More for Less: on Consumer Rationality and Bargaining Power on Telecommunication Markets

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***Abstract*—We analyse the large scale agent-based model of a prepaid telecommunication market with oligopolistic competition, heterogeneous calling patterns and different levels of agent rationality. We apply innovative implementation approach of utilizing high performance CUDA computing devices which allows us to consider population of up to 1 million consumers.**

We measure influence of a call graph structure, intra-family network choice coordination and agent rationality level on the market equilibrium. We discover that boundedly rational subscribers, who exploit simple decision heuristics to coordinate network choice within closed user groups, exert much stronger pressure on suppliers than fully rational ones. This leads to lower average calling costs, increased welfare and decreased monopolistic power of operators. We also observe asymmetry in operator margins and volume of on-net and off-net calls in accordance with empirical facts.

I. Introduction

THE breakup of national monopolies and liberalization of telecommunication markets in recent decades revived interest in economics of the sector. The seminal publications of Armstrong [1998] and Laffont et al. [1998a,b] became a cornerstone of the ongoing debate on nature of competition in the industry. The original model, subsequently known as the A-LRT framework, analysed a duopoly market with Bertrand style price competition and Hotelling-like network differentiation. In order to preserve its analytical tractability the authors made many simplifying assumptions, in particular they assumed a fully connected and uniform call graph between subscribers. Consequently consumers used operator's market share as a proxy for proportion of their peers using the network when considering which network to join.

It was soon noticed that the model failed to explain numerous stylized facts such as large on-net/off-net price differentials, high interconnection rates and asymmetry between on-net and off-net call volumes in equilibrium.[1] This motivated many researchers to extend it. One of the research threads focused on incorporating network heterogeneity into the framework. Some simple models of heterogeneity were proposed. Cherdron [2001] developed a model with two user groups and a subscriber calling pattern biased towards one of the groups. Dessein [2003, 2004] analysed models with call volume heterogeneity by dividing subscriber population into light and heavy users. In a similar manner Gabrielsen and Vagstad [2003] proposed a model with high and low demand user groups. These ideas were extended into a more general framework by Hahn [2004]. Gabrielsen and Vagstad [2008] introduced an idea of "calling clubs" the members of which call each other more often than the general population. More recently Hoernig et al. [2011] proposed a model of "calling circles" with non-uniform, concentrated calling patterns correlated with subscriber's network preferences. In contrast to the earlier model of "calling clubs" they allowed the circles to overlap. Finally to bridge a gap between theory and reality Harrison et al. [2009a,b] introduced a multiagent approach by explicitly simulating a call graph as a regular random graph.

All these papers however share the common simplifying assumption – a "representative" consumer owns a single telecommunication service and consumers do not coordinate their network choices. Clearly, this assumption is not met in practice. Families and companies can own more than one service and if they do they coordinate operator choices for their services as shown by Birke and Swann [2006, 2010]. Likewise Armstrong and Wright [2009] hypothesize that existence of "closed user groups" coordinating network membership choice may explain observed bias towards on-net calls and encourages on-net/off-net price discrimination by operators.

Modelling this kind of coordinated behaviour combined with network heterogeneity is a methodological challenge for purely mathematical approach. To overcome this limitation Kamiński and Łatek [2010] proposed a multiagent model implementing a concept of a *customer* – a network choice decision maker comprising multiple services. Alas, their results were not directly comparable to work of Laffont et al. [1998b]. Then Kamiński [2012a,b] proposed a multiagent model generalizing the model of Laffont et al. [1998b] beyond the duopoly case and replicating it as a special case. His work took into account call graph heterogeneity and coordination of subscriber choices within family-like groups. He operated on a relatively small population of 4096 fully-rational subscribers.

In this paper we extend the approach of Kamiński [2012a] to consider 2 classes of customers – fully and boundedly rational. Additionally we subdivide the latter class by using 2 different heuristics for network choice coordination among cooperating subscribers. Furthermore we introduce high performance CUDA computing technology to operate on a much larger

[1] see Harbord and Pagnozzi [2010, section 3.2] for recent empirical evidence from selected European markets

agent population (up to 1 million). This innovative approach allows us to: *(i)* take precise measurements for oligopoly markets with 3 and 4 operators which proven to be problematic with small agent population proposed by Kamiński [2012b]; *(ii)* compare the outcomes for fully rational subscribers with outcomes for subscribers characterized by bounded rationality; *(iii)* contribute to the research on bounded rationality by providing another example of *"less-is-more"* effect when simple heuristic decision rules prove to be superior to fully rational decision making.

The rest of the paper is organized as follows. In section II we describe the model used for the simulation and discuss the rationale for introducing the novel element of bounded rationality. In section III we describe simulation setup and present the results. We discuss the results in section IV. Section V concludes.

II. Model Description

In this section we describe the behaviour of agents (customers, subscribers and operators), method of generation of a call graph and subscriber families in the model. The specification strictly replicates model of Laffont et al. [1998b] for linear discriminatory pricing with extensions proposed by Kamiński [2012b] allowing for customer heterogeneity and oligopolistic competition. In the presentation of the base framework we follow description presented in Kamiński [2012a] with necessary additions and modifications.

Consider a telecommunication market on which there are K operators and N subscribers. A *subscriber* is a single calling contract with the operator. Each subscriber is owned and managed by a *customer* and a customer comprises one or more subscribers. Depending on context we may also call a subscriber – a *service*, and a customer – a *family*.[2] Subscribers call each other with non-uniform intensity – every subscriber maintains her list of "contacts" which is a subset of the subscriber population. The ensemble of subscribers' contacts constitutes the social communication network or a *call graph*.

The market is assumed to follow calling party pays regime and operators can discriminate between on-net and off-net calls. The pricing is linear i.e. there is no fixed fee for network membership and no "free minutes" allowance. There is also no switching costs for changing the operator. The market is mature – thus we take that the set of operators, customers, subscribers and a call graph remain constant.

Every operator $k \in \{1, ..., K\}$ sets a price for a unit of call time inside his network on_k and outside his network off_k. Let $s(i) \in \{1, ..., K\}$ be the operator chosen for subscriber i by a customer who owns it.

A. Subscriber and Customer behaviour

Subscriber call intensity depends on a call price. Let w_{ij} be the call intensity from subscriber i to subscriber j given the calling price equals 1 and q_{ij} be proportional change of this volume given actual market price. To determine the price paid by subscriber i calling subscriber j we have to check if they use the same operator. The price formula is defined as

$$p_{ij} = on_{s(i)}[s(i) = s(j)] + \mathit{off}_{s(i)}[s(i) = s(j)],$$

where expression $[\ell]$ evaluates to 1 if ℓ is true and to 0 if it is false. The rule states that if subscribers i and j belong to the same operator then on-net price is charged and otherwise off-net price is used.

Extending Laffont et al. [1998b] we define a subscriber's net surplus as

$$V_i\big(s(i)\big) = \sum_{j=1}^{N} w_{ij}\left(\frac{q_{ij}^{1-1/\eta}}{1-1/\eta} - q_{ij}p_{ij}\right) - \frac{X^i_{s(i)}\sum_{j=1}^{N} w_{ij}}{2\sigma},$$

where $\eta > 1$ is price demand elasticity, $X^i_{s(i)} \in [0,1]$ is measure of subscriber i preference towards operator $s(i)$ and $\sigma > 0$ is a measure of strength of this preference. We assume that subscriber preferences are independently uniformly distributed $\mathbf{X}^i = (X^i_1, \ldots, X^i_K) \in [0,1]^K$. In comparison to formula given by Laffont et al. [1998b] we need to add a normalizing factor $\sum_{j=1}^{N} w_{ij}$ to reflect the fact that a call graph need not be fully connected and uniformly weighted.

