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Modeling of Layering Ceramic Shell Mould

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Abstract. This paper presents the modeling the drying process of shell mould layering process for the ceramic shell mould fabrication process. As this process involves the repeated drying of the different wetted/saturated layers, therefore modeling of this phenomena involve a sequent approach to tackle the whole development of shell mould layering process. In this work an Ab-Initia approach was selected as the best simulation technique to map the drying mechanism. Saturation or moisture content was selected as the best parameter that will represent the drying of layering process. Using FEM with quadrilateral shape mapping of several interested points were selected to predict the moisture/saturation movement during the drying of shell layering stages. Standard drying time with 2 hours and early drying time were chosen to measure the moisture movement as layer added to the previous dried coated shell or layers. This complex mechanism of drying and penetration layering shell were then numerically solve with fully implicit backward time stepping scheme. Hopefully, this model and can be used to measure the complex movement of the main parameter i.e saturation/temperature in the drying process of multilayer system which is sometimes impossible directly measured under experimental technique

Introduction

In general producing a ceramic shell mould for investment casting process involves several critical stages starting from preparing the slurry for dipping process of the wax pattern until the drying and firing of the wet multilayered ceramic shell mould. The drying condition is very critical to shell manufacture due to the fact that its can lead to hygrothermal stresses and hence the shell can exhibit a tendency to crack[1,2,3,4]. In fact, within investment casting the drying process is one of the most important stages in ceramic shell buildup during which a number of ceramic layers are added to form the shell mould. Layer drying has a direct impact on process time and a number of layers are added in order to get sufficient strength. The ceramic shell attains its pre-final strength during the drying process (before firing during which the diffusion process forms bonds to achieve a hard and very strong body). Most investigation on drying only focus on the single layer system or only consider that the matrix is homogenous and uniform, without considering the multilayer system or different level of water saturation in the whole matrix system[5-7]. In fact, this consideration of different level of water is very critical for certain component such as for wall, building, tree etc. that always expose to the absorption and desorption of water . Therefore, mapping or modeling of this layering shell is not only can solve the cracking problem in the shell mould industry but also can be used to the other crucial application as mentioned above.

Theoretical formulation

The theoretical formulation of the heat and mass balance equations used here is based on the work by Ben Nasrallah[8], Whitaker[9] and others[5,6].

$$\frac{\partial(\phi\rho_l S_l)}{\partial t}+\frac{\partial(\phi\rho_v S_g)}{\partial t}=-\nabla(\rho_l v_l)-\nabla\left(\rho_v \dot{v}_v\right) \tag{1}$$

Water and gas velocity in its liquid and gas state can be easily derived from Darcy's law with incorporating the effect of gravity. Transport in the vapour state is occurred via diffusion. Meanwhile for the gas transport the equation is shown below:

$$\frac{\partial\{\phi S_g \rho_a\}}{\partial t} = -\nabla\left(\rho_a v_g - \rho_v v_v\right) \quad (2)$$

The energy equation employed considers the effect of conduction, latent heat and convection and is given below;

$$\frac{\partial\left(\left(1-\phi\right)C_p\rho_s + \sum_{i=a,l,v}\phi S_i\rho_i C_i\right)}{\partial t} = \nabla(\lambda\nabla T) - \nabla(\rho_v V_v + \rho_v V_g)L - \nabla\left(\sum_{i=a,l,v}(T-T_r)\rho_i C_{pi} V_i\right) \quad (3)$$

The general boundary condition formulation for convective mass and heat transfer are given by:

$$J_m = h_m\left(P_\infty^v - P_s^v\right) \quad (4)$$

$$J_h = h_T\left(T_\infty - T_s\right) \quad (5)$$

Solution of governing equations and numerical method

The coupled heat and mass transfer equations described above, in 2-dimensions can be written into a matrix form as follows;

$$[C(\Phi)]\frac{\partial}{\partial t}\{\Phi\} = \nabla\left([K_{cx}(\nabla\Phi)]i_x + [K_{cy}(\nabla\Phi)]i_y\right)\{\Phi\} + R(\nabla Z) \quad (6)$$

where $\{\Phi\}= \{P_w,T,P_g\}$ is the column of unknowns;[C],[K_{cx}]and [K_{cy}] are 3x3 matrices. Each element of the matrix, is a coefficient for the unknown $\{\phi\}$; i_x and i_y are the unit direction vectors. This simplified second order non-linear coupled partial differential equation, is then discretised using the FE method. By minimising the residual error using the Galerkin method, followed by the application of Greens theorem to the dispersive term involving second order derivatives, this simplified combined equation set can be expressed in the following form.

