Article citation info:
DUI H, CHEN L, WU S. Generalized integrated importance measure for system performance evaluation: application to a propeller plane system. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2017; 19 (2): 279–286, http://dx.doi.org/10.17531/ein.2017.2.16.

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GENERALIZED INTEGRATED IMPORTANCE MEASURE FOR SYSTEM PERFORMANCE EVALUATION: APPLICATION TO A PROPELLER PLANE SYSTEM

UOGÓLNIONA MIARA ZINTEGROWANEJ WAŻNOŚCI KOMPONENTÓW JAKO NARZĘDZIE OCENY WYDAJNOŚCI SYSTEMU: ZASTOSOWANIE W ODNIESIENIU DO UKŁADU ŚMIGŁOWCA

The integrated importance measure (IIM) evaluates the rate of system performance change due to a component changing from one state to another. The IIM simply considers the scenarios where the transition rate of a component from one state to another is constant. This may contradict the assumption of the degradation, based on which system performance is degrading and therefore the transition rate may be increasing over time. The Weibull distribution describes the life of a component, which has been used in many different engineering applications to model complex data sets. This paper extends the IIM to a new importance measure that considers the scenarios where the transition rate of a component degrading from one state to another is a time-dependent function under the Weibull distribution. It considers the conditional probability distribution of a component sojourning at a state is the Weibull distribution, given the next state that component will jump to. The research on the new importance measure can identify the most important component during three different time periods of the system lifetime, which is corresponding to the characteristics of Weibull distributions. For illustration, the paper then derives some probabilistic properties and applies the extended importance measure to a real-world example (i.e., a propeller plane system).

Keywords: *system performance, importance measure, Weibull distribution, transition rate.*

Miara zintegrowanej ważności (IIM) pozwala oceniać szybkość zmian wydajności systemu powstałych w wyniku przejścia elementu systemu z jednego stanu do drugiego. IIM pozwala rozważać scenariusze, w których szybkość przejścia elementu z jednego stanu do drugiego jest stała. Jest to jednak sprzeczne z założeniem degradacji, zgodnie z którym wydajność systemu obniża się, w związku z czym, szybkość przejścia może z upływem czasu ulegać zwiększeniu. Rozkład Weibulla opisuje żywotność danego elementu, co wykorzystuje się w wielu różnych zastosowaniach technicznych do modelowania złożonych zbiorów danych. W przedstawionej pracy, rozszerzono IIM uzyskując nową miarę ważności, która pozwala rozważać scenariusze, w których szybkość przejścia elementu z jednego stanu do drugiego w wyniku degradacji jest zależną od czasu funkcją rozkładu Weibulla. Przyjęto, że warunkowy rozkład prawdopodobieństwa elementu przebywającego w pewnym stanie jest rozkładem Weibulla, gdzie dany jest kolejny stan do którego ma przejść dany element. Badania nad nową miarą ważności umożliwiają identyfikację najważniejszych elementów podczas trzech różnych okresów czasu życia systemu, co odpowiada charakterystyce rozkładów Weibulla. Dla ilustracji, wyprowadzono pewne właściwości probabilistyczne i zastosowano rozszerzoną miarę ważności do analizy przykładu rzeczywistego układu śmigłowca.

Słowa kluczowe: *wydajność systemu, miara ważności, rozkład Weibulla, szybkość przejścia.*

Notation

$x_i(t)$	state of component i at time t, $x_i(t)=0,1,2,\ldots,M_i$
$X(t)$	$(x_1(t),x_2(t),\ldots,x_n(t))$: state vector of the components at time t
$\Phi(X(t))$	system structure function at time t and range $\{0,1,\ldots,M\}$, $\Phi(X(t))=\Phi(x_1(t),x_2(t),\ldots,x_n(t))$
$(\cdot_i,X(t))$	$(x_1(t),\ldots,x_{i-1}(t),\cdot,x_{i+1}(t),\ldots,x_n(t))$
$P_{i,ml}(t)$	$\Pr(x_i(t)=l\mid x_i(0)=m)$
$P_{i,m}(t)$	$\Pr(x_i(t)=m)$
$b_{i,ml}$	transition rate of component i from state m to state l
c_j	maintenance cost of improving the system from state j to state M.
$\rho_{i,m}(t)$	$\Pr\{x_i(t)<m\}$
$\rho_{i,ml}(t)$	$\Pr\{x_i(t)<l\mid x_i(0)=m\}$

1. Introduction

In reliability engineering, a bulk of research has been devoted to evaluate the contribution of components to system reliability and performance [14]. For example, Yang et al. [26] provided a method of simulating the reliability of degradation using Monte Carlo principle and cloud theory. Cheng et al. [6] proposed an approach to analyze the reliability evaluation based on fast Markov chain simulations. Leturiondo et al. [12] presented a model to evaluate the system performance with localized damage. Werbińska-Wojciechowska and Zając [21] used a delay-time concept to analyze the system maintenance performance.

