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A note on "The nearest symmetric fuzzy solution for a symmetric fuzzy linear system"

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Abstract

This paper provides accurate approximate solutions for the symmetric fuzzy linear systems in (*Allahviranloo et al.*[1]).

1 Introduction

The following section reviews basic definitions of fuzzy theory, which will be needed in the sequel:

Definition 1.1. *Let X be a universal set. Then, we define the fuzzy subset $\tilde{A}$ of X by its membership function $\mu_{\tilde{A}} : X \to [0,1]$ which assigns to each element $x \in X$ a real number $\mu_{\tilde{A}}(x)$ in the interval $[0,1]$; where the value $\mu_{\tilde{A}}(x)$ represents the grade of membership of x in $\tilde{A}$. A fuzzy set $\tilde{A}$ is written as:*

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X,\ \mu_{\tilde{A}}(x) \in [0,1]\}.$$

Definition 1.2. *A fuzzy set $\tilde{A}$ in $X = \mathbb{R}^n$ is convex fuzzy set if:*

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$$\forall x_1, x_2 \in X, \forall \lambda \in [0,1],$$
$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)).$$

Definition 1.3. *Let $\tilde{A}$ be a fuzzy set defined on the set of real numbers $\mathbb{R}$. $\tilde{A}$ is called normal fuzzy set if there exist $x \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x) = 1$.*

Definition 1.4. *A fuzzy number is a normal and convex fuzzy set, with its membership function $\mu_{\tilde{A}}(x)$ defined in real line $\mathbb{R}$ and piecewise continuous.*

Definition 1.5. *A fuzzy number $\tilde{A} = (a_1, a_2; \alpha, \beta)_{LR}$ is said to be an L-R fuzzy number, where its membership function satisfy*

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{a_1 - x}{\alpha}\right), & x \leq a_1 \quad \alpha > 0, \\ 1, & a_1 \leq x \leq a_2, \\ R\left(\frac{x - a_2}{\beta}\right), & a_2 \leq x \quad \beta > 0. \end{cases}$$

Where $a_1 \leq a_2$, and α and β are the left and right spreads, respectively; and the functions $L(.)$, $R(.)$, which are called left and right shape function, satisfying:

(1) $L(.)$, $R(.)$ are non–increasing from $\mathbb{R}^+$ to $[0, 1]$,

(2) $L(0) = R(0) = 1$, $L(1) = R(1) = 0$.

Also, if $\alpha = \beta$ and $L(x) = R(x)$ for all $x \in \mathbb{R}$ we say $\tilde{A}$ is a symmetric L-L fuzzy number.

Definition 1.6. ($Allahviranloo$ *et al.*[1]) *Let the shape functions $L(.)$, $R(.)$ are fixed. Consider two L-R fuzzy numbers as $\tilde{A} = (a_1, a_2; \alpha, \beta)$, and $\tilde{B} = (b_1, b_2; \gamma, \eta)$. We define the distance between $\tilde{A}$ and $\tilde{B}$ as follows:*

$$d\left(\tilde{A}, \tilde{B}\right) = \sqrt{\frac{[(a_1 - b_1) - (\alpha - \gamma)]^2 + [(a_2 - b_2) + (\beta - \eta)]^2 + (a_1 - b_1)^2 + (a_2 - b_2)^2}{4}}.$$

Definition 1.7. *A vector $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)$, where $\tilde{x}_i, 1 \leq i \leq n$ are L-R fuzzy numbers, is called an L-R fuzzy vector.*

Definition 1.8. ($Allahviranloo$ *et al.*[1]) *For two L-R fuzzy vectors $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)$, $\tilde{Y} = (\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_n)$ we defined*

$$D_p\left(\tilde{X}, \tilde{Y}\right) = \left(\sum_{i=1}^{n} d^p\left(\tilde{x}_i, \tilde{y}_i\right)\right)^{\frac{1}{p}}.$$

as distance between them, where $p \geq 1$.

2 Numerical examples

In this section we provide proposed solutions for the examples in [1].

