

IMPROVING CHAOTIC CRYPTOGRAPHIC PRIMITIVES BASED ON MAP'S COMPLEXITY AND PERIOD LENGTH OF THE CHAOTIC MAPS

AMIR AKHAVAN MASOUMI

UNIVERSITI SAINS MALAYSIA

2015

**IMPROVING CHAOTIC CRYPTOGRAPHIC
PRIMITIVES BASED ON MAP'S COMPLEXITY
AND PERIOD LENGTH OF THE CHAOTIC
MAPS**

by

AMIR AKHAVAN MASOUMI

**Thesis submitted in fulfillment of the
requirements for the degree of
Doctor of Philosophy**

AUGUST 2015

ACKNOWLEDGEMENTS

First and foremost, I would like to express my sincere gratitude to my dear supervisor Prof. Dr. Azman Samsudin for the continuous support of my study and research, for his patience, motivation, enthusiasm, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis. I am short in word to express his contribution to this thesis, with criticism, suggestions and discussions. I could not have imagined having a better supervisor and mentor for my Ph.D. study. I would like to make grateful acknowledgement to the Dean and all staff members of the School of Computer Sciences, USM. Indeed, I am fortunate to be surrounded by kind and helpful staff. There is no word to express my deep feeling to my lovely parents, sister and wife for their strong cooperation and inspirations. I am greatly thankful to God, that I am so much fortunate to have all good things together.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	ii
TABLE OF CONTENTS	iii
LIST OF TABLES	x
LIST OF FIGURES	xii
LIST OF ABBREVIATIONS	xvii
ABSTRAK	xix
ABSTRACT	xxi
CHAPTER 1: INTRODUCTION	1
1.1 Motivation	2
1.2 Problem Statement	3
1.3 Research Objectives	4
1.4 Research Scope	5
1.5 Research Methodology	5
1.6 Research Contribution	7
1.7 Outline of the Thesis	8
CHAPTER 2: BACKGROUND AND LITERATURE REVIEW	10
2.1 Introduction	10
2.2 Cryptographic Algorithms	10
2.2.1 Hash Functions	11
2.2.2 Encryption Algorithm	13
2.2.3 Pseudo Random Number Generator	14
2.3 Nonlinear Dynamic Systems and Chaos	14
2.4 Chaos-Based Cryptography	17

2.4.1	Symmetric Chaos-Based Cryptographic Algorithms	19
2.4.2	Asymmetric Chaos Based Cryptographic Algorithms	23
2.5	Drawbacks and Limitations of Chaos-Based Cryptography	24
2.6	Summary	28
CHAPTER 3: APPLIED CONCEPTS AND METHODS		29
3.1	Introduction	29
3.2	Nonlinear Dynamics	30
3.3	Discrete Chaotic Maps and Trajectories	31
3.4	Entropy and Phase Space	31
3.5	Statistical Complexity	33
3.6	Ergodicity and Chaos-based Cryptography	35
3.7	Bifurcation Diagram	36
3.8	Symbolic Dynamics	39
3.9	Lyapunov Exponent	40
3.10	Pseudo-Chaos and Conventional Cryptographic Algorithms	42
3.11	Summary	42
CHAPTER 4: ANALYSIS OF APPLIED CHAOTIC MAPS		45
4.1	Introduction	45
4.2	Higher Dimensional Chaotic Systems	46
4.2.1	Bifurcation Diagram of the 3D Chaotic Map	48
4.2.2	Lyapunov Exponent of the 3D Chaotic Map	51
4.2.3	Statistical Complexity of the 3D Chaotic Map	54
4.2.4	Transient Effect and Parameter-Sensitivity of the 3D Chaotic Map	56

4.3	Combination of Two Dynamical Systems	59
4.3.1	Bifurcation Diagram of the Combined Chaotic Map	61
4.3.2	Lyapunov Exponent of the Combined Chaotic Map	63
4.3.3	Statistical Complexity of the Combined Chaotic Map	65
4.4	Higher Precision Implementation of One Parameter Chaotic Maps with Two Fixed Points	67
4.4.1	Transient Effect Issue	68
4.4.2	Estimating Period Length versus Precision	69
4.4.3	Mean of Trajectory Decimal Digits in (ϕ^3)	71
4.4.4	Bifurcation Diagram of the Polynomial Chaotic Map	72
4.4.5	Lyapunov Exponent of the Polynomial Chaotic Map	73
4.5	Summary	74

CHAPTER 5: CRYPTOGRAPHIC PRIMITIVES BASED ON THE PROPOSED STRATEGIES

5.1	Introduction	76
5.2	Cryptographic Algorithms Based on Higher Dimension Chaotic Maps	76
5.2.1	Hash Function Based on 3D Chaotic Map	77
5.2.1.1	Proposed Hash Function Based on 3D Chaotic Map	77
5.2.1.2	Statistical Analysis of Hash Function Based on 3D Map	81
5.2.1.3	Collision Resistance and Birthday Attack Resistance	84
5.2.1.4	Mean of Absolute Difference for Hash Functions	88
5.2.1.5	Pseudo Collision of Hash Function Based on 3D Map	90
5.2.1.6	Key Space of the Hash Function Based on the 3D Map	91
5.2.1.7	Analysis of Speed for Hash Function Based on 3D Map	91

5.2.2	Image Encryption Algorithm Based on 3D Chaotic Map	92
5.2.2.1	Proposed Image Encryption Based on 3D Chaotic Map	92
5.2.2.2	Experimental Results of the Proposed Image Encryption	95
5.2.2.3	Information Entropy for Image Encryption Based on 3D Map	101
5.2.2.4	Analysis of Differential Attack Resistance	106
5.2.2.5	Key Space of the Image Encryption Algorithm	118
5.2.2.6	Analysis of Speed for Image Encryption Based on 3D Map	122
5.2.3	PRNG Algorithm Based on 3D Chaotic Maps	123
5.2.3.1	Proposed Algorithm for PRNG Based on 3D Chaotic Map	123
5.2.3.2	Statistical Analysis of the PRNG Based on 3D Chaotic Map	125
5.2.3.3	Simulation Results for PRNG Based on the 3D Chaotic Map	126
5.2.3.4	NIST Statistical Test Suite for PRNG Based on the 3D Chaotic Map	127
5.2.3.5	Marsaglia DIEHARD Test Suite for PRNG Based on the 3D Chaotic Map	128
5.2.3.6	ENT Test Package for PRNG Based on the 3D Chaotic Map	129
5.2.3.7	TestU01 Statistical Package	130
5.2.3.8	Key Space Analysis of PRNG Based on 3D Chaotic Map	132
5.2.4	A summary of the cryptographic primitives based on 3D chaotic maps	134
5.3	Cryptographic Primitives Based on Combination of Chaotic Maps	135
5.3.1	Hash Function Based on Combination of Chaotic Maps	135
5.3.1.1	Proposed Algorithm for Hash Function Based on Combination of Chaotic Maps	136
5.3.1.2	Hash Results of the Messages	138
5.3.1.3	Statistical Analysis of Hash Function Based on Combined Chaotic Maps	140

5.3.1.4	Analysis of Collision Resistance and Birthday Attack Resistance	143
5.3.1.5	Calculating Theoretical Values of Mean of Absolute Difference for Hash Functions	145
5.3.1.6	Key Space Analysis of Hash Function Based on Combined Chaotic Maps	147
5.3.1.7	Speed Analysis of Hash Function Based on Combined Chaotic Maps	147
5.3.2	Image Encryption Algorithm Based on the Combined Chaotic Maps	148
5.3.2.1	Proposed Algorithm for Encryption Algorithm Based on the Combined Chaotic Maps	149
5.3.2.2	Experimental Results of the Image Encryption Based on Combined Chaotic Maps	150
5.3.2.3	Histogram Analysis of the Image Encryption Algorithm Based on CCM	152
5.3.2.4	Correlation of Two Adjacent Pixels	153
5.3.2.5	Information Entropy of Image Encryption Based on Combined Chaotic Maps	158
5.3.2.6	Differential Attack Resistance of the proposed Image Encryption	160
5.3.2.7	Analysis of Speed for Image Encryption Based	165
5.3.3	PRNG Based on Composition of Chaotic Maps	166
5.3.3.1	Proposed PRNG Based on Combined Chaotic Maps	166
5.3.3.2	Statistical Analysis of PRNG Based on Combined Chaotic Maps	168
5.3.3.3	Simulation Results of PRNG Based on Combined Chaotic Maps	168
5.3.3.4	NIST Statistical Test Suite	168
5.3.3.5	Marsaglia DIEHARD Test	169

5.3.3.6	ENT Test Package	170
5.3.3.7	TestU01 statistical Package	171
5.3.4	A Summary of Cryptographic Algorithms Based on Combination of Chaotic Maps	172
5.4	Cryptographic Primitives Based on Higher Precision Realization of Chaotic Maps	172
5.4.1	Hash Function Based on Higher Precision Implementation of Chaotic Maps	173
5.4.1.1	Proposed Hash Function	173
5.4.1.2	Comparison of the Hash Digests	177
5.4.1.3	Statistical Analysis of Hash Function Based on Higher Precision Implementation of the Chaotic Maps	178
5.4.1.4	Collision and Birthday Attack Analysis	179
5.4.1.5	Mean of Absolute Difference for the Hash Function	180
5.4.1.6	Key Space Analysis of the Proposed Hash Function	181
5.4.1.7	Speed Analysis of Hash Function Based on Higher Precision Implementation	181
5.4.2	Image Encryption Based on Higher Precision Implementation	182
5.4.2.1	Proposed Image Encryption Algorithm	182
5.4.2.2	Experimental Results of the Image Encryption	184
5.4.2.3	Information Entropy of the Plain and Cipher Image	191
5.4.2.4	Analysis of Differential Attack Resistance	192
5.4.2.5	Key Space Analysis of Image Encryption Algorithm	192
5.4.2.6	Analysis of Chosen Plaintext Attack	193
5.4.2.7	Analysis of Speed for Image Encryption	194
5.4.3	PRNG Based on Higher Precision Implementation of Chaotic Maps	195
5.4.3.1	Proposed Algorithm for PRNG	196

