

MAGYAR TUDOMÁNYOS AKADÉMIA
Közgazdaság- és Regionális Tudományi Kutatóközpont



Centre for Economic and Regional Studies
HUNGARIAN ACADEMY OF SCIENCES

MŰHELYTANULMÁNYOK

DISCUSSION PAPERS

MT-DP – 2016/31

Retirement rules in Hungary: gainers and losers

TIBOR CZEGLÉDI – ANDRÁS SIMONOVITS

ENDRE SZABÓ – MELINDA TIR

Discussion papers
MT-DP – 2016/31

Institute of Economics, Centre for Economic and Regional Studies,
Hungarian Academy of Sciences

KTI/IE Discussion Papers are circulated to promote discussion and provoke comments.
Any references to discussion papers should clearly state that the paper is preliminary.
Materials published in this series may subject to further publication.

Retirement rules in Hungary: gainers and losers

Authors:

Tibor Czeglédi - Endre Szabó - Melinda Tir
research assistants at the Databank of the Research Centre for Economic
and Regional Studies - Hungarian Academy of Sciences
E-mails: czegledi.tibor@krtk.mta.hu, szabo.endre@krtk.mta.hu
and tir.melinda@krtk.mta.hu

András Simonovits
research advisor
Institute of Economics
Centre for Economic and Regional Studies, Hungarian Academy of Sciences
also Mathematical Institute of Budapest University of Technology, Budapest
E-mail: simonovits.andras@krtk.mta.hu

September 2016

ISBN 978-615-5594-64-9
ISSN 1785 377X

Retirement rules in Hungary: gainers and losers

Tibor Czeglédi – András Simonovits – Endre Szabó – Melinda Tir

Abstract

Though the Hungarian pension system has been suffering from many erroneous rules, in the present paper we confine our attention to the rules of retirement in Hungary since 1990. In every pension system, there exist two rules which determine how the lifetime contribution (which is approximately proportional to the years of contributions) and the retirement age influence the benefit amount, respectively. As a benchmark, we use the system of nonfinancial defined contributions, simulating a mandatory life insurance and life annuity system. More generally, we speak of flexible retirement if adding a year to the contributions or the retirement age strongly increases the retirement benefit, opening the way to the flexible choice of the retirement age. Due to erroneous concepts, flexibility has only functioned in a very imperfect form in Hungary. Before 2011/2012, an exemption rule completed the two foregoing rules: if somebody had above the critical value (35–40) of years of contributions, he/she could use early retirement without significant benefit reduction. Since 2011/2012 two other rules have completed these rules: (a) as an *exception*, since 2011, rule Females 40 has rewarded any woman who had at least 40 years of rights with a full benefit; (b) as a *rigid* rule, since 2012, nobody could have retired before reaching the statutory retirement age except for category (a). Taking into account the dependence of monthly benefits on the lifetime average valorized wages, we assess the gainers and losers of the past and the present systems.

Keywords: normal retirement age, early retirement, years of contributions, rights, flexible retirement

JEL classification: H55, I14, J26

Acknowledgements: We express our debt to the experts of the Central Administration of National Pension Insurance who created a very detailed dataset for the public; and to Nick Barr, Rudolf Borlói, Róbert Iván Gál, Gábor Kézdi, Ádám Reiff and an anonymous referee for their valuable comments. András Simonovits' research was supported by the Hungarian Research Fund OTKA 108668.

Nyugdíjba vonulási szabályok Magyarországon: nyertesek és vesztesek

Czeglédi Tibor – Simonovits András – Szabó Endre – Tir Melinda

Összefoglaló

A rendszerváltás utáni időszakra szorítkozva, a magyar nyugdíjrendszer számos szabálya bírálható, de ebben a cikkben csak a nyugdíjba vonulási szabályok okozta torzulásokat elemezzük. Minden nyugdíjrendszerben létezik két szabály, amely meghatározza, hogyan alakítja a kezdőnyugdíj a befizetett járulék (amely jó közelítéssel a szolgálati idővel arányos) és a nyugdíjba vonulási életkor. Kiindulásul az eszei számla (vagy egyszerűsített, lineáris változata) szolgálhat, amely egy életbiztosítási–életjáradékos kényszermegtakarítást utánoz. Általánosabban *rugalmas* korhatárról beszélünk, ha mind a szolgálati idő, mind a nyugdíjba vonulási kor emelése jelentősen növeli a nyugdíjjáradékot, s ezzel ösztönöz a nyugdíjba vonulás idejének rugalmas megválasztására. Hibás elképzelések miatt ez az ikerelv Magyarországon csak nagyon torz formában érvényesült és érvényesül. A fenti két szabályt korábban egy *engedékeny* szabály egészítette ki: átlag fölötti (legalább 35–40 éves) szolgálati idő esetén az előrehozott nyugdíjba vonulás nem csökkentette a nyugdíjat. Jelenleg két szabály egészíti ki a szabályokat. Az egyik a *kivételező*: 2011 óta a „Nők40” keretében minden nő, akinek a jogviszonya eléri a 40 évet, teljes (csökkentés nélküli előrehozott) nyugdíjat kap. A másik szabály a *merevítő*: 2012 óta az előző kategórián kívül senki sem mehet a korhatár alatt nyugdíjba – még csökkentett előrehozott nyugdíjjal sem. Bevonva a keresetek és a nyugdíj közti kapcsolatot létrehozó (harmadik) szabályt is, a rugalmashoz képest értékeljük a korábbi és a jelenlegi szabályok melletti nyerteseket és veszteseket.

Tárgyszavak: általános nyugdíjkorhatár, előrehozott nyugdíj, szolgálati idő, jogviszony, rugalmas korhatár

JEL kódok: H55, I14, J26

1. INTRODUCTION

The field of pension economics has been characterized by heated debates (e.g. the partial privatization of public pension systems or the introduction of basic pensions). In contrast, there are four issues of retirement rules in which there is a general agreement (Gruber and Wise, eds. 2007). To formulate them, we need to formulate two basic rules which determine the increase of the initial pension with the rise in (1) the lifetime contributions (which is approximately proportional to the length of contribution) and (2) the retirement age. As a benchmark we use the system of nonfinancial defined contributions, simulating a mandatory life insurance and life annuity system. More generally, we speak of flexible retirement if adding a year to the contributions or the retirement age strongly increases the retirement benefit, opening the way to the flexible choice of the retirement age.

The four pillars of the consensus are as follows: (i) to counterbalance the steeply rising life expectancy at age 60, it is important to raise the *normal* (or pensionable, statutory) retirement age; (ii) to raise the *expected retirement age*, it is important to make the system more flexible; (iii) the excessively permissive rule of *early* retirement does not reduce unemployment in the longer run; (iv) the introduction of *partial* (confusingly also called flexible) retirement (where between the full work and full retirement, an employee works in $x\%$ and is retired in $100-x\%$) would also increase flexibility.

Considering the post 1989 Hungarian pension system, a number of its features can be criticized. We shall, however, confine our attention to the retirement rules. Our major claim is as follows: *rather than applying the principle of flexible retirement age, more-or-less successful in the international practice, the various Hungarian governments modified/distorted it by very permissive or very rigid side rules.* (The principle that having a flexible retirement rule is socially advantageous in a wide range is a special case of the economic principle that the market is generally superior to an administrative distribution.)

Behind the two rules (concerning the role of the length of contribution and the retirement age) there is a third one, connecting the benefit to the average lifetime valorized earnings, the influence of which on the benefit can be proportional or progressive. (In valorization, the individual earnings of the various years are homogenized by an index of the nationwide earnings.) The fourth rule concerns the rise of pension in payment (indexation). In general, we skip the third and the fourth rules from now on.

In a number of countries, including Hungary, the first and the second rules have been frequently complemented by an exemption rule: with length of contribution above the critical value (at least 35–40 years), there is no reduction for early retirement (seniority pensions). In Hungary, rules 1 and 2 are now complemented by two side rules: (a) an *exceptional rule*:

since 2011, every women, who has accumulated at least 40 years of rights (somehow differing from the length of contribution), can retire within the framework *Females 40*, without any benefit reduction; (b) since 2012, *a stiffening one*: except (a), nobody can retire below the normal retirement age, even with reduced benefits.¹ We shall study the earlier exempting and the current exceptional/rigid retirement rules, paying particular attention to the induced income redistribution with respect to the ideal system of flexible retirement. We also mention that a large share of the workers do not know the pension rules (Simonovits, 2016, Chapter 12) and even if they knew them, they would not trust them.

