Life on the Range: Quine's Thesis and Semantic Indeterminacy

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1 Introduction

Philosophers of all stripes have, with little or no hesitation, entrusted the first-order quantifiers "there is" and "for all" with an extraordinary task — that of carrying the ontological commitments of theories, be they the informal theories implicit in everyday reasoning, the formal or semi-formal theories of science, or the lucubrations of metaphysicians. The practice rests on a popular simplification of Quine's thesis that "to be is to be the value of a variable" (Quine, 1948, pp. 34–35), which has come to be regarded, at least by some, as the basis for identifying the class of entities whose existence is necessary for the truth of a given body of propositions. Quine is, in fact, rather more careful in his characterization than contemporary philosophical practice takes him to be: not only does he specify that only *bound* variables are relevant here, as is obvious, but also that the criterion itself cannot be used to adjudicate between different ontologies, since it can only be used in "testing the conformity of a given ... doctrine to a prior ontological standard." This will be our conclusion as well, although by a different route.

Upon reflection, the Quinean thesis turns out to be remarkably more mysterious than it might appear at first. What, in fact, could it possibly mean to be *the value* (or even *a value*) of a bound variable? Strictly speaking, bound variables have *no* values: a point that was already recognized by Russell, when he pointed out that there is no (specific or — assuming the notion makes sense — generic) individual of which $\exists x Px$ predicates the property P. Quantified variables are purely *syncategorematic* devices, as the scholastics well knew, and too many of the moderns seem to have forgotten (see Dutilh Novaes, 2011). We can agree, though, that for an object a to be the value of a bound variable it means for a to satisfy a formula $\varphi(x)$, thereby functioning as a truth-maker for the corresponding existentially quantified sentence $\exists x \varphi(x)$. The other side of the Quinean equation is similarly obscure: it is not clear precisely what it means for a given theory to be ontologically committed to X. It turns out that

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explicating this notion is a somewhat subtler task than one might anticipate (see, for instance, Parsons (1970), or Rayo (2007) for a more recent treatment): in what follows we will assume that an intuitive understanding of what ontological commitment amounts to is available, and that will be enough for our purposes. Further, a case can be made that in its original form the Quinean thesis was meant to apply to languages that have been appropriately regimented in their form and interpretation. While this fact has often been neglected by the proponents of Quine's thesis, the points we are going to make apply equally well, in fact perhaps more clearly, to the regimented languages Quine had in mind.

Quine's characterization of ontological commitment as flowing from the semantics of the quantifiers is at the basis of his suspicion of second- and higher-order logic. For if higher-order quantifiers, like their first-order counterparts, also enact ontological commitments, then it seems there is no avoiding embracing a universe replete with predicates, relations, predicates of relations, and so on. Since the least ontologically extravagant way to implement such a universe is in set-theoretic terms, Quine's (1970) characterization of second-order logic as "set theory in sheep's clothing," readily follows: for second-order quantifiers rest on the set-theoretic notion of the power set, a notion that is quintessentially mathematical.

Much armchair philosophizing has ensued from Quine's proposed correlation of ontology with the semantics of first-order quantification. This is perhaps most evident in various branches of the philosophy of mathematics and mathematical philosophy: ascriptions of number, as in "the number of the planet is eight," for instance, not only employ numerical terms that purport to refer to a particular kind of abstract objects, *viz.*, numbers, but employ them in such a way that makes them available for existential generalization, allowing the inference to "something numbers the planets." But other branches of philosophy are far from immune from deploying such arguments: debates about the ultimate constituents of reality, which one would expect to be conducted with a modicum of input from science (as argued for instance by Thomasson (2008)), are instead at least sometimes cast as questions about first-order quantification — as though the answer to van Inwagen's (1987) famous question of whether there are tables, or just simples arranged table-wise, depended on the semantics of "there is" and "for all." In what follows we will distinguish the two halves of the Quinean thesis, *viz.*, the sufficiency thesis (being the value of a bound value is a sufficient condition for ontological commitment) and the converse necessity thesis (being such a value is a necessary condition of ontological commitment). The two halves, we will see, are importantly different.

Now, like all orthodoxies, the equivalence (if not the identification) of ontological commitment and first-order quantification has spawned more than its share of heretics, too (we will consider some of the arguments in Section 2). But the heretics have, in overwhelming majority, questioned the implications of Quine's thesis for the status of second- (and higher-) order logic, whereas it has come to be accepted as a matter of course that, in the *first-order* case, the semantics of the quantifiers really is a reliable guide to ontology. Accordingly, the main heretic arguments have, in one way or another, supported the conclusion that second-order quantifiers are really no more ontologically committed than the first-order ones. We will argue that the heretics are right that, in important ways, the first- and the second-order quantifiers are very much alike, but they have the direction of the analogy backwards. Our main thesis will be that, on a deeper understanding of the nature of the quantifiers as predicates of predicates, first-order quantifiers are just as dependent on the prior specification of a second-order domain. And while, as first pointed out by Henkin (1950), such second-order domains can vary in non-standard ways, likewise first-order quantifiers also admit of non-standard interpretations, a fact that has immediate repercussions for the Quinean thesis.

The development of the basic ideas underpinning such non-standard, or "general" interpretations will occupy the central parts of this paper. But it is important now to appreciate that just like the *semantic indeterminacy* of second-order quantifiers can be traced back to the existence of non-standard interpretations, the same is true in the first-order case. It has been known since Henkin's pioneering work that fixing the meaning of a second-order quantifier requires the selection, along with a first-order domain D_1 of objects, also of a second-order domain D_2 comprised of subsets of D_1, over which the quantifier is taken to range. Perhaps the central point of the present paper is that specification of the meaning of a *first-order* quantifier also requires — somewhat unexpectedly — the selection of a second-order domain, *even when* the first-order domain D_1 is already fixed. As a consequence, we will argue, first-order quantifiers are as semantically indeterminate as their second-order counterparts. Moreover, and perhaps just as importantly, such general or non-standard interpretations cannot be dismissed offhand, for there is a crucial sense in which non-standard interpretations are *transparent* from the point of view of language users, in that standard and non-standard interpretations cannot be set apart by consideration of which sentences are true or false in each.

