AIP Conference Proceedings

Analysis of performance measures with single channel fuzzy queues under two class by using ranking method

Zeina Mueen, Razamin Ramli, and Nerda Zura Zaibidi

Citation: AIP Conference Proceedings **1761**, 020077 (2016); doi: 10.1063/1.4960917
View online: http://dx.doi.org/10.1063/1.4960917
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1761?ver=pdfcov
Published by the AIP Publishing

Analysis of Performance Measures with Single Channel Fuzzy Queues Under Two Class by Using Ranking Method

Zeina Mueen [a)], Razamin Ramli and Nerda Zura Zaibidi

School of Quantitative Sciences, College of Arts and Sciences, Universiti Utara Malaysia, 06010 Sintok, Kedah, Malaysia

[a)]Corresponding author: mueen.zeiana@gmail.com

Abstract. In this paper, we propose a procedure to find different performance measurements under crisp value terms for new single fuzzy queue FM/F(H_1,H_2)/1 with two classes, where arrival rate and service rates are all fuzzy numbers which are represented by triangular and trapezoidal fuzzy numbers. The basic idea is to obtain exact crisp values from the fuzzy value, which is more realistic in the practical queueing system. This is done by adopting left and right ranking method to remove the fuzziness before computing the performance measurements using conventional queueing theory. The main advantage of this approach is its simplicity in application, giving exact real data around fuzzy values. This approach can also be used in all types of queueing systems by taking two types of symmetrical linear membership functions. Numerical illustration is solved in this article to obtain two groups of crisp values in the queueing system under consideration.

INTRODUCTION

Queueing models have a wide application in service organizations and one of such application area is real life situations having a policy of two class service channels. Different types of queuing models have been explored in [1,2] ranging from queuing models having constant crisp values. However, practical problems have the assumptions that certain parameters in the queueing model are not constantly known as assumed by classical model. Therefore, Zhadeh's principle [3] described as possibilistic having expressions such as “the average arrival rate is approximately 10” or “service times are approximately 20”, which is a more realistic way to represent these values. This makes fuzzy queueing models more practical than the classic queueing models in many real situations.

On the other hand, the conversion of fuzzy queues to crisp queues has also been extensively discussed in literature and a number of methods and approaches have been put in use. One of such methods is the ranking method [4]. Another method that dates further back into literature is the robust ranking method [5], which was also studied by [6,7]. The authors here adopted this method with single channel priority queuing models, while more recently studies [8-10] adopted priority queueing models with fuzzy queues. However, most previous studies have not considered two classes of arrival rates and two exponential service rates under arrival policy of first come first service (FCFS). Hence, this paper adopts one of the queueing models with method known as the left and right ranking method that obtains performance measures in terms of crisp value for fuzzy queuing model with two classes of arrival rates and mixture of exponential service rates.

The outline of this paper follows: Section 1 contains an introductory overview, Section 2 explains the basic concepts in fuzzy set theory, Section 3 describes the formulation of fuzzy queuing model with two classes, Section 4 introduces the left and right ranking method, while Section 5, 6 and 7 gives the numerical examples, discussion of results and conclusion respectively.

Proceedings of the International Conference on Applied Science and Technology 2016 (ICAST'16)
AIP Conf. Proc. 1761, 020077-1–020077-6; doi: 10.1063/1.4960917
Published by AIP Publishing. 978-0-7354-1419-8/$30.00

BASIC CONCEPTS OF FUZZY SET THEORY

A fuzzy set is specified by a membership function containing the components of a domain space or universe X in the interval [0, 1], that is $\widetilde{A} = \{(z, \mu_{\widetilde{A}}(z)); z \in Z\}$. Here, $\tilde{\mu}_{\widetilde{A}}: Z \to [0,1]$ is an interval called the degree of membership function of the fuzzy set $\widetilde{A}$ and $\mu_{\widetilde{A}}(z)$ represents the membership value of $z \in Z$ in the fuzzy set $\widetilde{A}$. Also, these membership degree are defined by $R \to [0,1]$.

A fuzzy set $\widetilde{A}$ of a universe of discourse X is called a normal fuzzy set if there exists at least $z \in Z$ such that $\mu_{\widetilde{A}}(z) = 1$.

A fuzzy set $\widetilde{A}$ is convex if and only if for any $z \in Z$ the membership function of $\widetilde{A}$ satisfies the condition $\mu_{\widetilde{A}}\{z_1 + (1-\lambda)z_1\} \geq \min\{\mu_{\widetilde{A}}(z_1), \mu_{\widetilde{A}}(z_2)\}, 0 \leq \lambda \leq 1$.

