On the Proper Treatment of Quantification in Contexts of Logical and Metaphysical Modalities*

Sten Lindström

We make a distinction between two intuitive interpretations of the operators □ and ◇ of alethic modal logic. First, there is the *logical* or *metalogical* interpretation:

> □φ: it is self-contradictory to assume that ¬φ is the case.
> ◇φ: it is not self-contradictory to assume that φ is the case.

Then, there is the *metaphysical* or *counterfactual* interpretation:

> □φ: φ and it could not have been the case that ¬φ.
> ◇φ: either φ or it could have been the case that φ.

The question now arises how the two kinds of alethic modalities relate to each other. The answer, of course, depends on how precisely we characterize the two notions. Suppose that we have a class $\mathcal{K}$ consisting of all Kripke models for a language of modal propositional or predicate logic with operators: □ for metaphysical necessity, L for logical necessity, and A for actuality. Each model is associated with a set of points (representing "possible worlds") of which one represents the "actual world". *Truth in a model* is defined as truth at the actual world of the model. *Logical truth*, or *validity*, is defined as truth in every model. We adopt the following semantic clauses for logical necessity, metaphysical necessity and actuality:

(i) $L\varphi$ is true at a point in a model iff φ is true at the actual world in every model.

(ii)   □φ is true at a point in a model iff φ is true at every point in the model.

(iii)   *A*φ is true at a point in a model iff φ is true at the actual world of the model.

The main purpose of the paper is to show how a semantics of this kind for logical and metaphysical necessity can be combined with individual quantification and identity in a way that allows for "quantifying in" across contexts governed by $\square$ as well as by *L*. A distinction is being made between two kinds of *rigid designators*: metaphysically rigid and logically rigid ones. For any model *M*, a *metaphysically rigid designator* refers to the same object at any point. A *logically rigid designator* refers to one and the same object for any model *M* and any point in *M*. In order to handle quantification into contexts governed by the logical necessity operator *L*, variables are assigned values independently of the models. In other words, they are treated as logically rigid designators. The range of quantification is allowed to vary from model to model and from world to world. The properties of the resulting modal logic are explored. It is suggested that this semantics gives us reasons to be skeptical of Quine's criticism of quantified modal logic—in particular of his criticism of quantifying into contexts governed by an operator for logical necessity. In an appendix, we consider model-theoretic explications of the notions of *analyticity* and *apriority*. Our approach to these notions is a development of the one given by Wlodek Rabinowicz in his contribution to the present volume, allowing for the introduction of special operators *An* and *Ap* for analyticity and apriority.

1 Two interpretations of the alethic modalities

By modalities in the narrow—or strict—sense, we understand the notions of necessity, possibility and impossibility. These are the *alethic* modalities of G. H. von Wright (1951), i.e., modes of being true. In addition, von Wright recognized the *epistemic* modalities (modes of being known: known to be true, known to be false, undecided), the *deontic* modalities (modes of obligation: obligatory, permitted, forbidden) and the *existential* ones (modes of existence: universality, existence, emptiness). In this connection, one should also mention the *temporal* modalities (past, present, future). In this paper, we will only be concerned with the alethic modalities. In particular, we discuss two kinds of interpretation of the alethic modalities standardly referred to as the *logical* interpretation and the *metaphysical* one. The logical interpretation can be traced all the way back to the origin of modern modal logic. In an article in *Mind* (1912), C. I. Lewis recommended that classical propositional logic be supplemented with a new "intensional" connective $\prec$ (*strict implication)*, which he regarded as a better formalization of the conditional 'if–then' than the truth-functional *material implication* of Russell and Whitehead. Lewis's intuitive idea was that a conditional of the form $\varphi \prec \psi$ should

be true just in case φ *logically implies* ψ. Thus, strict implication would bring about a much stronger (or "stricter") connection between the antecedent and the succedent of a conditional than material implication.

According to a charitable interpretation, Lewis wanted to introduce into the object language a connective that in a way reflects the metalinguistic relation of logical implication. On Quine's less charitable interpretation, Lewis was simply confused in conflating a connective (or operator) in the object language, 'if-then', with the metalinguistic predicate 'implies'. When speaking of one sentence φ implying another ψ, one mentions (or refers to) the sentences φ and ψ, while in a conditional "if φ, then ψ" the two sentences are used rather than mentioned. Thus, modal logic according to Quine was "conceived in sin, the sin of confusing use and mention". The confusion was supposed to have started with Russell when he named the standard truth-functional connective $\rightarrow$ material *implication*, and sometimes spoke of it as a kind of relation between sentences (or propositions):

> Lewis founded modern modal logic, but Russell provoked him to it. For whereas there is much to be said for the material conditional as a version of 'if-then', there is nothing to be said for it as a version of 'implies'; and Russell called it implication, thus apparently leaving no place open for genuine deductive connections between sentences. Lewis moved to save the connections. But his way was not, as one could have wished, to sort out Russell's confusion of 'implies' with 'if-then'. Instead, preserving that confusion, he propounded a strict conditional and called *it* implication.[1]

In spite of Quine's misgivings, the development in modal and intensional logic that Lewis had set in motion turned out to be irreversible. So the task became one of interpreting the new logics rather than fighting them.

Soon Lewis realized that his new binary connective $\rightthreetimes$ could be defined in terms of a unary intensional connective $\square$ of *logical necessity* together with material implication $\rightarrow$:

$$\varphi \rightthreetimes \psi =_{df.} \square(\varphi \rightarrow \psi).$$

Conversely, $\square$ and its dual $\Diamond$ (*logical possibility*) were definable from $\rightthreetimes$ as follows:

$$\square\varphi =_{df.} \neg\varphi \rightthreetimes \varphi$$
$$\Diamond\varphi =_{df.} \neg(\varphi \rightthreetimes \neg\varphi).$$

Hence, modal logic, narrowly conceived, could be pursued in terms of $\square$ (or $\Diamond$) rather than in terms of $\rightthreetimes$.

Lewis's original ambition was to find *the logic* of strict implication. Instead he found a multitude of different formal systems of propositional modal logic from which he was unable to select a unique one as being correct.

---

[1] Quine (1966).