5.4.3.2	Key space of the Proposed PRNG	198
5.4.3.3	Simulation Results for PRNG	198
5.4.3.4	Statistical Analysis for PRNG	199
5.4.3.5	Marsaglia DIEHARD Test Suite	199
5.4.3.6	NIST SP800-22 Statistical Test Suite	201
5.4.3.7	Fourmilab ENT Test Package	202
5.4.4	Summary of Higher Precision Implementation of Chaotic Maps	203
CHAPTER 6: DISCUSSION		204
6.1	Introduction	204
6.2	Discussion on Three Dimensional Chaotic Maps in Chaos-based Cryptography	204
6.3	Discussion on Combination of Chaotic Maps in Chaos-based Cryptography	206
6.4	Discussion on Higher Precision Implementation of Chaotic Maps in Chaos-based Cryptography	208
6.5	Summary of the discussion	210
CHAPTER 7: CONCLUSION AND FUTURE WORK		212
7.1	Introduction	212
7.2	Contributions	213
7.3	Recommendations and Future Work	214
REFERENCES		216

LIST OF TABLES

	Page
Table 3.1: Similarities of chaos and cryptography [70]	29
Table 4.1: Symbolic dynamic cycle look up	70
Table 5.1: Hash results of the sample message	81
Table 5.2: Number of bits changed (128-bit digest)	82
Table 5.3: Comparison of theoretical and experimental values of ω (10,000 times test)	86
Table 5.4: Total results of mean of absolute difference for $N=10,000$	89
Table 5.5: Analysis of speed	92
Table 5.6: Correlation Coefficient two adjacent pixels in plain and cipher image (Goldhill)	100
Table 5.7: Global entropy for the plain and cipher images	103
Table 5.8: Values of theoretical mean $\mu H(X)$ for different block sizes	105
Table 5.9: Local entropy for some of the sample images	105
Table 5.10: Critical point H^* value and block entropy for different K (Jellybeans 512×512)	106
Table 5.11: Results for NPCR analysis	111
Table 5.12: Results for UACI analysis	112
Table 5.13: Mean of absolute difference for red, green and blue values	121
Table 5.14: NIST SP800-22 test for 1,280,000 different randomly selected keys	128
Table 5.15: Marsaglia DIEHARD test results for PRNG based on 3D chaotic maps	129
Table 5.16: ENT package for 12 million bytes	130
Table 5.17: ENT package for 5 million bytes	130
Table 5.18: TestU01 results for the 32-bit proposed PRNG	131
Table 5.19: Mean of absolute difference for the modified sequences (ideal value=0.33)	133
Table 5.20: Hash values of the proposed hash function	139
Table 5.21: Number of bits changed (256-bit digest)	140
Table 5.22: 8-bit sub-blocks with the same value at the same location ($N = 10,000$)	144
Table 5.23: Total results of sum of absolute difference for $N=10,000$	146
Table 5.24: Speed comparison of the hash function based on CCM	148

Table 5.25:	Correlation Coefficient of two randomly chosen adjacent pixels	158
Table 5.26:	Global entropy for plain and ciphered sample images	158
Table 5.27:	Local entropy for 32 randomly selected blocks of size 512×512 pixels	159
Table 5.28:	NPCR for the four sample images with one bit modified	162
Table 5.29:	UACI for the four sample images with one bit modified	162
Table 5.30:	Results of the NIST SP800-22	169
Table 5.31:	Marsaglia DIEHARD test - PRNG based on the CCM	170
Table 5.32:	ENT test results for the proposed PRNG (10 Megabytes)	170
Table 5.33:	Results for big crush test on PRNG based on combination of two chaotic maps	171
Table 5.34:	Hash results of hash function (n is the location of the modified byte)	178
Table 5.35:	Number of bits changed (256-bit digest)	179
Table 5.36:	Collision resistance analysis results	179
Table 5.37:	Total results of mean of absolute difference for $N=10,000$	180
Table 5.38:	Correlation of two adjacent pixels (plain images and cipher images)	191
Table 5.39:	Global entropy for the sample plain and ciphered images	191
Table 5.40:	Local entropy for the sample plain and ciphered images	192
Table 5.41:	Results of the NIST test for Boat image	193
Table 5.42:	Results of the UACI test Boat image	194
Table 5.43:	DIEHARD test results for a file size of 12 million bytes	200
Table 5.44:	NIST SP800-22 test results for a stream size of 12 million bytes	202
Table 5.45:	ENT test results	203

LIST OF FIGURES

	Pages
Figure 1.1: Sub-problems and Research Objectives	4
Figure 1.2: Framework of the study	7
Figure 2.1: Lorenz attractor (using the Lorenz map)	17
Figure 2.2: Lorenz based chaotic circuit [160]	18
Figure 2.3: Number of publication on the Scopus database related to chaos and cryptography	20
Figure 2.4: Documents on Web of Science database related to chaos and cryptography	20
Figure 3.1: Ergodicity of Logistic map	35
Figure 3.2: Bifurcation diagram of Logistic map	36
Figure 3.3: Magnified bifurcation diagram of Logistic map [3.83, 3.87]	37
Figure 3.4: Magnified bifurcation diagram of Logistic map [3.847, 3.851]	38
Figure 3.5: Bifurcation diagram of one-dimensional map [0, 2]	39
Figure 3.6: Lyapunov exponent and Bifurcation diagram of Logistic map	41
Figure 4.1: Bifurcation diagram (normalized) for x, y and z with fixed α_2 and α_3	49
Figure 4.2: Bifurcation diagram (normalized) for x with fixed α_3	50
Figure 4.3: Bifurcation diagram for x, y and z versus α_2 (normalized values for comparison)	50
Figure 4.4: Bifurcation diagram for x, y and z with fixed α_3 (normalized values for comparison)	51
Figure 4.5: Lyapunov exponents for 3D chaotic map (Equation 4.1)	53
Figure 4.6: Lyapunov exponent (λ) of Lorenz map (Equation)	53
Figure 4.7: The complexity of polynomial map (thin line) vs Logistic map (thick line)	54
Figure 4.8: LMC statistical complexity for y_n dimension	55
Figure 4.9: LMC statistical complexity for z_n dimension	55
Figure 4.10: Results of transient effect analysis for 3D Chaotic map ($\alpha=1.2$)	56
Figure 4.11: Results of the transient effect analysis for the Logistic map ($\alpha=3.9$)	57
Figure 4.12: Transient effect (3D map, $\alpha=1.2$)	58
Figure 4.13: Transient effect (3D map, x_0 on Z)	58
Figure 4.14: Bifurcation diagram (x versus α_1) of the map presented in Equation 4.5	62

Figure 4.15:	Bifurcation diagram (y versus α_1) of the map presented in Equation 4.5	62
Figure 4.16:	Bifurcation diagram for the new generated chaotic map (Equation 4.11)	63
Figure 4.17:	Lyapunov exponent for the new generated chaotic mixed map (Equation 4.11)	64
Figure 4.18:	The 3D Lyapunov exponent, combination of two chaotic maps (Equation 4.11)	64
Figure 4.19:	Logistic map versus the composed chaotic map	65
Figure 4.20:	LMC complexity of the coupled chaotic map (Equation 4.5)	66
Figure 4.21:	LMC complexity of the coupled chaotic map (Equation 4.11)	66
Figure 4.22:	Transient effect elimination of the ϕ_3 versus minor changes in the initial conditions	68
Figure 4.23:	Difference of first 10,000 iteration from two trajectories with $x_0 - x'_0 = 10^{-400}$	69
Figure 4.24:	The mean of the three digits portion of 1 million trajectories	72
Figure 4.25:	Bifurcation diagram of the dynamical system ϕ_3	73
Figure 4.26:	Lyapunov exponent of the nonlinear dynamic system of ϕ_3	74
Figure 5.1:	Flowchart of the proposed hash function based on 3D chaotic map	80
Figure 5.2:	Binary representation of the Hash values in Table 5.1	81
Figure 5.3:	Values of B_i for the results of hash function based on 3D chaotic map	83
Figure 5.4:	Bit-Positions for all the 10,000 samples	84
Figure 5.5:	Collision probability for an ideal 32-bit hash function	84
Figure 5.6:	Number of 8-bit sub-blocks (same value, same location $L=128$, $N = 10,000$)	86
Figure 5.7:	Number of 8-bit sub-blocks (same value, same location $L=256$, $N = 10,000$)	87
Figure 5.8:	Number of 8-bit sub-blocks (same value, same location $L=512$, $N = 10,000$)	87
Figure 5.9:	Distribution of sum of absolute difference for 10,000 hash digests	89
Figure 5.10:	Flowchart of the image encryption algorithm	95
Figure 5.11:	(A) 512×512 pixels Peppers standard image (B) encrypted Peppers image	96
Figure 5.12:	Colored standard images and corresponding cipher images	96
Figure 5.13:	Grayscale standard images and the corresponding cipher images	97
Figure 5.14:	Encrypted black and white sample images	97
Figure 5.15:	Histogram of the Jellybeans image	98
Figure 5.16:	Histogram of the Jellybeans ciphered image	99
Figure 5.17:	Left: correlation of the gold hill image, Right: correlation the cipher image	101

