AGAINST HARMONY*

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Abstract Many prominent writers on the philosophy of logic, including Michael Dummett, Dag Prawitz, Neil Tennant, have held that the introduction and elimination rules of a logical connective must be 'in harmony' if the connective is to possess a sense. This *Harmony Thesis* has been used to justify the choice of logic: in particular, supposed violations of it by the classical rules for negation have been the basis for arguments for switching from classical to intuitionistic logic. The Thesis has also had an influence on the philosophy of language: some prominent writers in that area, notably Dummett and Robert Brandom, have taken it to be a special case of a more general requirement that the grounds for asserting a statement must cohere with its consequences. This essay considers various ways of making the Harmony Thesis precise and scrutinizes the most influential arguments for it. The verdict is negative: all the extant arguments for the Thesis are weak, and no version of it is remotely plausible.

Keywords Logical connectives, proof-theoretic harmony, proof-theoretic semantics, Inversion Principle, natural deduction, sequent calculi, Michael Dummett, Gerhard Gentzen, Dag Prawitz

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In both natural deduction and sequent formalizations of logical systems, each connective is associated with an introduction rule and an elimination rule. The introduction rule for the connective **C** is one which licenses the derivation of a formula dominated by **C**; the elimination rule is one which licenses the deduction of further conclusions from such a formula, often with other formulae as auxiliary premisses. In the sense of the term with which I shall be concerned, *harmony* is a particular relationship between the introduction rule and the elimination rule for a given connective. Whether the rules of a logical system are harmonious is certainly of great interest to proof theorists, but I am concerned with a philosophical claim about the notion. The *Harmony Thesis*, as I shall call it, says that a connective is defective unless its associated introduction and elimination rules are in harmony. It also says that a connective is defective if the logical principles which regulate its use go beyond a pair of harmonious introduction and elimination rules. Most proponents of the Harmony Thesis have, indeed, a particular defect in mind. On their view, a connective will not possess a sense—it will not have a coherent meaning—unless its logical behaviour is regulated only by a pair of harmonious introduction and elimination rules.

The Harmony Thesis is connected to wider claims in the philosophy of language and the philosophy of logic. According to *Inferential Role Semantics* (IRS), the meaning of a complete statement is determined by its role in inference, and the meanings of sub-sentential expressions are determined by their contribution to the inferential roles of complete statements in which they figure. As Julien Murzi and Florian Steinberger remark in their contribution to this volume, many adherents of IRS appeal to the Harmony Thesis in order to circumscribe the range of meaning-determining inferential roles. We have yet to see what harmony comes to, but it is also widely held that the classical introduction and elimination rules for negation violate the Thesis, so Harmony is often invoked in challenges to classical logic. Dummett's *The Logical Basis of Metaphysics* provides a good example of this. He holds that a connective's introduction and elimination rules must be in harmony if the connective is to make sense. So he takes the perceived lack of harmony between the classical rules for negation to be 'a strong ground for suspicion that the supposed understanding [of classical negation] is spurious' (Dummett 1991, 299).

In this essay, I want to scrutinize the most influential arguments which have been put forward for the Harmony Thesis. I find these arguments wanting, so my conclusion will be that the Thesis is not well supported. Any rejection of it must be provisional: tomorrow, someone may come up with a brilliant new argument which will persuade us all that there is a requirement of harmony on any intelligible connective. But I doubt it. As the analysis will reveal, the most popular arguments for the Harmony Thesis are not near-misses which might succeed with a bit of tweaking. Rather, they are superficially plausible arguments which turn out, on closer examination, to rely upon very dubious premisses from epistemology and the theory of meaning. My analysis will not challenge the claim that deductive systems exhibiting harmony have attractive proof-theoretical features; to the contrary, I regard that claim as obviously true. But a connective may possess a sense, and may in other ways be non-defective, without generating those nice features, so the elegance of a harmonious proof theory does not settle the philosophical questions I am addressing. I conclude by discussing briefly how the failure of the Harmony Thesis affects the prospects for Inferential Role Semantics.

1. The Inversion Principle

I have been using the term 'harmony', but what exactly does it mean? As we shall see, different parties mean rather different things by it. I start by expounding what I take to be the most prominent version of the Harmony Thesis.

Central to this version is the claim that a connective's introduction rule determines its sense or meaning. The claim goes right back to Gentzen, who wrote that 'the introduction rules represent, so to say, the "definitions" of the signs in question [i.e. the connectives and quantifiers], and the elimination rules are, in the final analysis, no more than consequences of these definitions…In eliminating a sign, we may use a formula whose main connective that sign is only with the meaning afforded it by the sign's introduction rule' (Gentzen 1935, 189 = Gentzen 1969, 80). Prawitz (see especially 1974), Dummett (see 1991), and Negri and von Plato (2001) also accept this claim.

What, exactly, are introduction rules and elimination rules? As we shall see, some delicate issues surround this question, but an initial answer runs as follows. While they may be extended to natural languages, the notions were originally applied to formalized languages, so let us consider for simplicity a propositional language L with sentence letters P_1, P_2,… and a collection of finitary connectives. A *sequent* is then defined to be an expression $X \Rightarrow A$, where A is a formula of L and X is a finite set (possibly empty) of such formulae.\[1]/ Where $n \geq 0$, consider the $(n+1)$-tuple of sequents $<X_1 \Rightarrow A_1,\ldots, X_n \Rightarrow A_n, Y \Rightarrow B>$. This determines an $(n+1)$-ary *rule for sequents* (for short, a *rule*), comprising all substitution instances of the $(n+1)$-tuple. This rule is understood as licensing the passage from any substitution instance of the first n sequents (the *premiss sequents*) to the corresponding instance of the last sequent (the *conclusion sequent*). We mark the division between the premiss and conclusion sequents with the solidus, /. When $n = 0$, we have a *rule of inference*, such as $\wedge$-introduction: $/ P_1, P_2 \Rightarrow P_1 \wedge P_2$ (when displaying particular rules, I shall often omit the angled brackets around tuples). When $n \geq 1$, we have a *rule of proof*, such as $\rightarrow$-introduction: $X, P_1 \Rightarrow P_2 / X \Rightarrow P_1 \rightarrow P_2$. A rule is *elementary* if all its members are substitution instances of a tuple $<X_1 \Rightarrow A_1,\ldots, X_n \Rightarrow \mathrm{A}_n /Y \Rightarrow B>$ such that (a) all the formulae in all the sequents are sentence letters except for at most one, which is of the form $\mathbf{C}(P_1,\ldots, P_k)$ for some k-place connective $\mathbf{C}$; and (b) this complex formula $\mathbf{C}(P_1,\ldots, P_k)$ (if it appears at all) occurs either on the left or the right of the conclusion sequent $Y \Rightarrow B$. If it occurs on the right of $\Rightarrow$, the elementary rule in question is an *introduction rule* for **C**. If it occurs on the left, it is an *elimination rule* for **C**.\[2]/

On this account, the rule of double negation elimination (*DNE*)—from $\neg\neg A$, infer A—does not qualify as an elimination rule. While this consequence may initially seem surprising, it is to be welcomed. Gentzen described *DNE* as an elimination rule: *DNE*, he wrote, ‘represents a new elimination of negation whose admissibility does not follow at all from our way of introducing the negation sign by the $\neg$-*I* rule’ (Gentzen 1935, 190 = Gentzen 1969, 81). Dummett followed him in this. Indeed, Dummett’s critique of classical logic in *The Logical Basis of Metaphysics* is really an extended elaboration of the sentence just quoted from Gentzen: the classical elimination rule for negation, viz. *DNE*, is not in harmony with its introduction rule. But *DNE* does not conform to Dummett’s own account of what an elimination rule is. In applying *DNE* we pass, of course, to a

[1] On my account, then, introduction and elimination rules are rules in a sequent calculus. Some of the writings to be examined below take harmony to be a relation between rules in natural-deduction formalizations of logic. In a rigorous treatment of this topic, however, it is best to work in a sequent framework, where the assumptions on which a conclusion depends are explicitly represented. The philosophical arguments for harmony proposed by those who prefer natural-deduction formalizations transpose to the sequent framework.

[2] For these definitions, cf. Humberstone and Makinson 2011, §2. As they remark (*op. cit.*, *n*.5), a rule which is elementary in the present sense will be both ‘pure’ and ‘simple’ in the terminology of Dummett 1991.

conclusion that contains two fewer occurrences of '$\neg$' than does the premiss, but that is not enough for it to count as an elimination rule. 'In the case of a logical constant, we may regard the introduction rules governing it as giving the conditions for the assertion of a statement of which it is the main operator, and the elimination rules as *giving the consequences of such a statement*' (Dummett 1981, 454-5; emphasis added). The *DNE* rule tells us nothing *in general* about the consequences of statements in the form $\ulcorner \neg A \urcorner$. It tells us something only about the very special case of statements in the form $\ulcorner \neg\neg A \urcorner$. It is to the good, then, that the proposed account does not classify *DNE* as an elimination rule.

