An elastic phenomenological material law for textile composites and it's fitting to experimental data

Dezső Hegyi[a], Marianna Halász[b], Kolos Molnár[b], Gábor Szebenyi[b], András A. Sipos[a]

[a]Dept. Mechanics, Materials and Structures, Budapest University of Technology and Economics, Budapest, Hungary
[b]Dept. Polymer Engineering, Budapest University of Technology and Economics, Budapest, Hungary

Abstract: Constitutive description of deformations in technical textiles mostly requires some highly nonlinear material law due to the interaction between the orthotropic yarns and the effect of the matrix. Phenomenological models aim to garb the overall (macro level) behavior required for engineering purposes. This paper introduces a new, two-dimensional phenomenological model for technical textiles accompanied with a data acquiring strategy to determine the material parameters involved in the model. It handles the nonlinear stress-strain relation observed in uniaxial tests and take the interactions in the two, orthogonal directions into account. As we aim to introduce a solution applicable at the service level of the loads, our model is inherently elastic, no time dependent or plastic behavior are introduced. The model can be solidly fit to the measured data.

Keywords: technical textiles, biaxial test, nonlinear constitutive law

1. Introduction

The observed deformation of textile composites under in-plane loading is nonlinear for several reasons: geometrical nonlinearity can be associated with the woven microstructure of the textile and the yarns contribute in a significant material nonlinearity. Furthermore - for specific materials - substantial time-dependent deformations may occur. This paper is intended to describe a data acquiring strategy and a novel nonlinear elastic constitutive model which takes the interaction between the two orthotropic directions into account.

The new phenomenological model aims to provide a simple and fairly accurate method for the structural analysis of textile materials at the service level of the loads. The ultimate failure of the structure is not in our interest. The widely used partial (safety) factors for technical textiles limit the service stress level to 30-40 % of the ultimate strength of the material. To reduce the effect of plasticity and viscous deformations, PTFE coated glass fiber woven textile was used in short-time uniaxial and biaxial tests. The glass fiber has an almost linear elastic behavior and essentially no viscous deformation at the targeted (moderate) stress level. Nevertheless, the measured data reflects the nonlinear effect of the PTFE coating.

Recent methods in the literature are based on the adequate involvement of the material's microstructure (Haan[12], Ballhause[4], Durville[10]). They construct the model from feasible components at the micro level via detailed constitutive laws of the yarns, the coating matrix and the interactions between these elements. The phenomenological model at the continuum level is reached via some homogenization technique or an appropriate strain-energy function is defined (dell'Isola[15]) that adequately describes the behavior of the fabric. Nevertheless, a detailed description of the internal geometry is also essential in this case. A shortcoming of such approaches is the high computational cost arising from the detailed description. Therefore, these methods are most often impractical for engineering problems.

There are phenomenological methods that describe the overall (macro) nonlinear behavior of the continuum by adequate nonlinear functions. Nevertheless, such a non-linear relation can be derived for models based on the microstructure mentioned above. However - as long as the elastic range is treated - a direct fit for macroscopic data might be preferable, since the assessment of numerous material parameters and the detailed geometry (needed for modeling at the fabric level) can be omitted. Apparently, there are several, solely macroscopic models in the literature. The dense net method uses two independent functions to be fitted to the uniaxial measurements in the two orthotropic directions (Ambroziak[1][2]). It uses nonlinear functions to represent the different slopes of the stress-strain curves (Chaboche[8], Bodner[5]), however it lacks to handle the interaction between the two orthogonal yarn directions, practically there is no transversal interaction in the model. Spline methods(Day[9], Bridgens[6][7])define the surface for the stress as a function of the two orthogonal elongations. By this method an accurate interpolation can be achieved to the measured data and the transversal interaction is accounted, too. Nevertheless, the usage of the power equations of the spline can lead to divergence in the nonlinear analysis because the power equations are hectic for extrapolation. During the nonlinear analysis of membrane extremely high strains occur, thus unreliable extrapolation should be avoided.

Our approach can be interpreted as a generalization of the classical elastic constitutive law for an orthotropic medium in two dimensions(Lampiere[16], Sipos[18]). To provide a better fit with the measured data in uniaxial and biaxial tests, exponential functions are used and the interaction of the two directions is modeled with a power function. The new elastic model gives a good approximation at the service load level of the structure and presents a proper extrapolation to have a stable numerical analysis. This is necessary to avoid the instability in the numerical analysis of the structure at stress localization zones (Sadd[17]).

2. Model development

2.1. Uniaxial behavior

A realistic model should simultaneously explain the material response in uniaxial and biaxial tests. First we introduce the terms reproducing the uniaxial behavior. Nevertheless, the uniaxial response is tested in the wrap and weft directions of the material. Two typical uniaxial tensile test curves are depicted in Fig. 1 and Fig. 2in the warp and in the weft directions, respectively.

In all measurements we determine the engineering stress: the strain is related to the original length and the stress is calculated by the original area of the cross section. In nonlinear calculations usually the Lagrange strain is used with the Second-Piola stress tensor to model large strains. In the case of membrane structures higher accuracy is required: the Biot strain and Cauchy stress tensors are practical (Hegyi[13][14]). The Biot strain is practically identical to the stretch tensor. In the further parameter analysis, we employ this phenomenon. During the analysis of the experimental data the stretch and the engineering stress can be easily calculated.