The informal reading of $\Box\varphi$ was "It is logically necessary that φ". Thus, according to Lewis's intended interpretation,

$\Box\varphi$ is true iff φ is logically true.
$\Diamond\varphi$ is true iff φ is logically consistent (i.e., if $\neg\varphi$ is not logically true).

This interpretation of the alethic modalities, necessity and possibility, should be distinguished from the one that is predominant today, largely due to the influence of Saul Kripke. According to the latter interpretation, the meaning of the modal operators is explained informally as follows:

$\Box\varphi$ is true iff φ and it could not have been the case that $\neg\varphi$.
$\Diamond\varphi$ is true iff (either φ or) it could have been the case that φ.

The former interpretation of $\Box$ and $\Diamond$ we refer to as the *logical* one. The latter interpretation we speak of as the *metaphysical* interpretation (or the *counterfactual* one). We also speak of two kinds of alethic modalities: the logical ones and the metaphysical (or counterfactual ones). In what follows, I shall usually use L and M for logical necessity and possibility, respectively, and reserve $\Box$ and $\Diamond$ for the metaphysical modalities.

The first modern logician to have clearly distinguished between the logical and the counterfactual interpretation of the modal operators seems to have been Saul Kripke. According to Kripke's own account, he first presented the distinction in seminars at Harvard during the academic year 1963–64.[2]

Intuitively, logical necessity appears to imply metaphysical necessity (but see some alleged counterexamples below), but the converse does not seem to hold in general. It does not appear self-contradictory to think, as the Greek did, that water is an element. But since water, as it turned out, is a compound of oxygen and hydrogen, it could not have been an element. There is, so to speak, no counterfactual situation, or possible world, where *water* is not a compound. So even if it is not logically necessary, it appears metaphysically necessary that water is a compound – or so the story goes.

The logical interpretation splits into semantic approaches that interpret the necessity operator in terms of logical validity, and syntactic ones that interpret necessity in terms of provability. First, there is what might be called the *validity interpretation*:

$L\varphi$ is true iff φ is logically valid (true in all models)
$M\varphi$ is true iff φ is satisfiable (true in some model).

Then, there is the *provability interpretation*:

$L\varphi$ is true iff φ is provable (informally or in some given formal system)

---

[2] Cf. Burgess (to appear).

$M\varphi$ is true iff $\neg\varphi$ is not provable.

I am not going to consider the provability interpretation in the following.[3]

The pioneers of modal logic, among them C. I. Lewis, Rudolf Carnap, and Ruth Barcan Marcus, were mainly concerned with the logical modalities. Thus, Carnap in 'Modalities and Quantification' (1946) proposed that the necessity operator be interpreted in terms of logical or analytic truth, which in turn he analyzed as truth in all state descriptions for a given language $\mathcal{L}$. Carnap's interpretation is explicitly metalinguistic:

> The guiding idea in our constructions of systems of modal logic is this: a proposition p is logically necessary if and only if a sentence expressing p is logically true. That is to say, the modal concept of the logical necessity of a proposition and the semantical concept of the logical truth or analyticity of a sentence correspond to each other.
> ...Therefore it seems natural to interpret 'N' [i.e., L] in such a way that the following convention is always fulfilled:

> **C1-1**. If '...' is any sentence in a system S containing 'N', then the corresponding sentence '$N(...)$' is to be taken as true if and only if '...' is L-true in S.

Here L-truth in S is explicated as truth in all state descriptions of S (the system S may contain meaning postulates imposing "analytical" constraints on the set of state descriptions).

It was not before the middle of the 1940's that Ruth Barcan-Marcus and Rudolf Carnap, independently of each other, presented systems of modal predicate logic. Carnap also provided his system with a formal semantics and discussed its intuitive interpretation. At roughly the same time, Quine started his famous critique of modal logic in general and quantified modal logic in particular. In his paper 'The problem of interpreting modal logic' from 1947, Quine formulates what one might call *Quine's challenge* to the advocates of quantified modal logic:

> There are logicians, myself among them, to whom the ideas of modal logic (e. g. Lewis's) are not intuitively clear until explained in nonmodal terms. But so long as modal logic stops short of quantification theory, it is possible ... to provide somewhat the type of explanation required. When modal logic is extended (as by Miss Barcan) to include quantification theory, on the other hand, serious obstacles to interpretation are encountered—particularly if one cares to avoid a curiously idealistic ontology which repudiates material objects.

---

[3] A sophisticated version of the provability interpretation is given in so-called *provability logic* (of Solovay, Boolos, and others), where the sentence letters are interpreted as ranging over sentences of some formal theory T (often taken to be (first-order) Peano arithmetic) and $L\varphi$ is translated into a formula of T saying that φ (or rather, its translation in T) is provable in T. A famous result in provability logic is *Solovay's completeness theorem* saying that the system GL of propositional modal logic contains precisely those schemata that are valid when $L\varphi$ is interpreted as φ being provable in Peano arithmetic.

What Quine demands of the modal logicians is nothing less than an explanation of the notions of quantified modal logic in non-modal terms. Such an explanation should satisfy the following requirements:

(i) It should be expressed in an extensional language. Hence, it cannot use any non-extensional constructions.

(ii) The explanation should be allowed to use concepts from the 'theory of meaning' like analyticity and synonymy applied to expressions of the metalanguage. Quine is, of course, quite skeptical about the intelligibility of these notions as well. But he considers it to be progress of a kind, if modal notions could be explained in these terms.

(iii) The explanation should make sense of sentences like:

$$\exists x(x \text{ is red} \wedge M(x \text{ is blue})),$$

in which a quantifier outside a modal operator binds a variable within the scope of the operator and the quantifier ranges over ordinary physical objects (in distinction from Frege's "Sinne" or Carnap's "individual concepts"). In other words, the explanation should make sense of "quantifying in" into modal contexts.

Quine (1947)—like Carnap before him—starts out from a metalinguistic interpretation of the necessity operator L in terms of the predicate '... is analytically true':

$$L\varphi \text{ is true iff } \varphi \text{ is analytically true.}$$

A sentence φ is analytically true in a language $\mathcal{L}$ if it is a logical consequence of the 'meaning postulates' of the language.

