Decision Making : A Quantum Mechanical Analysis Based On Time Evolution of Quantum Wave Function and of Quantum Probabilities during Perception and Cognition of Human Subjects.

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Abstract: We apply our theory of quantum time dynamics of cognitive entities to experimental data in order to calculate the time evolution of quantum wave function and of quantum probabilities during cognitive processes of the human subject. We introduce some criteria to evaluate the Time of Perceptual Decision of the human subject engaged with a task, and on this basis we estimate theoretical predictions that result in satisfactory agreement with the experimental data.

Keywords: decision making, quantum decision making, quantum mechanics, quantum cognitive process, quantum cognitive entities, wave function of mind entities, quantum probability, time evolution of quantum systems, quantum dynamics of cognitive entities, human perception, cognitive process.

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1. Introduction: Application of Quantum Mechanics to Cognitive Processes in Psychology

The aim of the present paper is to estimate for the first time the time evolution of the quantum wave function and of quantum probabilities during the process of perception-cognition of an human subject, and to give on this basis an explanation in quantum mechanical terms of such basic mind mechanism. We introduce some criteria to evaluate the Time of Perceptual Decision of the human subject engaged with a task, and on this basis we estimate theoretical predictions that result in satisfactory agreement with the experimental data.

The results are obtained on the basis of a previous performed experiment (Conte, 2009; Conte, 2009). The theory on this matter was formulated by us in 2005 (Conte, 2007). In a number of previous papers (Conte, 2002, 2003, 2006, 2008), we also exposed the general features of our formulation of mind entities that is based on the statement that quantum mechanics is a "physical" theory of cognitive processes of the mind.

2. The Theoretical Background

The experiments of perception-cognition were performed on a group of subjects as it was described in our papers (Conte, 2009; Conte, 2009). To summarize here, a quantum dichotomous mental observable, $B = \pm$, was measured and it was considered a quantum wave function of a superposition of states

$$\psi = c_+\varphi_+ + c_-\varphi_- \tag{2.1}$$

with

$$|c_+|^2 + |c_-|^2 = 1 \quad , \quad \varphi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad \varphi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.2}$$

The experimental probabilities were obtained

$$P_+ = P(B = +) = |c_+|^2 = 0.6667 \quad \text{and} \quad P_- = P(B = -) = |c_-|^2 = 0.3333 \tag{2.3}$$

The problem to study the time evolution of such a quantum system in psychology during cognition of an human subject, was previously considered by D. Aerts (Arts, 2003), by A. Khrennikov (Khrennikov, 2003), by J. R. Busemeyer (Busemeyer, 2006 and following papers). Based on a previous approach of C. Altafini (Altafini, 2003; Magnus, 1954), we developed in 2005 our theory of quantum time dynamics of cognitive entities. In appendix A we report an exposition of such theory. In this paper we are interested to the application of some basic formulas that are given in the following manner.

The hamiltonian H of the cognitive entity, as derived by this theory, is fully linear time invariant (see the (3,3)) and its exponential solution will take the following form

$$e^{t(\sum_{j=1}^{3}(a_j+b_j)A_j)} = \cos(\frac{k}{2}t)I + \frac{2}{k}\mathrm{sen}(\frac{k}{2}t)\left(\sum_{j=1}^{3}(a_j+b_j)A_j\right) \tag{2.4}$$

with $k=\sqrt{(a_1+b_1)^2+(a_2+b_2)^2+(a_3+b_3)^2}$. (2.5)

Still, it will result that

$$U(t)=\begin{pmatrix} \cos\frac{k}{2}t+\frac{i}{k}\mathrm{sen}\frac{k}{2}t(a_3+b_3) & \frac{1}{k}\mathrm{sen}\frac{k}{2}t[a_2+b_2+i(a_1+b_1)] \\ \frac{1}{k}\mathrm{sen}\frac{k}{2}t[-a_2-b_2+i(a_1+b_1)] & \cos\frac{k}{2}t-\frac{i}{k}\mathrm{sen}\frac{k}{2}t(a_3+b_3) \end{pmatrix} \tag{2.6}$$

and, obviously, it will result to be uni-modular as required. This is the matrix representation of time evolution operator for the considered cognitive entity.

The expression of the state $\psi(t)$, the quantum wave function of the cognitive entity at any time, will be given in the following manner

$$\begin{aligned}\psi(t)=&\left[c_+\left[\cos\frac{k}{2}t+\frac{i}{k}sen\frac{k}{2}t(a_3+b_3)\right]+c_-\left[\frac{1}{k}sen\frac{k}{2}t[(a_2+b_2)+i(a_1+b_1)]\right]\right]\varphi_+ + \\ &\left[c_+\left[\frac{1}{k}sen\frac{k}{2}t[i(a_1+b_1)-(a_2+b_2)]\right]+c_-\left[\cos\frac{k}{2}t-\frac{i}{k}sen\frac{k}{2}t(a_3+b_3)\right]\right]\varphi_-\end{aligned} \tag{2.7}$$

Finally, the two probabilities $P_+(t)$ and $P_-(t)$ that are expected for future selection and decision to $B=\pm$, as consequence of cognition measurement and context influence, will be given at any time t by the following expressions

$$P_+(t)=(A^2+B^2)\cos^2\frac{k}{2}t+\frac{1}{k^2}sen^2\frac{k}{2}t(P^2+Q^2)+\frac{senkt}{k}(AP+BQ)$$

and (2.8)

$$P_-(t)=(C^2+D^2)\cos^2\frac{k}{2}t+\frac{1}{k^2}sen^2\frac{k}{2}t(S^2+R^2)+\frac{senkt}{k}(RC+DS)$$

where

A= Re c_+ , B=Im c_+, C=Re c_- , D=Im c_- ,

$P=-D(a_1+b_1)+C(a_2+b_2)-B(a_3+b_3)$,

$Q=C(a_1+b_1)+D(a_2+b_2)+A(a_3+b_3)$, (2.9)

$R=-B(a_1+b_1)-A(a_2+b_2)+D(a_3+b_3)$,

$S=A(a_1+b_1)-B(a_2+b_2)-C(a_3+b_3)$.

