Cronfa - Swansea University Open Access Repository

Paper:

Valsamidis, A. & Reeve, D. (2017). Modelling shoreline evolution in the vicinity of a groyne and a river. *Continental Shelf Research, 132*, 49-57.
http://dx.doi.org/10.1016/j.csr.2016.11.010

Continental Shelf Research 132 (2017) 49–57

Contents lists available at ScienceDirect

Continental Shelf Research

journal homepage: www.elsevier.com/locate/csr

Research papers

Modelling shoreline evolution in the vicinity of a groyne and a river

Antonios Valsamidis, Dominic E. Reeve*

College of Engineering, Swansea University, SA2 8PP, UK

ARTICLE INFO

Keywords:
Coastal engineering
Coastal morphology
Mathematical models
Coastal processes
Analytical techniques
Coastal structures
Shoreline evolution
Time-varying hydrodynamic conditions

ABSTRACT

Analytical solutions to the equations governing shoreline evolution are well-known and have value both as pedagogical tools and for conceptual design. Nevertheless, solutions have been restricted to a fairly narrow class of conditions with limited applicability to real-life situations. We present a new analytical solution for a widely encountered situation where a groyne is constructed close to a river to control sediment movement. The solution, which employs Laplace transforms, has the advantage that a solution for time-varying conditions may be constructed from the solution for constant conditions by means of the Heaviside procedure. Solutions are presented for various combinations of wave conditions and sediment supply/removal by the river. An innovation introduced in this work is the capability to provide an analytical assessment of the accretion or erosion caused near the groyne due to its proximity to the river which may act either as a source or a sink of sediment material.

1. Introduction

1.1. Background

The interaction between nearshore wave climate and a source/sink of sediment discharge forms a rather complicated coastal system; meaning that the assessment of sediment transport along the beach and the consequent morphological changes for such a case is a challenging task. The sediment exchanges between tidal inlets and neighbouring beaches is a continuing area of scientific interest (e.g. van Lancker et al., 2004; Brown and Davies, 2009; Bever et al., 2011), as well as being of significant practical importance. In this study, an additional factor was added to this combination; that of an obstacle causing an abrupt blockage of longshore sediment transport at some distance from the source or sink. This obstacle could be a headland or a coastal construction such as a groyne or jetty. Consequently, an enhanced accretive or erosive trend is expected to occur in the vicinity of the obstacle due to the presence of a source or sink, respectively. An example which demonstrates the potentially severe consequences of this phenomenon is the siltation of ports caused by the presence of a nearby river source providing a sediment discharge. Clearly, this phenomenon may have serious navigational and economic impacts. Historical examples of coastal constructions which faced the adverse impacts of accretion due to the presence of a river-delta in their vicinity include the ancient port of Ephesus in Asia Minor and the port of Ostia near ancient Rome, both of which gradually silted up and became unusable (De Graauw, 2011). In modern times, the ports of Montevideo and Buenos Aires, located on the left and right side, respectively, of the estuary of Rio de la Plata which discharges in the Atlantic Ocean, need to be periodically dredged due to the sediment material coming from this estuary, (Sanchez and Wilmsmeier, 2007). Similarly, the port of Imeros in Thrace, Greece, suffers from sediment material which comes from the delta of the Lucius River, and is deposited near its entrance, (Delimani and Xeidakis, 2005). As a result, the approach of ships for anchorage in the port of Imeros is severely hindered (Fig. 1).

The description of the coastal phenomenon mentioned above, and consequently the mitigation of adverse impacts, requires the assessment of the accretion occurring near an artificial or natural obstacle, considering as primary cause factors the distance of the obstacle from the river-mouth and the sediment flow rate at the river-mouth. Therefore, the distance of a coastal structure from a river delta might be decided during its design phase so as to avoid having to take counter-measures after its construction. The opposite problem, that of erosion caused near a coastal structure or a natural obstacle due to the presence of a sink of sediment material in its vicinity might be expected near flood-dominated estuaries or estuaries that have been subjected to dredging operations. For instance, the dredging of the Avilès estuary in Spain was judged to have caused erosion problems in the neighbouring Salinas-El Espartal beach, and in addition, serious damage to the seafront promenade in the same area (Flor-Blanco et al., 2013).

The impact of an artificial blockage of longshore sediment transport on shoreline evolution has been investigated by Kraus and Harikai (1983) via numerical modelling. Frihy et al. (1991) identified patterns

* Corresponding author.
E-mail addresses: 667846@swansea.ac.uk (A. Valsamidis), D.E.Reeve@swansea.ac.uk (D.E. Reeve).

http://dx.doi.org/10.1016/j.csr.2016.11.010
Received 26 July 2016; Received in revised form 21 October 2016; Accepted 25 November 2016
Available online 30 November 2016
0278-4343/

Nomenclature

A	a parameter proportional to river-mouth's width, governing the width of the river-delta;
B	berm height;
d_{50}	median grain size;
h	water depth;
D_C	depth of closure;
D_B	berm height;
D	the sum of D_C and D_B
H	Heaviside function;
H_b	wave height at breaking position;
$H_{s,b}$	significant wave height at breaking position;
L	groyne's length
m_b	beach slope;
Q	longshore transport rate;
Q_k	longshore transport rate according to Kamphuis' formula;
Q_o	amplitude of longshore transport rate;
q	sediment transport rate per unit of beach from a source or a sink;
q_R	sediment discharge rate from the river or opposite flow rate in case of a sink;
s	the Laplace transform variable;
t	time;
T_p	peak wave period;
x	longshore distance relatively to a reference point;
x_o	distance of the river-mouth from groyne's position;
y	shoreline position;
a_b	the wave angle at breaking;
a_o	angle of breaking wave crests relatively to an axis set parallel to the trend of shoreline;
ε	diffusion coefficient;

of nearshore sediment transport along the Nile delta in Egypt, taking into account a complicated system of existing groynes and jetties in this area. However, the current work focuses on a description of the shoreline evolution through analytical means. Specifically, to predict the shoreline evolution which occurs in the vicinity of a coastal construction (e.g. a groyne) and a source or sink of sediment discharge considering an irregular sequence of wave events and sediment fluxes at the river mouth.

It is hypothesised that, in the case of net sediment supply from the river the time for a groyne to fill, and hence for sediment bypassing to commence, will be a function of the wave driven longshore transport rate, the length of the groyne, the rate of sediment supply and the distance between the groyne and the river. Consequently, one aim of the new analytical description is to be able to assess the time required for sediment bypassing to commence as this is a very significant parameter at the design stage of a coastal scheme. Specifically, once full, sand will by-pass the tip of the groyne so that sediment transport from its updrift to its downdrift side will resume.

1.2. Beach model background

A well-known but simplified model for predicting shoreline evolution is the one-line model. The basic assumption of this model is that the shape of any beach cross-shore profile is in equilibrium, in other words all the seabed contours are parallel. The model is formulated by coupling an equation defining the longshore transport and the equation for the continuity of sand (Eq. (1)):

$$\frac{\partial y}{\partial t} + \frac{1}{(D_C + D_B)}\frac{\partial Q}{\partial x}=0 \tag{1}$$

where y is the cross-shore position of the shoreline, t is time, x is the longshore distance, D_C is the depth of closure, D_B is the berm height and Q is the longshore transport rate (see e.g. Kamphuis, 2000; Dean and Dalrymple, 2002; Reeve et al., 2004; Bayram et al., 2007).