Now Quine argues that it is impossible to combine logical (or analytical) necessity with a standard theory of quantification (over physical objects). The argument (a variation of "the Morning Star Paradox") is based on the premises:

(1) L(Hesperus = Hesperus)
(2)   Phosphorus = Hesperus
(3) $\neg L$(Phosphorus = Hesperus),

where 'Phosphorus' and 'Hesperus' are two proper names (individual constants) and L is to be read 'It is logically necessary that'.[4] We assume that 'Phosphorus' is used by the language community as a name for a certain bright heavenly object sometimes visible in the morning and that 'Hesperus' is used for some bright heavenly object sometimes visible in the evening. Unbeknownst to the community, however, these objects are one and the same, namely, the planet Venus. 'Hesperus = Hesperus' being an instance of

---

[4] Although Quine's preferred interpretation of L was in terms of analytic (or broadly logical) necessity rather than logical necessity in the strict, or narrow, sense, for our discussion we can as well let L stand for strict logical necessity.

the Law of Identity is clearly a logical truth. It follows that the premise (1) is true. (2) is true, as a matter of fact. 'Phosphorus = Hesperus' is obviously not a logical truth, 'Phosphorus' and 'Hesperus' being two different names with quite distinct uses. So, (3) is true.

From (1), (2), (3) and the Law of Identity, we infer by sentential logic:

(4)   Phosphorus = Hesperus $\wedge \neg L$(Phosphorus = Hesperus),

(5)   Hesperus = Hesperus $\wedge L$(Hesperus = Hesperus).

Applying (EG) to (4) and (5), we get:

(6) $\exists x(x = \text{Hesperus} \wedge \neg L(x = \text{Hesperus}))$,

(7) $\exists x(x = \text{Hesperus} \wedge L(x = \text{Hesperus}))$.

As Quine (1947) points out, however, (6) and (7) are incompatible with interpreting $\forall x$ and $\exists x$ as objectual quantifiers meaning "for all objects x (in the domain D)" and "for at least one object x (in D)" and letting the identity sign stand for genuine identity between objects (in D). Because, under this interpretation, (6) and (7) imply that one and the same object, Hesperus, both is and is not necessarily identical with Hesperus, which seems absurd.

Quine's challenge to the modal logician, thus, is to show how the logical interpretation of the necessity operator can be combined in a sensible way with quantifying in.

2 Combining logical and metaphysical necessity

In this section, we describe a formal semantics (model theory) that combines operators for logical and metaphysical necessity. We are also going to show that these operators can be combined with quantifiers in a natural way.

We consider a language $\mathcal{L}$ of first-order modal predicate logic with the following vocabulary:

1. *Logical symbols*: (a) a denumerable sequence of individual variables; (b) the identity symbol =; (c) Boolean connectives $\neg$, $\rightarrow$; (d) the universal quantifier $\forall$; (e) intensional operators: L (logical necessity), $\square$ (metaphysical necessity); A (the actuality operator).

2. *Non-logical symbols*: (a) for each $n \geq 1$, a set of predicate symbols of degree (arity) n; (b) possibly a set of individual constants. $\mathcal{L}$ is actually a family of languages; we get a specific language by specifying which non-logical constants it contains. However, we will continue to speak as if $\mathcal{L}$ is a well-defined language.

An expression is a *term* of $\mathcal{L}$ iff it is a variable or an individual constant of $\mathcal{L}$. The set of *formulas* of $\mathcal{L}$ is defined in the standard way. A *sentence* is a closed formula, i.e., a formula in which no variables are free.

Next, we want to provide the language $\mathcal{L}$ with a formal semantics (model theory) that accords with our intuitions concerning the interpretation of the operators $\Box$, **L**, **A**, and allows for quantifying into contexts governed by these operators. Moreover, we want the treatment of quantification not to lead to unintuitive results.

The general idea is to define a fairly standard notion of a (Kripke) model for $\mathcal{L}$ and provide $\Box$, **L**, and **A** with semantic clauses in such a way that:

(i) $\Box\varphi$ is true at a point ("possible world") u in a model $\mathcal{M}$ iff φ is true at every point w in $\mathcal{M}$;

(ii)   **L**φ is true at a point u in a model $\mathcal{M}$ iff φ is true at the designated point ("actual world") in every model $\mathcal{N}$;

(iii)   **A**φ is true at a point u in a model $\mathcal{M}$ iff φ is true at the designated point ("actual world") in $\mathcal{M}$.

In order to handle quantification, especially quantification into contexts governed by the logical necessity operator **L**, we are going to modify the standard treatment of variables. One can say that variables are treated as *logically rigid designators*: i.e., they are assigned values that are independent, not only of worlds within models, but also of the models themselves. We now come to the technical details.

Definition.

(a) A *frame* (for $\mathcal{L}$) is an ordered triple $\mathcal{F} = <U, E, @>$ where (i) U is a non-empty set; E is a function which to each $u \in U$ assigns a non-empty set E_u, and $@ \in U$. Intuitively we think of matters thus: U is the set of *points* ("possible worlds"); for each point u, E_u is the set of individuals that exist at u; and $@$ is the designated point ("the actual world").

(b) Next, let us say that I is an *interpretation* in $\mathcal{F}$ if it is a family of functions I_u, where u ranges over U, such that I_u assigns a set of ordered n-tuples $I_u(P)$ to each *n*-ary predicate symbol P of $\mathcal{L}$ and an object $I_u(c)$ to each individual constant c of $\mathcal{L}$. Thus, I assigns appropriate *extensions* to the non-logical symbols of $\mathcal{L}$ relative to points in U. Observe that we have not required that $I_u(P) \subseteq (E_u)^n$, or even that that $I_u(P) \subseteq (\cup_{v \in U} E_v)^n$. That is, the *extension* of P in u may involve objects that do not exist in u, or even exist at any point in U. Similarly, we do not require that $I_u(c) \in E_u$, or that $I_u(c) \in \cup_{v \in U} E_v$.

