

Analysis of the calculation of the amplitude of accommodation

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ABSTRACT

The push-up method is routinely used to measure the amplitude of accommodation. In this method the diopter value corresponding to the nearest point that the eye can focus is determined, wearing his neutralizing lens, but not the value of the maximum diopter variation that makes the eye.

The aim of this paper is to review the calculation described in the push-up method taking into account the real position of the near point of the eye.

In the subjective push-up method, it is calculated the reciprocal of the distance from the lens until the test object at this position of first, slight, sustained blur.

We calculated the differences between this value and the ocular amplitude of accommodation, taking into account that the eye is really looking at the image of first blur through the neutralizing lens.

Experimental measurements were also taken to determine whether they were similar to theoretical values.

The results obtained by the two calculation methods compared were in general significantly different and the difference was greater for young people with high

ametropia. The theoretical results were in agreement with the experimental ones.

According to the optometric information required must select the appropriate calculation method, since the values obtained in each of them are not comparable.

KEY WORDS

Amplitude of accommodation, spectacle accommodation, ocular accommodation, push-up method, near point, far point, ametropia, neutralizing lens.

INTRODUCTION

The amplitude of accommodation (Am) is mathematically expressed as the dioptric difference between the far point (fp) and the near point (np) of the eye and it is related to the maximum power variation that can perform an eye.¹

The amplitude of accommodation is a measure of special interest in optometry because it is used to calculate the presbyopic addition and to establish if there is accommodative dysfunction. Therefore, it is an important magnitude for the optometrist when a lens type or vision therapy method must be decided upon.

Measurement of the amplitude of accommodation influences the diagnosis of insufficient accommodation because this measure declines in these patients.² In patients who have undergone refractive surgery it is also important to measure the amplitude of accommodation as this varies postoperatively, being especially relevant for presbyopic patients³. Finally, the amplitude of accommodation is a measure used to validate accommodative intraocular lenses.

As it is well known, the amplitude of accommodation of a person is not constant throughout his life because decreases gradually over the years.

The measurements taken by Donders (1864), and the Hofstetter's equations (1950), are still used today as normal reference values in the population.^{1,4}

However, it seems that the normal values from these authors were obtained from measurements of emmetropic subjects or low ametropias. Hence, it is appropriate to reconsider whether the values shown in these tables can be taken as standard for all observers, i.e. emmetropes and ametropes, based on the measurements of accommodation obtained from an optometric examination.⁵

Although there are different methods to determine objective and subjective values of the amplitude of accommodation, in optometry, generally, it used Donders subjective method, it is called the push-up method and it is routinely used. Subjective methods are used in clinical practice because they do not require specific instruments and are performed quickly and easily. Several pilot studies have been done to determine the value of the amplitude of accommodation by different methods, and found that the values obtained by the push-up method provided higher values than other methods.⁶⁻¹² It is therefore important to evaluate how precise in the subjective measurement of the amplitude of accommodation and, if necessary, refine it.

The push-up method determines the diopter value of the position of the point first, slight, sustained blur with neutralizing lens. With this method it is obtained the value named by some authors as the "spectacle accommodation" but does not correspond to the value of the ocular accommodation.¹³ Therefore, the push-up method does not provide information about the maximum diopter variation that the eye can perform, because it depends on the refraction.

The fact that both values can be considered the same or that both amplitudes can be similar could lead to misinterpretation. In this paper we will check what differences exist between both values and analyze the need for changes in the method of push-up depending on the measure that it is required obtain.

METHODS

1- Theoretical calculations

In the subjective push-up method, the amplitude of accommodation is measured with the ametropia corrected by spectacle lenses. In the measurement procedure, the patient observes a finely detailed test object which is brought closer to the patient's eye until the detail just begins to blur. The reciprocal of the distance from the test object at this position of first, slight, sustained blur to the spectacle plane in metres (np_D^G) represents the amplitude of accommodation (in diopters)¹⁴⁻¹⁸ or what some authors call "spectacle accommodation".¹³

$$Am_{\text{push-up}} = -\frac{1}{np_D^G} \quad \text{eq(1)}$$

In this method the fact that the patient is not accommodating at the plane where the test object is situated is not taken into account; the observer is actually accommodating at the distance where the lens forms the image of the test object. Therefore, the push-up method of calculation does not determine the maximum accommodation of the eye have, especially in ametropia.