The above equation can be solved for optimal call volume by a subscriber given operator prices. A utility maximizing subscriber will set $q_{ij} = p_{ij}^{-\eta}$ and will obtain surplus

$$V_i\big(s(i)\big) = \sum_{j=1}^{N} w_{ij}\frac{p_{ij}^{1-\eta}}{\eta - 1} - \frac{X^i_{s(i)}\sum_{j=1}^{N} w_{ij}}{2\sigma}.$$

Using this formula a subscriber can choose operator $s(i)$ so as to maximize the surplus. This is the standard analytical approach of Laffont et al. [1998b].

1) Fully rational customer: We extend the A-LRT approach and assume that a customer selects the *globally* optimal allocation of a family of subscribers to operators. Let $F \subset \{1, \ldots, N\}$ be a list of subscribers belonging to a single customer. Then a customer chooses the subscriber allocation $\vec{a} = \big(\forall i \in F : s(i)\big)$ that maximizes the aggregated family surplus

$$V_F(\vec{a}) = \sum_{i \in F} V_i\big(s(i)\big)$$

The optimal solution to the above allocation problem can be different from individual subscriber optimization because it allows a customer to *coordinate* operator choice among subscribers.[3] For example (as found in empirical data and later results of the model) one can expect that $on_k < \mathit{off}_k$. In such a case a customer may be better off by allocating all the subscribers to one operator to benefit from cheaper on-net

[2] to reflect that it corresponds to a "*friends & family*" circle in telecommunication jargon

[3] The fact of coordination within closed user groups is supported by empirical data – for example Birke and Swann [2006] find out in the examined sample that 68% of households with 2 mobile subscriptions use the same network operator

Algorithm 1 Find best subscriber allocation $\vec{a} \in \{1, .., K\}^{|F|}$

```
bestUtility ← (−∞)
for all x⃗ ∈ {1,..,K}^|F| do
  if V_F(x⃗) > bestUtility then
    bestUtility ← V_F(x⃗)
    a⃗ ← x⃗
    tieCounter ← 1
  else if V_F(x⃗) = bestUtility then
    tieCounter++
    if 1/tieCounter > runif(0,1) then
      a⃗ ← x⃗
    end if
  end if
end for
return a⃗
```

calls within a family circle.[4]

Please note that in the model customers choose operators non-strategically and take operator prices as given. Algorithm 1 describes details of the implementation.

2) Boundedly rational customer: One may argue that allocating services in a globally optimal way is too strong an assumption. Indeed, the number of possible allocations a fully-rational customer has to consider grows exponentially with family size as $K^{|F|}$. With 4 operators and 5 family members a total of 1024 allocations has to be considered. Computational burden with even larger families could be unbearable for a flesh & blood agent. Therefore we consider alternative scenarios with nearly-rational customers who optimize within a bounded set of possibilities and resort to a simple "*take-the-best*" heuristics in allocating their subscribers. These are:

1) *single operator optimization*: allocate the entire pool of subscribers to one of the operators or keep the current allocation – whichever best, where the initial allocation is randomly selected (see Algorithm 2);
2) *initial individual optimization*: maximize subscriber utilities individually taking other family member choices as given; repeat the procedure 3 times to allow for adjustment for other family members choices; then proceed as in Algorithm 2.

The constraints of human mind in processing large amounts of information laid foundations under the theory of bounded rationality [Simon, 1955]. Today there is abundance of evidence that individuals retreat to simplified reasoning and heuristic decision making when confronted with an overwhelmingly complex decision problem [Gigerenzer and Selten, 2002]. In context of telecommunications this is traditionally being related to plethora of available tariffs and their

[4]Obviously this type of uniform allocation needs not be globally optimal in every case as it depends on proportion of intra-family to extra-family call volume. In general finding the globally optimal subscriber allocation in case of a heterogeneous call graph is a non-trivial task and its completion as such can be guaranteed by the "brute-force" evaluation of *every* possible allocation. This is the approach taken in this work for fully rational customers – despite the obvious disadvantage of computational complexity of $O(K^{|F|})$.

Algorithm 2 Find best subscriber allocation $\vec{a}$: single operator optimization (heuristics 1)

```
a⃗ ← x⃗ ← {∀i ∈ F : s(i)} (initial subscriber allocation)
for k = 1 to K do
  for all i ∈ F do
    x_i ← k
  end for
  if V_F(x⃗) > V_F(a⃗) then
    a⃗ ← x⃗
    tieCounter ← 1
  else if V_F(x⃗) = V_F(a⃗) then
    tieCounter++
    if 1/tieCounter > runif(0,1) then
      a⃗ ← x⃗
    end if
  end if
end for
return a⃗
```

structural complexity. Bolle and Heimel [2005] observed that even intellectually sophisticated mobile phone users (university students) base their network membership decision on simple comparison of absolute levels of on-net/off-net price vectors failing to weigh properly proportion of on-net to off-net calls. This "fallacy of dominant price vectors" as they call it obviously leads to sub-optimal choice and higher average calling costs. The result was confirmed on an independent sample of students and faculty staff by Haucap and Heimeshoff [2011], who call the phenomenon a "price differentiation bias". Barth and Graf [2012] came to a similar conclusion in a discrete choice experiment. In a recent empirical study based on a large dataset from China Telecom Miao and Jayakar [2014] reported a vast majority of consumers to make non-optimal selection from the menu of available tariffs. They also found out probability of non-optimal selection to increase with complexity of a tariff plan. Likewise Lambrecht and Skiera [2006], Mitomo et al. [2009] and Gerpott [2009] observed a "flat rate bias" – propensity of consumers to overestimate savings provided by "unlimited" tariffs even if measured usage-based rates would result in lower average cost.

More generic evidence comes from extensive literature on bounded rationality. We consider studies of chess players to be particularity relevant to the subscriber coordination problem. The task of allocating services of a large family, say with 10 subscribers, to numerous available networks resembles the problem of a chess player in terms of immense quantity of available strategies. Chess player reasoning was studied by De Groot [1965] and later by Simon [1972] and Chase and Simon [1973a,b]. De Groot discovered that both beginners and master players apply similar simplified decision procedures (heuristics). At each move they analyse only a small subset of available strategies – no more than a dozen or so. The secret of course is to pick the right subset for analysis and this is where masters "master" as they know, through experience and learning, how to select the most promising ones.

Although the decision heuristics designed for our experiment may seem simplistic at first they follow the rules outlined by master chess players. It is easily seen that some allocation strategies, namely uniform allocation of services to one of the operators, bear higher potential for substantial gain in utility than any seemingly "random" strategy. When time or computational capacity is scarce it is best to focus on the most promising alternatives – as master chess players do. Such simplified optimization approach is also justified by anecdotal evidence – many if not most large organizations equip their staff with mobile phones from a single operator, although corporate contracts are somewhat more complex than a simple prepaid market model analysed here.

The second heuristics extends the first one by allowing subscribers to individually optimize their network choices prior attempting to assign all the family services to one operator. The rationale behind such approach is that a single operator optimization (heuristic rule 1) is beneficial mainly for large families. Firstly, the effort of fully rational allocation is huge for such families due to combinatorial explosion of the search space so heuristic decision making provides dramatic savings in this respect. Secondly, the larger a family gets the more likely volume of intra-family calls outweighs volume of extra-family calls. Small families on the other hand may be better off by *not* coordinating their choices and optimizing individually instead. They are also more likely to encounter a Pareto-optimal allocation within the 3-iteration adjustment schema we allowed for as size of their decision set is much smaller. Simply speaking the heuristic rule 2 allows small families to benefit from individual optimization and large families from a single-operator optimization at almost no computational overhead.

In the end it is worth noting that we model both rational and nearly-rational customers as having passive expectations i.e. they assume their neighbourhood to remain static when making network allocation decision for their subscribers. In fact the environment is not static as all the other customers make their subscriber allocation decisions at the same time and based on the same information set. Therefore we repeat allocation procedures for the entire customer population until the stable subscriber allocation is reached or changes are of simple cyclical nature i.e. no subscriber changes network allocation or some subscribers switch back and forth between networks in stable patterns.

