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An EWMA Chart for Sample Range of Weibull Data using Weighted Variance Method

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Abstract. This article proposes new EWMA chart in observing process standard deviation or dispersion with sample range of Weibull data using weighted variance method (WV). This control chart, called Weighted Variance EWMA sample range WV-EWMA$_{SR}$ chart hereafter. The proposed WV-EWMA$_{SR}$ chart compared with standard EWMASR of [7], skewness correction *R* chart (SC-*R*) suggested by[3]and Weighted Variance *R* chart (WV-*R*) proposed by [2], in the case of Type I and Type II errors when the data generated from Weibull distribution. Optimal parameters λ and k of the proposed WV-EWMASR and standard EWMASR are obtained via simulation using SAS program 9.4. The proposed WV- EWMASR control chart reduces to the standard EWMASR control chart of [7] when the process follow symmetric distribution. The proposed WV- EWMASR control chart has less Type I error than the standard EWMASR, SC-*R* and WV-*R* control charts, for Weibull distribution data. In case of Type II error, the proposed WV-EWMASR control chart is closer to EWMA chart with the exact limits than the standard EWMASR in [7].

Keywords: EWMA, control chart, Weibull data, process dispersion

INTRODUCTION

[8] First proposed EWMA chart of process sample mean. In his paper he also includes several ARL for different weighting values. EWMA chart for monitoring process dispersion also suggested by [11]. [10] Proposes two models for simultaneous monitoring mean and standard deviation of the control charts using EWMA.[7] used Exponential Weighted Moving Average to develop and evaluate of charts for observing process location and dispersion. Most of these EWMA chars proposed in the literature for monitoring process mean suggest or dispersion are considered under normality assumption. But, in many conditions, this assumption does no longer hold. [4] Proposed EWMA chart for monitoring the process mean using weighted standard deviation method. This chart has less false alarm than the basic EWMA in most levels of skewnesses. [7] Suggested EWMA control chart of skewed populations using weighted variance method for process location. This EWMA chart has better performance than the existing EWMA chart of skewed populations for monitoring process mean in case of Type I and Type II errors. The other control charts in literature that proposed the usage of heuristic method for monitoring process mean and dispersion for skewed population which are to be had consist of the $\bar{X}$ and *R* charts using the weighted variance (WV) by [2], the $\bar{X}$ and *R* charts using skewness correction (SC) approach suggested by [3], a multivariate synthetic control chart of the process mean vector of skewed populations using weighted standard deviations suggested by [5], a multivariate EWMA control chart using weighted variance method by [1].

REVIEW OF THE EXISTING CHARTS FOR DISPERSION

Weighted Variance (WV) *R* Chart

The weighted variance approach is primarily depends on the concept that a skewed distribution can be divided into two parts at its mean and each one of these parts is used for developing a new symmetric distribution. The weighted variance WV approach uses the two new symmetric distributions to construct the limits of the chart. The control limits of the WV- *R* chart are [2]

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$$\mathrm{UCL}_{\mathrm{WV}-R} = \mu_R + 3\sigma_R\sqrt{2P_X} \quad (1)$$

and

$$\mathrm{LCL}_{\mathrm{WV}-R} = \left[\mu_R - 3\sigma_R\sqrt{2(1-P_X)}\right]^+, \quad (2)$$

where μ_R and σ_R are the mean and standard deviation of the range *R*, respectively. Note that, $P_X = P(X \le \mu_X)$,is the probability that the characteristic X is less than or equal to its mean, μ_X .When $P_X = \frac{1}{2}$ the WV-*R* control chart will be reduced to the Shewhart *R* control chart.

Skewness Correction (SC-*R*) Chart

[3] Proposes the SC-*R* chart for skewed populations. The SC-*R* chart is depends on the following control limits [3]:

$$\mathrm{UCL}_{SC-R} = \mu_R + \left(3 + \frac{4\alpha_3(R)}{1+0.2\alpha_3^2(R)}\right)\sigma_R \quad (3)$$

and

$$\mathrm{LCL}_{SC-R} = \left[\mu_R + \left(-3 + \frac{4\alpha_3(R)}{1+0.2\alpha_3^2(R)}\right)\sigma_R\right]^+. \quad (4)$$

where, $\alpha_3(R)$ denotes the skewness of *R*.

