Luttinger Theorem for the Strongly Correlated Fermi Liquid of Composite Fermions

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While an ordinary Fermi sea is perturbatively robust to interactions, the paradigmatic composite-fermion (CF) Fermi sea arises as a nonperturbative consequence of emergent gauge fields in a system where there was no Fermi sea to begin with. A mean-field picture suggests two Fermi seas, of composite fermions made from electrons or holes in the lowest Landau level, which occupy different areas away from half filling and thus appear to represent distinct states. Using the microscopic theory of composite fermions, which satisfies particle-hole symmetry in the lowest Landau level to an excellent approximation, we show that the Fermi wave vectors at filling factors ν and $1-\nu$ are equal when expressed in units of the inverse magnetic length, and are generally consistent with the experimental findings of Kamburov *et al.* [Phys. Rev. Lett. 113, 196801 (2014)]. Our calculations suggest that the area of the CF Fermi sea may slightly violate the Luttinger area rule.

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A fundamental property of a Landau Fermi liquid is captured by Luttinger's theorem [1], according to which the volume occupied by the Fermi sea, appropriately defined, remains invariant when the interaction is switched on, so long as no phase boundary is crossed. A violation of this theorem signifies non-Fermi liquid behavior, which has motivated investigations [2–7] of its applicability for various strongly correlated systems, such as high-temperature superconductors. This Letter investigates the Luttinger theorem for an exotic emergent Fermi sea.

When two-dimensional electrons are subjected to a strong magnetic field, they exhibit the phenomenon of the fractional quantum Hall effect (FQHE) [8], which is understood in terms of topological particles called composite fermions [9–13]. Halperin, Lee, and Read [11] and Kalmeyer and Zhang [14] theoretically predicted that at Landau level (LL) filling factor $\nu = 1/2$, the external magnetic field is canceled, in a mean-field (MF) approximation, by the emergent gauge field carried by composite fermions, and they form a Fermi sea. The composite-fermion (CF) Fermi sea is special in the following sense. Ordinarily, we begin with the perfect Fermi sea of noninteracting fermions and then ask how interactions degrade or destroy it. In contrast, interactions are fully responsible for creating the CF Fermi sea (CFFS) in a system of electrons confined to the lowest LL (LLL) where, originally, there was no Fermi sea and, in fact, no kinetic energy and no composite fermions. The very existence of the CFFS thus is a manifestation of strong correlations. The essential validity of the CFFS has been confirmed in extensive detail in many experiments [15–23], and it also dovetails with the prominently observed sequences of fractions at $\nu = n/(2n \pm 1)$ [9].

Kamburov *et al.* [24] have recently made accurate measurements of the CF Fermi wave vector through commensurability effects in the presence of a periodic modulation. They have observed more commensurability oscillations than before and thus provided the most detailed confirmation of the CFFS state to date. Furthermore, the unprecedented accuracy of their measurement has revealed an intriguing puzzle. For electrons confined to the LLL, one can take two exactly equivalent starting points: One can define the problem in terms of either electrons at ν or holes at $1-\nu$. One can then go ahead and composite fermionize either electrons or holes to produce what we will label ^{e}CFs or ^{h}CFs, which experience an effective magnetic field given by $B^* = B - 2\rho\phi_0$, where $\phi_0 = hc/e$ and ρ is the density of composite fermions. (All CF quantities are marked by an asterisk.) For $\nu \neq 1/2$ the ^{e}CFs and ^{h}CFs have different densities, producing, *in the MF description*, different Fermi wave vectors for the fully spin-polarized CFFS state:

$$\text{MF for } {}^e\text{CFFS}: k_F^* = \sqrt{4\pi\rho_e} \leftrightarrow k_F^*\ell = \sqrt{2\nu},$$
$$\text{MF for } {}^h\text{CFFS}: k_F^* = \sqrt{4\pi\rho_h} \leftrightarrow k_F^*\ell = \sqrt{2(1-\nu)},$$

where $\ell = \sqrt{\hbar c/eB}$ is the magnetic length and the electron and hole densities are given by $\rho_e = \rho_\nu = \nu/(2\pi\ell^2)$ and $\rho_h = \rho_{1-\nu} = (1-\nu)/(2\pi\ell^2)$, respectively. The CFFS thus appears to have a split personality. This raises many interesting conceptual questions. At a given ν, do the ^{e}CFFS and ^{h}CFFS represent two distinct states, or are they dual descriptions of the same state? If the former is true, then which of these two, if either, occurs in real systems? If the latter is true, how does one reconcile the seemingly incompatible consequences of the MF picture, and how does one understand the violation of the Luttinger theorem for at least one of the two descriptions? In either case, what is the role of particle-hole (p-h) symmetry in the LLL? Finally, how do we understand the remarkable finding of Kamburov *et al.* [24] that the measured value of the CF Fermi wave vector is consistent with that expected from the smaller Fermi sea, namely, $k_F^*\ell = \min[\sqrt{2\nu}, \sqrt{2(1-\nu)}]$?