$$\underline{K}(\Phi)\underline{\Phi} + \underline{C}(\Phi)\underline{\dot{\Phi}} + \underline{J}(\Phi) = \{0\} \quad (7)$$

In which typical elements of the matrix are

$$\underline{K}_{ij} = \sum_{s=1}^{n}\int_{\Omega^e} K_{ij}\nabla N_r \nabla N_s d\Omega^e$$

$$\underline{C}_{ij} = \sum_{s=1}^{n}\int_{\Omega^e} C_{ij} N_r N_s d\Omega^e$$

$$\underline{J}_i = \int_{\Omega^e} K_{i4}\nabla N_r \nabla z d\Omega^e - \int_{\Gamma^e} N_r \underline{J}_i d\Gamma^e$$

(i, j=1,2,3) and ($\overline{n}$- outward normal vector to the boundary, Γof the domain Ω). The transient matrix and nonlinear second order differential equations above were then solved by using a fully implicit backward time stepping scheme[10] along with a Picard iterative method to account for non-linearity[11-12] .

Results and Discussions

The proposed model with coupled equation and numerically solved using the selected procedure (as mentioned in previous section which had been verified previously in other papers[6-10] and show a good agreement with other works[12-13], were used to model the drying effect of layering shell mould system. Based on this model, the simulation of multilayer shell mould with different level of water which referring to the water uptake in building the shell mould layering system were constructed and computed. The procedure of modeling this building of shell mould system will be described below.

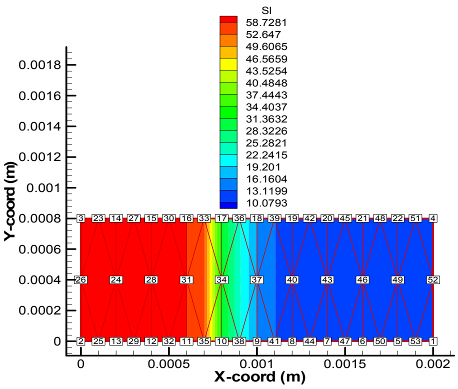
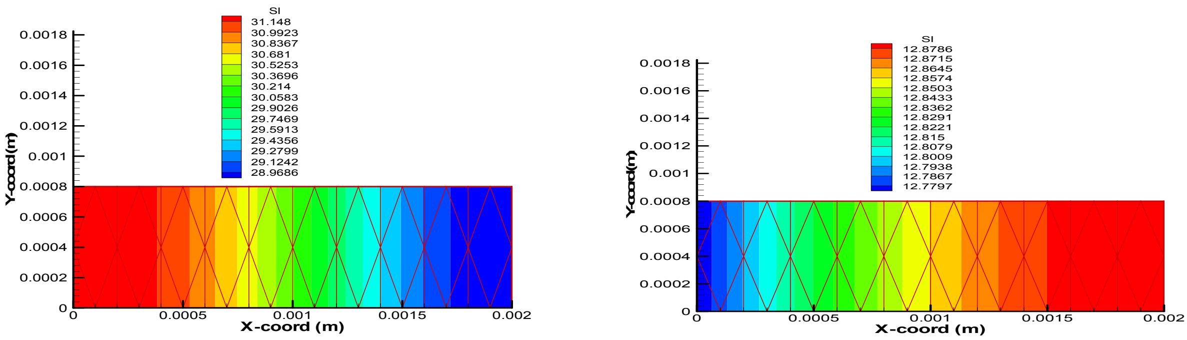


Fig. 1: Schematic of two layers of the shell showing the initial condition and including the selected nodes 1, 9 and 2.

