to final program development. The logic of the developed algorithm is also exposed.

In its original form the Dud algorithm of Ralston & Jennrich (1978) is closely allied to the Gauss-Newton method for estimating the minimum of a nonlinear function. Suppose that we have an observed data vector $\underline{y}$ which is related to such a nonlinear function by an equation of the type:

$$y_i = f_i(\underline{\Theta}) + e_i \quad i=1,2,\ldots,n \tag{A1}$$

where

y_i is the observed response at the ith data point,

$f_i(\underline{\Theta})$ is a known function of the parameter vector $\underline{\Theta}$, and

e_i is an error term (which could be zero).

The aim is to find a parameter vector $\underline{\Theta} = (\Theta_1, \Theta_2, \ldots \Theta_p)'$ which minimises the sum of squares

$$Q(\underline{\Theta}) = \sum_{i=1}^{n} (y_i - f_i(\underline{\Theta}))^2 \tag{A2}$$

The Gauss-Newton method proceeds iteratively by linearly approximating $f_i(\underline{\Theta}^*)$ in the region of $\underline{\Theta}^*$ (the current best estimate of $\underline{\Theta}$ in the sense just given) with a first order Taylor series expansion

$$f(\underline{\Theta}^*+\underline{\Delta\Theta}) \simeq f(\underline{\Theta}^*) + \Delta\Theta_1 f_1'(\underline{\Theta}^*) + \ldots + \Delta\Theta_p f_p'(\underline{\Theta}^*) \tag{A3}$$

(where $f_i'(\underline{\Theta}) = ((\partial/\partial\Theta_i) f(\underline{\Theta}))$, and this gives

$$\underline{y} - \underline{f}(\Theta^*) = \Delta\Theta_1 \underline{f}_1'(\underline{\Theta}^*) + \ldots + \Delta\Theta_p \underline{f}_p'(\underline{\Theta}^*) + \underline{e} \tag{A4}$$

A solution for $\underline{\Delta\Theta}$ which will minimise $Q(\underline{\Theta}^*)$ can be obtained by ordinary least squares, and $\underline{\Theta}^*$ can be replaced by $\underline{\Theta}^*+\underline{\Delta\Theta}$ until, hopefully, the whole process converges.

The Dud algorithm proceeds in very similar fashion, but uses a different approach to obtain local linear approximations of $f_i(\underline{\Theta}^*)$. It constantly updates a simplex of parameter vectors $\underline{\Theta}_1, \underline{\Theta}_2, \ldots, \underline{\Theta}_{p+1}$ (which will include $\underline{\Theta}^*$) and uses as its local approximation that linear function which passes through $f(\underline{\Theta}_1), f(\underline{\Theta}_2), \ldots, f(\underline{\Theta}_{p+1})$.

If we define a p by p matrix $\underline{\Delta F}$ whose i th column is given by

$$\underline{\Delta F}_i = \underline{f}(\underline{\Theta}_i) - \underline{f}(\underline{\Theta}_{p+1}) \qquad \text{(A5)}$$

then the local linear approximation to $\underline{f}(\underline{\Theta}^*)$, $\underline{\ell}(\alpha)$, is given by

$$\underline{\ell}(\alpha) = \underline{f}(\underline{\Theta}_{p+1}) + (\underline{\Delta F})\underline{\alpha} \qquad \text{(A6)}$$

The value of the coefficient vector $\underline{\alpha}$ which minimises

$$Q(\underline{\alpha}) = (\underline{y}-\underline{\ell}(\underline{\alpha}))'(\underline{y}-\underline{\ell}(\underline{\alpha}))$$

can be obtained once more by ordinary least squares:

$$\underline{\alpha} = (\underline{\Delta F}'\underline{\Delta F})^{-1}\ \underline{\Delta F}'(\underline{y}-\underline{f}(\underline{\Theta}_{p+1})) \qquad \text{(A7)}$$

and so a new estimate of $\underline{\Theta}^*$ is obtained at each iteration by the relation

$$\underline{\Theta}_{new} = \underline{\Theta}_{p+1} + \underline{\Delta\Theta}\,\underline{\alpha} \qquad \text{(A8)}$$

where $\underline{\Delta\Theta}$ is a p by p matrix whose columns are given by

$$\underline{\Delta\Theta}_i = \underline{\Theta}_i - \underline{\Theta}_{p+1} \qquad \text{(A9)}$$

The complete Dud algorithm thus has the following steps:

(a) Specify an initial simplex $\underline{\Theta}_1, \underline{\Theta}_2, \ldots \underline{\Theta}_{p+1}$. (A suitable method, the authors suggest, is to specify one vector, $\underline{\Theta}_{p+1}$, with nonzero elements. The other vectors, $\underline{\Theta}_1, \underline{\Theta}_2, \ldots, \underline{\Theta}_p$ are generated from $\underline{\Theta}_{p+1}$ by replacing Θ_{ii} with $h\Theta_{i(p+1)}$, h=0.1 being a reasonable choice for most circumstances).

(b) Reorder these vectors so that $Q(\underline{\Theta}_1) \geq Q(\underline{\Theta}_2) \geq \ldots \geq Q(\underline{\Theta}_{p+1})$.

(c) Compute $\underline{\Delta F}$ via equation A5 and $(\underline{y}-\underline{f}(\underline{\Theta}_{p+1}))$, and hence obtain $\underline{\alpha}$ via equation A7.

(d) Compute $\underline{\Delta\Theta}$ via equation A9, and obtain a new parameter vector $\underline{\Theta}_{new}$ via equation A8.

(e) Discard $\underline{\Theta}_1$. The old $\underline{\Theta}_2, \ldots, \underline{\Theta}_{p+1}$ are re-labelled $\underline{\Theta}_1, \ldots, \underline{\Theta}_p$, and $\underline{\Theta}_{new}$ becomes $\underline{\Theta}_{p+1}$.

(f) Test for convergence. If convergence has been obtained accept $\underline{\Theta}_{p+1}$ as the best estimate. Otherwise return to (c).

The convergence criterion suggested is that the following relationship should obtain on 5 successive iterations:

$$\frac{\left| Q(\underline{\Theta}_{new}) - Q(\underline{\Theta}_{p+1}) \right|}{Q(\underline{\Theta}_{p+1})} < T$$

where T is a small positive number, say 10^{-5}.