These observations have motivated two striking theoretical proposals that lead to experimentally testable predictions. Son has proposed [25] that viewing the composite fermion as a Dirac fermion allows a p-h symmetric description of the FQHE and the CFFS. Barkeshli, Mulligan, and Fisher [26] have taken the experimental observations to imply that the ^{e}CFFS and the ^{h}CFFS are distinct states of matter and a spontaneous breakdown of p-h symmetry within the LLL selects one of them. Within the Chern-Simons (CS) formulation of composite fermions [10,11], the MF Fermi wave vector $k_F^{*\mathrm{MF}}\ell$ is not expected to change to all orders in a perturbative treatment of the Coulomb and the gauge interactions, suggesting that the ^{e}CFFS and ^{h}CFFS are perturbatively disconnected, i.e., are topologically distinct, for any $\nu \neq 1/2$ and, by extension, also for $\nu = 1/2$. The CS formulation, however, does not implement the LLL constraint and hence does not satisfy the p-h symmetry, as has been stressed elsewhere in the literature [25,27].

We determine the CFFS area by using a different theoretical formulation of the CF paradigm, namely, the microscopic wave functions of composite fermions [9,12,13,28]. This theory (i) is explicitly restricted to the LLL, (ii) satisfies p-h symmetry, and (iii) does not assume, *a priori*, any specific value for $k_F^*\ell$. We show that $k_F^*\ell$ defined from Friedel oscillations in the pair-correlation function has the same value for states at ν and 1-ν related by p-h symmetry. We explicitly calculate $k_F^*\ell$ for filling factors in the vicinity of $\nu = 1/2$.

We define the Fermi wave vector through the Friedel oscillations in the pair-correlation function, for which we take the form [29]

$$g(\boldsymbol{r}) = 1 + A(r\sqrt{4\pi\rho_e})^{-\alpha}\sin(2k_F^* r + \theta), \tag{1}$$

where A, α, k_F^*, and θ are fitting parameters. We denote the particle coordinates by either $\boldsymbol{r}_j$ or $z_j = x_j - iy_j$ and set $\ell = 1$. This form is motivated by the observation that for noninteracting fully spin-polarized fermions in two dimensions at $B = 0$, the oscillatory part of $g(r)$ for large rk_F is given by $(4/\pi r^3 k_F^3)\sin(2k_F r)$. Let us consider a wave function ϕ_ν for a uniform density state at filling factor ν. Its pair-correlation function is given by

$$g_\nu(\boldsymbol{r},\boldsymbol{r}') = \rho_\nu^{-2}\langle\phi_\nu|\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r}')\hat{\Psi}(\boldsymbol{r}')\hat{\Psi}(\boldsymbol{r})|\phi_\nu\rangle, \tag{2}$$

$$\phi_\nu = \frac{1}{N!}\int\prod_{j=1}^{N} d^2\boldsymbol{r}_j\phi_\nu(\boldsymbol{r}_1,\ldots\boldsymbol{r}_N)\prod_{k=1}^{N}\hat{\Psi}^\dagger(\boldsymbol{r}_k)|0\rangle, \tag{3}$$

where $\phi_\nu(\boldsymbol{r}_1,\ldots\boldsymbol{r}_N)$ is the real space wave function, $\hat{\Psi}(\boldsymbol{r}) = \sum_{m=0}^{\infty}\eta_m(\boldsymbol{r})c_m$ is the electron annihilation operator in the LLL, and $\hat{\Psi}^\dagger(\boldsymbol{r})$ is the corresponding electron creation operator, with the single-particle LLL wave function defined as $\eta_m = (2\pi 2^m m!)^{-1/2}z^m\exp[-|z|^2/4]$. We can similarly define the pair-correlation function for electrons at $1-\nu$, with