The claim that first-order quantifiers are semantically indeterminate is of course not new. For instance, Hirsch (2002) first called attention to such indeterminacy under the rubric of "quantifier variance." Although Hirsch and his interlocutors are careful not to characterize quantifier variance in terms of varying domains, it is difficult to give a formally precise account of such variance in terms that, in some form of other, do not presuppose different domains of quantification (but see Sider (2007), for instance, for such an attempt, and Rossberg (2011) for some criticisms). This is not an issue that will occupy us in what follows, except to point out that even if quantifier variance is construed as mere variance in the domain of quantification, indeterminacy of first-order quantifiers persists *unless and until* an appropriate second-order domain is specified along with the first-order one, a feat that would appear to exceed the resources required for competent use of quantified expressions.

As we develep our account of the semantic indeterminacy of first-order quantifiers and its consequences for Quine's thesis, an issue that will play a role is the question of why the two quantifiers "there is" and "for all" (rather than others) occupy such a central position. The question is relevant from both the orthodox and the heretic point of view: an answer is crucial from the orthodox point of view if the two standard quantifiers are to carry the full weight of our ontological commitments; but an answer is due also from the heretic point of view, for it is of limited use to argue against the ontologically loaded nature of the standard quantifiers "there is" and "for all," if other, perhaps more fundamental quantifiers are standing at the ready, waiting to step in to do the grunt work of carrying ontological commitments.

2 The Quinean critics

The Quinean thesis establishing a tight connection between the semantics of the quantifiers and ontology has been variously challenged. Some of these challenges are particularly relevant for the account to be developed below, in that they highlight features of higher-order quantification that will turn out to be equally applicable at the first order level.

Arthur Prior was perhaps the first to point out, in characteristic iconoclastic fashion, that Quine's thesis "is just a piece of unsupported dogma" (Prior, 1971, p. 48). This conclusion occurs as part of an extensive defense of *non-nominal quantification* — the idea that expressions of any syntactic category, not just names, are available for the purpose of instantiating quantifiers of the corresponding kind. In fact, non-nominal quantification is ubiquitous in natural language. Consider the sentences:

> *I hurt him somehow.*
> *He's something I am not — kind.*

These sentences are obtained by existentially quantifying out the adverbial phrases in the following:

> *I hurt him by treading on his toes.*
> *He is kind, but I am not.*

But this fact does not by itself commit us to the existence of "ways" in which one can hurt or abstract entities such as attributes (see Rayo and Yablo, 2001, for a similar point). In other words — to steal yet another efficacious slogan from Quine (1970) — this is an instance of "logic in wolf's clothing."

What Prior's point really comes down to, is the idea that if ontological commitment is preserved by anything, it must be preserved by converse logical entailment. In other words,

> **Prior's insight:** If φ follows from ψ then φ's ontological commitments are also ψ's ontological commitments.

This is a principle that is supported by any conception that makes preservation of truth on any interpretation a necessary (and perhaps also sufficient) condition for logical consequence: if ψ entails φ, then φ is true on any interpretation on which ψ is true, so that the truth of φ cannot require (although of course it may allow) the existence of any further entities not already required by the truth of ψ. Hence, φ's ontological commitment is no more extensive than that of ψ. When applied to higher-order quantification, the argument neatly delivers that existential quantification over predicates carries along no more extensive ontological commitment than the sentence from which it follows. In particular, since Pa validly entails $\exists X\, Xa$, where P is an atomic predicate symbol, the second-order existential $\exists X$ is, on this account, ontologically neutral. (The case is not as clear-cut for the entailment from $\Phi(a)$ to $\exists X\, Xa$, where $\Phi(a)$ is a complex predicate expression, for then the validity of the argument depends on precisely which instances of the second-order comprehension axiom are available. But the atomic case is enough to establish Prior's point.) A version of the preservation of ontological commitment by converse entailment is also endorsed by Linnebo (2011, 2012), although Linnebo runs the implication in reverse, to conclude that indeed $P(a)$ is already committed to second-order entities (since its existential generalization is likewise committed).

A similar assessment of the tenuous connection between quantification and ontology is forcefully made in Wright (2007, p. 153):

> **Neutrality**: Quantification into the position occupied by a particular type of syntactic constituent in a statement of a particular form cannot generate ontological commitment to a kind of item not *already* semantically associated with the occurrence of that type of constituent in a true statement of that form.

The Neutrality Thesis embodies an important point, suggesting a partial decoupling of quantification and ontological commitment (Azzouni (2004) provides perhaps the most far-reaching argument for severing the connection of semantics and ontology). The part that is interesting for our purposes is the idea that to the extent to which quantification of any kind is connected to ontological commitment, such a connection is grounded on a prior selection of a domain of quantification. Thus we have a reversal in the order of dependence: rather than *enacting* such a commitment, quantification to a large extent *presupposes* it.

However, the main idea behind such reversal, important as it might be, is still not deployed in its full generality either in Prior's Insight or through the Neutrality Thesis, both of which share the narrow focus on the second-order case. Prior's argument, in its full generality, applies just as well at the first order as it does at the second order: since Pa validly entails $\exists x Px$, the ontological commitments of the latter do not exceed those of the former. And similarly the Neutrality Thesis shifts the burden of ontological commitment *away* from the first-order quantifiers just as much as it does for the second order quantifiers. Indeed, why *shouldn't* the Neutrality Thesis apply just as well at the first order?