(c) A *model* is a structure $\mathcal{M} = <U, E, @, I>$, such that $\mathcal{F} = <U, E, @>$ is a frame and I is an interpretation in $\mathcal{F}$. We say that $\mathcal{M}$ is a model *based on* the frame $\mathcal{F}$.

(d) An *assignment g* is any function with the set of variables as its domain. For each variable x, $g(x)$ is the *value* assigned to x by the assignment g.

Next we define the notion of *truth* for formulas relative to models, points, and assignments. We write $u \vDash_{\mathcal{M}} \varphi[g]$ with the meaning: φ is *true at u in M relative to g*. When it is clear from the context which model we are referring to, we often suppress the reference to $\mathcal{M}$.

Definition. Let $\mathcal{M} = <U, E, @, I>$ be a model and g an assignment. The value of a term t of $\mathcal{L}$ at the point u in $\mathcal{M}$ relative to g (in symbols, $|t[g]|_u$) equals to $I_u(t)$, if t is an individual constant; and $g(t)$, if t is a variable.

(i) $u \vDash_{\mathcal{M}} P(t_1,..., t_n)[g]$ iff $<|t_1[g]|_u,..., |t_n[g]|_u> \in I_u(P)$;

(ii) $u \vDash_{\mathcal{M}} (t_1 = t_2)[g]$ iff $|t_1[g]|_u = |t_2[g]|_u$;

(iii)   the clauses for $\neg$ and $\rightarrow$ are the obvious ones;

(iv) $u \vDash_{\mathcal{M}} \forall x \varphi[g]$ iff for every $a \in E_u$, $u \vDash_{\mathcal{M}} \varphi[g(a/x)]$, where $g(a/x)$ is the assignment which is just like g except for assigning the value a to x;

(v) $u \vDash_{\mathcal{M}} A \varphi[g]$ iff $@_{\mathcal{M}} \vDash_{\mathcal{M}} \varphi[g]$;

(vi) $u \vDash_{\mathcal{M}} \Box \varphi[g]$ iff for every $v \in U$, $v \vDash_{\mathcal{M}} \varphi[g]$;

(vii) $u \vDash_{\mathcal{M}} L \varphi[g]$ iff for every model $\mathcal{N}$, $@_{\mathcal{N}} \vDash_{\mathcal{N}} \varphi[g]$.

A formula φ is *true at a point u in M* (in symbols: $u \vDash_{\mathcal{M}} \varphi$) iff for every assignment g, $u \vDash_{\mathcal{M}} \varphi[g]$. We say that φ is *true in the model M under the assignment g* (in symbols: $\vDash_{\mathcal{M}} \varphi[g]$) if $@_{\mathcal{M}} \vDash_{\mathcal{M}} \varphi[g]$. φ is *true in the model M* (in symbols: $\vDash_{\mathcal{M}} \varphi$) iff for every assignment g, $\vDash_{\mathcal{M}} \varphi[g]$. φ is *logically valid* (in symbols: $\vDash \varphi$) iff for every model $\mathcal{M}$, $\vDash_{\mathcal{M}} \varphi$. That is, φ is logically valid if and only if φ is true at the designated point in every model.

Notice that for every model $\mathcal{M}$ and every point u in $\mathcal{M}$,

$$u \vDash_{\mathcal{M}} L\varphi \text{ iff } \vDash \varphi,$$

i.e. $L\varphi$ is true at a point u iff φ is logically valid. Hence,

$$\vDash_{\mathcal{M}} L\varphi \text{ iff } \vDash \varphi.$$

That is, for any model $\mathcal{M}$, $L\varphi$ is true in $\mathcal{M}$ if and only if φ is logically valid.

Of course, we have the usual principles for the system **S5** for L as well as for $\Box$. Here, we write up these principles for L:

(K) $\vDash L(\varphi \rightarrow \psi) \rightarrow (L\varphi \rightarrow L\psi)$
(T) $\vDash L\varphi \rightarrow \varphi$
(S4) $\vDash L\varphi \rightarrow LL\varphi$
(S5) $\vDash \neg L\varphi \rightarrow L\neg L\varphi$
(Nec)   If $\vDash \varphi$, then $\vDash L\varphi$.

The following schemata are also valid:

(1)   ⊨ *Lφ* iff ⊨ *φ*.
(2)   ⊨ ¬*Lφ* iff ⊭ *φ*.
(3)   ⊨ *Lφ* iff ⊭ ¬*Lφ*.

Of course, none of these schemata are valid for □.
    For identity we have:

(LI)   ⊨ *t = t*.   (*Law of identity*)
(I=)   ⊨ $\forall x \forall y (x = y \rightarrow (\varphi(x/z) \rightarrow \varphi(y/z)))$.
                                               (*Indiscernibility of identicals*)

In addition, we have:

$$\vDash M \exists x_1 ... \exists x_n (x_1 \neq x_2 \wedge ... \wedge x_1 \neq x_n \wedge x_2 \neq x_3 \wedge ... \wedge x_2 \neq x_n \wedge ... \wedge x_{n-1} \neq x_n),$$

for every $n \geq 2$. That is, it is logically valid that it is logically possible that there exist n distinct individuals.

Theorem: Suppose $\mathcal{L}$ contains at least one binary predicate symbol. Then, the set of logically valid sentences is not recursively enumerable. Hence there is no formal axiomatic system with this set as its theorems.

    Proof: Suppose the set the set of logically valid sentences of $\mathcal{L}$ is recursively enumerable. Let φ be any sentence of the non-modal fragment $\mathcal{L}_0$ of $\mathcal{L}$. Then, we have:

(i)   Either ⊨ *Lφ* or ⊨ ¬*Lφ*.
(ii)   If ⊨ *Lφ*, then ⊨ *φ*.
(iii)   If ⊨ ¬*Lφ*, then ⊭ φ.

Since, by assumption, there is an effective enumeration of the logically true sentences of $\mathcal{L}$, we can effectively decide which of ⊨ *Lφ* or ⊨ ¬*Lφ* holds. But, this means in virtue of (ii) and (iii) that we can effectively decide which of ⊨ φ or ⊭ φ that holds. By Gödel's completeness theorem for the predicate calculus, for any sentence φ of $\mathcal{L}_0$, ⊨ φ holds if and only if φ is a theorem of the first-order predicate calculus. But this is contrary to Church's theorem according to which the pure predicate calculus is undecidable. Q. E. D.