In what sense might an introduction rule for **C** be thought to *define* **C**? According to Prawitz and Dummett, it does so by specifying the *direct grounds* (alias the 'canonical' grounds) for asserting a formula dominated by **C**. Suppose that G_1 is a direct ground for asserting the interpreted formula A and that G_2 is a direct ground for asserting the interpreted formula B. Then the rule of $\wedge$-introduction tells us that the combination of G_1 with G_2 constitutes a direct ground for asserting the conjunctive formula $\ulcorner A \wedge B \urcorner$. Indeed, if this rule is to *define* the sense of '$\wedge$', it must be understood as telling us that the *only* direct grounds for asserting $\ulcorner A \wedge B \urcorner$ are those which combine a direct ground for A with a direct ground for B. The introduction rule for '$\vee$' is $<P_1/ P_1 \vee P_2> \cup <P_2/ P_1 \vee P_2>$. In a similar way, this rule is to be read as telling us that a direct ground for asserting a disjunctive formula $\ulcorner A \vee B \urcorner$ will be either a direct ground for A or a direct ground for B. As remarked, the rule of $\rightarrow$- introduction is a rule of proof, not a rule of inference, so here matters are less straightforward. But $\rightarrow$- introduction is understood as telling us that a direct ground for asserting $\ulcorner A \rightarrow B \urcorner$ is a method for transforming any ground for A into a ground for B. There are of course grounds for assertion—indeed, conclusive grounds for assertion—which are not direct. Thus we might assert $\ulcorner A \wedge B \urcorner$, not on the basis of the combination of G_1 with G_2, but on the strength of a deduction of $\ulcorner A \wedge B \urcorner$ from the premisses C and $\ulcorner C \rightarrow A \wedge B \urcorner$.

Any development of this theory of direct or canonical grounds clearly faces problems. For one thing, the method that constitutes a direct ground for asserting $\ulcorner A \rightarrow B \urcorner$ needs to be one that transforms *any* ground for A into a ground for B, so the specification of direct grounds would appear not to be straightforwardly compositional.\[3]/ It is, however, in the context of this conception of the meaning of the connectives that the present version of the Harmony Thesis belongs. For suppose that the meaning of a connective **C** is given by its introduction rule; then the elimination rule for **C** must be faithful to that meaning. In Gentzen's words, we may use **C** only with the meaning that the introduction rule affords it. On this view, the requirement of harmony does no more than spell out what such fidelity consists in. Gentzen conveys the requirement he has in mind only by way of an example: 'if we wished to use [the formula $A \to B$] by eliminating the $\to$-symbol…we could do this precisely by inferring B directly, once A has been proved, for what $A \to B$ attests is just the existence of a derivation of B from A' (Gentzen 1935, 189 = Gentzen 1969, 80-1, with incidental changes in the symbolism). Negri and von Plato, though, spell out the general principle to which Gentzen implicitly appeals. To find the elimination rule which is faithful to a given introduction rule, they write, 'we ask what the conditions are, in addition to assuming the major premiss derived, that are needed to satisfy the *Inversion Principle*:

> Whatever follows from the direct grounds for deriving a formula must follow from that formula (Negri and von Plato 2001, 6; I write 'formula' where they have 'proposition').

According to the present version of the Harmony Thesis, then, a non-defective logical connective must be regulated only by a pair of introduction and elimination rules which satisfy the Inversion Principle. This version of the Thesis is justified by the claim that the elimination rule for a connective must be faithful to the introduction rule that defines the connective's meaning. One finds similar, albeit less explicit, formulations of this version of the Thesis, and of the suggested justification for it, in Prawitz and Dummett.\[4]/

One merit which Negri and von Plato claim for their formulation is that it not only justifies a certain elimination rule, given an introduction rule, but 'actually determines what the elimination rules

3 In the first edition of *Elements of Intuitionism* (1977, 394-5), Dummett argued that the theory could be made compositional, all the same. For scepticism about his proposed way of achieving this, see Prawitz 1987, esp. 156-63, and Pagin 2009, esp. 724-34. Dummett entirely rewrote this passage for the second edition of *Elements*, and there concluded that the form of compositionality that could be justified was only 'a very thin one' (2000, 274).

4 On the history of inversion principles, with references to Lorenzen (1950, 1955) and Schroeder-Heister (1984) as well as to Gentzen and Prawitz, see Moriconi and Tesconi 2008.

corresponding to given introduction rules should be' (2001, xvi).\[5]/ Take disjunction as an example. The direct grounds for $\ulcorner A \vee B \urcorner$, we saw, are either direct grounds for A or direct grounds for B. The Inversion Principle is understood to say that whatever follows from *any* of the direct grounds for asserting a formula must follow from that formula. So we reach the $\vee$-elimination rule in the form: given a derivation of C from the assumption A, and another derivation of it from the assumption B, we may derive C from the disjunction $\ulcorner A \vee B \urcorner$. This is, in fact, the form of $\vee$-elimination that Negri and von Plato take their Inversion Principle to yield (2001, 7) and they go on to show how to excise from a derivation any deductive steps in which an instance of $\vee$-introduction is immediately followed by an instance of $\vee$-elimination. Suppose, for example, that we apply $\vee$-introduction to derive $\ulcorner A \vee B \urcorner$ from A, and then immediately eliminate $\ulcorner A \vee B \urcorner$ to reach the conclusion C. The derivation will then have the form

$$\dfrac{\begin{array}{c}\Delta \\ A \\ A \vee B\end{array} \qquad\qquad \begin{array}{c}A \\ \Delta' \\ C\end{array} \qquad \begin{array}{c}B \\ \Delta'' \\ C\end{array}}{C}$$

and we may simplify it by cutting out the occurrence of $\ulcorner A \vee B \urcorner$ entirely, thereby reaching

$$\begin{array}{l}\Delta \\ A \\ \Delta' \\ C.\end{array}$$

This is an example of what Prawitz labels a 'reduction step' and of what Dummett calls 'levelling a local peak'.

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5 Thereby gratifying a *desideratum* of Gentzen's: 'By making these ideas more precise it should be possible to display the *E*-inferences [i.e. the elimination rules] as unique functions [*eindeutige Funktionen*] of their corresponding *I*-inferences [introduction rules], on the basis of certain requirements' (Gentzen 1935, 189 = Gentzen 1969, 81).

There is, however, a problem here. The form of $\vee$-elimination that Negri and von Plato's Inversion Principle yields is the restricted version of the rule found in quantum logic, in which the conclusion *C* must be derived from *A*, and from *B*, without the use of any side premisses.\[6]/ However, only a few pages later in their treatise on proof theory (*op. cit.*, 15), they blithely reformulate the elimination rule in the form *with* side premisses: it is this stronger form of the rule that is found in classical, intuitionistic, and indeed minimal logic. It is hard to see what justifies the switch: in identifying the elimination rule that matches a given introduction rule, Negri and von Plato tell us, we are to 'ask what the conditions are…that are *needed* to satisfy the Inversion Principle' (2001, 6, with emphasis added): the 'needed' seems to imply that we are to select the *weakest* elimination rule which satisfies Inversion.\[7]/ I shall return to this problem in due course.

2. An argument for the Inversion Principle

First, though, we should consider the central question which this attempted justification of the Harmony Thesis raises: why should we accept the Inversion Principle?

At first blush, there seems to be a compelling argument for the Principle. As we have seen, it is to be read as saying: 'Whatever follows from *any* of the direct grounds for asserting a formula must follow from that formula.' Let *C* be a formula which follows from any of the direct grounds for asserting a formula *A*, and suppose that *A* is asserted. If *A* has been correctly asserted, one might think, then at least one of its direct grounds must obtain. *Ex hypothesi*, *C* follows from any such ground. So *C* must obtain if *A* has been correctly asserted. *C*, then, is a commitment of a correct assertion of *A*, and as such—one might think—*C* must follow from *A*. So far from being compelling, however, this argument faces two severe problems.

First, and most obviously, the argument implicitly rejects the view that consequence is a matter of the preservation (or necessary preservation) of *truth* in favour of a view whereby

[6] The same problem attends Stephen Read's requirement of 'general-elimination harmony'. See Read 2010, 566.

[7] As Negri and von Plato recognize, their Inversion Principle yields more general forms of $\wedge$-elimination and of $\rightarrow$-elimination than one usually finds in the textbooks (see 2001, 6-7 and 8-9). I do not object to this aspect of their theory, which might well be a bonus rather than a drawback. However, the inability to justify the unrestricted form of $\vee$-elimination is a difficulty.

consequence is a matter of the preservation (or necessary preservation) of correct *assertibility*. For suppose one did think that consequence was a matter of the preservation of truth. In that case, the proposed justification of Harmony would scarcely get started. On this view, in order to argue that *C* follows from *A*, we would need to show that *C* is true given only the assumption that *A* is *true*. From the assumption that *A* is true, however, it does not follow that any direct ground for asserting *A* obtains. Indeed, it does not follow that *any* ground for asserting *A* obtains. For all that has been said, the formula *A* might be true but unassertible. Only if consequence is understood to consist in the preservation of correct assertibility, then, does the mooted argument so much as get going. On that alternative view of consequence, in seeking to show that *C* follows from *A*, we *shall* start by assuming that *A* is correctly asserted.

A second problem confronts the argument, though, even if we do take consequence to consist in the preservation of correct assertibility rather than in the preservation of truth. On this view, in trying to show that *C* follows from *A* we shall start by assuming that *A* is correctly asserted, from which it follows that some ground for asserting *A* obtains. What we are given about *C*, though, is that it follows from any *direct* ground for asserting *A*. So in order to conclude from our assumption that *C* obtains, we shall need a premiss to the effect that whenever a ground for asserting a formula obtains, some *direct* ground for asserting it obtains. It is far from obvious what is supposed to support this additional premiss. Indeed, pending further explanation of what a 'direct' ground is, it is far from clear what the additional premiss means.