It is known that when an object is observed through a lens, the final image that forms on the retina is not obtained directly from the object, as can be seen in figure 1. First, the intermediate image of the object that the lens produces must be taken into account, since it is this image that will be the object for the eye.^{13,19}

Applying geometrical optics equations to the optical system of the eye, it is possible to calculate the exact position of the intermediate image (x_{eye}). Therefore, to determine the accommodation that the subject employs at this position, it is only necessary to apply the mathematical expression used to calculate the accommodation of the eye at a certain distance:¹

$$A = R - \frac{1}{x_{eye}} \quad eq(2)$$

R is the refraction of the eye and x_{eye} is the distance from the eye to the intermediate image in meters. If x_{eye} corresponds to the near point of the eye, the amplitude of accommodation of the eye may be calculated (ocular accommodation).

From the steps above, we can find a formula which allows the accommodation of a corrected eye at a given distance (A_N):¹

$$A_N = \frac{X(1 + \delta_v R)}{\delta_v^2 R X - 1} \quad eq(3)$$

In this equation X represents the reciprocal of the object distance from the eye in diopters ($X = 1 / x$) and δ_v is the distance from the lens to the eye in meters (see fig 1).¹

Applying the above expression taking x as the distance from the eye to the first blur point in meters (np_D), we can calculate the value of the ocular accommodation (Am_{eye}).

$$Am_{eye} = \frac{P_D(1 + \delta_v R)^2}{\delta_v^2 R P_D - 1} \quad eq(4)$$

P_D ($P_D = 1/np_D$) is the reciprocal of the distance of the first blur point from the eye in diopters determined by the push-up method.

Since in practice the distance from the eye to the test object when the observer reports the first blur can be directly measured, the amplitude of accommodation of the eye may be directly calculated using eq (4).

In order to quantify the differences in the calculation, the values obtained by the push-up method or “spectacle accommodation” (“ $Am_{push-up}$ ”) are compared with the values of ocular accommodation (Am_{eye}).

To perform the calculations we considered different values of neutralizing power of the lens (P_{NL}), associated with their corresponding values of refraction (R), and different values amplitude of accommodation of the eye.¹

$$P_{NL} = \frac{R}{1 + \delta_v R} \quad \text{eq(5)}$$

The values shown in the tables of results correspond to P_{NL} ranging for +10 D and -10D, in steps of 1 D. The values of Am_{eye} ranged between 15.5 D and 0.5 D, corresponding to patients aged between 10 and 65 years approximately.²⁰

2- Experimental measurements

In order to determine if the theoretical results obtained for the calculation of amplitude of accommodation were in agreement with experimental values, the following experiment was conducted on a total of 8 eyes with an VA=1 after their optical correction in patients between the ages of 20 and 25 years.

First, the position of the point first, slight, sustained blur with the neutralizing lens (ηp_D^G) was measured as described in Donder’s method. On the one hand, this distance was used to calculate Donder’s amplitude of accommodation ($Am_{push-up}$) (spectacle accommodation), that is, the dioptric value of this distance (eq(1)), and on the other, the amplitude of accommodation was found taking

into account that the eye is really looking at the image of first blur through the lens (Am_{eye}) (ocular accommodation) (eq(4)).

Then, the near point of the uncorrected eye was determined ($np_{no\ lens}$), measuring the distance from the corneal vertex to the test object the moment the patient reports the first blur. In this way, the experimental measurement is not affected by the lens. This value was used to calculate the amplitude of accommodation of the eye ($Am_{no\ lens}$) in the following equation:

$$Am_{no\ lens} = R - \frac{1}{np_{no\ lens}} \quad eq(6)$$

Monocular measurements were taken for myopic refractions.

RESULTS

1- Theoretical calculations

We compare the different methods of calculation, for a given value of R and Am_{eye} . To mathematically obtain the values of $Am_{push-up}$ from the values of Am_{eye} , the following calculations were performed:

a- Firstly, the position of the first point of blurring of the different observers was calculated from their refraction and ocular accommodation, eq(4).

b- Finally, we obtained the value of $Am_{push-up}$, calculating the reciprocal of the distance of the first blur point from the neutralizing lenses in metres, as described in the push-up method, eq(1).