B. Operator behaviour

Take that a_c is cost of initiating the call, b_c is cost of terminating the call and i_c is interconnection fee (paid by a call initiating operator to a terminating operator). By f_c we denote fixed cost of maintaining a subscriber by a chosen network operator. Under such notation operator k calculates profit π_k using formula (again it is a direct extension of Laffont et al. [1998b]):

$$\pi_k(on_k, \mathit{off}_k) = (on_k - a_c - b_c) on_k^{-\eta} \sum_{s(i)=k=s(j)} w_{ij} +$$

$$+(\mathit{off}_k - a_c - i_c)\mathit{off}_k^{-\eta} \sum_{s(i)=k\neq s(j)} w_{ij} +$$

$$+ \sum_{s(i)\neq k=s(j)} (i_c - b_c)\mathit{off}_k^{-\eta} w_{ij} +$$

$$-\{i : s(i) = k\} f_c \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{w_{ij}}{N}.$$

The formula comprises four terms taking into account optimal call intensity given prices ($q = p^{-\eta}$). In the first term we calculate operator profit from on-net calls. The second and third terms represent profits from outgoing and incoming off-net calls respectively. The last term counts fixed costs of customer maintenance, for example customer service and billing cost. Similarly to parameter σ in order to assure its consistency with the reference model it has to be normalized.

C. Equilibrium Prices

We compute equilibrium prices for both the reference A-LRT model and for its multiagent counterpart. Note that Laffont et al. [1998b] provide only implicit formula for equilibrium, so it is not possible to calculate the reference prices directly. Therefore following Kamiński [2012b] we applied an adaptation of numerical procedure proposed by Krawczyk and Zuccollo [2006]. We start from prices equal to marginal cost. In each step of the simulation operators find the best response prices in the neighbourhood of current prices and next they update their prices by moving halfway from current prices towards the best prices. In each iteration we narrow the neighbourhood radius. The procedure is repeated 2^{13} times – we confirmed experimentally that it is sufficient for obtaining accuracy of results in order of 10^{-3}.

Similar approach is taken for obtaining the multiagent model equilibrium prices. We start from prices in proximity of the reference A-LRT prices and with a random allocation of subscribers to networks. In each simulation step the elected operator finds the best response prices by testing if it is profitable to change its on-net and off-net prices by ± 0.01 or leave them unchanged. Then the new pricing is announced, other operators adjust their prices and consumer adjust their network membership choices. The procedure is repeated until the stationary state is reached i.e. the current prices being the best prices and stable allocation of subscribers to networks.

We assume operators to "anticipate" how customers will react to changed prices. This is achieved by an operator recursively sub-simulating the market response for new pricing as proposed by Łatek et al. [2009]. Because the multiagent simulation is non-deterministic there is a natural tendency towards instability of the optimal decisions. In order to pinpoint approximate equilibrium in each step prices are rounded to precision of 0.01. If optimal responses do not deviate more than this value from current prices then current prices are taken as the equilibrium approximation.

D. Call Graph Generation

We assume a call graph to have small-world property for consistency with structure of empirical telecommunication networks [Onnela et al., 2007a,b]. We generate it using Watts and Strogatz [1998] algorithm adapted to handle directed graphs. Graph generation is started from ring lattice with neighbourhood radius given as a parameter, next edges are rewired with defined probability. Because we assume the graph is directed, rewiring is done separately for each direction of initial connection between services.

A complementary part of a call graph generation is assignment of services to customers (families). We define parameter $family$ denoting maximal family size. Each family has random size that is drawn from Zipf distribution truncated at the parameter value level. In this way smaller families are more probable than large ones. Additionally we make services belonging to the same family a clique in a call graph which accounts for the fact that services belonging to the same family tend to call each other.

III. Simulation Results

In this section we first we describe the setup of the simulation experiment and then we present the results obtained.

A. Experiment Setup

The parameter range for the simulation is given in Table I. We follow recommendations of Kamiński [2012a] with minute modifications. We take a call termination cost $b_c = 1.0$ as the numéraire so that all the remaining parameters are expressed relative to it. It is natural to assume parameters a_c and i_c to be slightly larger than b_c and therefore we set their range to $[1.00; 1.50]$ and $[1.00; 1.75]$ respectively. The choice of the network substitutability parameter σ follows recommendations of Laffont et al. [1998b] where it is required not to be too large in order to ensure existence of a stable shared-market equilibrium. On the other hand if it was too small then the non-pricing component would dominate subscriber's utility which would be unrealistic. In order to balance these two effects and following Harrison et al. [2009a] and De Bijl and Peitz [2002] the range of σ was set to $[1.00; 2.00]$. In a similar manner following Laffont et al. [1998b] the value of elasticity parameter η should be greater than 1. On the other hand Ingraham and Sidak [2004] report that price elasticity of demand on telecommunication markets is not high, hence the choice of parameter range is $[1.25; 1.75]$. Next it is natural to assume that f_c should not be large. It is normalized not to exceed 10% of average customer calling costs when $on_k = off_k = 1$. Network radius parameter range was chosen following data from Wojewnik et al. [2011] and a graph rewire probability parameter spans full range of admissible values. The maximum family size parameter spans from 1 to 5 and the number of operators from 2 to 4.

We extend the span of parameter $family$ for boundedly rational customers to measure impact of family size more precisely. We put the cap of 5 on the parameter for fully rational customers due to computational complexity constraints (see section II-A2 for the discussion). Consequently we extend the span of parameter $radius$ to measure potential influence of interactions between the two.

TABLE I
Simulation Parameter Value Range

parameter	value (range)	parameter	value (range)
σ	$[1.0, 2.0]$	f_c	$[0.0, 0.1]$
η	$[1.25, 1.75]$	$rewire$	$[0.0, 1.0]$
b_c	1.0	N_{LRT2}	102400
a_c	$[1.0, 1.5]$	N_{EXT34}	531441
i_c	$[1.0, 1.75]$		
Fully rational		Boundedly rational	
$radius$	$\{3, 4, 5\}$	$radius$	$\{3, \ldots, 10\}$
$family$	$\{1, \ldots, 5\}$	$family$	$\{1, \ldots, 10\}$

B. Simulation Docking

We performed simulation docking to verify if it replicated the A-LRT model when the equivalent parametrization was used. For this purpose we used a fully connected subscriber graph with equal connection weights and single member families i.e. subscribers calling each other with uniform intensity and no intra-family coordination. We examined 4 different subscriber population sizes (15k, 100k, 500k and 1mln) and 3 different methods of probing subscriber preference space – equidistant grid, pseudo- and quasi-random. Mersenne-Twister and randomized Sobol were used as pseudo- and quasi-randomness sources respectively. We randomly selected 100 simulation parameter sets and for each set performed 32 simulation runs per model, per population size, per preference space probing method. The results of the docking exercise are summarized in Table II. Models are coded using 5 alphanumeric characters schema as follows:

- prefix LRT2 marks the standard duopoly model of Laffont et al. [1998b] with unit interval uniform preferences;
- models prefixed EXTn where n is number of operators are extended oligopoly models as proposed by Kamiński [2012b] with subscriber preferences independently uniformly distributed on hypercube $[0, 1]^K$;
- suffixes {S,R,Q} stand for equidistant grid, pseudo-random and quasi-random preference space probing respectively.

Columns *don* and *doff* are mean deviations from the numerically computed theoretical A-LRT equilibrium prices (on-net and off-net respectively). Columns *sdon* and *sdoff* are mean standard deviations of these values. Column *onshare* is mean share of on-net calls, the standard deviation of this value is 0 in all cases therefore it is not shown in the table.