In view of this unclarity, one might be tempted to delete the word 'direct' from the formulation of the Inversion Principle altogether, so that it now says simply: 'Whatever follows from any of the grounds for asserting a formula must follow from that formula'. The resulting position, however, does not at all fit the view we are considering, whereby the introduction rule for a connective is held to specify that connective's meaning. At least, it does not fit this view if the rules in question are to be the familiar introduction rules for the connectives. On this version of the view, the rule of $\vee$-introduction would imply that there are grounds for asserting the disjunction $\ulcorner A \vee B \urcorner$ if and only if there are grounds for asserting *A* or grounds for asserting *B*. And the 'only if' part of this claim is simply false, at least if the symbol '$\vee$' is supposed to have even roughly the same meaning as the English word 'or'. As Dummett noted in his early paper 'Truth' (1959), the claim is wholly unsustainable if we allow that the testimony of others can provide grounds for assertions. Reliable sources from the

Egyptian Fourth Dynasty tell us that the Pharaoh Cheops (whom Egyptologists now call 'Khufu') was either the son or the stepson of his predecessor on the throne, the Pharaoh Sneferu. Those sources, then, provide grounds for that disjunctive assertion. There are, though, no reliable grounds for asserting either disjunct. Indeed, there are other counterexamples to the 'only if' claim which do not rely on knowledge by testimony. If Inspector Morse knows (from the position of wounds on the victim) that the murderer is left-handed, and that Smith and Jones are the only left-handers among the possible culprits, then he has grounds for asserting 'Either Smith or Jones is the murderer'. In that circumstance, though, Morse may have no grounds for asserting either 'Smith is the murderer' or 'Jones is the murderer'. What we see, then, is that the present argument for the Inversion Principle depends upon finding a sense for the word 'direct' (or 'canonical') which treads a fine line. The sense has to be sufficiently generous to ensure that whenever a ground for asserting a formula obtains, a direct ground obtains. At the same time, it has to be sufficiently restricted to ensure that a direct ground for asserting $\ulcorner A \vee B \urcorner$ involves either a direct ground for A or a direct ground for B. (The introduction rules for the other connectives will impose corresponding restrictions on the acceptable sense of 'direct'.)

3. Problems with the argument

What are the prospects of solving these problems so that the present argument for Inversion can be vindicated? I address the problems in turn.

There is no doubt that the conception of consequence on which the argument rests deviates from the conception which has animated logic since its creation. The key mark of a valid argument is that its conclusion is true whenever all its premisses are true. At the heart of consequence, then, lies preservation of truth, not preservation of correct assertibility, or of knowability, or of anything other than truth. While disputes persist about the proper explanation of consequence, those disputes centre on what surrounds that heart: notably whether consequence involves the necessary preservation of truth (as Aristotle held) or whether actual preservation will do; and whether the sort of truth-preservation which is characteristic of logical consequence must be rooted in a formal relationship between premisses and conclusion. If an explanation of consequence in terms of the preservation of correct assertibility is going to be more than an eccentric misuse of the familiar notion, we must take ourselves to be in a dialectical context in which truth has already been 'dethroned' (as people used to say) from its usual place in that explanation. More particularly, we must presume that powerful arguments have already been given for explaining consequence in terms of the preservation of correct assertibility.

Supposing for a moment that some powerful arguments to this effect have been given, the foundations of logic will certainly need reconstruction. Logicians prove soundness theorems for various logical systems. That is, they show that, if the rules of a given system are followed, then the conclusion is true in every possible circumstance in which all the premisses are true. But what can soundness come to if truth has been dethroned? There must still be some standard against which individual deductions, rules, and indeed whole logical systems may be assessed. We still want to be able to say that someone who infers 'If Fred works hard, he will get a First; Fred will get a First; therefore Fred works hard' has reasoned unsoundly—that he has made a logical mistake. But in what does his mistake consist if not in the possibility that both premisses might be true when the conclusion is not true?

It might be answered that we can still give an account of why the reasoner is making a mistake in terms of correct assertibility. Our reasoner's argument is unsound because someone could be in a position correctly to assert both the premisses without being in a position correctly to assert the conclusion. But this just pushes the problem back: we shall then need to specify the conditions for correctly asserting the sentences or formulae of the relevant language. On any view, the introduction of logical connectives into a language that has hitherto lacked them is going to create new grounds for asserting formulae. This applies to atomic formulae as well as to molecules: once the language contains a conditional, for example, we can correctly assert an atomic formula *B* by (for example) deducing it from correct assertions of *A* and of ⌜If *A* then *B*⌝. But given that any logical rules are going to generate new grounds for assertions, we have to say what it is for *modus ponens* to constitute an acceptable expansion of those grounds while affirming the consequent does not. Moreover, the proponent of the present argument for Harmony has to give an account of this matter without falling back on the idea that a valid argument preserves truth.

The only developed account of this that I know relies heavily on the distinction between direct and indirect grounds for assertion. The thought is that, whilst logic certainly yields new indirect grounds for atomic assertions, its rules must be faithful to the *direct* grounds of formulae: we shall have an instance of consequence only if any direct grounds for the premisses could be transformed into a direct ground for the conclusion. This is the account of logical consequence shared by Prawitz (see especially his 1974) and Dummett (see especially his 1991). Instead of direct grounds for atomic formulae, Prawitz writes of valid 'closed' arguments for them. He duly 'defines a sentence *B* as a logical consequence of sentences $A_1,\ldots, A_n$ by the existence of an operation φ which for every choice *C* [of valid closed arguments] transforms any closed arguments for sentences $A_1,\ldots, A_n$ valid relative to *C* to a closed argument for *B* valid relative to *C*'(Prawitz 1974, 74-5). Dummett proposes essentially the same account. 'We regard [Euler's] proof as showing us, of someone observed to cross every bridge at Königsberg, that he crossed at least one bridge twice, *by the criteria we already possessed for crossing a bridge twice*' (1991, 219, emphasis in the original). 'If that is what deductive inference achieves', he continues, 'the requirement of harmony springs from its very nature. When an expression, including a logical constant, is introduced into the language, the rules for its use should determine its meaning, but its introduction should not be allowed to affect the meanings of sentences already in the language' (*op.cit.*, 220). By mastering logic we acquire new indirect grounds for making assertions. But the methods we master must be faithful to the meanings of the atoms in that they preserve their conditions of direct assertibility.

If consequence is to be explained in terms of the preservation of some form of correct assertibility, it is hard to think of any other account than the one which Prawitz and Dummett provide. That account, though, generates serious problems—problems which, I now argue, are so serious as to cast doubt upon the hypothesis that consequence *can* be explained in this way.

Euler's proof is said to show us, of someone observed to cross every bridge at Königsberg, that he crossed at least one bridge twice, *by the criteria we already possessed for crossing a bridge twice*. But that cannot mean that those criteria have actually been applied to verify that our promenader crossed a bridge twice. Perhaps they were so applied—perhaps an observer stationed on the Dombrücke, for example, saw him cross that bridge twice—but Euler's proof would not be refuted if the pre-existing criteria were not actually applied. The most that can be claimed is a counterfactual: had an observer been stationed on each bridge, with instructions to tick a box if, and only if, the promenader was observed crossing it twice, then at least one observer would have ticked his box.

This counterfactual claim, however, is susceptible to objections parallel to those which afflict putative counterfactual analyses of other categorical notions. Some philosophers used to say that an object is yellow if an observer with good eyesight, viewing it in white light, would perceive it as yellow. Saul Kripke objected that this account was inconsistent with something that is surely a metaphysical possibility—namely, the existence of *killer yellow*, a shade of yellow that kills any observer who looks at it in white light.\[8]/ In much the same way, Dummett's account of the validity of Euler's proof is inconsistent with the existence of *Königsberg ennui*, a strange neurological condition which ensures that anyone trying to observe whether a promenader has crossed a given bridge twice will fall into a catatonic state before any second crossing. Like killer yellow, Königsberg ennui is surely a metaphysical possibility. In a possible world where the denizens of Königsberg are afflicted by it, however, it will not be true that at least one of our observers would have ticked his box, had the promenader crossed every bridge. Even in such a world, however, 'The promenader crossed at least one bridge twice' still follows from 'He crossed every bridge'.

Even if we prescind from this rather general doubt, other worries press in fast, especially when we turn to our second main problem and reflect on the role which the distinction between direct and indirect grounds needs to play in the present argument for the Harmony Thesis. The notion of directness needs to be sufficiently generous, we said, that no ground for asserting a formula obtains unless a direct ground for asserting it could have obtained. Yet the direct grounds for asserting a complex formula are constrained to be those given by the introduction rule for the formula's main connective. Combining these points, we deduce that no ground for asserting a complex formula can obtain unless the assertion of that formula could have been justified by applying the introduction rule for its main connective. In other words, the present argument for the Harmony Thesis rests upon what Dummett calls the *Fundamental Assumption*.

Dummett is clear that the present argument does rest upon this Assumption. His discussion of the Assumption, though, does not inspire great confidence in its truth. The Assumption is tenable, I think, in the case of conjunction. If someone is entitled to assert $\ulcorner A \wedge B \urcorner$, then he is entitled to assert A and is also entitled to assert B, so his assertion of the conjunction could have been grounded in an application of the $\wedge$-introduction rule. For none of the other familiar sentential connectives, though, is the Fundamental Assumption remotely plausible.

[8] Kripke presented this case in lectures which remain unpublished, but Lewis 1997 contains a brief account of it. Kripke has long been on record as an opponent of counterfactual and dispositional accounts of colour; see *n*.71 to *Naming and Necessity* (Kripke 1980, 140).

In the case of disjunction, Dummett recognizes that the Assumption is quite untenable if we confine ourselves to the grounds available to an individual thinker. While I have a ground for asserting 'Cheops was either the son or the stepson of Sneferu', it is impossible for me to justify that disjunctive assertion by an argument which concludes with an application of the introduction rule for 'or'. The Assumption is only tenable, Dummett holds, if the grounds for making an assertion are taken to include those available to any of *us*, where 'whatever witnesses we trust must be included among "ourselves"' (1991, 266). Thus the ancient scribe who recorded that Cheops was either the son or the stepson of Sneferu is one of us, and the Fundamental Assumption tells us that his assertion is correctly made only if he knew which it was, or if he was himself told the disjunction by someone who knew which disjunct obtained.