$$\phi_{1-\nu} = \frac{1}{N!}\int\prod_{j=1}^{N} d^2\boldsymbol{r}_j\phi_\nu^*(\boldsymbol{r}_1,\ldots\boldsymbol{r}_N)\prod_{k=1}^{N}\hat{\Psi}(\boldsymbol{r}_k)|1\rangle, \tag{4}$$

where $|1\rangle$ is the state with the LLL fully occupied. Substituting into the expression for the pair-correlation function and noting $\langle 1|f(c_m,c_m^\dagger)|1\rangle = \langle 0|f(c_m\to c_m^\dagger, c_m^\dagger\to c_m)|0\rangle$ produces the relation

$$g_{1-\nu}(\boldsymbol{r},\boldsymbol{r}') = \rho_{1-\nu}^{-2}\langle\phi_\nu|\hat{\Psi}(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r}')\hat{\Psi}^\dagger(\boldsymbol{r}')\hat{\Psi}^\dagger(\boldsymbol{r})|\phi_\nu\rangle. \tag{5}$$

In terms of the LLL projected delta function [30,31]

$$\bar{\delta}(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{2\pi}\exp\left[-\frac{1}{4}(|\boldsymbol{r}-\boldsymbol{r}'|^2 - zz'^* + z'z^*)\right], \tag{6}$$

which satisfies $\bar{\delta}(\boldsymbol{r},\boldsymbol{r}') = [\bar{\delta}(\boldsymbol{r}',\boldsymbol{r})]^*$, we have $\{\hat{\Psi}(\boldsymbol{r}), \hat{\Psi}^\dagger(\boldsymbol{r}')\} \equiv \bar{\delta}(\boldsymbol{r},\boldsymbol{r}')$, $\langle\phi_\nu|\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r}')|\phi_\nu\rangle = \nu\bar{\delta}(\boldsymbol{r}',\boldsymbol{r})$, and $\langle\phi_\nu|\hat{\Psi}(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r}')|\phi_\nu\rangle = (1-\nu)\bar{\delta}(\boldsymbol{r},\boldsymbol{r}')$. Straightforward algebra gives the relation (assuming thermodynamic limit and translational invariance and setting $\boldsymbol{r}' = 0$)

$$g_{1-\nu}(r) = \frac{(1-2\nu)(1-e^{-r^2/2}) + \nu^2 g_\nu(r)}{(1-\nu)^2}, \tag{7}$$

where we have assumed the same magnetic lengths for ν and $1-\nu$. For $r \gg 1$, this reduces to $g_{1-\nu}(r) = 1 + [\nu/(1-\nu)]^2[g_\nu(r) - 1]$. The important point is that an oscillatory term $\sin(2k_F^* r)$ in g_ν implies identical oscillatory behavior for $g_{1-\nu}$, indicating that the states at ν and $1-\nu$ have the same $k_F^*\ell$. The "exact" $k_F^*\ell$ is thus the same at ν and $1-\nu$ and is independent of whether the problem is formulated in terms of electrons or holes.

We next determine the value of $k_F^*\ell$ in a microscopic calculation from the oscillations in $g(r)$ following Refs. [29,32]. Being an equal time correlation function, $g(r)$ can be evaluated from the knowledge of the microscopic wave functions for the ground state in the vicinity of $\nu = 1/2$. We concentrate on the fractions $\nu = n/(2n \pm 1)$ which approach $\nu = 1/2$ in the limit of sufficiently large n. The microscopic Jain wave functions for these states are given by [9]

$$\Psi_{n/(2n\pm 1)} = \mathcal{P}_{\mathrm{LLL}}\prod_{j<k=1}^{N}(z_j - z_k)^2\Phi_{\pm n}, \tag{8}$$