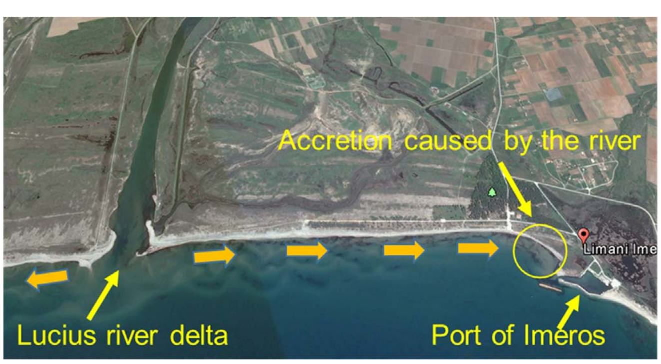

Fig. 1. Sediment discharge from Lucius river-mouth causes a littoral drift (desert sand arrows) towards the port of Imeros, 2.5 km away. Consequently, the sediment material is accumulated near the entrance of the port, on the right-hand side of the image.

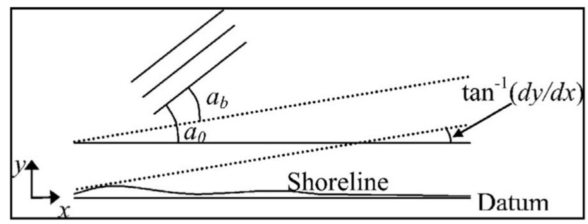

Fig. 2. Geometric characteristics of waves near the breaking point (Reeve, 2006).

The longshore transport equations in general use are empirical, being based on experimental studies. Some of the most well-known expressions are: the CERC equation (CERC, 1984); Kamphuis' formula (Kamphuis, 1991) and Bayram's equation (Bayram et al., 2007). In this study, Kamphuis' longshore transport equation has been used and this may be written as:

$$Q_k=7.\ 3H_{sb}^2T_p^{1.5}m_b^{0.75}d_{50}^{-0.25}sin^{0.6}(2a_b) \tag{2}$$

where m_b: beach slope, T_p: peak wave period, d_{50}: median grain size, $H_{s,b}$ is the significant wave height at breaking, a_b is the wave angle at breaking.

In particular, the wave angle at the breaking point is given as follows:

$$a_b = a_o - \arctan(\frac{\partial y}{\partial x}) \tag{3}$$

where α_o: angle of breaking wave crests relatively to an axis set parallel to the trend of shoreline, $\frac{\partial y}{\partial x}$ the local shoreline orientation (Fig. 2).

The one-line model can be solved either analytically or numerically. Analytical solutions are found on the premise of small angles of wave approach and smooth plan beach profile. In addition, the wave characteristics (height, period, direction of propagation) are considered uniform and constant. Under these assumptions, the combination of Eqs. (1)–(3) leads to a single equation for the evolution of the position of the shoreline that takes the form of a diffusion equation:

$$\frac{\partial y}{\partial t} = \varepsilon\frac{\partial^2 y}{\partial x^2} \tag{4}$$

where $\varepsilon=2Q_o/D$, Q_o is the amplitude of longshore transport rate (e.g. Larson et al., 1987) and is given by the following expression: $Q_o=2.\ 7H_{sb}^2T_p^{1.5}m_b^{0.75}d_{50}^{-0.25}$; $D=D_C+D_B$.

Through appropriate choices of boundary and initial conditions for Eq. (4) different shoreline situations can be modelled. The analytical solutions to Eq. (4) provide insight to the likely shoreline evolution in each case. The constraints of analytical models (constant and uniform wave conditions, smooth shoreline shape, and small breaking wave angle) limit their general applicability. That being said, analytical solutions have proven to be successful for an increasingly complex range of cases (e.g. Wind, 1990, Kamphuis, 1993; Larson et al., 1997; Reeve, 2006) and have demonstrated certain advantages over numerical modelling. Zacharioudaki and Reeve (2008) listed many of them, for example, the ability of analytical models to provide insight into the independent contribution of particular physical processes on the shoreline evolution. Moreover, the quick and accurate evaluation of analytical solutions is feasible without cumulative numerical errors related to repeated time stepping or numerical instability. Finally, analytical modelling can be used for independent validation of numerical codes.

The one-line modelling concept originated from Pelnard-Considère (1956) for the case of a single groyne, a constant wave forcing condition, and an initially straight shoreline, parallel to the axis of longshore distance *x*. Mathematically these conditions are expressed as follows:

Initial condition (IC): for $t = 0,\ y(x, t) = y(x, 0) = 0$ (5)

Boundary condition (BC) at $x = 0$(the location of the groyne)

$$: -\frac{\partial(0, t)}{\partial x} = \tan(a_b), \tag{6}$$

with *x* ranging in the domain: $x \in (0, \infty)$

By combining Eqs. (4)–(6) the solution to this problem is derived via Laplace transforms as:

$$y(x, t) = \tan(a_b)\left(2\frac{\sqrt{\varepsilon t}}{\sqrt{\pi}}\exp\left(-\left(\frac{x}{2\sqrt{\varepsilon t}}\right)^2\right) - \mathrm{xerfc}\left(\frac{x}{2\sqrt{\varepsilon t}}\right)\right) \tag{7}$$

Additional analytical solutions have been derived by other researchers in a similar manner. For instance, Grijm (1961) attempted to develop a littoral sediment transport equation capable of incorporating as input-data wave directions almost parallel to the orientation of the coast; while Le Mèhautè and Soldate (1977) developed Pelnard-Considere's analytical solution for several different cases. Larson et al. (1987, 1997) presented analytical solutions for a wide range of different problems based on Pelnard-Considère's theory while Wind (1990) used solutions to describe shoreline evolution considering the impact of coastal constructions.

One significant drawback of analytical models had been the constancy of wave conditions, a constraint imposed by the solution techniques. Larson et al. (1997) were partially successful in relaxing this restriction to a prescribed time variation in their solution for the cases of a single groyne and groyne compartment considering sinusoidally varying wave direction. Dean and Dalrymple (2002) were able to treat the effects of a sequence of arbitrary wave conditions by considering a specific form of spatial variation in beach plan (an individual Fourier component). Subsequently, Reeve (2006) provided a full analytical solution, via Fourier transform techniques, for a known arbitrary sequence of wave conditions, arbitrary source distribution and arbitrary initial shoreline position near a groyne. In general, the evaluation of the solution required numerical integration over the arbitrary sequence of driving conditions. The evaluation of this kind of solution has to be performed through numerical integration for all but the simplest cases, and is considered to be “semi-analytical”. Consequently, Zacharioudaki and Reeve (2008) used a similar technique to extended the range of analytical solutions to the cases of groyne compartment and a managed shoreline where the shoreline is actively controlled to within a certain tolerance and can be treated as a known function of time. Walton and Dean (2011) illustrated how a Heaviside-type technique could be used to emulate time varying wave conditions without recourse to numerical integration for the case of beach accretion near a single groyne. While effective for a small number of consecutive wave conditions the approach becomes unwieldy when long sequences of wave conditions need to be considered (Valsamidis et al., 2013).