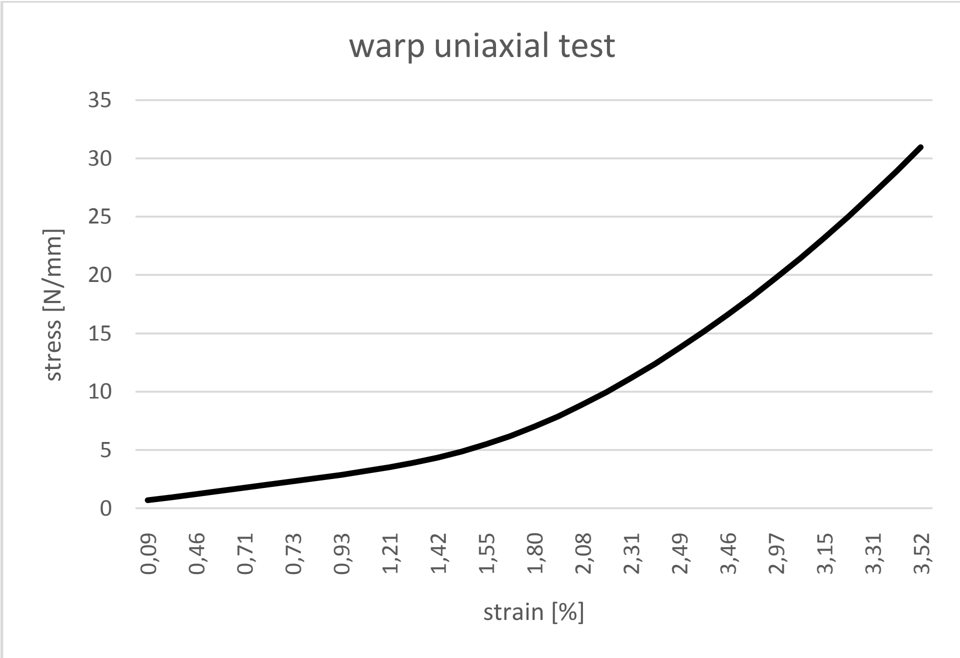

Figure 1. Measured stress, a typical stress-strain curve of a uniaxial tensile test in the warp direction.

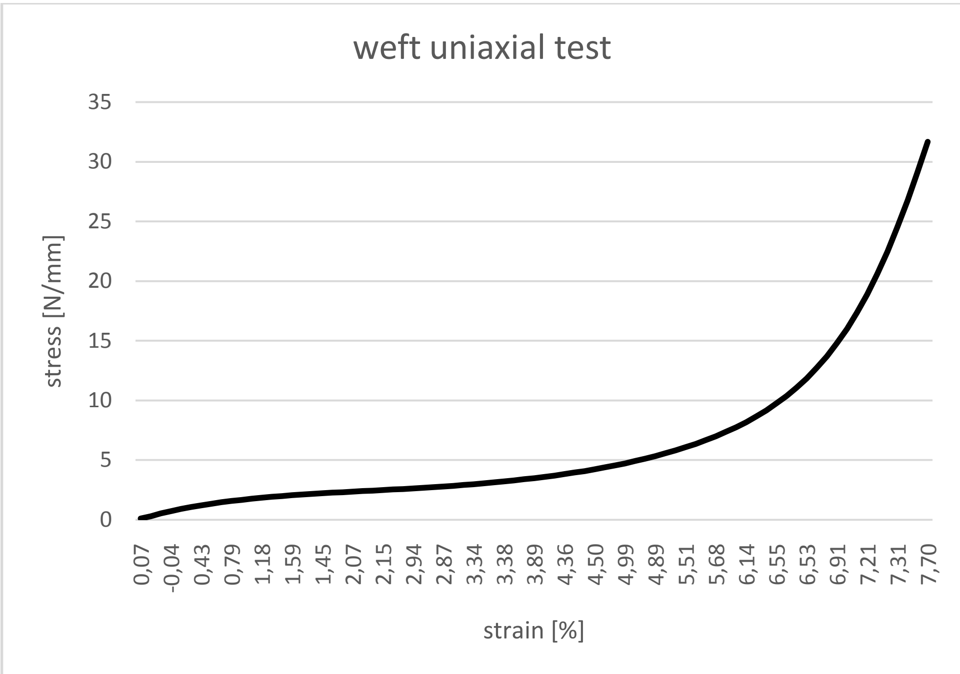

Figure 2. Measured stress, a typical stress-strain curve of auniaxial tensile test in the weft direction

It is worthy to note, that the stress-strain curves above hints to an inflection point along the curve. This feature seems to be typical among technical textiles. To reproduce the inflection in the uniaxial response a reasonable choice is a linear combination of two exponentials:

$$\sigma_{w,0} = f_{x1}(\varepsilon_w) + f_{x2}(\varepsilon_w) = a_1\varepsilon_w\left(1 - e^{-a_3\varepsilon_w^2}\right) + a_2\varepsilon_w\left(e^{-a_4\varepsilon_w^2}\right), \tag{3}$$

$$\sigma_{f,0} = f_{y1}(\varepsilon_w) + f_{y2}(\varepsilon_w) = b_1\varepsilon_f\left(1 - e^{-b_3\varepsilon_f^2}\right) + b_2\varepsilon_f\left(e^{-b_4\varepsilon_f^2}\right) \tag{4}$$

where $\sigma_{w,0}$ is the stress in the warp direction, ε_w is the strain in the warp direction and a_1, a_2, a_3 and $a_4$are material parameters. Similarly, $\sigma_{f,0}$ and ε_f denote the stress and strain in the weft direction, respectively. This expression fits well to the measured stress-strain diagrams: observe the resemblance of the σ_w curve in Figure 3(with parameters a_1=2,0 –a_2=0,5 – a_3=0,5 – a_4=10).and the output of the measurements in Figure 1 and Figure 2.

