Formal Analysis of Temporal Dynamics in Anxiety States and Traits for Virtual Patients

Abstract. This paper presents a temporal dynamic model of anxiety states and traits for an individual. Anxiety is a natural part of life, and most of us experience it from time to time. But for some people, anxiety can be extreme. Based on several personal characteristics, traits, and a representation of events (i.e. psychological and physiological stressors), the formal model can represent whether a human that experience certain scenarios will fall into an anxiety states condition. A number of well-known relations between events and the course of chronic fatigue are summarized from the literature and it is shown that the model exhibits those patterns. In addition, the agent model has been mathematically analyzed to find out which stable situations exist. Finally, it is pointed out how this model can be used in therapy, supported by a software agent.

Keywords. Anxiety Traits and States, Formal Analysis, Virtual Patient, Computational Psychology

1 Theoretical Background of Anxiety State and Traits

Anxiety can be defined as an unpleasant state of mental uneasiness or concern that causes physical and psychological discomfort. This unpleasant state may cause physical symptoms such as a racing heart and shakiness. There are various forms of anxiety disorders, including generalized anxiety disorder, phobic disorder, and panic disorder [1]. While each has its own characteristics and symptoms, they all include symptoms of anxiety. Modeling of cognitive behaviour is important for the development of agents that should exhibit human-like behaviour. For example, the development of virtual agents in games and virtual learning environments that should interact in a realistic manner has to incorporate the effect of its cognitive behaviours, especially on behaviour analysis and virtual training environment [2]. These kinds of agents can also be used to perform simulations of humans in particular situations to study their behaviour without having to perform real life experiments [3, 4, 5]. The application in a virtual patient used in this paper is an example of such type of applications. This paper is organized as follows. First, the basic functioning of the anxiety state model is explained at a high level in Section 2 and its mathematical formalization is introduced in Section 3. Next, the Section 4 gives a description of mathematical analysis performed with the model. In Section 5, the model has been verified by automated verification. Finally, Section 6 concludes the paper and discusses future work.

2 Theoretical Background of Anxiety State and Traits

According to Well's model (Meta-cognitive Model), problematic worry develops over time. It begins with a tendency to use worry as a coping strategy for real or imagined threats. When a person is initially faced an anxiety provoking event, positive beliefs about worry are compromised (known as Type 1 Worry). Worry is a weak cognitive attempt to avoid the aversive somatic and emotional experiences which naturally occur when being confronted with an episode of fear [1, 4]. Normally, it is caused by non-cognitive events (external situations or physical symptoms) and triggers anxiety responses. However, the responses may increase (or decrease) the anxiety state according to the initial condition of the problems [6]. During the course of Type 1 worry, coping strategy and individual's sensitivity will regulate the formation of short-term worry. Higher sensitivity increases the formation of beliefs about worry and reduces the ability to cope accordingly [7]. Engagement in ineffective coping strategies provides a chance about the belief that is uncontrollable. Effective coping results in adaptation, while ineffective coping results in maladaptation [8]. As a result, it escalates short-term worry [6, 8, 9, 10].

Individuals with anxiety traits and negative personality will later experience a negative reinforcement spiral experience of worry that further reinforces the worry [10]. It explains the condition where the individuals feel that the worry is uncontrollable or probably dangerous to them. This concept of "worry about worry" is known as Type 2 worry. An increased "worry about worry" is posited to lead to a spiralling of the worry emotion in a long run [8, 10, 11]. This later increases the long-term worry that will influence individual's thought control over negative events (triggers). This process explains that the high levels of anxiety. The intolerance to uncertainty serves to set off a chain of worrying, negative problem orientation, and cognitive avoidance. An increased IU therefore is posited to lead to a spiralling of the worry emotion [13, 15].

In short, the following relations can be identified from the literature: (1) a series of psychological and physiological stressor events can lead to the formation of anxiety; (2) low coping skills will increase the risk anxiety strait; (3) negative personality and personality traits factors aggravate the effect anxiety; (4) prolonged sensitivity will increase belief about worry; (5) good coping strategies and appraisal will reduce worry; (6) prolonged short-term worry will increase the risk of long-term worry in the future.