The seminal work of reliability importance measure is credited to Birnbaum, who introduce the well-known Birnbaum importance in 1969 [1]. Since then, a wide range of different importance measures have been introduced. For example, Wu and Chan [22] proposed the utility importance of component states in multi-state systems. Levitin et al. [13] generalized the importance measures for multi-state elements based on performance level restrictions. Wu and Coolen [24] extended the Birnbaum importance to a cost-based importance measure. Borgonovo et al. [2, 3] proposed differential importance measure, and time-independent reliability importance for the risk evaluation. Zhai et al. [27] presented a moment-independent importance to evaluate the safety probability. Tyrväinen [20] presented risk importance measures to analyze the dynamic reliability. Wu et al. [23] proposed a component maintenance priority importance to improve the system performance. Dutuit and Rauzy [9] extended the importance measures to complex components. Kuo and Zhu [11, 28] summarized the concepts of importance measures and their application in reliability and mathematical programming. Si, Dui et al. [17, 18] proposed the integrated importance measure (IIM) of component states, which evaluates how the transition of component states affects the system performance in multi-state systems. Si, Dui et al. [19] studied the IIM from component states to the component, which can identify the most important component for improving the system performance. Dui et al. [7, 8] studied the IIM in system lifetime and semi-Markov process to evaluate the change of the system performance, respectively.

The IIM evaluates the rate of system performance change due to a component changing from one state to another. The IIM simply considers the scenarios when the transition rate of a component from one state to another is constant. This may contradict the assumption of the degradation, based on which system performance is degrading and therefore the transition rate may be increasing over time.

On the other hand, the Weibull distribution is one of the most commonly used lifetime distributions in reliability modeling and lifetime testing [25]. It has been used in many different engineering applications to model complex data sets, such as life tests [15], fault diagnosis [4, 5], oral irrigators [16], et al. The Weibull distribution and its variants can accommodate increasing, constant or decreasing failure rates [10]. Thus, one may extend the IIM to a new importance measure that considers the scenarios where the transition rate of a component changing from one state to another as a time-dependent function. Typically, one may consider the conditional probability distribution of a component sojourning in a state is the Weibull distribution, given the next state that the component will jump to. On the basis of such consideration, this paper proposes a new importance measure. The research on the new importance measure can identify the most important component during three different time periods of the system lifetime, which is corresponding to the characteristics of Weibull distributions. The paper then derives some probabilistic properties. It also analyzes the properties of the proposed importance measure of the parallel-series systems and the series-parallel systems, respectively. A real-world example is borrowed to illustrate the proposed importance measure.

The rest of the paper is as follows. Section 2 extends the IIM. The corresponding properties of IIM in the Weibull distribution are analyzed for typical parallel-series system and series-parallel system structures in Section 3. An application is presented to illustrate the proposed method in Section 4. Section 5 gives the conclusion of this paper.

2. The extended IIM for system performance under Weibull distributions

In this paper, the state space of component i is $\{0,1,\ldots, M_i\}$ and the state space of the system is $\{0,1,\ldots, M\}$. State 0 represents the complete failure state and state M (M_i) is the perfect functioning state. The states are ordered from the complete failure state to the perfect functioning state.

We assume that the levels of maintenance cost and the system states ($c_0 \geq c_1 \geq \ldots \geq c_{M-1} \geq c_M = 0$) are inversely proportional. The expected maintenance cost is $C = \sum_{j=0}^{M-1} c_j \Pr(\Phi(X) = j) = \sum_{j=0}^{M-1} (c_j - c_{j+1}) \Pr(\Phi(X) \leq j)$. Si, Dui et al. [17] gave the following IIM, as in Equation (1).

$$\begin{aligned} IIM_{i,ml}(t) &= P_{i,m}(t) \cdot b_{i,ml} \sum_{j=0}^{M-1} (c_j - c_{j+1}) \left[\Pr\left(\Phi(m_i(t), X(t)) \leq j\right) - \Pr\left(\Phi(l_i(t), X(t)) \leq j\right) \right] \\ &= P_{i,m}(t) \cdot b_{i,ml} \sum_{j=0}^{M-1} c_j \left[\Pr\left(\Phi(m_i(t), X(t)) = j\right) - \Pr\left(\Phi(l_i(t), X(t)) = j\right) \right],\ m < l \end{aligned} \tag{1}$$

Then $IIM_{i,ml}(t)$ describes the rate of maintenance cost loss due to a component improving from state m to state l at time t.

In Equation (1), $b_{i,ml}$, which is the transition rate of component i from state m to state l, is a quantity independent of time t. Intuitively, $b_{i,ml}$ may be a time-dependent quantity as the component is a deteriorating/ageing unit. Hence, the IIM defined in Equation (1) is too restrictive. In order to attract wider applications, one may extend the IIM by introducing the following importance measure.

Definition 1. The extended integrated importance measure (EIIM) is given by:

$$EIIM_{i,ml}(t,y) = P_{i,m}(t) \cdot b_{i,ml}(y) \sum_{j=0}^{M-1} c_j \left[\Pr\left(\Phi(m_i(t), X(t)) = j\right) - \Pr\left(\Phi(l_i(t), X(t)) = j\right) \right], m < l. \tag{2}$$

In Equation (2), $b_{i,ml}(y)$ is a function of the sojourn time y. An interesting question is what form of function $b_{i,ml}(y)$ should be. In the following, we consider the case when the Weibull distribution is adopted.