Perhaps we can swallow *this* consequence of the Assumption. Other consequences of it, though, are far less palatable. Consider the assertion 'At the moment when Brutus first stabbed Caesar in Pompey's Theatre, there was either an odd or an even number of people in the Agora in Athens'. Let us assume that the space of the Agora has been precisely delimited, and that precise rules have been laid down for when a person counts as being *in* a space. Given that assumption, most of would think ourselves entitled to make the present disjunctive assertion. If we are so entitled, though, the Fundamental Assumption entails that someone—i.e., some one of us—could have been in a position either to assert 'At the moment when Brutus first stabbed Caesar, an odd number of people were in the Agora' or to assert 'At that moment, an even number of people were there'.

Dummett acknowledges, of course, that no one actually was in a position to make either of these claims. 'To interpret the fundamental assumption', he writes, 'we have to invoke the sense of "could have" which was used earlier to characterize what may be called the minimal undeniable concession to realism demanded by the existence of deductive inference' (1991, 267). In the case of statements about the past, he continues, this means 'that a sufficient condition for [an assertion's] correctness is that there exist effective means by which, at the relevant time, someone appropriately situated *could have* converted observations that were actually made into a verification of the statement asserted' (1991, 268). By the Fundamental Assumption, though, a closely related condition must be necessary: for an assertion to be correct, it is necessary that someone appropriately situated could, at the relevant time, have made observations which would have justified it. In the case of either of our two disjuncts, though, it is hard to see how this necessary condition could be satisfied. For where would an observer be ‘appropriately situated’? An observer in Pompey’s Theatre would have been well placed to notice when Brutus stabbed Caesar and to observe what was happening at that moment in that part of Rome; but he was not in a position to count how many people were then in the Athenian Agora. An observer situated in the Agora, by contrast, may have been in a position to make a count of those present in the square; but he would not know when to do so. What a direct ground for either disjunct needs is a *pair* of observers, with the first able to effect a practically instantaneous signal to tell the second when to make the count. But there was no effective means of sending such a signal ‘at the relevant time’: the necessary technology would not be invented for several centuries. Even if we gloss the Fundamental Assumption in the generous way that Dummett recommends, then, it is going to exclude many assertions that we take ourselves to be in a positon to make. Its hard-line adherents may swallow that consequence. The rest of us, though, will simply conclude that the Fundamental Assumption is false when applied to disjunctions.

Matters are no better when we turn to (indicative) conditionals. Dummett himself concedes that he cannot defend the Assumption for conditionals with disjunctive consequents (see 1991, 273) but in fact the problem conditionals present for it runs far deeper: the difficulty is that the standard introduction rule, Conditional Proof, is not a plausible codification of the circumstances in which we take ourselves to be entitled to assert English indicative conditionals. If Conditional Proof were the operative introduction rule for the vernacular ‘if…then, a direct ground for asserting a conditional would be a method for transforming any possible ground for the antecedent into a ground for the consequent, but this principle does not get the assertibility conditions of ordinary conditionals right. Variants of Moore’s Paradox provide one class of counterexamples. Consider the conjunction ‘It is raining but there are no grounds for asserting that it is raining’. It is plausible to hold that there are no possible grounds for asserting this conjunction: any grounds for asserting the first conjunct will falsify the second conjunct. Accordingly, we shall (vacuously) have a method for transforming any possible ground for this conjunction into a ground for asserting a self-evident absurdity, such as 0 = 1. Given the principle about conditionals, it follows that there is a ground—indeed a direct ground—for asserting ‘If it is raining but there are no grounds for asserting that it is raining, then 0 = 1’. But that conditional does not seem to be one that we shall wish to assert: in entertaining the supposition or hypothesis ‘It is raining but there are no grounds for asserting that it is raining’ we do not seem to be entertaining an absurdity but something which might well be the case.

The crucial point here is that in a conditional we conditionalize on the *truth* of the antecedent, not on its assertibility. Ironically, in some of his other writings Dummett makes this point very clearly. ‘In a sentence like “If you go into that room, you will die before nightfall”’, he remarks, ‘the event stated in the consequent is predicted on condition of the truth of the antecedent (construed as the future tense proper\[9]/), not of its justifiability’ (1990, 193). As a point about the meaning of conditionals in English this is clearly correct, and Dummett goes so far as to conjecture that it is when statements occur as antecedents of conditionals (and in related complex constructions) that we need to draw the distinction between truth and justifiability (*ibid.*). However that may be, the view that Conditional Proof specifies direct grounds for the assertion of ordinary conditionals is miles from the truth.

One might respond to this by saying that some *other* rule justifies such assertions; on the view we are considering, it will be this other rule which specifies the sense of the conditional. Even if it were possible to formulate an alternative rule, however, that would not help in the present context. For (a) we *do* seem to be prepared to eliminate vernacular conditionals using the rule of *modus ponens* while (b) it is *modus ponens* which stands as the inverse of Conditional Proof (for proof, see e.g. Negri and von Plato 2001, 8).\[10]/ In other words, whatever exactly they are, the rules which we actually go by in introducing and eliminating vernacular conditionals are not in harmony.

Severe problems also afflict the Fundamental Assumption as it applies to negated statements. According to the Assumption, we shall not be entitled to assert a statement in the form ⌜Not *A*⌝ unless we could have justified that assertion by applying the introduction rule for ‘not’. According to that introduction rule, we may assert ⌜Not *A*⌝ when we have derived a contradiction from our premisses along with the hypothesis *A*. In many circumstances where we take ourselves to be entitled to assert ⌜Not *A*⌝, however, it is hard to see what the appropriate premisses might be. I look out of the window and see that it is not raining. I am surely entitled to assert ‘It is not raining’, but what premisses does my observation deliver that would enable me to justify that assertion by applying the rule of ‘not’-introduction? In many circumstances of this kind, there is no plausible answer. In looking out of the

[9] Dummett contrasts the ‘proper’ or ‘genuine’ future tense with ‘the future tense used to express present tendencies’. ‘The latter occurs, e.g., in an announcement of the form “The wedding announced between *A* and *B* will not now take place”. Such an announcement cancels, but does not falsify, the earlier announcement, and is not itself falsified if the couple later make it up and get married after all; if this were not so, the “now” would be superfluous’ (Dummett 1972, 21).

[10] Vann McGee (1985) presents a case where, he thinks, we are not prepared to use modus ponens in drawing consequences from an indicative conditional; but see Rumfitt 2013, 176-8 and 185-6 for an alternative analysis of his case.

window, I might see that it is sunny, but being sunny is compatible with rain. The only specification of the content of my experience that is guaranteed to be incompatible with 'It is raining' is 'It is not raining', but while I can indeed see that it is not raining, the ensuing belief that it is not raining serves as a *premiss* in my reasoning. It is not a *conclusion* that has been reached by applying the rule of 'not'-introduction to some other premisses.

In fact, the situation with negation is even worse than that with disjunction and the conditional. In stating the introduction rule for negation, I said that ⌜Not A⌝ may be derived from some premisses when the combination of those premises with A yields a contradiction. But what is a contradiction? One answer might be: it is any statement in the form ⌜A and not A⌝—but we shall know that such a statement is contradictory only if we already know what 'not' means, so we cannot invoke this notion of a contradiction in a rule which purports to give the sense of 'not'. What is worse, if we understand the term 'introduction rule' in the way proposed in §1, it is demonstrable that there is no classically or intuitionistically correct introduction rule for '$\neg$'. More generally, let us follow Humberstone and Makinson in calling a k-place connective **C** *contrarian* if $\mathbf{C}(P_1,\ldots, P_k)$ is valued *False* when all of $P_1,\ldots, P_k$ are valued *True*. (Thus the *falsum* '$\bot$', conceived as a zero-place connective, and the unary connective '$\neg$'are both contrarian in this sense.) Then there is no classically or intuitionistically correct introduction rule for any contrarian connective.\[11]/ For let **C** be such a connective and suppose its introduction rule comprises all instances of the scheme $<X_1 \Rightarrow A_1,\ldots, X_n \Rightarrow A_n / Y \Rightarrow \mathbf{C}(P_1,\ldots, P_k)>$. Since the rule is an introduction rule, it is elementary, so all the formulae in the premiss sequents $X_1 \Rightarrow A_1,\ldots, X_n \Rightarrow A_n$ and in the set Y must be sentence letters. But then the rule cannot be classically correct. Consider the substitution instance got by replacing each sentence letter by a classical tautology: under this substitution, each premiss sequent becomes classically valid while the conclusion sequent has antecedents that are all true but a false succedent. Since every intuitionistically correct rule is also classically correct, there is no intuitionistically correct introduction rule for a contrarian connective either.

This result may seem bizarre: we teach our logic students sequent rules for '$\neg$', after all. On reflection, however, it is no surprise that sequent rules *of the form described* cannot characterize the logically relevant meaning of '$\neg$'. Such rules ensure the correctness of certain sequents—i.e., they

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[11] This result is the first 'Observation' in §3 of Humberstone and Makinson 2011.

ensure that *if* certain antecedents are true, then so are certain succedents. No collection of such rules can exclude the possibility that all the formulae in the language L are true, but we need to exclude that possibility in order to characterize '$\neg$', or indeed any contrarian connective. We need to ensure, for example, that $\ulcorner A \wedge \neg A \urcorner$ is *not* true.