Figure 5.18:	Global entropy of the gradient grayscale image is equal to 7.9921875	104
Figure 5.19:	Blocks randomly selected to calculate local entropy: ($k=32$) 32×32	106
Figure 5.20:	Sum of absolute difference for 100,000 pairs of ciphertext with modified keys	108
Figure 5.21:	Distribution of the sum of absolute difference	109
Figure 5.22:	One pixel difference between the two plain images I and I'	113
Figure 5.23:	Absolute difference of C and C' using proposed algorithm	114
Figure 5.24:	Absolute difference of C and C' using AES-256	114
Figure 5.25:	Block entropy of the decrypted values with one bit difference with the same keys	116
Figure 5.26:	(A) cipher image, (B) decrypted (C) 1-bit modified (D) 1-bit modified decrypted	117
Figure 5.27:	Range of initial conditions in a double precision variable in the chaotic map	118
Figure 5.28:	(A) Plane image, (B) Ciphered image and (C) Decrypted image	120
Figure 5.29:	Decrypted by modified (A) x_0 (B) y_0 and (C) z_0	120
Figure 5.30:	Decrypted by modified (A) α_1 (B) α_2 and (C) α_3	121
Figure 5.31:	Comparison of encryption speed (in Megabytes per second).	122
Figure 5.32:	The flowchart for PRNG based on 3D chaotic map	124
Figure 5.33:	Portions of a random number sequence	127
Figure 5.34:	Results for 106 big crush-TestU01 tests	131
Figure 5.35:	Distribution of the sequence R_{Original} (count versus random numbers)	133
Figure 5.36:	Mean of absolute difference test for $k=64$ random blocks	134
Figure 5.37:	Hash values under different conditions	139
Figure 5.38:	The ideal value for the bits changed for a 256-bit digest	141
Figure 5.39:	Values of B_i for 10,000 hash digests	142
Figure 5.40:	Variations of each of the bit positions for all the 10,000 samples	143
Figure 5.41:	Probability of reoccurrence (collision) for 256-bit ideal Hash function	143
Figure 5.42:	Repeated 8-bit sub-blocks $N = 10,000$. (Top) 256-bit, (Below) 512-bit digest	145
Figure 5.43:	Distribution of sum of absolute difference (512-bit hash digests) for 10,000 files	146
Figure 5.44:	Sample plain images (A) Barbara, (B) Flintstones, (C) Fingerprint and (D) Boat	151
Figure 5.45:	Sample cipher images (A) Barbara, (B) Flintstones, (C) Fingerprint and (D) Boat	151
Figure 5.46:	Histogram of ciphered (A) Barbara, (B) Flintstones, (C) Fingerprint and (D) Boat	152

Figure 5.47:	Hexadecimal representation of grayscale colors	153
Figure 5.48:	Correlation of Barbara standard image	154
Figure 5.49:	Correlation of Barbara standard ciphered image	154
Figure 5.50:	Correlation of Boat standard image	155
Figure 5.51:	Correlation of Boat standard ciphered image	155
Figure 5.52:	Correlation of Flintstones standard image	156
Figure 5.53:	Correlation of Flintstones standard ciphered image	156
Figure 5.54:	Correlation of Fingerprint standard image	157
Figure 5.55:	Correlation of Fingerprint standard ciphered image	157
Figure 5.56:	Blocks selected for the local entropy on the Boat ciphered image	159
Figure 5.57:	Mean of Absolute Difference for modified keys	161
Figure 5.58:	Effect of one bit change in the plaintext over ciphertext	163
Figure 5.59:	Local entropy (x axis: Samples, y axis value of local entropy)	164
Figure 5.60:	Sensitivity to cipher image in image encryption based on CCM	164
Figure 5.61:	Comparison of speed for image encryption based on CCM	165
Figure 5.62:	8-bit sub-blocks (same value, same location for $N = 10,000$ in 512-bit digest)	179
Figure 5.63:	Mean of absolute difference for 10,000, 512-bit sample digests	180
Figure 5.64:	Precision of the operations versus key space	181
Figure 5.65:	Sample images and their corresponding cipher images	184
Figure 5.66:	Histogram of plain images (A) Peppers, (B) hill, (C). Fingerprint and (D) Boat	185
Figure 5.67:	Histogram of the ciphered (A) Peppers, (B) hill, (C) Fingerprint and (D) Boat	185
Figure 5.68:	Correlation Coefficient of Lena plain image	187
Figure 5.69:	Correlation Coefficient of encrypted Lena plain image	187
Figure 5.70:	Correlation Coefficient of Fingerprint plain image	188
Figure 5.71:	Correlation Coefficient of encrypted Fingerprint plain image	188
Figure 5.72:	Correlation Coefficient of Goldhill plain image	189
Figure 5.73:	Correlation Coefficient of encrypted Goldhill plain image	189
Figure 5.74:	Correlation Coefficient of Peppers plain image	190
Figure 5.75:	Correlation Coefficient of encrypted Peppers plain image	190

Figure 5.76:	Analysis of speed for image encryption algorithm based on the HPCM	195
Figure 5.77:	Visualization of <i>100,000</i> 32-bit random numbers	198

LIST OF ABBREVIATIONS

AES	Advanced Encryption Standard
CCM	Combination of Chaotic Maps
DES	Data Encryption Standard (Name of a block cipher algorithm)
CCA	Chosen Ciphertext Attack
HPCM	Higher Precision realization of Chaotic Maps
IEEE	Institute of Electrical and Electronics Engineers
KS	Kolmogorov-Sinai, a type of entropy
LMC	López-Ruiz, Mancini and Calbet complexity measure
MAD	Mean of Absolute Difference
MD5	Message Digest Five
NIST	National Institute of Standards and Technology
NPCR	Number of Pixels Change Rate
NSA	National Security Agency
PRNG	Pseudo Random Number Generator
PWLCM	Piecewise Linear Chaotic Map
PWNLCM	Piecewise Nonlinear Chaotic Map
RC6	Rivest Cipher 6, is a symmetric key block cipher
RSA	Public-key encryption algorithm
SAD	Sum of Absolute difference

SHA	Secure Hash Algorithm (Name of a hash algorithm)
UACI	Unified Average Changing Intensity
USM	Universiti Sains Malaysia

MENINGKATKAN MUTU PRIMITIF KRIPTOGRAFI PETA CAMUK BERDASARKAN KERUMITAN DAN TEMPOH PANJANG PETA

ABSTRAK

Peta camuk berkentuan menghasilkan ciri-ciri seperti determinisme, ergodisiti, perlakuan seperti rawak, ketidaklinieran, aperiodisiti, entropi yang tinggi, imbalan, ketak kemerotan, korelasi maklumat yang amat rendah, dan kepekaan/kesensitifan yang amat tinggi terhadap perubahan yang amat kecil daripada keadaan awal dan parameter kawalan. Ciri-ciri ini amat sesuai untuk bidang kriptografi dan menjadikan peta camuk menarik bagi mereka bentuk primitif kriptografi. Berdasarkan ciri-ciri tersebut, sejak beberapa tahun kebelakangan ini, banyak peta camuk berdasarkan primitif kriptografi dicadangkan. Namun demikian, kebanyakannya didapati tidak begitu memuaskan atau mempunyai kekurangan seperti ruang kunci yang amat pendek, keberkesanan dan kelajuan yang rendah, kekuatannya yang tidak begitu baik serta sekuriti yang rendah. Justeru, kajian ini cuba mengkaji kekurangan tersebut dan keadaan awal, parameter kawalan dan trajektori sistem camuk digunakan dalam algoritma kriptografi berasaskan peta camuk dianalisis. Daripada hasil analisis, tiga kaedah baru bagi mereka bentuk primitif kriptografi berdasarkan peta camuk dicadangkan. Dalam usaha mengkaji kecekapan kaedah ini, satu set primitif kriptografi berasaskan peta camuk dicadangkan dengan menggunakan setiap kaedah tersebut. Kaedah pertama adalah aplikasi peta camuk berdimensi tinggi, di mana dimensinya adalah tiga dan ke atas. Kaedah kedua adalah berdasarkan kepada gabungan dua atau lebih peta camuk. Kaedah ini memanipulasi komposisi dua peta camuk untuk menjana satu peta camuk baru dengan ciri-ciri fizikal yang lebih baik dengan kerumitan statistik yang lebih tinggi. Dalam kaedah ketiga, merealisasikan peta camuk dalam ketepatan yang lebih tinggi adalah dicadangkan. Pada kebiasaannya, peta camuk direalisasikan dengan menggunakan ketepatan 32-bit hingga 64-bit, tetapi kaedah ini menggunakan ketepatan yang lebih tinggi (1024-bit hingga 2048-bit). Dalam usaha untuk menyiasat keselamatan dan kecekapan kaedah ketiga, primitif kriptografi berasaskan peta camuk (fungsi cincang, penyulitan imej dan penjanaan nombor pseudo-rawak) direka

berdasarkan ketepatan yang lebih tinggi pada peta camuk yang digunakan. Sekuriti algoritma yang dicadangkan dikaji menggunakan pelbagai jenis tekanan dan ujian sekuriti statistik biasa. Dapatan menunjukkan bahawa algoritma yang terhasil boleh memenuhi keperluan tahap sekuriti yang tinggi dengan halaju yang tinggi dan kos pengiraan yang rendah. Oleh itu, hasil kajian tesis ini menunjukkan bahawa peta serabut dengan cadangan penambahbaikan yang diutarakan boleh digunakan untuk aplikasi kriptografi dalam kehidupan sebenar.