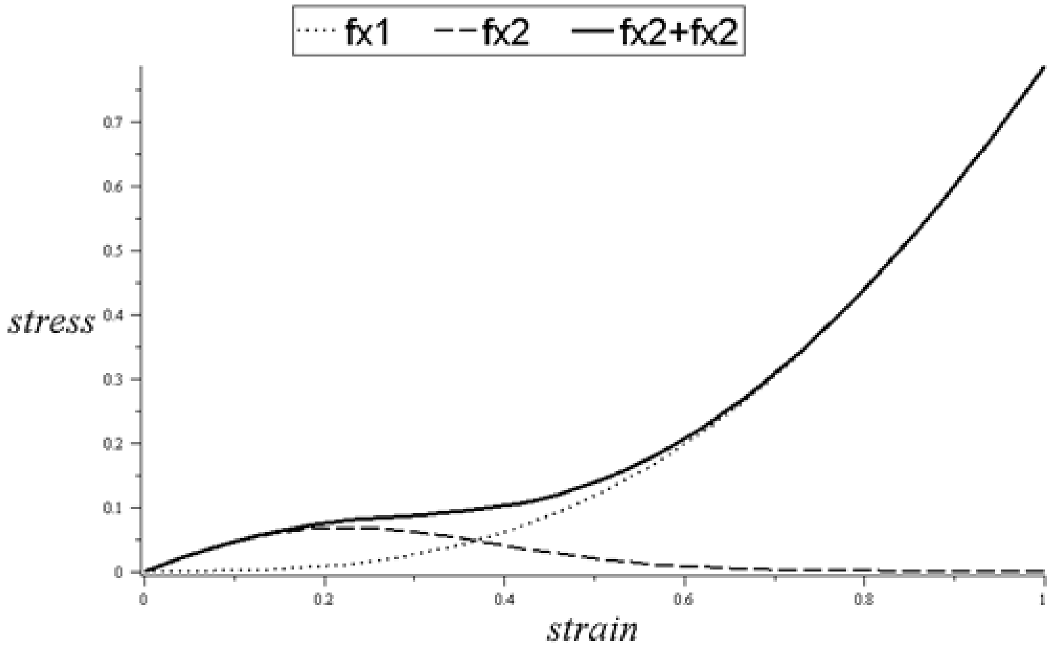

Figure 3. Calculated stress-strain diagram plotted by exponential functions.

2.2. The interaction between the two yarn directions

Any elastic constitutive law should satisfy the energy conservation law. It is well known, that this requirement manifests in a symmetric stiffness matrix. In other words, the following derivatives must be equal:

$$\frac{\partial\sigma_w}{\partial\varepsilon_f} = \frac{\partial\sigma_f}{\partial\varepsilon_w}. \tag{5}$$

where σ_w and $\sigma_f$are the stress in the warp and fill directions, respectively. Nevertheless, in the general case σ_w and $\sigma_f$are functions of two variables, ε_w and ε_f.

We seek a form that satisfy Eq. 5,is in accordance with the measured stresses in the biaxial tests and the transversal deformations in the uniaxial tests. This latest is an inherent consequence of the microstructure of the material. In specific, the yarns of textile composites are woven, they mutually bend each other. The matrix (PTFE in our samples) around the yarns are much softer than the yarns itself, thus the matrix contributes to the total deformation, too. This special arrangement has a significant effect on the transversal deformations.

For uniaxial tests Fig. 4 and Fig. 5depict the measured transversal deformation.

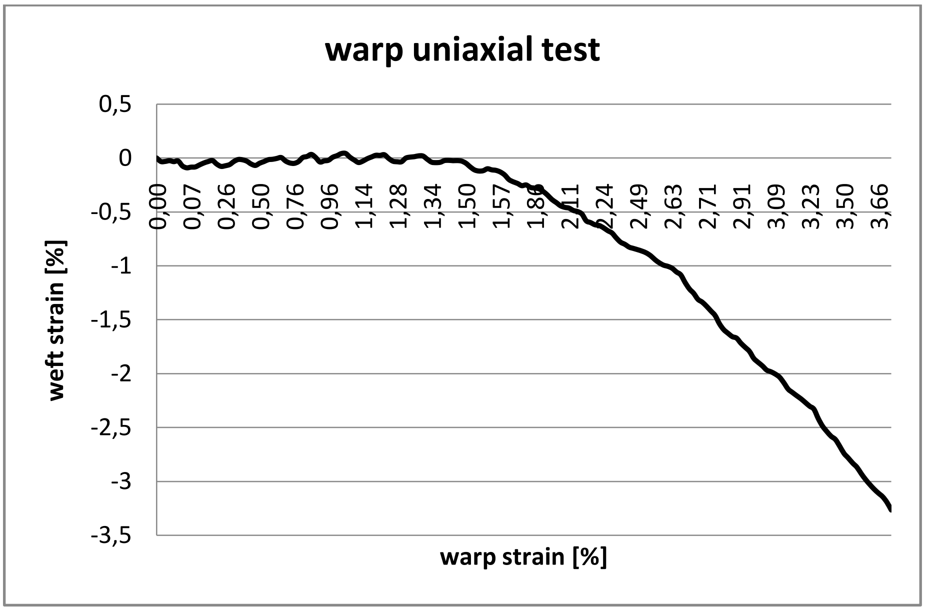

Figure 4. Transversal deformation, uniaxial test, warp loading.

