Triadic Balance and Closure as Drivers of the Evolution of Cooperation

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ABSTRACT

The prevalence of human cooperation continues to be one of the biggest puzzles for scientists. Structured interactions and clustering of cooperators are recognized mechanisms that help the dissemination of cooperative behavior. We analyze two dynamic micro structural mechanisms that may contribute to the evolution of cooperation. We concentrate on two mechanisms that have empirical justification: triadic closure and triadic balance. We study their relative efficiency under different parametric conditions, assuming that the structure of interactions itself might change endogenously as a result of previous encounters.

Keywords: evolution of cooperation; signed graphs; network dynamics; negative ties; triadic closure, triadic balance.

I. INTRODUCTION AND RELATED LITERATURE

It is difficult to justify the wide spread and extent of human cooperation. Cooperation is not the option that a calculative (rational) individual should choose in a social dilemma situation, such as the Prisoner's Dilemma (Axelrod 1984; Axelrod and Hamilton 1981). Structured interactions and the consequent clustering of cooperators, have been suggested as major mechanisms that support the emergence and dissemination of cooperative behavior (Hauert and Doebeli 2004; Lieberman et al. 2005; Nowak 2006; Ohtsuki et al. 2006; Santos et al. 2006). In fact, human interactions are seldom truly random: they are frequently repeated and they include partners in close spatial proximity or who are linked by a social network. The structure of social interactions also changes over time, sometimes endogenously, as a result of cooperation (Santos et al. 2006; Wang et al. 2013; Yamagishi and Hayashi 1996; Yamagishi et al. 1994). In this study, we consider a non-random structural dynamics and we use agent based modeling to explain the emergence and spread of cooperation in such context.

Social networks change in many different ways. The underlying mechanisms that govern their dynamics have just started to be characterized systematically (e.g., Ahuja et al. 2012). *Exogenous random changes* tend to diminish chances of cooperation diffusion as they introduce frictions to the establishment of clusters of cooperation (Durrett 2007). It is more realistic to consider *exogenous non-random changes*, such as preferential attachment and small world rewiring that describe observed patterns of dynamics and drive networks towards clustered topologies. These dynamics tend to result in more success for cooperative strategies (Wang et al. 2008). Finally, *endogenous topological changes* that reflect on previous play, are the most realistic as they highlight the interdependence between structure and behavior. Moreover, they are the most likely to speed up the evolution of cooperation.

In the current study, we concentrate on two endogenous mechanisms that have empirical justification in many social contexts: triadic closure and triadic balance. Both mechanisms are related to the concept of cognitive balance (Heider 1946), i.e, to the propensity of couples of individuals to align the way they feel about an object (or, in our case, a third person). *Triadic closure* is the tendency of "friends of friends" to become friend themselves or, from a network topology perspective, of triads to close (Fararo and Skvoretz 1987; Granovetter 1973; Rapoport 1953). *Triadic balance* is the tendency of people to maintain cognitive consistency in their relationships by changing the valence of their relationships in established triads so that the multiplication of signs turn positive and the relationships are structurally balanced (Cartwright and Harary 1956). In the context of our model, these mechanisms are chosen as they allow to endogenize both the relational sign evolution and the topological network update in an empirically justified manner.

Our aim is to study the efficiency of these dynamic network mechanisms for cooperation under different parameter conditions. We analyze their effect alone, but we also test if they have a synergic impact on cooperation, in addition.

II. THE MODEL

The agent-based model presented in this paper builds on our previous models (Righi and Takács 2013, 2014a,b,c). Our setup allows the coevolution of network structure, relational signs and agents strategies in the context of *signed networks*. In the current study, we introduce two empirically based

mechanisms (triadic closure and triadic balance) that guide the evolution of cooperation.

