Mathematics for teaching and deep subject knowledge
Voices of Mathematics Enhancement Course students in England.

Abstract

This paper reports an investigation into how students of a mathematics course for prospective secondary mathematics teachers in England talk about the notion of 'understanding mathematics in depth', which was an explicit goal of the course. We interviewed eighteen students of the course. Through our social practice frame, and in the light of a review of literature on mathematical knowledge for teaching, we describe three themes that weave through the students' talk: connectedness, reasoning and being mathematical. We argue that these themes illuminate privileged messages in the course, as well as the boundary and relationship between mathematical and pedagogic content knowledge in secondary mathematics teacher education practice.

Key words: mathematics for teaching, teacher education, deep subject knowledge, secondary, subject matter knowledge, pedagogic content knowledge.

1. Introduction

'Deep subject knowledge' or 'understanding mathematics in depth', is widely expressed as an important dimension of teachers' mathematical knowledge for teaching (e.g.Davis & Simmt, 2006; Krauss, Baumert, & Blum, 2008; Ma, 1999). In this paper we investigate and describe how students of a mathematics course for prospective secondary mathematics teachers in England, where 'understanding mathematics in depth' was an explicit goal, talk about this notion.

Our empirical field is a Mathematics Enhancement Course (MEC) in England, initiated in 2004. The MEC provides an alternate route into secondary mathematics teaching for graduates who do not have a mathematics related degree. Graduates with an A-level[1] (or perceived equivalent) in mathematics, who now wish to enter the teaching profession, can enrol for initial teacher training in England on the condition that they first successfully complete a six-month full time MEC. A range of institutions across England offer the MEC. As with any curriculum, the MEC will select from, and project a privileged orientation to, mathematics. Our interest in the MEC is because its students learn new, advanced level mathematics, and revisit/unpack mathematics they had previously learned, in ways deemed appropriate to their future careers as secondary mathematics teachers. A stated goal of the MEC is that its students learn 'mathematics in depth'. Insight into 'understanding mathematics in depth' in specifically designed mathematics teacher education courses, like the MEC, enables us to get some handle on what and how this construct and its interpretation plays out in various sites of practice. Furthermore, the MEC illuminates issues of knowledge selection into teacher education. In particular, it throws light onto the contestation over the boundary around subject matter knowledge (SMK), as described by Shulman (1986), and between SMK and pedagogical content knowledge (PCK) in secondary mathematics teaching. How this boundary is navigated impacts on planned curricula of all pre-service or initial teacher training programmes, including the MEC. Insight into how such programmes work, with and for their various participants, strengthens the field of both research and practice in mathematics teacher education.

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[1] "A Levels", or more formally, the General Certificate of Education Advanced Level, is the academic qualification offered by educational institutions in the UK (i.e. England, Wales and Northern Ireland) to students aged 17 – 18 years who are completing pre-university education. A-levels follow the General Certificate of Secondary Education (GCSE).

The research we report is part of a larger project (project name) and its study of what and how mathematical knowledge for teaching, or more simply 'mathematics for teaching' (cf Davis & Simmt, 2006), is taking shape in and across ranging contexts of mathematics teacher education. (Project name) work reported to date has focused on sites of mathematics teacher education, both pre- and in-service, predominantly in South Africa, and on the co-constitution of mathematics and teaching as dual objects of learning. We have examined pedagogic discourse as this unfolds in pedagogic practice across various courses so as to ascertain firstly whether mathematics and/or teaching is privileged in these offerings; and secondly to describe what then comes to be legitimated as mathematics for teaching and how this occurs (author).

The MEC is of particular interest as overall, it is a mathematics course, with a strong eye on the students' futures as practicing teachers. Mathematics is privileged, with selections and orientations shaped not only by mathematics, but also by anticipated or perceived demands of teaching secondary level mathematics. We describe it as a mathematics course where concern with mathematics teaching is simultaneously present, but back-grounded. In contrast to the South African work, our investigation of 'mathematics for teaching' in the MEC brings the prospective teachers themselves into focus, exploring their experiences of learning mathematics in the MEC, their progress into teaching through their initial teacher training course and their trajectories once in the profession[2]. The first of these explorations is the focus of this paper.

We have held in-depth interviews with a selection of MEC students from three different institutions at the end of their MEC. In addition to talking about what and how they learned mathematics in the MEC, we focused in on the students' notions of 'understanding mathematics in depth'. We noted this above as a common metaphor in use in the growing literature on mathematical knowledge for teaching, and officially a main aim of the MEC programme. The original MEC specification invited course providers to design a programme that developed "profound understanding of fundamental mathematics, emphasising deep and broad understanding of concepts, as against surface procedural knowledge" (Teacher Training Agency, 2003, p3). The specification called for development of "subject knowledge characterised by connectedness as against fragmentation... and by multiple perspectives – a flexible and adaptable understanding" (ibid., p3). There is an inevitable recontextualisation of knowledge and practices (Bernstein, 2000) in such curriculum design. We were thus interested in what and how the MEC students, as 'subjects' of mathematics pedagogy geared towards 'depth', projected its privileged elements, and how they traversed the boundary between SMK and PCK.

We begin with a brief description of the MEC, and the theoretical resources drawn on for this study. This includes a discussion of the literature base pertinent to the notions of 'deep subject knowledge' and 'mathematics for teaching'. This theoretical field provides a framing for the study and our description of what MEC students foreground in their talk. We conclude with discussions on the empirical, theoretical and practical contributions of our study to research on secondary mathematics teacher education.

2. The what and how of the MEC

The MEC was set up in response to secondary mathematics teacher shortages in England. It targets graduates from a range of non-mathematics degree backgrounds who are then required to develop their mathematical subject knowledge and deepen their understanding of mathematics prior to progressing to a one year post degree initial teacher training programme. The conceptualisation and development of the specifications for the MEC as a ***26-week full-time intensive mathematics***

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[2] We are in the process of analysing survey data of MEC graduates since 2004 from each of the institutions in our study. The survey explores their entry into, retention and progression in their professional posts, and thus the efficacy of the MEC with respect to recruitment and retention of secondary mathematics teachers in the UK.

course was done by a selected group of mathematics educators with experience in initial mathematics teacher training, together with mathematicians teaching undergraduate mathematics courses in universities, and mathematics schoolteachers. At the outset, the MEC was thus to draw from expertise in mathematics, teacher education and teaching. The directive was that graduates from the MEC should: know mathematical content equivalent to A-Level and beyond; revisit school mathematics; and in particular, they should have acquired a "profound understanding of fundamental mathematics, emphasising deep and broad understanding of concepts, as against surface procedural knowledge" (TTA, 2003, p. 3).