3. The Numerical Analysis of the Experiment

We may now evaluate the results of the experiment that was performed. We obtained the (2.1) with

$|c_+| = 0.8165$ and $|c_-| = 0.5773$

and

$$\cos\vartheta = -0.3563\,. \qquad (3.1)$$

Consequently, according to the (2.9), we have that

$$A = 0.8165\cos\vartheta\,;$$
$$B = 0.8165\, sen\,\vartheta\,; \qquad (3.2)$$
$$C = 0.5773\cos\vartheta\,;$$
$$D = 0.5773\, sen\,\vartheta\,;$$

We may now express the probabilities, $P_+(t)$ and $P_-(t)$, given in the (2.8), for $B = \pm$ as result of quantum evolution during cognition.

First write the Hamiltonian of the subject during perception-cognition. According to the (26) of Appendix A, we have that

$$H(t) = -\frac{1}{2}(a_1e_1 + a_2e_2 + a_3e_3) - \frac{1}{2}(b_1e_1 + b_2e_2 + b_3e_3) \qquad (3.3)$$

Note that the a_i $(i = 1,2,3)$ relate the inner neurological and psychological state or condition of the human subject while the b_i relate instead the interaction of the subject with the outsider stimulus that intervenes in his perception-cognition during the posed task.

Roughly and in a preliminary way we will consider in this paper two basic cases, that one with

(1) $a_i = b_i$ $(a_i = b_i = 1)$,

and the other interesting case of

(2) $a_1 << b_1$, $a_2 << b_2$, and $a_3 = b_3$, fixing

$a_1 = a_2 = 3 \quad ; b_1 = b_2 = 20 \quad ; a_3 = b_3 = 5$.

Obviously, we present here only some simple cases that instead require more careful consideration under the psychological and neurological profiles. In particular, the $b_i(t)$ relate the rate at which features of the task stimuli are integrated with human memorial representations and cognitive performance during the presentation of the task. Therefore, they must be analyzed with particular consideration. We give here only preliminary results .

The Hamiltonian in the case (1) becomes

$$H(t) = H = -\frac{1}{2}\begin{pmatrix} 1 & 1-i \\ 1+i & -1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 1 & 1-i \\ 1+i & -1 \end{pmatrix}$$

while in the case (2) it is

$$H(t) = H = -\frac{1}{2}\begin{pmatrix} 5 & 3-3i \\ 3+3i & -5) \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 5 & 20-20i \\ 20+20i & -5 \end{pmatrix}$$

The probabilities, given in the (2.8), may be now calculated in the case (1) and in the case (2), respectively.

In the case (1) we have the results given in Table 1, and in Figures 1, 2, 3.