We consider a population of size N, connected by a non-weighted, undirected network. The ties are assumed to be signed and are either positive or negative. Each agent i can play the Prisoner's Dilemma (PD) with peers in his current first order social neighborhood, and only with them. The social neighborhood of i is the subset of the population he shares network ties with, formally $\mathcal{F}_i^t \subset N$. The cardinality $|\mathcal{F}_i^t|$ (i.e., the degree of agents at time t) is assumed to be distributed according to some arbitrary probability mass function $f(k)$. For the sake of the preliminary simulations discussed in this manuscript, we considered an Erdős-Rényi *random network* with each link existing with an independent probability 0.10 at the setup. This type of network provides a useful benchmark against which to study results for other topologies.

Considering the existence of a signed link between two agents i and j, the strategy played by i in the PD can be of three types. (1) *Unconditional Cooperation* (hereby named UC): a strategy that always cooperates, regardless of the sign of the relationship between i and j. (2) *Unconditional Defection* (hereby named UD): a strategy that, symmetrically, always defects. (3) *Conditional Play* (hereby named COND): a strategy that prescribes cooperation for i if the link between i and j is positive, and prescribes defection if the link between i and j is negative.

Each dyadic game yields a payoff for the players, defined according to the classical PD in Table 1. When two agents cooperate with each other, each gets a reward (R). When they both defect, they are both punished (P). Finally, when one agent defects and the other cooperates, the first gets a temptation payoff (T) while his partner obtains the sucker payoff (S). The game is defined with payoffs $T > R > P > S$. In line with Axelrod (1984), we further assume that $T + S > R + P$. The dynamics of our model is summarized

TABLE 1: The Prisoner's Dilemma payoff matrix. The numerical payoffs used here are the same as those of Axelrod (1984).

	C	D
C	$(R = 3, R = 3)$	$(S = 0, T = 5)$
D	$(T = 5, S = 0)$	$(P = 1, P = 1)$

in Algorithm 1. At each time step, every pair of currently connected agents play the PD, and individual payoffs are calculated. As a consequence of the strategies played, tension emerges in a relationship, if a cooperator faces defection from the opponent. Tension can result (with probability P_{bal}) in an update of the status of the relationship which is hereby modeled through the empirically grounded mechanisms of balance. Moreover, in the absence of tension among agents, the mechanism of triadic closure mechanism is activated with probability P_{clo}; implying the closure of one triad involving the partners. Tension in social relationships in intended to model in a very simple way, the emotional consequences of partners behavior. In addition, in order to make meaningful

comparisons, we will also study the effect of exogenous triadic closure and balance mechanisms on cooperation, in which the structural change is independent of previous play, but depends on previous structure.

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for *each pair of individuals i and j connected at time $t - 1$* **do**

    Compute the social neighborhood $\mathcal{F}_i^{t-1}$ and $\mathcal{F}_j^{t-1}$;

    Play the PD with i and j and compute payoffs;

    if *the link between i and j is tense* **then**

        Update relational signs between i and j so to maximize triadic balance (with probability P_{bal});

    else

        Select an acquaintance k of either i or j who is not connected to the other and close the triad (with probability P_{clo});

        Assign a relational sign randomly to the new relationship;

        Delete one relation who does not include i or j that has the relational sign of the new relationship randomly;

    end

end

for *each agent i* **do**

    Compute average payoff of agent i;

    Observe the average payoffs of each agent $j \in F_i^{t-1}$;

    Adopt a random (strictly) better strategy (with probability P_{adopt});

end

Algorithm 1: Pseudo-code of dynamics of our model, repeated at each time step t. The relational update performed at time t, take effect only at time $t + 1$. The PD is therefore played in parallel by all agents. The implementation details about the closure and balance mechanisms are discussed below.

Let's define more precisely the procedure through which closure and balance are introduced in our model.