IMPROVING CHAOTIC CRYPTOGRAPHIC PRIMITIVES BASED ON MAP'S COMPLEXITY AND PERIOD LENGTH OF THE CHAOTIC MAPS

ABSTRACT

Deterministic chaotic maps possess profound characteristics such as determinism, ergodicity, random-like behavior, nonlinearity, aperiodicity, high entropy, balance, nondegeneracy, incredibly low correlation of information and extreme sensitivity to very small changes of the initial condition and control-parameters. These characteristics are very favorable for cryptography and make deterministic chaos an interesting candidate in designing of cryptographic primitives. Based on these characteristics, during the recent years many chaos-based cryptographic primitives have been proposed. Unfortunately, a vast portion of them had encountered drawbacks such as shortened key space, low speed, lack of robustness and low security. In this study, these drawbacks are studied and the initial conditions, control-parameters and trajectories of the chaotic systems used in the chaos-based cryptography algorithms are analyzed. Three new methods for the design of chaos-based cryptographic primitives are suggested based on the results of the analysis. In order to study the efficiency of these methods, a set of chaos-based cryptographic primitives are proposed. The first method is the application of higher dimensional chaotic maps, the dimension can be three and above. The second method is based on the combination of two or more chaotic maps. It manipulates the composition of two chaotic maps to achieve to a new chaotic map with physically better characteristics and higher statistical complexity. In the third method, realization of the chaotic maps in higher precision is proposed. Regularly, the chaotic maps are realized using 32-bit to 64-bit precision, whereas this method takes advantage of higher precision (1024-bit to 2048-bit). In order to investigate the security and efficiency of the third method, chaos-based cryptographic primitives (hash function, image encryption and pseudo random number generator) are designed based on the higher precision realization of the chaotic maps. The security of the proposed algorithms is examined

using different types of attacks and common statistical security tests. The results of the cryptanalysis indicate that the presented algorithms satisfy the requirements for a secure system. Therefore, the findings in this thesis indicate that chaotic maps with suggested improvements can be used for real life cryptography applications.

CHAPTER 1: INTRODUCTION

The study of chaotic systems and their application in cryptography has been an interesting subject for the researchers in computer science and physics. In the past two decades plenty of new algorithms, based on different sorts of chaotic maps have been proposed. Most of the algorithms rely on the sensitivity of the chaotic systems to their initial condition and control-parameters. Meanwhile, many of the presented algorithms had been cryptanalyzed and found to be vulnerable against different attacks, either because of dynamical behavior of the chaotic systems used in their structure or because of inefficient utilization of the dynamical system. Besides security problems, many of these algorithms suffer from low speed, high cost of computation and are difficult to implement in real life application.

In this study, the main goal is to find subtle points in the design of a chaos-based cryptographic algorithm using the existing literature and by trying to propose efficient strategies in order to achieve secure and practical algorithms. The study of the literature, suggested three different approaches in designing of secure chaos-based crypto-primitives. The proposed strategies in this work should be studied from both dynamics and security point of view. In order to study dynamical behaviors of chaotic systems, along with regular methods (such as Lyapunov exponent, phase space control, and Bifurcation diagram) an alternative test such as complexity, period length and transient effect analysis are applied. The new cryptographic primitives are designed based on each of these approaches, and the security of the new proposed algorithms are tested against known attacks. The test results are compared with the existing cryptographic algorithms, from security, speed and applicability aspects.

1.1 Motivation

With the fast growth of the Internet in the early years of twenty-first millennium, majority of the people started using internet in their daily lives. The demand for the new image encryption algorithms, with better performance on the bulk data lead to several studies on the design of more secure and flexible image encryption algorithms. Nonetheless, Chaos based image encryption algorithms provide good confusion and diffusion on the bulk data. Therefore, they became an interesting alternatives option for the image encryption purposes. The similarities between nonlinear dynamical systems, namely chaos, and the cryptographic systems have made chaos a natural candidate for designing cryptographic algorithms [1]. During the past few decades dozens of new chaos-based both symmetric [2]–[39] and asymmetric [40]–[54] algorithms have been proposed and several novel view points towards the issue has been suggested. However, there has been plenty of failures and drawbacks in the proposed chaos-based designs [55]–[79].

Moreover, bibliometric statistics, along with a review of the literature imply that there have been many chaos-based algorithms, either symmetric or asymmetric found to be weak against different types of attacks. Security of the chaos-based cryptographic systems, mostly rely on the possibility of recovering initial condition or control-parameters and consequently the information available from the ongoing chaotic orbit. In deterministic chaotic systems, discovering the current state of the system can help finding all the following states.

In many of the cryptanalyzed cases, there exist particular repeated patterns in the design and implementation of chaos-based cryptographic algorithm that are responsible for the security flaws. If these patterns refrained, cryptographic algorithms would be more secure against different types of known attacks. The modified Baptista type algorithm [80] is a good example, which was proposed in order to improve the security of the original Baptista type cryptosystem [81]. Thus,

by identifying the common problems and the corresponding reinforcement it is feasible that chaos cryptography remain as one of the main parts in modern cryptography.

1.2 Problem Statement

Since the mid-nineties, application of chaos in cryptography has attracted many researchers. The similarities between chaos and cryptography such as sensitivity to initial conditions and control-parameters have made chaos a good choice for designing new cryptographic primitives. Nonetheless, many of the presented chaos-based cryptographic algorithms have been cryptanalyzed and proved weak against different types of differential and statistical attacks [39], [55]–[77], [82]–[110]. The main weaknesses of these algorithms were consequence of several issues such as:

- Unexpected short period length of the digitalized discrete nonlinear chaotic systems in finite space [85].
- Low sensitivity of the chaotic maps to the changes of initial conditions and control-parameters in some areas of the phase space and control-parameter chaotic range [76].
- Unsecure and inefficient design of the existing chaos-based cryptographic algorithms [67], including careless implementation [39], lack of knowledge about the strange attractor's behavior [69] and slow speed of the chaos-based algorithms [111].

In order to resolve these issues, several strategies are proposed in the past decade, but most of these strategies were only suggestions and prescriptions and have rarely been applied in practice on cryptographic primitives, thus it is very hard to provide any comparison between them. In this study, in order to tackle the problems mentioned above, three strategies based on appropriate chaotic systems are suggested and their dynamical characteristics are carefully analyzed under the finite precision implementation. Based on each of these strategies, three chaos-based cryptographic primitives are carefully designed. Moreover, the security of each of the algorithms

is analyzed against statistical and differential attacks. The security analysis, speed and flexibility of each of the algorithms are compared to conclude the effectiveness study of each of the algorithms.

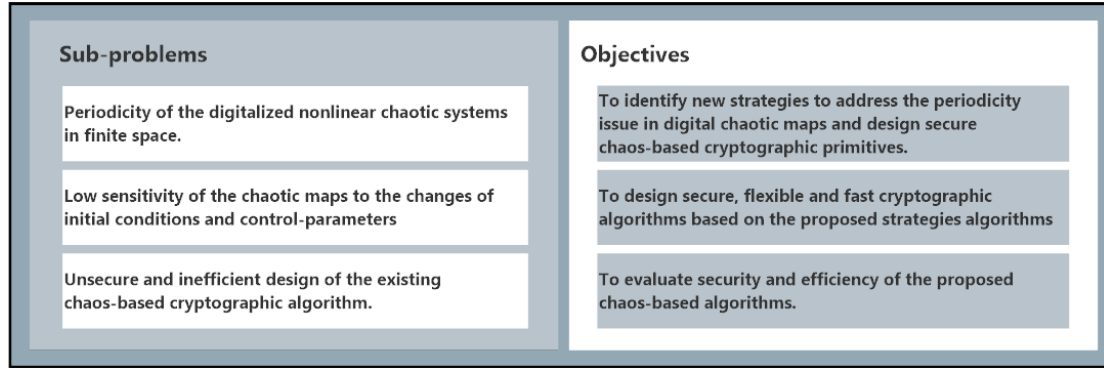


Figure 1.1: Sub-problems and Research Objectives

1.3 Research Objectives

According to the problem statement, the security, flexibility and speed of the chaos-based algorithms need to be improved. However, the proposed strategies and prescriptions in the past have not been applied in the newly proposed algorithms [4], [33], [39], [94], [112]. The main objective of this research is to propose strategies and design new cryptographic primitives based on each of the proposed strategies to improve the effectiveness, security, speed and flexibility of the chaos-based cryptographic algorithms. The objectives of this thesis can be categorized as below (Figure 1.1):

- To identify strategies to address the periodicity issue in digital chaotic maps.
- To design secure, flexible and fast cryptographic algorithms based on the proposed strategies.
- To evaluate security and efficiency of the proposed chaos-based algorithms.

1.4 Research Scope

The main scope of this study is to suggest strategies to improve the security of symmetric chaos-based cryptographic algorithms. In order to achieve this initial step is to find and investigate cryptanalyzed chaos-based cryptographic algorithms proposed in the past two decades. The investigation in this study mainly focuses on the discrete chaotic maps (unimodal, one dimensional, coupled map lattice and hyperchaotic maps, higher dimensional maps and finally quantum chaotic maps).