Notwithstanding this relative autonomy, the three institutions in our study (and, we would hold, other institutions in England that offer the MEC) all offered units covering Number, Calculus, Geometry Statistics, Mechanics, Discrete/Decision mathematics. These contents reflect the common secondary (for ages 11-18) mathematics curricula in England. All appeared to include material that was new to MEC students, and material that the students would have previously known. In other words, across units, there is content that is consistent with A-level specifications and beyond (i.e. advanced mathematics), and content focused on revisiting school mathematics (General Certificate of Secondary Education mathematics in England) – two components of mathematics for teaching advanced by others for secondary mathematics teachers (Zazkis, 2011; Zazkis & Leikin, 2010). While we are not suggesting that the mathematics material was presented in the same way across the institutions, interviews with tutors suggested that teaching sessions in all three institutions generally followed an interactive lecture-presentation format from tutors/lecturers followed by, or interspersed with, workshop time, during which participants tackled questions with tutor support. As with any selection of courses, employing a variety of tutors across several institutions, there is much teaching and learning which sits outside this general model.

Within this general framing of the course, and of interest in this paper and our wider study, what constituted 'deep and broad understanding of concepts' or 'deep' as opposed to 'superficial procedural knowledge' was open to interpretation by these institutions. We italicised the length and intensity of the course i.e. ***26-week full-time intensive mathematics course***, as this marks out its specificity. Unlike the typical structuring of courses in Higher Education, MEC students attend the course four days a week, six hours a day for six months where they are immersed in the study of mathematics. This intensity makes possible the development of a strong community of practice, where the object or focus of study is mathematics, and this mathematics is ultimately to be used in the practice of teaching.

There are additional similar features across institutions suggesting some convergence in how 'deep understanding' was interpreted in and for practice. Each of the institutions paid attention, though in different ways, to 'misconceptions' prevalent in key topics of mathematics learning, and to what in the specifications was referred to as 'fundamental' mathematics. This was interpreted as 'unpacking' key topics from the school mathematics curriculum. MEC students in all three institutions learn early on in their studies, that as mathematics teachers, it is not sufficient to operate at an instrumental level of understanding (Skemp, 1976). They are encouraged to think carefully about concepts and processes as they learn new mathematics, and as they revisit concepts located in elementary mathematics, which they may previously have taken for granted (Stevenson, forthcoming). This orientation to 'depth' – to fundamental mathematics, and upacking mathematical ideas - is exemplified by one of the institutions' tutors:

> We've been looking at a development of the definition of the locus of a parabola, defined as a point that moves () equidistant from a fixed point and fixed line. So we started off by going outside to the car park and I asked the students to arrange themselves as points on this locus. And here's the fixed point and there's the line [refers to example on paper]. … [I]t's not immediately obvious to them at all; they think they are going to be standing in a straight line. …. Then we come back to the classroom and we are laying out the cubes on a piece of flip chart to define this locus, um then we're saying, ok, let's change the rules; suppose the point is

> moving so it is twice as far from the line, () and this point [refers to example on paper] what shape are we going to get now? … Now that helps people to get to grips with something because they are really interacting with it, they are physically moving things about, they're measuring distances to get a rough sketch. Um, and then we can move into let's have a proper, or rather should call it a rigorous formal approach. So here is the point P, this is A, this is the foot of the perpendicular M, if A to P is equal to, if P to M is equal to () A, what does this give us? Then we'll bring in the formal textbook proof. … So we are starting somewhere different perhaps, we are dealing with it in a way which perhaps a traditional year 1 university module might not. They might just say conics, you know we're going to do the () parabola and here's an um definition. Um so next week I'll be moving on with that and going on to eccentricities and so on.

A common assessment rubric was provided in the specifications and guided assessment practices. The rubric indicated that all assessment needed to require demonstration of the following four dimensions of knowing mathematics: knowledge of mathematics; an ability to apply mathematical skills and techniques; an ability to make connections; and an ability to communicate mathematical ideas. Traces of interpretations of these dimensions are evident in the example above. That said, assessment forms, as with teaching approaches, varied within and across the three institutions, with a mixture of portfolio work and examinations, some of which are open book.

One of the institutions actually includes a course called "Fundamental Mathematics", and as implied above, deep understanding of mathematics is a goal that is *implicitly* threaded through the overall curriculum. The four dimensions of the assessment rubric give some content to this notion – it includes connections between and communication of mathematical ideas – and thus an expectation that MEC students are to expand their relationship to mathematics. They are to know more than specific content and skills. They need to appreciate and be able to use mathematics as a connected body of knowledge, and to become comfortable with communication of mathematical ideas. The question thus arises as to what and how this goal for deep subject knowledge and an expanded relationship to mathematics is enacted across the different institutions, and more specifically in this paper, how MEC students talk about this notion.

3. Locating the MEC and the students' talk – mapping the theoretical field

- **The MEC as a social practice**

As argued previously, mathematics teacher education programmes, wherever they are and however they are organised, are social practices (author). Others describe subject knowledge focused courses for teachers in England in a similar way (Crisan & Rodd, 2011). We can thus understand the MEC students as participants in a community of practice (Lave & Wenger, 1991; Wenger, 1998) where their shared goal, their mutual engagement, is *(re)learning mathematics* ***and*** *learning new mathematics* so that they are in a position to enter an initial teacher training programme where the focus will be on becoming a *secondary mathematics teacher*. As argued elsewhere (Author), programmes like the MEC offer a particular kind of apprenticeship. The MEC students are neither in schools apprenticing as teachers, nor are they in a mathematics departments in university where one might claim they are being apprenticed into mathematical practice per se. Nevertheless, it is useful to consider that they are learning to 'talk' both in and about mathematics, in a particular pedagogic setting and practice, with new or different boundaries around what counts in and as mathematics. Specifically, for the purposes of this paper, along with their revising and learning new mathematics, they are learning to talk in and about 'understanding mathematics in depth', as a valued orientation to mathematics in the MEC. What they say provides some insight not only into the messages communicated through the MEC, and the meanings they make of this notion, but also (1) whether and how *mathematics* and/or *teaching/ pedagogy* are foregrounded in their talk and (2) given the *implicit* nature of this notion, and the relative autonomy in institutions in their offerings, the *spread of meanings* that emerge and thus wider insight into the various ways in which this notion is privileged in MEC courses.