In the ideal system, there are no gainers or losers: everybody is treated fairly. The reality is, however, more complicated, therefore in a practical flexible system, there are inevitably gainers and losers, who are not individuals but classes of types defined by their common retirement age, length of contribution and gross/net average lifetime wage. To avoid any misunderstanding, we consider social insurance, therefore we are only concerned with statistical averages. In the main text (below Table 1) we give two alternative definitions for the gainers and in Appendix 1, do a third one. Here we only shortly refer to them: the alternative system gives gainers (i) higher monthly benefits, (ii) lower (i.e. negative) net balance and (iii) higher lifetime utility than the flexible system does. Note, however, that workers (not knowing even the basic rules) may not see their own situation correctly, whether they are gainers or losers.

A good government has to be satisfied with limiting rather than totally eliminating the gains and the losses. In general, we consider balanced systems. (If a system is unbalanced, then the workers are also gainers/losers.) The government does not know or does not want to know the dependence of life expectancy on the retirement age and the lifetime average wage (for example, within a given sex, it does not know that the retirement age signals the expected remaining life and with good reason, it does not want to know the mortality difference between males and females). Therefore the traditional insurance rules are not fair (see, for example, Diamond (2003), Eső and Simonovits (2002), Eső, Simonovits and Tóth (2011), and Krémer (2015)) but here we neglect this complication.

The Hungarian retirement rules have been changing quite erratically since the systemic change of 1990. This in itself contributed to the tendency that almost everybody retired as early as she/he could. Though the reforms have been introduced with short notice, the female and male normal retirement ages rose quite smoothly from 55 and 60 (1996) to 62 (in 2009 and 2001) respectively, and their common value is still rising to 65 (by 2022). The *minimum*

¹ Another stiffening rule which forbids working beyond normal retirement age in the public sector would require further analysis. The collapse of the aging spheres of healthcare, higher education and research could only be avoided by exempting the aged medical doctors, professors and researchers, respectively. Still another problem would be the treatment of work after retirement.

female and male retirement ages, and the deduction rates for early retirement have changed quite erratically between 1996 and 2015. The deduction decreased quite fast with the increase in the length of contribution and became zero for relatively low length (say 35-38 years) for a long period. The previous exempting system increased the budgetary burden, eventually paid by the workers. This laxity has disappeared by 2010 and it was expected that a good flexible system will be created, which is able to satisfy various individual needs and at the same time is sustainable. This is not the case, however, since 2012, there is no early retirement except for Females 40.

Due to the introduction of the two new rules (Females 40 and no early retirement), however, the shares of gainers and losers in the present Hungarian pension system are unnecessarily high, and similarly high are the relative values of the gains and losses. We only give two illustrative examples for the gainers and the losers, respectively. Since the introduction of Females 40, many women of age 58–60 have enjoyed early retirement without any deduction (they need not pay the reduction of 18–30% for 3–5 years with respect to the normal retirement age 62–63.)² According to Table 11 below, 12% of women of age 58–59 retiring in 2011, in the first year of the Females 40, had a relatively high net wage and pensions, 83.6% of the net nationwide wage vs. 69% of the average pensioner. At the same time, another large group of females and males of age 61 with pension rights of 39 years could not retire, even paying the due reduction. They are the losers. (For example, those 8.4% of the female cohort who retired at the normal retirement age had received only half of those benefits.) We wait with the presentation of the details, but we already emphasize that the further rise of the normal retirement age (from 63 to 65 by 2022) will strengthen the tensions induced by this system.

At the end of the Introduction, we shortly compare our results with the earlier Hungarian literature. We single out Augusztinovics (2005) and Augusztinovics–Köllő (2008) and (2009) who first documented the detrimental impact of fragmented working paths on the corresponding pension benefits. Cseres-Gergely (2008) studied the role of incentives in early retirement. The Hungarian public pension system was described in a number of publications (e.g. Augusztinovics, Gál, Matits, Máté, Simonovits, Stahl, 2002 and Simonovits, 2008). We call attention to the relevant data from Molnár D. and Mrs. Hollós Marosi (2015) on the dependence of life expectancy on the wage/benefit and the retirement age. In a comprehensive model, Freudenberg, Berki and Reiff (2016) considered the long-term impact of recent Hungarian pension reforms, including that of the retirement rules. (According to their estimates, the rise of the normal retirement age plus the elimination of early retirement reduce the pension deficit by 0.5% of the GDP, while the introduction of Females40 increases

² We do not consider as unworthy gainers those who started to work at age 15 and probably during their entire difficult working lives had very modest wages.

the deficit by the same amount.) Various Hungarian papers were published on the special problems of the system but we omit them.

We only sketch the vast international literature in a nutshell. Gustman and Steinmeier (1986) published a structural model of retirement from the US public pension system, assuming that retirees maximize their lifetime utility functions. Considering the interaction of the Social Security and the private pension system with defined benefit, Stock and Wise (1990) was able to explain the emergence of two peaks around the earliest and the normal retirement ages (62 and 65). Rust and Phelan (1997) analyzed a standard model of retirement behavior under the influence of social security and medicare and private wealth.

A new wave of papers concentrated on other dimensions of the retirement problem from the US as well as EU. For example, Chan and Stevens (2004) looked for an answer to the question: “How does job loss affect the timing of retirement?”. Staubli and Zweimüller (2013) and Manoli and Weber (2016) studied a similar question: “Does raising the early retirement age increase employment of older workers?” Perhaps the closest to our paper is Etgeton (2016) “Labor Market Frictions, Retirement and Inequality”. We only quote one observation (from the abstract): “widespread reform effectiveness is hampered by the heterogeneous availability of jobs.”

The structure of the remaining part of the paper is as follows: Section 2 models a flexible pension system (similar to the Swedish, German or the Slovakian one). Section 3 moves to the earlier permissive Hungarian system and Section 4 continues with the present (exceptional/rigid) one. Section 5 draws the conclusions. Five appendices complete the paper: Appendix 1 formally defines the welfare ranking of any two systems. Appendix 2 formulates the definition of the partial retirement. Appendix 3 illustrates the dependence of the correlation coefficient and the welfare on the fairness of the system on a numerical model. Appendix 4 shows illustrative US data on the dependence of the retirement age and of the remaining life expectancy at retirement on the income. Appendix 5 contains important general Hungarian statistics used to transform figures from absolute into relative numbers.

2. FLEXIBLE SYSTEM

First we outline the general framework and then analyze two types of flexible systems: the so-called nonfinancial defined contribution system and the linear benefit rule.

2.1 FRAMEWORK

In every pension system, three main individual variables eventually determine the benefit: the length of contribution S , the retirement age R and the average lifetime gross wages w ,

respectively. (In reality, when the ratio of own wage to average wage varies annually, the rules are more complicated, but we skip this complication.) To simplify the analysis, we work at constant prices and wages. We only consider a single cohort. We shall generally skip disability or survivor beneficiaries but in Tables 4 and 5, in addition to the old-age pensioners the former are also considered. We need a function, connecting average lifetime gross and net wages: $v = T(w)$. In Hungary, this function will be proportional: $v = \theta w$, ($\theta = 0.66$) in 40 years but now it is still strongly progressive (concave) and this special feature strongly influences the initial benefits and benefits in payment. We take into account that the *expected remaining life expectancy at age R* also depends on the gross average lifetime wage w : $e_{R,w}$. (We do not denote that this parameter characterizes a given cohort t , for example, those born in year t .) As is well-known, this life expectancy increases with the wage and decreases with the age, but more slowly than the age increases.

Assuming that the pension contribution is proportional to the gross wage (i.e. by neglecting the cap on the contribution base effective between 1993 and 2012), and denoting the time-invariant contribution rate by τ , the expected lifetime balance of type (R, S, w) is given by

$$(1) \quad d(R, S, w) = \tau w S - b(R, S, T(w)) e_{R,w},$$

where $b(R, S, T(w))$ is the corresponding benefit function.

2.2 NONFINANCIAL DEFINED CONTRIBUTION (NDC) SYSTEM

For an NDC (denoted by index I), the expected lifetime balance is zero for every type, therefore the pension rule is defined as

$$(2) \quad b_I(R, S, T(w)) = \tau w S / e_{R,w}, \text{ where } S \geq S_m \text{ and } R \geq R_m,$$

S_m = minimum length, R_m = minimum retirement age and the normal retirement age R^* does not play any role.