The assumption of uniform wave conditions still remains. This is a primary reason for developing computational solutions, which can also be adapted to include additional processes such as wave refraction and diffraction, and beach slope variation in an integrated package with a graphical user interface for ease of use (e.g. Gravens et al., 1991; Thomas and Frey, 2013).

Here, for the reasons alluded to above, we continue the development of analytical solutions that account for time-varying input data, and present a novel analytical solution to the problem of shoreline evolution in the vicinity of a groyne and a source of sediment discharge, considering time-varying wave action and sediment exchange between the beach and river mouth.

This paper is organised as follows. In Section 2 we describe the derivation of the new analytical solutions. In Section 3, evaluation of the analytical solution and results are presented. Section 4 contains a discussion and conclusions. Moreover, the hypothesis suggested in the end of Section 1.1. regarding the parameters involved in the assessment of time needed for a groyne to fill and subsequently sediment bypassing to commence is validated in Section 4.

2. Methodology

2.1. Derivation of the analytical solution

Consider a situation such as shown in Fig. 3, consisting of an undisturbed and straight initial shoreline, a single impermeable groyne of infinite length which constitutes its left boundary, a constant incident wave which propagates towards the shoreline and a source of sediment discharge. (Sediment extraction can be modelled simply by considering it as negative discharge).

The following extended version of Eq. (4) describes the consequent shoreline evolution, (e.g: Hanson, 1987):

$$\frac{\partial y}{\partial t} = \varepsilon\frac{\partial^2 y}{\partial x^2} + q \tag{8}$$

where *q(x,y,t)* is the sediment transport rate per unit area from a source or a sink and has units of m/s. We chose to describe the spatial distribution of *q* with a Gaussian function to mirror the approximate distribution of transport rate that might be seen at a river delta formation.

Specifically:

$$q = Be^{-[\frac{x - xo}{A}]^2} \tag{9}$$

$$B = \frac{q_R}{DA} \tag{10}$$

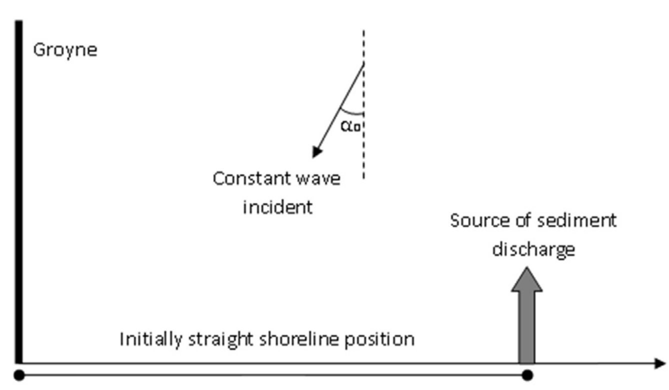

Fig. 3. Demonstration of the simplifications made for the derivation of the analytical solution.

q_R: sediment discharge rate from the river (m^3/h);
x_o: distance between the river-mouth and the groyne (m);

$D = D_C + D_B$

D_C: depth of closure (m);

D_B: berm height (m);
A: a parameter proportional to the width of the river-mouth (m).

Accordingly, Eq. (8) takes the form:

$$\frac{\partial y}{\partial t}=\varepsilon\frac{\partial^2 y}{\partial x^2}+Be^{-[\frac{x-xo}{A}]^2}\Leftrightarrow\varepsilon\frac{\partial^2 y}{\partial x^2}+Be^{-[\frac{x-xo}{A}]^2}-\frac{\partial y}{\partial t}=0 \tag{11}$$

The solution to Eq. (11) was performed via Laplace transforms, using the following initial and boundary conditions:

- Initial Condition (IC): $y(x,0)=0$; corresponding to initially undisturbed beach;
- Boundary Condition (BC1) at $x=0$: $-\frac{\partial(0,t)}{\partial x}=\tan(a_b)$; corresponding to zero transport, or equivalently an impermeable groyne of infinite length;
- Boundary Condition (BC2) at $x=+\infty$: $y(+\infty, t)=0$; implying negligible disturbance of the beach far from the groyne.

The Laplace transform of Eq. (11), combined with the initial and boundary conditions yields Eq. (12):

$$\bar{y}(s)=\frac{\tan(a_b)}{s\lambda}e^{-\lambda x}+2x_0\left[\frac{Be^{-\left(\frac{xo}{A}\right)^2}}{s^2A^2\lambda}\right]e^{-\lambda x}+\frac{B}{s^2}e^{-[\frac{x-xo}{A}]^2} \tag{12}$$

where $\bar{y}$ denotes the Laplace transformed variable of y, where: $\lambda=\sqrt{\frac{s}{\varepsilon}}$; s is the Laplace transform variable.

The inverse Laplace transform of Eq. (12) yields the solution to Eq. (11):

$$y(x,t)=\tan(a_b)\left[2\left(\frac{\varepsilon t}{\pi}\right)^{\frac{1}{2}}e^{-\frac{x^2}{4\varepsilon t}}-x\mathrm{erfc}\left(\frac{x}{2\sqrt{\varepsilon t}}\right)\right]+2x_o\left(\sqrt{\varepsilon}\frac{B}{A^2}e^{-(\frac{x_o}{A})^2}\right)\left((4t)^{\frac{3}{2}}i^3\mathrm{erfc}(\frac{x}{2\sqrt{\varepsilon t}})\right)+Be^{-\left[\frac{(x-xo)}{A}\right]^2}t \tag{13}$$

where i^n is an integration operator of order n.

A detailed description of the mathematical derivation of the solution in Eq. (13) can be found in Appendix A. Now, the solution of Eq. (13) consists of 3 terms. The first one:

$$y_1(x,t)=\tan(a_b)\left[2\left(\frac{\varepsilon t}{\pi}\right)^{1/2}e^{-\frac{x^2}{4\varepsilon t}}-x\mathrm{erfc}\left(\frac{x}{2\sqrt{\varepsilon t}}\right)\right] \tag{14}$$

is identical to the analytical solution which describes the shoreline evolution in the vicinity of a groyne due to the impact of constant wave forcing (Pelnard-Considere, 1956). The second term:

$$y_2(x,t)=2x_o\left(\sqrt{\varepsilon}\frac{B}{A^2}e^{-(\frac{x_o}{A})^2}\right)\left((4t)^{\frac{3}{2}}i^3\mathrm{erfc}(\frac{x}{2\sqrt{\varepsilon t}})\right) \tag{15}$$

encapsulates the trend for accretion near the groyne caused by the blockage of sediment coming from the source of sediment discharge. The third term describes the growth of the river-delta and reflects the shape of the source:

$$y_3(x,t)=Be^{-\left[\frac{(x-xo)}{A}\right]^2}t \tag{16}$$

Terms two and three can also describe the erosive trend appearing near a groyne due to the presence of a sink of sediment material in the vicinity of the groyne, corresponding to the case where the constant B is negative.