Like the Gauss-Newton algorithm, Dud sometimes fails to converge unless a step shortening procedure is used. That recommended is to make $\underline{\Theta}_q$ the new replacement vector instead of $\underline{\Theta}_{new}$:

$$\underline{\Theta}_q = d\underline{\Theta}_{new} + (1-d)\,\underline{\Theta}_1 \tag{A10}$$

where d is the first member of the sequence

$$d_i = \begin{cases} 1 & i = 0 \\ -(-\tfrac{1}{2})^i & i = 1, \ldots, m \end{cases}$$

which makes $Q(\underline{\Theta}_q) < Q(\underline{\Theta}_{p+1})$ (and if there is no such d_i then $d = d_m$ is used). Because of the extra function evaluations entailed by this step shortening procedure, it is suggested by the authors that Dud should be used initially with $m = 0$, only setting m to 5 if there is failure to converge.

It is quite possible for the matrix $\underline{\Delta F}'\underline{\Delta F}$ to become ill-conditioned as iterations proceed. A stepwise regression approach due to Jennrich & Sampson (1968) is employed to alleviate this problem. Gauss-Jordan pivots are used for matrix inversion, the pivot element at each stage corresponding to the variable which results in the largest decrease of $Q(\underline{\alpha})$. If the 'tolerance' of a pivot element fails to exceed a certain threshold, say 10^{-5}, then that element is not actually used for pivoting and the corresponding α coefficient is set to zero. (If $\bar{a}_{ii}$ is the ith diagonal element of a non-negative matrix (a_{ij}) after pivoting a number of other diagonal elements, the ratio $\bar{a}_{ii}/a_{ii}$ is called the 'tolerance').

Steps must also be taken to prevent the Dud routine from collapsing into a subset of the parameter space, which it will do if α_1 (which is the weight given to the discarded parameter vector) is zero. If $|\alpha_1|<10^{-5}$ the replacement routine is modified thus:

(a) $\underline{\Theta}_{new}$ (or $\underline{\Theta}_q$) replaces $\underline{\Theta}_i$, where α_i is the first component of $\underline{\alpha}$ whose absolute value exceeds 10^{-5}, and

(b) $\underline{\Theta}_1$ is replaced by $(\underline{\Theta}_1 + \underline{\Theta}_{new})/2$.

The Dud routine has been evaluated extensively against the now-traditional test functions (such as Rosenbrock's valley, Box's mixed exponentials, etc) and seems to have compared very favourably with even its derivative - requiring competitors, in terms both of the number of function evaluations required and of the final value of $Q(\underline{\Theta})$. However, in our context and with our model the published routine was not without its problems. Some of these problems were due to the nature of the fitted transfer function, but others seemed to stem mainly from the routine itself.

Difficulties arose with the fitted transfer function because the true optimum could lie quite close to model stability boundaries. When a trial parameter vector crossed these boundaries massive computational overflows would result and the whole iterative process would break down. This problem could quite easily be circumvented. Incipient instability could be detected in the subroutine which evaluated $Q(\underline{\Theta})$, and if a threshold value was exceeded while the elements of Q were successively computed and summed the subroutine would immediately return to the main program with an artificially large Q value. The step-shortening procedure would then come into play until finally a low $Q(\underline{\Theta})$ would result.

The problems with the routine itself seemed to be connected in part with a tendency - despite the precautions outlined - for the process virtually to collapse into a subset of the parameter space. The components of $\underline{\alpha}$ could in fact have widely varying absolute values, and so it was quite possible after a few iterations for the effect of one (or more) of the initial vectors almost to have disappeared. The matrix $\underline{\Delta F'\Delta F}$ would then indeed be nearly singular, and would tend to remain so in succeeding iterations. The routine might then continue moving along one hyperplane, or, with the limited computational accuracy due to shrinkage in one or more dimensions, it could behave erratically. In this latter event the step-shortening procedure would come into play, not always with beneficial results. Repetitive step-shortening could also be triggered by the stability problems previously mentioned, the routine repeatedly venturing across the stability boundary and then retreating to its immediate vicinity. Too small a simplex at an early stage in the search sequence inevitably brought rounding error and computational inaccuracy in its wake, leading generally to a poor directional choice, further contraction of the simplex, and hence, with a succession of very small steps and small changes in $Q(\underline{\Theta})$, to premature convergence.

Some remedial measures were clearly required, and it was felt that it might be profitable to extend the Dud algorithm in roughly the same way as Marquardt (1963) had extended the Gauss-Newton approach. Marquardt started by noting that the Gauss-Newton method not infrequently diverged due to the inadequacy of the local first order Taylor series approximation. Steepest descent algorithms, on the other hand, often made good initial progress, but final convergence tended to be painfully slow. The Gauss-Newton algorithm, as we have seen from equation (A4), computes modification vectors by the formula:

$$\underline{\Delta\Theta} = -\tfrac{1}{2}(\underline{f}'(\underline{\Theta})^T\underline{f}'(\underline{\Theta}))^{-1}\underline{Q}'(\underline{\Theta}) \qquad \text{(A11)}$$

$$= \underline{A}(\underline{\Theta})^{-1}\underline{Q}'(\underline{\Theta})$$

(where $\underline{Q}'(\underline{\Theta})=(\partial/\partial\Theta_i)Q(\underline{\Theta})$). The steepest descent method simply puts:

$$\underline{\Delta\Theta} = -b\underline{Q}'(\underline{\Theta}) \qquad \text{(A12)}$$

This method does not actually fix the length of the step, as opposed to its direction, and the value of b is generally determined by a further search in the direction of $\underline{Q}'(\underline{\Theta})$.

Marquardt's method replaces equation (A11) with:

$$\underline{\Delta\Theta} = (\underline{A}(\underline{\Theta}) + \lambda\underline{I})^{-1}\underline{Q}'(\underline{\Theta}) \qquad \text{(A13)}$$

(it being assumed that $\underline{A}(\underline{\Theta})$ is scaled in correlation matrix form). It will be seen at once that (A13) approaches the Gauss-Newton method a λ approaches zero. As λ increases, on the other hand, the formula approximates ever more closely to the steepest descent method and the size of the step decreases. Marquardt then completed his method with a set of procedural rules for modifying λ: starting with a relatively large value of λ (say 0.1); decreasing it so long as progress was satisfactory; but increasing λ if the routine should show signs of divergence.