The case for a conception of first-order quantification that is not entangled with ontological commitment is forcefully made by Routley in "On What There Is Not" (1982). Routley undertakes the project of laying the groundwork for an ontology replete with all sorts of objects, only some of which are characterized by *existence* or *being*. But more empirically or nominalistically inclined philosophers can still appreciate the strength of his arguments against the identification of existence with quantification (or "quantifiability"), an identification which according to Routley is "as false as it is simple" (1982, p. 169). Renouncing the identification goes hand in hand with a conception of first-order quantification that is existentially neutral, as Wright also would have it. The challenge is to make formal sense of such a notion of neutrality. It is relatively easy to provide an account of existentially neutral quantification in variable-domain modal contexts (in which quantifiers are taken to range over the union of all possible domains, with existence, i.e., existence at the *actual* world, to be settled separately; see Priest (2008, pp. 341–42) for instance). It is not quite as clear how to make sense of this notion in non-modal contexts, a question that will take center stage in what follows.

Before we start developing our account of the semantic indeterminacy of first-order quantification, it should be mentioned that there is a robust tradition originating with Boolos (1984) that also views second-order quantifiers as not more ontologically committed than their first-order counterparts. Second-order quantifiers like the one occurring in $\exists X\,\Phi(X)$ are analyzed as quantifying over *pluralities*, i.e., as asserting that "there are some x's such that Φ," where the plural locution "there are some x's such that ..." is in important cases demonstrably not paraphrasable at the first order. Insightful as this line of thought might be for philosophy of logic, it is not germane to the present concerns because it is based on highlighting special features of second-order quantifiers (*viz.*, that they refer to pluralities), features that they do not share with the first-order quantifiers, whereas the emphasis is here on the features that first-order quantifiers share with the second-order ones.

3 Quantifiers as second-order predicates

A formula such as $\varphi(x)$, in which the variable x occurs free, can be viewed as expressing a *predicate* over some given first-order domain D_1 of objects, and in particular as expressing the predicate under which all and only the objects fall that satisfy the formula $\varphi(x)$ in D_1 (we follow the terminological conventions of some neo-Fregean approaches in referring to subsets of the domain as "predicates" and the formulas that correspond to them as "predicate expressions"). For instance, the formula $Px \,\&\, \neg Qx$ expresses the predicate under which all and only the objects fall that have the property denoted by P but lack the property denoted by Q. Similarly, formulas with more than one variable express relations over D_1, etc. Plausible as this view might appear, it has nonetheless noteworthy implications for the proper understanding of the quantifiers.

The essential function of a quantifier symbol such as $\exists x$ or $\forall x$ is to combine with a formula $\varphi(x)$ to obtain a sentence. It is accordingly natural to characterize first-order quantifiers — *viz.*, the operators denoted by $\exists x$ or $\forall x$ — as second-order predicates, i.e., predicates of predicates. On this view, for instance, the existential quantifier applies to all and only the non-empty predicates, returning value *true* when, and only when, the predicate to which it is applied is non-empty; and dually the universal quantifier applies to a predicate if and only that predicate is equi-extensional with the whole domain D_1. This is in fact a view that was already advocated by Frege (1893–1903, §21), before being given rigorous mathematical formulation in the theory of generalized quantifiers initiated by Mostowski (1957), Lindström (1966), and Montague (1974). According to the theory, in fact, any permutation invariant second-order predicate (i.e., in purely extensional terms, any collection of subsets of D_1 containing a subset if and only if it contains any other subset equinumerous to it) gives rise to a quantifier. We have thus a vast collection of quantifiers beyond those denoted by $\exists$ and $\forall$, including for instance the quantifiers corresponding to “there are exactly k” (for $k \geq 0$), “there are infinitely many,” or “for all but finitely many” (and many others).

Once first-order quantifiers are construed as second-order predicates, Henkin’s development of non-standard interpretations for second-order logic can be transferred at the first-order level as well. On Henkin’s non-standard interpretations second-order quantifiers are taken to range over collections of predicates that may fall short of the true power-set of the domain. For instance, they may be taken to range over the collection of predicates that can be defined using a given, limited, stock of resources, etc. Such non-standard interpretations are referred to as “general,” in that they arise by relaxing, in a natural way, the requirement that $\forall X$ and $\exists X$ range over the collection of *all* predicates over the first-order domain D_1. On the general interpretation, second-order logic is co-interpretable with an appropriately designed multi-sorted first-order logic, and therefore its expressive power is vastly inferior to the standard case.

In spite of the fact that Henkin’s groundbreaking approach has been available for a long time, the theory of generalized quantifiers still shares the bias towards standard interpretations. The theory characterizes a first-order quantifiers as a predicate over the full power-set of the first-order domain D_1 (see, e.g., Peters and Westerståhl (2006)). But it is only natural to take this view one step further, and recognize that first-order quantifiers — just like their second-order counterparts — are open to general interpretations. Such interpretations would supply, beside a first-order domain D_1, also a second-order domain D_2 of subsets of D_1. The existential quantifier would then select, from among the members of D_2, those that are non-empty, and dually the universal quantifier would select the members of D_2 that are co-estensive with the whole domain D_1. The view neatly extends to other first-order quantifiers: the quantifier “there are infinitely many” would select those members whose cardinality is greater than or equal to the cardinality of the natural numbers, etc. Although non-standard interpretations for the first-order quantifiers have been around at least since Thomason and Johnson (1969) and Keisler (1970), this particular extension of Henkin’s general interpretations appears to have gone mostly unnoticed until Antonelli (2007).