2.2. Discretization in time of the analytical solution

So far the analytical solution describes the shoreline response to constant wave forcing and sediment discharge. To modify this solution to enable us to describe the shoreline evolution to time varying wave conditions and discharges requires an additional step. For this purpose, the Heaviside technique may be applied to incorporate time dependence, as described by Walton and Dean (2011). In this procedure a continuously time varying condition, (for example the incoming waves) is approximated as a piece-wise constant condition. The solution can be constructed from the accumulation of the constant conditions acting over finite time segments. For example, the solution for constant conditions has been found already in Eq. (13). The solution for the piece-wise case is found as follows. For the initial time segment the solution is given by Eq. (13). For the next segment we subtract the contribution due to the conditions pertaining to the first segment and add the contribution from the conditions in the second segment. For the third segment, we subtract the contributions pertaining to the conditions in the first and second segments and add the contribution from the conditions in the third segment, and so on. Considering just the first term in Eq. (13), the solution in the third time segment can thus be written as:

$$\begin{aligned}y^{(1)}{}_3(x,t)&=\tan(\alpha_1)f_1(\varepsilon_{t0,t},t-t_0)H(t-t_0)-\tan(\alpha_1)f_1(\varepsilon_{t1,t},t-t_1)H(t-t_1)\\&++\tan(\alpha_2)f_1(\varepsilon_{t1,t},t-t_1)H(t-t_1)-\tan(\alpha_2)f_1(\varepsilon_{t2,t},t-t_2)\\&H(t-t_2)++\tan(\alpha_3)f_1(\varepsilon_{t2,t},t-t_2)H(t-t_2)\end{aligned} \tag{17}$$

where α_1, α_2, α_3 are breaking wave angles corresponding to successive wave conditions during the corresponding time steps, H is the Heaviside function,

$$f_1(\varepsilon_{ti,t},t-t_i)=\left(2\frac{\sqrt{\varepsilon_{ti,t}(t-t_i)}}{\sqrt{\pi}}\exp\left(-\left(\frac{x}{2\sqrt{\varepsilon_{ti,t}(t-t_i)}}\right)^2\right)-x\mathrm{erfc}\left(\frac{x}{2\sqrt{\varepsilon_{ti,t}}(t-t_i)}\right)\right), \tag{18}$$

$\varepsilon_{ti,t}$ is the average of $\varepsilon(t)$ over the interval t_{i-1} to t_i and is given by:

$$\varepsilon_{ti,t}=\frac{\int_{ti}^{t}\varepsilon(t)dt}{\int_{ti}^{t}dt},\text{simplified to:}\varepsilon_{tj,tk}=\frac{\sum_{i=j}^{k}\varepsilon_i\Delta t}{t_k-t_j} \tag{19}$$

in case solutions are desired at the end points of equally spaced time intervals.

In a similar fashion, Eqs. (15) and (16) may be used to develop expressions corresponding to the 2nd and 3rd term of Eq. (13) respectively. Eq. (17) constitutes a solution to shoreline evolution near a groyne for time varying conditions through the application of the Heaviside technique. Indeed, Eq. (17), which is Walton & Dean's solution to the case of an isolated groyne, gives the first part of our extended solution. To get the full time varying solution we must apply the Heaviside technique to Eqs. (15) and (16) too. However, the time-varying parameter which is involved in these terms is not the breaking wave angle a_b but the river sediment discharge rate q_R. Thus the conjunction of Eqs. (10) and (15) yields:

$$y_2(x,t)=2x_o\left(\sqrt{\varepsilon}\frac{q_R}{DA^3}e^{-\left(\frac{x_o}{A}\right)^2}\right)\left((4t)^{\frac{3}{2}}i^3\mathrm{erfc}\left(\frac{x}{2\sqrt{\varepsilon t}}\right)\right) \tag{20}$$

In a manner similar to Eq. (18) we define f_2 as:

$$f_2(\varepsilon_{ti,t}, t-t_i) = \left(\sqrt{\varepsilon_{ti,t}}\frac{2x_o}{DA^3}e^{-\left(\frac{x_o}{A}\right)^2}\right)\left((4(t-t_i))^{\frac{3}{2}}i^3\mathrm{erfc}\left(\frac{x}{2\sqrt{\varepsilon_{ti,t}(t-t_i)}}\right)\right) \tag{21}$$

Thus, the piece-wise form of Eq. (15) which enables time-varying conditions to be taken into account is as follows (for 3 consecutive time-steps):

$$\begin{aligned} y^{(2)}{}_3(x,t) &= q_{R1}f_2(\varepsilon_{t0,t,t-t0})H(t-t_0) - q_{R1}f_2(\varepsilon_{t1,t,t-t1})H(t-t_1) + \\ &+ q_{R2}f_2(\varepsilon_{t1,t,t-t1})H(t-t_1) - q_{R2}f_2(\varepsilon_{t2,t,t-t2})H(t-t_2) + \\ &+ q_{R3}f_2(\varepsilon_{t2,t,t-t2})H(t-t_2) \end{aligned} \tag{22}$$

In the same way, the corresponding piece-wise form of Eq. (16) is:

$$\begin{aligned} y^{(3)}{}_3(x,t) &= q_{R1}f_3(\varepsilon_{t0,t,t-t0})H(t-t_0) - q_{R1}f_3(\varepsilon_{t1,t,t-t1})H(t-t_1) + \\ &+ q_{R2}f_3(\varepsilon_{t1,t,t-t1})H(t-t_1) - q_{R2}f_3(\varepsilon_{t2,t,t-t2})H(t-t_2) + \\ &+ q_{R3}f_3(\varepsilon_{t2,t,t-t2})H(t-t_2) \end{aligned} \tag{23}$$

where:

$$f_3(\varepsilon_{ti,t}, t-t_i) = \frac{1}{DA}e^{-\left[\frac{(x-xo)}{A}\right]^2}(t-t_i) \tag{24}$$

The sum of Eqs. (17), (22) and (23) provides a piece-wise analytical solution to the assessment of shoreline evolution in the vicinity of a groyne and a river, for a sequence of arbitrary wave conditions, and time-varying sediment discharge from a river.

3. Evaluation of the analytical solution

Without loss of generality, we consider a straight, north-facing beach whose normal is 0°N, with a shore normal groyne placed some distance to the west of a river mouth. A random sequence of wave events at the breaking point has been created for the evaluation of the analytical solution. This wave time-series consists of hypothetical weekly average conditions over a span of 10 years corresponding to 521 sequential steps. Specifically, a random sequence of wave heights (H_b) has been created, varying between: *0 m* (calm sea) and *1.30 m*, as well as a random sequence of periods (T) varying between *1.0 s* and *12.0 s*, and finally, a random sequence of directions (a_b) varying between *−0.13 rad* and *0.19 rad* (creating this way a predominant littoral drift towards the groyne for positive values of a_b). It should be noted that the conditions so created do not reflect any dependence structure between wave heights, periods or directions. In a similar fashion a random sequence of sediment discharges has been created. Regarding the new random sequences which were created, the average value of $H_b(t)$ is equal to *0.52 m* and its standard deviation is equal to *0.22 m* (Fig. 4 – upper panel). The corresponding average value and standard deviation for the wave direction a_b is *0.04 rad* and *0.05 rad*, respectively (Fig. 4 – middle panel). Similarly, the average value of $T(t)$ is equal to *5.93 s* and the associated standard deviation is equal to *2.02 s* (Fig. 4 – bottom panel). The predominant wave direction will cause a littoral drift towards the groyne (Fig. 5). Finally, the sediment discharge rate time-series $q_R(t)$ is characterized by an average value equal to $3.00\ m^3/h$ and a standard deviation equal to 3.03 m^3/h (Fig. 6).