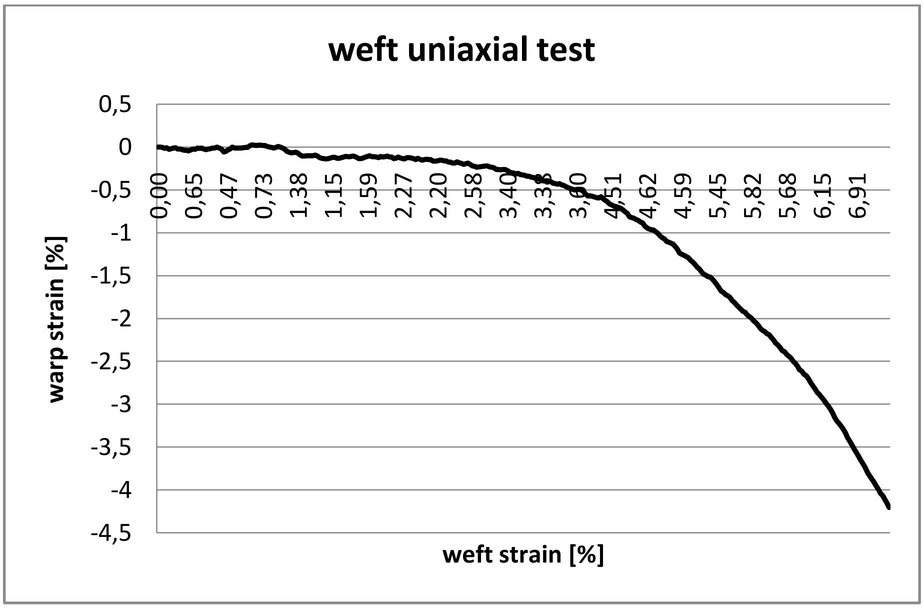

Figure 5. Transversal deformation, uniaxial test, weft loading.

The trend of the transversal deformations in the uniaxial tests (Fig. 4. and 5.) makes it clear that the terms expressing interaction between the two directions cannot be linear. This means, a simple orthotropic response is not sufficient in our case. With a slight generalization of the orthotropic model one arrives to the following expression (it includes the terms from Eqs. (8) and (9)):

$$\sigma_w = a_1\varepsilon_w\left(1 - e^{-a_3\varepsilon_w^2}\right) + a_2\varepsilon_w\left(e^{-a_4\varepsilon_w^2}\right) + c_1\varepsilon_f({\varepsilon^2}_w{\varepsilon^2}_f)^{c_2}, \quad (8)$$

$$\sigma_f = b_1\varepsilon_f\left(1 - e^{-b_3\varepsilon_f^2}\right) + b_2\varepsilon_f\left(e^{-b_4\varepsilon_f^2}\right) + c_1\varepsilon_w({\varepsilon^2}_w{\varepsilon^2}_f)^{c_2}. \quad (9)$$

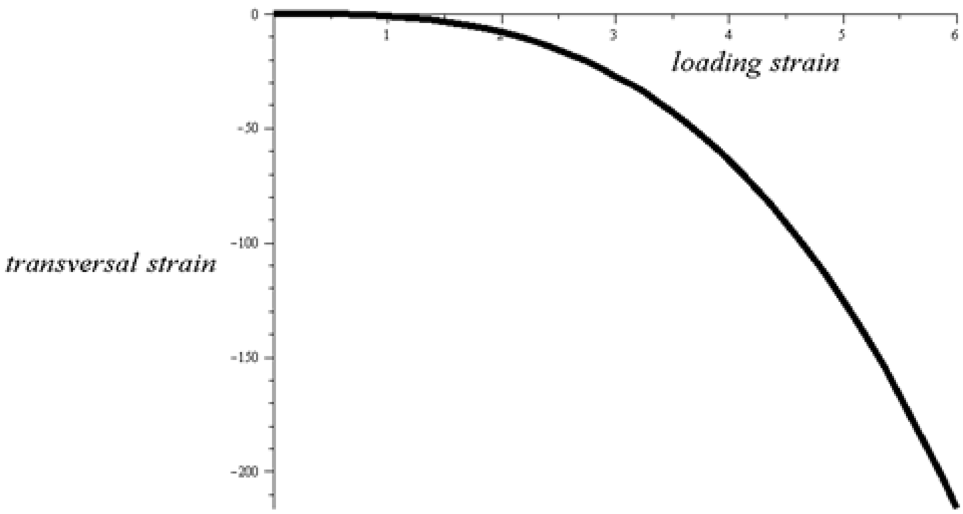

Figure 6. Calculated transversal deformation for uniaxial loading.

Figure 6 represent the calculated curve of the transversal deformation with parameters a_1=1, a_2=1, a_3=1, a_4=1, c_1=1, c_2=0,25.

Note, that c_1 and c_2 are material parameters. With $a_1=a_3=a_4=b_1=b_3=b_4=c_2=0$ we obtain the classical linear model of an orthotropic medium. Nevertheless, the symmetry condition in Eq. (5), and thus the energy conservation criteria is satisfied:

$$\frac{\partial\sigma_w}{\partial\sigma_f} = \frac{\partial\sigma_f}{\partial\sigma_w} = 2c_1c_2(\varepsilon_w\varepsilon_f)^{2c_2} + c_1(\varepsilon_w\varepsilon_f)^{2c_2}. \quad (10)$$

Observe the similarity between the measured and computed transversal deformations in Figs.4, 5 and 6.

A realistic model should contain a contribution from shear. At this stage of model development we neglect that contribution since the shear stiffness of technical textiles is about two order smaller respect to the normal stiffness in one of the orthogonal directions (Day[9]). Based on our experience, just about 1-2% of the initial slope of the normal strain can be used as a good approximation (Hegyi[14]). This observation significantly simplifies the experimental work, too.