Even in the NDC, the dependence of $e_{R,w}$ on w is neglected, rather it is simply assumed that $e_{R,w} = e_R$, therefore (2) is replaced by

$$(2') \quad b_I(R, S, T(w)) = \tau w S / e_R.$$

As a side remark, we note that the estimation of the expected life span at retirement is a very difficult problem and in the past decades e_R was typically underestimated.

Table 1 presents how freely chose the new Swedish retirees their retirement ages by the end 2013 between age 61 and 71. For example, among the oldest cohort (1938) 77% retired at the normal retirement age (65), among the youngest (1948), only 55%. The share of the

earliest retirees (aged 61) grew spectacularly: from 4 (1938) to 7% (1952). Though the main benefit rule strongly punishes them, the pension credit mitigates this effect.

Table 1.

The distribution of new Swedish retirees aged between 61 and 71, %

Cohort	61	62	63	64	65	66	67	68	69	70	71
1938	3.6	2.3	2.3	2.1	77.3	4.1	3.2	0.8	0.3	0.3	0.1
1939	3.9	1.9	2.1	2.3	75.6	6.5	2.3	0.8	0.3	0.3	0.2
1940	3.0	2.1	2.5	3.1	75.8	5.0	2.6	0.8	0.4	0.5	0.2
1941	2.9	2.2	3.0	3.7	73.2	6.3	2.8	0.8	0.5	0.4	0.2
1942	3.4	2.9	3.4	3.9	70.9	6.2	3.4	1.2	0.5	0.4	0.2
1943	4.0	3.1	3.6	5.3	66.4	7.1	4.4	1.2	0.4	0.5	
1944	4.7	3.4	4.7	5.9	63.2	7.9	4.0	1.1	0.5		
1945	5.1	4.2	5.3	6.1	61.7	7.2	4.0	1.3			
1946	6.0	4.8	5.5	6.7	59.4	6.7	4.3				
1947	6.4	4.6	6.0	7.5	57.2	7.0					
1948	6.1	5.0	6.7	7.9	55.4						
1949	5.9	5.5	7.0	8.8							
1950	5.9	5.5	7.8								
1951	6.6	6.4									
1952	6.9										

Remark. The ratio of newly retired to the potential size, end 2013.
Source: Swedish Pension System [2014, 26].

Before turning to imperfect systems, it is worth defining the gainers and losers in a system M with respect to the ideal system I. It would be difficult to neglect the dependence of outcomes on the effective mechanism M. Therefore we distinguish types in which the points of a sufficiently fine 3-dimensional grid are indexed: $i = 1, 2, \dots, n$; $(R_i(I), S_i(I), w)$ are compared to the alternative outcomes $(R_i(M), S_i(M), w)$.

We can give more than one *definition* for the gainers and losers.

1. With respect to the annual benefit, system M is better than system I for type i if and only if type i receives higher benefit for outcome $(R_i(M), S_i(M), w)$:

$$b_M(R_i(M), S_i(M), w) > b_I(R_i(M), S_i(M), w).$$

Remark. If the comparison is made on the basis of I rather than M, then the corresponding condition $b_M(R_i(I), S_i(I), w) > b_I(R_i(I), S_i(I), w)$ may not hold.

2. With respect to the net contribution, or lifetime balance, system M is better than system I for type i if and only if the net balance is negative for the outcome:

$$d_M (R_i(M), S_i(M), w) < 0.$$

Remark. As before it is not guaranteed that $d_M (R_i(I), S_i(I), w) < 0$ also holds.

The third definition will be given in Appendix 1 below.

2.3 LINEAR FLEXIBLE SYSTEM

Formula (2) is further simplified, namely *linearized* in other cases and gross wage I replaced by net wage. For a given earning and length of contribution, the net (i.e. after tax) benefit is an increasing linear function of the retirement age:

$$(3) \quad b(R, S, w) = [1 + \delta(R - R^*)] \gamma S T(w), \quad \text{where } R^*, \delta \text{ and } \gamma \text{ are positive constants.}$$

Their names are as follows: R^* is the normal retirement age, $R_m < R^* < R_M$, δ is the delayed/early retirement coefficient, γ is the constant *accrual rate* in terms of the net average wage. Here it is not self evident that the net balance is identically zero: $d(R, S, w) = 0$, therefore we have to require that their expected value be zero. Let $p_{R,S,w} > 0$ be the relative share of type (R, S, w) , their sum is being equal to 1. By definition, in a *balanced* pension system, the expected value of the net balances is zero:

$$(4) \quad \sum p_{R,S,w} d(R, S, w) = 0.$$

Inserting (1) and (3) into (4) yields a condition for the system is *balanced*:

$$(5) \quad \sum p_{R,S,w} \{ \tau w - [1 + \delta(R - R^*)] \gamma T(w) e_{R,w} \} S = 0.$$

We do not consider the problem of balance when it is taken for the whole population rather than a single cohort. Simplifying the previous approach, for the time being, we assume that the choice of the length of contribution and of the retirement age is independent of the system's parameter values. Then a simple equation is obtained for either the accrual rate γ or the contribution rate τ :

$$(6) \quad \tau \sum p_{R,S,w} w S = \gamma \sum p_{R,S,w} [1 + \delta(R - R^*)] T(w) S e_{R,w}.$$

To avoid arbitrariness, we have to stipulate that in a genuinely flexible system, the normal retirement age R^* lies years above the *minimum* retirement age R_m , years below the *maximum* retirement age R_M and the delayed retirement coefficient δ is several percent/year. Moreover, there may be a malus δ_1 and a bonus δ_2 for early and delayed retirement, respectively.

Table 2 displays the flexible benefits for selected employment lengths and retirement ages, calculating with normal retirement age 62 valid until 2012 with $\delta_1=0.03$ and $\delta_2=0.06$. For example, if somebody retires at age 60 with 38 years of service, he/she will receive 71.6%

of his/her net earning. The benefit in the cell (58, 40) is only 70.4%. We shall base our evaluation of the distortion caused in the exemptional/rigid system in Table 8 on this calculation.

Table 2.

Linear flexible benefits – retirement ages and length of contribution

Years of contribution, S Retirement age R	36	38	40	42	44
58	0.634	0.669	0.704	0.739	0.774
60	0.677	0.714	0.752	0.790	0.827
62	0.720	0.760	0.800	0.840	0.880
64	0.806	0.851	0.896	0.941	0.986
66	0.893	0.942	0.941	1.042	1.091

We have already mentioned that for simplicity, we generally neglect the impact of real earnings dynamics. Here we make an exception, and shortly discuss this issue. If in year t , a worker retires at age R with employment length S and average lifetime gross wage w_t , his/her benefit is equal to $b_t(R, S, T(w_t))$. Calculating with full employment for the last year, due to a delay of one year, his/her new entry benefit would be $b_t(R+1, S+1, T(w_{t+1}))g_{t+1}$, where g_{t+1} is the growth factor of the net wage from year t to $t+1$. Applying the pure price indexation, this should be compared to $b_t(R, S, T(w_t))$. The delay raises the yield of any extra year $g_{t+1} - 1 = 0.02$, except during the period 2013–2015, when the overindexation of pensions in payment amounted to 8%.

3. SYSTEM WITH EXEMPTION

We shall relate the system with exemption prevailing until 2012 to the foregoing flexible system. Unlike in (3), the value of δ was a sophisticated function of the length of contribution, and in a lot of cases, there was no deduction at all:

$$(3') \quad b(R, S, w) = [1 + \delta_S (R - R^*)] \gamma ST(w),$$

where δ_S is a non strictly decreasing function of the length of contribution: $(S = S_m, \dots, S_0)$, $\delta_{S_0} = 0$, where S_0 is the critical value of the length of contribution (e.g. 35 or 40 years), implying full benefits.

In such a system with exemption, almost every worker retired as soon as it was possible, i.e. when he/she reached the prescribed *critical* length of contribution S_0 . As is obvious, in

such a system, working until the normal retirement age hardly increases the benefit but lifts the net balance of contribution.

By the way, we can obtain a more precise description about the Hungarian pension system if we replace γS by a more complex series (c_s), representing the *accumulated accrual rates*. Table 3 displays the selected values, between which the function is a linear one (row 2). For example, $c_s = 0.02 S$ for $40 \leq S \leq 50$, but for $S \geq 50$, $c_s = 1$ (constant); below 40 its slope changes haphazardly. For example, for S lying between 36 and 40, $c_s = 0.74 + 0.015 (S - 36)$. Row 3 gives a hypothetical proportional scheme. In addition, the series of valorized net wages (v_t) and their progressive (concave) average also play a role.

Table 3.

