活动标形及其在点啮合齿轮传动误差分析中的应用[*]

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摘　要　活动标形法的论述首推苏步青译,佐佐木重夫著《微分几何学》,还有的著作从外微分形式引入活动标形.本文论证取曲面的正交曲率网为参数网时,曲面论的基本公式就是活动标形的微分形式,并用其分析了点啮合齿轮传动误差.

关键词　活动标形　接触分析　曲率网

Moving Frame Aand Its Application In The Error Analysis Of Point Contact Gearing

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Abstract　This paper demonstrates that the basic formulas of surface theory are just the differential formation of the moving frame, if the vertical curvature web is chosen as the parameter web. And applied the theory into the analysis of the transmission error of the point contact gearing.

Keywords　moving frame　contact analysis　curvature web

1　The moving frame and the fundamental theory of the differential geology

Suppose that the two parameter curves u and v have been chosen as the curvature web, then we will have the following equation $F=0, M=0$. On the surfaces there are 3 vectors which are perpendicular with each other,

$$\boldsymbol{e}_1=\boldsymbol{r}_u/\sqrt{E},\ \boldsymbol{e}_2=\boldsymbol{r}_v/\sqrt{G},\ \boldsymbol{e}_3=\boldsymbol{e}_1\times\boldsymbol{e}_2$$

The differentiate e´can be written in the form of linear combination of $\boldsymbol{e}_i$

$$(\boldsymbol{e}_i)_u=\sum a_{ij}\boldsymbol{e}_j,\ (\boldsymbol{e}_i)_v=\sum b_{kj}\boldsymbol{e}_j$$

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a_{ij} and b_{kj} formed a dissymmetric matrix respectively, that is $a_{kj}+a_{ji}=0, b_{ij}+b_{ji}=0$

$$e_{1u}=r_{uu}\sqrt{E}+(1/\sqrt{E})_u r_u,\ e_{1v}=r_{uv}/\sqrt{E}+(1/\sqrt{E})_v r_u$$

$$e_{2u}=r_{uv}\sqrt{G}+(1/\sqrt{E})_u r_v,\ e_{2v}=r_{vv}/\sqrt{G}+(1/\sqrt{G})_v r_v$$

Since that $F=r_u r_v=0, M=n r_{uv}=0$

$$a_{12}=-a_{21}=e_{2u}e_1=r_{uv}r_u/\sqrt{EF}=-E_V\sqrt{EF}$$

$$a_{13}=e_{1u}e_3=r_{uu}n/\sqrt{E}=L/\sqrt{E}$$

$$a_{23}=e_{2u}e_3=r_{uv}n/\sqrt{G}=0$$

by the same reason, $b_{12}=G_u/\sqrt{GE}$. $b_{13}=0, b_{23}=N/\sqrt{G}$

$$de=(e_1)_u du+(e_1)_v dv=\sum dk_{ij}e_j$$

$$dk_{ij}=a_{ij}du+b_{ij}dv=-dk_{ji}$$

$$dk_{12}=(-E_v du+g_u dv)/2\sqrt{eg}=k_{gq}dk_1+k_{g2}dk_2$$

In the equation, k_{g1} and k_{g2} are the geodetic distort curvatures of the curve u and v respectively.

Since that, $dk_1=\sqrt{E}du$ $dk_2=\sqrt{G}dv$

$$dk_{23}=N/\sqrt{G}=k_2 dk_2,\ dk_{13}=L/\sqrt{E}=k_d k_1$$

In the equation, k_1 and k_2 are the main curvatures of the curve u and v respectively. In the moving coordinate system that is fixed on the surface, the moving frame has the following differential form:

$$de_1=dk_{12}e_2+dk_{13}e_3$$

$$de_2=-dk_{12}e_1+dk_{23}e_3$$

$$de_3=-dk_{13}e_1-dk_{23}e_2$$

$$dr=dk_1e_1+dk_2e_2 \qquad (1)$$

What should be stressed is, the curvature web as parameter web exists in theory, but it probably can't be found in general situation. What's more, it is difficult to express in the form of apparent function. However, we needn't use such parameters in practical calculation in the example below.

2　The analysis of the movement of the point contact meshing by the Moving Frame

Suppose the $\sum^{(1)}$ and $\sum^{(2)}$ are two surfaces, establish the moving frames on the two surfaces respectively, according to equ. (1)

$$de_1^{(i)}=dk_{12}^{(i)}e_2^{(i)}+dk_{13}^{(i)}e_3^{(i)}$$

$$de_2^{(i)}=-dk_{12}^{(i)}e_1^{(i)}+dk_{23}^{(i)}e_3^{(i)}$$

$$de_3^{(i)}=-dk_{13}^{(i)}e_1^{(i)}+dk_{23}^{(i)}e_2^{(i)}$$

$$dr^{(i)}=dk_1^{(i)}e_1^{(i)}+dk_2^{(i)}e_2^{(i)} \qquad i=1,2, \qquad (2)$$

$$dk_1^{(i)}=\sqrt{E}^{(I)}du^{(i)}$$

$$d\mathrm{k}_2{}^{(i)} = \overline{G}^{(i)} dv^{(i)}$$
$$d\mathrm{k}_{(12)}{}^{(i)} = kg_1{}^{(i)} d\mathrm{k}_1{}^{(i)} + kg_2{}^{(i)} d\mathrm{k}_2{}^{(i)}$$
$$d\mathrm{k}_{(23)}{}^{(i)} = k_2{}^{(i)} d\mathrm{k}_2{}^{(i)}$$
$$d\mathrm{k}_{13}{}^{(i)} = k_1{}^{(i)} d\mathrm{k}_1{}^{(i)}$$