The application of the Marquardt principle to the Dud routine is obvious, equation (A7) (after suitable rescaling of $\underline{\Delta F}'\underline{\Delta F}$) being modified to:

$$\underline{\alpha} = (\underline{\Delta F}'\underline{\Delta F} + \lambda \underline{I})^{-1}\underline{\Delta F}'(\underline{y}-\underline{f}(\Theta_{p+1})) \tag{A14}$$

As one might expect, this modification (coupled with procedural rules for varying λ) did seem to alleviate those problems apparently stemming from ill-conditioned matrices. Also, as stated, a step shortening procedure via the modification of λ is implicit in the Marquardt routine, and this operated in a perfectly satisfactory manner during the early stages of the search. However, by its very logic Dud demands for final convergence a contraction of the simplex of search points, which leads in turn to relatively inaccurate step computations. If apparent divergence at a late stage should indicate the necessity for step shortening, then to increase λ would seem to be reverting to steepest descent methods at a most inappropriate time. In practice, then, it was found that it was beneficial to switch to a step-shortening routine similar to that used by Dud once λ had been attenuated below an arbitrary threshold (λ_{tol} , set to 10^{-5}), only increasing λ once more if this routine failed to have the desired effect. The version of Dud which was finally developed differed in quite a number of respects from the original. The complete method is described below:

(a) Specify an initial simplex $\underline{\Theta}_1$, $\underline{\Theta}_2$, ..., $\underline{\Theta}_{p+1}$. (The Dud method for generating this from a single vector was found to be satisfactory, although in our context a choice of h = 0.25 was preferred).

(b) Specify an initial value for λ, a factor μ (used for modifying λ - see (f) below), and the number of points, r, to be allowed in the simplex. (The option to retain additional search vectors was a feature incorporated in early stages of algorithm development to combat the ill-conditioning problem. After the Marquardt modification it was not often found helpful in practice). Suitable values for λ_{start} and μ are 0.1 and 10 respectively.

(c) Reorder the vectors so that $Q(\underline{\Theta}_1) \geq Q(\underline{\Theta}_2) \geq \ldots \geq Q(\underline{\Theta}_r)$.

(d) Compute $\underline{\Delta F}$ and $(\underline{y}-\underline{f}(\underline{\Theta}_r))$.

(e) Obtain $\underline{\alpha}$ via equation (A14) in conjunction with the Jennrich-Sampson stepwise regression approach. Hence obtain $\underline{\Theta}_{new}$.

(f) (i) If $Q(\underline{\Theta}_{new})<Q(\underline{\Theta}_r)$ put $\lambda = \lambda/\mu$ (or, if $\lambda > \lambda_{start}$, $\lambda=\lambda_{start}$). Go to (g).

(ii) If $Q(\underline{\Theta}_r) \leq Q(\underline{\Theta}_{new}) \leq Q(\underline{\Theta}_1)$ leave λ unaltered (or, if $\lambda>\lambda_{start}$, $\lambda=\lambda_{start}$). Go to (g)

(iii) If $Q(\underline{\Theta}_{new})>Q(\underline{\Theta}_1)$ and $\lambda>\lambda_{tol}$ put $\lambda=\lambda\mu$ and return to (e).

(iv) If $Q(\underline{\Theta}_{new})>Q(\underline{\Theta}_1)$ and $\lambda<\lambda_{tol}$ use the step shortening routine $\underline{\Theta}_{new} = (\underline{\Theta}_{new} + \underline{\Theta}_r)/2$ successively 4 times or until $Q(\underline{\Theta}_{new})<Q(\underline{\Theta}_1)$, whichever is the less. If $Q(\underline{\Theta}_{new})<Q(\underline{\Theta}_1)$ go to (g), but if this condition does not obtain after 4 cycles put $\lambda=\lambda_{tol}$ and return to (e).

(g) Replace one of the current search vectors with $\underline{\Theta}_{new}$ according to normal Dud rules. Test for convergence. Exit if the convergence criterion holds, otherwise return to (c).

It should be emphasised that the modified Dud algorithm has not been tested exhaustively against a variety of functions. Also worth noting is the limited word length (16 bits) of the Sperry-Univac V72 and V76 computers on which the routine was developed, tested and used. Although the crucial activities (such as matrix inversion and sums of squares and crossproducts accumulation) were carried out in double precision, this limited resolution could have contributed to the problems noted with Dud as it originally stood.

The last paragraph but one sketches in the working detail of ADUD. Transfer function evaluation was carried out using the recursive approach outlined in Section 4.3.5. The subroutines called are:

SSQ. Computes $Q(\underline{\Theta})$ for a specified $\underline{\Theta}$.

MATRIX. Computes $\underline{\Delta F}'\underline{\Delta F}$ and $\underline{\Delta F}'(\underline{y}-\underline{f}(\underline{\Theta}_{p+1}))$ for a specified set of parameter vectors $\underline{\Theta}_1, \underline{\Theta}_2, \ldots, \underline{\Theta}_{p+1}$.

A5.5 Program NOISE2

Fits the 'remnant' model specified in section 4.3.4 to the experimental tracking records of individual subjects. In each case the first step is to evaluate the determinate transfer function response via the approach outlined in section 4.3.5 (and using the model coefficients as determined by ADUD), and subtract it from the raw tracking data to leave the remnant. The maximum order, p_{max}, of the dependency scheme having been specified, the program evaluates the AR parameters (equation (35)) $\emptyset_{x1}, \emptyset_{x2}, \ldots, \emptyset_{xp}, \emptyset_{y1}, \emptyset_{y2}, \ldots \emptyset_{yp}$ for p=1,2,..., p_{max} via ordinary least squares, in each case producing an estimate of the residual sums of squares and computing the individual residuals when this dependency has been allowed for. The variance scheme outlined in equation (36) is then evaluated by fitting the σ^2 coefficients to the squared residual data. (As stated in section 4.3.4 various alternative variance schemes were examined, and various programs in the 'NOISE' series were produced as minor modifications of NOISE1 in order to pursue these investigations).

Subroutines used:

SLVE. Linear equation solver (Gauss-Jordan pivoting).

STEPX. Accurate computation of a vector of means and of a matrix of sums of squares and crossproducts.

A5.6 Program LASERR

Computes the tracking error variances for the two sampling points immediately subsequent to lase, and tests for significant relationships between the first of these and target movement characteristics (∇x, $\nabla^2 x$, ∇y, $\nabla^2 y$, $\nabla x \nabla y$). Also evaluates the regression of error at the second sampling point on error at the first.

Subroutines used:

SLVE. Linear equation solver (see A5.5 above).