Accumulated accrual rates

Length of employment (S) Replacement	20	25	32	36	40
Actual (c_s)	0.53	0.63	0.70	0.74	0.80
Proportional (γS)	0.40	0.50	0.64	0.72	0.80

In addition to these factors, due to changing rules, (3') also depends on the calendar year, but for the time being, this dependence is neglected. In the Hungarian practice, the length of contribution is downward rounded-off from month to years, but the retirement age is given in months. The benefit also depends on gender (f=female, m=male)).

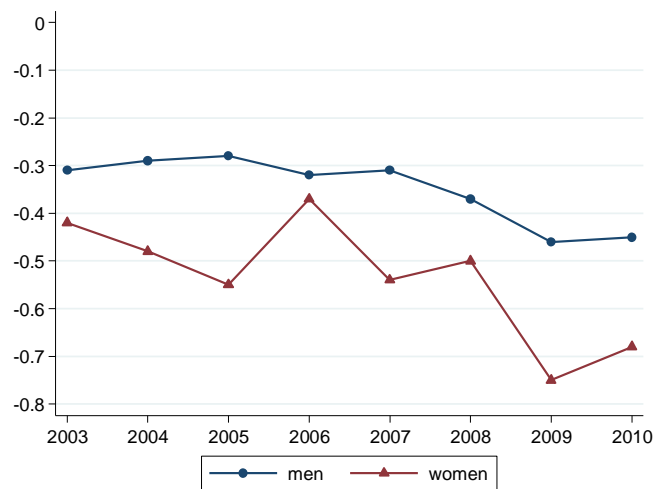
The simplest tool to characterize the distortion, due to the system with exemption, is the paradoxically negative correlation between the length of contribution and the retirement age. We recall the definition of correlation coefficient between two random variables. Let R and S be random variables of the length of contribution and of the retirement age, ER and ES their expected values, DR and DS their standard deviations, respectively. Then their coefficient of correlation is defined as $\rho(R, S) = \mathbf{E} [(R-ER) (S-ES)] / [DR DS]$. As is known, this index lies between -1 and 1 . For a negative correlation, the decrease in the index shows the strengthening of the correlation. In the usual one-dimensional framework, we expect strong positive correlation but in reality, the correlation is negative. (For details, see Appendix 3 below.)

Using the Data Bank data, we have constructed three figures (Figures 1 to 3). To make our figures, we relied on data base Nyugdme³, containing the aggregated pension decisions for the period 2003–2010.

According to Figure 1, the foregoing male correlation was “only” -0.3 in 2003, but it dropped to -0.45 by 2007. For females, the situation is even more paradox: it started from -0.4 and dropped to -0.7 by 2010. This is a sign of the strengthening impact of the exemption, neutralizing the rising normal retirement age.

Figure 1.

Correlation for those retiring above 54, between 2003 and 2010



To exclude outliers, we confine our attention to those who had at least 20 years of contribution (the recent minimum value). Similarly to Figure 1, Figure 2 also reports negative and time-decreasing correlation, only the values are less extreme.

Further delimiting the analysis, we only consider those who retired at or above the normal retirement age. Figure 3 still reports negative correlation but with low absolute values.

³ Database Nyugdme³ contains the retirement decisions concerning the period 1999–2010, aggregated according to birth year, retirement year and month, gender, length of contribution, average pension, and the valorized net earning.

Figure 2.

Correlation for those retiring above 54, with minimum 20 years of contribution, between 2003 and 2010

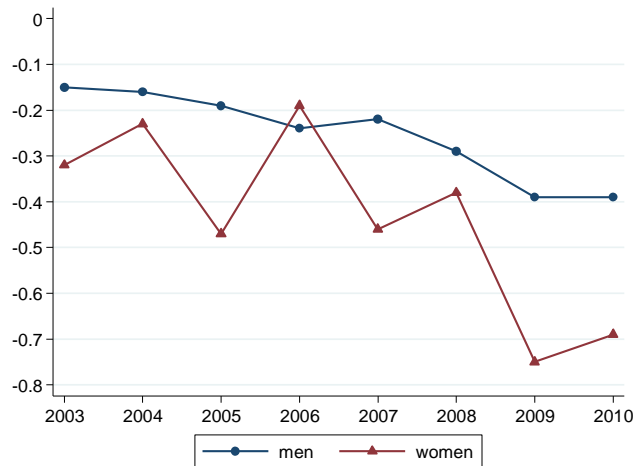
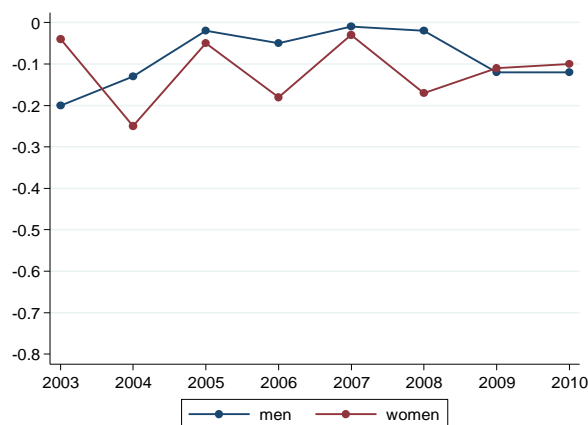


Figure 3.

Correlation for those retiring at or above the normal retirement age, between 2003 and 2010



We are moving from aggregate statistics to a somewhat disaggregated analysis. We expect that in a fair pension system, the benefits are smooth functions of the length of contribution and the retirement age. We shall see that this expectation only partly fulfilled.

Tables 4 and 5 display the relative size and relative benefit of those male and female groups who retired in 2011 on their own right, breaking down the data according to

retirement age and length of contribution. (Confining attention to old-age pensioners would yield similar results.) To have perspicuous tables, we drop the categories with extreme lengths of contributions. Note that the ratio of the number of all new male retirees to that of being in normal retirement age is about 97.4% and the *replacement rate* is equal to 71%, reaching its peak at retirement age of 61 with 99%.

Table 4.

**Relative size of male groups retired in 2011 and of benefits –
retirement age and length of contribution**

Age	Size–Length of contribution			Benefit–Length of contribution		
	35–39	40–44	(Full sample) Total	35–39	40–44	(Full sample) Average
54–55	0.012	0.005	0.033	0.710	0.768	0.627
56–57	0.012	0.019	0.044	0.769	0.802	0.708
58–59	0.019	0.110	0.160	0.797	0.826	0.807
60	0.047	0.239	0.378	0.555	0.753	0.732
61	0.005	0.021	0.031	0.567	1.118	0.983
62–63	0.030	0.030	0.160	0.559	1.246	0.667
Total	0.139	0.422	0.974	0.637	0.827	0.707

Source: ONYF [2012, p. 109]: Table 9.1.

The related numbers for females: retirement ratio is equal to 135% and the replacement rate is equal to 66%, reaching its peak at retirement age 60 with 74%. (Note that 2011 was a singular year when the Females 40 was already introduced but the permissive system of early retirement was valid. Furthermore, the minimal retirement age of female cohort 1952 just rose from 57 to 59.) We call the Reader’s attention to the bifurcation at length of contribution of 40.

Table 5.

**Relative size and benefits of female groups retired in 2011 –
retirement age and length of contribution**

Age	Size–Length of contribution				Benefit–Length of contribution			
	30–34	35–39	40–44	Total	30–34	35–39	40–44	Average
54–55	0.004	0.010	0.091	0.115	0.515	0.606	0.638	0.612
56–57	0.003	0.013	0.336	0.357	0.476	0.623	0.681	0.672
58–59	0.006	0.089	0.498	0.599	0.455	0.576	0.783	0.747
60	0.003	0.006	0.017	0.027	0.410	0.576	1.059	0.865
61	0.001	0.001	0.003	0.006	0.415	0.555	1.108	0.739
62–63	0.027	0.006	0.003	0.113	0.441	0.530	1.103	0.475
Total	0.065	0.131	0.949	1.347	0.486	0.581	0.739	0.661

Source: ONYF (2012, p. 109): Table 9.2.

4. EXCEPTIONAL/RIGID SYSTEM

We shall put the present system into a framework and then display its impact by a model and tables.

4.1. FRAMEWORK

The analysis of the current (exceptional/rigid) system is more important than the previous system (with exemptions). Since in the new system the distinction between females and males has reappeared, we should double all our equations correspondingly (generally omitted). The aggregated balance equations (4) and (5) would be obtained by the summation of the two variants.