Our assumption about the magnitudes of shear has another consequence: we take a nearly diagonal deformation gradient **F** associated with the measurements. The Biot strain in this special can be approximated as $\mathbf{E}^{\mathrm{Biot}}\approx\mathbf{U}-\mathbf{I}=\mathbf{F}-\mathbf{I}$, where **U** is the unitary matrix from the polar decomposition of **F** and **I** is the unit matrix in $\mathbf{R}^2$. It is worthy to note, that with this simplification the strain energy associated with the stresses in Eqs. (8) and (9) fulfill material objectivity.

3. Experimental

3.1. Experimental arrangements

To get proper parameters for the new constitutive law three experiment series were carried out: uniaxial tensile tests in warp and weft directions and biaxial tests. To record data, a Messphysik ME-46 full image video-extensometer were used in all cases. The measured area was a 30×30 mm square in the middle of the specimen. The longitudinal and the transversal elongations were measured in all cases.

For the uniaxial test standard specimens were used: their width was 50 mm, the grip distance was 200 mm. Displacement test were carried out, the speed of elongation was 0.50 mm/s in all tests. The tests terminated after the failure (break) of the specimen and 40% of the ultimate load was used for parameter identification.

For the biaxial tests a special equipment was used. It is a kind of pulley system developed by the authors(Bakonyi[3], Hegyi[14]). The testing configuration can be seen in Fig.7. An X shape specimen is folded through a pulley system: two "legs" are going up and two going down to the grips of a standard unidirectional tension testing machine. The pulleys' position determines a flat area for the proper measurement. The width of the "legs" are 100 mm, the radius at the intersections are 25 mm. Displacement test were carried out, the speed was 0.80 mm/s in all tests.

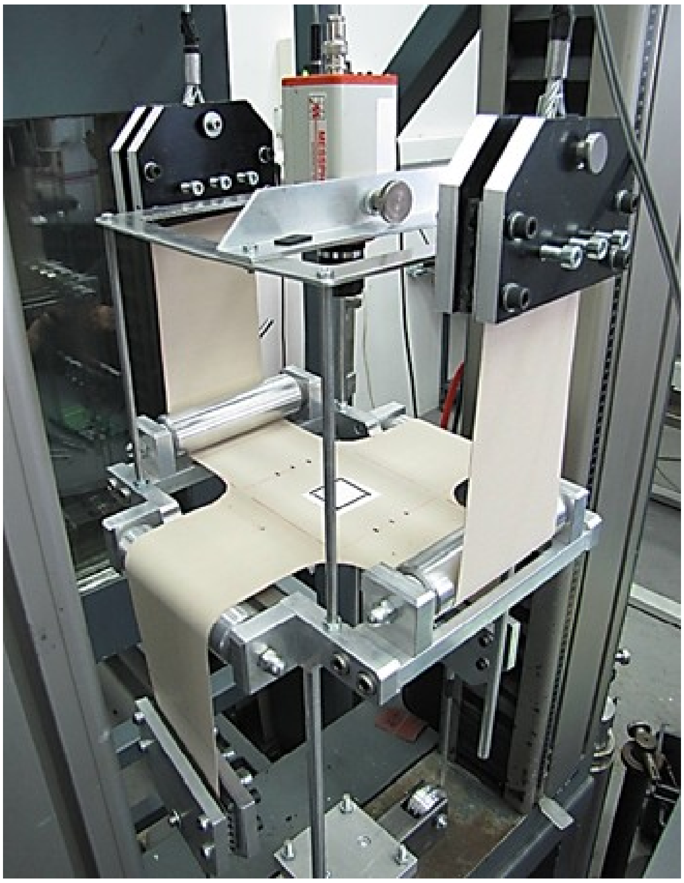

Figure 7. The bidirectional testing equipment.

The engineering stress in the optically measured area can be calculated as an average in case of the unidirectional test: the measured force of the testing machine is divided by the width of the specimen. To approximate the stress for the bidirectional tests a FEM analysis was needed. Unit stress were applied at the end of the "legs" of the virtual specimen. The stress level varies between 68.2-68.8% of the unit at the edge of the measuring area (Fig.8), so it is almost constant. Although it was a linear FEM analysis, we can expect almost the same distribution for the real specimen. From the measured force stress can be calculated to the "legs" of the specimen and the 68.5% of this stress were taken into account the measured area.