Of course, as a practice, identity work and relations of power are part of the. The dual identities of being mathematical and becoming a teacher are at play, and not all students are equally positioned with respect to the dominant messages in the course, including the notion 'deep understanding of mathematics'. These aspects of the MEC as a social practice are taken up elsewhere (Authors), and back-grounded in this paper. Here, we bring to the field ways of talking about deep subject knowledge by those immersed in 'it'. These are different voices of *mathematics for teaching* from those currently dominant in mathematics education research, particularly teacher-educator-researchers in Higher Education institutions.

- **Deep subject knowledge and its elaboration in the mathematics education literature base**

What then is 'understanding mathematics in depth' (expressed as a capability), or 'deep subject knowledge' (its reified form) for those in mathematics education research? Why is this kind of knowing/knowledge valued in and for secondary mathematics teaching, and where and how is this learned? We will engage with these questions through a review of selected pertinent research.

For some the notion of deep understanding is a capability, and linked explicitly to content knowledge, as in the MEC. Krauss et al (2008) and Baumert et al (2010), in their reports of the COACTIV[3] project, describe secondary teachers' "content knowledge" as an advanced perspective on secondary school mathematics.

> According to Shulman (1986), "the teacher need not only understand that something is so, the teacher must further understand why it is so (p. 9). Clearly, teachers' knowledge of the mathematical content covered in the school curriculum should be *much deeper than that of their students*. We conceptualised [content knowledge] as a *deep understanding* of the contents of the secondary school mathematics curriculum. (Krauss et al, 2008, p. 876). [*our emphasis*]

The example they provide as a measure of content knowledge asks: "Is it true that 0.99999… = 1? Please give detailed reasons for your answer" (p.889). They provide a possible answer as follows: "Let 0.999… = a, then 9.999 – 0.999 = 10a – a; hence a = 1". Thus, in their view, being able to *reason* this limiting condition is secondary school mathematics from an 'advanced standpoint' (Klein, in Krauss et al, p.876), and reflective of 'profound understanding of fundamental mathematics' in Ma's (1999) terms.

While describing 'deep understanding' as a capability, Krauss et al (2008, p. 876) simultaneously work with this content knowledge as a particular kind of mathematical knowledge. They propose a hierarchical classification where "a profound mathematical understanding of the mathematics taught at school" sits between "university-level mathematical knowledge that does not overlap with the content of the school curriculum" and "school-level mathematical knowledge that good school students have". Each of these is different from "the everyday knowledge that all adults should have", or what Ball, Thames, & Phelps (2008) refer to as "common content knowledge".

Zazkis & Leikin (2010), in contrast, describe Advanced Mathematical Knowledge (AMK) as "knowledge of the subject matter acquired during undergraduate studies at colleges or universities" (p. 263). While different from Krauss *et al's* notion of content knowledge (op cit), both align these interpretations of subject matter knowledge for secondary teaching with Ball *et al's* (2008) notion of "specialised content knowledge" (op cit), and a "deep understanding" of content. Zazkis & Leikin (2010) view AMK as "a necessary (although not a sufficient) condition for achieving this specialized knowledge for teaching at the secondary level". They examined teachers' descriptions of their use of AMK in teaching and report that the examples teachers

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[3] The COACTIV project is described as follows: Professional Competence of Teachers, Cognitively Activating Instruction, and Development of Students´ Mathematical Literacy

provided related to "meta-mathematical issues (proof, elegance of solution, rigor of language) or to pedagogical issues" (p.279). They summarise:

> … the majority of teachers focus on the purposes and advantages of their AMK for student learning, such as personal confidence, the ability to make connections …only a few provide content-specific examples (p.263).

Zazkis & Leikin's study has resonance with the research we report in this paper, as it presents practitioner perceptions - a view from practice. Interestingly, their perceptions are reflective of Ma's (1999) notion of profound understanding of fundamental mathematics (PUFM). Working from interviews with primary teachers in the US and China, Ma (1999) identified four key components to PUFM – connectedness, multiple perspectives, basic ideas and longitudinal coherence - that together constituted a knowledge that was "deep, broad and thorough" (p.122-123). She argued further that PUFM is "more than sound conceptual understanding of elementary mathematics", it includes the capacity to teach mathematics to students (p.124) i.e. it is mathematical knowledge attuned to and usable in teaching. Like the teachers in Zazkis & Leikin's (2010) study, content knowledge for teaching is situated, tied both to the context of teaching and the identity of being a teacher (Hodgen, 2011). Hence, connections 'between' aspects or topics of mathematics, rather than specific contents are foregrounded. Connectedness is central to PUFM. A teacher with PUFM makes connections "between concepts and procedures", and "among different mathematical operations and sub-domains", and the other three components above are "kinds of connections" that lead to different ways for students to make meaning of mathematics (p. 123) – and so a blurring of the boundary between subject matter and pedagogic content knowledge in Shulman's (1986) terms.

Krauss et al (2008) and Baumert et al (2010) are also clear that content knowledge is necessary but not sufficient for teaching. They clearly mark out content knowledge from PCK, which they conceptualise as "knowledge of explanations and representations, knowledge of students' thinking, and knowledge of multiple solutions to mathematical tasks" (Krauss et al, 2008, p.888). A measure of PCK they exemplify which involves 'representations' and 'multiple solutions', asks: "How the surface area of a square changes when the side length is tripled?" requiring respondents to "note down as many different ways of solving this problem as possible" (p. 889). They describe this as knowledge of mathematics and tasks, and hence PCK. Yet, is this not the kind of connectedness described above by teachers? Why are different ways of solving a problem, not 'deeper' understanding of the notion of area, for example? Implicit in the above PCK item is what others might refer to as flexible or connected knowledge of mathematics, being able to work within and between different representations. While most in the field would agree this is an important component of mathematical knowledge for teaching, whether this is PCK or content knowledge, and so part of 'deep' understanding or deep subject knowledge, is contested.