A6. Operator modelling: Monte Carlo simulation of experimental tracking data

A6.1 Program SIML

The Monte Carlo simulation of results of Experiments 1 and 2 is discussed in section 4.3.5 and of Experiments 4 and 5 in section 4.4.3, and the working methods are described there. SIML is appropriate to these last two experiments (which included 'lasing'). The number of runs to be simulated for each target course is first specified. The simulation is then run for as many individual subjects as desired. For each subject the program demands the lase variance, and the transfer function parameters (produced by ADUD) and remnant parameters (produced by NOISE2) are read from disc file. (SIM and SIM2 are further versions of SIML, the former being appropriate to the tracking only Experiments 1 and 2, but modelling a first order dependency scheme, and the latter being appropriate to the same experiments as SIML, but modelling a second order dependency scheme).

Subroutines used:

RAND. Random number routine (see program A4.1).

RNORM. Produces a pair of independent random normal deviates with mean zero and standard deviation 1. It is based on the algorithm given by Bell (1968).

A7. Prediction algorithm evaluation

A7.1 Program EVGEN

Generates the target position histories (in sightline axes) for 125 evaluation engagements, and also computes (if necessary) and places on disc file the firing delay; the time of flight which would be estimated on the basis of laser range finder information at 'cease track'; the true time of flight; the position of the target at nominal impact and at 'cease track'; and the position coordinates of the hull outline at nominal impact. Twenty of the engagements are taken to be straightline targets moving at 10m/sec, and for these the program demands a range, height difference, target aspect and sightline tilt. One hundred are assumed to be manoeuvring targets, for which range, aspect and estimated time of flight are required, plus the knot positions and the cubic spline coefficients which describe target motion (previously calculated in program A3.1). The 5 stationary targets are treated in the same way as the manoeuvring targets, but only the initial knot is specified and the spline coefficients are taken to be zero. Ten seconds-worth of target position data (held in differenced form stretching backwards at 0.2 sec intervals from 'cease track') is placed on discfile PDATA for each target, together with the other information mentioned.

Subroutines used:

LINE. Carries out the position calculations for the straightline targets.

TFLT. Computes time of flight as a function of range.

TASPCT. Computes coordinates of the tank hull and turret outlines in sightline axes (and taking the centre of the turret ring as its origin) as a function of aspect (angle of attack) and range.

A7.2 Program CIRCLE

Similar function to EVGEN, but places data for the 99 schematic engagements represented by target types A, B and C (section 6.5) on disc file PDATA. The program assumes a common firing delay of 0.435 sec, and requests target range at 'lase'. The target motion histories are fully described in section 6.5. The program uses the same subroutines as does EVGEN.

A7.3 Program PTOEV

Given the data on disc file PDATA previously computed by EVGEN or CIRCLE, this program calculates expected sight positions (as controlled by the gunner) and instantaneous standard deviations using the operator model which has been developed in this thesis, and which is reviewed and described in section 4.5, equations (40) to (48). The program asks the user to specify a common firing delay (and checks this for consistency with that already specified within PDATA); the length of the 'track' interval; and the operator model constants assumed (lase variance, transfer function coefficients and remnant coefficients). The parameters just mentioned, plus those pertaining to the terminal phase of the engagement (time of flight, target coordinates, etc, already present in PDATA), plus the expected sight positions and variances, are all placed on disc file EVDATA.

A7.4 Program CEVAL

This is the final program in the evaluation suite, computing hit probabilities for nominal engagements and for the modelled expected human tracking outputs during those engagements, utilising the predictor philosophy and with the other system error inputs outlined in this thesis. The program requests from the user: a set of predictor parameter coefficients, K_1 to K_6 (equations (80) and (81) in section 6.3); the target weighting scheme to be used (equation (82) in section 6.4.1); and the maximum number of evaluations to be permitted. If this latter is 1 the subroutine EVAL is called, which returns hit probabilities for all the targets represented in disc file EVDATA, plus the weighted composite hit probability as specified in equation (82). If more than 1 evaluation is permitted the non linear optimisation subroutine DUD is called (which calls EVAL in turn) which returns the optimal set of parameters in the sense of maximising the weighted composite hit probability just mentioned. In the optimisation mode the user specifies λ_{start}, μ and T (the Marquardt parameters and the convergence criterion, see program A5.4 ADUD). (Other programs in the EVAL series used the Powell (1964) function minimising scheme, or minimised other statistics).

Subroutines used:

DUD. The developed Dud algorithm (as described in section A5.4) in subroutine form.

EVAL. Reads tracking and other target data from disc file EVDATA and calls up the routine WEIGHT prior to computing results for any individual target, and the routines BVAR, TRUNC and HIT for each target in turn.

WEIGHT. Given the two parameters which determine the primary weights, computes the system of secondary weights as outlined in equations (51) to (60), section 6.1.

BVAR. Given secondary weights and expected values and variances (which in turn imply the covariances - see equation (48)) of sampled tracking errors, yields the expected values, variances and covariances of the predictor b coefficients before truncation.

TRUNC. Given the predictor threshold values computes the expected values, variances and covariances of the sets of predictor coefficients subsequent to truncation, plus the probability of utilising each of these sets.

HIT. Utilising the outputs of the other routines, plus the target information supplied from disc file EVDATA, evaluates the probability of hit for an individual target.

Subroutines SSQ and MATRIX (utilised by DUD) are similar to those described under program A5.4, and CDFN (utilised by HIT) is as described under program A2.2.

APPENDIX B. TARGET MOTION EQUATIONS FOR TRACKING EXPERIMENTS.

Experiments 1 and 2

Target motion for these experiments was specified in terms of third order splines with knots in each dimension at $t_1 = 0$, $t_2 = 5$, $t_3 = 9$, $t_4 = 13$ and $t_5 = 17$ sec. We can thus specify target position within the ith interval by equations of the type:

$$x = B_{x0} + B_{x1}(t - t_i) + B_{x2}(t - t_i)^2 + B_{x3}(t - t_i)^3$$

As absolute target position is arbitrary, and because transition from one interval to another must be smooth, the B_0 term provides us with no real information. The remaining coefficients are shown in Table B1, scaled to yield target position as seen by the eye (with x10 sight magnification).

TABLE B1. Target motion coefficients, Experiments 1 and 2.