A Proposed Weighted Variance EWMA SAMPLE RANGE (WV) WV-EWMASR Chart

Let the range of sample i dente as $R_{i.}$ Each value of R_i should be transformed into a corresponding EWMA z_i is done by the following statistic [7]:

$$z_i = \lambda R_i + (1-\lambda) z_{i-1} \quad (5)$$

The limits of the proposed WV-EWMASR are as following:

$$\mathrm{UCL}_{\text{WV-EWMASR}} = \mu_R + K\sigma_R\sqrt{\frac{\lambda}{2-\lambda}}(2P_X) \quad (6)$$

and

$$\mathrm{LCL}_{\text{WV-EWMASR}} = \mu_R - K\sigma_R\sqrt{\frac{\lambda}{2-\lambda}}(2P_X), \quad (7)$$

where λ and K are calculated via the simulation using SAS program 9.4 when the desired in-control ARL (false alarm rate) is given. Here, the WV-EWMASR control chart will be reduced to the standard EWMASR control chart of [8] , when $P_X = \frac{1}{2}$.

An assignable cause is issued by the WV-EWMASR chart at time *i* if $Z_i > \mathrm{UCL}_{\text{WV-EWMASR}}$ or $Z_i < \mathrm{LCL}_{\text{WV-EWMASR}}$.

PERFORMANCE EVALUATION OF THE WV-EWMASR CONTROL CHART

The proposed WV−EWMASR control chart is compared with the SC-*R* and WV-*R* control charts for Weibull data, in the case of Type I error. Therefore, in the case of Type II error, the proposed WV-EWMASR chart is compared with EWMA chart of the exact limits and standard EWMASR chart proposed by [7]. Simulation study is achieved using SAS 9.4 to compute the Type I and Type II errors. The charts that included in this paper are designed depend on an in-control Type I error of 0.0027.

$\sigma_1 = \delta\, \sigma_X$, is represents a shift of the standard deviation or variance in the process, where $\delta \in$ {1.5, 2, 2.5,3, 3.5, 4.0} is the level of a shift, in process standard deviation. The distribution considered in this paper, is Weibull, because is represent a variety of shapes from symmetric to highly skewed. This chart can be applied for any other skewed distributions. Beside Weibull distribution the standard normal distribution is also considered in this study. A scale parameter of one is considered for the Weibull distribution, for ease. Here, P_X for the Weibull distribution is:

$$P_X = 1 - \exp\left[-\left(\Gamma\left(1 + {}^{1}\!/\!_{\beta}\right)\right)^{\beta} \right] , \qquad (8)$$

where β, is the shape parameter of Weibull distribution. The skewness coefficients considered in the case of Type I error are $\alpha_3 \in$ {0.5, 1.0, 1.5, 2.0, 2.5, 3.0}, while skewness coefficient, α_3 =0.5,2 and 3 are considered in the case of the Type II error computations . The Type I and Type II are obtained based on 10000 simulation trials. Table 1,2 and 3 show that the proposed WV-EWMASR control chart has less Type I error than the standard EWMWSR ,SC-*R* and WV-*R* control charts for all levels of skewnesses and sample sizes, when the distributions is Weibull.

Table 4 shows that the Type II error of the proposed WV-EWMASR is close to EWMA chart of the exact limits than the standard EWMASR control chart for almost all levels of skewnesses and λ . In general, the proposed WV-EWMASR control chart provides good performances in the case of Type I and Type II errors for all levels of skewnesses, sample sizes and levels of the shift.

TABLE 1. Type I Error for Various charts Weibull Distributions Sample Size *n*=5

						EWMASR				***R* Charts**		
	λ	**0.05**		**0.1**		**0.2**		**0.3**				
	K	**2.451**		**2.689**		**2.863**		**2.954**				
Distribution	α_3	**WV EWMA**	**Standard EWMA**	**WV EWMA**	**Standard EWMA**	**WV EWMA**	**Standard EWMA**	**WV EWMA**	**Standard EWMA**	**Shewhart *R*-chart**	**SC *R*-chart**	**WV *R*-chart**
Normal	0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0046	0.0045	0.0045
Weib (2.2,1)	0.5	0.0020	0.0024	0.0019	0.0023	0.0018	0.0024	0.0017	0.0025	0.0043	0.0029	0.0037
Weib (1.564,1)	1	0.0021	0.0047	0.0020	0.0053	0.0020	0.0065	0.0022	0.0074	0.0139	0.0054	0.0057
Weib (1.211,1)	1.5	0.0027	0.0094	0.0025	0.0116	0.0027	0.0146	0.0032	0.0166	0.0302	0.0059	0.0075
Weib (1,1)	2	00032	0.0157	0.0030	0.0202	0.0034	0.0259	0.0041	0.0289	0.0477	0.0039	0.0089
Weib (0.863,1)	2.5	0.0036	0.0228	0.0035	0.0300	0.0039	0.0385	0.0049	0.0426	0.0631	0.005	0.01
Weib (0.77,1)	3	0.0039	0.0298	0.0038	0.0397	0.0042	0.0513	0.0054	0.0561	0.0754	0.0043	0.0108