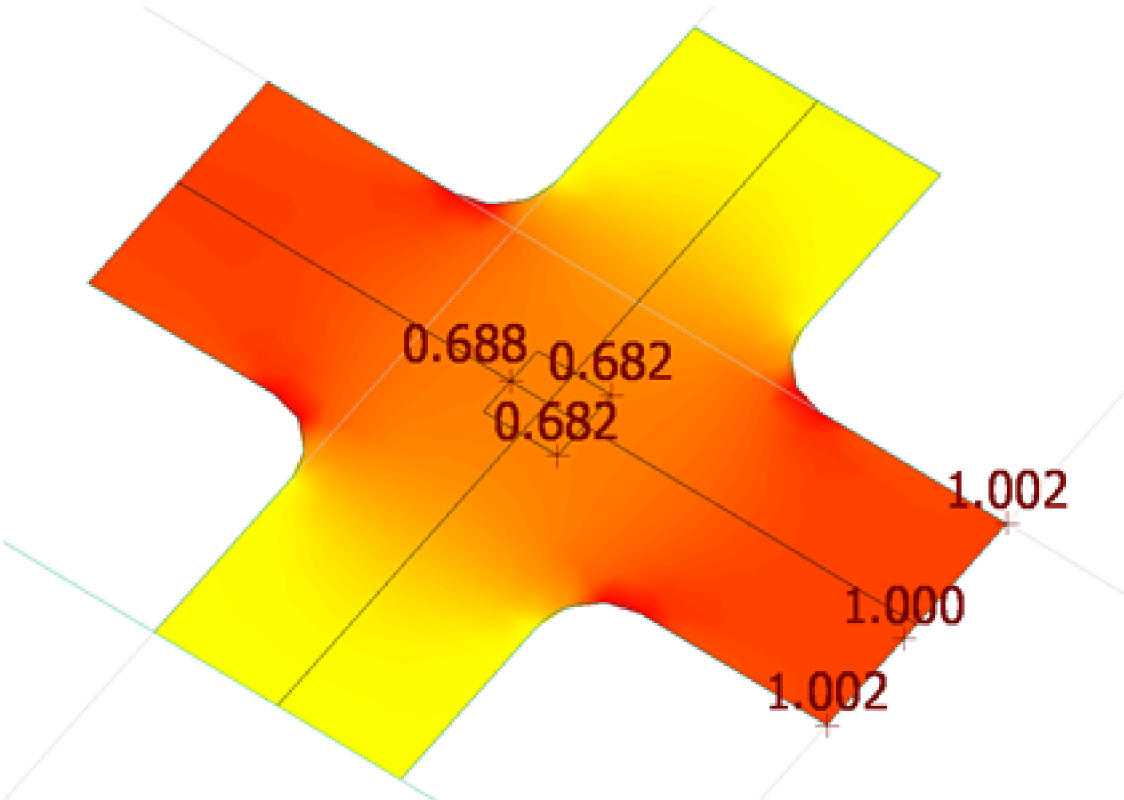

Figure 8. FEM analysis to approximate the stress level in the measured area.

3.2. Parameter identification

As it is general in the literature, the material parameters of the model might be approximated by a least square minimization from the measured data:

$$\Omega = \sum_{i=0}^{n} \left[\left(s_{wi} - \sigma_{w\left(\varepsilon_i^w, \varepsilon_i^f\right)} \right)^2 + \left(s_{fi} - \sigma_{f\left(\varepsilon_i^w, \varepsilon_i^f\right)} \right)^2 \right] \Rightarrow min \qquad (11)$$

where n is the number of available measurements and s_{wi} and s_{fi} denote the measured stresses in the warp and weft directions, respectively. The objective function Ω is the linear combination of the squared errors in the two directions. A serious shortcoming of such an approach is that due to the presence of the exponentials in the model (and the numerous material parameters) the objective function is far from being convex, it possesses many local minima. Even a slight change of the initial guess of the parameters can result in a significant shift in the parameter space. This is a widely known problem of regression models with exponentials (Golub[11]).

To obtain a reliable method, observe, that the following rearrangement of Eqs. (8) and (9)makes the third, interaction term vanishes:

$$S \coloneqq \sigma_w \varepsilon_w - \sigma_f \varepsilon_f = a_1 {\varepsilon_w}^2 \left(1 - e^{-a_3 \varepsilon_w^2}\right) + a_2 {\varepsilon_w}^2 \left(e^{-a_4 \varepsilon_w^2}\right) - b_1 {\varepsilon_f}^2 \left(1 - e^{-b_3 \varepsilon_f^2}\right) + b_2 {\varepsilon_f}^2 \left(e^{-b_4 \varepsilon_f^2}\right) \qquad (12)$$

Nevertheless, a counterpart of S, namely $T \coloneqq s_w e_w - s_f e_f$ can be computed from the measured stress (s_w and s_f) and strain data. To find the minimal deviation between S and T we use a MATLAB implementation of the variable projection method (O'Leary[19]).It determines all the linear and nonlinear parameters of the exponentials (i.e. a_1-a_4 and b_1-b_4) in our model. In numerical simulations we found, that this approach is much more robust for our data sets than a direct application of the least-square method in Eq. (11).

Having optimal values for the parameters of the exponentials we finally determine the parameters c_1 and $c_2$in the interaction term. Here least-squares is a perfect choice: we substitute the already computed values of a_1-a_4 and b_1-b_4 into our model and apply Eq.(11) to obtain optimal values for c_1 and c_2.

Note, that we expect positive reals for all parameters and in advance we expect c_2<1 to match our observations about the transversal behavior (Fig. 4 and 5). Our scheme produces parameters in accordance with these expectations.

The best-fit parameters for the material in our experiments are: a_1=7.11, a_2=6.97, a_3=0.831, a_4=1.94, b_1=1.51, b_2=3.07, b_3=0.268, b_4=0.727, c_1=0.537, c_2=0.335. It is worthy to note that the applied methodology is fairly robust: parameter fit just for the uniaxial or solely for the biaxial data result in close parameter values to the ones presented above. The results of the parameter fit are represented in Figures9 and 10.

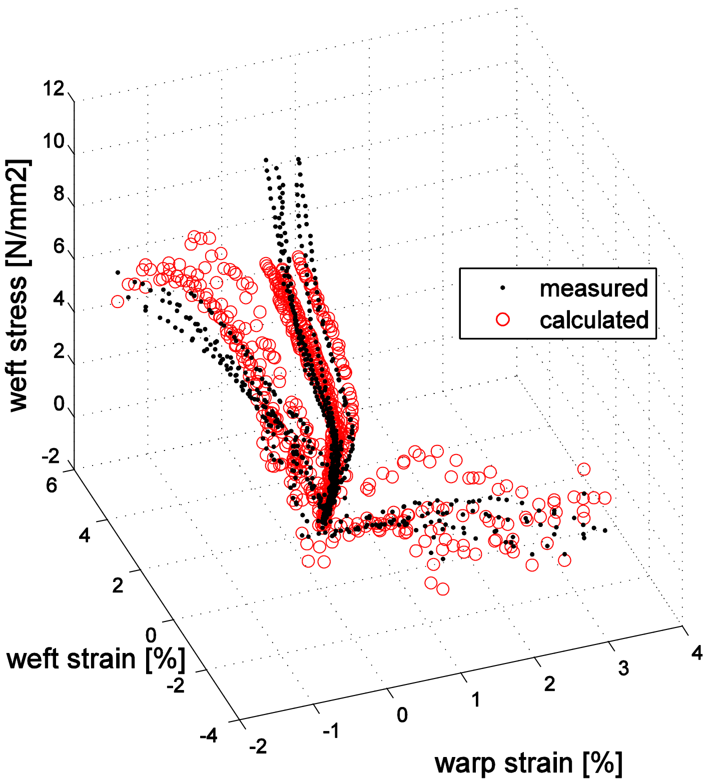

Figure 9. The result of the parameter identification for the weft direction.