The sediment discharge rate (q_R) is shown in Fig. 6.

The source of sediment discharge (river-mouth) was placed 1200 m away from the groyne (at x_o=*1200 m*) and the parameter A which governs the width of the river-delta was taken equal to *650 m*. Finally, the depth of closure and the berm height were set equal to: D_C=*7 m*; and D_B=*1 m*, respectively.

Having specified the values of all the parameters involved, the analytical solution may be evaluated. The results are shown in Fig. 7.

When the solution is evaluated the groyne is positioned at x=0 and the shoreline distance towards the river is considered as positive. Fig. 7a–c show the contributions of the first, second and third terms in Eq. (13) and Fig. 7d shows the combined solution due to the summation of the three terms. As expected, the predominant wave direction towards the groyne (Fig. 5) causes accretion in the vicinity of the groyne (Fig. 7a; and d). The river, acting predominantly as a source of sediment, contributes to a localised nourishment of the beach which spreads both updrift and downdrift over time. The downdrift transport is intercepted by the groyne which causes an enhancement of the accretion near the groyne.

The same analytical solution can be used for the case that the river acts mainly as a sink instead of a source of sediment discharge, in which case the sediment discharge rate q_R will be negative. To test this, the negative of the discharge time series was used with the same wave conditions as before. The corresponding shoreline evolution is shown in Fig. 8.

Fig. 8a–c show the contributions of the first, second and third terms in Eq. (13) and Fig. 8d shows the combined solution due to the summation of the three terms. In Fig. 8a the illustrated shoreline evolution is identical to that depicted in Fig. 7a since the same wave conditions were applied. In Fig. 8b is notable the erosive trend near the groyne due to the presence of the sink of sediment discharge *1.2 km* away. This is due to the drift towards the sink and the blockage of sediment transport at the groyne which could otherwise replace the lost sediment material. Fig. 8d illustrates that in the vicinity of the groyne, in this particular case, the wave-induced alongshore drift is sufficient to overcome the erosional tendency of the sink.

The analytical solution which corresponds to the accretive or erosive trend near the groyne due to the sediment material coming from the river-mouth or moving towards the river-mouth (Eq. (15)) respectively, constitutes an innovation since to date such an analytical approach has not been attempted for the description of this phenomenon. The solutions presented here show how the combination of a source/sink and a groyne can trigger a wavelike response in the beach planshape through simple linear superposition of different contributing factors.

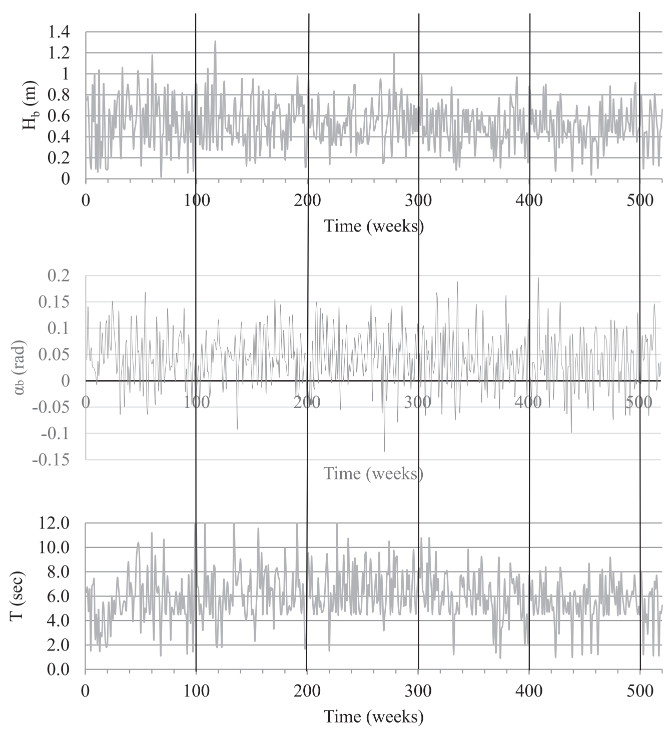

Fig. 4. Upper panel: Wave height (H_b) time-series; average value of H_b is equal to 0.53 m; standard deviation of H_b is equal to 0.31 m. Middle panel: Breaking wave angle (a_b) time-series; average value of a_b is equal to 0.041°; standard deviation of a_b is equal to 0.051°. Lower panel: Wave period (T) time-series; average value of T is equal to 5.93 s; standard deviation of T is equal to 2.02 s.

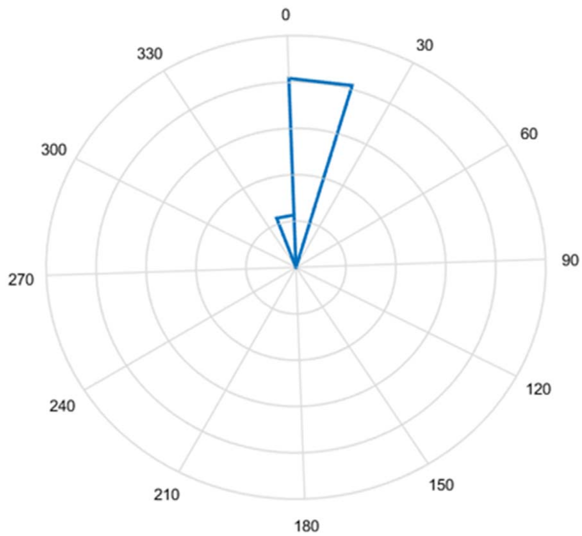

Fig. 5. Rose diagram showing the time-varying wave direction. The predominant wave direction has deliberately been chosen to cause littoral drift towards the groyne. The average wave angle α_b is equal to 0.04 rad or 1.2°, and the standard deviation of α_b is equal to 0.05 rad or 1.4°.

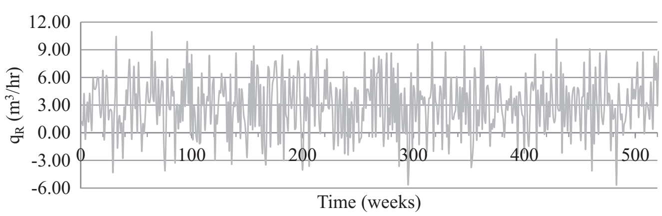

Fig. 6. Sediment discharge rate (q_R) time-series; average value of q_R is equal to 3.00 m^3/h; standard deviation of q_R is equal to 3.03 m^3/h.