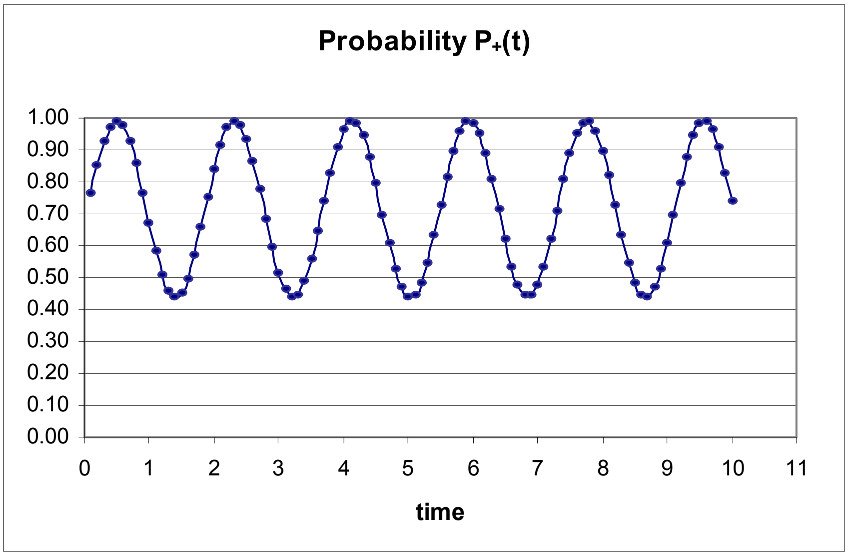

Figure 1

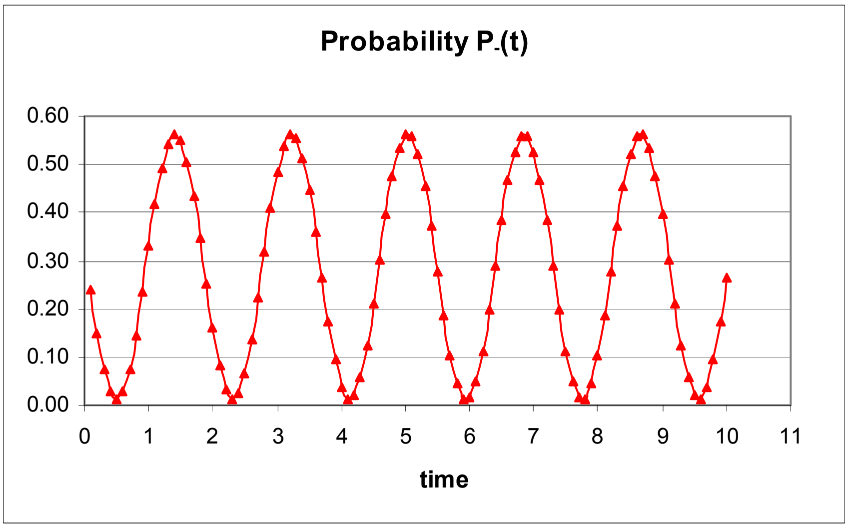

Figure 2

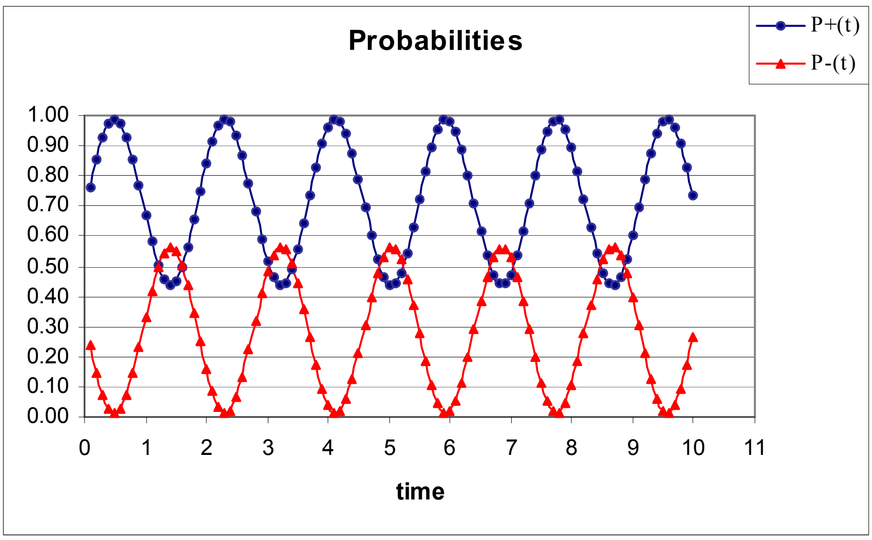

Figure 3

Table 1

time	$P_+(t)$	$P_-(t)$	$P_+(t)+P-(t)$	time	$P_+(t)$	$P_-(t)$	$P_+(t)+P-(t)$
0.1	0.7618	0.2381	0.9999	5.1	0.4434	0.5566	0.9999
0.2	0.8511	0.1489	0.9999	5.2	0.4799	0.5201	0.9999
0.3	0.9239	0.0760	0.9999	5.3	0.5440	0.4559	0.9999
0.4	0.9717	0.0283	0.9999	5.4	0.6283	0.3717	0.9999
0.5	0.9887	0.0113	0.9999	5.5	0.7225	0.2775	0.9999
0.6	0.9728	0.0271	0.9999	5.6	0.8156	0.1844	0.9999
0.7	0.9261	0.0738	0.9999	5.7	0.8964	0.1036	0.9999
0.8	0.8540	0.1459	0.9999	5.8	0.9554	0.0445	0.9999
0.9	0.7651	0.2348	0.9999	5.9	0.9856	0.0144	0.9999
1	0.6700	0.3299	0.9999	6	0.9833	0.0166	0.9999
1.1	0.5800	0.4200	0.9999	6.1	0.9489	0.0510	0.9999
1.2	0.5057	0.4943	0.9999	6.2	0.8865	0.1135	0.9999
1.3	0.4560	0.5440	0.9999	6.3	0.8033	0.1966	0.9999
1.4	0.4368	0.5632	0.9999	6.4	0.7094	0.2905	0.9999
1.5	0.4504	0.5496	0.9999	6.5	0.6159	0.3840	0.9999
1.6	0.4951	0.5049	0.9999	6.6	0.5339	0.4660	0.9999
1.7	0.5657	0.4343	0.9999	6.7	0.4731	0.5268	0.9999
1.8	0.6537	0.3462	0.9999	6.8	0.4408	0.5591	0.9999
1.9	0.7488	0.2512	0.9999	6.9	0.4408	0.5592	0.9999
2	0.8395	0.1604	0.9999	7	0.4731	0.5269	0.9999
2.1	0.9152	0.0847	0.9999	7.1	0.5338	0.4662	0.9999
2.2	0.9668	0.0331	0.9999	7.2	0.6158	0.3842	0.9999
2.3	0.9883	0.0117	0.9999	7.3	0.7093	0.2907	0.9999
2.4	0.9769	0.0230	0.9999	7.4	0.8032	0.1968	0.9999
2.5	0.9342	0.0657	0.9999	7.5	0.8863	0.1136	0.9999
2.6	0.8652	0.1348	0.9999	7.6	0.9489	0.0511	0.9999
2.7	0.7780	0.2219	0.9999	7.7	0.9833	0.0166	0.9999
2.8	0.6831	0.3168	0.9999	7.8	0.9856	0.0143	0.9999
2.9	0.5917	0.4083	0.9999	7.9	0.9555	0.0445	0.9999
3	0.5146	0.4853	0.9999	8	0.8965	0.1034	0.9999
3.1	0.4611	0.5388	0.9999	8.1	0.8157	0.1843	0.9999
3.2	0.4375	0.5625	0.9999	8.2	0.7226	0.2773	0.9999
3.3	0.4466	0.5534	0.9999	8.3	0.6284	0.3716	0.9999
3.4	0.4872	0.5127	0.9999	8.4	0.5442	0.4558	0.9999
3.5	0.5547	0.4453	0.9999	8.5	0.4800	0.5200	0.9999
3.6	0.6409	0.3590	0.9999	8.6	0.4434	0.5566	0.9999
3.7	0.7357	0.2643	0.9999	8.7	0.4388	0.5611	0.9999
3.8	0.8277	0.1723	0.9999	8.8	0.4668	0.5332	0.9999
3.9	0.9060	0.0939	0.9999	8.9	0.5240	0.4760	0.9999
4	0.9614	0.0385	0.9999	9	0.6036	0.3964	0.9999
4.1	0.9872	0.0127	0.9999	9.1	0.6961	0.3038	0.9999
4.2	0.9804	0.0195	0.9999	9.2	0.7906	0.2093	0.9999
4.3	0.9418	0.0581	0.9999	9.3	0.8759	0.1241	0.9999
4.4	0.8760	0.1239	0.9999	9.4	0.9418	0.0582	0.9999
4.5	0.7908	0.2092	0.9999	9.5	0.9804	0.0195	0.9999
4.6	0.6963	0.3037	0.9999	9.6	0.9873	0.0127	0.9999
4.7	0.6037	0.3963	0.9999	9.7	0.9615	0.0385	0.9999
4.8	0.5241	0.4759	0.9999	9.8	0.9061	0.0938	0.9999
4.9	0.4668	0.5331	0.9999	9.9	0.8278	0.1721	0.9999
5	0.4388	0.5611	0.9999	10	0.7358	0.2642	0.9999

In the case (2) we have the results that are given in Table 2, and in Figures 4, 5, 6.