3 Two kinds of rigidity

It is easy to infer the principles of *necessity of identity* and *necessity of distinctness* both for **L** and for □. We state the principles for metaphysical necessity:

(□=) $\forall x \forall y (x = y \rightarrow \Box(x = y))$
(□≠) $\forall x \forall y (x \neq y \rightarrow \Box(x \neq y))$

However, none of the following schemata:

(1) $c = d \rightarrow \Box(c = d)$
(2) $c \neq d \rightarrow \Box(c \neq d)$,

are valid, for arbitrary individual constants c, d.

Suppose now that we introduce a new syntactic category of *names* and require that the interpretation of a name $\boldsymbol{n}$ be constant over the set of all possible worlds in any model $\mathcal{M}$; formally,

$$I_u(\boldsymbol{n}) = I_v(\boldsymbol{n}),$$

for all $u, v \in U$. Then, if $\boldsymbol{n}$ and $\boldsymbol{m}$ are any names, then:

(3) $\boldsymbol{n} = \boldsymbol{m} \rightarrow \Box(\boldsymbol{n} = \boldsymbol{m})$
(4) $\boldsymbol{n} \neq \boldsymbol{m} \rightarrow \Box(\boldsymbol{n} \neq \boldsymbol{m})$,

are both valid. The proposed modification amounts to treating the elements of the new category of names as what is now known, after Kripke (1980), as *rigid designators*. However, even for names, the following principles fail:

(5) $\boldsymbol{n} = \boldsymbol{m} \rightarrow L(\boldsymbol{n} = \boldsymbol{m})$
(6) $\boldsymbol{n} \neq \boldsymbol{m} \rightarrow L(\boldsymbol{n} \neq \boldsymbol{m})$.

This suggests that we go even further and introduce another category of *logical names* (or *logically proper names*). Suppose that $\boldsymbol{k}$ is a logical name. We then require that:

> for some object a, the interpretation $I_u(\boldsymbol{k})$ is fixed once and for all to be a, for any model $\mathcal{M}$ and point u.

Then, if $\boldsymbol{k}$ and $\boldsymbol{l}$ are any logical names, then:

(7) $\boldsymbol{k} = \boldsymbol{l} \rightarrow L(\boldsymbol{k} = \boldsymbol{l})$
(8) $\boldsymbol{k} \neq \boldsymbol{l} \rightarrow L(\boldsymbol{k} \neq \boldsymbol{l})$,

are both valid. Since logical names are also assumed to be rigid (in the sense of Kripke) (3) and (4) hold also for logical names.

Is it intuitively meaningful to assume that there are any logical names? Perhaps certain names of abstract mathematical entities, like the symbol '$\emptyset$' for the empty set could be regarded as logical names.

From now on we assume that there are some (ordinary) names as well as some logical names in $\mathcal{L}$. In order to clarify the distinction between ordinary names and logical names, we might call the former *metaphysically rigid* and the latter *logically rigid*. Notice that we have treated variables as being logically rigid relative to assignments.

4 Quantifying in

With respect to the interplay between the quantifiers and the operator **L** it may be instructive to consider the following fallacious proof of the thesis that *everything there is exists by logical necessity*.

(1)	$\forall x \exists y(x = y)$	predicate logic
(2)	$\boldsymbol{L}\forall x \exists y(x = y)$	from (1) by (Nec)
(3)	$\forall x \exists y(x = y) \rightarrow \exists y(x = y)$	(US) (on variables)
(4)	$\boldsymbol{L}(\forall x \exists y(x = y) \rightarrow \exists y(x = y))$	from (3) by (Nec)
(5)	$\boldsymbol{L}\exists y(x = y)$	from (2) and (4) modal logic
(6)	$\forall x\boldsymbol{L}\exists y(x = y)$	from (5) by (UG)

According to our semantics, the fallacy is in line (3). The formula:

$$\forall x \exists y(x = y) \rightarrow \exists y(x = y)$$

is not valid. Suppose that $\mathcal{M} \vDash \forall x \exists y(x = y)[g]$. We can easily see to it that $\mathcal{M} \vDash \neg\exists y(x = y)[g]$ by letting $g(x)$ lie outside of the domain of the actual world in $\mathcal{M}$.

It is also of interest to point out that neither the Barcan formula nor its converse is valid for **L**:

(BF)	$\forall x\boldsymbol{L}\varphi \rightarrow \boldsymbol{L}\forall x\varphi$	(The Barcan formula)
(CBF)	$\boldsymbol{L}\forall x\varphi \rightarrow \forall x\boldsymbol{L}\varphi$	(The converse Barcan formula)

To see that (BF) fails consider a model $\mathcal{M}$ such that the empty set $\varnothing$ is the only object existing in the actual world of $\mathcal{M}$. Let $\boldsymbol{k}$ be a logical name denoting $\varnothing$. Then,

$$\mathcal{M} \vDash \forall x\boldsymbol{L}(x = \boldsymbol{k})[g],$$

but

$$\mathcal{M} \nvDash \boldsymbol{L}\forall x(x = \boldsymbol{k})[g]$$

Hence,

$$\mathcal{M} \nvDash \forall x\boldsymbol{L}(x = \boldsymbol{k}) \rightarrow \boldsymbol{L}\forall \mathrm{x}(x = \boldsymbol{k})[g].$$

Remark: If we allow free variables as parameters in the formula φ, we can use a variable y in the above argument instead of the logical name $\boldsymbol{k}$.

To see that (CBF) fails, we only need to notice that:

$$\vDash \boldsymbol{L}\forall x \exists y(x = y) \text{ but } \nvDash \forall x\boldsymbol{L}\exists y(x = y).$$

Of course, (BF) and (CBF) both fail for $\Box$.