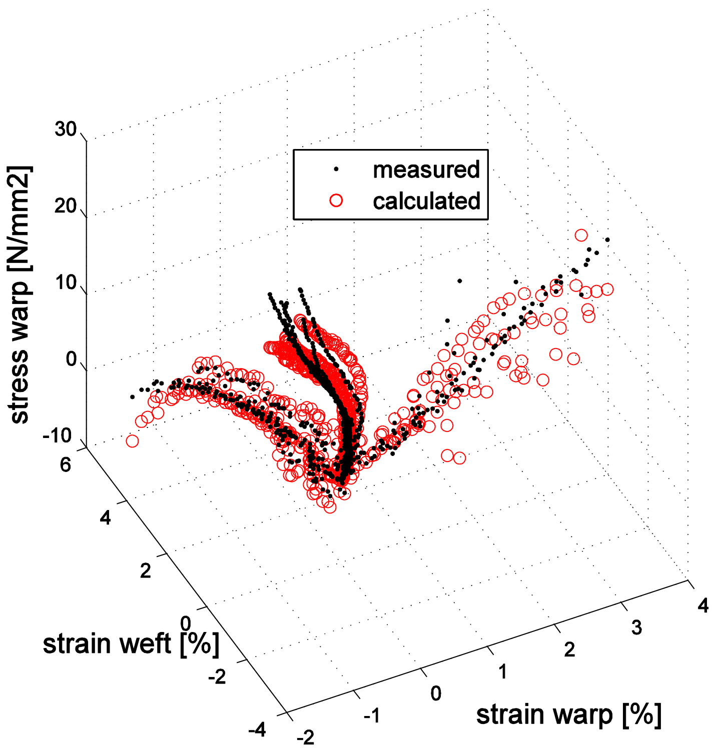

Figure 10. The result of the parameter identification for the warp direction.

5. Discussion and verification

To obtain the actual normal stresses from the bidirectional strains Eqs. (8) and (9) are applied. In practical numerical analysis of a structure the constitutive law is represented via the stiffness or the flexural matrices. A nonlinear constitutive law requires a nonlinear structural analysis. There are different strategies for nonlinear structures, for membrane structures the Total Lagrange Method (TLM) or the Updated Lagrange Method (ULM) can be regarded as typical.

In the case of TLM the deformation is calculated between the un-deformed, stress free state and the actual state. The secant of the stress-strain curve can be used:

$$D = \begin{bmatrix} \frac{\sigma_w}{\varepsilon_w} & \frac{\sigma_f}{\varepsilon_w} & 0 \\ \frac{\sigma_f}{\varepsilon_w} & \frac{\sigma_f}{\varepsilon_f} & 0 \\ 0 & 0 & G \end{bmatrix} > 0. \tag{14}$$

In the case of ULM the tangential stiffness matrix can be used:

$$D = \begin{bmatrix} \frac{\partial\sigma_w}{\partial\varepsilon_w} & \frac{\partial\sigma_f}{\partial\varepsilon_w} & 0 \\ \frac{\partial\sigma_f}{\partial\varepsilon_w} & \frac{\partial\sigma_f}{\partial\varepsilon_f} & 0 \\ 0 & 0 & G \end{bmatrix} > 0. \tag{15}$$

According to chapter 2.2 G=(a_1+b_1)/(2×100)=(7.11+1,51)/(2×100)=0,043 N/mm^{2}is used for further analysis.

Another important criteria for a stable constitutive material law to have a positive definite stiffness matrix:

$$|D| = \begin{vmatrix} D_{ww} & D_{wf} & 0 \\ D_{wf} & D_{ff} & 0 \\ 0 & 0 & G \end{vmatrix} = \begin{vmatrix} \frac{\partial\sigma_w}{\partial\varepsilon_w} & \frac{\partial\sigma_f}{\partial\varepsilon_w} & 0 \\ \frac{\partial\sigma_f}{\partial\varepsilon_w} & \frac{\partial\sigma_f}{\partial\varepsilon_f} & 0 \\ 0 & 0 & G \end{vmatrix} > 0, \tag{11}$$

where D is the stiffness matrix of the material, D_{ww}, D_{ff}, D_{wf} and G are the members of the stiffness matrix for normal stress, transversal effect and shear, respectively. G is a positive, so for positive definiteness it is enough to prove:

$$D' = \frac{\partial\sigma_w}{\partial\varepsilon_w}\frac{\partial\sigma_f}{\partial\varepsilon_f} - \left(\frac{\partial\sigma_f}{\partial\varepsilon_w}\right)^2 > 0. \tag{12}$$

With the identified parameters the D' function can be drawn as a two-variable function of the strains (Fig.11). The surface is above 0 for reasonable strains (the stress is represented in percent, and normally the strain is under 4-5% even in the ultimate load level). At larger strains and in moderate negative strain D' is negative. Fig.12 shows the border of the positive region, the zone containing the origin is positive. To have a stable numerical analysis the determinant should be controlled, but there is no problem in reasonable strain levels.