Let the two-parameter Weibull distribution $W(t;\theta,\gamma)$ be denoted by:

$$W(t;\theta,\gamma) = 1 - \exp\left[-(t/\theta)^{\gamma} \right], \theta, \gamma > 0, \tag{3}$$

where θ and γ are the scale and shape parameters, respectively. We can then obtain the distribution function of the sojourn time in state m and the transition rate for component i. Denote $\theta_{i,ml}$ the scale parameter of component i in state m, given that the next state is l, and γ_i the shape parameter of component i. Then the transition rate $b_{i,ml}(y)$ is $b_{i,ml}(y) = \left(\gamma_i / \theta_{i,ml}\right)\left(y / \theta_{i,ml}\right)^{\gamma_i - 1}$.

According to Equation (2), the expression of the EIIM under the Weibull distribution is as following:

Definition 2. The EIIM under the Weibull distribution is given by:

$$\begin{aligned} EIIM_{i,ml}(t,y) &= P_{i,m}(t)\left(\gamma_i / \theta_{i,ml}\right)\left(y / \theta_{i,ml}\right)^{\gamma_i - 1} \sum_{j=0}^{M-1} (c_j - c_{j+1})\left[\Pr\left(\Phi(m_i(t), X(t)) \le j\right) - \Pr\left(\Phi(l_i(t), X(t)) \le j\right)\right] \\ &= P_{i,m}(t)\left(\gamma_i / \theta_{i,ml}\right)\left(y / \theta_{i,ml}\right)^{\gamma_i - 1} \sum_{j=0}^{M-1} c_j \left[\Pr\left(\Phi(m_i(t), X(t)) = j\right) - \Pr\left(\Phi(l_i(t), X(t)) = j\right)\right]. \end{aligned} \tag{4}$$

It is known that with different shape parameters, the Weibull distribution becomes different distributions, such as the exponential distribution, the Rayleigh distribution, the normal distribution. Thus, with different shape parameters, Equation (4) can be converted into different expressions under different distributions.

When $M = 1$, the multi-state system reduces to a binary system. Equation (4) can be converted into:

$$EIIM_{i,01}(t,y) = P_{i,0}(t)\left(\gamma_i / \theta_{i,01}\right)\left(y / \theta_{i,01}\right)^{\gamma_i - 1} c_0 \left[\Pr\left(\Phi(0_i(t), X(t)) = 0\right) - \Pr\left(\Phi(1_i(t), X(t)) = 0\right)\right]. \tag{5}$$

It is obvious that the last term in the right hand side of Equation (5) is the Birnbaum importance measure (BM) of component i, as in Equation (6):

$$BM_i(t) = \Pr\left(\Phi(0_i(t), X(t)) = 0\right) - \Pr\left(\Phi(1_i(t), X(t)) = 0\right). \tag{6}$$

From Equations (5) and 6, the EIIM of component i is a generalization of BM based on the system performance. We will discuss the difference between BM and EIIM for the change of different parameters in section 4.

3. Characteristics of EIIM for typical system structures

We now discuss the properties of the EIIM for parallel-series system and series-parallel system structures.

Fig. 1 gives the structure of a typical parallel-series system, where [*ij*] represents the component that is located in row *i* and column *j*. The corresponding structure function of the system is $\Phi(X(t)) = \max\limits_{1 \le i \le N} \min\limits_{1 \le j \le N_i} \left\{X_{[ij]}(t)\right\}$.

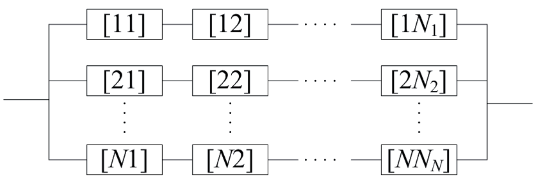


Fig. 1. A parallel-series system

The structure of a typical series-parallel system is as in Fig. 2, where [*ij*] represents the component which is located in column *i* and row *j*. The corresponding structure function of the system is $\Phi(X(t)) = \min\limits_{1 \le i \le N} \max\limits_{1 \le j \le N_i} \left\{X_{[ij]}(t)\right\}$.

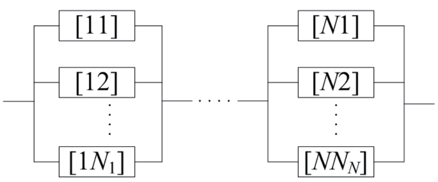


Fig. 2. A series-parallel system

Assume that the lifetime distribution function of component [*ij*] follows Weibull distribution $W\left(t; \theta_{[ij],ml}, \gamma_{[ij]}\right)$. We can obtain the following propositions:

Proposition 1. In a parallel-series system, assume $\gamma_{[i_1 j_1]} = \gamma_{[i_2 j_2]}$, then $EIIM_{[i_1 j_1],m(m+1)}(t,y) \geq EIIM_{[i_2 j_2],m(m+1)}(t,y)$ only when $\dfrac{P_{[i_1 j_1],m}(t)\cdot\left(1/\theta_{[i_1 j_1],m(m+1)}\right)^{\gamma_{[i_1 j_1]}}}{\alpha_m(i_1)(1-\rho_{[i_1 j_1],m}(t))} \geq \dfrac{P_{[i_2 j_2],m}(t)\cdot\left(1/\theta_{[i_2 j_2],m(m+1)}\right)^{\gamma_{[i_2 j_2]}}}{\alpha_m(i_2)(1-\rho_{[i_2 j_2],m}(t))}$.