Somewhat more formally, we can specify a general semantics for a first-order language (e.g., a language such as that of classical first-order logic with $\exists$ and its dual $\forall$), as follows. A model $\mathfrak{M}$ provides a non-empty first-order domain D_1 along with a collection D_2 of *non-empty* subsets of D_1. Truth on such an interpretation can then be defined by saying, for instance, that $\exists x \varphi(x)$ is true in $\mathfrak{M}$ if and only if the extension $\{x \in D_1 : \varphi(x)\}$ of $\varphi(x)$ is a non-empty member of D_2 (a rigorous definition will first proceed to define satisfaction of an open formulas by an assignment to the variables; see Antonelli (2013) for details). Notice that on this account if we were to define $\forall$ in a similar way, then duality with $\exists$ is lost. That is, if we define $\forall x \varphi(x)$ to be true in $\mathfrak{M}$ if and only if the extension of $\varphi(x)$ is a member of D_2 that is equi-extensional with D_1, then $\forall$ turns out no longer to be the dual quantifier of $\exists$. Duality can be retained by taking $\exists$ as primitive and abbreviating $\neg\exists x \neg\varphi$ by $\forall x \varphi(x)$: then $\forall x \varphi(x)$ is true in $\mathfrak{M}$ precisely when the extension $\{x \in D_1 : \neg\varphi(x)\}$ of $\neg\varphi(x)$ is a either not a member of D_2, or else such an extension is empty.

While the full technical details are explored in Antonelli (2013), we have enough to point out some of the ramifications of this account for the philosophical use of quantifiers as carriers of ontological commitment. These ramifications will be explored more in detail in later sections of this paper, but for now we point out that, as Quine's thesis can be decomposed into separate necessity and sufficiency claims, the possibility of providing general interpretations for the first-order quantifiers has different implications for each half.

According to the necessity thesis, being the value of a bound variable is necessary for ontological commitment: objects to which we are ontologically committed are available as values for bound variables and therefore ground the truth of the corresponding existentially quantified statement. According to the necessity thesis, if we are ontologically committed to a, and a satisfies $\varphi(x)$, then $\exists x \varphi(x)$ must be true, because of the commitment to one of its instances. But notice (a point to which we will return in more detail) that on the general interpretation of first-order quantifiers, the necessity thesis fails, for there will be interpretations in which $\varphi(x)$ is satisfied by a, but the extension $\{x \in D_1 : \varphi(x)\}$, while non-empty, will not be among the subsets in the collection D_2 used in stating the truth conditions for existentially quantified statements.

While the failure of the necessity thesis on the general interpretation is clear-cut, things are somewhat murkier in the case of the converse. According to the sufficiency thesis, if the quantified statement $\exists x \varphi(x)$ is true, then there must be objects instantiating $\varphi(x)$. This conclusion seems difficult to avoid, even on the general interpretation, for if the extension $\{x \in D_1 : \varphi(x)\}$ is a member of D_2 to which the quantifier applies, then in particular such an extension must be non-empty, and therefore there must be objects satisfying $\varphi(x)$ and instantiating the bound variable. We postpone further discussion of this point until later, when we will see that there is at least a way to make the notion of a failure of the sufficiency thesis coherent.

4 The case for "there is" and "for all"

We mentioned that Quine's linking of semantics and ontology in his characterization of ontological commitment presupposes a special role for the two quantifiers "there is" and "for all." This is true regardless of whether one approaches the characterization in order to support it or to undermine it. There is indeed a question as to why these two quantifiers, among many, have emerged to play such a central role, not only in philosophical disputes, but throughout the spectrum of human inquiry. And of course, any argument directed at explaining how that specific role bears upon ontological commitment runs the risk of being made irrelevant if other quantifiers are available to claim the same fundamental function. This section tells a story as to why these two quantifier are so prominent, a story that takes us on a detour through binary quantifiers and determiners (which, in this section, are only construed on their standard semantics).

We saw that from the point of view of the theory of generalized quantifiers, first-order quantifiers are to be identified with predicates over the power-set of the first-order domain D_1. But besides "unary" quantifiers applying to one predicate at a time, such as those expressed by $\exists$ and $\forall$, "binary" quantifiers applying to two predicates are also quite common, and in fact probably even more so, as they are ubiquitous in natural language, where they provide extensions for *determiners*. The Aristotelian quantifiers "All" and "Some" are in fact determiners, taking two predicates A and B as arguments (referred to as the "scope" and the "range" of the determiner, respectively) and returning propositions of the form "All A's are B's" or "Some A's are B's." But many more relations between predicates can be expressed by determiners, as revealed by even a cursory glance at the following partial list (from Peters and Westerståhl (2006, p. 120)):

> Some, a, all, every, no, several, most, neither, the, John's, at least 10, all but 10, infinitely many, about 200, an even number of, between 5 and 10, most but not all, either fewer than 5 or more than 100, John's but not Mary's, at least one of most students', neither the red nor the green, ...

The fact that determiners play such an extensive role in ordinary language and communication is a sign of their fundamental nature. Determiners express the fact that the scope and the range are related in some particular way: this is just another way of saying that their denotations are binary quantifiers. The Aristotelian determiner "All" expresses that the scope and the range are related by inclusion, "Some" expresses that the scope and the range have non-empty intersection, etc.