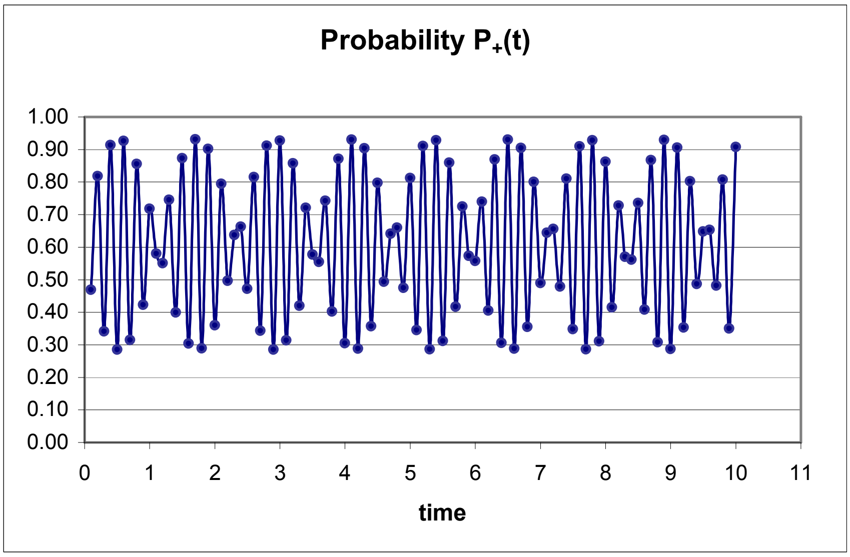

Figure 4

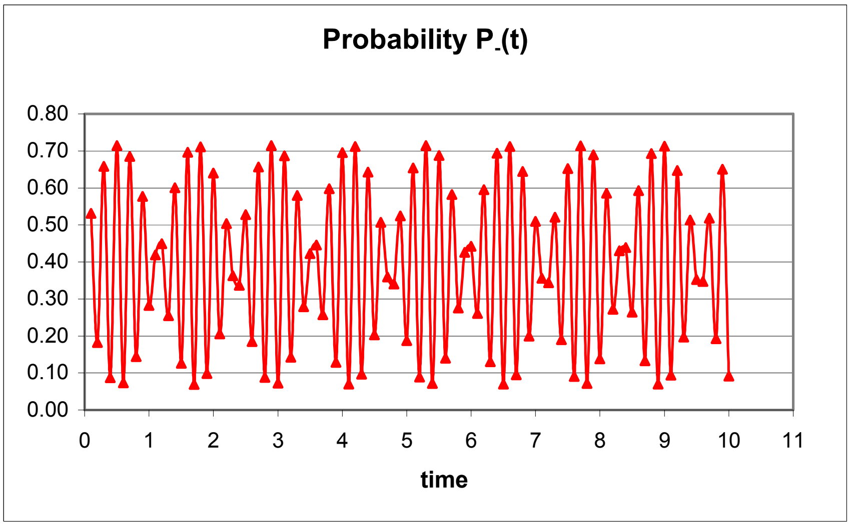

Figure 5

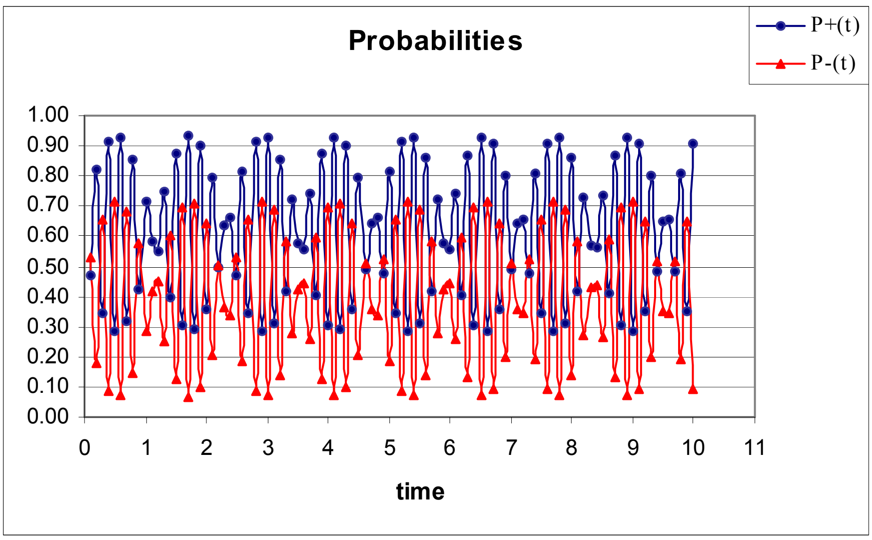

Figure 6

Table 2

time	$P_+(t)$	$P_-(t)$	$P_+(t)+P\text{-}(t)$	time	$P_+(t)$	$P_-(t)$	$P_+(t)+P\text{-}(t)$
0.1	0.4690	0.5310	0.9999	5.1	0.3455	0.6544	0.9999
0.2	0.8179	0.1821	0.9999	5.2	0.9106	0.0893	0.9999
0.3	0.3415	0.6585	0.9999	5.3	0.2858	0.7142	0.9999
0.4	0.9130	0.0869	0.9999	5.4	0.9279	0.0720	0.9999
0.5	0.2851	0.7148	0.9999	5.5	0.3121	0.6879	0.9999
0.6	0.9267	0.0732	0.9999	5.6	0.8598	0.1402	0.9999
0.7	0.3150	0.6849	0.9999	5.7	0.4174	0.5826	0.9999
0.8	0.8553	0.1446	0.9999	5.8	0.7244	0.2755	0.9999
0.9	0.4231	0.5768	0.9999	5.9	0.5736	0.4263	0.9999
1	0.7178	0.2821	0.9999	6	0.5580	0.4420	0.9999
1.1	0.5806	0.4193	0.9999	6.1	0.7390	0.2609	0.9999
1.2	0.5510	0.4489	0.9999	6.2	0.4049	0.5951	0.9999
1.3	0.7454	0.2545	0.9999	6.3	0.8694	0.1305	0.9999
1.4	0.3994	0.6005	0.9999	6.4	0.3060	0.6939	0.9999
1.5	0.8735	0.1264	0.9999	6.5	0.9300	0.0699	0.9999
1.6	0.3035	0.6964	0.9999	6.6	0.2878	0.7122	0.9999
1.7	0.9307	0.0692	0.9999	6.7	0.9046	0.0953	0.9999
1.8	0.2889	0.7110	0.9999	6.8	0.3551	0.6449	0.9999
1.9	0.9017	0.0982	0.9999	6.9	0.8000	0.1999	0.9999
2	0.3595	0.6404	0.9999	7	0.4899	0.5100	0.9999
2.1	0.7943	0.2056	0.9999	7.1	0.6441	0.3558	0.9999
2.2	0.4965	0.5035	0.9999	7.2	0.6563	0.3437	0.9999
2.3	0.6371	0.3628	0.9999	7.3	0.4786	0.5214	0.9999
2.4	0.6632	0.3367	0.9999	7.4	0.8097	0.1902	0.9999
2.5	0.4722	0.5278	0.9999	7.5	0.3476	0.6523	0.9999
2.6	0.8152	0.1848	0.9999	7.6	0.9093	0.0906	0.9999
2.7	0.3435	0.6565	0.9999	7.7	0.2862	0.7138	0.9999
2.8	0.9118	0.0881	0.9999	7.8	0.9285	0.0715	0.9999
2.9	0.2854	0.7145	0.9999	7.9	0.3107	0.6893	0.9999
3	0.9273	0.0726	0.9999	8	0.8620	0.1380	0.9999
3.1	0.3135	0.6864	0.9999	8.1	0.4146	0.5854	0.9999
3.2	0.8576	0.1424	0.9999	8.2	0.7277	0.2722	0.9999
3.3	0.4202	0.5797	0.9999	8.3	0.5701	0.4298	0.9999
3.4	0.7211	0.2788	0.9999	8.4	0.5614	0.4385	0.9999
3.5	0.5771	0.4228	0.9999	8.5	0.7358	0.2642	0.9999
3.6	0.5545	0.4454	0.9999	8.6	0.4076	0.5923	0.9999
3.7	0.7422	0.2577	0.9999	8.7	0.8673	0.1326	0.9999
3.8	0.4021	0.5978	0.9999	8.8	0.3073	0.6926	0.9999
3.9	0.8715	0.1285	0.9999	8.9	0.9296	0.0703	0.9999
4	0.3048	0.6952	0.9999	9	0.2873	0.7127	0.9999
4.1	0.9304	0.0696	0.9999	9.1	0.9060	0.0939	0.9999
4.2	0.2883	0.7116	0.9999	9.2	0.3529	0.6470	0.9999
4.3	0.9032	0.0968	0.9999	9.3	0.8028	0.1971	0.9999
4.4	0.3573	0.6427	0.9999	9.4	0.4866	0.5133	0.9999
4.5	0.7972	0.2028	0.9999	9.5	0.6476	0.3523	0.9999
4.6	0.4932	0.5068	0.9999	9.6	0.6528	0.3472	0.9999
4.7	0.6406	0.3593	0.9999	9.7	0.4818	0.5181	0.9999
4.8	0.6597	0.3402	0.9999	9.8	0.8070	0.1930	0.9999
4.9	0.4754	0.5246	0.9999	9.9	0.3497	0.6502	0.9999
5	0.8125	0.1875	0.9999	10	0.9080	0.0919	0.9999

4. The Evaluation of the Obtained Results

Let us start with examination of the results obtained in the case (1) that is

$$a_i = b_i \qquad (a_i = b_i = 1).$$