For others, deep understanding has been explored through the metaphor of 'unpacking' mooted by Ball et al, and suggested by Ma (1999) in how teachers organise and use their knowledge packages in teaching. (Authors), operationalized this notion through specifying its source in action, and the visibility of *chains of reasoning. "*Unpacking, be it in an operational sequence (e.g. in solving an equation), or in linking one mathematical idea to another, required reasoning – justification for a move from one step to the next" (p. XXX). That explicit mathematical reasoning is part of teachers' work is included in Ball et al's (2008) descriptions of tasks of teaching and their mathematical entailments, specifically providing robust explanations in forms meaningful for different levels of learners. Indeed, they separate reasoning from knowledge. Their identification of the mathematical work of teaching and its interpretation into measures, is that they require 'mathematical reasoning', and not other kinds of reasoning.

Ruthven's (2011) overview of the chapters in the book "Mathematical Knowledge in Teaching" distinguishes three approaches to subject knowledge for mathematics teaching. 'Subject knowledge differentiated' refers to approaches that categorise knowledge e.g. Ball et al (2008), Krauss et al (2008), and is distinguished from ‘subject knowledge situated’ (e.g. Hodgen, 2011), and from ‘subject knowledge mathematized’. Subject knowledge mathematized relates in some ways to Krauss et al’s (2008) description of content knowledge, linked more to the practices of mathematics rather than practices of teaching, and captured by the teachers in Zazkis & Leikin’s (2010) study as related to confidence, and orientation to proof, precision and language. This additional ‘voice’ links ‘deep understanding’ with mathematical practice.

Watson (2008) conceptualises mathematical knowledge as “a way of being and acting” (p1). Teachers’ knowledge develops and grows through “doing mathematics and being mathematical” (p1). Barton (2009) supports Watson’s (2008) ideas of knowledge as a way of being, and that being mathematical is essential. He recognises that *what* teachers know is important, but he contends that equally important is *how* teachers hold that knowledge. He argues that that “teachers must [sic] embody modes of mathematical enquiry themselves…Teachers must be mathematicians” (p 5). He suggests that a key to effective teaching is in the teacher’s attitudes and orientation towards mathematics. Notwithstanding the absence of focused empirical study, Watson and Barton (2011) drawing on extensive experience, contend that ‘teachers [must] enact mathematics’ (p67). It is the process of doing mathematics that is at the heart of teaching mathematics. Notwithstanding the imperatives and normative views here, this thread of literature in response to and as part of the literature on mathematics for teaching points to the issue of disposition, and one that has been noted elsewhere as an important ‘strand’ of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001).

Returning to the questions posed at the beginning of this review, notions of ‘understanding mathematics in depth’ (expressed as a capability), or ‘deep subject knowledge’ (its reified form) vary, not only across but also within primary and secondary studies of mathematical knowledge for teaching. This kind of knowledge, despite differences in interpretation, is widely valued as important for teaching. Recent large scale studies (Baumert et al., 2010; Hill, Rowan, & Ball, 2005) have reported positive correlations between this kind of knowledge, teaching quality and student achievement. Nevertheless, interpretations from all of the above into mathematics teacher education programmes and courses that focus on specialised or ‘deep subject knowledge’ like the MEC are likely to differ with respect to the boundary between and around each of content and pedagogical content knowledge. An interesting recent report on a similarly focused ‘capstone’ course in mathematics for prospective secondary mathematics teachers (Artzt, Sultan, Curcio, & Gurl, 2011) is described as linking college mathematics with school mathematics and pedagogy.

This turns to the question we posed about where and how such knowledge is learned? Unlike the capstone course above, those who see deep subject knowledge as situated, contend that it is largely learned in practice (e.g. Ma, 1999). For others, e.g. Zazkis & Leikin, (2010) it is embedded in university study of mathematics per se. Both positions pose questions as to the efficacy of a course like the MEC. Baumert et al (2010) discuss the socio-political ramifications of different mathematics teacher preparation programmes in Germany, and the different apprenticeships into academic mathematics provided. They suggest that knowledge of the structure and practices of mathematics that they believe is necessary for high quality secondary mathematics teaching does not necessarily imply a full undergraduate degree in mathematics.

Given the range of meanings attached to ‘deep understanding of mathematics” or “deep subject knowledge” in the field, the limited specification of this notion in the ‘official’ MEC outside of the assessment rubric, and its relatively short duration for further study of mathematics, our interest in this paper is the orientations to mathematics in the MEC as projected in student talk. Do they privilege advanced mathematics per se, in contrast to an advanced perspective (reasoning/proof and connectedness) on secondary mathematics? Is the mathematics in the MEC experienced as knowledge *for* pedagogy or does it include knowledge *of* pedagogy (i.e. activating learning of others)?

The following questions thus guided part of our interviews with the MEC students:

(a) What do MEC students foreground when they describe the MEC to others?
(b) Are there dominant threads in how students talk specifically about ‘understanding mathematics in depth’?
(c) To what extent and in what ways does the practice of ‘teaching’, or mathematics itself, legitimate their talk and ‘understanding mathematics in depth’.

4. The study and its methodology

The (Project name) UK project is a longitudinal study which commenced in 2008 and is currently on-going. It involves three institutions, which for anonymity we have named A, B and C. The empirical works for the study have spanned over three phases: The first phase of the study involved, initially, eliciting information from MEC tutors in March 2009 and then gathering more substantive data from students as they reached the end of their MEC course in June 2009. The second phase involved collecting data from the same MEC students towards the end of their initial teacher training course in June 2010. Currently, we are conducting a survey of MEC students in their teaching positions. Our concern in this paper is with the first phase data, and specifically the student interviews.

The student interviews

We selected six MEC students from each of the three institutions. The selection was purposive - guided by a set of criteria so that selection included a spectrum of students: with mathematical and non-mathematical backgrounds; different cultural and educational backgrounds; with ranging participations and performances on the course and included both male and female students. Specifically, 14 students were educated in the UK and four educated abroad (2 students from Nigeria, 1 from Cyprus, 1 from Pakistan). The point in their lives at which the students joined the MEC varied: 3 students joined straight after finishing their degrees at University. Four students had had a short career and 11 students had had a long career before entry into the programme. Only five students had some form of teaching career or teaching experience before joining the MEC. The subjects studied by the students at degree level ranged from: Educational Studies, Business, Computer Science, Engineering, to Sports Science. In regard to mathematical background and qualifications: 15 students had A level mathematics or its equivalent and three had below A level qualification in mathematics but with some mathematics related study or experience in their educational and work histories.

