

# Compact Coplanar Waveguide Band-pass Filter based on Coupled S-shaped Split Ring Resonators

A. K. Horestani<sup>1</sup>, M. Durán-Sindreu<sup>2</sup>, J. Naqui<sup>2</sup>, C. Fumeaux<sup>1</sup>, and F. Martín<sup>2</sup>

<sup>1</sup>School of Electrical & Electronic Engineering, The University of Adelaide, Adelaide, SA 5005, Australia

<sup>2</sup> Departament d'Enginyeria Electronica, Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain. Miguel Durán-Sindreu was with Universitat Autònoma de Barcelona when this research was performed.

Corresponding author: Ali.K.Horestani@ieee.org

**Abstract:** This letter is focused on the application of coupled single layer S-shaped split ring resonators (S-SRRs) to the design of highly compact bandpass filters in coplanar waveguide (CPW) technology. S-SRRs have been previously demonstrated as miniaturized resonators, particularly suited for applications in conjunction with coplanar line geometries. However, size reduction of CPW filters based on impedance inverters and S-SRRs is limited by the inverters. Therefore, this letter proposes an alternative geometry of CPW bandpass filters employing S-SRRs in a configuration based on the theory of coupled resonators. A highly compact third-order bandpass filter is designed using this principle, and the proposed approach is validated through experiment, demonstrating competitive filter performance achieved in an extremely small area.

## 1. Introduction:

The miniaturization of planar filters has been a subject of an intensive research for microwave engineers during many years. Within this context, it has been demonstrated that metamaterial-inspired resonators, such as split ring resonators (SRRs) and complementary SRRs (CSRRs), including their open counterparts (OSRRs and OCSRRs), can be efficiently used for size reduction in microwave filters [1]–[7]

The reason is that these resonators are electrically small, i.e., they are able to accommodate a significant capacitance and/or inductance in a certain area by virtue of their particular topology. An efficient approach to further reduce the electrical size of a SRR is to enhance its capacitance and/or inductance by modifying the resonator shape [8]. Thus, broadside coupled SRRs (BC-SRR) [9] and double-layer S-shaped SRRs (S-SRRs) [10], [11] can be by far electrically smaller than the conventional (edge coupled) SRRs. However, these topologies require two metal levels.

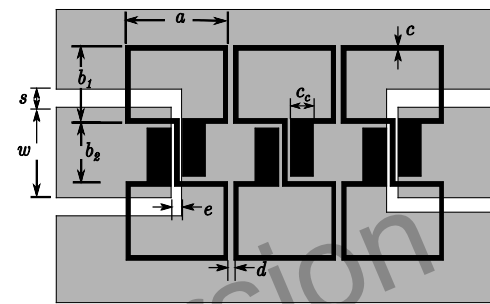
Recently, it was demonstrated by the authors that size reduction in planar resonators can be alternatively achieved by means of single-layer S-shaped geometries [12], [13]. Particularly, compact bandpass filters were implemented by loading a coplanar waveguide (CPW) with S-shaped SRRs (S-SRRs). The key to this level of miniaturization is that at the fundamental resonance frequency, the currents in the two connected rings of the S-SRR flow in the same direction, and the resonator inductance is large as compared to the inductance of a conventional SRR. In CPW technology, S-SRR excitation can be achieved by the contra-directional axial magnetic fields generated by the line, or by means of electric coupling. In the latter case, the S-SRR should include wide strip regions (acting as patch capacitors) to enhance the coupling. A compact bandpass filter based on S-SRRs coupled through impedance inverters was designed in [13]. However, less than 30% of the filter's unit cell was occupied by the S-SRR, whereas more than 70% of the unit cell length was required for the impedance inverters. The aim of this

letter is to achieve further miniaturization of CPW bandpass filters based on S-SRRs by eliminating the impedance inverters. To this end, this letter is focused on the application of the S-SRRs to the design of CPW bandpass filters based on the theory of coupled resonators.

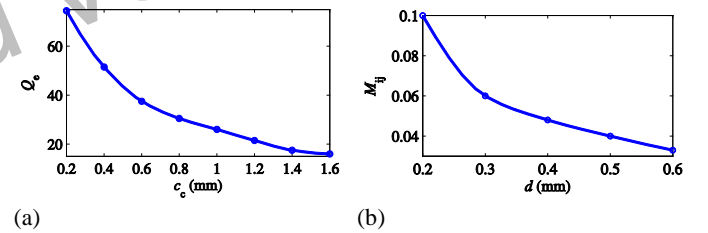
## 2. Bandpass Filter based on the Theory of Coupled Resonators:

In order to demonstrate the potential of the S-SRRs in conjunction with theory of coupled resonators to the design of highly compact bandpass filters, a third-order Chebyshev filter prototype is designed to meet the following specifications: center frequency  $f_0 = 1.1$  GHz, bandwidth of 80 MHz, and a maximum passband ripple of 0.2 dB.