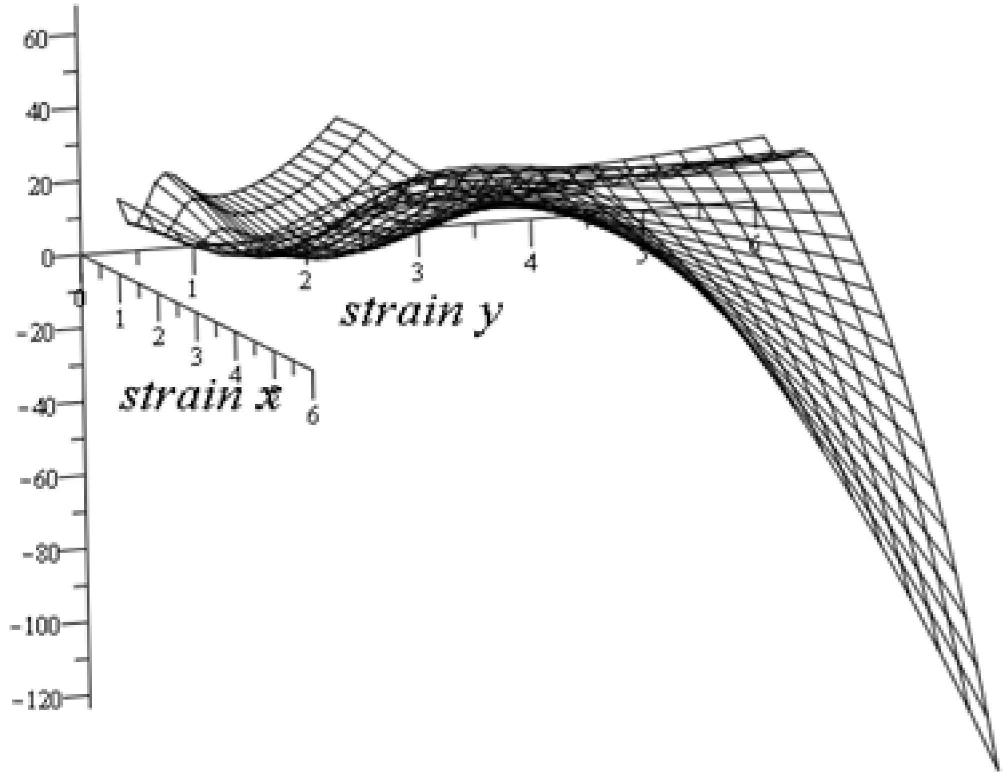

Figure 11. The surface of the D' function according to the strains.

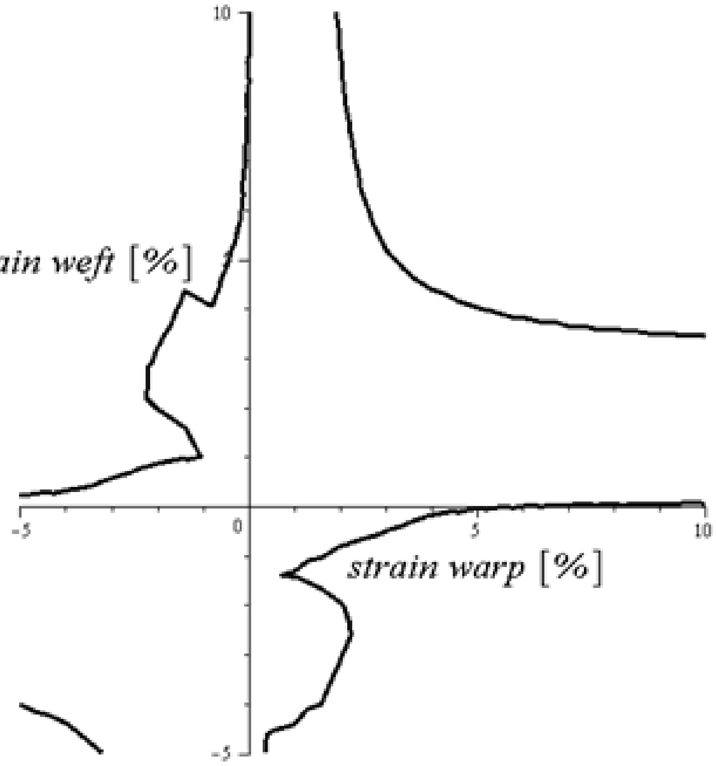

Figure 12. The intersection of the surface of the *D'* function with the 0 plan.

Fig. 12 shows, we are close to the border of the positive definite regime along the negative side of the axes, and over 5% of biaxial elongation. In classical Finite Element Method it can yield to instability, due to the requirement of the inverse of the stiffness. In any case for moderate strain the new constitutive model is stable. For extremely large strains the Dynamic Relaxation Method provides a proper strategy (Hegyi[13][14]). It does not use the inverse of the stiffness so positive definiteness is not an issue.

Both the power function methods(for instance the spline method) and the exponential functions we use have disadvantages at large strain. The benefit of the exponential approach introduced in this paper is the solid convergence to a slope instead of the hectic behavior of the power functions in extrapolation.

7. Conclusion

A new, elastic constitutive law for predicting service stresses of engineering textiles is introduced accompanied with a data acquiring strategy from uniaxial and biaxial tension tests. The constitutive law accounts for both the nonlinear behavior of the yarns and the geometric nonlinearity of the fabric. Exponential functions are used to avoid the divergence of the numerical scheme in the nonlinear structural analysis. The new constitutive law fulfills all the requirements for a real material: existence of a strain energy function, positive definiteness at reasonable strain levels.

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