The operative conception of an introduction rule needs to be liberalized, then, if a contrarian connective is to possess one. From a formal point of view, the simplest and most common liberalization permits a sequent to have a null succedent. We move, in other words, from *set/formula* sequents to *set/formula-or-empty* sequents. Such a sequent is correct if and only if the formula in its succedent is true whenever every formula in its antecedent is true. When the succedent is empty, i.e. when there is no formula in it, the sequent will be correct if and only if not every formula in its antecedent is true. The sequent $Q \Rightarrow \varnothing$, for example, will then be correct if and only if Q is not true. When the logical rules regulate set/formula-or-empty sequents, it is straightforward to give an introduction rule for '$\neg$', namely, $X \cup \{P_1\} \Rightarrow \varnothing \ / \ X \Rightarrow \neg P_1$.\[12]/ Indeed, as Makinson (2014) points out, in such a system we can give an introduction rule for any truth-functional connective apart from '$\bot$'. We should not expect '$\bot$' to have an introduction rule.\[13]/ On the theory we are considering, such a rule would specify the canonical conditions for asserting '$\bot$'; it would be surprising if there were conditions in which a speaker would be entitled to assert a formula which is understood always to be false. For any connective **C** not equivalent to '$\bot$', however, there will be at least one structure v where $v(\mathbf{C}(P_1, \ldots, P_k))$ is true. Where $P_{j1}, .., P_{jm}$ are those sentence letters evaluated as true under v and $P_{l1}, .., P_{ln}$ are those evaluated as false there, we have corresponding to v the rule

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[12] In *The Logical Basis of Metaphysics*, Dummett adopts a very different approach to the problem of finding an introduction rule for negation. He works in a language with an infinite collection $Q_1, Q_2, \ldots$ of atomic formulae. In our notation, his introduction rule for '$\neg$' is the infinitary rule whose underlying tuple is $<P \Rightarrow Q_1, P \Rightarrow Q_2, \ldots / \varnothing \Rightarrow \neg P >$ (Dummett 1991, 295). He also proposes a cognate introduction rule for '$\bot$': $< / Q_1, Q_2, \ldots \Rightarrow \bot >$. In the event that the atomic formulae of the language form a consistent set, his introduction rule for '$\neg$' allows A and $\ulcorner \neg A \urcorner$ both to be true. Similarly, his introduction rule for '$\bot$' allows '$\bot$' to be true in those circumstances. These features are surely weaknesses in his theory. Since he was not a dialetheist, an account which leaves it an open matter whether there can be true contradictions must be failing to characterize logically relevant aspects of the meaning of the negation sign. Similarly, an account which leaves open the possibility that the *falsum* might be true is not capturing the intended sense of '$\bot$'. Dummett may be right to say that in his system 'no logical laws could be framed that would entail' that not every atomic sentence can be true (*ibid.*), but that is a limitation of his system. In a system of set/formula-or-empty sequents, the rule $< / A, \neg A \Rightarrow \varnothing >$ entails that A and $\ulcorner \neg A \urcorner$ cannot both be true, and the infinitary rule $< / Q_1, Q_2, \ldots \Rightarrow \varnothing >$ excludes the possibility that $Q_1, Q_2, \ldots$ form a consistent set.

[13] It has, of course, an elimination rule: $< / \bot \Rightarrow \varnothing >$.

$<\varnothing \Rightarrow P_{j1},.., \varnothing \Rightarrow P_{jm}, P_{l1}\Rightarrow \varnothing,.., P_{ln} \Rightarrow \varnothing \,/\, \varnothing \Rightarrow \mathbf{C}(P_1,\ldots, P_k)>$.

The union of such rules for all v where $v(\mathbf{C}(P_1,\ldots, P_k))$ is true is then the introduction rule for **C**.\[14]/

Natural as this liberalization is from a formal point of view, it comes at a philosophical price. As remarked at the outset, many adherents of the Harmony Thesis are also adherents of Inferential Role Semantics. As such, they are ambitious to characterize an expression's meaning by the rules that regulate its inferential *use*. The move from set/formula sequents to set/formula-or-empty sequents, however, involves a retreat from direct engagement with the way logical expressions are used in inference. A set/formula sequent represents an actual argument, in which a reasoner passes from a set of premisses to a conclusion. Hence the correctness of such a sequent can be related to the intuitive acceptability of the corresponding inferential passage. Where a speaker fails to reach a conclusion, however, we do not have an inference; we merely have a list of statements. Accordingly, we cannot explain the correctness of a set/formula-or-empty sequent directly in terms of the intuitive acceptability of an inference. We shall need instead to give a metalogical account of correctness, such as that in the previous paragraph. This takes us further away from what, for an IRS theorist, is foundational.

There are, to be sure, alternative ways of liberalizing the formal system which cleave more closely to the ideal that its rules should record the way we use connectives. I expounded one of these in my essay '"Yes" and "No"'.\[15]/ The operational logical rules given there are 'bilateral' principles which regulate deductive transitions between premisses and a conclusion in each of which a yes-no question is followed by one of its expected answers, as in 'Is Fred in Berlin? No. So is it the case that he is either in Paris or is not in Berlin? Yes'. But even if we find a way of remaining faithful to this ideal, the present strategy for justifying the Harmony Thesis has reached a dead end. Dummett conceded that his 'examination of the fundamental assumption has left it very shaky' (1991, 277), and with this conclusion we can only concur. A theory of the meaning of the connectives which passes muster for 'and', but which fails for 'or', 'if…then' and 'not'—which is committed, indeed, to counting these ubiquitous expressions as meaningless—is not doing well.

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[14] We may liberalize introduction and elimination rules to those governing set/formula-or-empty sequents while retaining the requirement that such rules must be elementary. If we do this, we shall exclude the introduction and elimination rules that Stephen Read proposes for his paradoxical zero-place connective 'bullet', a proof-conditional Liar sentence (Read 2000, 140-42). Those who regard the bullet as meaningless will wish to retain the requirement of elementariness.

[15] Rumfitt 2000. When I wrote that paper, I still thought there might be something in the Harmony Thesis, so I was concerned to show how the operational rules of my system conformed to an analogue of the harmony requirement. I no longer see any grounds for requiring such conformity.

One might wonder how Dummett felt entitled to pursue his project of justifying the laws of intuitionistic logic, while de-legitimating classical logic on the ground of its alleged violations of the Harmony Thesis, when he admitted that the grounds for the Thesis were so shaky. His reason is interesting. The laws of intuitionistic logic, he says, ‘are not going to be called into question by any uncertainties over the scope or status of the fundamental assumption, precisely because the classical logician will admit that assumption, interpreted in terms of an ideal observer’ (1991, 279). His thought is this. At this point in the dialectic, we shall have been so completely persuaded—perhaps by Dummett’s own ‘manifestation argument’, or his argument about the acquisition of language—that truth needs to be dethroned from its place in the traditional explanation of consequence, that we shall accept the Prawitz-Dummett account of that notion in terms of preservation of direct grounds. It is simply that those people willing to assert (for example) the disjunction ‘At the moment when Brutus first stabbed Caesar, there was either an odd or an even number of people in the Agora’ will do so because they are prepared to postulate a God-like ideal observer who was able to make a count of the people in the Agora in Athens at the very moment when the knife went in in Rome. The upshot of our discussion, however, is that this is quite the wrong moral to draw. There is no space to rehearse Dummett’s arguments against the intelligibility of a notion of truth that goes beyond the existence of a verification. To put the point at its mildest, though, those arguments are far from conclusive, and our analysis suggests that a real strike against their conclusion is the immense difficulty we shall then face in trying to forge a notion of consequence to replace the familiar one cast in terms of truth-preservation.

More precisely, what we have seen are some of the difficulties in forging an alternative account of consequence that will sustain the Harmony Thesis. At every turn, the traditional account in terms of preservation of truth cries out to be restored. Of course, there is no suggestion that resurrecting the traditional account is going to open any direct path towards justifying classical logic: for one thing, one can adopt the traditional account of consequence without postulating the Principle of Bivalence. But the arguments we have so far considered for imposing a harmony requirement, and for deviating from classical logic because it violates that requirement, lead only into a morass. If this is the best that can be said in favour of the Harmony Thesis, the classical logician has nothing to fear from it.

4. Arguments from the 'innocence' of logic

The argument for the Harmony Thesis just analysed assumes that the introduction rule for a connective specifies its meaning. That assumption is in any case far from compelling. It is more plausible to take ordinary competence with the indicative conditional, for example, to be manifest in applications of its elimination rule—*modus ponens*—than in mastery of whatever rule regulates its introduction.\[16]/ The arguments for Harmony that I consider next do not assume any semantic priority for the introduction rules. Indeed, they do not assume that any one sort of rule—whether it be the introduction rules or the elimination rules—will by itself specify the meanings of the connectives.\[17]/

The first such argument rests on a premiss about the nature of logic. Dummett sometimes writes as though the Inversion Principle follows from a general requirement of harmony that applies between the grounds and consequences of any meaningful statement. Florian Steinberger, *per contra*, argues as follows:

> Whatever misgivings one may have about Dummett's wider project, a strong case can be made for a logic-specific harmony requirement. The reason for this stems from the role logic plays in our assertoric practices. On the use-theoretic view [of meaning], the meanings of non-logical sentences (sentences not containing any logical operators) are thought to be given

[16] See again *n*.10 above on purported counterexamples to *modus ponens*.

[17] Recognizing the difficulties confronting his Fundamental Assumption, Dummett briefly canvassed an alternative theory whereby every connective's meaning is given by its elimination rule. On this view, justifying the Harmony Thesis 'will depend upon an inverse fundamental assumption, namely, that any consequence of a given statement can be derived by means of an argument beginning with an application of one of the elimination rules governing the principal operator of that statement, in which the statement figures as the major premiss. This assumption is open to fewer intuitive objections than the fundamental assumption on which our original justification procedure rested. It is more plausible that we derive simpler consequences from complex statements only when those consequences follow logically than that we assert such statements only when they follow logically from simpler statements we have previously accepted' (Dummett 1991, 281).