We were interested in patterns of talk across the student spectrum, and across the institutions. We conducted in-depth semi-structured interviews that included questions related to the students’ entry into, and their experiences on the MEC. This paper focuses on two particular interview questions interviews and the students’ responses, as we work to answer the research questions posed above:

1. Imagine a prospective student comes to you and asks you as an ‘outgoing’ MEC student: “*I am thinking of shifting to teaching, and will need to do the MEC programme. What can I expect to be doing in the programme?”* What would you tell such a prospective student about the MEC course?
2. In the MEC documentation, and all your introductory documents, an emphasis is placed on ‘understanding mathematics in depth’. Can you describe what this means for you now that you have nearly completed the programme?

Data analysis

Following our presentation of the MEC as a social practice, and the students of the MEC as participants in this practice, we understand that students will acquire ways of talking and acting in the manner of the practice. We thus examined their talk, so as to describe patterns – regularities - in how they represented the course on the one hand and the notion of ‘deep understanding’ on the other. Our content analysis of responses to Question 1 focuses on whether the MEC was talked about predominantly as a mathematics course, or as a course about learning to teach mathematics. The following questions guided the analysis: Is the MEC a mathematics course for them? If so, how do they describe and exemplify it? How do they legitimate their descriptions and examples: from a perspective of becoming more mathematically knowledgeable, or from a concern with their future roles as teachers? Interview utterances describing the MEC that referred directly to mathematics, to levels, topics, concepts were coded as *mathematical*. Utterances describing the MEC that referred to aspects of teaching and learning e.g. *learning about specific misconceptions* were coded as representing the MEC as a course about *teaching*. This content analysis provides a context for interpreting their responses to Question 2, and what counts as mathematics in the MEC.

5. Our analysis with respect to Question 2 focuses on how students talked about 'understanding mathematics in depth'. Here we developed a language with which to systematically analyse all the data, working as is necessary in educational research between our empirical and theoretical fields (Dowling & Brown, 2010). Our first level of analysis was inductive, and then as themes emerged, we re-examined all the data typologically (Hatch, 2002), using the language of description we constructed. **Presenting and interpreting the students' voices**

5.1. The MEC is an 'intense' mathematics course

In their descriptions of what a future student of the MEC could expect, fifteen of the eighteen students interviewed described the MEC in mathematical terms. Emphasis was placed on both revisiting school level topics comprehensively, as well as moving onto A-Level mathematics and beyond. This articulation was common across the three institutions, and reflects the explicit intentions of the MEC to be a *mathematics course* that precedes the students' formal teacher training. Some descriptions included reference to the demands of the course as well as its content, indicating that levels of mathematical competence were important for coping with the course from the start, and that hard work was required.

We distinguish the student quotes by their unique ids. This consists of a number assigned to each student and the letter of their respective institution A, B or C. For example, A4 below is the fourth student interviewed in Institution A. We deliberately include students from across the institutions.

> I would say *a lot of maths*, [laughs] um yeah *a lot of maths*. (A4)

> Em, it's a *lot more intense* than a standard undergraduate degree, so if you're coming into this with the opinion that it's just another university course, I mean, you should dispel that notion straight away... because *you are expected to do an awful lot. A lot of it is to be done off your own back*... (C5)

> *It's a lot more intensive* than I thought it was going to be um [you] basically hit the ground running. Why I find this is maybe because my maths knowledge, I, I, find that my maths knowledge is quite a bit below people that are on the course. Um, the typical day, start a new topic um and you can kind of get it but personally [I] get it till lunch time, after lunch time its just [laughs] () *it's just that the level is stepped up quite a bit. (B6)*

> I just knew it was *going to be Maths. But I didn't know how much, how in depth*. But you, your first, the first thing I felt was really good at getting all the basics down. … And, em, so and they just, it got rid of all the cracks and the problems and like it just brought in like all your algebra, your trigonometry. Like it just, the first term is all about securing what, in theory, is called 'the basics'. I mean, it's

> not the basics, because some of it was at A-level ..further GCSE, lower A-level, AS stuff. …. (B5)
>
> Well the stuff you do a lot of it isn't like school level maths … *It's kind of more like degree level maths*. (A5)
>
> Obviously, *it's a high level of maths that you're doing. It's A Level and above,* em, but nothing far from what you should have already done at A Level, nothing too far from that. *Like you've learnt a topic at A Level but then you're moving on to work on it a little bit further, a bit more in-depth.* (A2)

While for all these students, the mathematical focus is clear, their descriptions of the kind and level of mathematics contained less certainty. This hedging in their talk (i.e. it's kind of more like degree level, a little bit further) is interesting, and clearly an assertion of identity and positioning with respect to mathematics (cf. Author et al, 2013). Nevertheless, the MEC is marked out in their talk as not being a match with GCSE nor A-level maths learned before, nor with university mathematics. You did 'more' with the maths you were learning, both new and revisited. As exemplified and legitimated below, the 'more' is not about *content* per se, but about 'depth' or 'underlying trends'/linkages' or 'proof', legitimated by the (envisaged) demands of teaching.

> I guess I, the way I describe it is we're effectively, we're training to be teachers up to the standard of A-level and therefore what the MEC in many ways does is like if you imagine every topic that you might have to come across at A level... to a certain level so you're comfortable... and confident with them and then as well as that they're all linked in together, you know, underneath that, so there's the, there's the underlying trends... that run through all of them, but it's basically, it's not... my sister's done a maths degree and, er, you know, I don't know some of the things that she knows by any stretch of the imagination but for the intents and purposes of becoming a maths teacher it's kind of, it's almost perfect really, because it takes you to the level that you're going to be teaching it at, and slightly above, so that you're comfortable when you've got the understanding of all the areas that you'll be teaching. (B3)
>
> ...it's almost like, you know, a rusty machine where the cogs are starting to work.... so it took me a good few weeks to, you know, almost get up to speed, get into that way of thinking... you're maybe revisiting subject areas you've done before...but you'll certainly learn, you'll see those subjects in a different context, em, you'll see proofs … (A6)

A common story across all three institutions and students in them could be retold as follows: We are going to be teachers. So in the MEC you revisit school mathematics (particularly GCSE, and also some A-level maths) in new ways. You also learn new mathematics, some of it at a high level, but closer to what you have done before as you have done A-levels though some time ago. You will work very hard, and do a lot of mathematics, but in a different context – mathematics that is not like school nor university maths.