3 Formal Model

This section explains the details of the model in a mathematical specification. The implemented relations between different concepts are based on earlier findings in literature on anxiety state and disorder. The general structure of the formal model for anxiety state is shown in Fig. 1. In this figure, it can be seen that the model consists of several interrelated nodes.

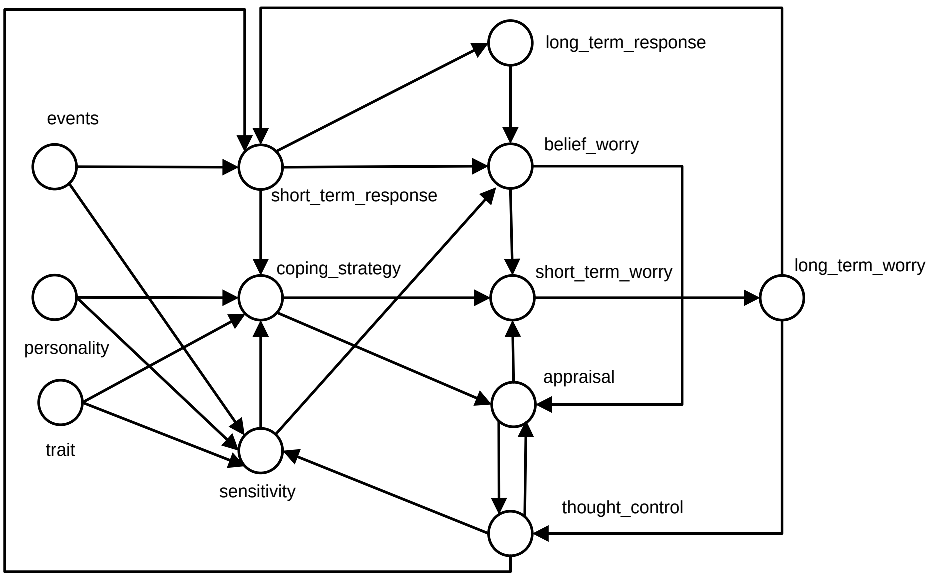

Fig. 1. Global Relationships of Variables Involved in the Formation of Worry

Once the structural relationships in the model have been determined, the model can be formalized. In the formalization, all nodes are designed in a way to have values ranging from 0 (low) to 1 (high). This model involves a number of instantaneous and temporal relations, which will be discussed in greater detailed below.

3.1 Instantaneous Relationships

Physical event (*Pe*) is influenced by the amount of threatening event (*Te*). Coping skills (*Cs*) is proportional to the sensitivity (*Sy*), personality (*Ps*) and short-term response (*Sr*). Short-term response (*Sr*) is represented by a level of physical event, long-term worry (*Lw*) and personal traits (*Tr*).

$$Pe(t) = \sigma_p . Te(t) \quad \textbf{(1)}$$
$$Cs(t) = \left[\gamma_c . \left(1 - Sy(t)\right) + (1 - \gamma_c) . Ps(t)\right] . (1 - Sr(t)) \quad \textbf{(2)}$$
$$Sr(t) = [\alpha_s . Pe(t) + (1 - \alpha_s) . Lw(t)] . (1 - Tr(t)) \quad \textbf{(3)}$$

The impact of belief about worry (*Bw*) are dependent on low and high values from short-term response, long-term response (*Lr*) and sensitivity. Short-term worry (*Sw*) itself is increased by low level of coping skills (*Cs*), appraisal (*Ap*) and a high level of belief about worry. Appraisal decreases the thought control (*Tc*) while long-term worry (*Lw*) increases the effect of thought control.

$$Bw(t) = \alpha_b . [\beta_b . Sr(t) + (1 - \beta_b) . Lr(t)] + (1 - \alpha_b) . Sy(t) \quad \textbf{(4)}$$
$$Sw(t) = [1 - \left(\left(\theta_s . Cs(t) + (1 - \theta_s) . Ap(t)\right)\right) . \left(1 - Bw(t)\right)] \quad \textbf{(5)}$$

$$Tc(t) = (1 - Ap(t)).Lw(t)) \quad \textbf{(6)}$$

Sensitivity (*Sy*) can be described through the proportional contribution of personality, thought control, normal sensitivity level (Sy_{norm}) and physical event.