Target Course	t_i (sec)	B_{x1}	B_{x2}	B_{x3}	B_{y1}	B_{y2}	B_{y3}
1	0	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.0	25.0	0.0	0.0	5.0	0.0
	9	200.0	−12.5	0.0	40.0	−2.5	0.0
	13	100.0	6.25	0.0	20.0	0.0	0.0
	17	150.0	−6.25	0.0	20.0	−1.25	0.0
2	0	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.0	25.0	0.0	0.0	0.0	0.0
	9	200.0	−12.5	0.0	0.0	0.0	0.0
	13	100.0	6.25	0.0	0.0	1.25	0.0
	17	150.0	−6.25	0.0	10.0	1.25	0.0
3	0	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.0	0.0	0.0	0.0	5.0	0.0
	9	0.0	0.0	0.0	40.0	−2.5	0.0
	13	0.0	6.25	0.0	20.0	−1.25	0.0
	17	50.0	0.0	0.0	10.0	0.0	0.0
4	0	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.0	0.0	0.0	0.0	0.0	0.0
	9	0.0	0.0	2.0	0.0	0.0	0.0
	13	96.0	−6.25	0.0	0.0	0.0	0.0
	17	46.0	0.0	0.0	0.0	0.0	0.4
5	0	25.0	0.0	0.0	5.0	0.0	0.0
	5	25.0	0.0	0.0	5.0	0.0	0.0
	9	25.0	0.0	0.5	5.0	0.0	0.0
	13	49.0	12.0	0.5	5.0	0.0	0.0
	17	169.0	0.0	0.0	5.0	−0.625	0.0
6	0	25.0	0.0	0.0	0.0	0.0	0.0
	5	25.0	−3.125	0.0	0.0	0.0	0.0
	9	0.0	0.0	0.0	0.0	0.0	0.0
	13	0.0	20.0	0.0	0.0	0.0	0.0
	17	160.0	0.0	0.0	0.0	0.8	0.0
7	0	50.0	0.0	0.0	10.0	0.0	0.0
	5	50.0	0.0	0.0	10.0	2.5	0.0
	9	50.0	0.0	1.0	30.0	0.0	−0.4
	13	98.0	15.0	0.0	10.8	0.0	0.0
	17	218.0	−5.0	0.0	10.8	−1.0	0.0
8	0	50.0	0.0	0.0	0.0	0.0	0.0
	5	50.0	6.25	0.0	0.0	0.0	0.0
	9	100.0	0.0	0.0	0.0	0.0	0.0
	13	100.0	0.0	0.0	0.0	2.5	−0.2
	17	100.0	0.0	0.0	10.4	−2.5	0.2
9	0	75.0	0.0	0.0	15.0	0.0	0.0
	5	75.0	6.25	0.0	15.0	−1.25	0.0
	9	125.0	−25.0	2.0	5.0	0.0	0.0
	13	21.0	0.0	0.0	5.0	0.0	0.0
	17	21.0	−3.125	0.0	5.0	−1.25	0.0
10	0	75.0	0.0	0.0	0.0	0.0	0.0
	5	75.0	−9.375	0.0	0.0	0.0	0.0
	9	0.0	0.0	0.0	0.0	0.0	0.0
	13	0.0	25.0	−2.0	0.0	0.0	0.0
	17	104.0	0.0	0.0	0.0	5.0	−0.4

TABLE $B1_2$ (continued)

Target Course	t_i (sec)	B_{x1}	B_{x2}	B_{x3}	B_{y1}	B_{y2}	B_{y3}
11	0	100.0	0.0	0.0	20.0	0.0	0.0
	5	100.0	−6.25	0.0	20.0	2.5	0.0
	9	50.0	0.0	0.0	40.0	−5.0	0.0
	13	50.0	−3.125	0.0	0.0	0.625	0.0
	17	25.0	0.0	0.0	5.0	0.0	0.0
12	0	100.0	0.0	0.0	0.0	0.0	0.0
	5	100.0	−3.125	0.0	0.0	0.0	0.0
	9	75.0	−9.375	0.0	0.0	0.0	0.0
	13	0.0	0.0	0.0	0.0	0.0	0.0
	17	0.0	25.0	0.0	0.0	2.5	0.0
13	0	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.0	12.5	−1.0	0.0	0.0	0.0
	9	52.0	12.5	0.0	0.0	0.0	0.0
	13	152.0	−12.5	0.0	0.0	5.0	−0.833
	17	52.0	0.0	0.0	0.0	0.0	0.0
14	0	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.0	20.0	0.0	0.0	0.0	0.0
	9	160.0	−25.0	2.0	0.0	0.0	0.0
	13	56.0	−12.5	1.0	0.0	0.0	0.0
	17	4.0	0.0	0.0	0.0	0.0	0.0
15	0	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.0	25.0	0.0	0.0	0.0	0.0
	9	200.0	−25.0	4.167	0.0	0.0	0.0
	13	200.0	−25.0	0.0	0.0	5.0	0.0
	17	0.0	12.5	0.0	40.0	−2.5	0.0

Experiment 3

For this experiment the five target courses indicated in Table B2 were replaced with courses in which the target was stationary, apart from a step change in position at t = 9 sec. The size of this step in each dimension is shown in Table B2. All other courses remained the same as those in Experiments 1 and 2.

TABLE B2. Step input targets. Size of step (milliradians at the eye).

Target Course	Step size x	Step size y
2	150	0
7	−75	15
8	0	90
12	18	−45
13	75	45

Experiments 4 and 5

These experiments retained Target Courses 6 and 15 from the initial set (but utilising only that portion from t = 5 sec onwards). The other target courses were generated from the AMSAA data, and were depicted as cubic splines (the knot positions in the x and y dimensions not being identical). Since absolute target position is arbitrary, the knot positions and the first and second order spline coefficients are sufficient to specify the target motion equations. Taking the 'lase' time as zero, target velocity (once more as measured at the eye) within the ith interval can be specified by equations of the type:

$$\dot{x} = C_{x1} + C_{x2}t$$

The relevant coefficients and knot positions (t_i) are shown in Table B3.

TABLE B3. Target motion coefficients, Experiments 4 and 5.