This last sentence is pivotal – the MEC is a different social practice with a particular or specialised orientation to mathematics, where it is dealt with intensely, and in depth. From the description of the course above, this orientation is reflected in the assessment rubric where connections and communication on the one hand, and fluent procedures and conceptual understanding on the other are valued. These resonate, to some degree, with a kind of mathematics that is neither like school nor university mathematics, so perhaps 'in between' these; content knowledge in Kraus et al's terms, or 'depth, breadth and thoroughness', PUFM, in Ma's terms. So what then, is this 'a bit more', 'in-depth maths' and 'different way of learning it' for the MEC students? We turn now to the students' direct responses in the interview on 'understanding mathematics in depth'

5.2. Understanding mathematics in depth as reasoning, as connectedness, and as developing mathematical disposition

As noted earlier, from a social practice theory perspective, the students' articulations reflect privileged practices in the MEC, and their learning to talk within and about mathematics in the course (Lave & Wenger, 1991). The first and most prevalent theme noted in our inductive analysis was that of ***'reasoning'***. This was identified in statements that related to *communicating*, for example, *'why'* a procedure worked, how a statement could be *proved*, or how an idea could be *'unpacked'* and so shown 'where it was coming from'.

The second theme was that of ***'connectedness'***, and identified in statements that referred to *making links*, be these between mathematics and the non-mathematical, or between topics/concepts in mathematics, or between concepts and related algorithms. The less prevalent, yet worth noting, and our third theme, was what in the literature has been referred to as **'*mathematical disposition*'** (Kilpatrick et al, 2000). We identified *dispositional* talk when students talked about becoming 'more mathsy' or '*seeing more mathematics in the world*'. Kilpatrick et al (op cit) are clear that the strand of 'disposition' in their construct of mathematical proficiency is slippery. We agree, but posit, nevertheless that the presence of talk of "becoming more mathematical' is indicative of an orientation to mathematics in the course, and thus worth noting here. We attend more systematically to this notion in the work on identity mentioned above.

We found it interesting that in many cases, the students resorted to *negation* to explain what understanding mathematics in depth meant to them. Their elaboration of this notion was through a description of what it is *not*. For example, the connectedness and reasoning themes are evident in the following: [understanding mathematics in depth is …] 'not just touching on the topic', 'not looking at things in complete isolation', 'not just rules'.

Of further interest and a pattern across student uttterances was the strong association between each of the themes and their significance for teaching. Understanding mathematics in depth as a kind of mathematical knowledge and way of knowing mathematics was important if you were going to teach; important for being able to explain mathematics to others.

Our movement between the data and the literature is apparent in our illumination of connectedness, reasoning and disposition as notions of mathematics for teaching in the literature review above. We now turn to evidence the emergence of these themes in and across the MEC students' talk. We select voices that strongly exhibit the particular themes as well as illuminate the spread of meanings attached to a theme. Discussion of these findings follows in section 6.

Understanding mathematics in depth as reasoning

Of the 18 students interviewed, the majority (12/18) spoke of understanding mathematics in depth as some form of proof or mathematical reasoning such as being able to work from first principles, providing a proof, knowing why and where 'things are coming from'.

To illuminate our analysis, we have annotated the text according to the following key::

i. **Statements identified with the theme are highlighted in bold typ**
ii. *Negations are italicised*
iii. <u>Statements that legitimate this kind of knowledge as important for teaching are underlined</u>

For most of the students who emphasised reasoning, understanding mathematics in depth is being able to break down knowledge (unpack it), understand its sources, work from first principles and construct proofs. What is NOT allowed, or regarded as insufficient knowledge in the course is 'just rules'. In addition, motivation and egitimation for these aspects of deep understanding lay in teaching, and the importance of being able to explain to (and reason with/for) others, particularly learners whose level of mathematics was 'lower' (than the students'), or who see things differently:

> [understanding mathematics in depth is] … [knowing] **where it comes from, historically; so it is having the history of the topic and a proof of the topic; … like being able to split the graph into tiny little bits …I use the example of Pythagoras ... stands out for me as the most basic proof** that I can't believe no one's ever told me; You need background to it ... Having that understanding behind you to go on to the classroom is, could be very powerful, em, and getting that across in all different topic areas as well. *You can't just give "here's the rules".* (A3)

> ... there are different levels to understanding maths in depth, I think. I think to start with, **it's the first principles and working from first principles to**, particularly for, em … the early maths learners. ... people who are maybe at a lower level and need to take things more slowly. I think that was really important to go through those **first principles** and to explain the, the **basics** and be able to explain the basics**.** (C6)

> **Where something comes from ... understanding what particular parts of the formula mean or the rule; how you prove them … You can say this rule is in a textbook, but knowing why that works or how that works is the most important thing**

> If I learn it in a deeper way it will definitely help teach it in the future …. **I'll have like the basics of why certain things are the way they are**… so it will help me find different ways of teaching kids who are actually struggling and for kids who learn in a completely different way (A4).

Some students emphasised reasoning through Skemp's (1976) well known distinction between instrumental and relational understanding. For these students, understanding mathematics in depth was about being able to r***eason,*** and about ***knowing why.*** Just knowing rules, getting right answers, or passing exams was not reflective of deep subject knowledge. Here, teaching was not always explicitly invoked. For example,

> **You are learning why that's the case, throughout the course on different modules**. *You are not just touching on a topic and "Here's a set of rules, learn them";* **You are understanding the reasoning behind them.** (A2)

Reasoning, as evidenced in the students' talk, thus included a range of meanings and legitimations. It was not tied to specific topics, or modules, nor to a single institution. It threaded through and across the three institutions' MEC courses. Given its strong legitimation in teaching, it is reasonable to suggest that this valued emphasis in the various MECs developed in response to the dimension of 'communication' in the assessment rubric.

- **Understanding mathematics in depth as connectedness**

Half of the students (9/18) spoke of deep subject knowledge as being connected, flexible knowledge, and deep understanding as reflected in the ability to make links across different areas of mathematics, and to the real world, and through this being able to explore different kinds of mathematical problems.