It is natural to ask what a most generic, or *weakest*, determiner would look like: such a determiner would simply be expressing the fact that the scope and the range are related *some way or other*, i.e., expressing the existence of a relation having the scope as its domain and the range as co-domain. Let us define the *most general* determiner as denoting the binary quantifier holding between A and B precisely when there is some relation R relating objects in A to objects in B. Given some widely accepted set-theoretic assumptions, the existence of such a relation R is equivalent to the existence of a function f mapping objects in A to objects in B (clearly a function is a special kind of relation, and conversely any relation can be refined to a function by selecting for each $a \in A$ a unique $b \in B$ such that Rab). In order to simply matters, we will accordingly denote the most general determiner by Q^f, where the binary quantifier Q^f holds of A and B if and only if there exists $f : A \to B$.

It would indeed appear that the denotations of many, perhaps all, natural-language determiners can then be obtained by placing further restrictions on the function f mapping the scope into the range of the determiner. For instance the Aristotelian determiner "Some" can be characterized in this way by saying that "Some A's are B's" is true precisely when there is a function $f : A \to B$ having a fixed point, i.e., an object $a \in A$ such that $f(a) = a$. Similarly, "All" can be characterized by saying that there is a function $f : A \to B$ such that $f(a) = a$ for each $a \in A$. The conjecture that many, perhaps all, natural language determiners can be obtained in this way (that is, by placing appropriate conditions on the function mapping the scope into the range) lends plausibility to the characterization of Q^f as a most general, or weakest, determiner, for then other determiners can be obtained by refining and strengthening it.

It would seem natural to conjecture that the expressive power of a quantifier varies in accordance with the strength of the restrictions imposed on the function f relating the scope and the range of the corresponding determiner. This amounts to saying that if quantifiers Q_1 and Q_2 are defined by imposing given restrictions Φ_1 and Φ_2 on f:

$$\begin{aligned}
\mathsf{Q}_1(A,B) &\iff \exists f\left[f : A \to B \,\&\, \Phi_1(f)\right],\\
\mathsf{Q}_2(A,B) &\iff \exists f\left[f : A \to B \,\&\, \Phi_2(f)\right],
\end{aligned}$$

and $\Phi_1(f)$ implies $\Phi_2(f)$ for any f, then the expressive power of Q_1 is at least as great as that of Q_2. We will not specify precisely how expressive power is to be measured, but a necessary condition for the expressive power of Q_1 to be at least as great as that of Q_2 would seem to be that Q_1 interprets Q_2 (relative to some given background language). But this conjecture fails, along with its converse, showing that the expressive power of the quantifier is independent of the strength on the condition Φ_1 or Φ_2. Consider the following three binary quantifiers obtained by imposing increasingly stronger restrictions on the function f:

$$\begin{aligned}
\mathsf{Q}^f(A,B) &\iff \exists f : A \to B,\\
\mathsf{Q}^1(A,B) &\iff \exists f : A \to B \,\&\, f \text{ injective},\\
\mathsf{Q}^=(A,B) &\iff \exists f : A \to B \,\&\, \forall a \in A : f(a) = a.
\end{aligned}$$

Clearly $\mathsf{Q}^=(A,B)$ implies $\mathsf{Q}^1(A,B)$, since identity is injective, and $\mathsf{Q}^1(A,B)$ implies $\mathsf{Q}^f(A,B)$. But $\mathsf{Q}^1(A,B)$ is strictly stronger than the other two, and by far. To see this, observe that the *weakest* of the three quantifiers above, *viz.*, Q^f, is equivalent, over a weak logic comprising an identically empty predicate $\varnothing$ (i.e., a name for the empty set, such as $x \neq x$) and Boolean operators, to each of the ordinary quantifiers $\exists$ and $\forall$. For clearly, there exists a function $f : A \to B$ (which is what Q^f expresses) precisely when $B = \varnothing$ implies $A = \varnothing$, and obviously "if B is empty then so is A" is expressible using $\exists$ or $\forall$. Conversely, $\exists x\, Ax$ can be expressed as $\neg\mathsf{Q}^f(A,\varnothing)$ and dually $\forall x\, Ax$ as $\mathsf{Q}^f(\neg A,\varnothing)$. Thus the logic with Q^f as its only quantifier is essentially the same as ordinary first-order logic. Moreover, $\mathsf{Q}^=$ is just the Aristotelian determiner "All A's are B's," or $A \subseteq B$, which is also expressively equivalent to first-order logic, as one easily sees. Thus the logic with $\mathsf{Q}^=$ is also equivalent to first-order logic. However, the intermediate quantifier Q^1 is much more expressive than standard first-order logic. In fact Q^1 interprets well-known cardinality quantifiers such as Rescher's or Härtig's, and so it can be used, e.g., to provide a categorical axiomatization of arithmetic (a result that goes back to Yasuhara (1969); see also Antonelli (2010)).

This detour on binary quantifiers and determiners was gave us a characterization of the fundamental role played by the unary quantifiers $\exists$ and $\forall$. Our story identifies the general form of the determiner as expressing the existence of a functional relation between its two arguments, the scope and the range. When no further constraints are imposed on such a functional relation, we have a basic, most general, and arguably most natural form of the determiner. But as shown above such a form, in turn, is expressively equivalent to the two quantifiers, “there is” and “for all,” thereby giving us at least the beginning of an insight into their fundamental nature and crucial role.