$$Sy(t) = \Psi_s.Sy_{nom}(t).[1 - Ps(t)] + (1 - \Psi_s).[\lambda s_1.Sy_{nom}(t) + \lambda s_2.Pe(t)].(1 - Tc(t)) \quad \textbf{(7)}$$

3.2 Temporal Relationships

Long term response (*Lr*) is primarily contributed the accumulation exposure towards short term response, while the accumulated short-term worry produces long-term worry (*Lw*). The formation of long-term appraisal is modelled using the presence coping skills, thought control and belief about worry.

$$Lr(t + \delta t) = Lr(t) + \beta_L.[Pos(Sr(t) - Lr(t)).(1 - Lr(t)) - Pos(-(Sr(t) - Lr(t)).Lr(t)].\delta t \quad \textbf{(8)}$$

$$Lw(t + \delta t) = Lr(t) + \phi_L.[Pos(Sw(t) - Lw(t)).(1 - Lw(t)) - Pos(-(Sw(t) - Lw(t)).Lw(t)].\delta t \quad \textbf{(9)}$$

$$Ap(t + \delta t) = Ap(t) + \rho_a.[Pos(Sg(t) - Ap(t)).(1 - Ap(t)) - Pos(-(Sg(t) - Ap(t)).Ap(t)].\delta t \quad \textbf{(10)}$$

where $Sg(t) = [Ws_1.Cs(t) + Ws_2.Tc(t)].(1 - Bw(t))$

Note that the change process is measured in a time interval between t and $t + \delta t$. In addition to all this, the rate of change for all temporal specifications are determined by flexibility rates β_L, ϕ_L, and ρ_a. Using all defined formulas, a simulator was developed for experimentation purposes; specifically to explore interesting patterns and traces that explain the behaviour of the human agent model related to anxiety states.

4 Mathematical Analysis

For the mathematical verification, equillibria analysis is used to describe situations in models where the values (continuous) approach a limit under certain conditions and stabilize (the trajectories do not change too much under small perturbations) [12, 14]. It means, if the dynamics of a model is described by a differential equation, then equilibria can be estimated by setting a derivative (or all derivatives) to zero. One important note that an equillibria condition(s) is considered stable if the model always returns to it after small disturbances. For example, using the autonomous equation,

$$\frac{dy}{dx} = f(y)$$

the equillibria or constant solutions of this equation are the roots of the equation

$$f(y) = 0$$

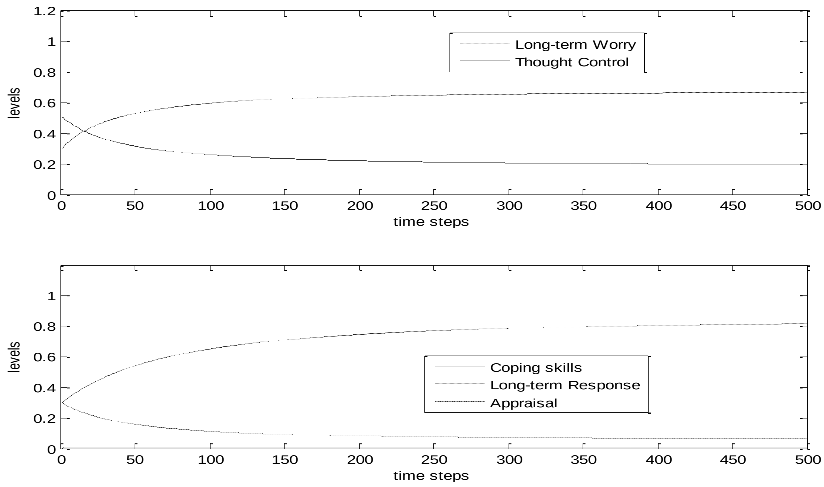

Fig. 2. Equillibria Conditions for Selected Variables in the Model

If unstable, the system will move away from its point when slightly disturbed. These equillibria conditions are interesting to be explored (as depicted in Fig. 2), as it is possible to explain them using the knowledge from the theory or problem that is modelled. As such, the existence of reasonable equilibria is also an indication for the correctness of the model. To obtain possible equilibrium values for the other variables, first the temporal equations are described in a differential equation form,