4. Discussion and conclusions

A new analytical solution has been developed and presented in this paper which provides a mathematical description of the shoreline evolution in the vicinity of an estuary which acts either as a source or sink of sediment discharge, and an obstacle such as a groyne, placed near the estuary, blocking the littoral drift. The shoreline evolution attributed solely to a source of sediment discharge can be assessed by the summation of 2 independent terms corresponding to the influence of the groyne on longshore distribution of sediment from the river and the growth of the river delta. The impact of the wave action on shoreline evolution is represented in a separate term familiar from earlier studies. Since Eqs. (15) and (16) do not incorporate the wave angle α_b, the shoreline evolution solely due to a source or sink of sediment material near a groyne is independent of the wave direction and consequently the direction of the littoral drift.

A key element of the new solution is its capability to incorporate arbitrarily time varying input conditions of both the local wave climate and the sediment flux at the river-mouth. Consequently, the new analytical solution can be used as a means to validate computational models which simulate such cases. It can be evaluated very quickly since no numerical integrations or loop procedures take place.

Next, the sensitivity of the new analytical solution to the choice of temporal resolution was tested. Since field data were not available, the shoreline plan shape obtained using the shortest temporal step, (one week), was used as a reference. Next, the analytical solution was evaluated for the following temporal steps: 2 weeks; 4 weeks; and 8 weeks. Finally, the resulting shoreline curves were compared with the reference solution. The differences between the reference solution and the other solutions were measured using the root mean square error, (RMSE), and are presented in Table 1:

The comparison shows that as the temporal step decreases the resulting shoreline curves converge to the one corresponding to temporal step equal to 1 week, indicating computational convergence.

In passing it may be noted that, in common with earlier analytical solutions to the one-line model, wave conditions are considered to be

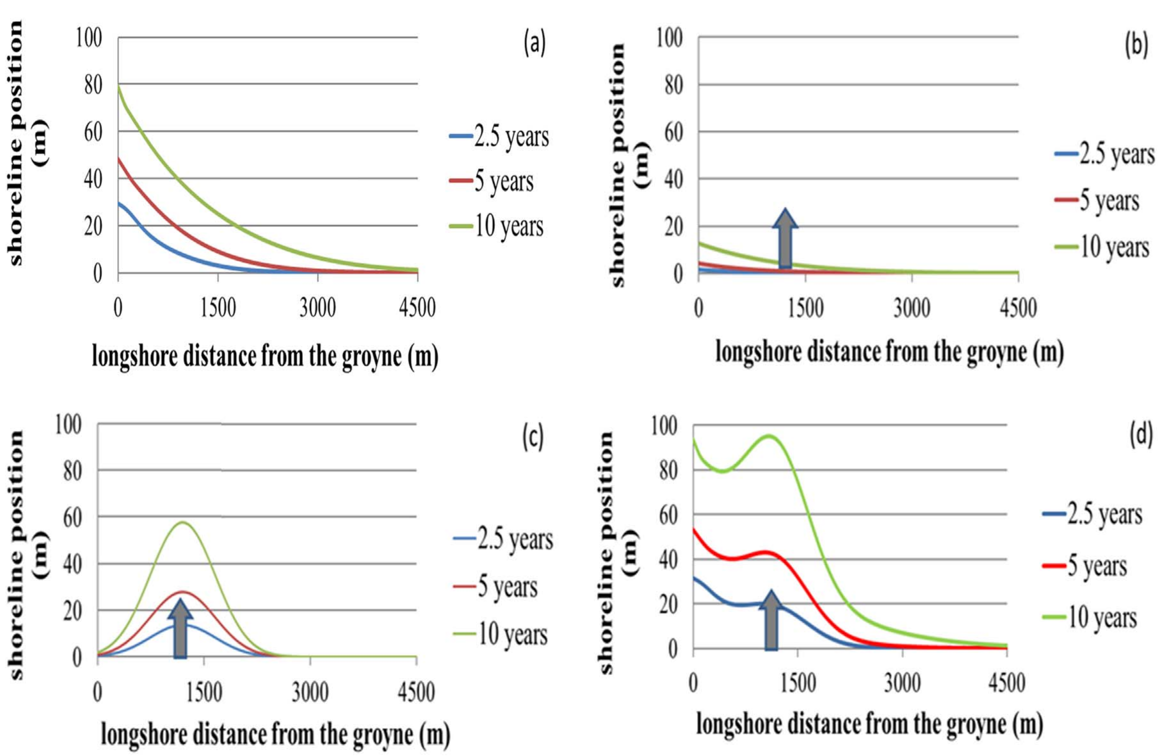

Fig. 7. (a) The impact of wave action on shoreline evolution (Eq. (17)). (b) Accretion caused near the groyne due the impact of the source of sediment discharge (Eq. (22)). (c) The gradual growth of the river-delta (Eq. (23)). (d) The combined impact of the aforementioned factors is used for the assessment of the final shoreline position.

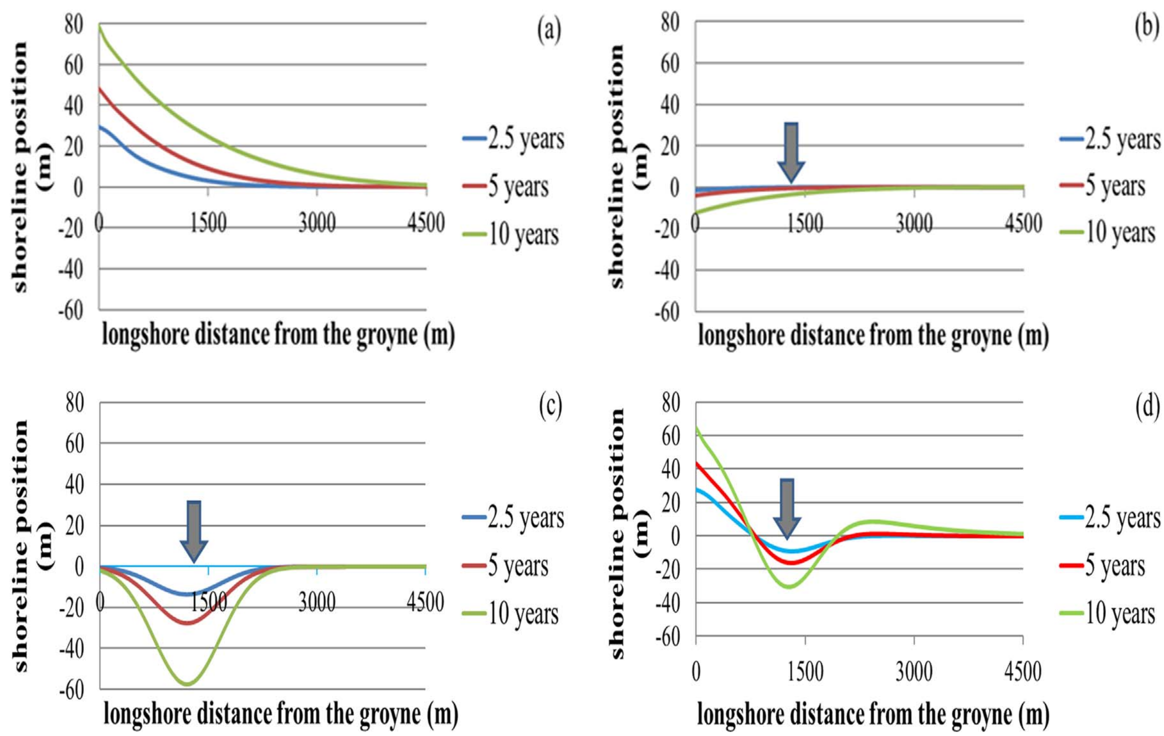

Fig. 8. (a) The impact of wave action on shoreline evolution (Eq. (17)). (b) Erosion caused near the groyne due the impact of the sink of sediment discharge (Eq. (22)). (c) The gradual erosion caused by the sink of sediment discharge (Eq. (23)). (d) The combined impact of the aforementioned factors is used for the assessment of the final shoreline position.