The filter design process is based on the control of coupling  $M_{12}$  and  $M_{23}$  between resonators, as well as the coupling between the filter feed lines and the first and last resonators, which are characterized with the external quality factors  $Q_{e1}$  and  $Q_{e3}$ .



**Fig. 1** Layout of the proposed third-order coupled resonator bandpass filter based on S-SRRs.



**Fig. 2** (a) The external quality factor of the S-SRR resonator coupled to an open-ended CPW for different values of  $c_c$ , and (b) the coupling coefficient of a pair of S-SRRs versus the space between the two resonators  $d$  for a 0.635 mm thick Rogers RO3010 material with relative permittivity of 10.2.

Given the filter specifications, the coupling coefficients  $M_{12}$  and  $M_{23}$  and the external quality factors  $Q_{e1}$  and  $Q_{e3}$  are obtained from [14]

$$Q_{e1} = \frac{g_0 g_1}{FBW}, \quad Q_{e3} = g_3 \frac{g_4}{FBW}$$

$$M_{12} = \frac{FBW}{\sqrt{g_1 g_2}}, \quad M_{23} = \frac{FBW}{\sqrt{g_2 g_3}}$$

where  $g_i$ 's are the Chebyshev lowpass prototype elements and FBW denotes the fractional bandwidth of the filter. Using these equations, the filter specifications are achieved using the external quality factors  $Q_{e1} = Q_{e3} = 17$  and the coupling coefficients  $M_{23} = M_{12} = 0.061$ .

Figure 1 illustrates the geometry of the filter, which is composed of two open ended CPWs (shown in gray shading) which are coupled through three S-SRRs etched on the backside of the substrate (shown in black color). In this configuration the coupling

coefficients can be adjusted by changing the space between the S-SRRs, denoted as  $d$  in the figure, and the required external quality factor can be achieved by changing the area where the open-ended CPW and the S-SRRs are overlapped, i.e., by slightly displacing the full S-SRR either on the left or the right of the CPW open end. Note that since in this configuration open-ended CPWs are used, the dominant coupling mechanism between the CPW and the first and the last resonators is an electric coupling. Thus, alternatively the S-SRR can be horizontally centered onto the open end of the CPW and the width of the capacitive parts of the S-SRR  $c_c$  can be adjusted to achieve the desired external quality factor. The latter approach is applied in this example and the external quality factor and coupling coefficients are characterized through electromagnetic simulations by using [14]

$$Q_e = \frac{2\pi f_0 \tau_{11}(\omega_0)}{4}$$

Where  $f_0$  is the resonant frequency and  $\tau_{11}(\omega_0)$  is the group delay of  $S_{11}$  at resonance frequency, and

$$M_{ij} = \frac{f_2^2 - f_1^2}{f_2^2 + f_1^2}$$

where,  $f_2$  and  $f_1$  are the two resonance frequencies of the pair of coupled resonators while they are weakly coupled to the open-ended CPWs.

The dimensions of the CPW are  $w = 4.8$  mm and  $s = 1$  mm, which corresponds to a characteristic impedance of  $50 \Omega$  on a  $0.635$  mm thick Rogers *RO3010* material with relative permittivity of  $10.2$ . Dimensions of the S-SRRs are as follows:  $a = 6.2$  mm,  $b_1 = 4.2$  mm,  $b_2 = 3.2$  mm,  $c = 0.2$  mm, and  $e = 0.8$  mm. The simulated external quality factor  $Q_e$  as a function of the S-SRR's dimension  $c_c$ , and the coupling coefficient  $M_{ij}$  as a function of the distance between a pair of S-SRRs  $d$  are depicted in Fig. 2 (a) and (b), respectively. The figures show that in order to meet the required external quality factor  $Q_{e1} = Q_{e2} = 17$  and coupling coefficients  $M_{12} = M_{23} = 0.061$  required by the filter specifications  $c_c$  needs to be  $1.4$  mm, and the space between the resonators  $d$  needs to be set to  $0.3$  mm.