In order to compare the two values, for each refraction, the dioptric difference between the Am_{eye} and $Am_{push-up}$ was found as well as the percentage difference.

Figures 2 and 4 show $Am_{\text{push-up}}$ as a function of Am_{eye} (figure 2 for myopic refractions and figure 4 for hyperopic refractions) and figures 3 and 5 show the percentage differences between both methods (figure 3 for myopic refractions and figure 5 for hyperopic refractions).

1.1- Myopic refractions. In figure 2 the values of amplitude of accommodation calculated by the push-up method are the same as the real values of the eye when $P_{\text{NL}}=0$, since these represent the measurements taken for an emmetropic eye without lenses.

The difference in diopters is as much as 9.02D. For an average value of $R= -8.91\text{D}$ which corresponding a neutralized power lens $P_{\text{NL}}= -10\text{D}$, a difference of up to 9.02D is found for a $Am_{\text{eye}}=15.5\text{D}$. For low refractions the differences are smaller than in the case of high refractions, although for $P_{\text{NL}}= -1\text{D}$ ($R= -0.99\text{D}$) there are variations in the values of the amplitude of accommodation ranges from 0.02 to 4.06D (for values of Am_{eye} of 0.5D to 15.5D).

Figure 4 shows that in myopic refractions the percentage change between the values obtained ranges from 3.3% at a $P_{\text{NL}}= -1\text{D}$ and $Am_{\text{eye}}=0.5\text{D}$ to 36.8% for $P_{\text{NL}}= -10\text{D}$ and $Am_{\text{eye}}=15.5\text{D}$. Most of the calculated values have a percentage variation of not less than 20%.

Therefore, in the case of myopic subjects the differences between the values obtained with the two methods of calculation are generally important. Regardless of the value of refraction in myopia, the amplitude of accommodation calculated using the subjective push-up method is greater than the value of Am_{eye} . In general, we can see that when there is high myopic refraction the difference between the values obtained by the two methods of

calculation increases. Similarly, it may be seen that the higher the amplitude of accommodation compared the greater the difference found.

1.2- Hyperopic refractions: It should be noted that in the case of positive refraction, the $Am_{Donders}$ may be higher or lower than Am_{eye} depending on the refraction and amplitude of accommodation of the eye in question. Overall, in the case of low refraction and high amplitude, the amplitude of accommodation is overestimated by the push-up method. Conversely, when the refraction increases and the amplitude of accommodation of the eye decrease the calculated value is underestimated. However, about half of the measurements have percentage differences of over 10%.

In this case, the biggest difference is of 3.04D, for $Am_{eye}=15.5D$ and $R=1.01D$ which corresponding a neutralized power lens $P_{NL}=1D$. Whereas in most cases dioptric differences of under 1D are found, and in some cases there is no variation in the value. The maximum percentage difference found was 27.5%.

Comparing the results for the two types of refraction it can be seen that, overall, the differences obtained with the two methods of calculation are significantly higher in myopic refractions than in hyperopic refractions.

Therefore, the calculation procedure described in the push-up method may be said to provide inaccurate values in ametropia if the aim is to determine the maximum refractive variation as the eye can make.

However, it is true that it is a method that is easy to apply in clinical practice due to its simplicity and speed. It would not be practical if, in addition to the set of tests done in an optometric examination, it were necessary to perform complex calculations.

Consequently, we have created tables that enable the practitioner of optometry to quickly obtain the amplitude of accommodation of the eye ($A_{m_{eye}}$) (ocular accommodation) by simply obtaining the dioptric power of the neutralizing lens (P_{NL}) of the observer and the distance from the lens to the first blur point, as is normally done in the push-up technique.

In tables 1 and 2, the rows represent different values of P_{NL} and the columns the distance of the first blur point measured from the lenses (x^G). The optometrist would simply need to locate the cell at which the values of P_{NL} and x^G absolute value intersect to find the patient's amplitude of accommodation, without having to perform any type of calculations.