Table 1
The RMSE between the reference solution for a temporal step of one week and the solutions obtained with larger steps.

Temporal Step:	2 weeks	4 weeks	8 weeks
RMSE:	6.04 m	8.72 m	10.69 m

spatially uniform. Moreover, the new analytical solution can be extended to the left-hand side half-plane, for $-\infty < x \le 0$, in the case of an impermeable groyne of infinite length at $x=0$. The solution is anti-symmetric about the line $x=0$ so the shoreline in the region $0 < x \le +\infty$ is the negative of that shown in Fig. 3.

It is also worth noting that the solution given by Eq. (13) can be used to provide approximate answers to practical problems. For example, when designing a groyne in such a case it is often required to estimate the time before the beach reaches the end of the groyne. At this point, sediment will bypass the end of the groyne and the littoral drift to the downdrift side will recommence. The time for the groyne to fill can be estimated from Eq. (13) as follows. Let the groyne have length L and be positioned at $x=0$. Substituting these values into Eq. (13) together with appropriate values for the other parameters yields an equation that can be solved for time. That is, the time at which the beach position at $x=0$ is equal to the length of the groyne. Specifically, if t_L is the time required for the sediment material to fill the updrift side of a groyne having length L, up to its tip, then at the groyne ($x=0$) the application of Eq. (13) gives:

$$L \equiv y(0,t_l)=2\times\tan(a_b)\sqrt{\left(\frac{\varepsilon t_L}{\pi}\right)} + 2x_o\left(\sqrt{\varepsilon}\frac{B}{A^2}e^{-\left(\frac{x_o}{A}\right)^2}\right)\left(\frac{1}{6}\frac{1}{\sqrt{\pi}}(4t_L)^{3/2}\right)+Be^{-\left(\frac{x_o}{A}\right)^2} t_L \quad (25)$$

By setting $\omega=t_L^{1/2}$ and after some rearrangements Eq. (25) may be expressed as a 3rd order polynomial equation:

$$\frac{8}{3}x_o\left(\sqrt{\frac{\varepsilon}{\pi}}\frac{B}{A^2}e^{-\left(\frac{x_o}{A}\right)^2}\right)\omega^3 + Be^{-\left(\frac{x_o}{A}\right)^2}\omega^2+2\times\tan(a_b)\sqrt{\left(\frac{\varepsilon}{\pi}\right)}\omega - L = 0 \quad (26)$$

Eq. (26) demonstrates that the hypothesis presented in Section 1.1 about the parameters involved in the assessment of the required time for a groyne to be filled with sediment material, considering a source of sediment discharge in the groyne's vicinity, was correct. There are several ways available (algebraic and numerical) to find the roots of Eq. (26). For the cases considered here the parameters A, B, x_O and a_b are positive and so the coefficients of the first three terms in Eq. (26) are all positive. According to Descartes rule of signs, Eq. (26) has only one real positive root ω_o, so that $t_L=\omega_o^2$, since only one change of signs occurs between its successive terms. Next an example regarding the evaluation of Eq. (26) is presented corresponding to the case discussed in Section 3. As far as the time varying parameters are concerned such as the wave characteristics and the sediment discharge rate at the source, their mean values were taken into account. Subsequently, by substituting the following values in Eq. (26): $L=200\,m$, $A=650\,m$, $x_O=1200\,m$, $H_b=0.52\,m$, $T=5.93\,s$, $a_b=0.04\,rad$, $D=8\,m$, and $q_R=3.00\,m^3/h$, the following 3rd order polynomial expression was derived:

$$0.0321x10^{-5}\omega^3+3.2087x10^{-5}\omega^2+0.1716\omega-200 = 0 \quad (27)$$

A numerical solution of Eq. (27) via MATLAB yields: $\omega_o=637$, thus $t_L = \omega_o^2 = 637^2\ h = 405,769\ h \approx 46\ yr$. Therefore, about 46 years are required for the sediment material to fill the updrift side of the groyne and consequently, to bypass the groyne's seaward tip and start nourishing its downdrift side. The time t_L which was calculated above is considered reasonable given the fact that the length of the groyne was taken: $L=200\,m$.

In concluding we note that Eq. (26) demonstrates that our original hypothesis is indeed correct within the limitations imposed by the theoretical development leading to Eq. (13). This new analytical solution represents an advance and an important addition to the range of cases that can be used to validate numerical models. Although it cannot treat the case of an initially arbitrary shoreline position, and is constrained by the usual restrictions of analytical models, it can be used for the validation of numerical models which simulate simplified cases of coastal problems. These types of applications are a necessary step before incorporating the full complexity of a coastal site into a numerical model (Hanson, 1987). The methods presented here could be used in the preliminary study of the morphological changes expected to occur due to the presence of coastal constructions near river deltas or estuaries.

Acknowledgements

Antonios Valsamidis would like to thank Esteban Lopez for the fruitful discussions they had about analytical modelling.

Appendix A. Solution for shoreline evolution near an impermeable groyne and source of sediment discharge with the application of Laplace transforms)

The governing equation may be written as:

$$\varepsilon\frac{\partial^2 y}{\partial x^2} + Be^{-[\frac{x-xo}{A}]^2} - \frac{\partial y}{\partial t}=0 \tag{A1}$$

where

ε: diffusion coefficient $[l^2/t]$
A: a parameter proportional to river-mouth's width $[l]$
D: closure depth $[l]$
q_R: sediment transport rate $[l^3/t]$
$B = \frac{q_R}{DA}$
x_o: position of river mouth on the longshore axis

(l: length unit; t: time unit).
The initial and boundary conditions are:

$y(x,0)=0$ corresponding to initially undisturbed, straight beach;
$-\frac{\partial(0,t)}{\partial x} = \tan(a_b)$; corresponding to zero transport at $x=0$;
$y(+\infty, t) = 0$ corresponding to zero disturbance far from the domain of interest.