This is the case in which we admit that it exists a strong balance between the inner psychological condition of the human subject at the moment of the submitted task and the interaction that it is established with his mind state when the task is given. From the data we deduce that both $P_+(t)$ and $P_-(t)$, at the moment of perception and cognition, start to fluctuate in time with $P_+(t)$ oscillating between a minimum value about 0.45 and a maximum value about 0.95 while $P_-(t)$ oscillates between a minimum value approaching zero and a maximum value about 0.55. As in a "quantum random walk", the mind of the subject oscillates with regularity between such different values of $P_+(t)$ and $P_-(t)$ at each time step, integrating, at each time step, the features of the task stimulus with his memorial representation and cognitive performance that is based on his mind-structure and his fixed threshold criteria. Fluctuations of $P_+(t)$ result greater of fluctuations for $P_-(t)$ at each time step with the only but fundamental exception of the time steps corresponding to maximum uncertainty for the subject. In this case there is overlap. On the basis of such regular mechanism of fluctuations in the values of probabilities and, according to the previously mentioned threshold criteria, the subject performs his final decision in a given "response time" or "reaction time", RT.

In the case (2) that is

$a_1 << b_1$, $a_2 << b_2$, and $a_3 = b_3$,

with

$a_1 = a_2 = 3 \;\; ; b_1 = b_2 = 20 \;\; ; \;\; a_3 = b_3 = 5$

the strong balance between the inner psychological condition of the human subject at the moment of the submitted task and the interaction that it is established with his mind state when this task is given, is violated and it is assumed instead that a strong unbalancing is realized with the outsider interaction greater than the inner psychological condition of the subject. From the data we deduce that we have a strong different behavior in time for both $P_+(t)$ and $P_-(t)$. We have short time steps in which $P_+(t)$ fluctuates between a minimum value about 0.30 and a maximum value about 0.95 with corresponding fluctuations for $P_-(t)$ from a minimum of about 0.05 to a maximum about 0.70. This time dynamical regime is followed from brief time intervals in which the fluctuations of $P_+(t)$ and $P_-(t)$ are strongly reduced as well as their maximum and minimum values oscillate now between a minimum value of about 0.55 and a maximum value of 0.75 for $P_+(t)$ and a minimum value of 0.25 and a maximum value of 0.45 for $P_-(t)$. They never tend to overlap.

In addition to a basic difference in the maximum and minimum values for $P_+(t)$ and $P_-(t)$, respect to the case of balancing condition, we have that the fluctuations in the values of probabilities oscillate now irregularly, and still the regions of overlap between $P_+(t)$ and $P_-(t)$ strongly increase. The condition of strong unbalancing induces a more evident condition of uncertain in human subject decision.

5. A Quantum Analysis of the Experiment

We intend to deepen our analysis under the profile of quantum mechanics.