$$\frac{dLr}{dr} = \beta_L.[Pos(Sr(t) - Lr(t)).(1 - Lr(t)) - Pos(-(Sr(t) - Lr(t)).Lr(t)] \quad \textbf{(11)}$$

$$\frac{dLw}{dt} = \phi_L.[Pos\big(Sw(t) - Lw(t)\big).\big(1 - Lw(t)\big) - Pos\big(-\big(Sw(t) - Lw(t)\big).Lw(t)\big]. \quad \textbf{(12)}$$

$$\frac{dAp}{dt} = \rho_a.[Pos(Sg(t) - Ap(t)).(1 - Ap(t)) - Pos(-(Sg(t) - Ap(t)).Ap(t)] \quad \textbf{(13)}$$

Next, the equations are identified that describe:

$$\frac{dLr}{dt} = \frac{dLw}{dt} = \frac{dAp}{dt} = 0$$

Assuming all parameters are non-zero, this provides the following equilibrium equations;

$$Pos\big(Sr(t) - Lr(t)\big).\big(1 - Lr(t)\big) = 0$$
$$-Pos(-(Sr(t) - Lr(t)).Lr(t) = 0$$

Notice that Pos(x) > 0, so this equilibrium equation is equivalence to;

$$Pos(Sr - Lr).(1 - Lr) - Pos(-(Sr - Lr).Lr) = 0 \quad \textbf{(14)}$$

This later provides cases

$$(Sr \le Lr \wedge Sr \ge Lr) \vee (Sr \le Lr \wedge Lr = 0) \vee$$
$$(Lr = 1 \wedge Sr \ge Lr) \vee (Lr = 1 \wedge Lr = 0)$$

The latter case cannot exist, and as $0 \le Lr \le 1$ the other three cases are equivalent to $Sr=Lr$. Similar the cases for equations (13) and (14), the equillibria state occurs when $Sw = Lw$ and $Sg = Ap$ respectively. Note that for each of the distinguished cases, further information can be found about the equilibrium values of other variables using the other non-dynamic-equations.

Case #1: *Sr = Lr*

$$Cs = [\gamma_c.(1 - Sy) + (1 - \gamma_c).Ps].(1 - Lr) \quad \textbf{(15)}$$

$$Bw = \alpha_b.[\beta_b.Lr + (1 - \beta_b).Sr] + (1 - \alpha_b).Sy \quad \textbf{(16)}$$

Case #2: *Sw = Lw*

$$Sr = [\alpha_s.Pe + (1 - \alpha_s).Sw].(1 - Tr) \quad \textbf{(17)}$$

$$Tc = (1 - Ap).Lw \quad \textbf{(18)}$$

Case #3: *Sg = Ap*

$$Sw = [1 - \big((\theta_s.Cs + (1 - \theta_s).[Ws_1.Cs + Ws_2.Tc].(1 - Bw))\big).(-Bw)] \quad \textbf{(19)}$$

$$Tc = (1 - [Ws_1.Cs + Ws_2.Tc].(1 - Bw)).Lw) \quad \textbf{(20)}$$

Case #4: Lr =1 ^ Sr = Lr

$$Cs = 0 \quad \textbf{(21)}$$

$$Bw = \alpha_b.[\beta_b + (1 - \beta_b)] + (1 - \alpha_b).Sy \quad \textbf{(22)}$$

Assuming the proportional contribution of $\beta_b = \alpha_b = 0.5$, therefore,

$$Bw = 0.5 + 0.5Sy \quad \textbf{(23)}$$

5 Temporal Trace Analysis

In addition to this verification process, the results of the experiment and a number from properties from the literature have been analysed in more detail by converting them into formally specified traces and checking relevant properties. These properties were translated as temporal logical expression, against these traces. To this end, a number of properties were logically formalized in the Temporal Trace Language (TTL). This predicate logical language is built on atoms referring to states of the world, time points and traces. States are related to state properties via the satisfaction relation denoted by the prefix predicate holds (or by the infix predicate |=): holds(state(γ,t), *p*) (or state(γ,t) |= *p*), which denotes that state property *p* holds in trace γ at time point t. In general, TTL terms are constructed by induction in a standard way from variables, constants and function symbols typed with all before-mentioned TTL sorts. Transition relations between states are described by dynamic properties, which are expressed by TTL-formulae [16]. The set of well-formed TTL-formulae is defined inductively in a standard way using Boolean connectives (such as ¬,∧,∨,⇒,∃,∀) , and quantifiers over variables of TTL sorts. An example of the TTL formula, which describes observational belief creation of a virtual agent, is given below:

"In any trace, if at any point in time t1 the virtual agent *A* observes that it is raining, then there exists a point in time t2 after t1 such that at t2 in the trace the virtual agent *A* believes that it is raining."

∀γ ∀t1 [holds(state(γ,t1),observation result(itsraining)) ⇒
∃t2 > t1 holds(state(γ,t2),belie f(itsraining))]

As an input for this analysis technique either a simulation or a formalized empirical trace(s) is/are provided. A trace is represented by a finite number of state atoms, changing their values over time a finite number of times, i.e., complies with the finite mathematical specifications (formal properties) as defined in Section 3. For each of the properties, first an informal description is given, and next the formal description that has been used for the automated checking software.

VP1: Low Trait and Positive Personality will Reduce Anxiety State

Individuals with less negative personality and low anxiety trait develop lesser chance of having long-term worry.

VP1≡ ∀γ:TRACE, t1, t2, t3 :TIME, v1,v2,w1,w1,h1, h2:REAL
[state(γ, t1)|=personality(v1) &
state(γ, t1)|=personal_trait(w1) &
state(γ, t1)|=long_term_worry(h1) &
state(γ, t2)|=personality(v2) &
state(γ, t2)|=personal_trait(w2) &
v2 < v1 & w2 < w1] ⇒ ∃t3:TIME > t2:TIME &
t2:TIME > t1:TIME [state(γ, t3)|= long_term_worry(h2) & h1 > h2]

VP2: Higher Sensitivity Increases Worry

Individual's sensitivity is related to the risk of long term worry.

VP2 ≡ ∀γ: TRACE, ∀t1, t2:TIME, ∀F1,F2,H1,H2, d:REAL
[state(γ,t1)|= sensitivity(F1) &
state(γ,t1)|= long_term_worry(H1) &
state(γ,t2)|= sensitivity(F2) &
state(γ,t2)|= long_term_worry(H2) &
t2 ≥t1 +d & F1< F2] ⇒ H2 > H1

VP3: Monotonic Decrease of Long-term Worry for Any Individual When Sensitivity and Belief about Worry, are Reduced

When a person manages to control his or her perception (sensitivity) and belief about the negative consequences of the experienced events throughout time, then the person will reduce the level of long-term worry in future.

VP3 ≡ ∀γ:TRACE,t1,t2:TIME,D1,D2,E1,E2, H1, H2:REAL
[state(γ, t1)|= sensitivity(D1) &
state(γ, t2)|= sensitivity(D2) &
state(γ, t1)|= belief_about_worry(X,E1) &
state(γ, t2)|= belief_about_worry(X,E2) &
state(γ, t1)|= long_term_worry(X,H1) &
state(γ, t2)|= long_term_worry(X,H2) &
t2 > t1 & D2 ≥ D1 & E1 ≥ E2] ⇒ H2 ≤ H1

VP4: Good Coping Strategy Decreases Worry

A good coping skill (e.g. problem-focused coping) is a better option to reduce worry.

VP4 ≡ ∀γ: TRACE, ∀t1, t2:TIME, ∀F1,F2,H1,H2, d:REAL
[state(γ,t1)|= coping(F1) &
state(γ,t1)|= long_term_worry(H1) &
state(γ,t2)|= coping(F2) &
state(γ,t2)|= long_term_worry(H2) &
t2 ≥t1 +d & F1< F2] ⇒ H2 > H1

VP5: Monotonic Increase of Variable, v for Worry Amplifies Future Response over Negative Events

For all time points t1 and t2 between tb and te in trace γ if at t1 the value of v is x1 and at t2 the value of v is x2 and t1 < t2, then x2 ≥ x1