Dummett's account of this alternative 'pragmatist' theory is sketchy, although Prawitz (2007), Queiroz (2008), and Litland (forthcoming) have developed it further. In particular, Litland (*op. cit.*, §4) corrects various mistakes in Dummett's sketch, and shows that a cleaned up Inverse Assumption justifies precisely the intuitionistic introduction rules for the connectives, given the intuitionistic elimination rules for them. It is good to know where this approach leads. In later writings, however, Dummett came to doubt if the sort of pragmatist theory of meaning that the Inverse Fundamental Assumption requires could be coherently elaborated (see especially Dummett 2007). In Rumfitt 2016, I identify a number of foundational problems that pragmatist theories of meaning must face, and criticize extant attempts to solve them.

> by their I- and E-principles.\[18]/ Logic, in addition to the direct grounds for assertion given by the appropriate I-principles, offers indirect grounds for asserting non-logical sentences: we may assert a non-logical sentence if it can be correctly deduced from a set of accepted premisses. But for these indirect deductive routes to assertibility to be not only legitimate but to have the unassailable reliability we require of logical inference, our logical modes of inference must respect the conditions under which the (direct) assertion of non-logical sentences is justified. That is, logical inference alone may not license the assertion of non-logical sentences that we should not have been in a position to assert directly (at least in principle). Let us call this the *principle of innocence*: it should not be possible, solely by engaging in deductive logical reasoning, to discover hitherto unknown (atomic) truths that we would have been incapable of discovering independently of logic….How can we make sure that innocence obtains? This is where harmony comes in. The primary purpose of harmony is to secure the innocence of logic (Steinberger 2011, 619-20).

Steinberger further contends that harmony is the best way—perhaps the only way—of securing innocence:

> A moment's reflection reveals not only that harmony is an adequate measure, but that it seems entirely natural that any measure designed to guarantee the holding of the requirement of innocence should take the form of a harmony requirement. After all, our aim is to ensure that the meanings of the logical constants are fixed in such a way as not to perturb the non-logical regions of language. The best way to do this (at a local level) is by requiring that the introduction and elimination rules that govern the meanings of logical constants be exactly commensurate in strength. Why? Well, because when such an equilibrium between I-rules and E-rules obtains, we can rest assured that our deductive practices will not, as it were, create novel grounds for asserting non-logical sentences (as in the case, for example, of [Prior's invented connective] *tonk*). The requirement of harmony thus seems to be an eminently reasonable and natural safeguard for the principle of innocence (*op. cit.*, 620).

[18] In Steinberger's terminology, the 'I-principles' pertaining to a sentence *A* state the conditions in which a speaker of the relevant language is entitled to assert *A*. The corresponding 'E-principles' state what a speaker who asserts *A* is thereby entitled to do. See *op. cit.*, 618.

Steinberger spells out as follows the equilibrium he has in mind: ‘nothing more and nothing less may be deduced from an assertion of $A\$B$ via \$-E than can already be deduced from the premisses of the corresponding I-rules. Put another way,…the E-rules ought to exploit *all* and *only* the inferential powers that the I-rules have bestowed upon it’ (*ibid.*). His requirement of equilibrium, then, demands the satisfaction of Negri and von Plato’s Inversion Principle, and something more besides. As we saw in §1, their Inversion Principle is satisfied when the standard introduction rule for ‘$\vee$’ is paired with its quantum-logical elimination rule, but Steinberger insists that that elimination rule fails to exploit *all* the inferential power bestowed by the introduction rule (see his discussion of ‘E-weak disharmony’, *op. cit.*, 621). If it works, then, Steinberger’s argument justifies a form of the Harmony Thesis that is even more exacting (as far as the logical connectives are concerned) than that proposed by Prawitz and Dummett.

Does his argument work, though? There are strong reasons to doubt it.

First, the principle of innocence is far less compelling than Steinberger supposes. Since Mill’s *A System of Logic*, with its notorious claim that ‘nothing ever was, or can be, proved by syllogism which was not known, or assumed to be known, before’ (Mill 1891, II iii 1), a central problem in the philosophy of logic has been to reconcile the conclusiveness of correct deduction with its ability to expand our knowledge. Part of the explanation of how deduction generates new knowledge is that premisses founded on different sources of knowledge can entail a conclusion that no source founds by itself. A trusty informant tells me that John is either in the common room or the library; I see that he is not in the common room; I deduce that he is in the library. I know the first premiss through testimony and I know the second as result of observation. But whilst I come to know the conclusion, my knowledge of it does not stem from either testimony or observation alone: I was not told that John is in the library, nor did I see him there. Of course, this case is not itself a counterexample to Steinberger’s principle of innocence. Although I did not in fact see John in the library, in principle I could have done. In our ordinary deductive practice, however, we are fully prepared to splice together different sources of knowledge to deduce conclusions that could not be founded on any ‘direct’ evidence, even in principle. Suppose I know—through astronomical theory, and appropriate observations—that a body *B* is either in region *R* of the Andromeda Galaxy or is in a black hole. Suppose I make some further observations, and come to know that *B* is not in region *R*. I may then deduce that *B* is in a black hole. It is (we may assume) impossible even in principle to discover by direct observation whether a body is in a black hole. For all that, this second deduction appears to be just as cogent as the first. In each case, the deduction yields knowledge of its conclusion, even though in the second case that knowledge could not have been attained directly, even in principle. *Contra* Steinberger's principle of innocence, then, by engaging in deduction we can discover hitherto unknown atomic truths that we would have been incapable of discovering without logic. The principle of innocence is far from innocent. Were it accepted, it would seriously restrict the use we actually of make of deductive logic in enlarging our knowledge.

Second, even if we grant the principle, it may be secured by a weaker requirement than that of equilibrium between introduction and elimination rules. As Steinberger acknowledges, innocence will be secured if the non-logical regions of language are unperturbed by the logical rules—i.e., if those rules create no novel grounds for asserting non-logical sentences. This condition will be met if the logical rules *en bloc* create no such novel grounds—a 'global' condition in his terms. To ensure innocence, then, we need not descend to the 'local' level and require the introduction and elimination rules of each individual connective to be in equilibrium.

There is, in fact, a natural way of making the global condition more precise. The direct grounds for asserting non-logical formulae will induce a consequence relation R^- on the language L^- that comprises only such formulae. Innocence will be secured if the expanded consequence relation R that is induced on the full language L when the logical rules are added is a *conservative extension* of R^-. That is, R as restricted to L^- does not extend R^-. Steinberger is well aware that this global condition may be satisfied even when the introduction and elimination rules of a certain connective are not in equilibrium (*op cit.*, 625 and 634-8). The problem is that his principle of innocence only sustains the global condition. The conservative extension requirement is enough to ensure that the 'meanings' of atomic formulae—or better: the consequential relations between them—are left unperturbed. It also excludes Prior's rogue connective 'tonk', whose introduction rule is $<P_1/ P_1 \text{ tonk } P_2>$ and whose elimination rule is $<P_1 \text{ tonk } P_2/ P_2>$ (Prior 1960). 'Tonk' is indeed a runabout inference ticket, which licenses the move from one formula to any other, so its rules will violate conservativeness unless the pre-logical consequence relation R^- is already total (see Belnap 1962).

Robert Brandom's conception of logic in similar to Steinberger's, but he is more circumspect about its implications for harmony (Brandom 1994, 2000). On Brandom's view, what characterizes the logical notions is their role in 'making explicit' the relations between non-logical sentences that are traced out in material inferences and in recognitions of incompatibility. A good material inference takes a thinker from 'Edinburgh is north of London' to 'London is south of Edinburgh'; we may express our acceptance of that inference by asserting the conditional 'If Edinburgh is north of London, then London is south of Edinburgh'. Similarly, we express our recognition of the incompatibility between the table's being red all over and its being green all over by saying 'If the table is red all over, then it is not green all over'. What Brandom takes to follow from this doctrine is, simply, the conservative extension requirement:

> Unless the introduction and elimination rules are inferentially conservative, the introduction of the new vocabulary licenses new material inferences, and so alters the contents associated with the old vocabulary. So if logical vocabulary is to play its distinctive expressive role of making explicit the original material inferences, and so conceptual contents expressed by the old vocabulary, it must be a criterion of adequacy for introducing logical vocabulary that no new inferences involving the old vocabulary be made appropriate thereby (Brandom 2000, 68-9; see Brandom 1994 123-30 for a fuller exposition of the same argument).