First of all, let us analyze the two probabilities $P_+(t)$ and $P_-(t)$ as obtained in the (2.8). It is seen that at time t=0 the two probabilities give respectively $(A^2 + B^2)$ and $(C^2 + D^2)$, respectively, as it is required. Starting from this initial time, in the following time steps we have quantum interference

for the wave function, given in the (2.7), and in probabilities given in the (2.8). The meaning of such quantum interference must be intended here in the sense that, during time evolution of the perceptive–cognitive process of the human subject, the quantum probability amplitude (c_-) competes with the quantum probability amplitude (c_+) in order to determine the time value of $P_+(t)$ and at the same time the quantum probability amplitude (c_+) competes with quantum probability amplitude (c_-) in order to determine the time value of the probability $P_-(t)$.
The same interference like behavior we find in the expression of the quantum wave function given in the (2.7).
Actually, the human subject, owing to the interaction terms, fixed in the Hamiltonian representation by the $b_i(t)$, perceives the visual stimuli that is given by the ambiguous figure, representing the given task, and aims to pursue a perceptual decision. Such decision follows a conflicting psychological path that in psychological terms is based on perceptual reversals that the human subject perceives and elaborates, and, in the formal counterpart, instead, is based on the quantum interference like terms that we point out in the two formulas of probabilities, $P_+(t)$ and $P_-(t)$ given in the (2.8).
In addition to such interesting feature, the fundamental step is that the human subject has a proper timing of perceptual decisions and our formulation should be able to predict such time value.
We may reason in the following manner.
Starting with the initial time of the experiment ($t = 0$) the human subject is submitted to the visual ambiguous stimulus and a decision is asked him, $B = \pm$, on the basis of the posed question. We have seen in the previous sections that the probabilities, $P_+(t)$ to give answer $B = +$, and $P_-(t)$ to give answer $B = -$, fluctuate in the time steps, starting from time $t = 0$, according to the (2.8) and to the Figures 1, 2 and 4, 5 previously discussed. In the previous section we anticipated that the human subject reaches his decision on the basis of a prefixed threshold criteria. In detail, the human subject is able to perform his proper perceptual decision when the fluctuations of such probabilities cease. In other terms, the human subject reaches his decision when a "steady state" for probabilities is reached. Consequently, we are able to calculate the timing of perceptual decision for each subject if

$$\frac{dP_+(T)}{dt} = 0$$

and (5.1)

$$\frac{dP_-(T)}{dt} = 0$$

In this case, we may estimate the timing of perceptual decision, T, by solving the previous set (5.1).
Using the (2.8) in the (5.1), we introduce the following notations

$$a_1 + b_1 = X$$
$$a_2 + b_2 = nX \qquad (5.2)$$
$$a_3 + b_3 = qX$$

where (n) and (q) are real numbers. After calculations we obtain that

$$T = \frac{arctgR}{\sqrt{1+n^2+q^2}\,X} \qquad (5.3)$$

where

$$R=\frac{\lambda}{\frac{1}{2}\sqrt{1+n^2+q^2}\,(A^2+B^2)-\frac{\omega}{2\sqrt{1+n^2+q^2}}} \tag{5.4}$$

with

$$\lambda = BC - AD + nBD + nAC \tag{5.5}$$

and

$$\omega=(n^2+1)C^2+(n^2+1)D^2+q^2A^2+q^2B^2+2qAC+2qBD+2nqAD-2nqCB \tag{5.6}$$

in the case of

$$\frac{dP_+(T)}{dt}=0 \tag{5.7}$$

and

$$R=\frac{\lambda}{\frac{1}{2}\sqrt{1+n^2+q^2}\,(C^2+D^2)-\frac{\omega}{2\sqrt{1+n^2+q^2}}} \tag{5.8}$$

with

$$\lambda=-BC-nAC+qCD+AD-nBD-qCD \tag{5.9}$$

and

$$\omega=(A-nB-qC)^2+(-B-nA+qD)^2 \tag{5.10}$$

in the case of

$$\frac{dP_-(T)}{dt}=0 \tag{5.11}$$

We call T the Time of Perceptual Decision.
Let us look now to the neurological correlate of our problem.
To this purpose let us consider the excellent paper of Thomas J. McKeeff and Frank Tong published in 2006 (T. J. McKeeff, 2006). In this paper the problem is posed on the manner in which the brain determines what might be present in the physical world when incoming sensory signals are weak, variable, or ambiguous. The possible answer, formulated by these authors, is that the brain must analyze, integrate, and interpret the relevant sensory signals to form a perceptual decision, which can then be used to guide the behavior. Forming a perceptual decision is intended to involve the classification of sensory signals and the conversion of this information into a representational format that can guide the action. Therefore, an important question concerns how perceptual decisions are represented in the brain. In particular, McKeeff and Tong pose the question on the manner in which the critical neural processes take place that determine the outcome and the timing of perceptual decisions. It arises from this study and several previous studies (for references the reader is invited to look at the work of McKeeff and Tong that in fact contains a long quotation of previous studies) that parietal and prefrontal areas, implicated in attentional selection and motor planning, have a critical role in the formation of perceptual decisions Taken together, the above studies demonstrate the importance of high-level areas in forming decisions when weak sensory

signals must be integrated over time to minimize perceptual uncertainty. McKeeff and Tong used event-related functional magnetic resonance imaging (fMRI) to measure the time course of cortical activity while subjects were required to make perceptual decisions about ambiguous Mooney stimuli over a prolonged time period. Event-related fMRI analyses were analyzed by these authors in order to investigate relationships between the time of the subject's decisional response and the timing of fMRI responses from multiple sites along the sensory-motor pathway. Timing of perceptual decisions were experimentally calculated, and it resulted that they may vary from 0.700 to 11.700 sec. Clearly, they must change in function of different variables as in particular the complexity of the given visual, ambiguous stimuli and still other factors. This is the neurological context concerning our work.

Let us remember now that the data to which the present analysis is related, regard an experiment that was performed by us on ambiguous figures given to the perception-cognition of an human subject (Conte 2009; Conte 2009). In such kind of experiments psychologists often determine an experimental parameter that is of interest. A lot of studies was devoted in the past to the experimental determination of the time of perceptual reversals. We will not mention here all such studies but we will consider a paper in which such parameter was analyzed. It was published by Sheree T. et al. (Sheree T.Kwong See et al. 2006). The times of perceptual reverse were estimated to vary from 0.900 to 2.200 sec. in the cases of our interest. The corresponding frequencies result 1.111 Hz and 0.454 Hz, respectively.