Proof. According to the structure function in the parallel-series systems and Equation (4), the right hand side of Equation (4) can be converted into:

$$\begin{aligned}
&\sum_{j=0}^{M-1}(c_j - c_{j+1})\Pr\left(\Phi(m_i(t),X(t))\leq j\right)\\
&=\sum_{j=0}^{M-1}(c_j - c_{j+1})\Pr(\max\{\min_{1\leq h\leq N_1}\{X_{[1h]}(t)\},\ldots,\min_{1\leq h\leq N_{i-1}}\{X_{[(i-1)h]}(t)\},\\
&\min\{X_{[i1]}(t),\ldots,X_{[i(j-1)]}(t),m,X_{[i(j+1)}(t),\ldots,X_{[iN_i]}(t)\},\min_{1\leq h\leq N_{i+1}}\{X_{[(i+1)h]}(t)\},\ldots,\\
&\min_{1\leq h\leq N_N}\{X_{[N_N h]}(t)\})\leq j\\
&=\sum_{j=0}^{M-1}(c_j - c_{j+1})\Pr(\min_{1\leq h\leq N_1}\{X_{[1h]}(t)\}\leq j,\ldots,\min_{1\leq h\leq N_{i-1}}\{X_{[(i-1)h]}(t)\}\leq j,\\
&\min\{X_{[i1]}(t),\ldots,X_{[i(j-1)]}(t),m,X_{[i(j+1)}(t),\ldots,X_{[iN_i]}(t)\}\leq j,\min_{1\leq h\leq N_{i+1}}\{X_{[(i+1)h]}(t)\}\leq j,\ldots,\\
&\min_{1\leq h\leq N_N}\{X_{[N_N h]}(t)\}\leq j).
\end{aligned}$$

For $k\in\{1,2,\ldots,N\}, k\neq i, j\in\{0,1,\ldots,M-1\}$, we can obtain:

$$\begin{aligned}
&\Pr(\min_{1\leq h\leq N_k}\{X_{[kh]}(t)\}\leq j) = 1-\Pr(\min_{1\leq h\leq N_k}\{X_{[kh]}(t)\}> j)\\
&=1-\Pr(X_{[k1]}(t)>j,\ldots,X_{[kN_k]}(t)>j) = 1-\prod_{h=1}^{N_k}(1-\rho_{[kh],j}(t)).
\end{aligned}$$

$\min\{X_{[i1]}(t),\ldots,X_{[i(j-1)]}(t),m,X_{[i(j+1)]}(t),\ldots,X_{[iN_i]}(t)\}\leq m$, so for $k=i, j\in\{m,\ldots,M-1\}$, we have: $\Pr(\min\{X_{[i1]}(t),\ldots,X_{[i(j-1)]}(t),m,X_{[i(j+1)]}(t),\ldots,X_{[iN_i]}(t)\}\leq j)=1$, and for $k=i, j\in\{0,\ldots,m-1\}$, we have:

$$\begin{aligned}
&\Pr(\min\{X_{[i1]}(t),\ldots,X_{[i(j-1)]}(t),m,X_{[i(j+1)]}(t),\ldots,X_{[iN_i]}(t)\}\leq j)\\
&=1-\Pr(\min\{X_{[i1]}(t),\ldots,X_{[i(j-1)]}(t),m,X_{[i(j+1)]}(t),\ldots,X_{[iN_i]}(t)\}> j)\\
&=1-\Pr(X_{[i1]}(t)>j,\ldots,X_{[i(j-1)]}(t)>j,X_{[i(j+1)]}(t)>j,\ldots,X_{[iN_i]}(t)>j)\\
&=1-\prod_{h=1,h\neq j}^{N_k}(1-\rho_{[ih],j}(t)).
\end{aligned}$$

So we can get:

$$\begin{aligned}
&\sum_{j=0}^{M-1}(c_j - c_{j+1})\Pr\left(\Phi(m_i(t),X(t))\leq j\right)\\
&=\sum_{j=0}^{m-1}(c_j - c_{j+1})\prod_{k=1,k\neq i}^{N}[1-\prod_{h=1}^{N_k}(1-\rho_{[kh],j}(t))]\cdot[1-\prod_{h=1,h\neq j}^{N_i}(1-\rho_{[ih],j}(t))]+\sum_{j=m}^{M-1}(c_j - c_{j+1})\prod_{k=1,k\neq i}^{N}[1-\prod_{h=1}^{N_k}(1-\rho_{[kh],j}(t))].
\end{aligned}$$