The reader may notice that equidistant grid preferences (type S) delivered the most accurate results for the A-LRT equivalent setup (LRT2). The accuracy however drastically deteriorated for an extended Kamiński [2012b] model specification. The reason was that equidistant probing of multidimensional preference space caused heavy discretization of the model response surface.[5] This resulted in large bias. For the extended specification pseudo-random (R) and quasi-random (Q) probing worked much better and with comparable accuracy. One may also see that accuracy of models with 500k

[5] the problem known as "*curse of dimensionality*" in numerical analysis

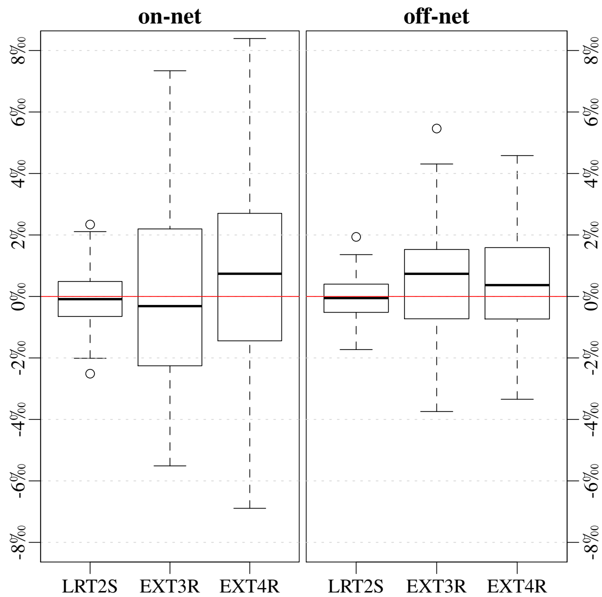

Fig. 1. Simulated to theoretical price deviations for the A-LRT equivalent configuration

and 1mln subscribers is very close. It was natural therefore to choose the smaller population for further analysis as having lower computational capacity requirements.

We used accuracy of docking results to select models for further analysis and the actual simulation run. We have arbitrarily chosen the subscriber population sizes to be $N_{LRT2} = 320^2 = 102400$ for the duopoly A-LRT equivalent setup and $N_{EXT34} = 81^3 = 729^3 = 531441$ for the extended triopoly and 4-network oligopoly as providing the best accuracy to performance ratios. For the preference space probing we have chosen uniform grid (S) and pseudo-random preferences (R) for the duopoly and oligopoly models respectively. The quasi-random approach (Q) delivered slightly lower variance of the results but we opted for pseudo-randomness as more reliable for standard error estimates.

Fig. 1 shows simulated to theoretical price deviations of the selected models for the A-LRT equivalent configuration (averaged observations per point). One may see the multiagent models reproducing theoretical prices with high accuracy – deviating by no more than 0.9% and 0.5% for on-net and off-net prices respectively in case of model EXT4R. The small discrepancies came from two sources: (*a*) rounding errors (we used 4 and 2 decimal places for computing the theoretical A-LRT and multiagent equilibrium prices respectively) and (*b*) stochastic nature of discrete subscriber preferences in the multiagent setup for the oligopoly models (EXT3R, EXT4R).

C. Results

The results of the multiagent simulation were obtained using 10131 random parametrizations for the duopoly (LRT2) model and 7745 random parametrizations for the triopoly (EXT3) and 4-network oligopoly (EXT4) models. For each parametrization there were in average 32 simulation runs for the duopoly model and 19 simulation runs for the oligopoly models. For each run of EXT3 and EXT4 models a new set of random preferences was drawn. Additionally a new random call and family graphs were generated every eight run. In total 712041 simulation runs were performed.

We applied linear regression metamodeling approach to ensure simple interpretation of the results. Even though the relationship between parameters and equilibrium prices is in fact non-linear, within the chosen paremeter range such approximation is acceptable as suggested by R^2 values of the regression model close to 1. The regression results for the average calling cost are presented in tables III and IV. Network choice coordination classes are coded as "FamilyCN" where C denotes fully rational (F), heuristic type 1 (H) and type 2 (G) coordination procedures respectively, and $N \in \{1, \ldots, 10\}$ denotes a maximum family size.

Fig. 2 shows, *ceteris paribus*, average on-net and off-net margin differentials between the reference A-LRT model and the simulated multiagent models with network heterogeneity and different levels of agent rationality for the duopoly market. The theoretical prediction of Laffont et al. [1998b] is that in equilibrium on-net and off-net profit margins are equal. As can be seen simulated operator margins deviate significantly from the theoretical results. When no customer coordination is taken into account $(family = 1)$ the simulated on-net and off-net margins are in average *higher* by 5 and 1 percentage points respectively. This represents the pure effect of the network heterogeneity and confirms that for non-uniform call graphs tariff mediated network effects are internalized by operators in form of higher profits. This experimental result is in accordance with recent theoretical predictions of Hoernig et al. [2011] and is independent of the number of competing operators as seen on Fig. 2, 5 and 6.

Customer *coordination effect* counteracts the network heterogeneity effect and quickly outweighs it as the family size grows. In the duopoly setup with large families the fall in on-net margins is stronger than rise in off-net margins. Overall the average calling costs are decreased as seen on Fig. 3. The coordination effect is strongest for the duopoly market and fades as the number of networks grows (Fig. 5 and 6). This is intuitively understandable – with more operators, peer services are more dispersed among them and coordination becomes more intricate. Also operator margins go down due to intensified competition so there is less room for price adjustment anyhow. For oligopoly markets with 3 and 4 operators and fully rational customers the fall in on-net margins is just about enough to offset the rise in off-net margins. This causes costs to remain constant regardless of a family size.

Quite unexpectedly the fall in margins and average calling costs is dramatically larger in case of boundedly rational customers as compared to fully rational ones. Moreover in this case both on-net and off-net margins go down *simultaneously*. One may see for example that in case of a duopoly market with up to 5 member families fall in on-net margin is more than 2 and 3 times deeper for heuristic rule 1 and 2 respectively as compared to fully rational case (Fig. 2). Again the magnitude

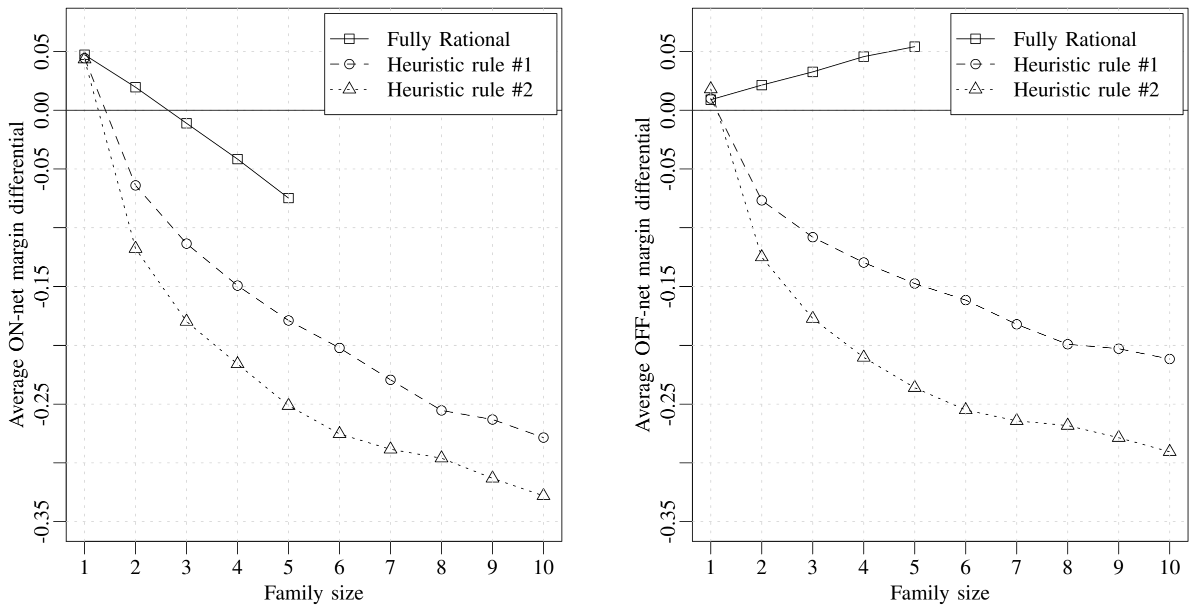

Fig. 2. Simulated to theoretical margin differentials for the duopoly model (LRT2S)

of the effect fades as number of operators grows for the same reason as above, but its direction remains the same for both on-net and off-net prices.