(1) is replaced by the equation of the favored females:

$$(7) \quad b(R, S, w) = \gamma S T(w) \quad \text{if } S \geq 40 \text{ and } R < R^*.$$

(1) is modified into the equation below for all the others, retiring in the rigid system:

$$(8) \quad b(R, S, v) = [1 + \delta(R - R^*)] c_s T(w) \quad \text{if } R \geq R^*.$$

There is a further complication: years spent in vocational schools and higher education are counted in (8) but neglected in (7). Therefore passing the normal retirement age, the female accrual rate jumps by the quantity corresponding to 3–5 years exempted, amounting to 7.5–12.5%. This is neglected here.

Currently (2016), the parameter values of the Hungarian system are as follows: $R^* = 63$ years: $\delta = 0.06$ and $\gamma = 0.02$, $R_m = R^*$, early retirement is only allowed for women, if $S \geq 40$ but without any deduction. $S_m = 20$ years.

4.2. FEMALES 40

Table 6 displays the program Females 40 as it stood in 2013. The largest cohort is of 1955, its average retirement age is about 58 years, and its average length of contribution is about 41 years. The majority retired with 40 years of rights but 15 and 11% with 42 and 43 years, respectively.

Table 6.

Data of females 40, 2013.

Birth year	Size distribution, %	Average retirement age	Average length of employment	Size distribution according to employment length, %				
				40	41	42	43	44
1953	4.9	60.0	41.5	37.7	29.4	18.4	4.9	5.1
1954	26.6	59.0	41.1	59.7	16.1	8.5	8.5	4.4
1955	32.9	58.2	41.1	61.4	9.3	15.2	10.5	1.7
1956	17.7	57.1	41.7	31.2	17.4	37.8	11.3	0.0
1957	9.3	56.1	40.7	65.6	23.6	7.2	0.0	*
1958	4.7	55.2	40.3	87.1	9.7	*	*	*
Average	100.0	57.9	41.1	56.3	14.8	15.9	8.2	2.0

Source: ONYF (2014, 111–112. o., Table 6.9)

Table 7 shows the relative benefits of the same categories. The original table also demonstrates that those newly retired who were born in 1951 (0.3%) had an average net valorized wage of 118%, while those born in 1952 (1.3%) had only 103.5%. Our censored table displays that those born in 1956 or later had still lower average net valorized wage of 83.3%.

Table 7.

Relative benefits of Females 40, retiring in 2013

Birth year	Relative average earning, %	Average entry benefit	Average length of rights	Size distribution according to employment length, %				
				40	41	42	43	44
1953	0.938	0.771	40.5	70.6	18.2	6.9	2.3	1.2
1954	0.954	0.776	40.2	86.9	10.4	1.8	0.6	0.2
1955	0.954	0.775	40.2	90.2	8.0	1.2	0.4	0.2
1956	0.793	0.655	40.2	89.8	8.8	1.0	0.3	0.2
1957	0.792	0.639	40.2	91.7	7.6	0.6	0.0	*
1958	0.760	0.609	40.1	95.0	5.0	*	*	*
Average	0.897	0.731	40.2	88.2	9.1	1.5	0.6	0.3

Source: ONYF [2014, 111–112. o., Table 6.9] censored.

To make the tables shorter, we cut out the less important very early and very late birth years (–1952 and 1959+, respectively) and the similarly extremely short and long retirement ages (–39 and 45+, respectively), the displayed shares do not add up to 1. Similarly, the averages refer to the whole population.

Less important very early and very late birth years (–1952 and 1959+, respectively) and the similarly extremely short and long retirement ages (–39 and 45+, respectively), the displayed shares do not add up to 1. Similarly, the averages refer to the whole population.

To widen the analysis, we cite a number of important data from 2012 about the dependence of life expectancy on the earning in Hungary from the path-breaking study of Molnár D. and Hollós-Marosi (2015). Dropping the lowest decile of pensions (to avoid complications stemming from partial pensions received by emigrants), the foregoing paper divided the remaining nine deciles into four equal parts. For example, for males, the lowest benefits were between 43.2 and 61.2% of average net earning, while their life expectancy at 60 was equal to 17.1 years. The highest benefits started at 104.3% and the respective life expectancy was four years longer. The female earnings were uniformly lower and the life expectancy hardly depended on the earning.

The foregoing study presents interesting data on the dependence of life expectancy and the retirement age, too. Unfortunately, the categories are too large, therefore only slight differences arise. For example, in 2012 those Hungarian males who retired before reaching age 59, had a remaining life expectancy of 14.9 years, while those retiring older than 61, had another 16.1 years. (The corresponding demographic numbers are 16.7 and 15.4 years.) For

females, those retiring between 50 and 54 years had another 22.5 years, while those retiring beyond 61 years, live only slightly longer: 23.1 years. (Note that the unofficial data of Esó, Simonovits and Tóth (2011) cited much larger differences for those who died in 2004.)

One of the main issues of the present paper is as follows: what is the impact of the elimination of early retirement except for Females 40? Table 8 translates the model calculations of Table 2 to the exceptional/rigid system. Rows 4-6 are dropped, since they are identical to those of Table 2. Returning to our earlier examples: 0 benefit stands in cell (60, 38), while for females, the cell (58,40) jumps from 70.4 to 80%! This is obviously unfair.

Table 8.

**Exceptional/rigid benefits – retirement ages and length of contribution
(females)**

Years of contribution, <i>S</i> Retirement age <i>R</i>	36	38	40	42	44
58	0	0	0.80	0.84	0.88
60	0	0	0.80	0.84	0.88

4.3. REAL OUTCOMES

It is worth presenting some data on real outcomes. Sampling well-known statistics, first Table 9 displays the characteristics of females, females 40 and males between 2006 and 2014. The outcome is chaotic. The relative size of newly retired cohorts developed erratically. For example, in 2010, the number of newly retired females was equal to 20% of the number of those females of normal retirement age, while in 2011, it jumped to 119%. In 2007, 101% of males retired, while in 2014, only 54%. Of course, everything can be explained by the erratic developments of the rules. In 2010, females delayed their retirement until the much more favorable era starting in 2011. In 2007, males (and females) surpassed the sudden decrease of 8% in the entry pensions announced for 2008, in 2014, the normal retirement age rose by 1/2 year.

Table 9.

Retirement ages and benefits: females, females 40, and males

Year	Females		Females 40		Males	
	Average retirement age (year)	Relative size	Average retirement age (year)	Relative size	Average retirement age (year)	Relative size
2006	57.5	0.888			59.9	0.544
2007	57.8	1.233			59.7	0.844
2008	57.3	0.829			59.8	0.400
2009	59.9	0.298			59.7	0.582
2010	60.7	0.247			60.2	0.541
2011	58.5	1.479	57.6	0.769*	60.3	0.608
2012	59.1	0.889	57.8	0.374	62.0	0.299
2013	59.5	0.670	57.8	0.329	62.2	0.289
2014	59.3	0.599	58.2	0.360	62.2	0.201

Source. Fazekas–Varga (2015, p. 262, Table 11.5).

*Oral communication: The number for Females 40 in 2011 also contains those who retired earlier but were reclassified in 2011.

4.4. DETAILED OBSERVATIONS

Due to the Attached Administrative Data Base, we can obtain a more precise picture on the situation of the newly retired females between 2002 and 2011. Taking into account our topic, we shall distinguish three types of old-age retirement: early retirement, Females 40 and ‘normal’ retirement (ironically referring to the rare retirement at the normal retirement age). We shall compare now the three groups (with respect to fragmentation of career, earning before retirement and the entry pension).

A lot of statistics attest that the share of early retirees was always quite high; moreover, normal retirement is an exception rather than the rule. The changes in the average retirement age only follow the changes in the law. Already commenting Table 8 we called the Reader’s attention to the critical role of length of contribution 40, and to the inequalities in benefits (present between those who retire with 35-39 and 40-44 years of contribution, respectively).

FURTHER DETAILS

Applying our administrative data, we were unable to take into account the length of contribution but we relied on the entry pensions and the earnings before retirement.

According to Table 10, the participants in Females 40 had as benefits 77.2% of the average net wage, somewhat lagged behind those of early retirees (82,2%) but by far surpassed those of the 'normal' retirees (43.2%).

Table 10.