Taking the Laplace transform of (A1) with respect to t yields:

$$\varepsilon\frac{d^2\bar{y}}{dx^2} - s\bar{y} + \frac{B}{s}e^{-[\frac{x-xo}{A}]^2}=0 \tag{A2}$$

where s is a Laplace transform variable and $\bar{y}$ is the Laplace transform of y.
Then the general solution may be written as the sum of the complementary solution $\bar{y}_c$ and a particular solution $\bar{y}_p$:

$$\bar{y} = \bar{y}_c + \bar{y}_p = A_1e^{\lambda x} + A_2e^{-\lambda x} + \frac{B}{s^2}e^{-[\frac{x-xo}{A}]^2} \tag{A3}$$

By applying the boundary conditions, the coefficients A_1 and A_2 may be determined leading to the following solution for $\bar{y}$:

$$\bar{y} = \frac{tana_b}{s\lambda}e^{-\lambda x}+2x_0\left[\frac{Be^{-\left(\frac{xo}{A}\right)^2}}{s^2A^2\lambda}\right]e^{-\lambda x} + \frac{B}{s^2}e^{-[\frac{x-xo}{A}]^2} \tag{A4}$$

The solution to Eq. (1) is found by taking the inverse Laplace transform of the solution for $\bar{y}$. This may be done term by term and from tables may be shown to be:

$$y = tana_b\left[2\left(\frac{\varepsilon t}{\pi}\right)^{1/2}e^{-\frac{x^2}{4\varepsilon t}} - xerfc\left(\frac{x}{2\sqrt{\varepsilon t}}\right)\right]+2x_o\left(\sqrt{\varepsilon}\frac{B}{A^2}e^{-(\frac{x_o}{A})^2}\right)\left((4t)^{\frac{3}{2}}i^3erfc(\frac{x}{2\sqrt{\varepsilon t}})\right) + Be^{-\left[\frac{(x-xo)}{A}\right]^2}t \tag{A5}$$

where $i^3erfc(z)$ is the 3rd integral of $erfc(z)$ and can be written as:

$$i^3erfcx = \frac{1}{6}(x^2+1)\left(\frac{1}{\sqrt{\pi}}e^{-x^2} - xerfcx\right) - \frac{x}{12}erfcx$$

References

Bayram, A., Larson, M., Hanson, H., 2007. A new formula for the total longshore sediment transport rate. Coast. Eng. 54, 700–710.
Bever, A.J., McNinch, J.E., Harris, C.K., 2011. Hydrodynamics and Sediment-Transport in the Nearshore of Poverty Bay 31. Observations of Nearshore Sediment Segregation and Oceanic Storms: Continental Shelf Research, New Zealand, 507–526.
Brown, J.M., Davies, A.G., 2009. Methods for medium-term prediction of the net sediment transport by waves and currents in complex coastal regions. Cont. Shelf Res. 29, 1502–1514.
CERC, 1984. Shore protection manual: Vicksburg. Coastal Engineering Research Center, U.S., Corps of Engineers.
Dean, R.G., Dalrymple, R.A., 2002. Coastal Processes with Engineering Applications. Cambridge University Press, Cambridge, 488.
De Graauw, A., 2011. Silting-up of harbours: Ancient Ports - Ports Antiques, The Catalogue of Ancient Ports. ⟨http://www.ancientportsantiques.com/⟩ (Accessed 20/10/2016).
Delimani, P., Xeidakis, G.S., 2005. Environmental problems in the coastal zone of East Macedonia – Thrace. Geographies 9, 58–73.

Flor-Blanco, G., Flor, G., Pando, L., 2013. Evolution of the Salinas-El Espartal and Xago beach/dune systems in north-western Spain over recent decades: evidence for responses to natural processes and anthropogenic interventions. Geo-Mar. Lett. 33, 143–157.
Frihy, O.E., Fanos, A.M., Khafagy, A.A., Komar, P.D., 1991. Patterns of nearshore sediment transport along the Nile Delta, Egypt. Coast. Eng. 15, 409–429.
Gravens, M.B., Kraus, N.C., Hanson, H., 1991. GENESIS: Generalized Model for Simulating Shoreline Change: Report 2, Workbook and System User's Manual. U. S. Army Corps of Engineers.
Grijm, W., 1961, Theoretical forms of shoreline. In: Proceedings of the 7th International Conference on Coastal Engineering, ASCE, pp. 219–235.
Hanson, H., 1987. GENESIS-A Generalized Shoreline Change Numerical Model for Engineering Use. Lund Inst. of Tech/.Univ. of Lund, Lund.
Kamphuis, J.W., 1991. Alongshore sediment transport rate. J. Waterw. Port Coast. Ocean Eng. 117, 624–640.
Kamphuis, J.W., 1993. Effective modelling of coastal morphology. In: Proceedings of the 11th Australian Conference on Coastal Engineering: Sydney, Australia, Institute of Engineers of Australia, pp. 173–179.
Kamphuis, J.W., 2000. Introduction to Coastal Engineering and Management. World Scientific Publishing Co. Pte. Ltd, Kingston.
Kraus, N.C., Harikai, S., 1983. Numerical model of the shoreline change at Oarai Beach. Coast. Eng. 7, 1–28.
Larson, M., Hanson, H., Kraus, N.C., 1987. Analytical solutions of the one-line model of shoreline change. Technical Report CERC-87-15, USAE-WES, Coastal Engineering Research Cente, Vicksburg, Missisippi.
Larson, M., Hanson, H., Kraus, N.C., 1997. Analytical solutions of one-line model for shoreline change near coastal structures. J. Waterw. Port Coast. Ocean Eng. 123, 180–191.
Le Mèhautè, B., Soldate, M., 1977. Mathematical modelling of shoreline evolution. In: USAE-WES (Ed.), CERC Miscellaneous Report. Coastal Engineering Research Center, Vicksburg, Mississippi.
Pelnard-Considère, R., 1956. Essai de theorie de 1′evolution des forms de rivages en plaged e sablee t de galets: 4th Journees de l′ Hydraulique, les Energiesd e la Mer, Question III, v. Question III, Rapport No. 1, pp. 289–298.
Reeve, D.E., Chadwick, A.J., Fleming, C., 2004. Coastal Engineering: Processes, Theory and Design Practice. Spon Press, London, 552.
Reeve, D.E, 2006. Explicit expression for beach response to non-stationary forcing near a Groyne. J. Waterw. Port Coast. Ocean Eng. 132, 125–132.
Sanchez, R.J., Wilmsmeier, G., 2007. The River Plate Basin - A Comparison of Port Devolution Processes on the East Coast of South America. Devolution, Port Governance and Port Performance. Elsevier Ltd., Oxford, UK, 661.
Thomas, R.C., Frey, A.E., 2013. Shoreline Change Modeling Using OneLine Models: General Model Comparison and Literature Review: ERDC/CHL CHETN-II-55. US Army Corps of Engineers⟨www.dtic.mil/cgi-bin/GetTRDoc?AD=ADA591362⟩, (accessed 5/7/16).
Valsamidis, A., Cai, Y., Reeve, D.E., 2013. Modelling beach-structure interaction using a Heaviside technique: application and validation. J. Coast. Res. 65, 410–415.
van Lancker, V., Lanckneus, J., Hearn, S., Hoekstra, P., Levoy, F., Miles, J., Moerkerke, G., Monfort, O., Whitehouse, R., 2004. Coastal and nearshore morphology, bedforms and sediment transport pathways at Teignmouth (UK). Cont. Shelf Res. 24, 1171–1202.
Walton, T.L., Jr, Dean, R.G., 2011. Shoreline change at an infinite jetty for wave time series. Cont. Shelf Res. 31, 1474–1480.
Wind, H.G., 1990, Influence functions. In: Proceedings of the 21th International Conference on Coastal Engineering. pp. 3281–3294.
Zacharioudaki, A., Reeve, D.E, 2008. Semianalytical solutions of shoreline response to time-varying wave conditions. J. Waterw. Port Coast. Ocean Eng. 134, 265–274.