In the equation, $k_{g_1}{}^{(i)}$ and $k_{g2}{}^{(i)}$ are the geodetic distort curvatures of the curve $u^{(i)}$ and $v^{(i)}$ respectively and $k_1{}^{(i)}$ and $k_2{}^{(i)}$ are the main curvatures of the curve $u^{(i)}$ and $v^{(i)}$ respectively.

When the two bodies turn about their own axes, the differential form of the two moving frame in the fixed coordinate systems are respectively (that is, the relationship between relative differentiation and absolute differentiation).

$$d\boldsymbol{E}_j{}^{(i)} = d\boldsymbol{e}_j{}^{(i)} + \boldsymbol{a}^{(i)} \times \boldsymbol{e} d\mathrm{H}^{(i)} \qquad d\boldsymbol{R}^{(i)} = d\boldsymbol{r}^{(i)} + \boldsymbol{a}^{(i)} \times \boldsymbol{R}^{(i)} d\mathrm{H}^{(i)} \tag{3}$$

The condition for the two surfaces to contact is

$$d\boldsymbol{R}^{(1)} = d\boldsymbol{R}^{(2)} \qquad \boldsymbol{e}_3{}^{(1)} = \boldsymbol{e}_3{}^{(2)} \tag{4}$$

Put the equation (3) into the equation (4), both sides is multiplied by $\boldsymbol{e}_3$, considering $d\boldsymbol{r}^{(i)}\boldsymbol{e}_3 = 0$, the instantaneous ratio is:

$$d\mathrm{H}^{(1)} / d\mathrm{H}^{(2)} = (\boldsymbol{a}^{(1)}, \boldsymbol{R}^{(1)}, \boldsymbol{e}_3) / (\boldsymbol{a}^{(2)}, \boldsymbol{R}^{(3)}, \boldsymbol{e}_3) \tag{5}$$

Differentiate once more, considering equation (4), we have:

$$d^2\mathrm{H}^{(1)} / (d\mathrm{H}^{(2)})^2 = \frac{(\boldsymbol{a}^{(1)}, d\boldsymbol{R}^{(1)}/d\mathrm{H}^{(1)}, \boldsymbol{e}_3) + (\boldsymbol{a}^{(1)}, \boldsymbol{R}^{(1)}, d\boldsymbol{e}_3/d\mathrm{H}^{(1)}) - [(\boldsymbol{a}^{(2)}, d\boldsymbol{R}^{(2)}/d\mathrm{H}^{(1)}, \boldsymbol{e}_3) + (\boldsymbol{a}^{(2)}, \boldsymbol{R}^{(2)}, d\boldsymbol{e}_3/d\mathrm{H}^{(1)}]d\mathrm{H}^{(1)}/d\mathrm{H}^{(2)})}{(\boldsymbol{a}^{(2)}, \boldsymbol{R}^{(2)}, \boldsymbol{e}_3)} \tag{6}$$

Put equation (2) into equation (3), we have:

$$d\boldsymbol{R}^{(i)} / d\mathrm{H}^{(1)} = d\mathrm{k}_1{}^{(i)} / d\mathrm{H}^{(1)} \boldsymbol{e}_1{}^{(i)} + d\mathrm{k}_2{}^{(i)} / d\mathrm{H}^{(1)} \boldsymbol{e}_2{}^{(i)} + \boldsymbol{a}^{(i)} \times \boldsymbol{R}^{(i)} \tag{7}$$
$$d\boldsymbol{E}_3 / d\mathrm{H}^{(1)} = -k_1{}^{(1)} d\mathrm{k}_1{}^{(i)} / d\mathrm{H}^{(1)} \boldsymbol{e}_1{}^{(i)} - d\mathrm{k}_2{}^{(i)} / d\mathrm{H}^{(1)} \boldsymbol{e}_2{}^{(i)} + \boldsymbol{a}^{(i)} \times \boldsymbol{e}_3 \tag{8}$$

The equation (4) can be written as:

$$d\mathrm{k}_1{}^{(1)} \boldsymbol{e}_1{}^{(1)} + d\mathrm{k}_2{}^{(1)} \boldsymbol{e}_2{}^{(1)} + \boldsymbol{a}^{(1)} \times \boldsymbol{R}^{(1)} d\mathrm{H}^{(1)} = d\mathrm{k}_1{}^{(2)} \boldsymbol{e}_1{}^{(2)} + d\mathrm{k}_2{}^{(2)} \boldsymbol{e}_2{}^{(2)} + a^{(2)} \times \boldsymbol{R}^{(2)} d\mathrm{H}^{(2)} \tag{9}$$