Female pensioners of 2011-ben: average last earning and initial benefit

Type of retirement	Relative average benefit	Share %	Relative average earning 3 months before retirement	Share of employed 3 month before retirement
Females 40 (54-59)	77.2	43.5	128.9	39.7
Early pensioners	82.2	17.8	156.5	10.3
Females retiring at the normal retirement age	43.2	6.1	80.2	1.3

Table 11 breaks down into three age-groups those pensioners who retired in the first year of starting Females 40. With the rise of the retirement age, not only the replacement rates but also the relative value of pre-retirement net wage with respect to the average net wage grew steeply: 76.5% (aged 54-56) vs. 94.8% (aged 58-59).

Table 11.

The pension and pre-retirement date of Females 40 in three age-groups

Retirement age	Relative average benefit	Relative size	Ratio of gross wage to the average one	The size of workers 3 months before retirement
54-56	0.701	0.184	0.765	0.173
57	0.744	0.163	0.802	0.149
58-59	0.836	0.260	0.948	0.236

Looking at Figure 4 (obtained by combined administrative database⁴) it is evident that the earlier (i.e. at the younger age) one benefited from Females 40, the lower her benefit and last earning. Comparing the beneficiaries of Females 40 with non-retired workers of the same age, we find the following difference: until age 57, the latter earn more than the former, but at age 58, the situation is reversed. Knowing this tipping point, the birth-cohort dependence of pension and earnings depicted in Table 6 becomes clear. We also note that those retiring at the normal retirement age are even at worse situation.

Figure 4.

Monthly average wages in May 2010 at an annual percentage of the average wage in 2010.

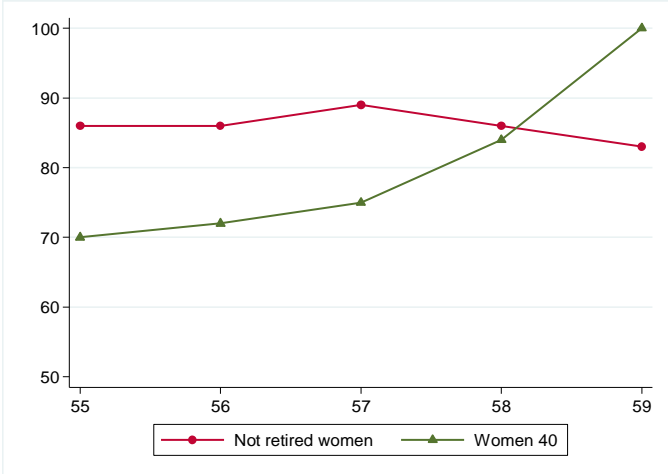


Table 12 summarizes the data of females retired in period 2012–2014. We emphasize that in all the three years, the number of those retiring in Females 40 was 1.5-3 times higher than those at or above the normal retirement age; they were 4-5 years younger and the difference between the contribution lengths of the two categories dropped from 14 to 11 years. Through the zigzagged accrual schedule, the loss of the second category was somewhat lower than suggested by Tables 2 and 8, it remains severe.

⁴ The combined data base was created by the combination of five administrative organizations' data. It contains the labor force data of 50% of the Hungarian population between 15 and 74 years, in a monthly breakdown of the period 2003–2011. To make our calculations, in addition to labor force and transfer data, we analyzed the NYUFIG data. We have created a detached database, which contains the benefit decision and payment, furthermore, it unifies the benefit amount on an annual base.

Table 12.

The most important characteristics of female retirees, 2012–2014

Year	Type	Size	Average age (year)	Average length of contribution (year)	Average replacement
2012	Reaching NRA	0.144	62.6	26.0	0.450
	Females40	0.389	57.8	40.7	0.772
2013	Reaching NRA	0.195	62.3	28.1	0.485
	Female40	0.324	58.0	40.7	0.731
2014	Reaching NRA	0.130	62.8	30.2	0.522
	Female40	0.374	58.3	40.9	0.742

Source: ONYF (2016, pp. 51–53, Table 4.1).

5. CONCLUSIONS

In this Section we summarize the results obtained and draw some conclusions. It is almost a commonplace that in a fair system—in addition to the average lifetime earning and within wide limits—the entry benefit is a strongly increasing function of the length of contribution and of the retirement age. (In fact, considering the pension contribution as forced saving, in the resulting life annuity, both the lengths of saving and dissaving periods are important.) This principle is obvious in most countries but not in Hungary: either the employment length or the retirement age is decisive. Before 2010, every man or woman with sufficiently long employment was allowed to retire with full benefit. Since 2011, having accumulated 40 years of rights, every woman can retire without any deduction. On the other hand, except Females 40, nobody can retire before reaching the normal retirement age. For example, in 2016, even 39 years of right does not allow a 62 year-old woman to retire—even ‘paying’ serious deduction.

Both the public statistics and our administrative data reveal the significant distortion implied by the neglect of either the retirement age or the length of contribution in the calculation of benefits. Our data underline the surprising fact that in the system with exemption, there is a negative correlation between these two variables. Typically, the later one retires, the shorter is his employment. This was only possible because a large part of the workers—frequently unintended—worked with long breaks. We conjecture that this negative correlation survived after 2011 but we cannot document it.

The beneficiaries of Females 40 form a basically heterogeneous group. Breaking down the group by age (and the underlying education level) one can distinguish two subgroups: one is disadvantaged (aged 54-56) and another is advantaged (aged 58-59), the latter's members resemble the early retirees. Already in 2016, every woman with a university diploma will have the 40-year of right at the age 63 and then the rigid prohibition of early retirement does not affect her. True, there remains a 5 year-long difference between the lengths of contribution and of the right, diminishing by 12.5% of the extra benefit at reaching the normal retirement age. But this would be too little to deter joining Females 40 when the normal retirement age rises to 65.

Using the data of the Central Statistical Office and the Central Administration of National Pension Insurance we analyzed the post 2010 situation. We emphasized that the Females 40 gives a significant advantage to a lot of women and causes also significant and unfair deduction to others. The elimination of early male retirement has already produced strong tensions and due to further significant rise in the normal retirement age, it will become unbearable. In addition, the special favor does not apply to those females whose careers are fragmented and whose average net earning is below those of the beneficiaries.

The new and *ad hoc* Females 40+ (promulgated just in January 2016) tries to mitigate this problem by an awkward way: for several months, the government is ready to pay the full compensation of those unemployed and previously low-paid females to accumulate 40 years of rights, who deserve help—even by the government's evaluation. The fair solution is so obvious: actuarial reduction of benefits at early retirement!

It would be socially optimal to close down Females 40 and introduce the flexible rules which are satisfactory to the employees and the government. The longer the Hungarian government insists on the exceptional/rigid system, the stronger tensions will be accumulated. In our opinion, even having introduced a flexible system, following a rigid social norm, a lot of workers would still retire as early as possible, even if they received reduced pensions. At the same time, we hope that with carefully designed parameter values the foregoing tendency can be limited and a socially optimal system can be created which provides room for individual choice within certain limits and at the same time, sustainable.

APPENDIX 1. WELFARE COMPARISON OF DIFFERENT MECHANISMS

In the main text the gainers and the losers were defined without relying on lifetime utility functions. Only direct comparisons were made: the gainers are those who obtain higher benefits or have lower net balances in the alternative mechanism than in the flexible one. In contrast, Appendix 1 outlines the basics of welfare comparison. Let $i = 1, 2, \dots, n$ be the index of various types, f_i be the share in the population, M be the alternative system (e.g. flexible, exceptional, rigid). Let $U_i(R_i, S_i, b_i)$ be the lifetime utility of type i , and V be the utilitarian social welfare function:

$$V = \sum_i f_i U_i(R_i, S_i, b_i).$$

To compare two mechanisms M_1 and M_2 , we define M_1 is better than M_2 if the first provides higher welfare than the second. Formally:

$$V(M_1) > V(M_2).$$

We conjecture in general and show numerically in particular in Appendix 3 below that in a well-calibrated model the flexible mechanism typically provides higher welfare than either that with exemption or the rigid; therefore the former is better than the latter.

APPENDIX 2. PARTIAL RETIREMENT

International experts have known for a long period that even the so-called flexible retirement system is not flexible enough, at least with respect to the system of *partial* retirement. Though the international experiences are not yet encouraging, we hope that not the idea but only the practice is bad. Appendix 2 contains a formal description. There are two rather than one retirement ages: R_1 and R_2 , those of the partial and of the full retirement and two rather than one length of contribution: S_1 and S_2 . The net lifetime balance is given as

$$(1') \quad d(R_1, R_2, S_1, S_2, w) = \tau w [S_1 + (1 - \alpha) S_2] - [\alpha e_{R_{12}, w} + e_{R_2, w}] b(R_1, R_2, S_1, S_2, T(w)),$$

$e_{R_{12}, w}$ being the number of expected years spent in interval $[R_1, R_2)$. Of course, if $R_1 = R_2$ or $\alpha = 1$, then the partial retirement reduces to the flexible one. For $d=0$, the NDC is obtained.