Queueing models have a wide application in service organizations and one of such application area is real life situ

Triangular Fuzzy Number

A triangular fuzzy number $\widetilde{A}(z)$ can be represented by $\widetilde{A} = (a, b, c; 1)$ where, a, b and c represent the points inside the interval with the membership function $\mu_{\widetilde{A}}(z)$ defined by

$$\mu_{\widetilde{A}}(z) = \begin{cases} \dfrac{(z-a)}{(b-a)}, & a \leq z \leq b \\ 1, & z = b \\ \dfrac{(z-c)}{(b-c)}, & b \leq z \leq c \\ 0, & o.w \end{cases} \tag{1}$$

Trapezoidal Fuzzy Number

A trapezoidal fuzzy number $\widetilde{A}(z)$ can be represented by $\widetilde{A} = (a, b, c, d; 1)$ where, a, b, c and d represent the points inside the interval. The membership function $\mu_{\widetilde{A}}(z)$ is defined by:

$$\mu_{\widetilde{A}}(z) = \begin{cases} \dfrac{(z-a)}{(b-a)}, & a \leq z \leq b \\ 1, & b \leq z \leq c \\ \dfrac{(z-d)}{(c-d)}, & c \leq z \leq d \\ 0, & o.w \end{cases} \tag{2}$$

Generalized Fuzzy Number

Generalized fuzzy number is a fuzzy set of $\widetilde{A}$, described in the universal set of real numbers represented R, also if the membership function characterized as $\mu_{\widetilde{A}}: R \to [0, w]$ is continuous $\mu_{\widetilde{A}}(z) = 0$ for all $z \in (-\infty, a_1) \cup [a_4, \infty)$, then:

- $\mu_{\widetilde{A}}(z)$ is strictly increasing on $[a_1, a_2]$; also strictly decreasing on $[a_3, a_4]$;
- $\mu_{\widetilde{A}}(z) = w$ for all $z \in [a_2, a_3]$;

  where $0 < w \leq 1$.

Left and Right Type Generalized Fuzzy Number

Generalized left and right fuzzy number is represented as $(a, b, c, d; w)_{LR}$. This is said to be left and right type generalized fuzzy number using triangular and trapezoidal fuzzy numbers.

Generalized Triangular Fuzzy Number

Generalized triangular fuzzy number $\widetilde{A}(z)$ can be represented by $\widetilde{A} = (a, b, c; w)$ under membership function $\mu_{\widetilde{A}}(z)$ defined by

$$\mu_{\widetilde{A}}(z) = \begin{cases} \dfrac{w(z-a)}{(b-a)}, & a \le z \le b \\ w \quad , & z = b \\ \dfrac{w(z-c)}{(b-c)}, & b \le z \le c \\ 0, & o.w \end{cases} \tag{3}$$

Generalized Trapezoidal Fuzzy Number

Generalized trapezoidal fuzzy number as $\widetilde{A}(z)$ can be represented by $\widetilde{A} = (a, b, c, d; w)$ under membership function $\mu_{\widetilde{A}}(z)$ defined by

$$\mu_{\widetilde{A}}(z) = \begin{cases} \dfrac{w(z-a)}{(b-a)}, & a \le z \le b \\ w \quad , & b \le z \le c \\ \dfrac{w(z-d)}{(d-c)}, & c \le z \le d \\ 0, & o.w \end{cases} \tag{4}$$

FUZZY QUEUEING MODEL WITH TWO CLASSES

Consider a single channel fuzzy queueing model with two classes FM/F(H1,H2)/1/FCFS with no priorities in arrival rates, where FM denotes the fuzzy arrival rates as a Poisson process while $F(H_1,H_2)$ denotes the fuzzified hyper exponential service time rates with two classes in a First Come First Serve (FCFS) manner, noting that the system capacity and population size is infinite.

In this model, customers arrived in groups by single channel represented by $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}_1$ and $\tilde{\mu}_2$ respectively. Let $\emptyset_{\tilde{\lambda}_1}(w), \emptyset_{\tilde{\lambda}_2}(x), \emptyset_{\tilde{\mu}_1}(y)$, and $\emptyset_{\tilde{\mu}_2}(z)$ then the fuzzy sets are represented by four sets as:

$$\tilde{\lambda}_1 = \left\{\left(w, \emptyset_{\tilde{\lambda}_1}(w)\right) \middle| w \in W\right\}, \tag{5}$$

$$\tilde{\lambda}_2 = \left\{\left(x, \emptyset_{\tilde{\lambda}_2}(x)\right) \middle| x \in X\right\}, \tag{6}$$

$$\tilde{\mu}_1 = \left\{\left(y, \emptyset_{\tilde{\mu}_1}(y)\right) \middle| y \in Y\right\}, \tag{7}$$

$$\tilde{\mu}_2 = \left\{\left(z, \emptyset_{\tilde{\mu}_2}(z)\right) \middle| z \in Z\right\}. \tag{8}$$

where W, X, Y and Z are crisp universal group of the arrival rate and service rate. Let f(w,x,y,z) denote the system particular of interest. Hence w, x, y and z are fuzzy numbers and likewise f(w,x,y,z) are fuzzy numbers.