Target Course	t_{xi} (sec)	C_{x1}	C_{x2}	t_{yi} (sec)	C_{y1}	C_{y2}
1	0.000	−36.43	−19.61	0.000	1.93	2.58
	1.733	−179.26	62.80	2.893	31.55	−7.66
	4.252	79.83	1.87	5.560	−61.51	9.08
	5.992	580.62	−81.71	8.562	61.29	−5.26
	8.272	−418.13	39.03			
2	0.000	113.87	19.63	0.000	−2.58	4.80
	1.412	198.43	−40.24	2.105	45.46	−18.03
	3.824	−75.06	31.28	3.250	−57.59	13.68
	6.484	143.93	−2.50	4.084	−5.59	0.95
	11.222	565.41	−40.06	14.105	106.62	−6.98
	13.946	−315.70	23.12			
3	0.000	138.71	33.79	0.000	−19.11	−5.80
	3.008	235.65	1.56	4.098	−65.13	5.43
	6.867	641.58	−57.55	12.375	8.43	−0.51
	12.139	−278.33	18.23			
4	0.000	−10.16	−16.94	0.000	−2.33	1.31
	0.600	−29.87	15.90	8.027	23.67	−1.94
	6.008	50.20	2.57	12.358	−15.64	1.24
	8.445	215.84	−17.04			
	12.796	−17.23	1.17			
5	0.000	111.69	0.46	0.000	22.81	−0.37
	5.317	215.04	−18.98	4.849	35.41	−2.97
	9.704	28.06	0.29	10.381	−5.14	0.94
	15.908	191.38	−9.98	15.984	37.67	−1.74
7	0.000	−80.97	8.29	0.000	10.80	−2.19
	2.387	−118.04	23.83	2.514	22.45	−6.83
	4.731	−2.39	−0.62	5.295	−32.95	3.64
	8.657	−233.87	26.12	8.645	18.16	−2.28
	13.127	114.41	−0.41	13.782	−45.53	2.35
8	0.000	28.23	1.53	0.000	−0.32	−0.33
	5.498	52.83	−2.95	5.268	−6.25	0.80
	7.321	25.35	0.80	7.341	0.85	−0.17
9	0.000	5.11	2.84	0.000	−0.16	0.02
	11.240	135.56	−8.77	13.653	8.12	−0.59
10	0.000	16.31	−0.81	0.000	−2.26	−0.51
	0.950	9.83	6.02			
	5.333	40.80	0.21			
	12.565	−20.36	5.07			
11	0.000	53.51	−33.33	0.000	6.00	−3.91
	3.341	−173.43	34.60	3.552	−25.13	4.86
	9.086	176.24	−3.88	6.988	−3.01	1.69
	11.665	520.02	−33.36	11.125	56.31	−3.64
12	0.000	165.05	−8.96	0.000	11.37	1.11
	7.718	55.26	5.26	3.573	21.02	−1.59
	10.945	345.89	−21.29	7.866	−16.16	3.14
	13.058	477.04	−31.34	9.474	38.96	−2.68
13	0.000	−87.44	43.50	0.000	−8.16	3.83
	4.864	80.35	9.01	2.465	2.88	−0.65
	10.084	346.78	−17.41	5.506	−29.78	5.28
				9.608	35.97	−1.56
14	0.000	7.64	−20.58	0.000	−1.20	−3.59
	3.002	−56.54	0.80	2.749	−17.04	2.17
	6.487	−308.59	39.66	10.722	17.37	−1.04
	9.603	−54.55	13.20			
	12.987	606.03	−37.66			

APPENDIX C. TRACKING MODEL PARAMETERS FITTED TO INDIVIDUAL SUBJECTS

TABLE C1. Transfer function parameters.

Subject	b_{x1}	b_{x2}	b_{x3}	b_{x4}	b_{x5}	b_{x6}	b_{x7}	b_{y1}	b_{y2}	b_{y3}	b_{y4}	b_{y5}	b_{y6}	b_{y7}
Experiment 1 (Movement joystick)														
01S	−0.30	−0.01	3.91	−3.02	1.03	−0.16	−0.16	0.01	−0.24	15.77	−7.80	−0.29	0.69	−0.01
02S	1.08	0.01	4.15	−3.15	0.47	0.47	−0.41	9.46	0.07	5.75	−3.81	0.36	0.59	−0.48
03S	−2.83	−0.02	4.87	−3.16	0.32	−0.15	−0.06	2.55	−0.26	8.05	−6.17	0.05	0.39	−0.27
04S	2.65	−0.03	4.50	−3.37	0.46	0.35	−0.51	−0.01	0.01	0.55	2.68	2.00	−1.34	0.30
06S	0.88	0.00	3.79	−2.47	0.71	−0.01	−0.17	−0.49	−0.10	5.05	−2.48	0.45	0.78	−0.73
07S	−2.17	0.04	4.44	−2.21	0.36	0.36	−0.39	−0.69	−0.15	3.16	−6.41	1.20	−0.33	−0.15
08S	−1.96	0.07	4.47	−2.87	0.16	0.39	−0.37	−0.63	−0.03	4.53	−2.90	0.11	0.58	−0.19
09S	−5.42	0.05	5.15	−2.49	0.26	0.57	−0.57	−0.18	−0.04	2.92	−2.07	1.47	−0.90	0.27
11S	−0.71	−0.02	3.70	−1.85	0.62	0.16	−0.22	0.42	−0.33	8.71	−6.45	0.12	0.21	0.11
12S	−1.16	0.02	3.12	−1.78	1.19	−0.89	0.26	−0.21	−0.01	1.29	2.40	1.48	−0.70	0.08
13S	−2.75	0.00	2.70	−0.48	0.56	0.47	−0.44	1.04	−0.05	4.42	−2.25	0.51	0.77	−0.68
14S	0.32	0.04	0.67	2.01	1.55	−0.94	0.18	0.19	−0.07	4.06	−1.82	0.42	0.65	−0.58
Experiment 1 (Pressure joystick)														
01S	−1.44	0.06	3.02	−2.09	1.13	−0.47	−0.03	−0.23	−0.21	4.02	−4.60	0.22	0.59	−0.11
02S	−0.77	0.00	4.11	−3.65	0.89	−0.49	0.10	7.08	−0.06	3.46	1.02	0.59	0.61	−0.61
03S	−2.69	0.03	4.69	−3.84	0.50	−0.22	−0.04	2.11	−0.15	7.96	−5.33	0.02	0.60	−0.09
04S	−1.11	0.14	4.61	−2.30	0.16	0.35	−0.45	−0.03	0.02	4.37	−4.83	1.11	−0.69	0.16
06S	−1.10	0.07	3.88	−2.58	0.89	−0.50	0.16	−1.43	0.02	9.71	−4.72	−0.44	0.52	0.05
07S	−3.62	0.11	5.10	−3.56	0.15	0.19	−0.34	−1.88	0.03	5.77	−3.61	0.04	0.22	0.23
08S	−0.67	0.02	0.77	1.23	1.86	−1.47	0.47	−1.10	−0.03	5.34	−4.52	0.04	−0.08	0.58
09S	−2.93	−0.03	3.56	−1.72	0.14	0.78	−0.37	−0.16	−0.04	2.21	6.02	0.75	−0.16	0.26
11S	−1.53	−0.01	4.41	−3.16	0.53	−0.24	−0.13	0.26	−0.11	4.50	−4.12	0.59	0.50	−0.63
12S	−2.47	0.04	3.87	−2.58	0.39	0.04	−0.18	−0.74	−0.16	4.34	−6.82	0.16	0.09	0.03
13S	−4.98	0.07	4.68	−4.11	0.82	−0.75	0.29	0.26	0.11	6.29	−2.48	0.25	0.52	−0.41
14S	−0.78	0.01	2.42	−1.96	0.28	0.36	−0.52	0.09	−0.38	2.91	2.71	0.74	0.51	−0.59
Experiment 2 (Movement joystick)														
01a	−1.25	0.03	3.98	−1.48	0.08	0.35	−0.31	−0.30	−0.09	5.40	−2.62	−0.02	0.58	−0.17
01b	−1.37	0.05	4.43	−2.01	0.04	0.38	−0.38	−0.50	−0.02	7.11	−2,97	−0.04	0.42	−0.18
02	0.19	0.02	4.47	−2.54	0.21	0.11	−0.25	0.35	−0.04	4.44	−1.20	0.47	0.38	−0.31
03	−1.04	0.02	4.20	−2.96	0.17	0.24	−0.34	−0.62	−0.03	5.70	−3.43	0.20	0.49	−0.25
04	2.34	0.03	4.54	−3.22	0.13	0.01	−0.26	0.19	0.04	5.33	−2.21	0.34	0.62	−0.43