Example 2.1. (*Allahviranloo et al.*[1])
According to [1], the symmetric exact solution for S-L-FLS is:

$$\tilde{X}_v = \begin{bmatrix} (x_1^1, x_2^1; \alpha_x^1, \alpha_x^1) \\ (x_1^2, x_2^2; \alpha_x^2, \alpha_x^2) \\ (x_1^3, x_2^3; \alpha_x^3, \alpha_x^3) \end{bmatrix} = \begin{bmatrix} (1, 2; 2, 2) \\ (-1, 1; 1, 1) \\ (2, 4; 3, 3) \end{bmatrix}.$$

But $\tilde{X}_v$ does not correspond to the system, for instance if $b_1^1 = 2$ is examined in vector $\tilde{B}$, we get :

$$(-1)(2) + (-1)(1) + (1)(2) = -1,$$

By using Definition 1.8. we produce $D_2\left(A\tilde{X}_v, \tilde{B}\right) = D_1\left(A\tilde{X}_v, \tilde{B}\right) = 3.$

However, the symmetric exact solution corresponds to the system by solving the associated linear system is as follows:

$$\tilde{X}_e = \begin{bmatrix} \left(-\frac{5}{4}, -\frac{1}{4}; 2, 2\right) \\ (-7, -5; 1, 1) \\ \left(-\frac{13}{4}, -\frac{5}{4}; 3, 3\right) \end{bmatrix},$$

$D_p\left(A\tilde{X}_e, \tilde{B}\right) = 0, \forall p \geq 1.$

Example 2.2 (*Allahviranloo et al.*[1])
According to[1], the nearest symmetric approximate solution is:

$$\tilde{X}_v = \begin{bmatrix} (x_1^1, x_2^1; \alpha_x^1, \alpha_x^1) \\ (x_1^2, x_2^2; \alpha_x^2, \alpha_x^2) \\ (x_1^3, x_2^3; \alpha_x^3, \alpha_x^3) \end{bmatrix} = \begin{bmatrix} (2, 2, ; 2, 2) \\ (0.8333, 3.1667; 1, 1) \\ (0.5, 0.5; 1, 1) \end{bmatrix},$$

$$\text{then } A\tilde{X}_v = \begin{bmatrix} \left(b_1^1, b_2^1; \alpha_b^1, \alpha_b^1\right) \\ \left(b_1^2, b_2^2; \alpha_b^2, \alpha_b^2\right) \\ \left(b_1^3, b_2^3; \alpha_b^3, \alpha_b^3\right) \end{bmatrix} = \begin{bmatrix} (-3.8334, 0.8334; 5, 5) \\ (-4.6667, -2.3333; 4, 4) \\ (-2.6667, -0.3333; 6, 6) \end{bmatrix},$$

$$D_1(A\tilde{X}_v, B) = 1.1666,$$
$$D_2(A\tilde{X}_v, B) = 0.763763.$$

In fact, there are many nearer (symmetric or non-symmetric) approximate solutions based on the distance metric function in Definition 1.8.

In this note, we illustrate two cases for approximate fuzzy solutions.

Case 1: Symmetric approximate solution

The following L-L fuzzy vector $\tilde{X}_1$ is a symmetric approximate solution for the system, with distance metric function smaller than distance of solution $\tilde{X}_v$ in [1].

Given

$$\tilde{X}_1 = \begin{bmatrix} (2, 2; 1, 1) \\ (0.75, 3.25; 0.75, 0.75) \\ (0.5, 0.5; 2.5, 2.5) \end{bmatrix}, \text{ then } A\tilde{X}_1 = \begin{bmatrix} (-4, 1; 5, 5) \\ (-4.75, -2.25; 4.25, 4.25) \\ (-2.75, -0.25; 5.25, 5.25) \end{bmatrix},$$

and the following result is obtained using Definition 1.8.

$$D_1(A\tilde{X}_1, \tilde{B}) = 0.707107,$$
$$D_2(A\tilde{X}_1, \tilde{B}) = 0.559017.$$

Case2: Non-symmetric approximate solution

The following L-R fuzzy number vector $\tilde{X}_2$ is a non-symmetric approximate solution for the system, with distance metric function smaller than distance of solution $\tilde{X}_v$ in [1].

Given

$$\tilde{X}_2 = \begin{bmatrix} \left(\frac{13}{6}, \frac{13}{6}; \frac{7}{6}, \frac{5}{6}\right) \\ \left(\frac{5}{6}, \frac{10}{3}; \frac{5}{6}, \frac{2}{3}\right) \\ \left(\frac{1}{2}, \frac{1}{2}; \frac{5}{2}, \frac{5}{2}\right) \end{bmatrix}, \text{ then } A\tilde{X}_2 = \begin{bmatrix} (-4, 1; 5, 5) \\ \left(-5, -\frac{5}{2}; 4, \frac{9}{2}\right) \\ \left(-3, -\frac{1}{2}; 5, \frac{11}{2}\right) \end{bmatrix},$$

and we produce the following results

$$D_1(A\tilde{X}_2, \tilde{B}) = 0.809017,$$
$$D_2(A\tilde{X}_2, \tilde{B}) = 0.612372.$$

Note:

Our new solutions are obtained by using distance metric function which not only provides L-L fuzzy number vector, but also L-R fuzzy number vector.

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