Another interesting simulation outcome is bias towards on-net calls. In the A-LRT framework in equilibrium the share of on-net calls equals the market share of the operator. This is at odds with empirical data as on-net volume share observed on real markets is much higher [Harbord and Pagnozzi, 2010]. As seen on Fig. 4 this effect emerges from the multiagent model. Again we see impact of the heterogeneity effect (when $family = 1$) and the coordination effect – on-net volume share grows as the family size grows. Interestingly the differential to the A-LRT model gets larger as number of operators grows as seen on Fig. 5 and 6. As previously the effect is much stronger for boundedly rational customers as compared to fully rational ones.

Finally it is worth pointing out that the effect of the interconnection charge (i_c) on the average consumer cost is systematically larger in the multiagent setup than in the A-LRT equivalent setup, despite monopolistic power of operators in the former being weakened by the coordination effect. This shows that models ignoring network graph heterogeneity underestimate influence of this parameter on market prices.

IV. Discussion

Both the magnitude and the direction of influence of agent rationality level on the market equilibrium came as a surprise. Conventional wisdom perceives boundedly rational behaviour as inferior substitute to fully rational one and expects it to deliver results that are second-order worse at minimum. Moreover one would expect that in a large population small individual level deviations from Pareto-optimal choices (i.e. nearly-rational behaviour) cancel each other out and do not cause significant distortion of the aggregated outcome.

At least two lines of research questioned such reasoning. Gigerenzer [2004] provides examples of heuristic decision making leading to results superior to fully rational one – the "*less-is-more*" effect. Akerlof and Yellen [1985] show how seemingly insignificant second-order deviations from perfect rationality cause unexpected first-order systemic effects. Contributing to this line of thinking the results of our model provide an example of less mental effort leading to the more desirable outcome – increased welfare of agents. They also show how nearly-optimal behaviour may lead to substantial deflection from the theoretical results.

One should however be careful in drawing generalized conclusions. In an attempt to explain the phenomenon we need to consider whether it is an artifact of the model, the emergent property of the system being modelled or maybe both. We argue that the latter is the case. Part of the explanation lies in the way we model subscriber preferences. We inherit the A-LRT approach with preferences uniformly distributed on a generalized unit interval $[0, 1]^K$. Notice that when we bundle i.i.d. subscribers into customers and as the family size grows the aggregated customer preferences approach the normal distribution and concentrate around the marginal subscriber. Hence the population of customers (who are decision makers in our model) is more indifferent between operators than population of subscribers. As a result the competition between operators intensifies and prices go down. This effect is equivalent to increasing value of σ – the substitutability parameter. The above is not the complete rationale however. If it were there would not be visible discrepancies between fully-rational and boundedly rational outcomes as the σ-effect occurs for *any* type of subscriber coordination.

Strong network effects seem to be the hidden driving force.

A single subscriber switching operators may trigger a cascade of followers who find it beneficial to do the same in the modified network constellation. This ignites a chain-like reaction in the entire population. This induced churn effect is at work for both fully and boundedly rational setups however it is orders of magnitude stronger in the latter case. To see why consider that in the fully rational setup families will most likely *not* choose uniform network membership while in the boundedly rational setup they will most likely choose it. Than in the latter case entire subscriber families (customers) switch operators as opposed to mostly subscriber level switches in the former case. Let us remind here that customers in the model choose networks non-strategically i.e. they take the environment as given and do not anticipate far-reaching consequences of their decision on the systemic level. Paradoxically this causes fully rational customers to overlook potential gains in utility reaped by boundedly rational agents as a cumulative side effect of their simplified reasoning.

Another considerable fact is divergence between outcomes of the two apparently similar heuristic rules. It was already explained in section II-A2 but we may repeat for clarity that the heuristic rule 1 is beneficial mostly for large families. Small families do better by *not* coordinating on a single network as volume of their intra-family calls is unlikely to outweigh volume of extra-family calls. Such families are better off when subscribers make network choices individually. Heuristic rule 2 allows for such diversity of choice and hence fits the environment better. Consequently it leads to implicit market segmentation as small and large families act differently. Overall customers are able to extract more surplus for themselves. Since in our model operators are not allowed to discriminate pricing on customer classes they respond by decreasing prices yet again to give up more of the surplus for "smarter" customers. This affects off-net prices more than on-net prices as seen on Fig. 5 and 6.

To what extent the subscriber coordination effect is at work on real markets is an open question that requires further research, especially if you consider some unrealistic assumptions of the model. For example subscriber mobility is assumed to be frictionless as there is no cost associated with network switching. So you may have a lot of network "jumpers". Nevertheless consistency of the results with theoretical predictions on the one hand and the stylized facts on the other indicate the effect as a plausible explanation to many enigmatic market phenomena, one complementary to hypothesis of "call externalities" prevailing in the theoretical literature. Furthermore diversity of the results and sensitivity of the model suggest that scrutinizing actual network choice coordination procedures among consumers is crucial for understanding the mechanics of telecommunication markets, particularly in context of the long-lasting regulatory discussion on the extent of monopolistic power of network operators. Our research demonstrates that it depends heavily on micro-foundations as apparently insignificant differences in agent behaviour lead to substantially different market outcomes.

IMPLEMENTATION NOTE

The simulation was implemented as a standalone Java application with the compute intensive parts ported to C with NVidia CUDA extensions. We used Java CUDA bindings to glue the pieces together. CUDA converts mass-market graphical processing units (GPUs) into massively parallel high performance computing devices. It has been used for scientific computing in many domains [Owens et al., 2008, Nickolls and

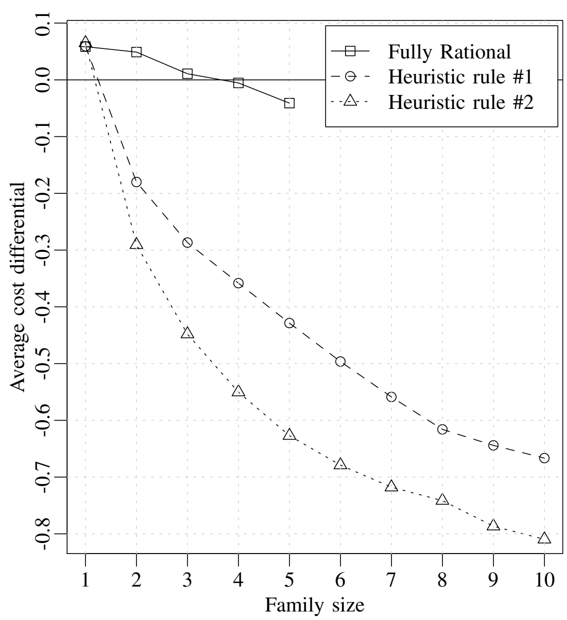

Fig. 3. Simulated to theoretical average cost differential for the duopoly model (LRT2S)

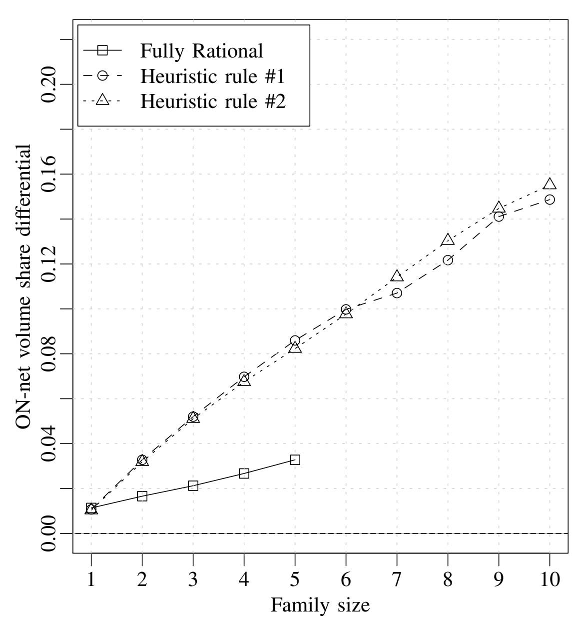

Fig. 4. Simulated to theoretical on-net volume share differential for the duopoly model (LRT2S)

Dally, 2010].