Brandom is right, I think, to claim that his expressivist account of logic demands the conservative extension requirement. The important point for present purposes, however, is that it *only* demands that: conservative extension may be satisfied even if the rules for the connectives do not satisfy the Principle of Inversion, i.e., even if the Harmony Thesis is false.\[19]/

This comes out clearly, indeed, in the case of the classical propositional calculus. As noted earlier, the introduction rule for negation is *Simple Reductio*: $X \cup \{P_1\} \Rightarrow \varnothing \,/\, X \Rightarrow \neg P_1$. The elimination rule which is in harmony with this is *Ex Contradictio Quodlibet*: $\{P_1, \neg P_1\} \Rightarrow P_2$. While these two rules together characterize the intuitionistic logic of negation, its classical logic demands a further principle. There are many additional principles which will do. For definiteness, let us add the rule form of Excluded Middle, *EM*: $X \cup \{A\} \Rightarrow B, Y \cup \{\neg A\} \Rightarrow B \,/\, X \cup Y \Rightarrow B$. (Adding assumptions to a sequent is often called 'thinning', so Harold Hodes aptly calls EM a 'thickening' rule: it allows the

[19] Peregrin (2008) argues that intuitionistic logic is the strongest logic that makes inferences explicit. He reaches this conclusion, however, by importing a number of contentious assumptions into the explanation of what it is to make an inference explicit.

deduction of a sequent from other sequents with the same succedent whose antecedents include formulae which do not appear in the antecedent of the conclusion (Hodes 2004, 148)). In the resulting system, the Harmony Thesis is violated: the logical behaviour of '$\neg$' is not regulated *only* by a pair of harmonious introduction and elimination rules. But the classical consequence relation conservatively extends whatever pre-logical consequence relation obtains among the non-logical formulae: where $P_1, \ldots, P_n$ and Q are atoms, there is a classically admissible truth-value assignment which assigns *True* to all of $P_1, \ldots, P_n$ and False to Q, so if Q is a consequence of $P_1, \ldots, P_n$, that must be because the pre-logical consequence relation determines it as such.\[20]/

5. Tennant's argument for harmony

Neil Tennant is an adherent of the Harmony Thesis who distinguishes sharply between harmony proper and the conservative extension requirement.\[21]/ He also holds that the introduction rule and elimination rule for a connective *jointly* determine its sense, so that the Thesis cannot be justified by requiring the rules of one sort to keep faith with the meanings already laid down by rules of the other sort.\[22]/ Rather, he holds, its justification arises from a requirement of coherence between the introduction and elimination rules for a given connective. As Prior's 'tonk' shows, not every pair of introduction and elimination rules succeeds in endowing the connective it purports to characterize with a sense. It is by spelling out the requirements for coherence that Tennant aims to justify the Thesis.\[23]/

[20] If the introduction and elimination rules of a new connective are in harmony, will the resulting system conservatively extend the pre-existing consequence relation? Prawitz (1994, 374) argued not: the natural introduction and elimination rules for the truth-predicate are in harmony, but the result of adding a truth-predicate to Peano Arithmetic is not a conservative extension of it. See, however, Hodes (2004, 148-50) and Steinberger (2011, 635-7) for reasons to doubt whether the scope of introduction and elimination rules should be extended to encompass predicates as well as operators.

[21] See especially chapter 10 of Tennant 1987, which patiently untangles passages in Dummett's early writings on the topic (1975a, 1975b) that mix up the two requirements.

[22] At least, he does in his book *Anti-Realism and Logic* (Tennant, 1987). In *The Taming of the True*, he holds that the introduction rules give the meanings of the connectives as they are used in *a priori* science whereas the elimination rules give their meanings as they appear in empirical discourse. Harmony is then needed to ensure that there is no equivocation between the two sorts of occurrence (Tennant 1997, 23). Unfortunately, I lack the space to analyse this argument here.

[23] See especially p.94: 'There is another kind of equilibrium, which would be of interest even to one who refuses to acknowledge the asymmetric division of rules into those that are constitutive and those that are merely explicative of meaning. This way is to regard the rules of introduction and elimination

The precise form of harmony that Tennant defends is subtly different from that in Negri and von Plato. To state it accurately we need some terminology. Let us say that a formula is a *maximally strong F* if it is *F* and entails any formula that is *F*; and let us say that a formula is a *maximally weak F* if it is *F* and is entailed by any formula that is *F*. Tennant's First Principle of Harmony may then be stated as follows. A formula whose main connective is **C** is to be

(a) a maximally strong statement that can stand as conclusion of the introduction rule for **C**, given the elimination rule for **C**; and

(b) a maximally weak statement that can stand as major premiss of the elimination rule for **C**, given the introduction rule for **C**.

We may illustrate this First Principle with the case of '$\vee$'. To show part (a) for this connective, let X be any formula that can stand as the conclusion of $\vee$-introduction, with A and B as premisses. We then have that A entails X and B entails X. By $\vee$-elimination, it follows that $\ulcorner A \vee B \urcorner$ entails X, showing that $\ulcorner A \vee B \urcorner$ is a maximally strong statement that can stand as conclusion of $\vee$-introduction. To show part (b), let Y be any formula that can stand as the major premiss of $\vee$-elimination. Then, whenever A entails C and B entails C, Y entails C. By $\vee$-introduction, A entails $\ulcorner A \vee B \urcorner$, as does B, so Y entails $\ulcorner A \vee B \urcorner$. Thus $\ulcorner A \vee B \urcorner$ is a maximally weak statement that can stand as major premiss of $\vee$-elimination.

As this demonstration shows, Tennant's First Principle is satisfied whether the $\vee$-elimination rule has its usual form, in which the use of side premises is permitted, or takes the restricted form it has in quantum logic, in which C may be inferred from $\ulcorner A \vee B \urcorner$ only if it follows from A alone and from B alone. The First Principle, then, fails to distinguish between the two forms of the rule. For this reason, Tennant also lays down a Second Principle. When a pair of introduction and elimination rules **CI** and **CE** for a connective **C** meets conditions (a) and (b), let us say that the pair is in harmony (with a small 'h'). We further say that the pair is in Harmony (with a capital 'H') if **CE** is the strongest elimination rule with which **CI** is in harmony and **CI** is the strongest introduction rule with which **CE** is in harmony. Tennant's Second Principle requires the rules for a meaningful connective to be in Harmony. The form of $\vee$-elimination which permits side premises is stronger than the form which does not: it

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as equally involved in specifying or constituting meaning, but to demand harmony as a *conditio sine qua non* of their doing so. The thought would be that not just any set of rules will do in order to confer determinate meaning on a logical operator'.

allows us to derive more conclusions from a given disjunction. It is the unrestricted form of the elimination rule, then, which is in Harmony with the rule of $\vee$-introduction, so the restricted form falls foul of Tennant's Second Principle. It is good to have a criterion which improves on Negri and von Plato's Inversion Principle in excluding the restricted form of $\vee$-elimination. We shall, however, need a justification for requiring rules to be in Harmony as well as harmony.

Tennant contends that satisfaction of both his Principles of Harmony is 'a *conditio sine qua non* of' rules specifying or constituting a coherent meaning for the connective in question (1987, 94). He further claims that this condition has revisionary implications for logic. 'The correct consequence relation, insofar as it should arise solely from the meanings of the logical constants, is, naturally, the least relation with respect to which the Harmony of the rules governing those constants can be sustained' (*op.cit.*, 97). The least such relation, Tennant thinks, is that characterized by the system he calls intuitionistic relevant logic. 'I intend thereby to reveal as unjustifiable excrescences the extra ingredients in the consequence relation of classical logic that have earned the generic labels (a) the fallacies of relevance, and (b) the classical laws of negation' (*ibid.*). We need, then, to consider the arguments Tennant advances for two theses. The first thesis is his claim that satisfaction of the two Principles of Harmony is a necessary condition for a connective to possess a coherent meaning. The second is the claim that any logical principles that go beyond a Harmonious pair of introduction and elimination rules are 'unjustifiable excrescences'. I shall contend that neither of these theses is well supported.\[24]/

How does Tennant argue for the first thesis? The precise course of his reasoning is somewhat hard to follow, but we are told that 'the requirement for harmony emerges clearly if one follows a philosophical method that has the appearance of empirical speculation about the origins of language, but is actually designed to focus on constitutive features of meaning. This is the method of enquiring after the *aetiology of entrenchment* of expressions in a language and of conventions governing their use' (1987, 77). The general idea is that we shall be unable to explain how the logical connectives could have become entrenched in a language—that is to say, how they could have acquired a stable meaning—unless their introduction and elimination rules are in Harmony. The claim that Harmony is

[24] It is Tennant's second thesis that justifies his claim that the correct laws of logic are confined to the rules of intuitionistic *relevant* logic. This logic yields the *least* consequence relation that satisfies his two Principles of Harmony. Litland (op. cit. Part II) shows in effect full intuitionistic logic is the strongest logic that satisfies the two Principles.

a *conditio sine qua non* for such rules to constitute or specify a connective's meaning duly comes at the end of a long passage describing how meanings for the connectives might have become entrenched.

What is Tennant's account of entrenchment? It certainly has the appearance of empirical speculation about the origins of language. We are asked to imagine a community of speakers who start off using only atomic statements; Tennant then asks how connectives could be added—one at a time—to their dialect. He suggests that we would be unable to understand how this could happen unless the rules governing those connectives satisfy his two Principles of Harmony.

This account exemplifies a genre which one might call the *Just So Story*. We find it hard to imagine how a meaningful connective could have been added to a language unless certain conditions are met. So we take those conditions to be necessary for the connectives to have a meaning. Of course, my name for the genre carries a warning. Kipling's account of how the elephant got its trunk does have a certain explanatory charm. Few people today, though, would regard it as even a remote approximation to the truth. So if we are, O Best Beloved, to venture forth to the philosophical tributary of the great grey-green, greasy Limpopo River, all set about with normalized proof trees, we shall need to take care. We shall need to make sure that if a condition is imposed on the connectives, the condition really is necessary for them to have a meaning and does not simply express a philosopher's preconceptions about how language ought to work. If we read Tennant's account with that warning in mind, we shall find his story even less persuasive than Kipling's.