Then the right hand side of Equation (4) is:

$$\begin{aligned}
&\sum_{j=0}^{M-1}(c_j - c_{j+1})\left[\Pr\left(\Phi(m_i(t),X(t))\leq j\right)-\Pr\left(\Phi(l_i(t),X(t))\leq j\right)\right]\\
&=\sum_{j=m}^{l-1}(c_j - c_{j+1})\prod_{k=1,k\neq i}^{N}[1-\prod_{h=1}^{N_k}(1-\rho_{[kh],j}(t))]-\sum_{j=m}^{l-1}(c_j - c_{j+1})\prod_{k=1,k\neq i}^{N}[1-\prod_{h=1}^{N_k}(1-\rho_{[kh],j}(t))]\cdot[1-\prod_{h=1,h\neq j}^{N_i}(1-\rho_{[ih],j}(t))]\\
&=\sum_{j=m}^{l-1}(c_j - c_{j+1})\prod_{k=1,k\neq i}^{N}[1-\prod_{h=1}^{N_k}(1-\rho_{[kh],j}(t))]\cdot\prod_{h=1,h\neq j}^{N_i}(1-\rho_{[ih],j}(t)).
\end{aligned}$$

At last, we can obtain:

$$EIIM_{[i_1 j_1],m(m+1)}(t,y) = P_{[i_1,j_1],m}(t)\left(\gamma_{[i_1 j_1]} / \theta_{[i_1 j_1],ml}\right)\left(y / \theta_{[i_1 j_1],ml}\right)^{\gamma_{[i_1 j_1]}-1} \times$$
$$\left\{(c_m - c_{m+1}) \prod_{k=1,k\neq i_1}^{N}\left[1-\prod_{h=1}^{N_k}(1-\rho_{[kh],m}(t))\right]\cdot \prod_{h=1,h\neq j_1}^{N_{i_1}}(1-\rho_{[i_1 h],m}(t))\right\},$$

$$EIIM_{[i_2 j_2],m(m+1)}(t,y) = P_{[i_2,j_2],m}(t)\left(\gamma_{[i_2 j_2]} / \theta_{[i_2 j_2],ml}\right)\left(y / \theta_{[i_2 j_2],ml}\right)^{\gamma_{[i_2 j_2]}-1} \times$$
$$\left\{(c_m - c_{m+1}) \prod_{k=1,k\neq i_1}^{N}\left[1-\prod_{h=1}^{N_k}(1-\rho_{[kh],m}(t))\right]\cdot \prod_{h=1,h\neq j_1}^{N_{i_1}}(1-\rho_{[i_1 h],m}(t))\right\}.$$

Then we have:

$$EIIM_{[i_1 j_1],m(m+1)}(t,y) \geq EIIM_{[i_2 j_2],m(m+1)}(t,y)$$
$$\Leftrightarrow \frac{P_{[i_1,j_1],m}(t)\left(\gamma_{[i_1 j_1]} / \theta_{[i_1 j_1],ml}\right)\left(y / \theta_{[i_1 j_1],ml}\right)^{\gamma_{[i_1 j_1]}-1}\prod_{h=1}^{N_{i_1}}(1-\rho_{[i_1 h],m}(t))}{\left[1-\prod_{h=1}^{N_{i_1}}(1-\rho_{[i_1 h],m}(t))\right](1-\rho_{[i_1 j_1],m}(t))} \geq \frac{P_{[i_2,j_2],m}(t)\left(\gamma_{[i_2 j_2]} / \theta_{[i_2 j_2],ml}\right)\left(y / \theta_{[i_2 j_2],ml}\right)^{\gamma_{[i_2 j_2]}-1}\prod_{h=1}^{N_{i_2}}(1-\rho_{[i_2 h],m}(t))}{\left[1-\prod_{h=1}^{N_{i_2}}(1-\rho_{[i_2 h],m}(t))\right](1-\rho_{[i_2 j_2],m}(t))}$$
$$\Leftrightarrow \frac{P_{[i_1,j_1],m}(t)\left(\gamma_{[i_1 j_1]} / \theta_{[i_1 j_1],ml}\right)\left(y / \theta_{[i_1 j_1],ml}\right)^{\gamma_{[i_1 j_1]}-1}}{\alpha_m(i_1)(1-\rho_{[i_1 j_1],m}(t))} \geq \frac{P_{[i_2,j_2],m}(t)\left(\gamma_{[i_2 j_2]} / \theta_{[i_2 j_2],ml}\right)\left(y / \theta_{[i_2 j_2],ml}\right)^{\gamma_{[i_2 j_2]}-1}}{\alpha_m(i_2)(1-\rho_{[i_2 j_2],m}(t))}$$
$$\overset{\gamma_{[i_1 j_1]}=\gamma_{[i_2 j_2]}}{\Leftrightarrow} \frac{P_{[i_1,j_1],m}(t)\cdot\left(1/\theta_{[i_1 j_1],m(m+1)}\right)^{\gamma_{[i_1 j_1]}}}{\alpha_m(i_1)(1-\rho_{[i_1 j_1],m}(t))} \geq \frac{P_{[i_2,j_2],m}(t)\cdot\left(1/\theta_{[i_2 j_2],m(m+1)}\right)^{\gamma_{[i_2 j_2]}}}{\alpha_m(i_2)(1-\rho_{[i_2 j_2],m}(t))}.$$

□

Since the parallel system and the series system are dual systems, according to Proposition 1, we can obtain Proposition 2.