5 Varieties of general interpretations

In this section we return to non-standard interpretations for first-order quantifiers, and specifically for those denoted by $\exists$ and $\forall$. As we will see, such interpretations come in many different kinds. For our present purposes by “first-order logic” we mean the first-order language obtained from a given stock of extra-logical symbols by means of truth-functional connectives and the quantifier $\exists$ (with $\forall$ defined as the dual of $\exists$). The contrast here is with first-order languages including arbitrary first-order quantifiers besides (or perhaps even instead of) these two, a general case that is given fuller treatment in Antonelli (2013). We have seen that on the general semantics for first-order logic, a model $\mathfrak{M}$ supplies a non-empty domain D_1 along with a collection D_2 of subsets of D_1. The existential quantifier is interpreted relative to the second-order domain D_2: $\exists x \varphi(x)$ is true in $\mathfrak{M}$ if and only if the extension $\{x \in D : \varphi(x)\}$ is a non-empty member of D_2, and since we take $\forall$ as the dual quantifier, the definition at the same time fixes the truth conditions for universally quantified sentences as we saw in Section 3.

Consideration of general interpretations can help shed light on at least half of the Quinean thesis, that instantiating a bound variable — i.e., falling under a predicate which in turn falls under the quantifier — is necessary for ontological commitment. General interpretations make it clear that in this respect necessity fails: we can be committed to entities having certain properties without such entities being available as values of — truth-makers for — the corresponding existentially quantified sentences. In order to bring general interpretations to bear also on the other half of the Quinean thesis, sufficiency, we need to consider a class of interpretations that appear at first sight to be of quite a different kind.

There is a deep connection between non-standard models satisfying certain further constraints (to be specified in a moment) and the kind of inner-outer domain interpretations originally proposed for positive free logic (see Leblanc and Thomason, 1968). Positive free logic was developed by Lambert (1963) to include non-referring terms in such a way that not all atomic predications involving such terms are automatically false. Inner-outer domain interpretations accomplish this by providing *two* domains of objects: a possibly empty *inner* domain I and an *outer* domain O, with $I \subseteq O$. The inner domain is intended to comprise “existing” objects, those towards which we carry ontological commitment, whereas the outer domain provides denotations for non-referring terms. The extension of an atomic predicate might comprise objects from either domain, thereby allowing some predications involving non-referring terms to be true. Of course, the quantifiers $\exists$ and $\forall$ are then taken to range over the inner domain.

The connection between outer-domain models and general interpretations can be stated as follows: for every inner-outer domain model $\mathfrak{M}$ there exists a generalized model $\mathfrak{N}$ verifying the same sentences. In fact, for a given inner-outer domain model $\mathfrak{M} = (I, O)$ the corresponding generalized model can be obtained by putting $D_1 = O$ and defining D_2 to consist of all those subsets of D_1 that contain at least one member of the inner domain I. It follows that if a sentence φ is true in all general models then it is true in all outer-domain model, and hence the logic of general models is contained in positive free logic. The converse is not true: not all general models are equivalent to inner-outer domain models; however, it is possible to identify a condition on which a general model is, in fact, equivalent to an inner-outer domain model. The condition simply requires that if D_2 contains a predicate P having non-empty overlap with a predicate Q, then Q is also a member of D_2 (details are given in Antonelli, 2013). We will return to this fact in the next section.

It is interesting perhaps that some non-standard interpretations of the two quantifiers "there is" and "for all" recover, in fact, the standard reading (the one on which they are takes to range over the full power set of D_1), without having the quantifiers apply to the full power-set of the first-order domain. Consider for instance a standard interpretation $\mathfrak{M}$ for a first-order language, with domain D_1, and let D_2 consists of all non-empty subsets of D_1 that are *first-order definable* in $\mathfrak{M}$ by a formula φ (possibly involving parameters). We can then form the "hull" of $\mathfrak{M}$, i.e., the non-standard interpretation $\mathfrak{N}$ having D_1 and D_2 as its first- and second-order domains, respectively (i.e., the hull of $\mathfrak{M}$ has the same first-order domain, but a "lean" second-order domain consisting only of the subsets that are definable in $\mathfrak{M}$. It is not obvious that the quantifiers have the same meaning in the original model as they do in the leaner hull. It is therefore somewhat unexpected that exactly the same formulas are satisfied by the same members of D_1 in the two interpretations (see Antonelli, 2013). This could be construed as evidence that the standard interpretation "overshoots" the target by requiring that the quantifiers range over the full power set, when in fact much less than that is sufficient to recover the ordinary construal.

The fact that any standard first-order model is equivalent to its hull shows that there is some kind of "reflective equilibrium" that seems to characterize genuinely first-order notions *vis-à-vis* second-order ones. For we have seen that the difference between first- and second-order notions is *not* in the fact that, as one might be tempted to assume, the second-order notions, but not first-order ones, are semantically sensitive to interpretations whose second-order domains falls short of the full power set. Rather, a potentially useful criterion for demarcating first-order and second-order notions is whether these notions can be characterized as invariant upon transition to hulls, i.e., upon restricting the interpretations to those having a second-order domain comprised only of definable subsets. Here of course "definable" means "definable using those very notions:" this apparent circularity points in fact to a the reflective equilibrium characterizing first-order notions.

Now, of course, one could worry about the circularity, and question the significance to be attached to the results just mentioned, because after all, in order to be able to specify the class D_2 of all first-order definable subsets of D_1 (i.e., those definable on the standard interpretation), the general interpretation obtained depends on a *prior* understanding of the standard meaning of the quantifiers. But there are ways around this, although they require some fair amount of detail in building up the collection of definable subsets "from below," as it were, rather than by the roundabout device of identifying such subsets wholesale in terms of the standard satisfaction relation. The details of the procedure are given in the companion paper (Antonelli, 2013) already mentioned, but the main idea is to identify the appropriate collection of Gödel operations (first introduced in Gödel (1940) in connection with the constructible universe) and then characterize the collection of all definable subsets of the domain as the closure of the class of primitive relations over D_1 (including identity) under those functions. The definable subsets are thus identified *directly* without resorting to the standard notion of the quantifier.