APPENDIX 3. THE IMPACT OF THE RETIREMENT RULES ON THE CORRELATION AND WELFARE

In this Appendix a simple model is constructed where the impact of the retirement rules on the correlation coefficient and welfare (numerically represented by relative efficiency) can be studied theoretically. We shall show that as we move smoothly from the rigid/lean retirement rules to the flexible retirement, the foregoing correlation grows from -1 to 1 and the welfare rises.

THEORY

The starting point is that there are groups in the population whose working careers are differently fragmented. Let integer $n > 1$ be the number of groups, $k = 1, 2, \dots, n$ be the generic group index. Let L be the common age when people start working (possibly including the years in higher education) and D be the common age when they die. Denote R_k and S_k the retirement age and the years of employment, respectively, and $1 - \varphi_k$ be the *degree of fragmentation* of type k 's career. Then by definition, $S_k = \varphi_k (R_k - L)$. We shall index the groups in an increasing order of fragmentation: $\varphi_k > \varphi_{k+1} > 0$, and $\varphi_1 = 1$ (nonfragmented career). Let $f_k (> 0)$ be the population share of group k with $\sum_k f_k = 1$.

To avoid confusion between the individual based approach in Section 2 and the category-based approach here, we repeat some definition from Section 2. We need the expected years of employment and expected retirement age, respectively:

$$\mathbf{ER} = \sum_k f_k R_k \text{ and } \mathbf{ES} = \sum_k f_k S_k$$

and their variances:

$$\mathbf{D}^2\mathbf{R} = \mathbf{E}(\mathbf{R} - \mathbf{ER})^2 \text{ and } \mathbf{D}^2\mathbf{S} = \mathbf{E}(\mathbf{S} - \mathbf{ES})^2 .$$

Finally we define the correlation coefficient between \mathbf{R} and \mathbf{S} :

$$\rho(\mathbf{R}, \mathbf{S}) = \mathbf{E}((\mathbf{R} - \mathbf{ER})(\mathbf{S} - \mathbf{ES})) / (\mathbf{DR} \mathbf{DS}) \text{ if } \mathbf{DR} > 0 \text{ and } \mathbf{DS} > 0 .$$

As is known, $-1 \leq \rho(\mathbf{R}, \mathbf{S}) \leq 1$, and the equalities hold if and only if $\mathbf{S} = \mathbf{A}\mathbf{R} + \mathbf{B}$, with $\mathbf{A} < 0$ and $\mathbf{A} > 0$, respectively. (Note that if all the degrees of fragmentation were close to each other, then $\rho(\mathbf{R}, \mathbf{S}) \approx 1$ but this is not the case.)

The simplest way to model the retirement rules is the following. There is a normal retirement age (\mathbf{R}^*) and there are two critical values: a critical length of employment (\mathbf{S}_0) and a critical retirement age (\mathbf{R}_0). To make the model meaningful, it is assumed that type 1 (with full employment) has at least the critical length of employment if (s)he retires at the normal retirement age: $\mathbf{S}_0 \leq \mathbf{R}^* - \mathbf{L}$. It is also assumed that the critical retirement age is at most as high as the normal: $\mathbf{R}_0 \leq \mathbf{R}^*$.

In case of sufficiently long employment, the benefit is proportional to the years of employment and the net wage $1 - \tau$, where τ is the contribution rate and γ is the proportionality factor (the accrual rate):

$$b(\mathbf{R}, \mathbf{S}) = \gamma \mathbf{S} (1 - \tau) \text{ if } \mathbf{S} \geq \mathbf{S}_0 .$$

In case of sufficiently late retirement but still below the normal retirement age and shorter than critical employment length (but at least as long as the minimal length \mathbf{S}_m), the worker can retire with an annual deduction δ_1 :

$$b(\mathbf{R}, \mathbf{S}) = \gamma \mathbf{S} (1 - \tau) [1 + \delta_1 (\mathbf{R} - \mathbf{R}^*)] (1 - \tau) \text{ if } \mathbf{R}_0 \leq \mathbf{R} \leq \mathbf{R}^* \text{ and } \mathbf{S}_m \leq \mathbf{S} < \mathbf{S}_0 .$$

After reaching the normal retirement age, the second rule gives credit rather than deduction:

$$b(R, S) = \gamma S (1-\tau)[1+\delta_2 (R-R^*)](1-\tau) \text{ if } R \geq R^* \text{ and } S \geq S_m.$$

Otherwise no retirement is allowed. For simplicity, we introduce the notation

$$b[R_k] = b(R_k, \varphi_k (R_k - L)).$$

Note that our scheme contains the two extreme systems: (i) the permissive and rigid with $R_o = R^*$ and $S_o < R^* - L$ and (ii) flexible with $R_o < R^*$ and $S_o = R^* - L$.

To derive the retirement ages as a function of the retirement rules, we posit a standard lifetime utility function. It consists of three terms: the utility enjoyed while (i) working, (ii) being idle and (iii) being retired:

$$U[R_k] = [\log (1-\tau) - \varepsilon] \varphi_k (R_k - L) + [\log C - \varepsilon] (1-\varphi_k)(R_k - L) + \log b[R_k] (D - R_k),$$

where ε is the per-period utility loss due to work or unemployment and C is the value of social income.

Finally, we define the per worker balance of the system, i.e. the difference between contributions and benefits:

$$B = \tau ES - C(ER - ES - L) - E[b[R] (D - R)].$$

Numerical calculation

Turning to numbers, we choose three types: $n = 3$ and Table A3.1 shows the three types' parameter values.

Table A3.1.

Parameters of the three types, normal case

Types	1	2	3
Shares f_k	0.6	0.3	0.1
Fragmentation φ_k	1.0	0.9	0.8
Disutility ε_k	1.0	1.3	1.7

Other parameter values are as follows: $L=20$, $R^*=62$, $D = 77$, $C = 0.25$. For $\gamma = 0.03$, the balanced value of c varies around 0.358. The exceptional/rigid system is characterized by $S_o = 40$ and $R_o = 62$ and the flexible $S_o = 42$, $R_o = 60$ and $\delta_1=0.06$. We can achieve a smooth transition between the two extreme systems with the following equations:

$$S_o = 40+0.5x, R_o = 62-0.5x \text{ and } \delta_1=0.015 x, x=0, 1, 2, 3 \text{ and } 4.$$

To avoid the ambiguity of the social welfare functions, we introduce the concept of relative efficiency. Mechanism y 's *relative efficiency* with respect to that of x is a positive real number

ω , if multiplying the wages and benefits by ω in x yields the same welfare as the original y . In formula:

$$V[y] = V[x] + (D-L)\log \omega, \text{ i.e. } \omega = \exp \{ (V[y] - V[x]) / (D-L) \}.$$

Table A3.2 shows the results. As we claimed in the introduction, during a smooth transition from the exceptional/rigid system to the flexible one, the correlation coefficient grows from -1 to 1 and the relative efficiency grows from 1 to 1.011 .

Table A3.2.

Transition from the exceptional/rigid system to the flexible one, normal case

Annual deduction δ_1	Critical		Retirement age for type			Correlation coeff. $\rho(R, S)$	Relative efficiency ω
	length of employment, S_0	retirement age, R_0	1 R_1	2 R_2	3 R_3		
0.000	40.0	62.0	60.1	62.0	62.0	-0.875	1.000
0.015	40.5	61.5	60.6	61.5	61.5	-0.895	1.004
0.030	41.0	61.0	61.0	61.0	61.0	0.001	1.008
0.045	41.5	60.5	60.9	60.5	60.5	0.914	1.007
0.060	42.0	60.0	62.0	60.4	60.0	1.000	1.011

In summary, we consider types with various degrees of fragmentation, when the system operates with a critical length of contribution and a critical retirement age plus an adjustment rate. As we raise the critical length and the deduction rate, while diminish the critical age, we move from the exceptional/rigid system toward the flexible system, and the signed correlation coefficient between the length and age increases from -1 to 1 . This signals the improvement of fairness as well.

A COUNTEREXAMPLE

To show a case where the exceptional/rigid system is more efficient than the flexible one is, we choose parameter values where there is no fragmentation, the labor disutilities 1 and 3 change places and the start and exit ages are steeply rise with the lifespan.

Table A3.3.