Proposition 2. In a series-parallel system, assume $\gamma_{[i_1 j_1]} = \gamma_{[i_2 j_2]}$, then $EIIM_{[i_1 j_1],m(m+1)}(t,y) \geq EIIM_{[i_2 j_2],m(m+1)}(t,y)$ only when

$$\frac{P_{[i_1 j_1],m}(t)\cdot\left(1/\theta_{[i_1 j_1],m(m+1)}\right)^{\gamma_{[i_1 j_1]}}}{\beta_m(i_1)\rho_{[i_1 j_1],m}(t)} \geq \frac{P_{[i_2 j_2],m}(t)\cdot\left(1/\theta_{[i_2 j_2],m(m+1)}\right)^{\gamma_{[i_2 j_2]}}}{\beta_m(i_2)\rho_{[i_2 j_2],m}(t)}.$$

Proof. Similarly to Proposition 1, the proof can be established.

In Proposition 1, if we consider the situation that there is only one row *i*, the parallel-series system reduces a series system. In Proposition 2, if we consider the situation that there is only one column *i*, the series-parallel system reduces a parallel system. According to Propositions 1 and 2, we have Corollaries 1 and 2.

Corollary 1. In a parallel-series system, assume $\gamma_{[ij_1]} = \gamma_{[ij_2]}$, then $EIIM_{[ij_1],m(m+1)}(t,y) \geq EIIM_{[ij_2],m(m+1)}(t,y)$ only when

$$\frac{P_{[ij_1],m}(t)\cdot\left(1/\theta_{[ij_1],m(m+1)}\right)^{\gamma_{[ij_1]}}}{1-\rho_{[ij_1],m}(t)} \geq \frac{P_{[ij_2],m}(t)\cdot\left(1/\theta_{[ij_2],m(m+1)}\right)^{\gamma_{[ij_2]}}}{1-\rho_{[ij_2],m}(t)}.$$

Proof. Similarly to Proposition 1, the proof can be established.

Corollary 2. In a series-parallel system, assume $\gamma_{[ij_1]} = \gamma_{[ij_2]}$, then $EIIM_{[ij_1],m(m+1)}(t,y) \ge EIIM_{[ij_2],m(m+1)}(t,y)$ only when

$$\frac{P_{[ij_1],m}(t)\cdot\left(1/\theta_{[ij_1],m(m+1)}\right)^{\gamma_{[ij_1]}}}{\rho_{[ij_1],m}(t)} \ge \frac{P_{[ij_2],m}(t)\cdot\left(1/\theta_{[ij_2],m(m+1)}\right)^{\gamma_{[ij_2]}}}{\rho_{[ij_2],m}(t)}.$$

Proof. Similarly to Proposition 2, the proof can be established.

From Propositions 1, 2 and Corollaries 1, 2, in a parallel-series system or series-parallel system, if the transition rate function of component *i* from state *m* to an adjacent state is larger than that of component *j*, then the effect of component *i* on the system performance is larger than that of component *j*. This also means that we can identify the most important component in the system when considering the effects of improvement between adjacent states of a component on system performance, at time *t*. If two components are in the same degrading state, then the component with larger importance value should be maintained first in the operation of a parallel-series system or series-parallel system to get the larger improvement of system performance.

4. Application to a propeller plane system

In this section, we use a real-world example to illustrate the application of the EIIM.

A propeller plane mainly consists of engines 1, 2, 3, 4 (components 1, 2, 3, 4), control panel (component 5), and wings 1, 2 (components 6, 7), as shown in Fig. 3. The four components (ie., engines) constitutes a 2-out-of-4: G system. That is, in order to ensure safety flight, at least two engines must be working.

We assume that when the propeller plane system fails, the maintenance cost is 1 million CNY. In order to analyze the difference between BM and EIIM, we assume that the lifetime of all the components follow the Weibull distribution with the same scale parameters and different shape parameters $\gamma_i = 0.1, \gamma_i = 1, \gamma_i = 3$, and $\theta_{i,01} = 1$. Fig. 4 illustrates the difference between BM and EIIM of engines and other components for different shape parameters.

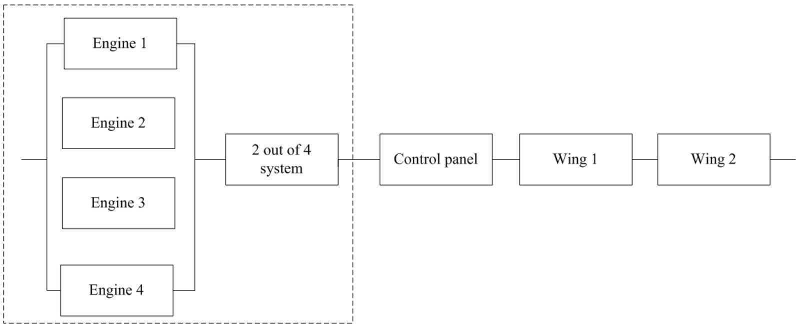
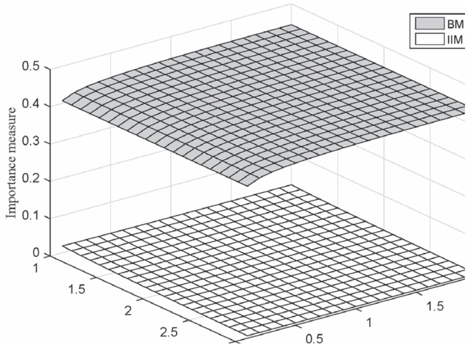