The root of the problem is that Tennant's account of how the connectives get entrenched does not explain how they come to have their actual meanings. The difficulty comes out clearly in the case of negation—a case which is of course central to the choice between classical and intuitionistic logic. Our signs for negation, Tennant hypothesizes, originate in the need one speaker may have to contradict or challenge an atomic assertion by another: 'dialogue, not monologue is where negation first flourishes' (1987, 83). Let us accept this for the sake of argument. 'The challenger', he goes on, 'must have information to the contrary, rather than be merely playing the role of the *uninformed* doubter' (*op. cit.*, 84). Let us accept this too. Tennant further contends that a speaker who challenges *A* by saying $\ulcorner$Not *A*$\urcorner$ is 'saying something *about the same subject matter*' as *A* (*ibid.*, emphasis in the original). If a speaker who says $\ulcorner$Not *A*$\urcorner$ were merely doubting 'the existence of a warrant for [*A*], then the challenge would be self-warranting, for nothing could serve as better evidence for such a claim than its own making' (*ibid.*). Again, this seems right. Tennant infers from this that the sort of challenge to *A* that is expressed by ⌜Not *A*⌝ 'must be conceived of as possessing warrants that are as open to independent public assessment as are the warrants of the assertions challenged' (*ibid.*). We may accept this too. 'Denial of *A*,' he concludes, 'has the force "I have good reason to believe that there is no warrant for *A*" rather than the weaker "I have no reason to believe (apart from your asserting it) that you have any warrant for *A*". Denial… carries with it no in-built guarantee of excluded middle' (*op. cit.*, 85).

That final conclusion, though, does not follow from the considerations that are adduced to support it, and in any case it seriously misrepresents the way ordinary speakers actually use signs for negation. Of course Tennant is right to claim that someone who says ⌜Not *A*⌝ is saying more than 'I have no reason to believe that you have any warrant for *A*'. As Heyting pointed out long ago, if this were the right account of the meaning of 'not', then someone who said 'Not every even number greater than two is the sum of two primes' would be making an autobiographical statement, not a mathematical one. But Tennant's account equally misrepresents the content of that negated claim. On his view, someone who makes the claim is saying 'I have good reason to believe that there is no warrant for the claim that every even number greater than two is the sum of two primes'. Now in certain circumstances that might be a perfectly sensible thing to say, and if one understands negation in this way, then the law of excluded middle will indeed fail to be valid. It is not, however, the way most of us understand negation. On Tennant's account, it would be correct to say 'Not every even number greater than two is the sum of two primes' if the Goldbach Conjecture were unprovable. For most of us, though, it would be correct to say as much only if the Conjecture were false—i.e., only if some even number greater than two were not the sum of two primes.

It may be replied that to object in this way is to fail to take seriously the possibility that classical logic might need to be revised. Not so: the objection is simply to Tennant's argument for revising it. It is, we are told, impossible to understand how the use of 'not' could have become entrenched unless it originated in the way Tennant describes. But the story he tells fails to explain the patterns of use which have actually become entrenched. In this respect, his Just So Story is less persuasive than Kipling's, for Kipling was at least offering an explanation for something that is actually the case. Elephants, after all, do have trunks.

So much for Tennant's first thesis. What about his claim that any logical principles that go beyond a Harmonious pair of introduction and elimination rules are 'unjustifiable excrescences'? As far as I can discern, the only argument he gives for this second thesis is in a parenthesis: 'The correct consequence relation, insofar as it should arise solely from the meanings of the logical constants, is, naturally, the least relation with respect to which the Harmony of the rules governing those constants can be sustained' (*op. cit.*, 97). That 'insofar as' clause is doing all the work. Tennant seems to take it to be obvious that the logical laws regulating a connective will arise solely from its meaning, but he gives no argument for this claim, which is in truth very far from obvious. Certainly, it is not obvious that only introduction and elimination rules may regulate a connective's logical behaviour. A classical logician might hold the position sketched at the end of §4, whereby the introduction and elimination rules for negation are the intuitionistic ones—namely, the rules of *Simple Reductio* and *Ex Contradictione Quodlibet*—but where an additional rule concerning negation—the thickening rule *EM*—is nevertheless valid.[25] There is no question of trying to justify *EM* by way of harmony considerations: it has the form neither of an introduction rule nor of an elimination rule. However, in the absence of any argument for the claim that the only valid principles that concern a connective are a Harmonious pair of such rules, there is no basis for Tennant's claim that *EM* is an 'unjustifiable excrescence'.

Tennant may protest that the case of 'tonk' shows that there must be some constraints on the introduction and elimination rules for a meaningful connective. Many people believe that those constraints amount to Harmony. Adding extra logical rules for a connective threatens to destabilise the equilibrium that Harmony guarantees, and thereby deprive the connective in question of a coherent sense. But there is a far better explanation of why 'tonk' is meaningless than that it violates Harmony. It is not meaningful, because a formula whose main connective it is does not say anything; such a formula does not say anything because it does not have truth-conditions. Thus consider the formula '2 is prime tonk 4 is prime'. This formula follows by 'tonk'-introduction from '2 is prime', which is true,

[25] This is, indeed, Hodes's position in his 2004. He holds that only introduction and elimination rules can constitute the sense of a connective (147), and requires that the elimination rule should be the maximum inverter of the introduction rule and that the introduction rule should be the maximum invertee of the elimination rule (156). (This amounts to Tennant's requirement of Harmony.) Hodes defines the 'basic logic' of a language to be that comprising only the sense-constituting rules for the connectives (151). Given his requirement of Harmony, he takes the basic logic for English to be first-order intuitionistic logic (*ibid.*). However, he allows that other sorts of rule, including *EM*, are fully justified (154), so that the 'total logic' for ordinary mathematical English is classical.

Hodes advances no argument for the Harmony requirement: he simply presents it as a conjecture whose implications are worth tracing out. Given that he allows the legitimacy of *EM*, though, his acceptance of the Harmony Thesis is in any case somewhat half-hearted. On his view, *EM* is a legitimate part of our inferential practice with negation. From an IRS perspective, then, it *is* part of the meaning of 'not'. I do not see the point of saying that, because *EM* is neither an introduction nor an elimination rule, it is not part of the *sense* of that word.

so it must be itself true. Yet '4 is prime', which is not true, follows by 'tonk'-elimination from it, so the formula cannot be true. No coherent truth-condition can be assigned, then, to '2 is prime tonk 4 is prime', and since both components do have truth-conditions, the culprit is clearly 'tonk'. As an explanation of why 'tonk' is meaningless, this explanation is superior to Belnap's, according to which 'tonk' is defective because it non-conservatively extends the pre-existing consequence relation. We shall sometimes want to do that, as when we add a truth-predicate to a mathematical theory (see *n*.20 above), but we shall never want a declarative formula to lack truth-conditions. If a formula succeeds in saying something, it will have a truth-condition, viz., the condition that is satisfied if, and only if, things are as the formula says they are. So the only declarative formulae that lack truth-conditions are those that fail to say anything.\[26]/

6. Harmony and Inferential Role Semantics

Our examination of three prominent arguments for the Harmony Thesis has left it without any justification. Supposing it is false, does that threaten Inferential Role Semantics? I think not. Our discussion of the Dummett-Prawitz argument for Harmony revealed the huge difficulties that confront the project of trying to explicate the notions of consequence and validity *directly* in terms of the rules which, for the IRS theorist, constitute the meanings of the connectives. But the IRS theorist is free to take an indirect approach. He might take the rules that characterize a connective's inferential role as specifying its sense, but allow that it also has a reference, or a semantic value. This semantic value will be the contribution the connective makes to the truth-conditions of a formula in which it occurs. Once we have a specification of truth-conditions for formulae of the relevant language, we can apply the traditional account of consequence in terms of the preservation (or necessary preservation) of truth.\[27]/

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[26] This account of what is wrong with 'tonk' is essentially that proposed by J.T. Stevenson in his reply to Prior (Stevenson 1961). Stevenson's reply was rather eclipsed by Belnap's, which appeared the following year and started the harmony hare running. Brilliant as Belnap's paper is, I think it was Stevenson who gave the better explanation of why 'tonk' fails to have a sense.

[27] In the Dewey Lectures which he delivered at Columbia University in 2002 (published as Dummett 2006), Dummett retreated to this position. 'The proponent of a truth-conditional theory of meaning', he wrote, 'must argue that [the] use [of sentences] cannot be described without appeal to the conditions for the truth of statements…To an important degree, such an argument would be correct' (Dummett 2006, 29). Truth is 'indispensable' in describing how sentences are used because 'a salient part of using a language is to give arguments in support of some conclusion', so that a full description of their

This oblique approach plainly requires an account of *how* the inferential roles determine semantic values—in Fregean terms, how sense determines reference. That is, it requires what Christopher Peacocke calls a 'determination theory'. In his paper 'Understanding Logical Constants: A Realist's Account' (1987), Peacocke begins to develop such a theory for the connectives, and Hodes (2004) has pursued the matter further. The best hope of an IRS theory of meaning lies, I think, with this approach, and the determination theory goes more smoothly if the inferential roles played by the connectives are characterized by the 'bilateral' rules mentioned at *n*.15 above, rather than the more familiar 'unilateral' rules. The kernel of any determination theory for the connectives will be the principle that the rules the reasoner goes by (or ought to go by) must preserve the correctness of sequents. For unilateral sequents, correctness is in turn a matter of preserving truth. Even given Bivalence, however, this constraint on the classical sequent rules fails to ensure that ⌜Not *A*⌝ is true whenever *A* is false (see Peacocke 1987, 164 and Hodes 2004, 162). By contrast, that fact about the semantic value of 'not' may be 'read off' the intuitive correctness of the bilateral sequent rule exemplified by 'Is Fred at home? No. So is it the case that Fred is not at home? Yes'.

Whether a fully satisfactory determination theory can be given for the connectives is an open question—one of the most interesting and pressing in the philosophy of logic and language. The verdict on the immediate issue, though, is clear. Some people like Górecki's Third Symphony but few would say that it is a patch on Beethoven's. One reason is that Beethoven knew better when to leaven harmony with dissonance. As in music, so in logic: there is no universal requirement of harmony.

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use 'needs a notion of truth, as that which is guaranteed to be transmitted from premisses to conclusion of a deductively valid argument' (*op.cit.*, 29, 31, 32).

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