where $\mathcal{P}_{\mathrm{LLL}}$ denotes LLL projection and Φ_n is the wave function of n filled LLs, with $\Phi_{-n} = [\Phi_n]^*$. At first it may appear that the above-mentioned dichotomy is present also in the microscopic theory of composite fermion, as we illustrate by considering the fraction $\nu = [(n+1)/(2n+1)]$. [Similar considerations apply to $\nu = n/(2n+1)$.] According to the CF theory, there are two ways of constructing a FQHE state at this fraction: (i) As the electron partner of the $\nu^* = n$ integer QHE (IQHE) of ^{h}CFs in positive B^*, with the wave function given by $C_{p\text{-}h}\mathcal{P}_{\rm LLL}\prod_{j<k}(z_j - z_k)^2\Phi_n$, where $C_{p\text{-}h}$ represents p-h transformation; or, (ii) as the $\nu^* = n+1$ IQHE of ^{e}CFs in a negative B^*, with the wave function given by $\mathcal{P}_{\rm LLL}\prod_{j<k}(z_j - z_k)^2[\Phi_{n+1}]^*$. *A priori*, these appear to represent two distinct FQHE states, and one may ask which one applies to the real system. However, explicit evaluations [33,34] have demonstrated the nontrivial result that these two descriptions represent the same state. They predict identical quantum numbers for the ground state and the excitations (see Supplemental Material [35]), and there is an almost perfect overlap between the two wave functions wherever it has been evaluated (e.g., for the ten-particle 2/3 state, the two wave functions have overlaps of 0.996 and 0.994 with the exact Coulomb state [33]). We have evaluated the $g(r)$'s of $\Psi_{(n+1)/(2n+1)}$ and $\Psi_{n/(2n+1)}$ and found them to be related by p-h symmetry to a very high accuracy (see Supplemental Material [35]). The wave functions $\Psi_{n/(2n\pm1)}$ produce, in the limit of $n \to \infty$, the same CFFS from either side, because $\Phi(B^* = 0)$ is real [36]. Furthermore, Rezayi and Haldane [37] have directly constructed the wave function for the CFFS on a torus and found that, for $N = 16$ particles, it has an overlap of 0.9994 with its hole conjugate and 0.9925 with the exact p-h symmetric Coulomb ground state [37]. The degree to which the microscopic wave functions of the CF theory satisfy the p-h symmetry may seem surprising but is a by-product of the fact that these are excellent approximations of the exact Coulomb states in the LLL which satisfy p-h symmetry exactly.

The understanding of FQHE at $\nu = (n+1)/(2n+1)$ as $\nu^* = n+1$ IQHE of ^{e}CFs in a negative B^* becomes essential when we consider the spin degree of freedom, because it is the only known way to explain the nonfully spin-polarized FQHE states here, e.g., the spin singlet state at $\nu = 2/3$. (Recall that, for spinful states, p-h symmetry relates ν to $2-\nu$.) Extensive experimental [38–50] and theoretical [33,34,51–57] literature on spin phase transitions has validated the explanation of the $\nu = (n+1)/(2n+1)$ as $\nu^* = n+1$ IQHE of ^{e}CFs.

The validity of $\Psi_{n/(2n\pm1)}$ for the *incompressible* states has been established by extensive numerical studies [12,33,36,58–60]. We will make the *assumption* that $\Psi_{n/(2n\pm1)}$ remain valid to arbitrarily high n, i.e., that the compressible region around $\nu = 1/2$ consists of unresolved IQHE states of composite fermions. We stress that we cannot rule out the possibility that the ^{e}CFFS and ^{h}CFFS are in reality topologically distinct, as proposed in Ref. [26], and a spontaneous breaking of the p-h symmetry selects one of them. This would happen, for example, if the half filled ground state were unstable to CF pairing [61–64] and ^{e}CFFS and ^{h}CFFS are the normal states of the topologically distinct Moore-Read Pfaffian and anti-Pfaffian paired-CF states [65–68]. Nonetheless, the presently known facts do admit the possibility of a p-h symmetric CFFS, and our aim here is to deduce its properties, so experiments may distinguish between the different proposals.

We have calculated the pair-correlation function for $\nu = n/(2n+1)$ up to 7/15 using the Metropolis Monte Carlo method. For technical reasons, we find it convenient to use the standard spherical geometry [69]; see Supplemental Material for details [35]. The results extrapolated to the thermodynamic limit apply to the planar geometry of the experiments. The spherical analogs of the above wave functions, as well as the details of LLL projection, can be found in the literature [12,70,71]. All wave functions considered below have uniform density and are translationally invariant (i.e., have orbital angular momentum $L = 0$ on the sphere). The $g(r)$'s for the largest systems in our study are shown in Fig. 1(a). For incompressible systems the pair-correlation function is expected to decay in a Gaussian manner in the limit of $r \to \infty$, but there is a range of intermediate r where it exhibits well-defined oscillations from which a Fermi wave vector can be extracted. In fitting $g(r)$ to Eq. (1), we avoid very small r (where short distance correlations are important) and very large r (where curvature effects become non-negligible). From the results for finite systems, we obtain the thermodynamic limits for the $k_F^*\ell$ (see [35]). We find that very large systems ($N > 100$) are needed for a satisfactory thermodynamic extrapolation of $k_F^*\ell$. The thermodynamic limits are shown in Fig. 1(b). [We have assumed exact p-h symmetry, which implies that the $k_F^*\ell$ at $\nu = (n+1)/(2n+1)$ is the same as that at $\nu = n/(2n+1)$.] The range of $k_F^*\ell$ includes uncertainly in the fits [estimated by linear and quadratic fitting in $1/N$ for $g(r)$] as well as uncertainty due to the curvature of the spherical geometry (estimated by considering fits with r chosen as the chord or the arc distance). For reference, Fig. 1(b) also shows the values $k_F^{*\rm MF}\ell = \sqrt{2\nu}$ and $k_F^{*\rm MF}\ell = \sqrt{2(1-\nu)}$ as expected from a MF picture for the ^{e}CFFS and ^{h}CFFS.