6 The Quinean thesis: necessity and sufficiency

We now have the tools necessary to re-assess the Quinean thesis that "to be is to be the value of a bound variable," as composed by the two distinct claims: that being the value of a bound variable is *sufficient* for ontological commitment, and the converse claim that being the value of a bound variable is *necessary* for ontological commitment.

The possibility of non-standard interpretations for first-order quantifiers, and particularly for $\exists$ and $\forall$, makes it clear that the necessity thesis fails in a crucial sense. This is because on the general interpretation of the first-order quantifiers, $\exists$ and $\forall$ apply only over a subset of the true power set of D_1. There might then be members of D_1 that are not captured by the two quantifiers: they, so to speak, lie beyond the quantifiers' reach. This can happen, as mentioned, if a formula $\varphi(x)$ is satisfied by some object in D_1 but the extension $\{x \in D_1 : \varphi(x)\}$ is not a member of the second-order domain D_2, thus making $\exists x \varphi(x)$ false on the interpretation: in such a case the necessity thesis fails. Thus, on the general interpretation of the quantifiers, being the value of a variable is not necessary for ontological commitment, in that some members of D_1 are not available as possible instantiation of the variable in $\exists x \varphi(x)$ and $\forall x \varphi(x)$. As an immediate consequence of this fact, we see that inference patterns that take the form of existential generalization fail on this semantics. That is because instances of the form $\varphi(a)$ might be true, but their existential generalizations, $\exists x \varphi(x)$, might not, as we just observed, so that existential generalization is not truth-preserving.

We also saw that there is a deep connection between the general interpretation of the quantifiers "there is" and "for all" and the inner-outer domain semantics: every inner-outer domain corresponds to an equivalent general model, and conversely every general model satisfying the condition given in Section 5 is equivalent to an inner-outer domain model. But this technical fact should *not* be taken to mean that the logic of the generalized "there is" and "for all" is in fact just a variant of free logic. In fact the two are approaches have very different inspirations and outcomes. Free logic was originally developed as the logic of *non-referring terms*, be they atomic terms such as "Pegasus" or "Bellerophon," or complex terms such as the empty descriptions "the winged horse" or "the golden mountain." On the contrary, the general interpretation of the quantifiers has nothing to do with "non-existent" objects (of which, we submit, there are none), but rather with the possibility that some particular objects (on some interpretations) might lie beyond the reach of the first-order quantifiers. This would appear to be a more natural construal of the failure of existential generalization than the one given by free logic, at least in its positive version, requiring formulas to be satisfied by objects that are, by the lights of the logic itself, "non-existent."

A parallel analysis can be given in the second-order case. We mentioned that second-order quantifiers can be given a general interpretation on which they are taken to denote some subset of the double power-set of D_1, as first pointed out by Henkin. And in fact, any second-order model in which some instance of second-order comprehension *fails* must be of this kind, for then there will be complex predicates expressions $\Phi(x)$ that fail to correspond to some subset X of D_1 falling within the range of the second-order quantifiers $\exists X$ or $\forall X$. This is possible because although of course the extension of $\Phi(x)$ will be among the members of the true power-set of D_1 it will not, in general, be among those over which the second-order quantifiers range (systems with restricted second-order comprehension play an important role in the foundations of mathematics). It is of course possible to regard such complex predicates — in analogy to the first-order case — as *non-referring*: on this account second-order logic, on the general interpretation, could be viewed as the free logic of these non-referring predicates. But it is more natural, and customary, to regard the general interpretation as allowing predicates that exceed the grasp of the second-order quantifiers. The parallel construal of first-order quantifiers is similarly more natural than the extravagant ontology required by positive free logic.

We now turn to the other half of the Quinean thesis, sufficiency. Are there any more insights to be gained from considering the sufficiency thesis in the light of this understanding of the quantifiers, comparable to those delivered by consideration of the necessity thesis? As we have seen in Section 2, several arguments have traditionally been put forward to defuse the sufficiency thesis, but only as it applies to *second-order* quantifiers. Both Prior's Insight and Wright's Neutrality Thesis are squarely aimed at the second-order case, when as it should now be evident there is nothing specifically second-order about either one of them, and they equally well could — and *should* — be applied at the first order. This is especially clear in the case of what we called Prior's Insight: if ontological commitment is preserved through converse logical consequence, then the commitment of $\exists x \varphi(x)$ cannot exceed that of $\varphi(a)$, from which it follows, and since the latter is not ontologically committed, the sufficiency thesis fails.

A challenge remains, though. Given the generalized semantics of the quantifiers, is there any sense to be made of the failure of the sufficiency thesis? It would seem that the sufficiency thesis can only really fail in the presence of extravagant ontologies — be they Routley's rich ontology of non-existents or the more mathematically precise ontology of variable-domain quantified modal logic. But if we want to avoid the former while remaining in a non-modal context we need some account of how exactly the sufficiency thesis is supposed to fail: being the value of a bound variable, while clearly not necessary, would seem to be at least sufficient for *actual* existence. In other words, even if the non-standard semantics makes it clear that objects in D_1 might outstrip the grasp of $\exists$ and $\forall$, it still *does* follow that the truth of $\exists x \varphi(x)$ carries with it commitment to the extension of $\varphi(x)$'s being non-empty. For $\exists x \varphi(x)$ is true on an interpretation precisely when the extension of $\varphi(x)$ is among the predicates in D_2 and such that the quantifier applies to it, and this last condition requires that some object from D_1 must be a member of the extension of $\varphi(x)$.