Parameters of the three types, counterexample

Types	1	2	3
Shares f_k	0.6	0.3	0.1
Disutility ε_k	1.0	1.3	1.7
Start working L_k	16.0	20.0	22.0
Age at death D_k	70.0	75.0	80.0

Now the outcome of Table A3.4 is totally different from that of Table A3.2. The short-lived worker with heavy labor disutility can take early retirement, while the long-lived worker with light burden cannot. The correlation coefficient remains strongly negative, while the relative efficiency first stagnates then it diminishes.

Table A3.4.

Transition from the exceptional/rigid system to the flexible one, counterexample

Annual deduction δ	Critical		Retirement age for type			Correlation coeff. $\rho(R, S)$	Relative efficiency ω
	length of employment, S_0	retirement age, R_0	1 R_1	2 R_2	3 R_3		
0.000	40.0	62.0	57.0	62.0	62.0	-0.877	1.000
0.015	40.5	61.5	57.0	61.5	61.5	-0.886	1.000
0.030	41.0	61.0	57.0	61.0	61.4	-0.921	1.001
0.045	41.5	60.5	57.6	60.5	62.0	-0.990	0.999
0.060	42.0	60.0	58.1	60.4	62.0	-1.000	0.997

APPENDIX 4. SOME RELEVANT DATA ON THE US SOCIAL SECURITY

For an international comparison, Appendix 4 cites two relevant tables of the US Social Security from Bosworth, Burtless and Zhang (2016). Table A4.1 demonstrates that even in the United States, where the employment rate is quite high, about the majority retires at the earliest age (62, 4 years below the normal retirement age, being 66) and their share slightly decreases with earning. At first sight, it appears that if 42% of the highest third retires as soon as possible, then it is excessive, though the reduction is very high: cc. 25%.

Table A4.1.

The shares of earliest and of normal retirement in the US

Retirement	Income thirds (%)		
	lowest	middle	upper
At the earliest age	56.1	48.3	42.3
Normal retirement age	13.8	18.7	28.3

Table A4.2 displays the dependence of life expectancy at age 50 on income for two cohorts: born in 1920 and 1940, females and males, the lowest and highest deciles. As is the case in general, males live much shorter than females, but this is especially true for the poor.

Table A4.2.

Life expectancy at 50 by gender and income, USA

Cohort	Females (year)		Males (year)	
	poorest	richest	poorest	richest
1920	80.4	84.1	74.3	79.3
1940	80.4	90.5	76.0	88.0

Appendix 5. Hungarian labor and pension statistics

Appendix 5 contains some basic statistics used in converting absolute Hungarian data into relative ones.

Table A5.1.

Employment rate of population aged 15-74 by selected age groups, percent

Year	Males			Females		
	55-59	60-64	Total	55-59	60-64	Total
2010	56.3	16.5	54.2	46.6	9.5	43.6
2011	56.9	17.4	55.0	49.9	11.0	43.7
2012	61.2	17.0	55.7	49.7	11.2	44.9
2013	64.9	21.1	57.4	51.4	11.1	45.4
2014	70.6	26.9	60.8	56.8	13.4	48.0

Source: Fazekas and Varga (2015, p. 221, Tables 4.13 and 4.14).

Table A5.2.

Nominal and real earnings

Year	Gross earnings 000 HUFs	Net earnings 000 HUFs	Consumer price index (previous year)
2010	202.5	132.6	104.9
2011	213.1	141.2	103.9
2012	223.1	144.1	105.7
2013	230.7	151.1	101.7
2014	237.7	155.7	99.8

Source: Fazekas–Varga (2015, 2015, p. 241, Table 6.1).

Table A5.3.

Number of males and females at normal retirement age (2006–2014)

Year	Number of ('000)	
	males	females
2006	51.9	60.8 ^a
2007	50.3	60.3 ^b
2008	47.4	64.3 ^b
2009	51.0	63.7
2010	55.0	68.8
2011	57.4	71.1
2012	60.3	73.7
2013	60.6	74.7
2014	59.4 ^c	73.6 ^c

Remarks. a)-b) The female normal retirement age was only 60 in 2006 and only 61 in 2007 and 2008. c) Unisex normal retirement age was already somewhat higher than 62 in 2014.

REFERENCES

- Augusztinovics, M.; Gál, R. I.; Matits, Á.; Máté, L.; Simonovits, A.; Stahl, J. (2002): “The Hungarian Pension System Before and After the 1998 Reform”, in E. Fultz (ed.), *Pension Reform in Central and Eastern Europe*, Vol. 1–2, Geneva, International Labour Office.
- Augusztinovics, M. and Köllő, J. (2008): “Pension System and Fragmented Labor Market Careers”, in, Gál, R. I.; Iwasaki, I; Széman, Zs., eds. 2008. *Assessing Intergenerational Equity: An Interdisciplinary Study of Aging and Pension Reform in Hungary*, Budapest, Akadémiai Kiadó, 154–170.
- Augusztinovics, M. and Köllő, J. (2009): “Decreased Employment and Pensions. The Case of Hungary”, In: Holzmann R., MacKellar L., Repansek J. (eds.), *Pension Reform in Southeastern Europe. Linking to Labor and Financial Market Reform*. Washington, D.C.: World Bank, 2009. 89–104.
- Bosworth, B.; Burtless, G. and Zhang, K. (2016): *Later Retirement, Inequality in Old-age and the Growing Gap in Longevity between Rich and Poor*, Washington, D.C., Brookings Institution.
- Chan, S. and Stevens, A. H. (2004): “How does Job Loss Affect the Timing of Retirement?”, *The B.E. Journal of Economic Analysis and Policy*, 3:1, 1–26.
- Cseres-Gergely, Zs. (2008): “Incentive Effects in the Pension System of Hungary”, *The Hungarian Labor Market, Review and Analysis*, eds. Fazekas, K., Cseres-Gergely, Zs. and Scharle, Á, 103–115.
- Eső, P. and Simonovits, A. (2002): Eső, P.–Simonovits A.: “Designing Optimal Benefits for Retirement”, *Northwestern University, Discussion Papers* 1353.
- Eső, P., Simonovits A. and Tóth, J. (2011): “Designing Benefit Rules for Flexible Retirement: Welfare and Redistribution”, *Acta Oeconomica*, 61, 3–32. o.
- Etgeton, S. (2016): “Labor Market Frictions, Retirement and Inequality”, lecture at EEA-ESEM 2016 Congress.
- Fazekas, K. and Varga, J., eds. (2015): *The Hungarian Labor Market 2015*, Budapest, IE-CERS-HAS.
- Freudenberg, Ch.; Berki, T. and Reiff, Á. (2016): “A Long-term Evaluation of Recent Hungarian Reforms”, *Magyar Nemzeti Bank WP* 2.
- Gustman, A. L. and Steinmeier, T. L. (1986): “A Structural Retirement Model”, *Econometrica*, 54, 555–584.
- Krémer, B. (2013): “Why is it So Frightening if People Live Longer?” *Sociological Review*, 23:3, 51–83.
- Manoli, D. and Weber, A. (2016): “The Effect of the Early Retirement Age on Retirement Decisions”, IZA Discussion Paper No. 10154.
- ONYF (2012): *ONYF Statistical Yearbook, 2011*, (in Hungarian) Budapest, Central Administration of National Pension Insurance, for short CANPI.
- ONYF (2013): *ONYF Statistical Yearbook, 2012* (in Hungarian) Budapest, CANPI
- ONYF (2014): *ONYF Statistical Yearbook, 2013* (in Hungarian) Budapest, CANPI
- Rust, J. and Phelan, C. (1997): “How Social Security and Medicare Affect Retirement Behavior in a World of Incomplete Markets”, *Econometrica*, 65:4, 781–831.

- Simonovits, A. (2008): “The Pay-as-you-Go System and the Permanent Reform: The First Pillar,” *Assessing Intergenerational Equity* eds. Gál, R., Iwasaki, I and Széman, Zs., *Assessing Intergenerational Equity*, Budapest, Akadémia, 71–86.
- Simonovits, A. (2016): Intra- and Intergenerational Transfers: Simple Paternalistic Models, typescript.
- Staubli, S. and Zweimüller, J. (2013): “Does Raising the Early Retirement Age Increase Employment of Older Workers?”, *Journal of Public Economics* 108, 17–32.
- Stock, J. H. and Wise, D.A. (1990): “Pensions, the Option Value of Work and Retirement”, *Econometrica* 58:5, 1151–1180.
- Swedish Pension System (2014): The Orange Report, 2013, Stockholm.