In order to investigate adequacy and efficiency of the proposed strategies, new chaos-based cryptographic primitives following the suggested strategies are designed. The proposed algorithms are under the category of hierarchy of polynomial chaotic maps (one dimensional, coupled map and 3D chaotic maps). The asymmetric cryptographic algorithms, stream ciphers, watermarking and steganography are not in the scope of this study. Meanwhile only Tent map and a class of hierarchy of polynomial chaotic maps are used in the structure of the proposed algorithms and other chaotic maps are not in the scope of this research.

1.5 Research Methodology

The methodology used in this research is conducted by finding the evidence available in the literature and analyzing the existing data in order to achieve the expected results such as design of new cryptographic algorithms with higher capabilities and faster operation time. The literature review consists of few major sections, such as identifying major literature from the main publishers and famous authors, tracking down, storing and reviewing all the closely related documents to the topic/scope.

According to the reviewed literature, groups and categories from the chaos-based cryptographic algorithms with the same concepts are categorized. Moreover, the problems and drawbacks of each category is listed. In order to overcome the problems and eliminate the

drawbacks, new strategies are proposed. According to the objectives of this study, the proposed methods are applied in order to design new chaos-based cryptographic algorithms. Finally, the security and efficiency of the proposed chaos-based cryptosystems are analyzed and the results of the analysis are compared with other chaotic and conventional cryptographic algorithms. Figure 1.2 provides an overview of the steps of the research. The major steps in deciding the strategies are based on the following issues:

- Selecting and modifying chaotic maps to fit the requirements of a highly sensitive chaotic map with long period length.
- Study the effect of realization of the chaotic maps in finite precision.
- Analyze the dynamical behaviour of the chaotic maps with respect to the evolution of the trajectories.
- Design new algorithms based on the structure and characteristic of the chaotic maps.
- Design experiments to investigate statistical characteristics and speed of the proposed algorithms.
- Design experiments to analyze security of the algorithms against different types of attacks.

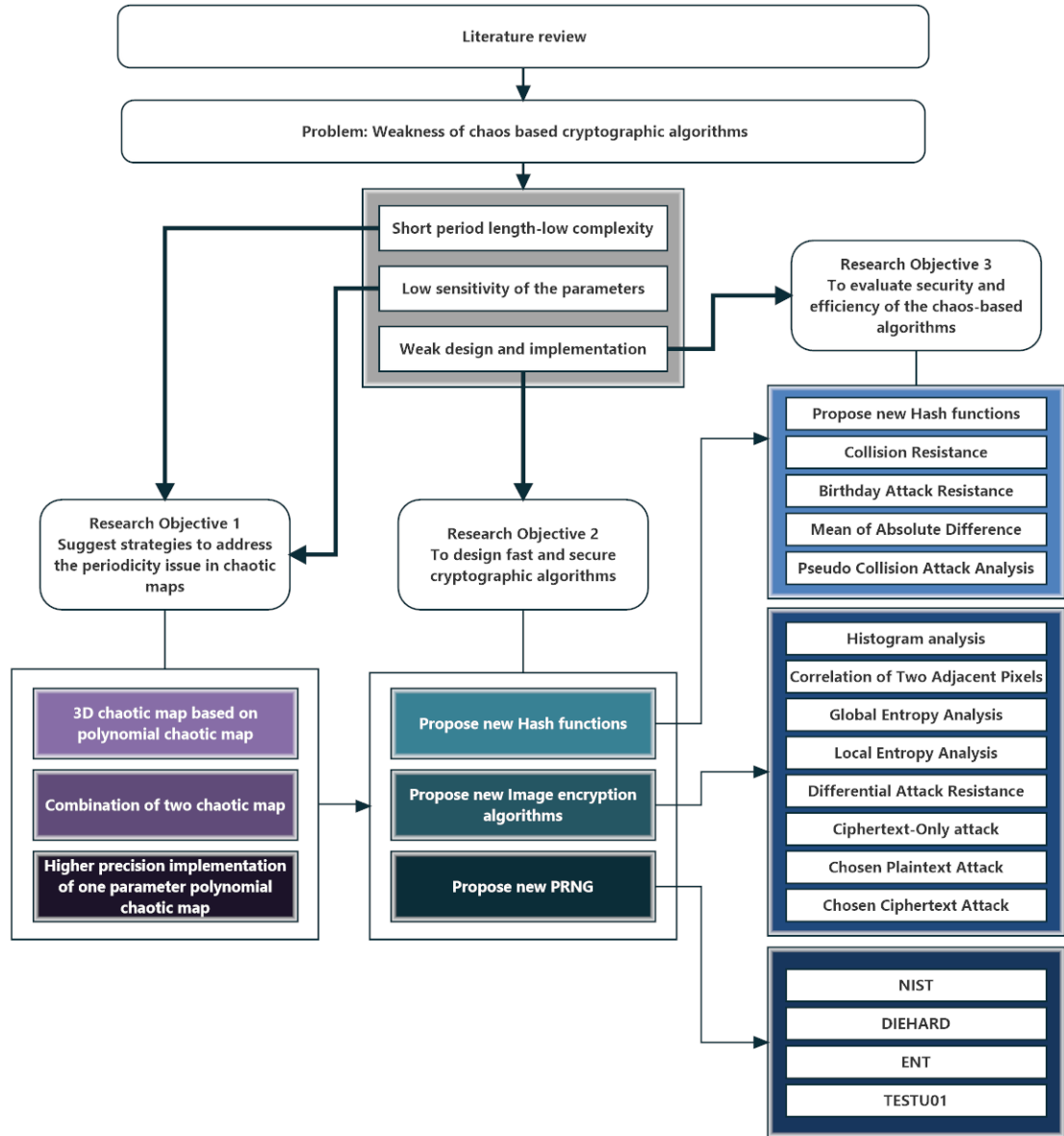


Figure 1.2: Framework of the study

1.6 Research Contribution

In the process to achieve the objectives of the thesis, three major contributions are made. The main contribution of this thesis is design of new chaos-based cryptographic primitive based on the suggested strategies. The algorithms can be categorized as below:

- 1) Application of higher dimensional chaotic maps in cryptography:

- Parallel hash function on application of higher dimensional chaotic maps
- Image encryption algorithm based on application of higher dimensional chaotic maps
- Pseudo random number generator (PRNG) based on application of higher dimensional chaotic maps

2) Application of combinational chaotic maps in cryptography:

- Hash function based on application of combinational chaotic maps
- Image encryption algorithm based on application of combinational chaotic maps
- PRNG based on application of combinational chaotic maps

3) High precision realization of one-dimensional polynomial chaotic maps:

- Parallel hash function based on application of higher precision realization of chaotic maps
- Image encryption algorithm based on application of higher precision realization of chaotic maps
- PRNG based on application of higher precision realization of chaotic maps

There are new original methods applied in each of the algorithms in order to achieve speed and flexibility along with the security, which will be described in detail in the following chapters.

1.7 Outline of the Thesis

The presented thesis is organized in seven chapters. The outline of each of the chapters can be described as below:

Chapter 1 provides a brief description about the concepts, problems, objectives and the contributions of the research. Chapter 2 reviews the related literature and presents a description about the concepts and tools used in this research. Chapter 3 provides a review of methods used in designing of the cryptographic algorithms. Chapter 4 studies the characteristics of the chaotic

maps applied in this study. Meanwhile, in this chapter the following issues are discussed: the transient effect, precision change effect and sensitivity to initial conditions and also control parameters. In addition, a new combinational chaotic map is proposed in this chapter. Chapter 5 discusses the application of higher dimensional chaotic maps, newly generated combinational chaotic map and higher precision implementation of the hierarchy of polynomial chaotic maps. This chapter also provides details about designing and analysis of three cryptographic primitives based on each strategy. Moreover, several tests and details are presented for the security and speed analysis of each of the proposed algorithms. Chapter 6 provides a discussion on the research results. In this chapter the security and speed of the proposed algorithms in the previous chapters are pointed out. Chapter 7 presents the summary, conclusion and future work of this study.

CHAPTER 2: BACKGROUND AND LITERATURE REVIEW

2.1 Introduction

Cryptography has an ancient history, which can go as far as Ancient Egypt [113]. This offers the idea that the desire for concealing writings has been a demand for thousands of years. With the fast improvement in the information technology and emergence and rapid growth of the internet, the demand for secure communication increased dramatically. Thus, design of faster and more secure algorithms gained extra importance. Therefore, numerous new algorithms were developed in a short span and chaos-based cryptography was one of the most controversial methods suggested in the meantime.

Chaos-based cryptography is a relatively new field of study. However, it has been a very active field during the past two decades according to the bibliometric information. Chaos and cryptography are very similar to each other. Both conventional methods of cryptography and chaos-based cryptographic algorithms are in favor of the maximum possible entropy [114]. Chaotic maps in most of the scenarios of chaos-based cryptography algorithms are considered as means of providing confusion and diffusion [115]. The unpredictable and random like behavior of the chaotic systems along with their strong sensitivity to changes in the control-parameters and initial conditions makes them very interesting candidates for the purpose of cryptography.

2.2 Cryptographic Algorithms

In general, in order to provide a secure communication between two parties on two sides of a channel (usually called as Alice and Bob), the message has to be converted, concealed or modified (i.e.: scrambled, transformed or permuted). This operation should be carried out in such a way that a third party in the middle (Eve or Eavesdropper) cannot find access to the content of the information or modify it without receiver noticing it. According to the Kerckhoffs's

principle, the man in the middle should not be able to recover the message even if he has access to the ciphering and deciphering algorithms [116].