TABLE C1 (continued)

Subject	b_{x1}	b_{x2}	b_{x3}	b_{x4}	b_{x5}	b_{x6}	b_{x7}	b_{y1}	b_{y2}	b_{y3}	b_{y4}	b_{y5}	b_{y6}	b_{y7}
Experiment 2 (Movement joystick)														
01a	−1.25	0.03	3.98	−1.48	0.08	0.35	−0.31	−0.30	−0.09	5.40	−2.62	−0.02	0.58	−0.17
01b	−1.37	0.05	4.43	−2.01	0.04	0.38	−0.38	−0.50	−0.02	7.11	−2.97	−0.04	0.42	−0.18
02	0.19	0.02	4.47	−2.54	0.21	0.11	−0.25	0.35	−0.04	4.44	−1.20	0.47	0.38	−0.31
03	−1.04	0.02	4.20	−2.96	0.17	0.24	−0.34	−0.62	−0.03	5.70	−3.43	0.20	0.49	−0.25
04	2.34	0.03	4.54	−3.22	0.13	0.01	−0.26	0.19	0.04	5.33	−2.21	0.34	0.62	−0.43
Experiment 2 (Pressure joystick)														
01a	−0.26	0.03	0.78	1.37	1.38	−0.68	0.05	−0.49	0.07	4.26	−2.68	0.24	0.19	−0.40
01b	−1.22	0.13	3.12	−1.41	0.11	0.36	−0.46	−0.39	0.05	4.92	−2.28	0.06	0.49	−0.49
02	−0.60	0.11	3.48	−1.61	0.26	0.34	−0.38	−0.86	0.09	5.55	−3.13	0.16	0.12	−0.16
03	−1.02	0.18	3.44	−1.79	0.23	0.35	−0.46	−0.54	0.05	2.18	1.86	0.69	0.07	−0.73
04	1.99	0.13	3.93	−2.01	0.01	0.44	−0.53	−0.67	0.15	5.97	−4.05	0.49	0.15	−0.39
Experiment 3 (Movement joystick)														
01a	−0.66	0.02	5.64	−1.39	1.58	−0.93	0.16	−1.16	−0.07	6.30	−3.61	−0.01	0.17	0.07
01b	0.20	0.08	3.31	−1.68	0.13	0.32	−0.46	−1.09	−0.07	5.93	−3.81	0.14	−0.05	0.23
02	0.29	0.07	4.71	−2.87	0.30	−0.13	0.01	−0.84	−0.03	5.75	−3.99	0.22	0.45	−0.30
04	0.77	0.04	3.35	−1.35	0.28	0.46	−0.54	−0.29	0.04	4.22	−3.05	1.09	−0.60	0.13
05	2.03	0.08	4.34	−3.17	0.20	0.03	−0.41	0.51	0.00	5.14	−2.57	0.07	0.72	−0.29
Experiment 4 (Movement joystick)														
01a	1.00	0.08	3.12	−1.99	0.28	0.24	−0.37	0.43	−0.01	4.95	−3.89	0.05	0.61	−0.02
01b	1.13	0.10	3.70	−2.57	0.22	0.17	−0.39	0.18	−0.01	1.22	−0.24	0.97	0.22	−0.33
02	1.74	0.05	0.65	1.34	1.84	−1.48	0.55	0.24	0.00	1.98	−0.85	0.88	0.32	−0.38
03	−4.93	0.03	3.14	−1.67	0.22	0.10	−0.34	−1.95	−0.03	6.06	−6.35	0.20	−0.09	0.22
05	−1.70	0.08	3.13	−1.23	0.12	−0.02	−0.60	1.70	0.02	5.51	−2.12	0.25	0.56	−0.42

TABLE C1 (Continued)

Subject	b_{x1}	b_{x2}	b_{x3}	b_{x4}	b_{x5}	b_{x6}	b_{x7}	b_{y1}	b_{y2}	b_{y3}	b_{y4}	b_{y5}	b_{y6}	b_{y7}
Experiment 5 (Movement joystick)														
01G	0.08	0.00	0.28	2.28	1.72	−1.11	0.32	−0.28	0.01	3.92	−1.88	0.43	0.63	−0.46
02G	−0.19	0.03	5.38	−2.18	0.02	0.26	−0.37	0.54	−0.06	3.91	−7.99	0.45	0.58	−0.34
03G	−0.82	0.04	3.64	−1.45	0.36	0.15	−0.36	0.47	−0.12	4.88	−8.80	0.26	0.21	−0.02
04G	2.06	0.04	5.00	−3.60	0.21	0.29	−0.30	1.05	−0.05	4.34	−5.29	0.54	0.57	−0.31
05G	1.83	0.07	3.24	−1.12	0.58	0.23	−0.41	−0.02	0.03	2.84	−3.93	1.04	−0.18	−0.14
06G	1.65	0.01	3.77	−2.07	0.26	0.36	−0.42	−0.06	−0.04	1.78	−2.26	0.94	0.22	−0.35
07G	−0.04	0.00	3.59	−2.48	0.56	−0.16	−0.16	0.24	−0.04	3.75	−5.39	0.77	−0.06	−0.07
08G	0.54	0.09	3.67	−1.86	0.19	0.10	−0.33	0.24	−0.03	6.09	−5.23	0.04	0.56	−0.08
09G	−0.32	−0.01	3.77	−1.82	0.33	0.17	−0.31	1.30	−0.09	8.05	−11.94	0.01	0.48	−0.18
10G	1.11	0.01	5.01	−2.52	0.16	0.39	−0.36	0.95	−0.02	3.00	−5.13	0.52	0.50	−0.34
11G	−0.66	0.06	4.68	−3.21	0.23	0.21	−0.31	−0.25	−0.01	3.45	−5.10	0.66	0.57	−0.35
12G	−1.52	0.06	3.82	−2.63	0.38	0.24	−0.22	0.37	−0.08	4.20	−5.29	0.54	0.57	−0.31