In order to address this point, let us first observe that the notion of a failure of the sufficient thesis is at least *coherent*, in that it can be realized in a model. In fact, such a model can be obtained by combining the general interpretation of the first-order quantifiers with a device derived from the realization that existence is no longer the exclusive purview of the quantifier, i.e., that existence is an extra-logical and extra-semantical notion (as also Azzouni (2004) would have it), and that therefore it needs to be expressed by extra-logical means. We can implement this idea by recourse to a form of outer-domain semantics, as follows. Given a first-order domain D_1, identify a subset I of D_1 as the inner domain of individuals to whose existence we are ontologically committed, and let the existential quantifier $\exists$ range over some collection of non-empty subsets of D_1 (note: *not* necessarily subsets of I), and dually for $\forall$. This means that a model is now identified with a triple (D_1, D_2, I), where $I \subseteq D_1$ and $D_2 \subseteq \mathscr{P}(D_1)$. In such a model, then, being the value of a bound variable is neither necessary nor sufficient for ontological commitment. It is not necessary because, as before, some member of D_1 instantiate a formula whose extension fails to be in D_2, so that the existential closure of that formula fails to be true. But the sufficiency thesis fails as well, in that some formula $\varphi(x)$ might have a non-empty extension in D_2 comprised only of members of D_1 that fall outside of I. The corresponding existential $\exists x \varphi(x)$ would then be true, but its instances would not meet the extra-logical standard needed for ontological commitment. Therefore, to the extent to which the inner domain I represents the ontological commitments of the interpretation, being the value of a bound variable does not guarantee being the object of such a commitment.

But this argument against the sufficiency thesis is importantly different from the one we gave against the necessity thesis. The latter is a "for all we know" argument, in the sense that, *for all we know*, there might really be objects exceeding the grasp of our ordinary first-order quantifiers. On *some* interpretations these "outlying objects" will then be logically inert, thereby making the particular interpretation of the quantifiers completely transparent to us. In particular, this will be the case for those interpretations in which quantification provides the only route providing access to the objects of the domain. But there will also be interpretations in which these objects will *not* be logically inert: one lesson we can learn from the proponents of free logic, is that singular terms — atomic or complex — can also provide an access route to objects that do not fall within the reach of the quantifiers. We need not characterize these objects as "non-existent," but we need to recognize that ontological commitment to such objects is prior to, and independent of, the semantics of the quantifiers. This is also the reason why on-standard interpretations cannot be ruled out, at least not on purely linguistic grounds: they might be completely transparent to users of the language, who would then need to resort to other extra-semantical means to assess questions of existence.

On the other hand, the argument establishing the coherence of the notion of a failure of the sufficiency thesis only provides a model. The model is not quite viable as a model of anything resembling actuality, unless one subscribes to extravagant ontologies (which we are trying to avoid). Non-existent objects should be just that — non-existent — and therefore no model that countenances such objects or their *simulacra* can lay any claim to realistic plausibility. But the model in which sufficiency fails still plays a role, perhaps a crucial one. For the model makes clear what the direction of the explanation ought to be: it's not the quantifier that *enacts*, so to speak, the ontological commitment, but rather it's the semantics for the quantifier that depends in the first instance on a *prior* selection of a domain D_1 of quantification, *as well as* an accompanying a second-order domain D_2, just as Wright's Neutrality thesis would require, once properly extended to include first-order quantifiers.

7 Conclusion

Being the value of a bound variable is neither necessary nor (in an importantly different sense) sufficient for ontological commitment. Consideration of non-standard models for the existential and universal quantifiers reveals that the possibility that there might be objects that lie beyond the reach of such quantifiers cannot be ruled out, or at least not on purely semantical grounds. Whether the language so interpreted has the expressive means to discriminate such cases depends on the details of the interpretation (a rich enough interpretation might be indistinguishable from the standard one — as the one comprised of definable subsets shows). Thus the necessity thesis indeed easily fails.

But the sufficiency thesis is also far from unassailable, in the sense that its failure is at least a coherent notion. This is apparent by turning on its head the free logician's expedient of an outer domain: instead of constraining the quantifiers to range over the inner domain, as free logicians do, we allow the truth of some existentially quantified sentences whose only witnesses lie outside the inner domain, but retain membership in the inner domain as a measure of ontological commitment. Thus the ontological commitments that accompany our use of the first-order quantifiers, such as they are, are in fact *dependent* upon our prior selection not just of a first-order domain, D_1, but also of a second-order domain D_2.

Of course, this conclusion still leaves open the question of exactly *how* those ontological commitments are to be established in the first place, i.e., how we go about the relevant domain selections. We are thus brought back to a broader Quinean conception according to which ontological commitments are in some way bundled up in a more or less holistic manner with our linguistic practices (a conception that Quine himself would have found appealing, as we noticed at the beginning). It's just the atomistic attempt to individuate one particular linguistic component — the quantifiers — as the specific *locus* of the commitments that fails. The result is that we are left facing a radical indeterminacy in the semantics of the quantifier, not dissimilar to the indeterminacy of numerical notions brought about by non-standard model for arithmetic. In both cases determination of whether language suffices to pin down the intended model requires access to an external, independent viewpoint unavailable within the limited expressive resources of first-order languages.

From a formal point of view, the upshot of the discussion is that first-order quantifiers are just as semantically sensitive to general interpretations with a non-standard second-order domain as their second-order counterparts. The realization cuts both ways, though. On the one hand it makes clear that indeterminacy in the semantics of the first-order quantifiers cannot be addressed simply by fixing a first-order domain D_1: a second-order domain D_2 needs to be specified as well, just as for second-order logic. But on the other hand, any reservations one might have concerning the ontological commitments of second-order logic can be assuaged by the fact that those are the same as in the first-order case, which has long been considered ontologically innocent; and this last realization can contribute to the establishment of second-order logic on the same safe footing as first-order logic.

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