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Bounded Confidence, Radical Groups, and Charismatic Leaders

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***Abstract*—By few simple extensions it is possible to model radical groups, charismatic leaders and processes of radicalization in the bounded confidence framework. In the resulting model we get a lot of surprising (non-)monotonicities. In certain regions of the parameter space more radicals or more 'charismaticity' may lead to less radicalisation.**

I. Introduction

There are some stylized facts about charismatic leaders, radical groups and processes of radicalisation. Among them we presumably find the following three: *First*, a radical group has compared to 'normal' agents a comparatively stable in-group consensus on an extreme opinion. *Second*, a charismatic leader counts for 'normal' agents that are under his/her influence much more than other 'normal' agents. *Third*, in a process of radicalisation people tend to get less and less open-minded. The three facts inspire an applied and modified version of the well known bounded confidence model as introduced by Hegselmann and Krause in [1]. The modified version is still an extremely simple conceptual model. Under some assumptions the whole parameter space can be analysed. The model shows some surprising results and mechanism that inspire new possible explanations, new perspectives for empirical studies, and new ideas for prevention policies.

II. Basics of the Bounded Confidence Model (BC-model)

The basic assumptions of the BC-model are:

- There is a set of n individuals; $i, j \in I$.
- Time is discrete; $t = 0, 1, 2, \dots$.
- Each individual starts with a certain opinion, given by a real number; $x_i(t_0) \in [0, 1]$.
- The profile of all opinions at time t is $X(t) = x_1(t), \dots, x_i(t), x_j(t), \dots, x_n(t)$.
- Each individual i takes into account only 'reasonable' others. Reasonable are those individuals j whose opinions are not too far away, i.e. for which $|x_i(t) - x_j(t)| \leq \epsilon$, where ϵ is the *confidence level* that determines the size of the *confidence interval*.
- The set of all others that i takes into account at time t is:

$$I\big(i, X(t)\big) = \{j \big| |x_i(t) - x_j(t)| \leq \epsilon\} \qquad (1)$$

- The individuals update their opinions. The next period's opinion of individual i is the average opinion of all those which i takes seriously:

$$x_i(t+1) = \frac{1}{\#\big(I\big(i, Xt)\big)\big)} \sum_{j \in I\left(i, X(t)\right)} x_j(t) \qquad (2)$$

III. Modifications of the BC-model

For the modified BC-model we now assume that there are *two* groups of agents: The first group, the *normals*, have opinions from the interval $[0, 1]$, they all have a positive $\epsilon^{normals} > 0$, and they update according to equation (2). The second group, the *radicals*, have all the opinion R, with R again from the unit interval, but more or less close to the upper bound, e.g. $R = 0.9$. The radicals' confidence level $\epsilon^{radicals}$ is constantly and homogeneously 0. Consequently, they update according to

$$x_i^{radicals}(t+1) = x_i^{radicals}(t) = R \qquad (3)$$

Figure 1 shows single runs with the same uniform start distribution for 50 normal agents with an $\epsilon^{normals} = 0.2$. In the left figure there are no radicals. Light grey vertical lines between *neighboring* opinions indicate that their distance is not greater than $\epsilon^{normals}$. In the figure in the centre a group of 5 radicals is added. Their opinion is $R = 0.9$. The black horizontal line is their trajectory. Dark grey vertical lines indicate the chain of direct or indirect (i.e. via a chain of others) influence of radicals on normals. In period 4 that chain breaks. The dark grey area indicates that part of the opinion space in which all normals, given the size of their confidence interval, are under the direct influence of the radicals. (The right figure will be explained below.)

Charismatic leaders can be covered by *reinterpretation*: We consider a radical group with m members as *one* person that counts m-times for all normal agents that have the charismatic leader within their confidence interval. Thereby, the radicals' group size m turns into a kind of *degree of charismaticity*. Assuming that R is the same in all cases, the conceptual model covers it all: a radical group, a charismatic leader, or any combination of both.