Let $L_q^{(1)}$ and $L_q^{(2)}$ represent the conventional equation in the classical single queueing model as:

$$L_q^{(1)} = \frac{\lambda_1\left({}^{\rho_1}/_{\mu_1} + {}^{\rho_2}/_{\mu_2}\right)}{1-\rho}, \tag{9}$$

$$L_q^{(2)} = \frac{\lambda_2\left(\rho_1/\mu_1 + \rho_2/\mu_2\right)}{1-\rho}. \tag{10}$$

Also, the stability steady state is $\rho \equiv \rho_1 + \rho_2 < 1$ and $0 < \rho < 1$. The other performance measurements are defined by:

$$W_q^{(i)} = \frac{L_q^{(i)}}{\lambda_i}, \tag{11}$$

$$W_s^{(i)} = W_q^{(i)} + \frac{1}{\mu_i}, \tag{12}$$

$$L_s^{(i)} = \lambda_i W_s^{(i)}\ ;\ i = 1,2 \tag{13}$$

LEFT AND RIGHT RANKING METHOD

In this section the steps to convert the fuzzy numbers into crisp numbers is explained, whether triangular of trapezoidal fuzzy numbers by using the left and right ranking procedure which is represented by $F(R) \to R$. To start mapping the algorithm for this method, consider the triangular fuzzy numbers $\widetilde{A}(z) = (a, b, c; w)$, then the left and right ranking is defined by:

$$R(\widetilde{A}) = \int_0^w \frac{L^{-1}(z) + R^{-1}(z)}{2}, \tag{14}$$

where $L^{-1}(z) = a + \left(\frac{b-a}{w}\right)z$ and $R^{-1}(z) = b + \left(\frac{c-b}{w}\right)$.

$$R(\widetilde{A}) = \frac{w(a + 2b + c)}{4}. \tag{15}$$

In the same way, consider the trapezoidal fuzzy numbers $\widetilde{A}(z) = (a, b, c, d; w)$

$$R(\widetilde{A}) = \frac{w(a + b + c + d)}{4} \tag{16}$$

NUMERICAL EXAMPLE

Consider a line production of machine receives two types of arrival customers $\tilde{\lambda}_1$,$\tilde{\lambda}_2$ and the service time can be represented as a mixture of the exponential distribution $\tilde{\mu}_1$ and $\tilde{\mu}_2$ respectively, noting that all parameters are in a fuzzy environment and the management wants to compute the mean of queue length for each class. Triangular and Trapezoidal fuzzy numbers are illustrated for the method as shown in the following subsections.

The Triangular Fuzzy Number

Assume that both arrival rate with two classes and service rates are given as $\tilde{\lambda}_1 = [20,30,40]$, $\tilde{\lambda}_2 = [60,70,80]$, $\tilde{\mu}_1 = [110,120,130]$ and $\tilde{\mu}_2 = [150,160,170]$. Through equation (15) the ranking of class one and class two are given as:

$$R(\tilde{\lambda}_2) = R(20,30,40;1) = \frac{(20 + 60 + 40)}{4} = 30 \tag{17}$$

$$R(\tilde{\lambda}_2) = R(60,70,80;1) = \frac{(60 + 140 + 80)}{4} = 70 \tag{18}$$

$$R(\tilde{\mu}_1) = R(110,120,130;1) = \frac{(110 + 240 + 130)}{4} = 120 \tag{19}$$

$$R(\tilde{\mu}_2) = R(150,160,170;1) = \frac{(150 + 320 + 170)}{4} = 160 \tag{20}$$

According to equation (9) and (10)

$$L_q^{(1)} = \frac{30\left(\frac{30}{120}/120 + \frac{70}{160}/160\right)}{1 - \frac{30+70}{120+160}} = 0.225 \tag{21}$$

$$L_q^{(2)} = \frac{70\left(\frac{30}{120}/120 + \frac{70}{160}/160\right)}{1 - \frac{30+70}{120+160}} = 0.524 \tag{22}$$