In the presence of logical names, it is also easy to show that the schema:

$$\exists x\boldsymbol{L}\varphi \rightarrow \forall x\boldsymbol{L}\varphi$$

fails to be valid. Let k be a logical name in $\mathcal{L}$. Consider any model $\mathcal{M}$ in which the denotation of k exists in the actual world. Then,

$$\mathcal{M} \vDash \exists x L(x = k).$$

However, if the actual world of **M** contains more than one element, then

$$\mathcal{M} \nvDash \forall x L(x = k).$$

5 The relationship between logical and metaphysical necessity

In our semantics, logical necessity does not imply metaphysical necessity. For example, every instance of the following schema is valid:

(1) $L(\varphi \leftrightarrow A\varphi)$,

although the corresponding schema for metaphysical necessity fails (in both directions):

(2) $\square(\varphi \leftrightarrow A\varphi)$.

We can easily construct models $\mathcal{M}$ for which (2) fails.

Thus it appears, as Zalta (1988) has argued, that logical necessity does not imply metaphysical necessity. There are logical truths that are metaphysically contingent. However, this claim may appear to be counterintuitive. There are various ways of avoiding the conclusion that logical truth does not imply metaphysical necessity. One may, for one reason or another, refuse constructions like 'actually', that make reference to special worlds, the status of logical constants.

Another option is to modify the notion of logical truth. The notion of logical truth that we have employed is what might called *real-world validity*. It is the notion according to which a statement is logically true (valid) iff it is true at the actual world in each model. However, there is an alternative notion, *general validity*, according to which a statement is logically true iff it is true at each world in each model.[5]

Let us write $\vDash$ and $\vDash^*$ for real-world validity and general validity, respectively. The two notions are related as follows: For any statement φ,

(1) $\vDash \varphi$ iff $\vDash^* A\varphi$.
(2) $\vDash^* \varphi$ iff $\vDash \square\varphi$.

The operator L was introduced by "reflecting" the meta-linguistic notion of real-world validity into the object language. We can also introduce an

---

[5] Cf. Humberstone (2004, 22–24), for a comparison between the two concepts of logical truth (validity) and for the history of the distinction between the two.

operator L^* corresponding to the notion of general validity. The semantic clauses for L (*real-world logical necessity*) and L^* (*general logical necessity*) are:

(vii) $u \vDash_M L\varphi[g]$ iff for every model $\mathcal{N}, @_{\mathcal{N}} \vDash_{\mathcal{N}} \varphi[g]$.

(viii) $u \vDash_M L^*\varphi[g]$ iff for every model $\mathcal{N}$ and every point v in $\mathcal{N}, v \vDash_{\mathcal{N}} \varphi[g]$.

That is, L corresponds to truth at the actual world in each model and L^* corresponds to truth at every world in each model. The two notions of logical necessity are interdefinable:

(1) $\vDash^* L\varphi \leftrightarrow L^*A\varphi$
(2) $\vDash^* L^*\varphi \leftrightarrow L\square\varphi$.

Moreover, we have:

(3) $\vDash^* L^*\varphi \rightarrow \square\varphi$,

although, as we have seen, the corresponding implication does not hold for real-world logical necessity, i.e., for L.

Metaphysical necessity does not imply logical necessity. Thus, the statement:

(1)   Water is a compound

is metaphysically necessary (assuming that "water", is a rigid designator), but it is not logically necessary.

In conclusion, we can say that real-world logical necessity (L) neither implies nor is implied by metaphysical necessity ($\square$). General logical necessity (L^*), on the other hand, implies metaphysical necessity, but is not implied by it.

Appendix. A note on analyticity à la Rabinowicz and apriority

We consider Wlodek Rabinowicz's proposal—made in his contribution to the present volume—of explicating the notion of *analyticity* in model-theoretic terms. Rabinowicz also considers—without endorsing it—a model-theoretic approach to the notion of *apriority*. In this appendix, we expand on Rabinowicz's treatment of analyticity and apriority by introducing *operators* corresponding to these notions. Hence, we assume that the language $\mathcal{L}$ has been enriched with two new operators $\mathcal{An}$ and $\mathcal{Ap}$ for *analyticity* and *apriority*, respectively.

Following Rabinowicz, we start out from the idea that a sentence φ of an interpreted formal language being *analytically true* if it is true solely in vir-

tue of its meaning. Rabinowicz makes the observation that the kind of Kripke semantics that we have considered so far is ill suited for the purpose of defining analyticity. The notion of an *interpretation* that we have considered so far is only defined relative to a frame $\mathcal{F}$ consisting of a set $U_\mathcal{F}$ of points representing the possible worlds and a designated point $@_\mathcal{F}$ representing the actual world. In order to define analyticity we would like to have a notion of interpretation that we can keep constant while varying the frame. Rabinowicz's idea is to define a more general type of interpretation, namely a function *m* which assigns to every (admissible) frame $\mathcal{F}$ an interpretation of the old type in $\mathcal{F}$. Let us call an interpretation of the new kind a *meaning function*. To be precise, a meaning function is as a function m having a set $\mathcal{K}$ of frames as its domain and assigning to every frame $\mathcal{F}$ in $\mathcal{K}$ an interpretation $m(\mathcal{F})$ in $\mathcal{F}$. Relative to a meaning function *m*, we can say that a sentence φ is *analytically true* if for every frame $\mathcal{F}$ in the domain of *m*, φ is true relative to the frame $\mathcal{F}$ and the interpretation $m(\mathcal{F})$.[6]

If X is a term or a formula of $\mathcal{L}$, $\mathcal{F}$ is a frame in $\mathcal{K}$, and *u* is a point in $\mathcal{F}$, then we let $m_{\mathcal{F},u}(X)$ be the *extension* of X at the point *u* relative to $\mathcal{F}$ and *m*. That is, if X is a term, then $m_{\mathcal{F},u}(X)$ is the object that X denotes at *u* relative to $\mathcal{F}$ and $m(\mathcal{F})$; and if X is a sentence, then $m_{\mathcal{F},u}(X)$ is the truth-value of X at *u* relative to $\mathcal{F}$ and $m(\mathcal{F})$. The *intension of X relative to $\mathcal{F}$ and m*, in symbols $m_\mathcal{F}(X)$, is the function which to every point *u* in $\mathcal{F}$ yields the value $m_{\mathcal{F},u}(X)$. The *meaning of X relative to m*, in symbols $m(X)$, is the function assigning to every admissible frame $\mathcal{F}$ the intension $m_\mathcal{F}(X)$. Hence, *intensions* are functions from points in frames to extensions and *meanings* are functions from frames to intensions. According to Rabinowicz definition, a sentence φ with meaning $m(\varphi)$ is *analytically true* if for every frame $\mathcal{F}$ in the domain of *m*, φ is true in $\mathcal{F}$ relative to the interpretation $m(\mathcal{F})$. According to this definition, analyticity does not imply metaphysical necessity. Sentences of the form, $A\varphi \leftrightarrow \varphi$ are analytically true, but not in general necessary truths.