Cryptology, on the other hand, is the science of information security and privacy. It deals with both selection of the tools and methods for concealing information and also evaluation of the achieved encryption systems [114]. The first stage of cryptology, which deals with selecting suitable methods and frameworks, is called *cryptography*. The term *cryptography* is taken from the Greek language, meaning “secret writing” and the second stage concentrates on the security of the proposed methods (*the cryptanalysis*).

Cryptographic algorithms can also be divided into several groups based on their concept of the work and demands. The first group and perhaps most used method is *encryption algorithm*, which are designed to cipher message in one side so that the other party can decipher it. The second group is called *one-way hash function*. Hash function, practically acts like a compression algorithm, while the compressed information cannot be retrieved anymore. In this study, the term hash function is used as MAC (Message Authentication Code).

2.2.1 Hash Functions

Hash functions are commonly used for information integrity, message authentication and storing sensitive information in databases such as usernames and passwords. A hash function should have several key characteristics, while weakness in any of them can lead to an unsecure design and as a result can be broken. A hash function has to be strongly sensitive to the message, irreversible, collision resistant and it has to be infeasible to find a message with the same corresponding hash value [117]. The terms “digest” and “checksum” are sometimes used alternatively for the hash value of a message. Size of the hash value may vary between 64 to 512-bit for different hash functions based on their structure and purpose, whereas the message size is

arbitrary. Amongst all the conventional hash functions, SHA-1 is the most famous hash function with 160 bits digest size.

Hash functions are unkeyed and if they come with keys they are usually called MAC (Message Authentication Code) [118], and are used both in cryptography and other fields of computer science such as data mining, genetic algorithms and neural networks. A keyed hash function (MAC) can be defined as below:

$$H_k: M \rightarrow D \quad 2.1$$

where $K = \{k_1, k_2, \dots, k_l\}$ is the set of the keys, $M = \{m_1, m_2, \dots, m_n\}$ is the set messages to be digested using the hash function H_k and the key for the hash function $k \in K$ and the resulting hash value $D = \{d_1, d_2, \dots, d_m\}$. $l = |D|$ is cardinality of the digest size, which is normally a fixed value between 64 to 512-bits and $n = |M|$ is the cardinality of the arbitrary message size.

Cryptographic hash functions are frequently used in the structure of more complicated algorithms such as digital signatures, fragile watermarking algorithms and secure network layers, therefore their security is of great importance. Almost all of the secure transactions and communications on the internet rely on the security and correlation resistance of the hash functions. SHA1 and MD5 are the most commonly used hash functions in the structure of internet protocols and it can be claimed that they have been the two of the most used algorithms in history of online transactions. Although SHA1 and MD5 have been cryptanalyzed by Wang et al. [119], [120] and found weak against collision attack but they are still widely used in different types of protocols. However, SHA1 and MD5 should be replaced and therefore design of new hash functions with larger digest size and stronger correlation resistance for higher security has been of great importance. During the last decade numerous conventional and chaotic cryptographic hash functions have been proposed [11], [27], [94], [108], [118], [121]–[134].

2.2.2 Encryption Algorithm

While hash functions are one-way operations, encryption algorithms are designed to work two sided, meaning that an encrypted message with a key “ K_e ” can be decrypted by a decryption algorithm and key “ K_d ”. In the case that the encryption and decryption keys are the same, the encryption algorithm is symmetric otherwise it is asymmetric. Until the mid-seventies and the invention of public key cryptography by Diffie and Hellman in their famous paper “*New directions in cryptography*” [135] all the encryption algorithms were symmetric and the sender and the receiver had to share a secret key through a secure channel. After that many other public key cryptographic algorithms such as Knapsack, RSA and El-Gamal [40] are introduced, however most of them rely on the idea of infeasibility of solving a hard problem to access private key. Based on this idea, several chaos-based cryptographic public key algorithms using semi-group characteristics of the Chebyshev polynomial based chaotic maps are suggested that most of them have been cryptanalyzed [40]–[54], [136]. An encryption algorithm can be defined as below:

$$E_{k_e}: P \rightarrow C, \quad 2.2$$

E_{k_e} is an encryption function, $k_e \in K_e$ and K_e is the key space for this mapping. Key space defines all the possible keys for this particular mapping, which maps plaintext $P = \{p_1, p_2, \dots, p_n\}$ to $C = \{c_1, c_2, \dots, c_m\}$, $n = |P|, m = |C|$. P and C are the plaintext space and ciphertext space respectively.

Recovering message can be carried out using following function:

$$D_{k_d}: C \rightarrow P, \quad 2.3$$

D_{k_d} is the decryption function, $k_d \in K_d$ and K_d is the key space for this mapping. In symmetric encryption both encryption and decryption keys are the same. A symmetric cryptosystem can be

either *block cipher* or *stream cipher*. One of the main differences between a block cipher and a stream cipher cryptosystem is the size of the data chunks encrypted in each round, usually if the size of the data chunk is larger than 64 bits; it is considered as block cipher. In many cases size of the processes chunk by the stream ciphers are as small as one byte [114].

2.2.3 Pseudo Random Number Generator

Pseudo random number generators (PRNG) are used in many of the research fields. A PRNG is a deterministic algorithm, which generates deterministic sequences of numbers. A close relative of PRNGs are the pseudo random bit generators (PRBG). A PRNG can be defined with five tuples $\{S, R, \varphi, \psi, P_A\}$ involving a finite set of generator states S , a set of possible outputs R , a state function ($\varphi: S \rightarrow S$), an output function ($\psi: S \rightarrow R$), and probability metric of random distribution of the seed (P_A) [137].

A cryptographically secure pseudo random number generator is a type of random number generator that the attacker cannot guess the next random number having access to a long stream of previously generated numbers. Such an algorithm can be used in the cryptographic applications such as stream ciphers. Several statistical tests such as Runs test, Parking lot test and Crabs test can be applied to investigate the randomness of a PRNG. Some randomness test batteries such as NIST SP800-22 [137], Diehard [138], TestU01 [139] and ENT tests are designed to perform set of several statistical tests on the random numbers generated by the PRNG. In between these batteries, TestU01 has a larger set of statistical tests with three major tests (Small Crush, Crush and Big Crush) [139].

2.3 Nonlinear Dynamic Systems and Chaos

Dynamics as an interdisciplinary subject was originally a branch of Physics [140] and began in the seventeenth century with Newton's invention of differential equation, discovery of laws of

motion and universal gravitation and their combination with Kepler's law of planetary motion [141]. Newton also could solve the two-body diagram, motion of the earth and sun. Later many mathematicians and physicists tried applying the same idea for three-body problem. However, it turned out to be more difficult until late nineteenth century that Poincare introduced a new viewpoint and suggested a geometric approach towards analyzing the idea. Consequently, the modern approach towards dynamics came into existence. In his work, he also mentioned the concept of chaos, in which a deterministic system is aperiodic and very sensitive to initial conditions and control-parameters and therefore it is almost impossible to predict the state of the system in the long term. In the geometric way of studying the nonlinear dynamics, pictures are usually more helpful than the formulas [140]. Poincare also suggested a new method of drawing phase space diagram and bifurcations for cases with higher than three dimension using a sectioning method, which was later called Poincare sectioning [142]. This method is very useful while studying a chaotic system of more than one dimension as the phase space exceeds three dimensions and makes it very hard to visualize the geometric diagrams for the phase space.

Field of nonlinear dynamics has a very wide scope and there are many chaotic systems which some of them have been modeled from the nature. The neglected points in the design of the chaos-based symmetric cryptographic primitives are the concern of this study. In order to come up with a secure cryptosystem, during this research, complexity of the chaotic systems used in the field of discrete chaos-based symmetric cryptography is analyzed and the results are used as to refrain from faulty implementations. Nonlinear dynamics tries to simulate aperiodic nonlinear dynamical systems in the real world in different fields such as population growth, mechanical systems, biology, chemistry, weather forecast, planetary motions and several others subjects, also the mathematical equations suggested based on differential equation modeling of the systems can be considered as a source of high entropy information [143].

It is almost two decades that chaotic systems have been used for designing secure communication methods and cryptography algorithms and based on the literature review, there exist many chaos-based algorithms that have been proved to be either unsecure, slow, hard to implement or in general inefficient [55]–[79], [82]–[104], [106]–[110], [144]. The behavior of chaotic maps is very appealing for researchers in the field of cryptography and that might be because cryptographic algorithms, especially the symmetric algorithms with nonlinear characteristics, are actually chaotic deterministic systems without a particular differential equation describing their chaotic behavior. It can be claimed that all the nonlinear systems in the real world are algorithmic pseudo chaotic systems [140].

Until the last few decades that computer realization of the chaotic systems became possible, the field of nonlinear dynamics was not very active [145]. Lorenz in 1963 proposed a model for the thermal convection, which were three ordinary differential equations (Equation 2.4) [146]. In the same document, he has mentioned, “lack of periodicity is very common in natural systems”. This sentence, eventually points to existence of chaotic non-periodic characteristics of natural systems. Furthermore, in an interview, Lorenz mentioned the metaphor of the “Butterfly effect”, a flap of butterfly’s wing in Brazil, could influence a hurricane several weeks later in Texas.

$$\begin{aligned}\dot{x} &= \sigma(y - x), \\ \dot{y} &= rx - xz - y, \\ \dot{z} &= xy - \beta z.\end{aligned}\tag{2.4}$$

The phase space for the Equation 2.4 if drawn, the famous Lorenz attractor (the Lorenz Butterfly) would be generated. The parameters (r , σ and β) are the control-parameters and the values for the (x , y and z) are the initial conditions. Figure 2.1 demonstrates Lorentz attractor for initial conditions ($x=8$, $y=3$, $z=33$) with the time frame of 100 and precision of 10^{-5} and control-parameters ($\sigma=10$, $r=28$, $\beta=2.66666666$).