To include that in a process of radicalisation normals get *less and less open-minded*, requires a simple, but substantial modification of the original BC-model: We apply the BC

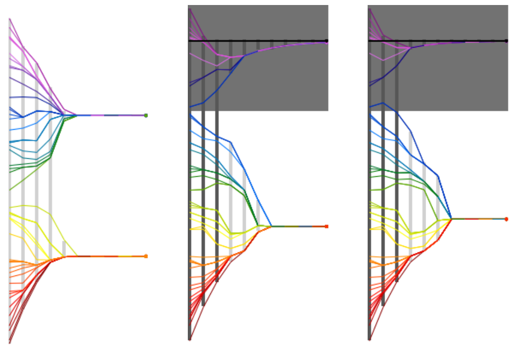

Fig. 1. 50 normals, the same start distribution of normals in all pictures, $\epsilon = 0.2, R = 0.9$. *Left*: no radicals. *Centre*: 5 radicals, no confidence dynamics. *Right*: 5 radicals, with confidence dynamics.

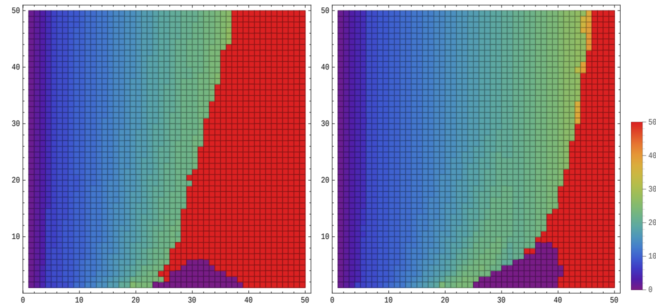

Fig. 2. x-axis: the confidence level increases in 50 steps of size 0.01 from 0.01 to 0.5. y-axis: the number of radicals increases from 1 to 50. Colours indicate the number of normals that end up at the radical position which is here assumed to be $R = 0.9$. The total number of normals is always 50. *Left*: without a confidence dynamics. *Right*: with a confidence dynamics.

mechanism, i.e. averaging over elements within one's confidence interval, on *both*, opinions *and* the confidence levels. That modification has no effect, if ϵ is the same for all agents. However, in our context radicals have an $\epsilon^{radicals} = 0$, while normals start with an $\epsilon^{normals} > 0$. As a consequence, the confidence level of normals will shrink if they are directly or indirectly influenced by radicals. Figure 1 right shows such a dynamics for the same start distribution as in figure 1 centre. Note that under this *confidence dynamics* (as we will call it in the following) one normal less ends up at the radical position.

A bit more formally: A confidence dynamics makes the confidence level $\epsilon^{normals}$ an individualised and time dependent $\epsilon_i^{normals}(t)$ and the set of agents j that i takes seriously changes from (1) to

$$I\big(i, X(t), \epsilon_i(t)\big) = \{j \big| |x_i(t) - x_j(t)| \leq \epsilon_i(t)\}. \tag{4}$$

Under a confidence dynamics normals, then, update according to *two* equations:

$$x_i^{normals}(t+1) = \frac{1}{\#\Big(I\big(i, X(t), \epsilon_i(t)\big)\Big)} \sum_{j \in I\big(i, X(t), \epsilon_i(t)\big)} x_j(t), \tag{5}$$

and, additionally, with regard to the confidence level by

$$\epsilon_i^{normals}(t+1) = \frac{1}{\#\Big(I\big(i, X(t), \epsilon_i(t)\big)\Big)} \sum_{j \in I\big(i, X(t), \epsilon_i(t)\big)} \epsilon_j(t). \tag{6}$$

The radicals stick to their confidence level 0 and 'update' accordingly:

$$\epsilon_i^{radicals}(t+1) = \epsilon_i^{radicals}(t) = 0. \tag{7}$$

(2) together with (3) defines a system *without* a confidence dynamics. (3) together with (5) to (7) is the corresponding systems *with* a confidence dynamics of normals.[1]

[1]Note that in the model the radicals or a charismatic leader, respectively, are simply *given*. Baurmann, Betz and Cramm present in [2] a model in which charismatic opinion leaders can *evolve*.