For $\nu = 1/2$, we have estimated $k_F^*\ell$ by an extrapolation, in the spherical geometry, of filled shell CF systems at zero effective flux [36] occurring at $N = n^2$. For technical reasons, we are not able to go to systems with $N > 81$ (which requires filling the tenth Landau-like level of composite fermions, where the numerics become unstable). We have therefore also studied the CFFS in the torus geometry [37,72,73], where we can go up to $N = 153$, and find that the results are consistent with our spherical results. Results in the torus geometry are presented in [35].

For ν away from 1/2, our calculated $k_F^*\ell$ is close, but not equal, to the smaller of $\sqrt{2\nu}$ and $\sqrt{2(1-\nu)}$. Both from extrapolation of the results from the sequences $n/(2n\pm1)$

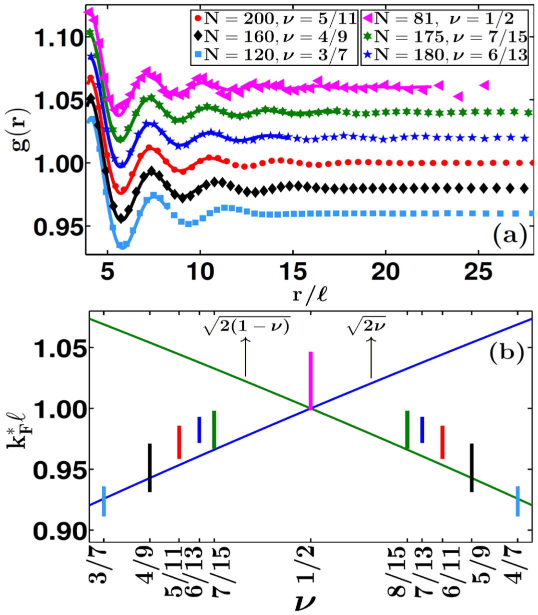

FIG. 1 (color online). (a) Pair-correlation function $g(r)$ as a function of r/ℓ, where r is the arc distance on the sphere. The projected wave functions in Eq. (8) have been used for its evaluation. The solid lines are fits using Eq. (1) for the initial oscillations. For clarity, the curves (except for 5/11) have been shifted up or down by multiples of 0.02. (b) The calculated thermodynamic values of $k_F^*\ell$ as a function of ν. The mean-field values $\sqrt{2\nu}$ and $\sqrt{2(1-\nu)}$ are also shown for reference.

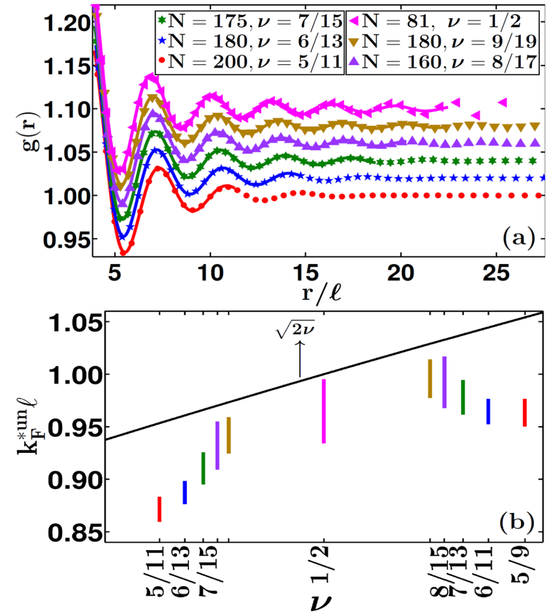

FIG. 2 (color online). The same as in Fig. 1 but for the unprojected wave functions $\Psi^{\mathrm{un}}_{n/(2n\pm1)}$. Also shown for reference is $k_F^{*\mathrm{MF}}\ell = \sqrt{2\nu}$ corresponding to the Chern-Simons mean-field theory for ^{e}CFs.

and from calculations directly at $\nu = 1/2$, our calculations suggest that the CFFS area at $\nu = 1/2$ slightly deviates from the value expected from the Luttinger rule.