VP5 ≡ ∀γ: TRACE, ∀t1, t2:TIME, ∀X1,X2:REAL
[state(γ,t1)|= has_value(v, X1) &
state(γ,t2)|= has_value(v, X2) &
tb ≤ t1 ≤ te &
tb ≤ t2 ≤ te &
⇒ x2 ≥ x1

6 Conclusion

A computational model of anxiety dynamics (traits and states) has been presented that incorporates concepts from general theories about anxiety traits and states. This model has been used to simulate different scenarios in which personal characteristics determine the effect of related traits and states on the anxiety level of a person. A mathematical analysis illustrated the different equilibriums of the model for persons with different characteristics. By formally checking properties of the simulation traces, the adherence of the model to the most important ideas in the theories was internally validated. This work provides the first step in the development of an intelligent software agent or robot to support individuals with anxiety traits and states in a personal manner. Future work of this agent and model integration will be specifically focus how interactions and sensing properties can be further developed and enriched, to promote a better way to fluidly embedded this into any monitoring and health informatics system

References

1. D.E, Aikins, M.G. Craske, Cognitive Theories of Generalized Anxiety Disorder, Psychiatr Clin North Am, vol. 24, pp. 57-74, (2001).
2. A.A. Aziz, M.C.A. Klein, Computational Modeling of Therapies Related to Cognitive Vulnerability and Coping, 2nd International Conference on Biologically Inspired Cognitive Architectures (BICA 2011), IOS Press, USA. pp.16-25, (2011)
3. A. A. Aziz, M.C.A. Klein, J. Treur, An Agent Model of Temporal Dynamics in Relapse and Recurrence in Depression. Proceedings of the 22nd International Conference on Industrial, Engineering & Other Applications of Applied Intelligent Systems, LNAI, Springer Verlag, pp. 36-45, (2009).
4. G.I. Barbash, S.A. Glied, New Technology and Health Care Costs: The Case of Robot-Assisted Surgery, The New England Journal of Medicine, vol. 363, pp. 701-70, (2010).
5. C. Bartneck, T. Suzuki, T. Kanda, T. Nomura, The Influence of People's Culture and Prior Experiences with Aibo on Their Attitude Towards Robots, AI & Society, vol. 21 (1-2), pp. 217-230, (2007).
6. G.E Berrios, Anxiety Disorders: A Conceptual History, Journal of Affect Disorder, vol. 56(3), pp. 83-94, (1999).
7. J. F. Brosschot, S. Pieper, J. F. Thayer, Expanding Stress Theory: Prolonged Activation and Preservative Cognition, Psychoneuroendocrinology, vol. 30(10), pp. 1043-1049, (2005).
8. G.P. Chrousos, P.W. Gold, The Concepts of Stress and Stress System Disorders Overview of Physical and Behavioural Homeostasis. J. Am. Med. Assoc. 267, pp. 1244–1252, (1992).
9. S. Dickerson, S. Gruenewald, S., T. Kemeny, Psychobiological Responses to Social Self -Threat: Functional or Detrimental? Self and Identity, Vol. 8(2-3), pp. 270-285, (2009).
10. C.K. Gale, M. Oakley-Browne, Generalized Anxiety Disorder, Am Fam Physician, vol. 67, pp. 135-138, (2003).
11. M.F. Gliatto, Generalized Anxiety Disorder, Am Fam Physician, vol. 62(7), pp. 1591-1600, (2000).
12. D. Mareschal, S.C. Thomas, Computational Modelling in Development Psychology, IEEE Transaction on Evolutionary Computation, Vol. 11(2), pp. 137-150, (2007).
13. G. Miller, E. Chen, E. Zhou, E, If it goes up, Must it Come Down? Chronic stress and the Hypothalamic-Pituitary-Adrenocortical Axis in Humans, pp. 25-45, (2006).
14. F. Richardson, M.S.C., Thomas, The Benefits of Computational Modelling for the study of Development Disorders, Developmental Sci., Vol. 9, pp. 151-155, (2006).
15. D. Stein, The Cognitive-Affective Neuroscience of Depression and Anxiety Disorders, (2003).
16. A.Sharpanskykh and J. Treur, A temporal trace language for formal modelling and analysis of agent systems, in Specification and verification of multi-agent systems, Springer, pp. 317–352, (2010).