In defining the initial condition, the cross section with the straight plain structure coupled with a relatively coarse mesh was used to prescribe a moisture gradient between the two different layers. At first the layer will be described as two layer system which reflects the dipping of second layer on the first dried layer, as shown in Figure 1. The drying period is then simulated and the final results are averaged over the shell thickness that has been built up.



(a) (b)

Fig. 2 : Saturation contour at 6 minutes and 2 hours drying duration

The selected nodes that can represent two different areas with different moisture content were selected to give a clear penetration or absorption of water content along the shell thickness as the drying proceeds. At the beginning of the drying time (in Figure 2(a)), there is a balancing or transition that shows a movement of the moisture content from the wet layer to the inner dried layer within 1 minute. This penetration of moisture to the inner layer is also strongly influenced by the properties that initially exist within the previously dried inner layer(s). As the initial transient is finished, then the moisture gradient starts to decrease and the moisture distribution across the domain levels out(in Figure 2(b)). The complete evolution of the moisture transition for this drying process also represent in the plotting form for the measurement at the selected node, as shown in Figure 3 (a) and (b).

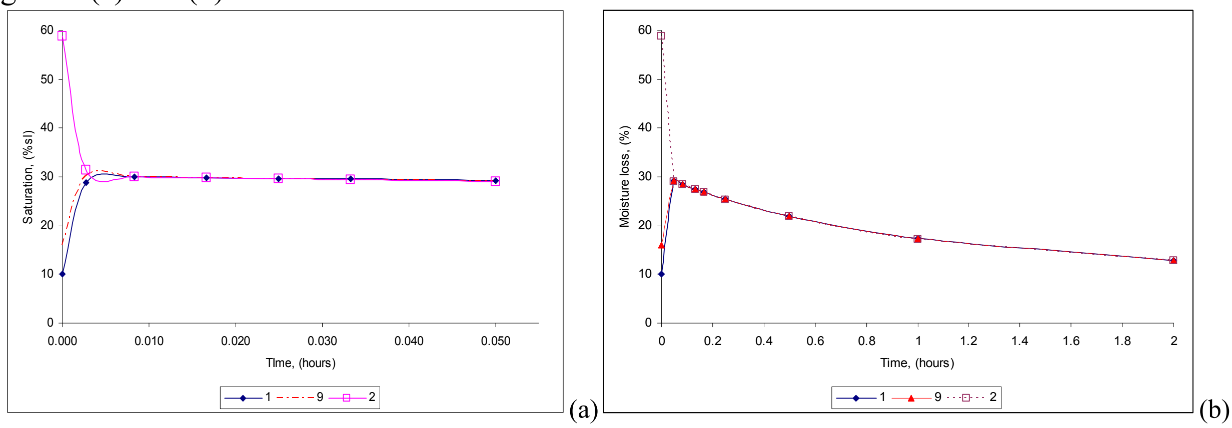
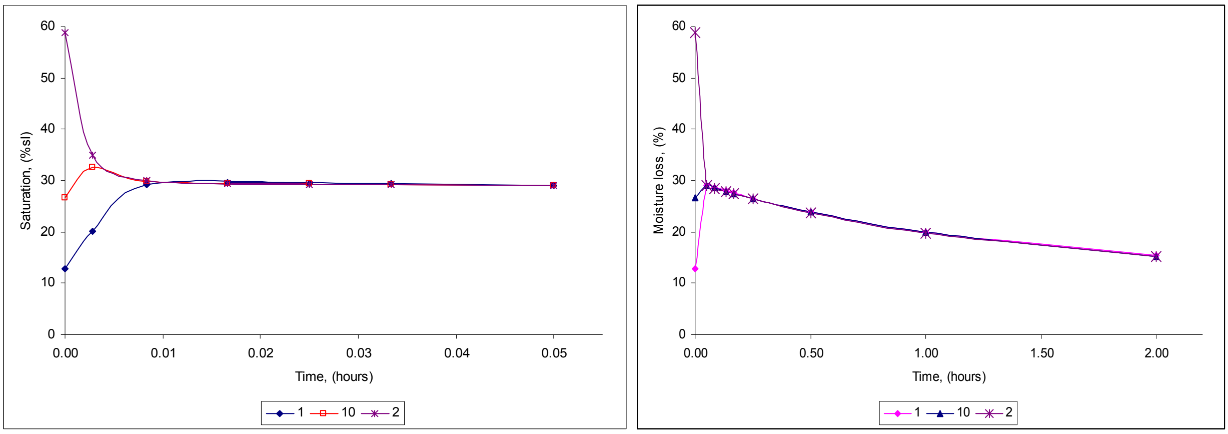