IV. First Results

There are many interesting questions about final results, e.g. the number or frequency of normals that finally end up at the radical position R, the mean and median of the normals' opinions, or – as one of several possible distance measures – something like a root mean square deviation of the normals' opinions with regard to R. It is very natural to think that final results crucially depend upon the number of radicals (or the degree of charismaticity, respectively) compared to the number of normals, the confidence level at $t = 0$, and the radical Position R. All the questions mentioned above, can be answered by systematic simulations that cover the whole parameter space—admittedly, under some simplifying assumptions.

A first step in this direction is documented in *figure* 2. The figures show, indicated by color, the number of normals that finally end up at the radical position R, which in this example is assumed to be $R = 0.9$. To the left are the results without, to the right the results with a confidence dynamics. The x-axis gives the value of the normals' confidence level at $t = 0$. It is the same for all normals. In 50 steps of size 0.01 the confidence level $\epsilon^{normals}$ increases from 0.01 to 0.5. The y-axis gives the number of radicals (or the degree of charismaticity, respectively). In 50 steps that number increases from 1 to 50. As to the normals we always assume 50 agents with opinions all over the opinion space. For each step combination x, y we run simulations until the dynamics is (almost) stable. Then we count the number of normals that (almost) ended up at the radical position R. *Figure* 2 shows, indicated by colors, the number of normals that finally end up at the radical position.

There are at least *three lessons* that we can take away from *figure* 2:

1) With or without a confidence dynamics, for a certain range of confidence levels an *increasing* number of radicals leads to *less* radicalisation in the sense that less normals end up at the radical position R. Thus, for a certain range of confidence levels radicalisation is mildly though clearly *monotonically decreasing* with respect to the number of radicals.

2) In another range of confidence levels, again with or without a confidence dynamics, it holds: With only a few radicals *no* normal ends up with the radical position R. Above a certain number of radicals suddenly *all* normals end up with the radical position. But if we further increase the number of radicals, then, suddenly, we get again *less* radicalisation. Thus, in that range of confidence levels radicalisation is *not monotonic* with respect to the number of radicals.
3) For any number of radicals, *with* a confidence dynamics the sudden transition to a state in which all normals end up radical occurs, compared with the dynamics without a confidence dynamics, only for much bigger confidence levels. In general: The confidence dynamics lets the confidence level of at least some normals shrink. But that leads to *comparatively less* radicalisation.

Obviously, in our model works a complicated and sometimes counteracting interplay of increasing number of radicals and/or an increasing size of the confidence level. As a consequence we get some *counterintuitive (non-)monotonicities*. All the effects can be explained by an analysis of single runs.

V. Next Steps

The simulations documented in *figure* 2 show, indicated by color, the number of normals that finally end up at the radical opinion, which was assumed to be $R = 0.9$. To get a complete overview we will run simulations for $R = 1.0, 0.99, \ldots, 0.5$. That will be done for both, without and with a confidence dynamics. The results will be visualised by two animations of 51 pictures of the type used in *figure* 2. There is much more that can be analysed and visualised in the same style: for instance, the mean or the median of the normals' opinions, the final cluster structure and their distances to the radical position.

However, there are two major *caveats* with regard to the preliminary results: *First*, it has to be checked, whether or not the results crucially depend upon the ratio *or* the absolute numbers of normals and radicals. And, *second*, a kind of confession: The results in *figure* 2 are *not* based on repeated runs with random start distributions for each x, y-combination. They are based on just *one* run for each combination. In *all* runs the same very special, but in a certain sense 'typical' start distribution of n normals is used: An opinion profile is an *ordered* profile iff for all $i \leq (n-1)$ it holds that $x_i(0) \leq x_{i+1}(0)$. In all runs of *figure* 2 we use an ordered start profile in which the i^{th} normal opinion is $i/(n+1)$. In such an ordered and equidistant start profile the i^{th} opinion is exactly there where it will be at the average over infinitely repeated uniform *random* distributions. On the one side, that distribution is therefore very 'typical'. On the other side, the regular structure of the profile may make us blind for important effects that are caused by the typical density fluctuations of single random distributions. Whether or not the use of our start distribution is a problem or a solution of many problems has still to be checked.

Acknowledgment

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