The application have run on mixture of commodity Intel and NVidia hardware under control of Linux operating system. Access to hardware has been provided courtesy NVidia Test Drive program by Boston Limited and Megware Computer Vertrieb und Service GmbH. Access to the source code will be provided by authors upon an e-mail request.

V. Concluding Remarks

We have built the large scale multi-agent model of a mobile telecommunication market. It extends the standard model of Laffont et al. [1998b] by introducing customer heterogeneity, intra-family network choice coordination and different levels of customer rationality.

We confirmed that network heterogeneity plays an important role in shaping equilibrium prices on telecommunication markets. We observed the heterogeneity surplus being internalized by operators in form of higher profits as predicted by Hoernig et al. [2011]. We have shown the subscriber *coordination effect* to counteract the heterogeneity effect and offset or outweigh it leading to significant fall in operator margins and lower average calling costs.

Most importantly we discovered that the level of customer rationality has very strong influence on the market equilibrium. We have shown that, due to unanticipated network effects, boundedly rational customers extract significantly more of the surplus than fully rational ones. Surprisingly, tariff mediated network effects caused by termination based price discrimination turned against operators confronted with nearly rational, network choice coordinating subscribers.

We also proposed an innovative approach to implementation by utilizing high performance CUDA computing devices. This allowed us to operate on large scale with population of up to 1 million agents. We demonstrated how mass-market, commodity GPU devices can be used by social scientists as an inexpensive alternative for traditional data centres. We conclude that massive parallelism of CUDA devices makes it a perfect fit for multigent simulations characterized by high level of inherent parallelism.

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APPENDIX

TABLE II
MODEL DOCKING RESULTS

Model	Population	don	sdon	doff	sdoff	onshare
EXT2Q	15625	0.0091	0.0551	0.0028	0.0582	0.5000
EXT2R	15625	0.0178	0.1291	0.0198	0.1342	0.5000
EXT2S	15625	-0.1973	0.1236	-0.2891	0.1297	0.5000
EXT3Q	15625	0.0032	0.0747	0.0114	0.0701	0.3333
EXT3R	15625	0.0098	0.0971	0.0165	0.1072	0.3333
EXT3S	15625	2.3996	0.1624	2.8022	0.1763	0.3339
EXT4Q	14641	0.0166	0.1008	0.0093	0.0741	0.2500
EXT4R	14641	0.0143	0.0942	0.0116	0.0877	0.2500
EXT4S	14641	3.0507	0.3256	3.5281	0.3550	0.2545
LRT2Q	15625	-0.0012	0.0226	0.0004	0.0212	0.5000
LRT2R	15625	0.0200	0.1275	0.0213	0.1320	0.5000
LRT2S	15625	-0.0014	0.0064	-0.0012	0.0063	0.5000
EXT2Q	117649	-0.0007	0.0264	0.0010	0.0235	0.5000
EXT2R	117649	0.0000	0.0486	0.0017	0.0481	0.5000
EXT2S	117649	0.1061	0.0290	-0.0689	0.0315	0.5000
EXT3Q	117649	-0.0014	0.0291	0.0024	0.0193	0.3333
EXT3R	117649	-0.0000	0.0411	0.0038	0.0390	0.3333
EXT3S	117649	0.2762	0.1364	0.3696	0.1499	0.3335
EXT4Q	104976	0.0043	0.0409	0.0017	0.0215	0.2500
EXT4R	104976	0.0056	0.0469	0.0032	0.0326	0.2500
EXT4S	104976	2.3398	0.2231	2.7599	0.2514	0.2517
LRT2Q	117649	-0.0004	0.0073	-0.0004	0.0061	0.5000
LRT2R	117649	0.0010	0.0489	0.0028	0.0477	0.5000
LRT2S	117649	-0.0003	0.0022	-0.0002	0.0029	0.5000
EXT2Q	531441	0.0003	0.0110	0.0008	0.0088	0.5000
EXT2R	531441	0.0002	0.0252	0.0010	0.0233	0.5000
EXT2S	531441	0.0089	0.0291	0.1784	0.0772	0.5000
EXT3Q	531441	-0.0005	0.0149	0.0013	0.0084	0.3333
EXT3R	531441	-0.0004	0.0220	0.0018	0.0195	0.3333
EXT3S	531441	-0.4257	0.0359	-0.4090	0.0398	0.3334
EXT4Q	531441	0.0016	0.0167	0.0011	0.0079	0.2500
EXT4R	531441	0.0016	0.0229	0.0015	0.0157	0.2500
EXT4S	531441	0.8856	0.5223	1.0829	0.6278	0.2508
LRT2Q	531441	-0.0006	0.0028	-0.0005	0.0020	0.5000
LRT2R	531441	-0.0015	0.0253	-0.0009	0.0236	0.5000
LRT2S	531441	-0.0006	0.0003	-0.0003	0.0006	0.5000
EXT2Q	1000000	0.0004	0.0085	0.0007	0.0070	0.5000
EXT2R	1000000	0.0004	0.0189	0.0005	0.0178	0.5000
EXT2S	1000000	0.0052	0.0215	0.1359	0.0492	0.5000
EXT3Q	1000000	-0.0007	0.0116	0.0010	0.0067	0.3333
EXT3R	1000000	0.0000	0.0169	0.0019	0.0147	0.3333
EXT3S	1000000	-0.3708	0.0369	-0.3536	0.0383	0.3334
EXT4Q	1048576	0.0011	0.0125	0.0010	0.0057	0.2500
EXT4R	1048576	0.0016	0.0182	0.0006	0.0118	0.2500
EXT4S	1048576	0.1410	0.2017	0.2083	0.2511	0.2505
LRT2Q	1000000	-0.0006	0.0018	-0.0006	0.0011	0.5000
LRT2R	1000000	-0.0013	0.0193	-0.0005	0.0179	0.5000
LRT2S	1000000	-0.0005	0.0000	-0.0006	0.0000	0.5000

TABLE III
REGRESSION OF THE AVERAGE CALLING COST – THE A-LRT EQUIVALENT SETUP

	LRT2S				EXT3R				EXT4R			
	Estimate	Std.Err	t value	Pr(>$\|t\|$)	Estimate	Std.Err	t value	Pr(>$\|t\|$)	Estimate	Std.Err	t value	Pr(>$\|t\|$)
(Intercept)	0.1602	0.0020	78.2743	0.0000	-0.3248	0.0027	-121.6703	0.0000	-0.3694	0.0026	-142.6373	0.0000
sigma	-0.7627	0.0004	-1755.5613	0.0000	-0.6095	0.0006	-1065.1016	0.0000	-0.4984	0.0006	-897.0317	0.0000
eta	1.1506	0.0009	1302.9708	0.0000	1.1148	0.0012	952.3294	0.0000	1.0114	0.0011	889.8822	0.0000
ic	0.1041	0.0007	144.1288	0.0000	0.3243	0.0009	366.7979	0.0000	0.3353	0.0009	390.5088	0.0000
ac	1.9433	0.0009	2231.9662	0.0000	1.7474	0.0012	1518.8540	0.0000	1.6243	0.0011	1452.6669	0.0000
f	7.0465	0.0044	1601.4341	0.0000	6.8455	0.0058	1172.4098	0.0000	6.5418	0.0057	1153.1944	0.0000
	RSE: 0.0736 on 336598 DF, Adj.R^2: 0.9739				RSE: 0.0747 on 198339 DF, Adj.R^2: 0.9678				RSE: 0.0725 on 197958 DF, Adj.R^2: 0.9640			