Let us consider now that, as explained in detail in Appendix A and in the previous sections of the present paper, our theoretical formulation, based on time quantum evolution analysis of mind entities during perception-cognition of human subjects, introduces two fundamental sets of parameters and variables, that are respectively the a_i and the $b_i(t)$ $(i = 1,2,3)$. In this formulation, the a_i represent the proper inner frequencies characterizing the mind entity while the $b_i(t)$ represent the proper frequencies or the coupling frequencies that are established from the mind entity in the moment in which it interacts with the outside. In detail, it characterizes the proper frequency of the human subject at the moment in which the visual stimulus is given to his attention and a decision is asked. In the (5.2) we have introduced the new variables X, nX, qX in order to account for the combined contributions of the a_i and $b_i(t)$ during the quantum evolution of mind entities of the subject leading to a final decision. In the case of our experimentation, the basic frequency X represents the frequency of perceptual reversals that were observed by Sheree T. et al. (Sheree T.Kwong See et al. 2006) and ranging between 1.111 Hz and 0.454 Hz. Inserting such values in the (5.2)-(5.11) we may arrive to estimate the Time of Perceptual Decision, T, as given by the (5.3), and compare the predictions of our theoretical formulation with the experimental data. In fact, in the course of the experiments we estimated such Times of Perceptual Decision for the different subjects employed in the task, and we obtained times varying from about 1.000 to 2.500 seconds.

Fixing the values of the parameters n and q to $n = q = 0.3$ we obtain from the (5.3) that the Times of Perceptual Decision, T, estimated theoretically by our formulation, result respectively $T = 1.090\,\text{sec.}$(corresponding experimental value $T \approx 1.000\,\text{sec.}$) and $T = 2.688\,\text{sec.}$(corresponding experimental value $T = 2.500\,\text{sec.}$), plus possibly the multiples. The agreement between experimental and theoretical data results to be satisfactory.

In conclusion, it seems that we have given a preliminary but satisfactory formulation of decision making based on quantum mechanics at least in the case of human subjects during the perceptive-cognitive processes.

Appendix A

In abstract and formal terms we may say that we have to introduce a dynamical evolution operator U(t), time dependent, that acts on the initial state of the cognition entity. In the most simple case of the superposition given in ψ in (2.1), if we indicate such state of cognitive entity by ψ_0 to express that it is related to the initial time 0, we will write that the state of the cognitive entity at any time t, will be given by

$$\psi(t) = U(t)\psi_0 \quad \text{and} \quad \psi_0 = \psi(t=0) \tag{20}$$

The entity starts its cycle and, if left unmeasured by some cognitive measurement, remains statistically in its undifferentiated superposition state of potentialities. If, during such dynamical evolution, some cognitive measurement will start, the dynamical evolution of the superposition state will be interrupted and a final state will be selected among the ontological possibilities and yielded to be actualized on the basis of the intrinsic features of the entity and of its interaction and context. Before of all, we would examine the nature of the dynamical time evolution expressed by the (20). We have to attribute a physical meaning to the time t before the actualization will be performed owing to cognitive measurement and acting context. We will call it the time of the temporal evolution of the cognitive entity. Essentially, a Hamiltonian H must be constructed such that the evolution operator U(t), that must be unitary, gives $U(t) = e^{-iHt}$.

It is well known that, given a finite N-level quantum system described by the state ψ, its evolution is regulated according to the time dependent Schrödinger equation

$$i\hbar \frac{d\psi(t)}{dt} = H(t)\psi(t) \quad \text{with } \psi(0) = \psi_0. \tag{21}$$

Let us introduce a model for the hamiltonian H(t). It is the hamiltonian of the cognitive entity. We express by H_0 the free hamiltonian of the cognitive entity, and we consider it as a constant- internal hamiltonian component that resumes all the basic mental, historical, social features of the considered entity. We than add to H_0 an external time varying hamiltonian, $H_1(t)$, representing the interaction of the cognitive entity with the control fields, intending by this term all the mind and also brain influences that will act on the cognitive entity during the evolution of the initial superposition state indicated by ψ_0 as induced from the task. Thus, for the first time, we attempt here to give an unitary representation of a cognitive entity including in the time varying also the term $H_1(t)$ representing the mental contributions as well as synchronous contributions deriving from mind-brain relation when a stimulus or a task is posed to the perception-cognition of the subject. In conclusion we write the total hamiltonian as

$$H(t) = H_0 + H_1(t) \tag{22}$$

so that the time evolution of the state of the cognitive entity will be given by the following Schrödinger equation

$$i\hbar \frac{d\psi(t)}{dt} = [H_0 + H_1(t)]\psi(t) \tag{23}$$

and $\psi(0) = \psi_0$. We have that

$$\psi(t) = U(t)\psi_0 \tag{24}$$

where U(t) pertains to the special group SU(N). We will write that

$$i\hbar \frac{dU(t)}{dt} = H(t)U(t) = [H_0 + H_1(t)]U(t) \qquad \text{and } U(0)=I \tag{25}$$

Let $A_1, A_2, \ldots, A_n$, ($n=N^2-1$), are skew-hermitean matrices forming a basis of Lie algebra SU(N). Assuming semiclassical approximation for external acting fields $H_1(t)$, and following the previous papers developed by Altafini (Altafini, 2003), one arrives to write the explicit expression of the hamiltonian H(t) of the cognitive entity. It is given in the following manner

$$-iH(t) = -i[H_0 + H_1(t)] = \sum_{j=1}^{n} a_j A_j + \sum_{j=1}^{n} b_j A_j \tag{26}$$

where a_j and $b_j = b_j(t)$ are respectively the constant components of the free hamiltonian and the time-varying control parameters characterizing the interaction of the cognitive entity during the task and thus the human perception-cognition of the human subject. If we introduce T, the time ordering parameter, still following in detail the previous work given in (Altafini, 2003), we arrive also to express U(t) that will be given in the following manner

$$U(t) = T\exp(-i\int_0^t H(\tau)d\tau) = T\exp(-i\int_0^t (a_j + b_j(\tau))A_j d\tau) \tag{27}$$

that is the well known Magnus expansion (Altafini,2003; Magnus,1954). Locally U(t) may be expressed by exponential terms as it follows (Altafini, 2003; Magnus,1954)

$$U(t) = \exp(\gamma_1 A_1 + \gamma_2 A_2 + ... + \gamma_n A_n) \tag{28}$$

on the basis of the Wein-Norman formula

$$\Xi(\gamma_1, \gamma_2, ..., \gamma_n)\begin{pmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_2 \\ ... \\ \dot{\gamma}_n \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ ... \\ a_n + b_n \end{pmatrix} \tag{29}$$

with Ξ n x n matrix, analytic in the variables γ_i. We have $\gamma_i(0) = 0$ and $\Xi(0) = I$, and thus it is invertible, and we obtain

$$\begin{pmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_2 \\ ... \\ \dot{\gamma}_n \end{pmatrix} = \Xi^{-1}\begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ ... \\ a_n + b_n \end{pmatrix} \tag{30}$$

The present elaboration has reached now some central objectives that seem to be of considerable interest.