### 3. Experimental Results:

In order to validate the design, a prototype of the filter is fabricated and tested, and the measured results are compared with the full-wave electromagnetic simulation results. Figure 3 depicts the photographs of the top and bottom view of the fabricated sample. The active area of the prototype has a longitudinal length of  $19.2$  mm and transverse length of  $11.6$  mm, i.e.  $0.12 \lambda_g$  by  $0.08 \lambda_g$ , where  $\lambda_g$  is the guided wavelength. Figure 4 compares the simulated and measured transmission and reflection coefficients of the filter. The figure shows a good agreement between the simulated and measured results. Note that the electromagnetic simulation results correspond to the case of a lossless device, confirming that radiation losses are not significant.

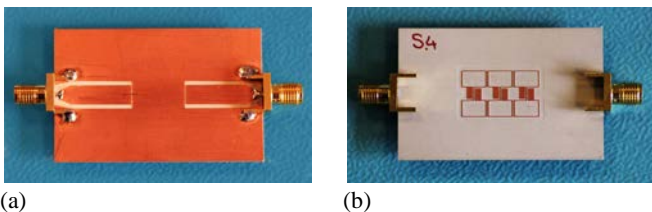


Fig. 3 Photographs of (a) top and (b) bottom view of the fabricated third-order coupled S-SRR bandpass filter.

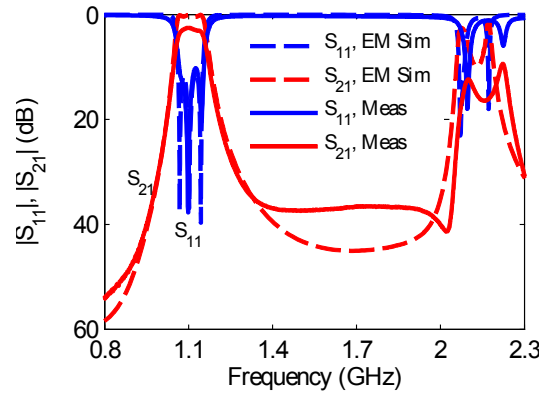


Fig. 4 Comparison between measured and lossless electromagnetic simulated transmission and reflection coefficients of the third-order coupled S-SRR filter of Fig. 3. The results show that there is no important feature below or above the presented frequency band up to the second harmonic.

In order to evaluate the performance of the proposed filter, Table 1 provides a comparison between the measured characteristics of the proposed filter to those of state-of-the-art narrow bandpass CPW filters.

The table shows that in comparison to the bandpass filters with similar order and fractional bandwidth, the proposed third-order filter based on coupled S-SRRs benefits from a good in-band insertion loss of  $2.5$  dB as well as a highly compact size.

Table 1: A Comparison of Various Narrow Bandpass CPW Filters.

Ref	n	$f_0$ (GHz)	FBW (%)	IL (dB)	Electrical Size	Electrical area ( $\lambda_g^2$ )
[15]	3	4.5	3.5	2.8	$0.50\lambda_g \times 0.25\lambda_g$	0.125
[16]	4	2.86	7	2.8	$0.84\lambda_g \times 0.46\lambda_g$	0.385
[17]	2	4.92	3.8	2.77	$0.46\lambda_g \times 0.20\lambda_g$	0.093
[18]	2	5	5.4	2.7	$0.57\lambda_g \times 0.20\lambda_g$	0.115
[19]	4	1.75	9.6	2.15	$0.37\lambda_g \times 0.33\lambda_g$	0.122
[20]	3	7	3.5	4	$0.41\lambda_g \times 0.30\lambda_g$	0.123
[13]	3	1.25	12	2.4	$0.31\lambda_g \times 0.07\lambda_g$	0.022
This work	3	1.1	7.2	2.5	$0.12\lambda_g \times 0.08\lambda_g$	0.0096

### 4. Conclusion:

It has been demonstrated that very compact narrow- and moderate-band bandpass filters can be implemented by means of coupled S-SRRs fed by open-ended CPWs. In particular, a third-order Chebyshev bandpass filter with  $7.2\%$  fractional bandwidth (centered at  $1.1$  GHz) and  $0.2$  dB in-band ripple has been designed and fabricated. Filter performance has been found to be competitive, and filter size is as small as  $0.12\lambda_g \times 0.08\lambda_g$  ( $\lambda_g$  being the guided wavelength at the central frequency).

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