TABLE IV
REGRESSION OF THE AVERAGE CALLING COST – THE MULTIAGENT SETUP

	LRT2S				EXT3R				EXT4R			
	Estimate	Std.Err	t value	Pr(>$\|t\|$)	Estimate	Std.Err	t value	Pr(>$\|t\|$)	Estimate	Std.Err	t value	Pr(>$\|t\|$)
(Intercept)	0.2803	0.0032	87.1028	0.0000	-0.1486	0.0033	-45.5957	0.0000	-0.2323	0.0030	-76.9718	0.0000
sigma	-0.6718	0.0006	-1054.9123	0.0000	-0.5333	0.0007	-812.9612	0.0000	-0.4406	0.0006	-725.1634	0.0000
eta	1.0671	0.0013	829.2190	0.0000	0.9994	0.0013	745.3509	0.0000	0.9145	0.0012	736.1419	0.0000
ic	0.1772	0.0011	164.5358	0.0000	0.3630	0.0010	347.3133	0.0000	0.3704	0.0010	382.5382	0.0000
ac	1.8330	0.0013	1443.8558	0.0000	1.6700	0.0013	1264.9617	0.0000	1.5677	0.0012	1280.5955	0.0000
f	6.7904	0.0064	1059.2387	0.0000	6.5881	0.0067	984.8478	0.0000	6.3448	0.0062	1023.1532	0.0000
rewire	-0.1735	0.0006	-270.4930	0.0000	-0.1208	0.0007	-181.5881	0.0000	-0.0879	0.0006	-142.5061	0.0000
radiusr=4	0.0122	0.0005	24.7803	0.0000	0.0055	0.0006	9.7988	0.0000	0.0031	0.0005	6.0398	0.0000
radiusr=5	0.0195	0.0005	39.0220	0.0000	0.0063	0.0006	11.0646	0.0000	0.0026	0.0005	5.0346	0.0000
radiusr=6	0.0182	0.0011	17.1324	0.0000	0.0074	0.0009	8.3679	0.0000	0.0039	0.0008	4.7156	0.0000
radiusr=7	0.0123	0.0011	11.5272	0.0000	0.0115	0.0009	12.9661	0.0000	0.0084	0.0008	10.2043	0.0000
radiusr=8	0.0225	0.0010	21.6842	0.0000	0.0157	0.0009	18.0055	0.0000	0.0094	0.0008	11.6464	0.0000
radiusr=9	0.0135	0.0013	10.7262	0.0000	0.0050	0.0010	4.7174	0.0000	0.0016	0.0010	1.6472	0.0995
radiusr=10	0.0125	0.0012	10.1877	0.0000	0.0041	0.0010	3.9528	0.0001	0.0018	0.0010	1.8935	0.0583
FamilyF1	0.0585	0.0012	48.5098	0.0000	0.0461	0.0012	39.2199	0.0000	0.0364	0.0011	33.3879	0.0000
FamilyF2	0.0489	0.0012	39.2481	0.0000	0.0535	0.0012	43.4216	0.0000	0.0456	0.0011	39.9165	0.0000
FamilyF3	0.0108	0.0012	9.1054	0.0000	0.0384	0.0011	33.6400	0.0000	0.0360	0.0011	34.0611	0.0000
FamilyF4	-0.0054	0.0012	-4.5254	0.0000	0.0457	0.0012	39.2590	0.0000	0.0462	0.0011	42.8909	0.0000
FamilyF5	-0.0410	0.0012	-33.9068	0.0000	0.0370	0.0012	31.3832	0.0000	0.0431	0.0011	39.5773	0.0000
FamilyH1	0.0592	0.0014	41.3713	0.0000	0.0597	0.0015	39.8899	0.0000	0.0480	0.0014	34.6196	0.0000
FamilyH2	-0.1800	0.0015	-122.6138	0.0000	0.0022	0.0015	1.5027	0.1329	0.0216	0.0014	15.9309	0.0000
FamilyH3	-0.2868	0.0014	-206.1914	0.0000	-0.0540	0.0015	-35.9890	0.0000	-0.0202	0.0014	-14.5722	0.0000
FamilyH4	-0.3583	0.0014	-253.9186	0.0000	-0.1258	0.0015	-84.8218	0.0000	-0.0802	0.0014	-58.3866	0.0000
FamilyH5	-0.4286	0.0014	-301.5214	0.0000	-0.1935	0.0015	-130.8469	0.0000	-0.1359	0.0014	-99.2570	0.0000
FamilyH6	-0.4963	0.0018	-276.1554	0.0000	-0.2456	0.0015	-166.1425	0.0000	-0.1771	0.0014	-129.3945	0.0000
FamilyH7	-0.5588	0.0018	-302.8385	0.0000	-0.3003	0.0015	-198.9627	0.0000	-0.2220	0.0014	-158.8458	0.0000
FamilyH8	-0.6159	0.0018	-341.0998	0.0000	-0.3453	0.0015	-232.6603	0.0000	-0.2571	0.0014	-187.1531	0.0000
FamilyH9	-0.6442	0.0017	-368.2016	0.0000	-0.3774	0.0014	-261.0040	0.0000	-0.2825	0.0013	-211.0761	0.0000
FamilyH10	-0.6666	0.0018	-367.2946	0.0000	-0.4012	0.0015	-268.7229	0.0000	-0.3000	0.0014	-217.0025	0.0000
FamilyG1	0.0650	0.0019	33.3858	0.0000	0.0524	0.0016	31.8987	0.0000	0.0424	0.0015	27.8968	0.0000
FamilyG2	-0.2907	0.0020	-147.9344	0.0000	-0.0688	0.0016	-42.3793	0.0000	-0.0253	0.0015	-16.7978	0.0000
FamilyG3	-0.4481	0.0021	-213.2274	0.0000	-0.1828	0.0017	-104.7305	0.0000	-0.1109	0.0016	-68.6169	0.0000
FamilyG4	-0.5503	0.0021	-267.0427	0.0000	-0.2655	0.0017	-152.7928	0.0000	-0.1763	0.0016	-109.8182	0.0000
FamilyG5	-0.6271	0.0020	-318.2227	0.0000	-0.3259	0.0016	-199.0975	0.0000	-0.2232	0.0015	-147.2806	0.0000
FamilyG6	-0.6789	0.0020	-340.4438	0.0000	-0.3771	0.0017	-224.4127	0.0000	-0.2643	0.0016	-169.6662	0.0000
FamilyG7	-0.7178	0.0019	-374.4083	0.0000	-0.4181	0.0016	-261.1374	0.0000	-0.2977	0.0015	-200.7579	0.0000
FamilyG8	-0.7415	0.0020	-365.6167	0.0000	-0.4421	0.0017	-263.4054	0.0000	-0.3162	0.0016	-203.3334	0.0000
FamilyG9	-0.7866	0.0020	-400.7365	0.0000	-0.4847	0.0017	-290.5156	0.0000	-0.3538	0.0015	-229.1157	0.0000
FamilyG10	-0.8096	0.0019	-425.6128	0.0000	-0.5034	0.0016	-311.9743	0.0000	-0.3676	0.0015	-245.9765	0.0000
	RSE: 0.1072 on 336565 DF, Adj.R^2: 0.9559				RSE: 0.0854 on 198306 DF, Adj.R^2: 0.9613				RSE: 0.0791 on 197925 DF, Adj.R^2: 0.9588			