TABLE 2. Type I Error for Various charts Weibull Distributions Sample Size *n*=7

						EWMASR				***R* chart**		
	λ	**0.05**		**0.1**		**0.2**		**0.3**				
	K	**2.442**		**2.686**		**2.875**		**2.965**				
Distribution	α_3	**WV EWMA**	**Standard EWMA**	**WV EWMA**	**Standard EWMA**	**WV EWMA**	**Standard EWMA**	**WV EWMA**	**Standard EWMA**	**Shewhart *R*-chart**	**SC *R*-chart**	**WV *R*-chart**
Normal	0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0044	0.0045	0.0044
Weib (2.2,1)	0.5	0.0015	0.0023	0.0013	0.0023	0.0011	0.0022	0.0011	0.0024	0.004	0.0044	0.0036
Weib (1.564,1)	1	0.0017	0.0052	0.0015	0.0059	0.0015	0.0069	0.0016	0.0078	0.0146	0.0076	0.0055
Weib (1.211,1)	1.5	0.0022	0.0110	0.0020	0.0136	0.0020	0.0169	0.0025	0.0191	0.0328	0.0083	0.0071
Weib (1,1)	2	0.0028	0.0187	0.0025	0.0240	0.0027	0.0307	0.0033	0.0344	0.0526	0.007	0.0085
Weib (0.863,1)	2.5	0.0033	0.0269	0.0031	0.0358	0.0033	0.0463	0.0040	0.0519	0.067	0.004	0.0094
Weib (0.77,1)	3	0.0036	0.0351	0.0033	0.0474	0.0036	0.0619	0.0046	0.0672	0.0836	0.0039	0.0102

TABLE 3. Type I Error for the various charts Weibull Distributions Sample Size *n*=10

		EWMASR								*R* Chart		
	λ	0.05		0.1		0.2		0.3				
	K	2.448		2.686		2.877		2.976				
Distribution	α_3	WV EWMA	Standard EWMA	WV EWMA	Standard EWMA	WV EWMA	Standard EWMA	WV EWMA	Standard EWMA	Shewhart *R*-chart	SC *R*-chart	WV *R*-chart
Normal	0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0043	0.0041	0.0043
Weib (2.2,1)	0.5	0.0011	0.0022	0.0010	0.0021	0.0008	0.0022	0.0008	0.0022	0.004	0.0059	0.0038
Weib (1.564,1)	1	0.0013	0.0056	0.0012	0.0066	0.0011	0.0077	0.0012	0.0086	0.0158	0.0092	0.0055
Weib (1.211,1)	1.5	0.0018	0.0123	0.0017	0.0157	0.0016	0.0197	0.0019	0.0220	0.0361	0.0108	0.007
Weib (1,1)	2	0.0023	0.0213	0.0021	0.0282	0.0021	0.0362	0.0026	0.0405	0.0586	0.0092	0.0081
Weib (0.863,1)	2.5	0.0027	0.0307	0.0026	0.0415	0.0026	0.0547	0.0033	0.0617	0.0801	0.0058	0.009
Weib (0.77,1)	3	0.0030	0.0400	0.0028	0.0550	0.0029	0.0728	0.0037	0.0827	0.0993	0.0041	0.0096

TABLE 4. Type II Risks of WV-EWMASR Chart compare with Exact and standard EWMASR charts ,Sample Size *n*=5