1) we have learned as to write explicitly the hamiltonian of a cognitive entity.
2) Still, we have learned how to write explicitly the time evolution unitary operator U(t) regulating the dynamic time evolution of a cognitive entity in absence (b_j=0) of external influences or when mental and brain influences are present.
3) Finally, we have evidenced that, by direct experimentation conducted on cognitive entities, we may arrive to express not only the Hamiltonian H(t) of a cognitive entity and evolution operator U(t), but we may arrive also to estimate the fundamental parameters a_j , b_j (t) that reproduce the basic features of the cognitive entity. We may also express differential equations for such parameters and variables by the γ_j introduced in the previous considered systems of differential equations (29) or (30). In brief, we have arrived to express a formalism that enables to give for the first time a satisfactory characterization of the basic cognitive and neurological features that may pertain to a cognitive entity.

In the mean while we may also see how we may render still more explicit the previously obtained results.

To reach this objective we must consider a simple case of cognitive entity based on the superposition of only two states as we considered it in the (2.1). As we outlined, we have

$$\psi = [y_1, y_2]^T \qquad \text{and} \qquad |y_1|^2 + |y_2|^2 = 1 \tag{31}$$

As previously said, we have here an SU(2) unitary transformation. Select the skew symmetric basis for SU(2). We will have that

$$e_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \;,\; e_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \;,\; e_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{32}$$

Now we will consider the following matrices

$$A_j = \frac{i}{2} e_j \quad , j = 1,2,3 \tag{33}$$

We are now in the condition to express H(t) and U(t) in some cases of interest. In this paper we will take in consideration only the most simple case, that one of fixed and constant control parameters b_j. In subsequent papers we will take in consideration more complex and also non linear behaviors. According to (Altafini,2003), the hamiltonian H of the cognitive entity will become fully linear time invariant and its exponential solution will take the following form

$$e^{t(\sum_{j=1}^{3}(a_j+b_j)A_j)} = \cos(\frac{k}{2}t)I + \frac{2}{k}\mathrm{sen}(\frac{k}{2}t)\left(\sum_{j=1}^{3}(a_j + b_j)A_j\right) \tag{34}$$

with $k = \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2 + (a_3 + b_3)^2}$. In matrix form it will result

$$U(t) = \begin{pmatrix} \cos\frac{k}{2}t + \frac{i}{k}\mathrm{sen}\frac{k}{2}t(a_3 + b_3) & \frac{1}{k}\mathrm{sen}\frac{k}{2}t[a_2 + b_2 + i(a_1 + b_1)] \\ \frac{1}{k}\mathrm{sen}\frac{k}{2}t[-a_2 - b_2 + i(a_1 + b_1)] & \cos\frac{k}{2}t - \frac{i}{k}\mathrm{sen}\frac{k}{2}t(a_3 + b_3) \end{pmatrix} \tag{35}$$

and, obviously, it will result to be unimodular as required. This is the matrix representation of time evolution operator for the considered cognitive entity.

Starting with this matrix representation of time evolution operator U(t), we may deduce promptly the dynamic time evolution of the state of cognitive entity at any time t writing

$$\psi(t) = U(t)\psi_0 \tag{36}$$

On the general case of a dichotomous quantum variable , we are considering here that c_+ states for c_{true} and c_- for c_{false} .On this general plane, we have for ψ_0 the following expression

$$\psi_0 = \begin{pmatrix} c_{true} \\ c_{false} \end{pmatrix} \tag{37}$$

having assumed for the true and false states the following matrix expressions

$$\varphi_{true} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \varphi_{false} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{38}$$

Finally, one obtains the expression of the state $\psi(t)$ of the cognitive entity at any time

$$\begin{aligned} \psi(t) = & \left[c_{true}\left[\cos\frac{k}{2}t + \frac{i}{k}\mathrm{sen}\frac{k}{2}t(a_3 + b_3)\right] + c_{false}\left[\frac{1}{k}\mathrm{sen}\frac{k}{2}t[(a_2 + b_2) + i(a_1 + b_1)]\right]\right]\varphi_{true} + \\ & \left[c_{true}\left[\frac{1}{k}\mathrm{sen}\frac{k}{2}t[i(a_1 + b_1) - (a_2 + b_2)]\right] + c_{false}\left[\cos\frac{k}{2}t - \frac{i}{k}\mathrm{sen}\frac{k}{2}t(a_3 + b_3)\right]\right]\varphi_{false} \end{aligned} \tag{39}$$

As consequence, the two probabilities $P_{true}(t)$ and $P_{false}(t)$ that are expected for future selection to, true or false, as consequence of cognition measurement and context influence, will be given at any time t by the following expressions

$$P_{true}(t) = (A^2 + B^2)\cos^2\frac{k}{2}t + \frac{1}{k^2}\mathrm{sen}^2\frac{k}{2}t(P^2 + Q^2) + \frac{\mathrm{sen}\,kt}{k}(AP + BQ)$$

and (40)

$$P_{false}(t) = (C^2 + D^2)\cos^2\frac{k}{2}t + \frac{1}{k^2}\mathrm{sen}^2\frac{k}{2}t(S^2 + R^2) + \frac{\mathrm{sen}\,kt}{k}(RC + DS)$$

where

A= Re c_{true}, B=Im c_{true}, C=Re c_{false}, D=Im c_{false},

$P=-D(a_1+b_1)+C(a_2+b_2)-B(a_3+b_3)$,

$Q=C(a_1+b_1)+D(a_2+b_2)+A(a_3+b_3)$, (41)

$R=-B(a_1+b_1)-A(a_2+b_2)+D(a_3+b_3)$,

$S=A(a_1+b_1)-B(a_2+b_2)-C(a_3+b_3)$

As it is seen, our initial purpose to introduce an abstract quantum formalism in order to describe the time dynamics of a cognitive entity has been now fully reached. By using proper experimentation we are now in the condition to analyze cognitive behavior in simple cases of control fields as well as in cases of more complex and non linear dynamical conditions. In any case the finality will be to analyze cognitive dynamics and its basic interactions by establishing with the experiments the correct behavior of the constant parameters a_j and of the time dependent functions $b_j(t)$ that regulate the time dependent behavior of the acting control fields during the dynamics of the cognitive process.

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