Fig. 3. A propeller plane system

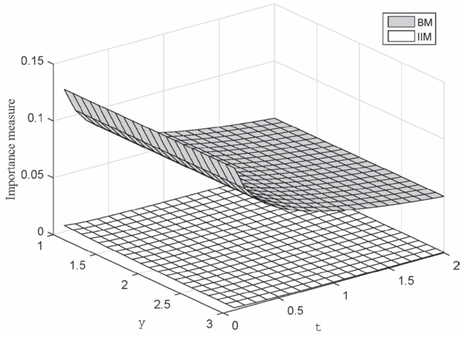
From Fig. 4, we have the following findings.

- If the shape parameter of a component is less than 1, the transition rate decreases with time, the BM of a component is bigger than its EIIM.
- If the shape parameter of a component is equal to 1, the transition rate is $b_{i,01}(y) = 1/\theta_{i,01} = 1$ based on Proposition 2, and $IIM_{i,01}(t,y) = P_{i,0}(t)BM_i(t)$ which is independent of the sojourn time.
- If the shape parameter of a component is more than 1, the transition rate increases with time, and component EIIM is also higher than its BM.

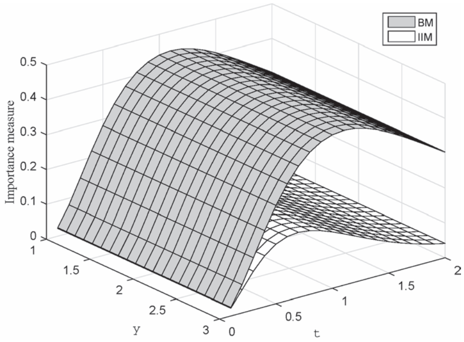
When fixing y=1 and $\theta_{i,01} = 1$, we can obtain the BM and EIIM of engines with the change of shape parameter as in Fig. 5, which is corresponding to Figs. 4 (a), (c), and (e) for $\gamma_i = 0.1, \gamma_i = 1, \gamma_i = 3$. When the shape parameter becomes bigger, the value of EIIM also be-



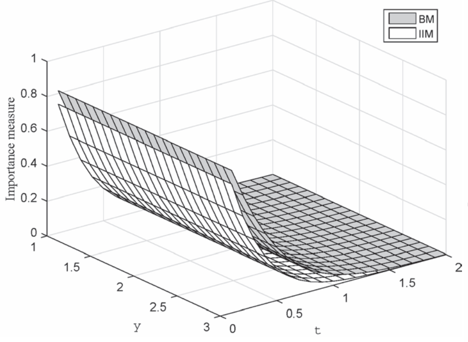
(a) Importance of engines when $\gamma_i = 0.1$



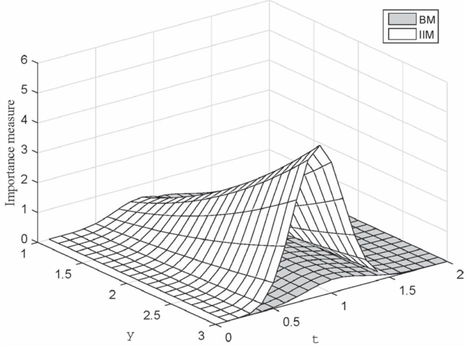
(b) Importance of other components when $\gamma_i = 0.1$ and other components



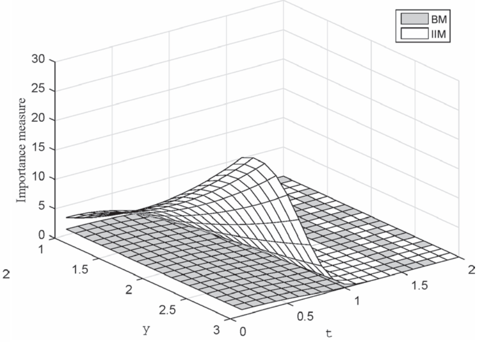
(c) Importance of engines when $\gamma_i = 1$