The physics of the CFFS at $\nu = 3/2$ is analogous to that at $\nu = 1/2$ once the B dependence of the density of either ^{e}CFs or ^{h}CFs in the spin-reversed LL is accounted for [23,74]. Near $\nu = 1/4$, both $n/(4n+1)$ and $n/(4n-1)$ are understood only in terms of ^{e}CFs, and thus one expects $k_F^*\ell \approx \sqrt{2\nu}$ (as opposed to $k_F^*\ell \approx \sqrt{2(1-\nu)}$), as observed experimentally [24,74] and also in our calculations [35]. Analogous consideration for the CFFS at $\nu = 3/4$ gives $k_F^*\ell \approx \sqrt{2(1-\nu)}$.

We next investigate how robust the CFFS area is to LL mixing. LL mixing requires a formulation in terms of electrons (rather than holes of the LLL), and the LLL electronic wave functions in Eq. (8) can be used as a starting point to address this issue [75]. A realistic treatment of LL mixing is outside the scope of the current study, but we have considered the "unprojected" Jain wave functions $\Psi^{\mathrm{un}}_{n/(2n\pm1)} = \prod_{j<k=1}^{N}(z_j - z_k)^2\Phi_{\pm n}$, which contain some amplitude outside of the LLL [29,76]. Even though they do not give a realistic account of LL mixing, it is likely that they are adiabatically connected to the projected wave functions (as explicitly demonstrated for $\nu = 2/5$ [77]) and hence to the actual Coulomb ground states. For these wave functions, the $g(r)$'s of $\nu = n/(2n-1)$ and $\nu = n/(2n+1)$ are identical for a given N when plotted in units of the sphere radius, which in the thermodynamic limit implies the relation

$$(k_F^{*\mathrm{un}}\ell)_{n/(2n-1)} = \left(\frac{2n+1}{2n-1}\right)^{1/2}(k_F^{*\mathrm{un}}\ell)_{n/(2n+1)}. \tag{9}$$

The calculated values of $k_F^{*\mathrm{un}}\ell$ using the ^{e}CF description (see Supplemental Material for details) are shown in Fig. 2. Our calculations thus provide evidence that $k_F^*\ell$ depends on LL mixing. Another approximate wave function with LL mixing is the CS MF state $\Psi^{\mathrm{MF}}_{n/(2n\pm1)} = \prod_{j<k=1}^{N}[(z_j - z_k)/|z_j - z_k|]^2\Phi_{\pm n}(B^*)$. Given that its $g(r)$ coincides with that for $\Phi_{\pm n}(B^*)$, we get $k_F^{*\mathrm{MF}} = \sqrt{4\pi\rho_e}$, i.e., $k_F^{*\mathrm{MF}}\ell = \sqrt{2\nu}$ for all $\nu = n/(2n\pm1)$. For the unprojected or the CS-MF wave functions, $k_F^*\ell$ does not obey particle-hole symmetry, as expected in the presence of LL mixing.

In summary, we have shown that, within the microscopic theory of composite fermions, it is valid to consider electron- (or hole-) based composite fermions for $\nu < 1/2$ as well as $\nu > 1/2$. We have calculated the CF Fermi wave vector in the vicinity of $\nu = 1/2$ and find that it is closer, but not equal, to the smaller of $\sqrt{4\pi\rho_e}$ and $\sqrt{4\pi\rho_h}$. In terms of electron-based composite fermions, this implies that the Luttinger theorem is slightly (substantially) violated for $\nu < 1/2$ ($\nu > 1/2$). At $\nu = 1/2$, our results suggest, but do not conclusively demonstrate, that the k_F^* differs slightly (by a few percent) from the value $\sqrt{4\pi\rho_e}$ predicted by the Luttinger area rule. We also provide evidence that k_F^* varies as a function of LL mixing.

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Note added.—Since the completion of this work, several other articles have appeared addressing the nature of the CFFS and the role of p-h symmetry [78–82].

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