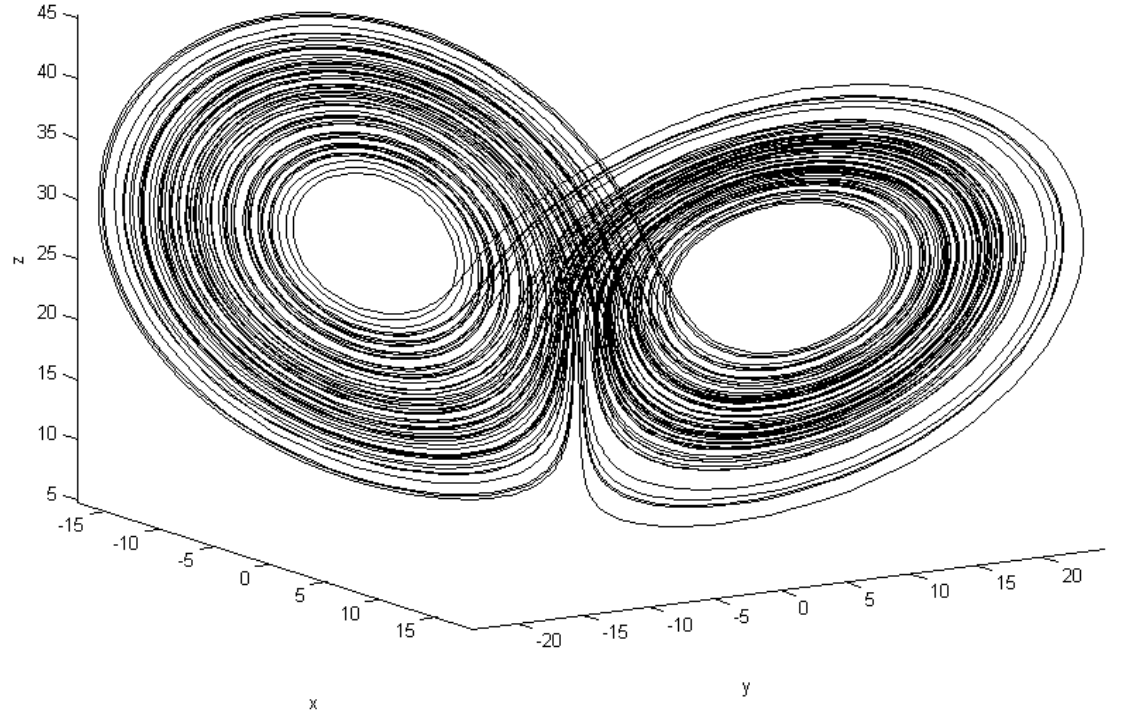


Figure 2.1: Lorenz attractor (using the Lorenz map)

The Lorenz map, (Equation 2.4) was first used for generating chaotic signals using a circuit (Figure 2.1) and a message, non-chaotic containing meaningful information, was masked by the chaotic signal and unmasked by an identical system which was synchronized and the message was retrieved in the receiver side [140].

2.4 Chaos-Based Cryptography

Although the main factors attracting researchers to chaos-based cryptography algorithms were almost the same and limited to few mutual characteristics, but there has been numerous different efforts and approaches towards this field [4], [9], [11], [16], [17], [19], [20], [23]–[26], [28], [33], [34], [36], [37], [39], [122], [126], [147]–[156]. These approaches can be categorized as chaos-based symmetric [2], [4], [5], [9], [15], [16], [20], [26], [27], [29], [30], [37], [118], [147], [150], [157], [158] and chaos-based asymmetric cryptographic algorithms [40]–[54], [136].

Analog chaos-based secure communication method was first introduced in the mid-1980s and in 1990 it was developed by Pecora and Carrol [159]. Their method was mostly based on chaos masking, the data as analog signals were masked under a very high frequency of chaotic signals generated by a circuit [140], [160]. In [160] the Lorenz map chaotic circuit was represented and used to generate chaotic mask for the wave carrying information.

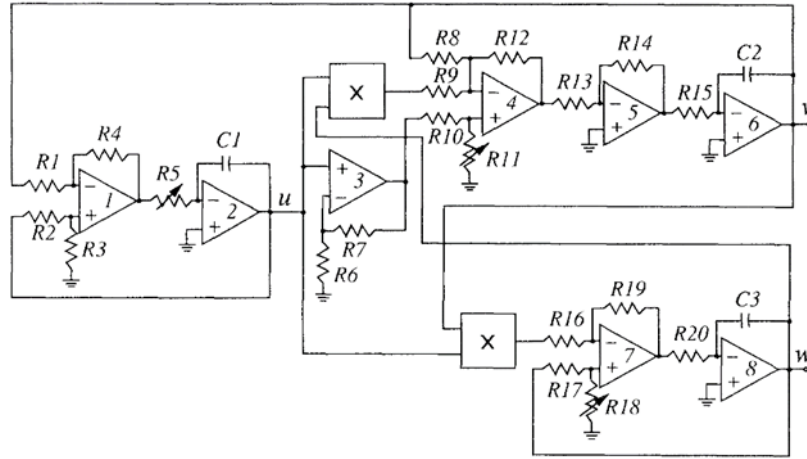


Figure 2.2: Lorenz based chaotic circuit [160]

Most of the analog chaotic cryptosystems, apart from masking mostly are based on chaos synchronization [102], [140], [160]. The basic idea of analog synchronization secure communication methods rely on synchronization of two or more chaotic systems in the transmitter and the receiver side [161]. However, the digital chaotic cryptosystems usually concentrate on the deterministic characteristics of the chaotic systems. In the other words, analog chaotic secure communication schemes and digital chaotic cryptosystems are based on two different techniques [102], while both of them depend on sensitivity to initial condition, control-parameters, complex behavior, unpredictability and determinism of chaotic systems. The first one uses the synchronization methods whereas the second one relies on the deterministic sensitivity of the chaotic trajectories to the initial conditions and control parameters.

The digital chaotic cryptosystems are mostly based on the discrete chaotic systems. They generally try to utilize discrete dynamical systems and encrypt the messages using the generated chaotic trajectories. Just few years after the emergence of analog secure communication methods based on chaotic systems, the idea of using discrete chaotic maps for encryption of digital data or as mentioned in [162], digital communication was developed. The idea was very simple and straightforward, both chaos and cryptography have mutual counterparts, such as sensitivity to initial conditions, control-parameters in chaos, random like behavior, unpredictability and finally determinism, which are very favorable for cryptographers and researchers in physics community.

2.4.1 Symmetric Chaos-Based Cryptographic Algorithms

Cryptography can in large scale be divided into two major categories, symmetric and asymmetric [163]. Chaos-based cryptographic schemes can be categorized as asymmetric and symmetric systems. Habutsu [164] was the first to introduce the chaos-based asymmetric cryptographic algorithm. His method was cryptanalyzed at the same conference by Biham [89].

In the last two decades a lot of research has been conducted on the symmetric chaos-based cryptographic algorithms and many new algorithms are proposed [2]–[39], [118], [122], [147]–[150], [157], [158], [165]–[169]. Figure 2.3 and Figure 2.4 demonstrate the number of articles found on the Scopus and Web of Science databases that are directly related to chaos and cryptography. The solid line represents all articles (both cryptography and cryptanalysis documents) and the hashed line demonstrated the cryptanalysis related documents. It can be seen that there has been a sudden increase in the number of publication from mid-nineties.

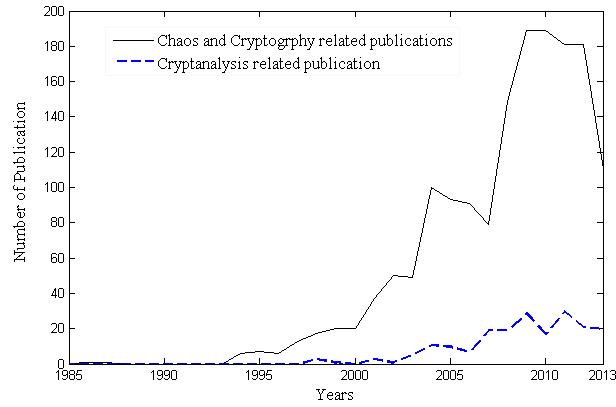


Figure 2.3: Number of publication on the Scopus database related to chaos and cryptography