(d) Importance of other components when $\gamma_i = 1$ and other components



(e) Importance of engines when $\gamma_i = 3$ and other components



(f) Importance of other components when $\gamma_i = 3$ and other components

Fig. 4. Difference between BM and EIIM of engines and other components

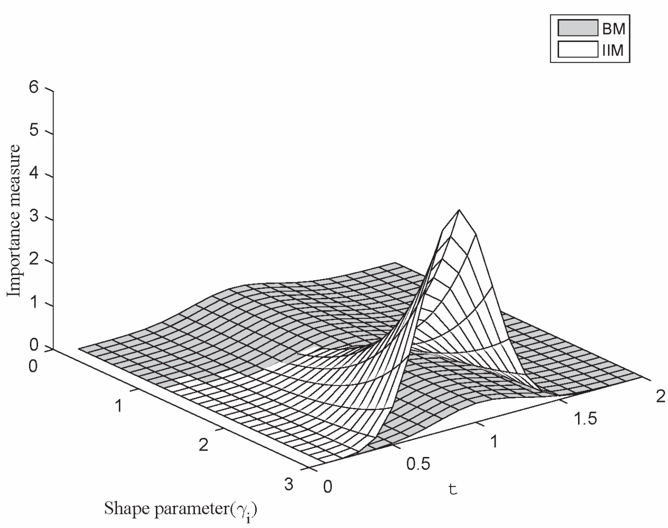


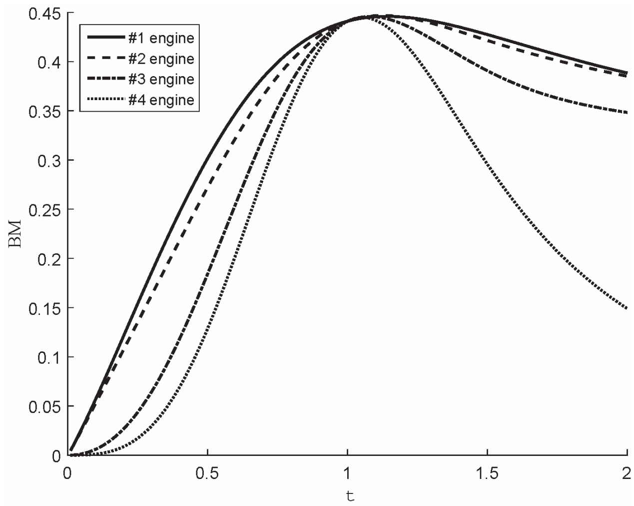
Fig. 5. Importance of engines with the change of shape parameter

comes bigger than BM more and more. This is because that $\gamma_i / \theta_{i,01}$ becomes bigger, and the EIIM is related to shape parameters.

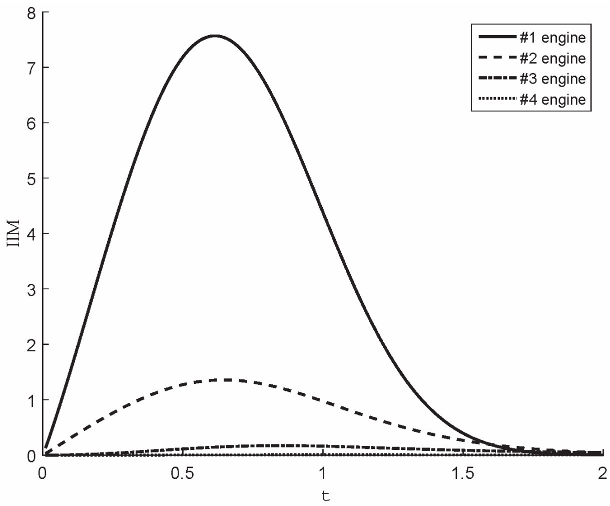
As discussed above, the four engines constitute a 2-out-of-4: G system. In case all the four engines follow different Weibull distributions, how do the engines affect the system performance? To find the answer, we will analyze the difference of importance among engines.

We assume $\theta_{i,01} = 1, y = 3$, $\gamma_5 = \gamma_6 = \gamma_7 = 3$, and for the shape parameter of engines, $\gamma_1 = 3$, $\gamma_2 = 2, \gamma_3 = 1, \gamma_4 = 0.1$. The difference between the BM and the IIM among engines is shown in Fig. 6.

From Fig. 6, the order of BM and EIIM of four engines is engine 1 > engine 2 > engine 3 > engine 4. When $\theta_{i,01} = 1$ and t=1, according to Equation (3), $W\left(t;\theta_{i,01},\gamma_i\right) = 1 - \exp\left[-\left(t/\theta_{i,01}\right)^{\gamma_i}\right] = 1 - e^{-1}$. That is to say that the reliabilities of all engines are the same. So BM of all engines are the same when t=1, as in Fig. 6(a). When $\theta_{i,01} = 1$ and y=3, the value of EIIM of engines increases with the increment of shape parameters, which is corresponding to Fig. 6(b).



(a) BM of engines



(b) EIIM of engines

Fig. 6. Difference of importance among engines

5. Conclusions and future work

This paper extends the integrated importance measure to a new measure and studies the propositions of the new measure. The results are useful for maintenance managers to evaluate which component state generates the most improvement in providing the system performance. It typically considers the conditional probability distribution of a component sojourning at a state is the Weibull distribution, given the next state that component will jump to. Then the difference between the Birnbaum importance and the integrated importance measure of a component with different shape parameters is discussed. If the shape parameter is smaller than 1, the transition rate decreases with time, and the Birnbaum importance measure of a component is bigger than its extended integrated importance measure. If the shape parameter of a component equals to 1, the transition rate is constant, which is independent of the sojourn time of a component in a state. If the shape parameter of a component is greater than 1, the transition rate increases with time, and component extended integrated importance measure is higher than its BM.

In the future work, we will discuss the other distributions when component maintenance cost changes with the failure rate and time. The cost can take different types into consideration, such as the constraint of other resources (finance, external factor), opportunity cost, and so on.

Acknowledgements

The authors gratefully acknowledge the financial supports for this research from the National Natural Youth Science Foundation of China (Nos. 71501173, 61401403, 71571168).

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