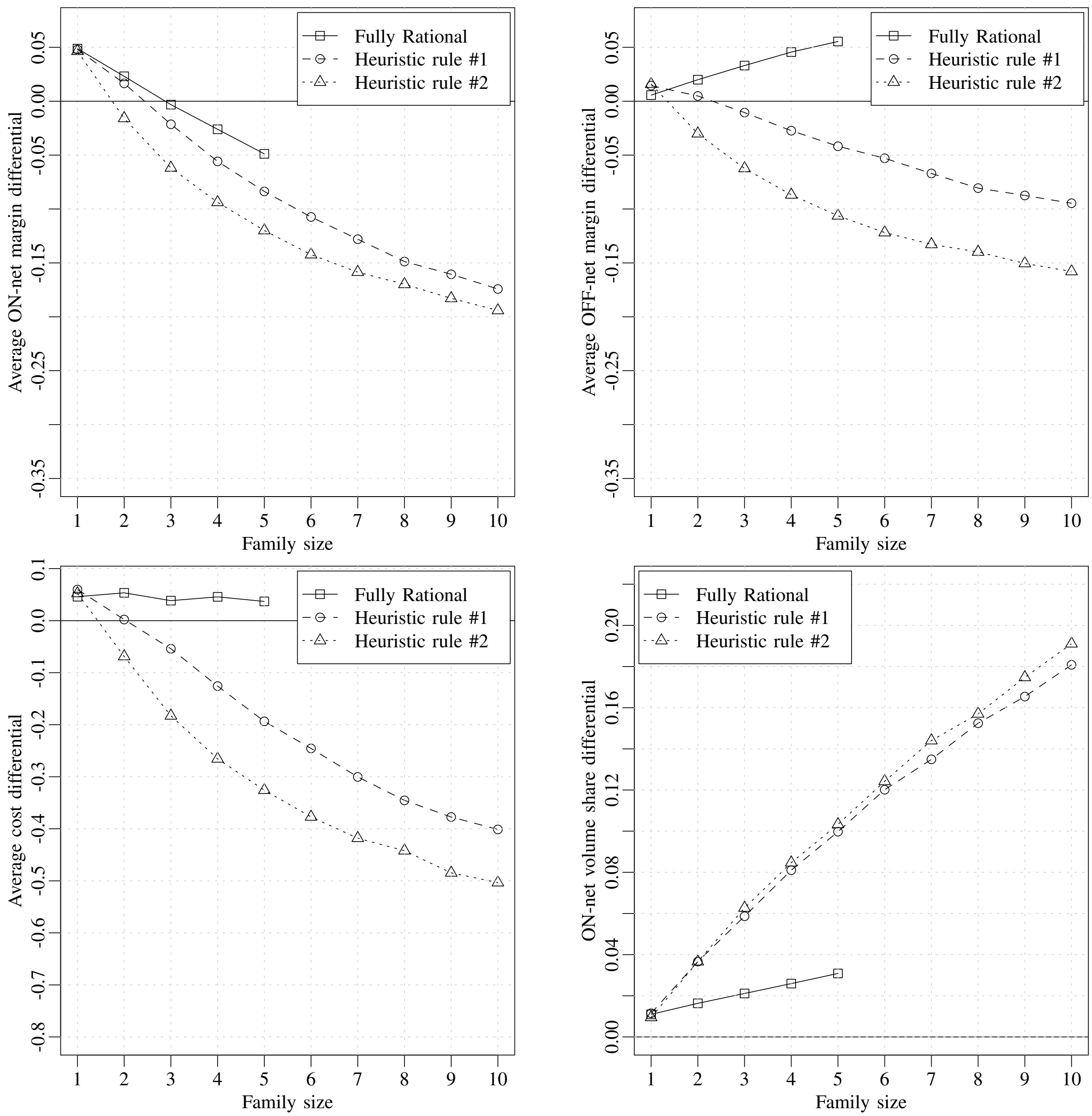

Fig. 5. Results for the triopoly model (EXT3)

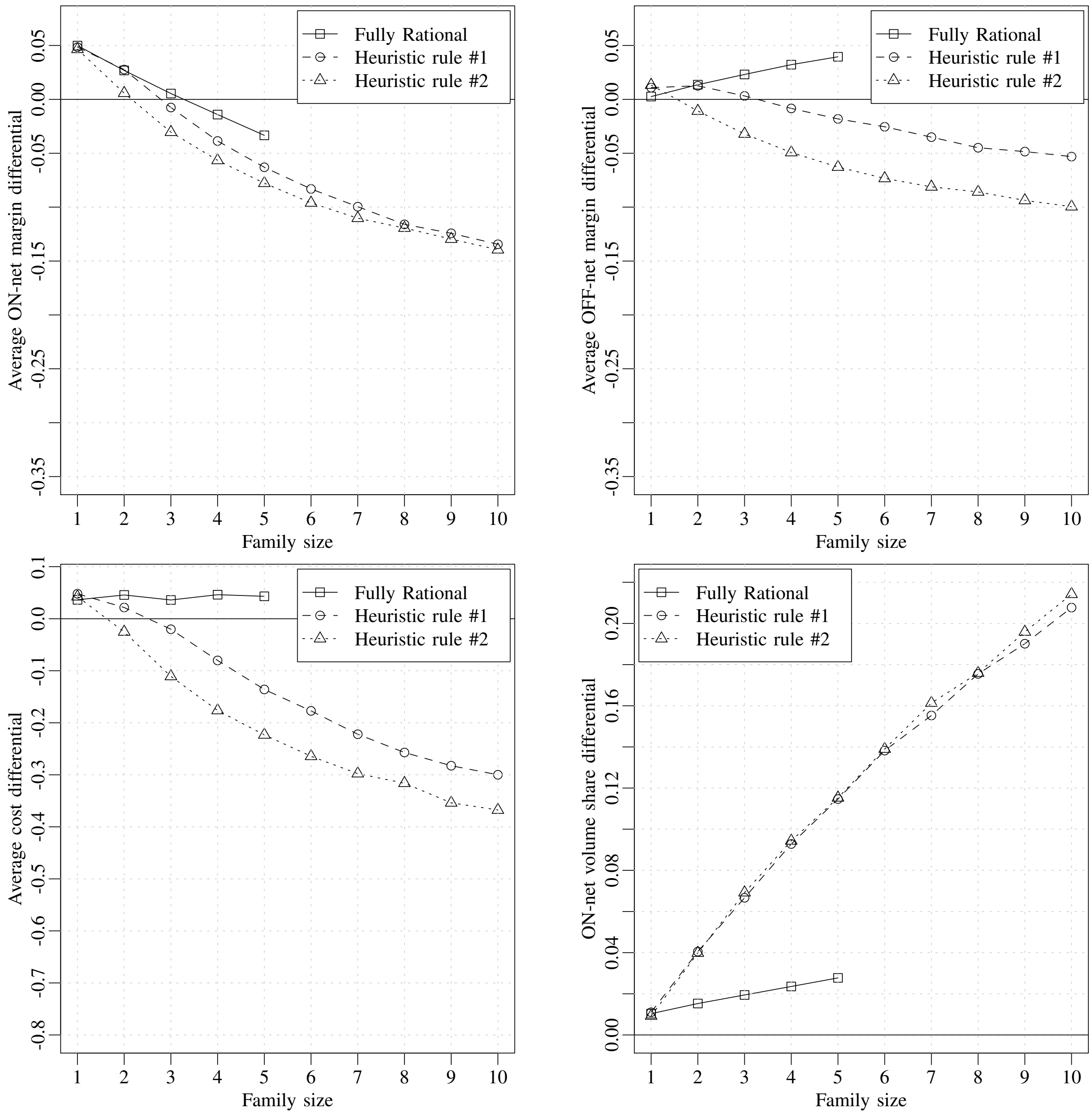

Fig. 6. Results for the 4 network oligopoly model (EXT4)