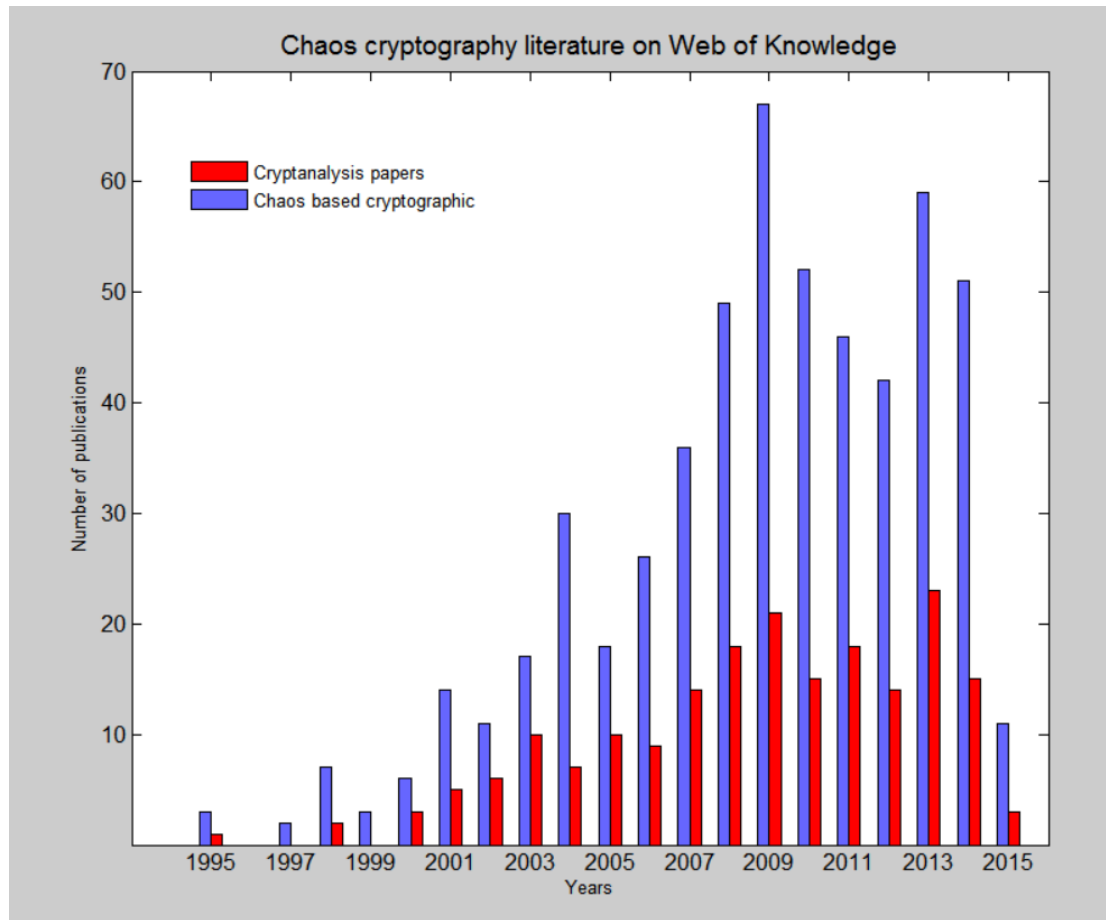


Figure 2.4: Documents on Web of Science database related to chaos and cryptography

Unfortunately, a large portion of these algorithms are cryptanalyzed and are proved to be unsecure [57], [59]–[62], [65], [67], [69]–[72], [74], [78], [82]–[84], [86], [87], [89]–[104], [106],

[110], [144]. The big amount of attention, accompanied with plenty of number of cryptanalyzed algorithms, suggests that, there exist some typical problems in the algorithms. This common problem makes them weak against different types of attacks. On the other hand, some of these algorithms not only are unsecure but also are very slow and inefficient. Most of the mentioned algorithms are image encryption algorithms and the rest are one-way hash functions and pseudo random number generators. Without loss of generality, it can be said that, just like symmetric cryptography, chaos-based symmetric cryptography can be technically categorized into two categories: block cipher algorithms and stream cipher.

The available chaos-based symmetric algorithms can be divided into five major categories: First category of algorithms, which are more primitive, manage to apply the random like behavior of nonlinear dynamic systems (namely chaotic systems) to shuffle the position of the data blocks (either as small as one bit or as large as several bytes). Most of the early image encryption algorithms that are proposed as premiers in the field follow the same idea. Their main strategy is shuffling or permuting the pixels (of a grayscale image) or the bits (of a black and white image) position to reach a scrambled output [170].

The second category of algorithms are the image encryption algorithms that are based on skewing the image files in several rounds with Arnold or Cat map. The second type algorithms are very famous for their direct connection between the image pixels and nonlinear systems. Besides security issues, most of the algorithms in this category have slow speed of encryption and need several rounds of encryption to provide a random output without any visible patterns in it. One of the famous algorithms of the second type is the Fridrich type algorithms, mainly based on Fridrich method proposed in his paper in 1998 [171]. In this method a two dimensional chaotic map (the Baker map) is applied to generate a block cipher algorithm mainly for image encryption. Later this type of algorithms were cryptanalyzed by Solak et al. [59].

The algorithms in third category are usually referred as Baptista type, which are mostly inspired by the creative encryption algorithm first proposed by Baptista [81]. The main concern in this algorithm is the number of iteration. Therefore, encryption of each block of data needs several iterations and consequently the encryption speed decreases dramatically. Many modified methods were presented based on Baptista method [55], [66], [107]–[109], [172] and most of them were also cryptanalyzed [67], [82], [173]–[175].

The forth type of the chaos-based cryptosystems possesses more complex structure and are stronger from the security point of view and clearly try to reconstruct the traditional cryptographic algorithm by the aid of chaotic maps. Xiang et al. [176] proposed a new chaos-based cryptographic algorithm, which applies a two dimensional chaotic map in order to generate S-Box for the cryptosystem [176]–[179], meanwhile some of these algorithms found to be not secure enough and were cryptanalyzed [79], [144].

The fifth type and the final type are the image encryption algorithms and one-way cryptographically secure hash functions based on a hybrid combination of control-parameters initial conditions and the plaintext. These methods provide more security because of their nature. They possess a complex structure and take advantage from the ergodic characteristics of the chaotic maps along with the mixing property and sensitivity to initial conditions in several directions [35], [37], [147], [166]. Some of the algorithms designed on this strategy were cryptanalyzed because of the problems in the applied chaotic maps in their structure [62], [65].

$$x_{n+1} = \alpha x_n(1 - x_n) \quad 2.5$$

Chaos-based stream cipher cryptosystems and also chaos-based pseudo random number generators as other classes of symmetric chaos-based cryptographic algorithms have also been discussed a lot in the last couple of decades [14], [72], [92], [149], [180]–[193]. Most chaotic stream cipher algorithms directly depend on the pseudo-random number/bit generators based on

chaotic maps. One of the most famous methods of generating a chaos-based sequence of random bits is the symbolic dynamic representation of the Logistic map. In this particular method, a threshold is chosen (usually $x_t=0.5$) and the chaotic Logistic maps is iterated while in order to avoid attractors, the values of the initial conditions are normalized to stay in the safe zone, apart from 0.0 , 0.5 and 1.0 . The initial condition (x_0) and control-parameter (α) are typically taken as the secret key and in the simplest condition, the Logistic map (Equation 2.4) the generated bits are XORed with the plaintext values to generate the ciphertext. In more advanced stream cipher methods, a set of several chaotic maps are manipulated in order to generate higher complexity and reduce the possibility of predicting the bits.

2.4.2 Asymmetric Chaos Based Cryptographic Algorithms

Asymmetric cryptography has fundamental differences with the symmetric cryptography and therefore chaos-based asymmetric cryptographic algorithms also manipulate different characteristics of the chaotic maps [64]. First time in 1976, Diffie and Hellman proposed “*New Directions in Cryptography*” [135], which gave the revolutionary idea of key exchanging method. Later on 1977 Ron Rivest, Adi Shamir and Leonard Adleman proposed the RSA public key cryptography algorithm based on difficulty of factoring large integers.

Most of the asymmetric chaotic cryptography schemes, try to get advantage from the semi-group characteristics of the Chaotic-Chebyshev polynomial maps [41], [44], [45], [47], [48], [53] while most of the algorithms base the traditional asymmetric cryptographic schemes are based on the hard mathematical problems [114]. Some of the asymmetric chaos-based cryptographic algorithms have tried to take advantage of both the hard mathematical problems and semi-group characteristics of the Chaotic-Chebyshev polynomial maps simultaneously. In 1993, a chaos-based public key algorithm based on a method similar to ElGamal asymmetric encryption algorithm was presented [194]. But there was a problem in making it practical, as there is no

practically efficient fast method to iterate up to the n -th value of the chaotic map presented in it while the value of n is extremely large [64]. Later on, Kocarev et al [41] established a new method based on the commutative characteristics of Chaotic-Chebyshev polynomials over real numbers. But very soon was cryptanalyzed by Bergamo et al. [63]. It was proved unsecure as an adversary can find the plaintext from a given ciphertext. The main problem in asymmetric chaos-based cryptographic algorithms arises from inappropriate use of the real number implementation [63].

In order to improve the cryptosystem against the suggested attack, in another research, Kocarev et al. [48] modified the algorithm and manipulated the Chebyshev polynomials over the finite field Z_N , also Lima et al. [48] with another modification proposed a new algorithm with prime finite field [48], [195]. In 2005, Xiao et al. proposed a new key agreement protocol based on chaotic maps [43]. This protocol was supposed to resist the known attacks, which were earlier suggested by Bergamo et al. [63]. Nevertheless, very soon Xiang et al. [68] and Han et al. [77] suggested a new attack which proved protocol proposed by Xiao et al. is unsecure [43]. In 2011, Lee et al. proposed a new extension to chaotic maps-based key agreement protocol. It took advantage of the semi-group characteristics of the Chebyshev polynomials to overcome all the attacks presented for the previous chaos-based public key cryptosystems [51].

2.5 Drawbacks and Limitations of Chaos-Based Cryptography

There have been many attempts in the field of cryptography with chaos in the last two decades, some of the methods and ideas although very brilliant and innovative have been successfully cryptanalyzed and their security has been proved to be practically inefficient [55]–[79], [82]–[104], [106]–[110], [144]. Nonetheless, the similarities between chaos and cryptography, such as ergodicity, aperiodic system evolution, sensitivity to initial conditions and control-parameters together with easy and quick implementation capabilities have been so strong