The distances used to draw up the tables ranged from 5 cm (myopic) and 7 cm (hyperopic) to 80 cm (myopic) and 100 cm (hyperopic) at intervals of 0.5 cm up to a distance of 16 cm. The subsequent intervals then increased following the criterion that the differences between the measurements at the intermediate steps did not exceed 0.25D. For distances less than 7 cm only in some values, the differences are somewhat greater than 0.25D, but not exceeding the value of 0.5D.

Moreover, taking into account Hoffstetter's equations, the selected distances correspond to accommodation values in people aged between 10 and 70 years. As for the dioptric value of the neutralizing lenses considered to prepare the tables, dioptric variations of 1D, from 1 to 10D were taken. The criterion in this case was the same as above, ie, dioptric variations between the amplitude of accommodation values calculated did not exceed 0.25D.

2- Experimental measurements

As may be seen in figure 6, the experimental measurements taken as described in Donders's method always gave the highest amplitude of accommodation values ($Am_{\text{push-up}}$). This is true if we compare them with the values obtained using the position of first blur when the eye is uncorrected ($Am_{\text{no lens}}$) and with those obtained with the corrected eye (Am_{eye}).

The Am_{eye} and $Am_{\text{no lens}}$ values are very similar and in most cases do not differ in more than 0.50D. This result is logical because both are the same magnitude but experimentally obtained in different ways. However, the $Am_{\text{push-up}}$ values differ in as much as 4D or more, which in most cases represents a difference of over 25%.

The experimental values obtained show, as do the theoretical results, that the differences between $Am_{\text{push-up}}$ and Am_{eye} increase as the P_{NL} value, that is, the observer's R, increases. In the same way, when Am_{eye} increases, the differences between the amplitude of accommodation values obtained are greater.

DISCUSSION AND CONCLUSIONS

Donders's method for measuring the amplitude of accommodation in optometric visual examinations, as described in the references, does not determine the amplitude of accommodation of the eye (ocular accommodation). The method of calculation described does not take into account the real distance at which the eye is accommodating and the distances are measured from the corrective lenses to the test.

As can be seen in our study, the amplitude of accommodation values obtained by the two calculation methods compared are, generally, significantly different,

especially in the case of negative refraction. In these cases, the difference increases as the refraction and amplitude of accommodation of the observer, increases. That is, the differences are greater for young people with high ametropia.

As showed in the results, the calculation method used in the push-up method provides, in most cases, higher values than the amplitude of accommodation, according to different studies.⁶⁻¹²

Keep in mind that the normal range usually applied was obtained from the method of calculation of Donders in emmetropic subjects, if these values are compared with ametropic subjects to determine if the amplitude of accommodation that is obtained is normal, you should perform the calculation considering the effect of neutralizing lens ($A_{m_{eye}}$).

Therefore, we can conclude that it is important to exercise great care when taking measurements especially in myopic subjects, because in these cases we obtained the largest differences when performing calculations as described in the push-up method.

In order to obtain more precise estimates of the amplitude of accommodation of the eye, we have built the Tables 1 and 2 in which its value is obtained from the refraction and the first blur point ($A_{m_{eye}}$). Although the differences are not the same for all cases of ametropia, if the optometrist obtains the value of the amplitude of accommodation of the eye, optometric information available will be more complete and the resulting measurement will provide him a more reliable basis for the analysis of the optometric case in question.

It is useful to know what information is needed depending on the purpose of the optometric examination to perform the most convenient calculation. It is very

important to know the origin of the differences obtained by the two methods of calculation and also the optometric information obtained from each of them. As we can see, to assume that the amplitude of accommodation in lenses is equal or similar to the amplitude of accommodation of the eye is a significant error.

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FIGURE LEGENDS

Figure 1. Images of an object formed by the lens-eye optical system. x_{eye} represents the distance at which the eye is actually accommodating.

Figure 2. $Am_{push-up}$ as a function of Am_{eye} for myopic subjects.

Figure 3. Percentage difference between $Am_{push-up}$ and Am_{eye} as a function of Am_{eye} for myopic subjects.

Figure 4. $Am_{push-up}$ as a function of Am_{eye} for hyperopic subjects.