Triadic Closure. Two agents, sharing a stable positive relationship (i.e. in the absence of tensions due to past behavior) tend to increase the number of common friends. This triadic closure mechanism is adopted with probability P_{clo}. When this is the case, agents (say i and j) select one of the acquaintances of either i or j, who is not an acquaintance of the other (denoted here with k). Agent k is then connected with the other agents so to obtain a complete triad. The new link is assigned a random sign. [1] Finally, to keep the overall density constant, one old link, not involved in any triad between i and j is selected randomly and eliminated. As we want to keep triadic closure neutral with respect to the evolution of relational signs, we further assume that if a new negative relationship is created, then an old negative one is deleted, and a new

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[1] One could assume instead that the new link takes a sign such that the triad that results is in balance. However, we choose to use the random sign allocation rule in order to separate more effectively the role of triadic closure from that of triadic balance.

positive sign implies the deletion of an old positive sign. This mechanisms allows network topology and agent's strategies to co-evolve endogenously. Indeed, structural changes are induced by the absence of tension, which results from previous positive interactions. The probability P_{clo}, is assumed to be equal for the whole population and non-strategic.

Triadic Balance. When tension emerges in a dyad, due to the divergence of agents' behavior in the PD, it can be resolved through a balance mechanism. This is assumed to occur with probability P_{bal}. In this case, we consider the signs involved in all triads where both i and j are members. The relational sign between i and j is then changed so to maximize the number of balanced triads that involve them.

As discussed, each individual can only play the PD with other agents in his own local neighborhood. Following most of the literature on evolutionary games played on networks, we assume the *average* of the payoffs obtained in dyadic interactions as the measure of individual fitness. Due to this assumption, it is important that the order in which dyads are selected for interaction, does not matter for the outcome. For this reason, each dyad interacts and updates strategies and link signs observing only the previous step status quo. Moreover, changes in network topology, relational signs and strategy updates take effect from the following time step. We thus assume that updates are made in parallel.

Finally, the **evolutionary mechanism** that we adopt in this paper is relatively simple. After all agents performed their round of social interactions, each observes his average payoff as well as the ones of the agents he played with in his social neighborhood. Thus each agent is able to measure the relative local efficiency of his strategy. If a subset of agents in $\mathcal{F}_i^t$ has a payoff higher than his own, then agent i adopts the strategy played by one of them, selected uniformly at random. Evolutionary update happens, for each agent, with probability P_{adopt} which is assumed equal for all agents.

III. Preliminary Results and Discussion

In this section, we introduce some preliminary results on this model. In particular, we study the effect on cooperation of one of the mechanisms discussed: triadic closure. In the simulations reported, we fixed P_{bal} to 0.15. This is a first step in the direction of a more comprehensive analysis that we are in the process of developing. What is the impact of triadic closure on the level of cooperation observed in the model? In Figure 1 and 2 we show that, at any level of P_{adopt}, in the absence of a closure mechanism, no cooperation survives. Increasing the probability of closure to occur progressively, our setup suddenly enters a short but intense phase of instability, where simulation results differ widely (hence the high standard deviations). Subsequently, the proportion of negative ties, surviving at the end of the simulation, suddenly drops and the proportion of conditional and unconditional cooperators jumps to values significantly larger than zero.

Moreover, the level of closure required for the emergence of cooperation increases with the probability of adoption of a better strategy. Indeed, increasing the evolutionary pressure tends to favor the evolutionary stable strategy: defection.

IV. Final Remarks and Further Work

In this extended abstract we introduced a model aimed at studying the emergence of cooperation in a system where network topology coevolves with agents strategies and with relational signs. Building on our previous work, we introduced two mechanisms (and we began to study the impact of one of them) with the objective of understanding their impact on cooperation on signed networks.

In this paper we only provide some exploratory results showing the role of triadic closure on the outcome of the simulations. Our results show that, at any level of evolutionary pressure the introduction of enough triadic closure leads to the possibility of emergence of cooperation. The emergence of cooperation is not gradual but happens right after a sharp *phase transition*. On this regard our result is similar to the one proposed by Santos et al. (2006), which shows how rewiring can provide a mechanism for the emergence of cooperation. However we provide the sociological micro-foundation that justify the rewiring (and its effect on cooperation) and we extend the study to signed networks.