The Trapezoidal Fuzzy Number

Assume that both the arrival rate with two class and service rates are trapezoidal fuzzy numbers defined as $\tilde{\lambda}_1 = [20,30,40,50], \tilde{\lambda}_2 = [60,70,80,90]$, $\tilde{\mu}_1 = [110,120,130,140]$ and $\tilde{\mu}_2 = [150,160,170,180]$. In the same way, using equation (16), the ranking for trapezoidal numbers is obtained as $R(\tilde{\lambda}_1) = 35,\ R(\tilde{\lambda}_2) = 75,\ R(\tilde{\mu}_1) = 125$ and $R(\tilde{\mu}_2) = 165$ respectively. This leads to the computation for $L_q^{(1)}$and $L_q^{(2)}$ for trapezoidal fuzzy numbers. Recall equations (11), (12) and (13) to evaluate the system via another performance measurements, the results obtained are given in Table 1, which explain the different values for each class for the two types of membership functions; triangular and trapezoidal fuzzy numbers.

Table 1. Different Performance Measurements of Type I and Type II

Type	$L_q^{(1)}$	$L_q^{(2)}$	$W_q^{(1)}$	$W_q^{(2)}$	$W_s^{(1)}$	$W_s^{(2)}$	$L_s^{(1)}$	$L_s^{(2)}$
I.	0.225	0.524	0.007	0.007	0.015	0.031	0.47	0.91
II.	0.282	0.604	0.008	0.008	0.016	0.050	0.56	0.75

DISCUSSIONS

In this section the results are discussed and the system evaluated. It is clear the ranking method gives various sets of real values such as arrival rates and service rates for each class. Likewise, different performance measurements are obtained which are given and is seen to converge between two classes in the whole system. It is also seen from Table 1 that all performance measurements of class one are less than performance measures of class two in the system for both types of fuzzy numbers (triangular and trapezoidal). Also, using both fuzzy number types led to obtaining more real data and flexible choices in the system.

CONCLUSIONS

In this paper fuzzy set theory is shown to be a strong tool when dealing with real applications in queuing models with two classes such as manufacturing production line. The ranking approach adopted is also seen to be effective when transforming fuzzy queues into crisp queues, evaluating the system by conventional performance measurements such as the expected queue length of customers in the queues and system for both classes of arrivals. Also, the expected waiting time of customers in queue and in the whole system too. Therefore the manager can take the best values and make optimal decisions. Another advantage of using ranking method index is obtaining exact values inside closed crisp interval, while also providing more than one solution of values in the queueing system with different types of membership functions. For future work in this area can investigate the effectiveness of this approach to other queueing models and other linear membership functions.

REFERENCES

1. U. N. Bhat, *An Introduction to Queueing Theory* (Springer Science and Business Media, Birkhauser Boston, 2008).
2. D. Gross, J. F. Shortie, J. M. Thompson, and C. M. Harris, *Fundamentals of Queueing Theory* (John Wiley & Sons Inc, Hoboken New Jersey, 2008).
3. L. F. Zadeh, "Fuzzy Sets As a basic for a Theory of Possibility," Fuzzy Sets Syst. **1**, 3-28 (1978).
4. R. Jain, "Decision Making in the presence of fuzzy variables," IEEE Trans. Fuzzy Syst. **1**, 698–703 (1976).
5. R. Yager, "A procedure for ordering fuzzy sub sets of the unit interval," Inf. Sci. (Ny) **24(2)**, 143–161 (1981).
6. R. Ramesh, and S. K. Ghuru, "Analysis of Performance Measures of Fuzzy Queueing with Unreliable Server Using Ranking function Method," Int. J. of Adv. in Comp. Sci. and Tech. **3(10)**, 442-445 (2014).
7. B. Palpandi and G. Geetharamani, "Computing Performance Measures of Fuzzy Non-Preemptive Priority Queues Using Robust Ranking Technique," Appl. Math. Anal. **7(102)**, 5095–5102 (2013).
8. J. Devaraj and D. Jayalakshmi, "A Fuzzy Approach to Priority Queues," Int. J. Of Fuzzy Math. And Syst. **2(4)**, 479-488 (2012).
9. J. Devaraj, "A Fuzzy Approach to Non-Preempted Priority Queues," Int. J. Of Math. Archive. **3(7)**, 2704-2712 (2012).
10. R. Ramesh and S. K. Ghuru, "Priority Discipliened Queueing Models With Fuzzy Parameters," J. of Math. and Comp. Sci. **4(3)**, 594-602 (2014).