Figure 5. Percentage difference between $Am_{push-up}$ and Am_{eye} as a function of Am_{eye} for hyperopic subjects.

Figure 6. Comparison of the amplitude of accommodation calculations for each of the eyes measured.

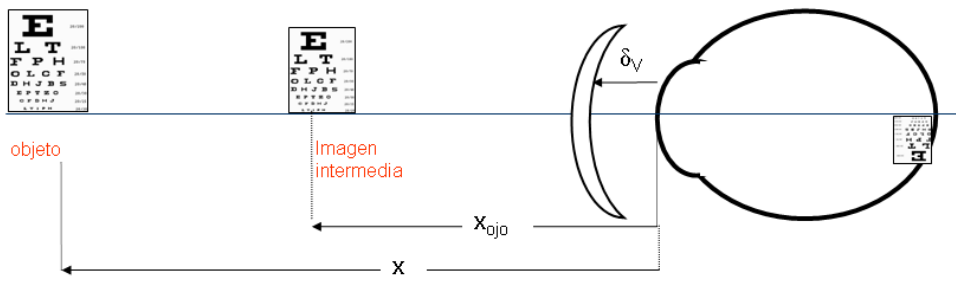


Figure 1

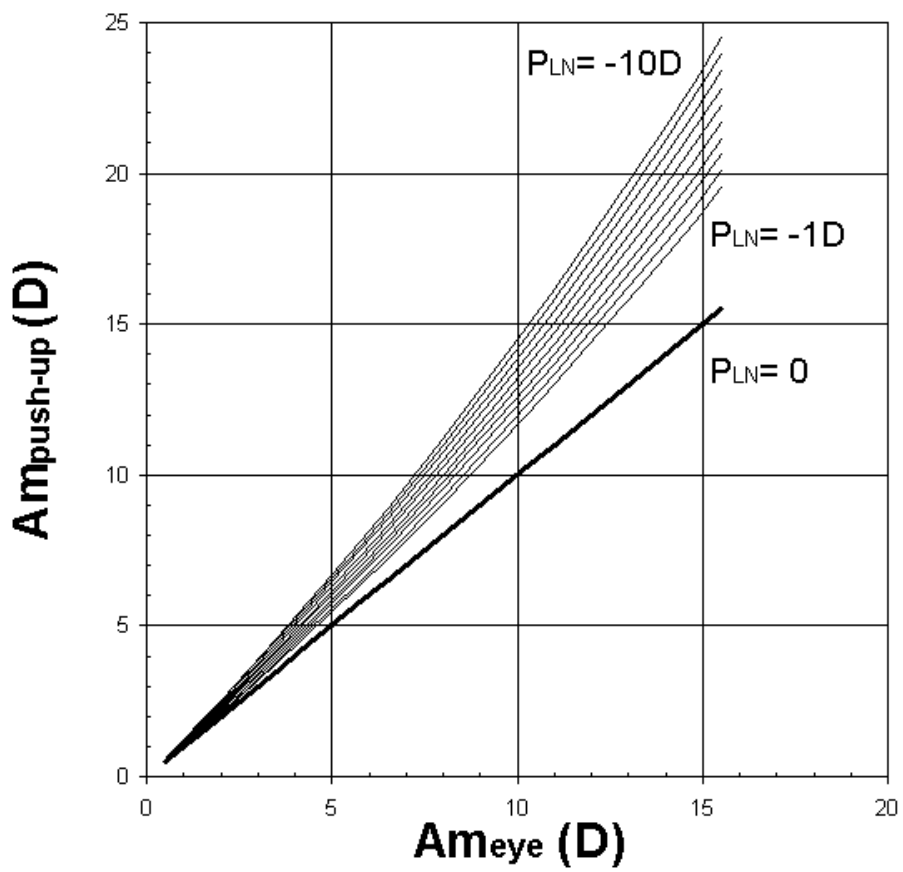


Figure 2

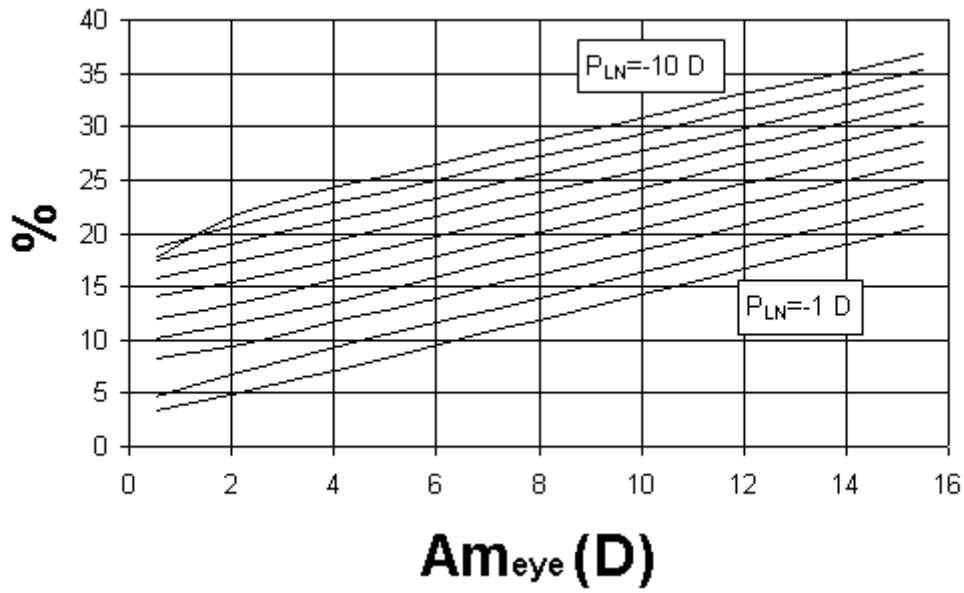