There is an alternative definition according to which analyticity implies metaphysical necessity. Let us say that a sentence φ with meaning $m(\varphi)$ is *strongly analytic* if for every frame $\mathcal{F}$ in the domain of *m*, φ is true at every point in $\mathcal{F}$ relative to the interpretation $m(\mathcal{F})$. In other words, φ is *strongly analytic* if $\square\varphi$ is analytic. Of course, strong analyticity implies analyticity (à la Rabinowicz), but the converse does not hold.

Intuitively, the truth of a (modal) statement depends on three factors: the meaning of the sentence φ, the structure of modal reality, and finally on which world is the actual one (i.e. represents how things actually are). This can be compared with the way the truth of a temporal statement depends on

---

[6] Stig Kanger defined a statement as being *analytic* if it was true in every domain under a fixed interpretation (see Kanger 1957a, 1957b and Kanger 1957c). This definition can be viewed as a precursor to Rabinowicz definition of analyticity within modern possible worlds semantics.

three factors: the meaning of the sentence, the structure of time, and the position of the present moment ("now") in the temporal structure. Within the present framework, we say that a sentence φ is *apriori true* if it is true and its truth is independent on which world is the actual one. φ is *analytically true* if its truth is independent not only on which world is the actual one, but on the nature of modal reality as well. A sentence is *necessary true* if given its meaning, the nature of modal reality, as well as the identity of the actual world, it is true not just in the actual world but in every possible world. Given these definitions all logical truths are analytic, all analytic truths are apriori, but apriority does not entail analyticity. Thus, there is within this framework room for synthetic apriori truths.

In order to give semantic clauses for the new operators $\mathcal{A}n$ and $\mathcal{A}p$, we need to modify the notion of a frame and that of a model. Therefore, we introduce the notion of a *generalized frame* $\mathcal{GF} = \langle\mathcal{K}, \mathcal{F}_0\rangle$, where $\mathcal{K}$ is a set of frames in the previous sense (the *admissible frames* of $\mathcal{GF}$) and $\mathcal{F}_0$ is a member of $\mathcal{K}$ (the *designated frame*). We assume that the set of admissible frames of $\mathcal{GF}$ satisfies Rabinowicz' condition of closure:

Closure: If $\langle U, E, @\rangle \in \mathcal{K}$ and $u \in U$, then $\langle U, E, u\rangle \in \mathcal{K}$.

We say that $\langle U, E, u\rangle$ is an *actual-world variant* of $\langle U, E, @\rangle$, i.e., two frames are actual-world variants of each other, if they differ only in which world is the actual one.

A *generalized model* for $\mathcal{L}$ is a structure $\mathcal{GM} = \langle\mathcal{K}, \mathcal{F}_0, m\rangle$ such that $\langle\mathcal{K}, \mathcal{F}_0\rangle$ is a generalized frame and m, the *meaning assignment* of $\mathcal{GM}$, is a function assigning to every frame $\mathcal{F} = \langle U, E, @\rangle$ in $\mathcal{K}$ a model $\mathcal{M} = \langle U, E, @, I\rangle$ based on $\mathcal{F}$.

Let $\mathcal{GM} = \langle\mathcal{K}, \mathcal{F}_0, m\rangle$ be a generalized frame for $\mathcal{L}$ and g an assignment. We define what it means for a formula φ to be true at a point u in a frame $\mathcal{F} \in \mathcal{K}$ relative to g (in symbols: $\langle\mathcal{F}, u\rangle \vDash_{\mathcal{GM}} \varphi[g]$):

(i)   if φ is atomic, $\langle\mathcal{F}, u\rangle \vDash_{\mathcal{GM}} \varphi[g]$ iff $u \vDash_{m(\mathcal{F})} \varphi[g]$;

(ii)   the clauses for $\neg$ and $\rightarrow$ are the obvious ones;

(iii) $\langle\mathcal{F}, u\rangle \vDash_{\mathcal{GM}} \forall x\varphi[g]$ iff for every $a \in E_u$:
$\langle\mathcal{F}, u\rangle \vDash_{\mathcal{GM}} \varphi[g(a/x)]$,
where $g(a/x)$ is the assignment which is just like g except for assigning the value a to x;

(iv) $\langle\mathcal{F}, u\rangle \vDash_{\mathcal{GM}} A\varphi[g]$ iff $\langle\mathcal{F}, @_{\mathcal{F}}\rangle \vDash_{\mathcal{GM}} \varphi[g]$.

The semantic clauses for $\square$ and L are essentially the same as before:

(v) $\langle\mathcal{F}, u\rangle \vDash_{\mathcal{GM}} \square\varphi[g]$ iff for every $v \in U_{\mathcal{F}}$, $\langle\mathcal{F}, v\rangle \vDash_{\mathcal{GM}} \varphi[g]$;

(vi) $\langle\mathcal{F}, u\rangle \vDash_{\mathcal{GM}} L\varphi[g]$ iff for every model $\mathcal{N}$: $@_{\mathcal{N}} \vDash_{\mathcal{N}} \varphi[g]$, where $@_{\mathcal{N}}$ is the designated element of $\mathcal{N}$.

For the new operators $\mathcal{A}n$ and $\mathcal{A}p$, we have the following semantic clauses:

(vii) $<\mathcal{F}, u> \vDash_{\mathcal{GM}} \mathcal{A}n\varphi[g]$ iff for every frame $\mathcal{G}$ in $\mathcal{K}$:
$<\mathcal{G}, @_{\mathcal{G}}> \vDash_{\mathcal{GM}} \varphi[g]$;

(viii) $<\mathcal{F}, u> \vDash_{\mathcal{GM}} \mathcal{A}p\varphi[g]$ iff for every actual world variant $\mathcal{G}$ of
$\mathcal{F}$: $<\mathcal{G}, @_{\mathcal{G}}> \vDash_{\mathcal{GM}} \varphi[g]$.