		0.05			0.1			0.2			0.3		
Distribution	Shift	Exact EWMA	WV EWMA	Standard EWMA	Exact EWMA	WV EWMA	Standard EWMA	Exact EWMA	WV EWMA	Standard EWMA	Exact EWMA	WV EWMA	Standard EWMA
Weibull (2.2,1)	1.5	0.8785	0.8813	0.8723	0.8623	0.8643	0.8544	0.8513	0.8503	0.8373	0.8527	0.8487	0.8329
	2	0.7518	0.7565	0.7434	0.7087	0.7120	0.6963	0.6625	0.6609	0.6415	0.6393	0.6335	0.6116
	2.5	0.6423	0.6484	0.6312	0.5789	0.5830	0.5631	0.5068	0.5049	0.4818	0.4664	0.4601	0.4357
	3	0.5465	0.5538	0.5333	0.4673	0.4721	0.4488	0.3796	0.3778	0.3542	0.3332	0.3272	0.3043
	3.5	0.4604	0.4687	0.4457	0.3702	0.3753	0.3515	0.2797	0.2779	0.2563	0.2354	0.2301	0.2017
	4	0.3823	0.3912	0.3668	0.2890	0.2941	0.2708	0.2042	0.2027	0.1845	0.1659	0.1617	0.1462
Weibull (1,1)	1.5	0.9351	0.9340	0.8803	0.9354	0.9283	0.8611	0.9442	0.9251	0.8394	0.9548	0.9254	0.8275
	2	0.8561	0.8540	0.7643	0.8434	0.8319	0.7256	0.8417	0.8109	0.6825	0.8539	0.8035	0.6596
	2.5	0.7838	0.7811	0.6648	0.7592	0.7444	0.6108	0.7435	0.7053	0.5521	0.7489	0.6867	0.5208
	3	0.7185	0.7152	0.5780	0.6840	0.6666	0.5149	0.6568	0.6134	0.4475	0.6544	0.5856	0.4119
	3.5	0.6907	0.6554	0.5035	0.6170	0.5978	0.4345	0.5803	0.5336	0.3629	0.5716	0.4992	0.3261
	4	0.6052	0.6011	0.4388	0.5570	0.5364	0.3670	0.5132	0.4648	0.29533	0.4994	0.4262	0.2599
Weibull (0.77,1)	1.5	0.9556	0.9529	0.8856	0.9588	0.9498	0.8660	0.9689	0.9473	0.8421	0.9758	0.9461	0.8278
	2	0.8963	0.8918	0.7813	0.8926	0.8778	0.7473	0.9036	0.8657	0.7097	0.9180	0.8619	0.6885
	2.5	0.8408	0.8347	0.6907	0.8206	0.8097	0.6456	0.8335	0.7850	0.5970	0.8477	0.7746	0.5706
	3	0.7897	0.7823	0.6132	0.7707	0.7476	0.5601	0.7679	0.7115	0.5043	0.7787	0.6945	0.4749
	3.5	0.7428	0.7340	0.5460	0.7174	0.6917	0.4885	0.7080	0.6462	0.4287	0.7150	0.6235	0.3977
	4	0.6993	0.6900	0.4880	0.6690	0.6410	0.4273	0.6538	0.5881	0.3662	0.6567	0.5605	0.3350

CONCLUSION

In this paper, we have proposed the WV-EWMASR control chart for Weibull data. This proposed chart depends on the weighted variance (WV) method. The WV-EWMASR chart shows a good performance compare to the standard EWMASR and all existing charts considered in this paper for Weibull data, in the case of the Type-I and Type-II errors.

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REFERENCES

1. A.M.A. Atta, M.H.A. Shoraim and S.S.S. Yahaya, Int. Res. J. of Sci. & Engg., 2 (6):191-202 (2014).
2. D. S. Bai, and I. S. Choi, *Journal of Quality Technology*, 27, 120 – 131 (1995).
3. L. K. Chan and H. J. Cui, *Naval Research Logistics*, 50, 555 – 573 (2003).
4. Y. S. Chang and D. S. Bai, *Quality and Reliability Engineering International*, 17, 397 –406 (2001).
5. M. B. C. Khoo, A. M. A. Atta, and Z. Wu, communications in statistics – simulation and computation, 38: 1493 – 1518 (2009).
6. M.B.C. Khoo, A.M. A. Atta, Proceedings of the International Conference on Industrial Engineering and Engineering Management, 218-223, (2008).
7. C.H. Ng and K.E. Case, *Journal of Quality Technology,* 242-250 (1989).
8. S. W. Roberts, Technometrics, 239-250 (1959).
9. A. L. Sweet, Institute of Industrial Engineers, Transactions, 26-33 (1986).
10. A.W.Wortham and L.J.Ringer, the Transportation and Logistic Review 33