Figure 3

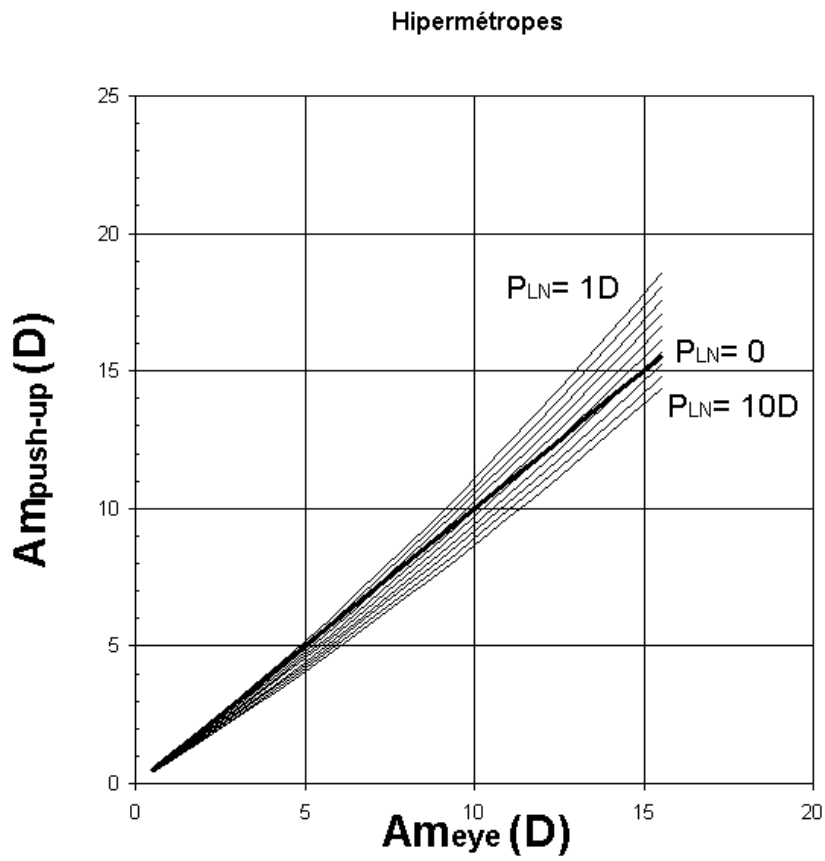


Figure 4

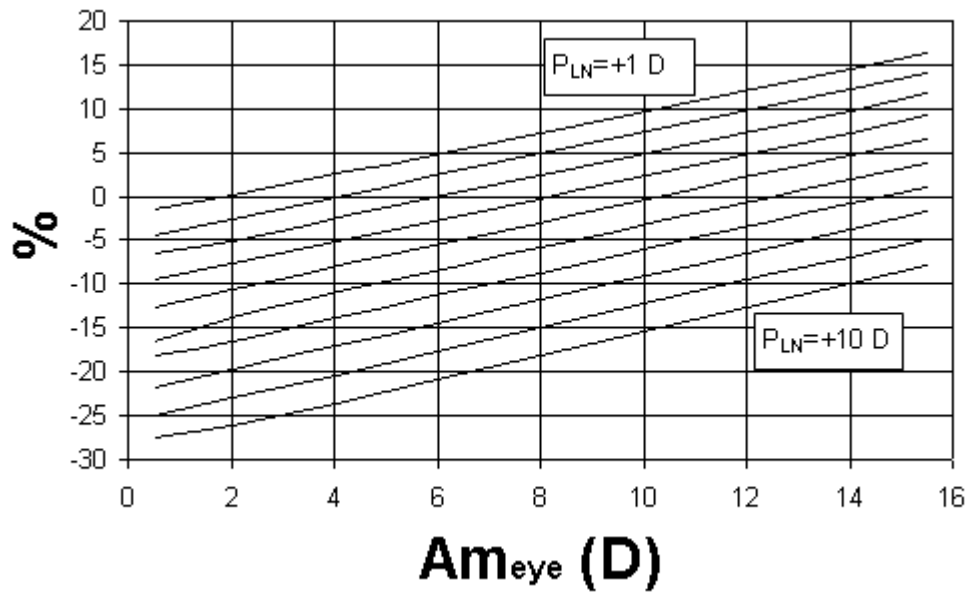


Figure 5

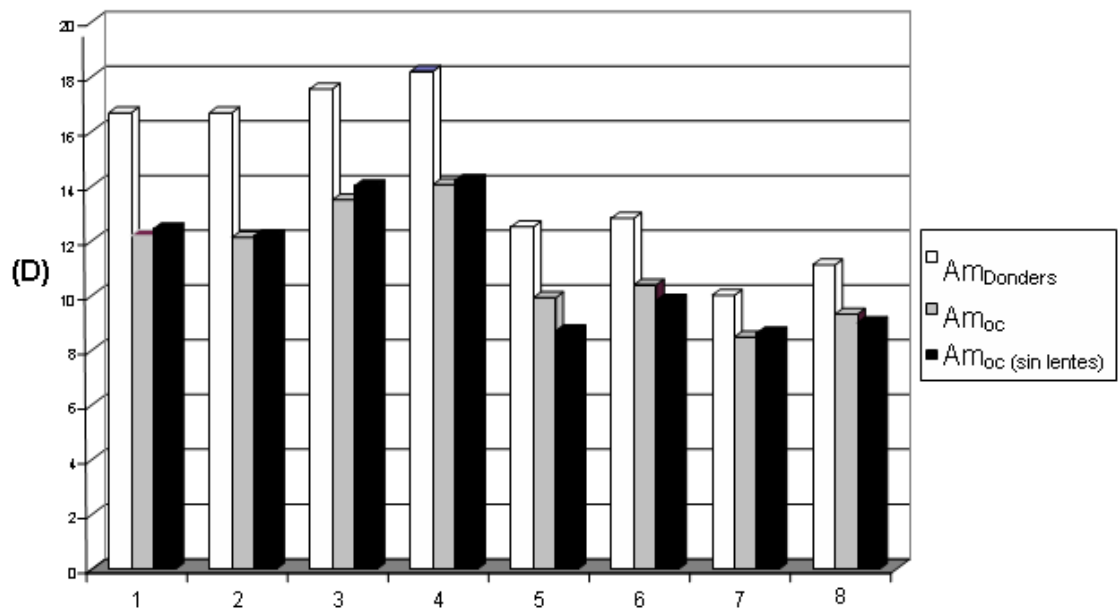


Figure 6