Following the same key as above, **statements indicating the theme of connectedness are in bold type.** Deep subject knowledge, like an open box, enables you to choose appropriately what to do and when, what links can be made and how. If you understand mathematics in depth you can approach problems, flexibly and solve them. Negations are italicised. hat is 'not allowed' cue-based behaviour – this is a manifestation of fragmented, inert knowledge.

> [understanding mathematics in depth is] Different concepts … how they originated ... **how it can be applied** ... **how it is interconnected with other aspects of mathematics**; **it's like an open box**; *We don't just look at the problem and say "Oh, this is calculus" ... no,* **You can try different methods, be flexible; see what applies**; you are not going to be scared of anything because you know there is a solution and you can attain it. (B1)

The talk of connectedness was, interestingly, not explicitly tied to teaching. Focus was more on the importance of connected knowledge as what is means to know and be able to do mathematics. Understanding mathematics in depth is an appreciation of this connectedness of mathematics and thus also an orientation to mathematical activity. For example,

> **It's a link … I used mechanics in my calculus coursework**. I came across a problem and saw "**Hey, I know how to do this. It's a maximum. I know if I set it to zero and differentiate, it is going to give me what I need".** (B3)

> … **depth of understanding is that … you are acquiring a facility to work with certain building blocks**, … … I think that's what I've been getting most excited about, because a lot of what we've done, apart from the decision maths, isn't new to me, **it's starting to kind of look at those linkages and... starting to think about.... 'Yes, how does that fit in with this?' and, 'Oh, so there are different ways to...' for example, if you've got a series and you're trying to find out what the function is that fits that, you know, there are some heavy duty sort of calculus approaches, but there are some simpler ones that you can use with induction**. ... understanding that some of the concepts that you get taught at a particular level might then be turned on their heads... to be aware of the assumptions and the kind of constraints that relate to a particular subject area. (C3)

> Understanding the reasons why ... You might be able to use it at a superficial level ... say it was a formula, you can use that formula fine, **then it is going to the next level, you know how to use it in the real world, when you shouldn't use it, how it can link to other parts or areas of mathematics or other subjects as well. You're not looking at everything in just complete isolation …** (A6)

As with the talk on reasoning, connectedness in the students' talk reflected a range of emphases. The presentation of mathematics as a connected body of knowledge threaded within and across the institutions and the MEC courses, indicating its valued emphasis. It is reasonable to suggest a further link to the explicit requirement of 'connections' in the assessment specifications. The MEC students did not only talk about making connections: some easily exemplified their claims.

- **Understanding mathematics in depth as developing a mathematical disposition**

For a few students, becoming more mathematical, or developing a mathematical disposition was in the foreground. For example, in bold type below the **dispositional** is evident in an emphasis on coming to see the world through mathematical eyes. Embedded is the connectedness theme or the underlying web of linked concepts in Ma's terms.

> [understanding mathematics in depth s] The patterns you never used to start noticing, **at this point in my life, I definitely think about mathematics and I think quite different to how I did what I was doing my GCSEs; ..**. I think I, as a child when you're doing maths, I think you just, you see the question and it seems to fill your mind, em, you**, I suppose I just see numbers in a different way,** when you, and, er, having to extract the maths from a question where it's given in English is, em, seems a lot easier and more automatic, em. … **I don't think it's something that can really be taught, and I, maybe it's part of the reason why the MEC does pile everything on top of you, doing three separate, em, modules simultaneously**, is that you do notice, em, the similarities between the different branches, **em. Em, I mean, I suppose maths has a bit of a mystery quality to it, you kind of, the more you, the deeper you go into it the more is kind of unveiled to you...** (C2)

> **the more you know about something the more you want to know** the more you want to impart that knowledge (B6)

> **You can try different methods, be flexible; see what applies**; *you are not going to be scared of anything because you know there is a solution and you can attain it.* (B1)

Having evidenced the three themes in the students' talk, acknowledging the limitations to the theme of disposition, we move on to elaborate these further, relating back to our research questions and then research in the field.

6. **Discussion**

In this paper our focus has been on how MEC students responded to two particular questions in our study. The first question relates to how they would describe the MEC to prospective students and the second relates to "understanding mathematics in depth". We have shown that the students overwhelmingly consider the MEC to be an intensive mathematics course. They describe it in terms of a) the level of mathematics that they do on the course - GCSE, A level and beyond; and b) how they learn new topics and re-visit old topics. The course entails doing "a lot" of mathematics, that is different from the hierarchical mathematical knowledge you would develop through a full undergraduate mathematics degree. This 'more' mathematics learned is further projected as pertinent to their particular practices i.e. that they are all training to become mathematics teachers. In other words, the MEC is a mathematics course *for* pedagogy (privileging SMK in Shulman's terms); the students do not project it as including specific or explicit knowledge *of* pedagogy. The students thus reflect a particular boundary around mathematics in the MEC, a function perhaps of the set-up of the MEC as preceding an initial teacher training course, where knowledge *of* pedagogy becomes more explicit.

As our second question, we explored the MEC students' talk about the notion of 'understanding mathematics in depth', so as to describe the specificity of this 'more' within the MEC course. We posited that this provides another empirical dimension to the notion of mathematical knowledge for teaching, emerging as it does from those who were directly involved in its learning. Of interest are the ways in which they emphasised and legitimated the three themes that were dominant in their interview responses. The students' legitimations for being able to reason, knowing why, where parts of mathematics come from, and being able to prove theorems, was deemed important for being a mathematics teacher – and for enabling the learning of others: if you were going to teach, you needed to be able to explain mathematics to others. In contrast, connections in particular, and developing a mathematical disposition were more about what it means to do mathematics and so being able to do mathematics for yourself. Being able to connect various parts of mathematics, being flexible in this, connecting mathematics to the world i.e. approaching mathematical problems with flexibility and confidence were about becoming more mathematical.