The angle bwtween the main directions of the two surfaces is ψ, then:

$$\boldsymbol{e}_1{}^{(2)} = \cos \mathrm{j}\, \boldsymbol{e}_1{}^{(1)} + \sin \mathrm{j}\, \boldsymbol{e}_2{}^{(1)}$$
$$\boldsymbol{e}_2{}^{(2)} = -\sin \mathrm{j}\, \boldsymbol{e}_1{}^{(1)} + \cos \mathrm{j}\, \boldsymbol{e}_2{}^{(1)} \tag{10}$$

Put equation (10) into equation (9), we have:

$$d\mathrm{k}_1{}^{(2)} / d\mathrm{H}^{(1)} + (\boldsymbol{a}^{(1)}, \boldsymbol{R}^{(1)}, \boldsymbol{e}_1{}^{(2)}) d\mathrm{H}^{(2)} / d\mathrm{H}^{(1)}$$
$$= d\mathrm{k}_1{}^{(i)} / d\mathrm{H}^{(1)} \cos \mathrm{j} + d\mathrm{k}_2{}^{(i)} / d\mathrm{H}^{(1)} \sin \mathrm{j} + (\boldsymbol{a}^{(1)}, \boldsymbol{R}^{(1)}, \boldsymbol{e}_1{}^{(2)}) \tag{11}$$
$$d\mathrm{k}_2{}^{(2)} / d\mathrm{H}^{(1)} + (\boldsymbol{a}^{(2)}, \boldsymbol{R}^{(2)}, \boldsymbol{e}_1{}^{(2)}) d\mathrm{H}^{(2)} / d\mathrm{H}^{(1)}$$
$$= -d\mathrm{k}_1{}^{(i)} / d\mathrm{H}^{(1)} \sin \mathrm{j} + d\mathrm{k}_2{}^{(i)} / d\mathrm{H}^{(1)} \sin \mathrm{j} + (\boldsymbol{a}^{(1)}, \boldsymbol{R}^{(1)}, \boldsymbol{e}_2{}^{(2)}) \tag{12}$$

Differentiate the second equation of (4)

$$-d\mathrm{k}_{13}{}^{(1)} \boldsymbol{e}_1{}^{(1)} - d\mathrm{k}_{(23)}{}^{(1)} \boldsymbol{e}_2{}^{(1)} + \boldsymbol{a}^{(1)} \times \boldsymbol{e}_3 d\mathrm{H}^{(1)}$$
$$= -d\mathrm{k}_{13}{}^{(2)} \boldsymbol{e}_1{}^{(2)} - d\mathrm{k}_{13}{}^{(2)} \boldsymbol{e}_2{}^{(2)} + \boldsymbol{a}^{(2)} \times \boldsymbol{e}_3 d\mathrm{H}^{(2)}$$

Finally put the equation (10) into it:

$$-k_1^{(2)} d\mathrm{k}_1^{(2)}/d\mathrm{H}^{(1)} + \boldsymbol{a}^{(2)}\boldsymbol{e}_2^{(2)} d\mathrm{H}^{(2)}/d\mathrm{H}^{(1)}$$
$$= -k_1^{(1)} d\mathrm{k}_1^{(1)}/d\mathrm{H}^{(1)}\cos\mathrm{j} - k_2^{(1)} d\mathrm{k}_2^{(i)}/d\mathrm{H}^{(1)}\sin\mathrm{j} + \boldsymbol{a}^{(1)}\boldsymbol{e}_2^{(2)} \qquad (13)$$
$$-k_2^{(2)} d\mathrm{k}_2^{(2)}/d\mathrm{H}^{(1)} + \boldsymbol{a}^{(2)}\boldsymbol{e}_1^{(2)} d\mathrm{H}^{(2)}/d\mathrm{H}^{(1)}$$
$$= k_1^{(1)} d\mathrm{k}_1^{(1)}/d\mathrm{H}^{(1)}\sin\mathrm{j} - k_2^{(1)} d\mathrm{k}_2^{(i)}/d\mathrm{H}^{(1)}\cos\mathrm{j} - a^{(1)}\boldsymbol{e}_1^{(2)} \qquad (14)$$

Solve the equatoins (1), (12), (13) and (14) instantaneously, we can determind

$d\mathrm{k}_1^{(1)}/d\mathrm{H}^{(1)}$,

$d\mathrm{k}_2^{(2)}/d\mathrm{H}^{(1)}$,

$d\mathrm{k}_1^{(2)}/d\mathrm{H}^{(1)}$,

$d\mathrm{k}_2^{(2)}/d\mathrm{H}^{(1)}$.

Then take them into equation (7)、(8), and (6), we can fine the acceleration of the of driven gear.

参考文献

[1] 苏步青译．佐佐木重夫著．微分几何学, 1980.

[2] 吴大任．微分几何与啮合原理．科学出版社, 1989.

[3] L. Baxter. Jr,: Second- order Surface Generation. Industrial Mathematics, 1973, 23(2): 1- 10.

[4] M· L· Baxter· Jr· Basic Geometry and Tooth contact of Hypoid Gears· Industrial Math· 11(2), 1961, 1- 8.