We are currently designing the simulations to study the impact of the triadic balance mechanisms on cooperation. Furthermore we are exploring the emerging meso-level mechanisms that produce the results discussed. Our further analysis will focus in particular on two aspects of our model: (1) the effects on cooperation of the interaction between triadic balance and triadic closure for different levels of probability of these two mechanisms and (2) the implications, for the emergence of cooperation, of the existence (or the absence) of an explicit causal link between the strategy played by agents and network/relational signs updates (endogenous vs exogenous dynamics).

One potential limitation of our approach is that both the social interactions and the social mechanisms are fully local. A further direction of research that we intend to explore will address this limitation studying the impact of closure and balance in a context where the PD can also be played among non connected agents.

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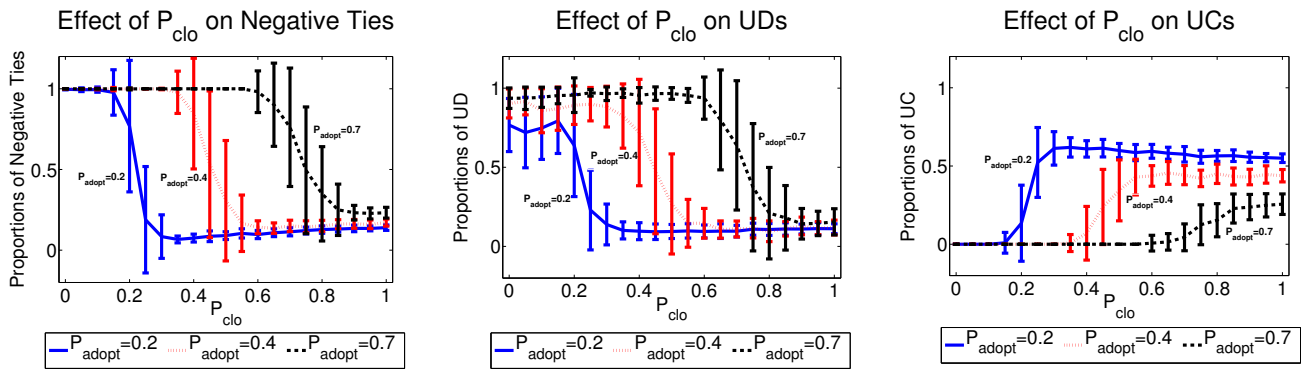Durrett, R. (2007). *Random graph dynamics*, volume 20. Cambridge university press.

Fig. 1: Effect of the *Triadic Closure* probability at different levels of probability of adoption of a better strategy (P_{adopt}). Results provided for $P_{adopt} = 0.2$ (solid blue line) for $P_{adopt} = 0.4$ (dotted red line) and for for $P_{adopt} = 0.7$ (dashed black line). Data for the final proportion of negative ties in the network (Left Panel), of UDs (Central Panel) and of UCs (Right Panel) are displayed. Each data-point represents the average of 50 simulations. For each simulations $N = 200$ and network signs are randomly initialized with equal probability. All populations are initialized as equally divided among the three agent types. The social networks are initialized as *random network* with each link existing with the independent probability 0.10.
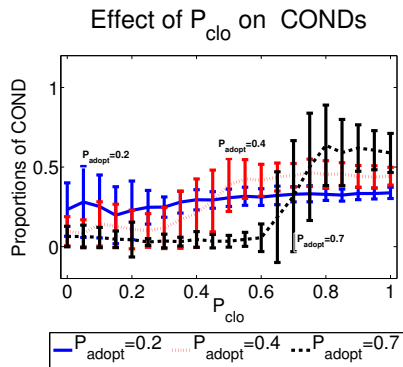

Fig. 2: Effect of the Triadic Closure probability on the proportion of CONDs, at different levels of probability of adoption of a better strategy (P_{adopt}). Results provided for $P_{adopt} = 0.2$ (solid blue line) for $P_{adopt} = 0.4$ (dotted red line) and for for $P_{adopt} = 0.7$ (dashed black line). Parameter values are the same as those in Figure 1.

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