From the perspective of the MEC as a social practice, and the voices of students a reflection of recontextualising processes at work, our data suggests that whatever the intentions of the MEC, students' experience of 'reasoning, proof and proving' in their learning of mathematics was more about being able to reason for others, than developing these mathematical practices per se. In most cases, talk about mathematical reasoning and proof did not foreground nor exemplify the nature of proof. On the other hand, being flexible and making connections between various parts of mathematics, seeing it like an 'open box' was what was needed to do mathematics per se, for yourself. This strengthens our earlier suggestion that the specification for connections as necessary in assessment meant that students were required to demonstrate this in their mathematical activity. In turn, the students could recognise their increasing facility with this.

Of course, there is a fine boundary between doing mathematics for oneself, and in then in a way that enables others to learn. What is emergent here is an interesting perspective on how the MEC shifts the boundary around mathematics, so that mathematics as an internally connected body of knowledge is privileged, together with its applications, and a related disposition. This is an expanded relationship with mathematics, pointing to what was accomplished (as projected by students) through the intensive and specific orientation to mathematics in the MECs, their content selections and pedagogies.

The prominence of reasoning and connectedness are also interesting in the UK context within the influential works of Skemp (1976) and Askew et al (1997). Skemp's (1976) early work focused on the distinction between relational and instrumental understanding with the former oriented towards connections and relations between mathematical concepts and the latter towards mathematical performance and procedures. Skemp highlighted then that school mathematics tends to emphasise the instrumental over the relational, and this theme has continued to have influence particularly in teacher education programmes, and mathematics education research. At a more empirical level, and thus differently powerful and influential, Askew et al,'s (1997) study found a link between quality of teaching and connectedness of teachers' knowledge – again with impact on teacher education programmes. It is thus not surprising that reasoning and connectedness emerge as part of 'deep understanding' for prospective teachers in the UK within the MEC students' talk.

The emphasis on connections in the MEC as reflected by the students resonates with the wider literature that reports on courses or programmes designed for 'mathematics for teaching'. For example, Vale et al's (2011, p.209) in-service course for -'out-of-field' (non-specialist mathematics teachers is geared to 'positioning (practicing) teachers of secondary mathematics as learners', providing opportunities to make connections in mathematics. The students' emphasis on revising GCSE mathematics has resonance with the description of the capstone course for prospective secondary mathematics teachers discussed by Artzt et al (2012), where revisiting secondary school curriculum topics is included, and structured 'from an advanced perspective'.

The advanced perspective on secondary mathematics brings the COACTIV project (Baumert et al., 2010, Krauss et al., 2008) back into focus. In our presentation of the students' talk about the MEC, and what they have learned, we argued that this was, for them, a mathematics course. Mathematics, and not teaching/pedagogy, is privileged in the course. And in this mathematics course, they revisited GCSE mathematics, some A-level mathematics, and some of the mathematics they did went 'beyond' A levels. This course is perhaps an interesting empirical example of a course where Content Knowledge for teaching as described by Krauss et al, i.e. specialised knowledge for secondary teaching was in focus, together with some university or college mathematics. The students' description of their mathematics in the MEC, however, suggested something 'more'. It was not only what mathematics they learned, but also how they learned this, and so how they were able (in their view) to hold and use their mathematics.

Of additional interest to us were the ways in which many of the students' descriptions of the mathematics in the MEC and of understanding mathematics in depth were supported by negation- by strong statements about what it 'is' and what it is 'not'; by 'what is not legitimate or allowed'. We posit here that this negation suggests on the one hand, that 'mathematics for teaching' as a new discourse, remains 'fledgling', with descriptors of what it is requiring further development. On the other hand, negation and the relatively strong statements by students also suggest a new orthodoxy, and the implications of this for students' mathematical and teaching identities are explored further in (Authors).

7. Conclusions and implications

Our contribution to the literature, research and practice related to mathematics for teaching is two-fold. Firstly, in exploring the voices of the MEC students we add to the voice of participants in mathematical knowledge for teaching – importantly recognising that these voices are of a group who began their journey as non-specialists in mathematics . From a social practice perspective, these voices are a recontextualisation of the MEC, projected by the students. Through their talk we see their experience of a retraining programme where mathematical development is made possible, while simultaneously turning the practices towards students' futures as teachers. Key elements of this practice include both revisiting and new more advanced mathematics; and assessment guidelines that specify at least at some level, the expected and valued mathematical practices.

Secondly, we set out to describe what and how students who have been learning mathematics in a relatively new course that has 'deep subject knowledge' as an explicit goal, talk about this notion. We have shown that three themes thread through their talk. While these are not surprising, indeed a social practice perspective anticipates such projections, the way these were emphasised is illuminating. Deep subject knowledge is connected knowledge, enabling flexible thinking and problem-solving or doing of mathematics. Their talk about connectedness appeared to be more of a statement about how they were expected to approach the doing of mathematics themselves, and how seeing the connections between different aspects of mathematics was critical to this. Connected knowledge was largely described in terms of what it is, e.g. like an open box (i.e. not closed), and strongly tied to a positive disposition towards mathematics. Many of the students, across the three institutions, spoke about connectedness in this way. In contrast, when the talk emphasised reasoning, relating reasoning to explaining why, working with first principles and proof, it was largely legitimated as necessary for the practice of enabling others to learn mathematics. Talk about reasoning was also marked out by the prevalence of the students simultaneously stating what it is not. The students' talk suggests that the specification of connectedness in the MEC assessment guidelines is fruitful. Their talk suggests too that reasoning and proof are valued practices. It is thus interesting that these are not explicitly indicated in assessment guidelines.

Finally, the students project the MEC as unambiguously a mathematics course, yet the discourses of 'mathematics for teaching' are strongly prevalent within it. They learn more than mathematics. They also learn an orientation to mathematics, one that is valued in the emerging field of mathematical knowledge for teaching. That said, some disclaimers are necessary by way of conclusion. We cannot and do not wish to claim that the students' talk in any way tells us what they know and are able to do in practice. It is of interest, however, that they are able to talk about and exemplify these valued aspects of mathematics pertinent to teaching. Others, Ma (1999) for example, argue that 'profound understanding' develops in the context of teaching, and thus in service, rather than in pre-service. We are also not claiming that students 'have acquired' or 'know' this specialised knowledge. Rather, as active participants in a version of this, and a particulat social practice they have begun to acquire its features. Finally, we are not arguing in this paper, that the constructed bias and focus of the MEC is necessarily productive of "good" or "effective" mathematics teachers. This question is the object of following forthcoming work as we track the graduates from the MEC through their PGCE and into schools.

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Blinded

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