A formula φ is *true at a point u* in a frame $\mathcal{F}$ in $\mathcal{GM}$ (in symbols: $<\mathcal{F},-u>$-$\vDash_{\mathcal{GM}} \varphi$) iff for every assignment g, $<\mathcal{F}, u> \vDash_{\mathcal{GM}} \varphi[g]$. We say that φ is *true in the generalized model* $\mathcal{GM}$ *under the assignment g* (in symbols: $\vDash_{\mathcal{GM}}-\varphi[g]$) if $<\mathcal{F}_0, @> \vDash_{\mathcal{GM}} \varphi[g]$. φ is *true in the generalized model* $\mathcal{GM}$ (in symbols: $\vDash_{\mathcal{GM}}-\varphi$) iff for every assignment g, $\vDash_{\mathcal{GM}}-\varphi[g]$. Thus, φ is true in $\mathcal{GM}$ if and only if φ is true in the actual world in the designated frame in $\mathcal{GM}$. φ is *logically valid* (in symbols: $\vDash-\varphi$) iff for every generalized model $\mathcal{GM}$, $\vDash_M \varphi$. That is, φ is logically valid if and only if φ is true at the designated frame in every generalized model.

We end with some examples. Consider,

(1)   Phosphorus = Hesperus,

where 'Phosphorus' and 'Hesperus' are two names (rigid designators) for the planet Venus. Suppose 'Phosphorus' and 'Hesperus' are synonymous with rigidified definite descriptions 'the object x such that Actually P(x)' and 'the object x such that Actually Q(x)'. Even if (1) is true from the perspective of the actual world, it might not have been true had another world been the actual one. That is, the truth of (1) is not independent of which world is the actual world. Hence, (1), although being necessarily true, is not apriori.

Consider the sentence:

(2)   1 meter = the length of the standard meter in Paris.

We suppose that 1 meter is conventionally fixed to be the actual length of the standard meter in Paris. We also assume that 'the standard meter in Paris' is a rigid designator. Then, the sentence (2) is apriori true: it is true regardless of which possible world is the actual one. Presumably, it is also analytically true: it is true by stipulation, so it is true in all admissible frames. On the other hand, as Kripke (1980) points out, (2) is not necessarily true. The standard meter could have been longer than it actually is. But then, (2) would have been false. Since, (2) is not necessarily true, it is not strongly analytic either.

Consider,

(3)   All bachelors are unmarried
(4)   Whatever is red is colored.

On the intended readings, these sentences are strongly analytical: in every admissible frame, they are true in every possible world.

Next, consider,

(5)   Cats are animal

(6)   Cats are animals → □Cats are animals.

Kripke (1980) argues that (5) is empirical, and hence not apriori. (6) on the other hand, could, perhaps, be established by apriori reasoning. So (6) appears to be apriori. However, it is dubious whether (6) is analytically true. So, perhaps, (6) would be an example of a *synthetic apriori truth*.

Finally, consider

(7)   1 is the successor of 0.

Assuming that '0' and '1' are logically rigid, (7) is presumably strongly analytic.

It is left for another occasion to explore to what extent the model-theoretic definitions of analyticity and apriority that are presented here are intuitively reasonable.

References

Burgess, J. P., To appear, 'Kripke models', To appear in Cambridge University Press Contemporary Philosophers series, volume on Saul Kripke, edited by Alan Berger.

Carnap, R., 1946, 'Modalities and quantification', *The Journal of Symbolic Logic* 11, 33–64.

Carnap, R., 1947, *Meaning and Necessity: A Study in Semantics and Modal Logic*, Chicago: University of Chicago Press. Second edition with supplements, 1956.

Holmström-Hintikka, G, Lindström, S., Sliwinski, R. (eds.), 2001a, *Collected Papers of Stig Kanger with Essays on his Life and Work*, Volume I, Synthese Library Vol. 303, Dordrecht: Kluwer.

Humberstone, L., 2004, 'Two-dimensional adventures', *Philosophical Studies* 118, 17–65.

Kanger, S., 1957a, *Provability in Logic,* Acta Universitatis Stockholmiensis, *Stockholm Studies in Philosophy* 1, Stockholm: Almqvist & Wiksell. Reprinted in Holmström-Hintikka et al. (2001a).

Kanger, S., 1957b, 'The morning star paradox', *Theoria* 23, 1–11. Reprinted in Holmström–Hintikka et al. (2001a).

Kanger, S., 1957c, 'New Foundations for Ethical Theory', mimeographed, Stockholm University. Reprinted in Holmström-Hintikka et al. (2001a).

Kripke, S., 1963a, 'Semantic considerations on modal logic', in *Proceedings of a Colloquium on Modal and Many-Valued Logics,* Helsinki, 23–26 August, 1962. *Acta Philosophica Fennica* 16, 83–94. Reprinted in Linsky (1971).

Lewis, C. I., 1912, 'Implication and the algebra of logic', *Mind* 21, 522–531.

Lindström, S. and Segerberg, K. (to appear) 'Modal logic and Philosophy', in van Benthem, Blackburn, P., and Wolter F. (eds.) *Handbook of Modal Logic*, Elsevier.

Quine, W. V., 1947, 'The problem of interpreting modal logic', *The Journal of Symbolic Logic* 12, 43–48.

Quine, W. V., 1966, 'Reply to Professor Marcus', in *The Ways of Paradox and Other Essays,* Cambridge, Mass.: Harvard University Press.

Rabinowicz, Wlodek, 2006, 'Analyticity: An unfinished business in possible-world semantics', this volume.

Zalta, E., 1988, 'Logical and analytic truths that are not necessary', *The Journal of Philosophy*, Vol. 85, 57-74.

von Wright, G. H., 1951, *An Essay in Modal Logic*, Amsterdam: North-Holland.