TABLE C2. Remnant parameters

Subject	ϕ_x	σ^2_{x0}	σ^2_{x1}	σ^2_{x2}	σ^2_{xlase}	ϕ_y	σ^2_{y0}	σ^2_{y1}	σ^2_{y2}	σ^2_{ylase}
Experiment 1 (Movement joystick)										
01S	0.83	39.54	0.03	0.28		0.82	8.07	0.01	0.07	
02S	0.77	67.40	0.12	0.05		0.84	7.64	−0.01	0.12	
03S	0.60	26.16	0.14	0.01		0.73	14.14	0.04	0.11	
04S	0.57	36.74	0.14	0.20		0.69	7.47	0.04	0.09	
06S	0.60	33.82	0.16	0.30		0.58	12.86	0.01	0.16	
07S	0.63	30.97	0.15	0.09		0.86	3.69	0.00	0.10	
08S	0.43	21.72	0.11	0.17		0.52	3.29	0.01	0.14	
09S	0.73	25.90	0.08	0.10		0.81	2.38	0.01	0.10	
11S	0.79	38.19	0.05	0.05		0.86	5.10	0.03	0.03	
12S	0.69	13.90	0.09	0.14		0.53	5.18	0.04	0.19	
13S	0.65	28.59	0.10	0.20		0.73	6.95	0.07	0.08	
14S	0.53	13.57	0.10	0.21		0.61	3.32	0.05	0.05	
Experiment 1 (Pressure joystick)										
01S	0.71	34.19	0.01	0.13		0.78	17.78	0.04	0.10	
02S	0.82	22.18	0.07	0.03		0.66	31.68	0.03	0.07	
03S	0.50	32.37	0.14	0.07		0.42	35.55	0.07	0.13	
04S	0.52	24.28	0.09	0.21		0.43	16.92	0.05	0.08	
06S	0.59	14.48	0.21	0.17		0.51	18.89	0.03	0.14	
07S	0.50	12.17	0.11	0.12		0.65	4.10	0.02	0.11	
08S	0.41	21.93	0.08	0.11		0.29	9.39	0.02	0.06	
09S	0.70	7.14	0.22	0.07		0.77	6.17	0.03	0.16	
11S	0.72	20.84	0.09	0.17		0.70	13.54	0.06	0.08	
12S	0.63	13.63	0.12	0.11		0.41	12.67	0.04	0.16	
13S	0.76	40.41	0.04	0.10		0.68	29.58	0.05	0.09	
14S	0.79	27.69	0.07	0.12		0.68	12.05	0.02	0.11	
Experiment 2 (Movement joystick)										
01a	0.59	13.75	0.15	0.28		0.56	2.91	0.02	0.12	
01b	0.51	11.51	0.13	0.16		0.57	2.77	0.02	0.09	
02	0.57	−0.51	0.42	0.23		0.65	1.11	0.04	0.28	
03	0.67	8.87	0.08	0.10		0.68	2.30	0.02	0.08	
04	0.76	10.26	0.07	0.04		0.78	2.53	0.01	0.08	

TABLE C2 (Continued)

Subject	ϕ_x	σ^2_{x0}	σ^2_{x1}	σ^2_{x2}	σ^2_{xlase}	ϕ_y	σ^2_{y0}	σ^2_{y1}	σ^2_{y2}	σ^2_{ylase}
Experiment 3 (Movement joystick)										
01a	0.64	8.31	0.09	0.12		0.65	2.57	0.02	0.13	
01b	0.63	8.26	0.11	0.06		0.66	2.45	0.01	0.12	
02	0.60	13.11	0.09	0.18		0.72	1.87	0.01	0.08	
04	0.56	11.65	0.12	0.22		0.74	2.45	0.00	0.09	
05	0.58	10.96	0.13	0.09		0.86	1.78	0.01	0.04	
Experiment 4 (Movement joystick)										
01a	0.57	18.08	0.15	0.41	44.29	0.67	2.08	0.02	0.24	12.09
01b	0.59	19.96	0.14	0.16	54.99	0.69	1.93	0.01	0.24	12.19
02	0.58	24.94	0.13	0.30	58.78	0.59	2.89	0.00	0.23	6.39
03	0.53	17.71	0.23	0.24	111.16	0.56	2.43	0.01	0.33	39.05
05	0.52	27.41	0.19	0.25	79.98	0.64	4.19	0.00	0.15	0.10
Experiment 5 (Movement joystick)										
01G	0.53	16.64	0.18	0.44	108.29	0.61	2.91	0.02	0.23	33.18
02G	0.61	24.27	0.14	0.11	157.58	0.72	4.13	0.01	0.15	34.09
03G	0.56	20.40	0.21	0.20	38.11	0.65	4.61	0.02	0.19	11.75
04G	0.61	28.48	0.15	0.14	94.08	0.70	5.92	0.01	0.16	28.81
05G	0.71	18.37	0.13	0.28	282.08	0.64	4.30	0.01	0.17	93.71
06G	0.70	23.47	0.13	0.14	50.98	0.70	4.54	0.02	0.11	11.02
07G	0.64	12.02	0.14	0.12	113.73	0.65	2.80	0.00	0.23	25.38
08G	0.43	39.77	0.17	0.45	204.72	0.63	4.46	0.02	0.05	32.27
09G	0.55	48.34	0.16	0.49	429.58	0.70	6.29	0.03	0.13	49.29
10G	0.78	17.83	0.07	0.13	22.85	0.76	3.52	0.01	0.10	8.99
11G	0.74	25.79	0.07	0.21	206.00	0.83	1.58	0.00	0.20	85.01
12G	0.61	1.54	0.47	−0.08	126.63	0.77	7.46	0.00	0.12	23.87

N.B. Target position inputs in these experiments were specified in terms of the angle subtended at the eye, and so the units are 0.1 milliradians in simulated real space. Tracking outputs were recorded in terms of the minimum digital element which could be resolved by the computer: 0.05 milliradians in simulated real space. All the constants in Tables C1 and C2 are quoted in a form appropriate to the scaling.

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