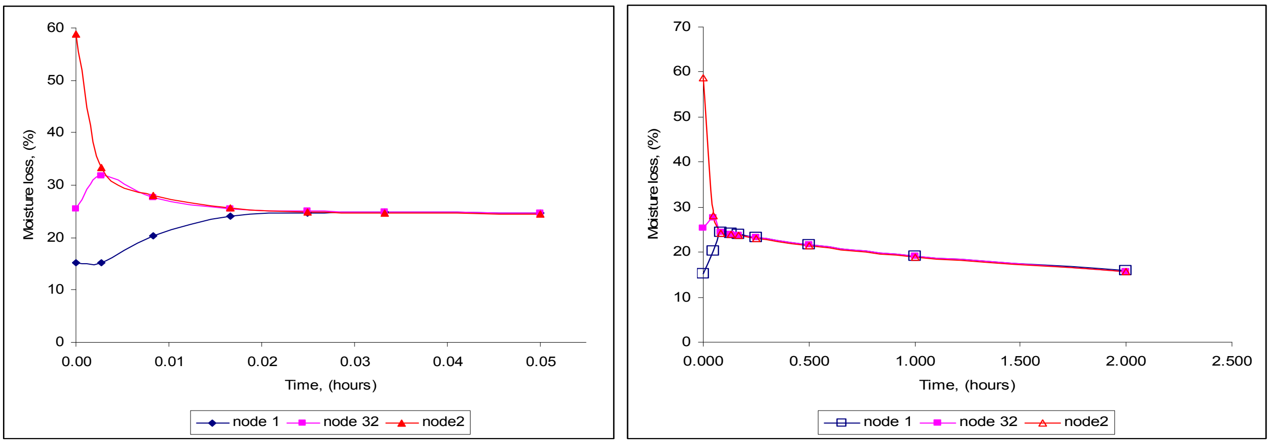
Fig. 3 : Saturation level of coat 2 at the selected nodes over a 3 minutes(a) and 2 hours(b)

Again, the simulation was carried out on the simple linear section for the third or next shell coat. To continue this simulation the excess moisture from the previous layer is assigned as the initial condition for the next dried layer after that. As shown in Figure 4 the initial condition for a three layer system where the excess moisture in the dried layer is equal to a saturation level of 12 % (which is the excess moisture that is present in the combined first and second layers after two hours drying time).



(a) (b)

Fig. 4 : Saturation level of coat 3 at the selected nodes over a 3 minutes and 2 hours.



(a) (b)

Fig. 5: Saturation level of coat 4 at the selected nodes over a 3 minutes (a) and 2 hours (b) drying.

The same condition is exhibited in the case of the three and four layer systems as described in the previous two layers system, where again the results show the transition in moisture constant takes place over the early stages. These are shown in Figure 5 (a) and Figure (b). In general there is only a slightly difference in the moisture change as each layer is added. As can be seen from Figure 4 and Figure 5, the water penetration (which reflects also the water soak back mechanism) take a longer time as layers are added and the thickness of dried layer increase. This shows the effect of thickness build up as layers are added and the effect of previous layers that have an increasing excess moisture content due to the soak back mechanism.

Summary

This work represent the modeling of drying ceramic porous material governs by the coupled equations of heat and mass transfer. Due to highly nonlinearity of transport variables in this complex drying process and the existence of multilayer structure therefore fully implicit backward time stepping scheme coupled with picard iterative method were used. Combination of this governing equation and numerical solution showed the appropriateness of the measured variables and other critical transport properties in the layering process of shell mould build up for investment casting

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