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DETERMINATION OF SYNCHRONOUS MOTOR VIBRATIONS  
DUE TO ELECTROMAGNETIC FORCE HARMONICS

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ABSTRACT

In this paper the authors present a method of computation of the magnetic vibrations produced by electric motors. The method is based first on the calculation of the electromagnetic field in the motor. Then the fluctuations of the magnetic forces applied on the teeth of the stator are evaluated. Finally harmonics of these magnetic forces are calculated to lead by means of a mechanical finite element analysis to the motor vibrations. The method is applied to compare vibrations produced by two permanent magnet synchronous motors of different magnetic structures.

INTRODUCTION

Magnetic vibrations are specific to electrical machines. In an electric motor they are generated by the fluctuations of magnetic forces applied on the stator.

These vibrations have been the subject of several investigations [1] [2]. Researchers employed analytical and conformal methods to calculate the electromagnetic field in electrical machines. These methods have the advantage of being simple and allow one to obtain the frequencies of the resulting magnetic forces. But they cannot be employed to evaluate accurately the amplitudes of these forces and of the mechanical vibrations generated by them. Besides, these methods allow one to calculate only radial force density and not tangential force density ; therefore, the vibrations produced by the torque ripple cannot be known.

The authors proposed a computation method of these vibrations based on an electromagnetic field finite difference package and on a mechanical structure finite element analysis. The work that the authors have recently carried out consisted of making an existing electromagnetic field software package suitable for the computation of the electromagnetic field evolution, implanting in this numerical package magnetic force calculation based on the Maxwell's stress tensor as described in [3], and creating a finite element program for mechanical structure analysis. In this paper, the methods employed are described and some of the results obtained are presented and discussed.

ELECTROMAGNETIC FIELD CALCULATION

Because of the low values of frequencies used in electrotechnics, displacement currents can be neglected and Maxwell's equations can be reduced to the magneto-dynamic equations [4] :

$$\nabla \times \vec{H} = \vec{J}, \quad \nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad (1)$$

Electric motors possess moving magnetic materials on their rotors. Characteristic equations of the different magnetic materials, and particularly the moving magnetic materials, must be added to equations (1) :

$$\vec{B} = \mu \vec{H} + \vec{H}, \quad \vec{J} = \sigma \vec{E} + \sigma \vec{V} \times \vec{B} + \vec{J}_{ext} \quad (2)$$

In equations (2),  $\sigma$  and  $\mu$  are respectively conductivity and permeability of the material.  $\vec{J}_{ext}$  is the current density provided by an external power supply.  $\vec{V}$  is the velocity of the material. In synchronous motors, with no damper windings, induced currents represented by the term :

$$\vec{J}_i = \sigma \vec{E} + \sigma \vec{V} \times \vec{B} \quad (3)$$

can be neglected. Therefore magneto-static equations only have to be taken into account :

$$\nabla \times \vec{H} = \vec{J}_{ext}, \quad \nabla \cdot \vec{B} = 0, \quad \vec{B} = \mu \vec{H} + \vec{H} \quad (4)$$

In conclusion the evolution of electromagnetic field inside synchronous motors can be calculated by assimilating their dynamic operation to a succession of magnetic states governed by magnetostatic equations. For each state, which is characterized by a position of the rotor relatively to the stator, the current density is calculated by means of characteristics of the external power supply and the electrical characteristics of the motor ; the magnetic field is computed by means of a numerical field analysis. For this study we use an electromagnetic field finite difference analysis called DIFIMEDI [5] which can solve steady state magnetic field problems with linear, non linear and permanent magnet materials.

We have calculated with this procedure the no load back e.m.f. of a radially oriented permanent magnet type synchronous motor with no skewing. Excellent agreement between calculated and measured voltage wave forms has been achieved. The back e.m.f. waveform of this motor is shown on figure 1.

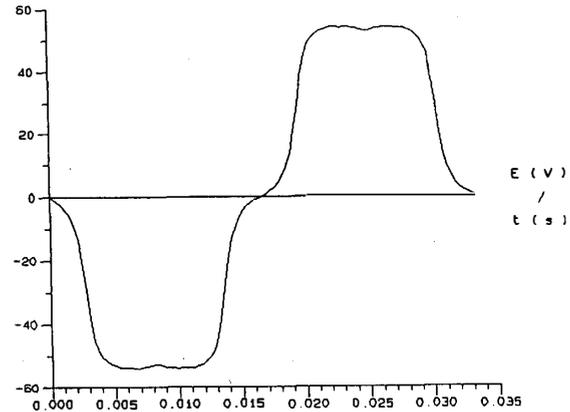


Figure 1 : No-load e.m.f. of synchronous motor with no skewing.

MAGNETIC FORCES CALCULATION

Only magnetic forces applied on stators are calculated. Vibrations due to magnetic forces applied on rotor are neglected.

Magnetic forces are applied on the conductors in slots and on the teeth of stators. Magnetic forces exerted on conductors are calculated by the LAPLACE's law and magnetic forces on teeth are calculated by the MAXWELL's stress tensor. In a previous paper [3] the authors showed experimentally that integration of the surface force density given by the MAXWELL's stress tensor over a surface covering partially a tooth of a motor leads to magnetic force applied on this tooth.

Calculations of forces on conductors and on teeth give a

good evaluation of the distribution of the magnetic forces along stators of electric motors. To obtain the evolution of forces in the time domain these calculations are done for each state selected for the electromagnetic field computation.

The modeling approach described here, and given in details in [5], is applied to compare vibrations produced by two permanent magnet synchronous motors [7]. These motors have been designed to have the same speed, torque and stator. They differ only in the magnetic structure of their rotors. The first motor has tangentially oriented magnets and the second motor has radially oriented magnets. By means of ferromagnetic materials placed between two tangentially oriented magnets it is possible to obtain a very high value of the mean flux density in the air gap of the first motor. This technique is utilized to have high value of the torque per ampere. In the second motor the mean value of the magnetic flux density in the air gap is about one and a half times lower than in the first motor. Each motor has 18 poles, 3 slots per pole. The current frequency is 48 Hz and the nominal torque is 500 N.m.

To compare vibrations produced by the two motors it is not necessary to make a structural analysis because they are working at the same speed and have the same stator. Magnetic forces calculation is sufficient to compare vibrations levels.

Figure 2 and figure 3 show the time evolution of the radial and the tangential magnetic forces applied on the three teeth of a pole of the stator. Forces on conductors are not shown because they have very weak value. The frequency of the magnetic force is equal to twice the current frequency  $f_c$ . Forces calculated on the teeth of the first motor are higher than the forces on the teeth of the motor with radially magnetized magnets. Table 1 compares the amplitude of each harmonic force applied on a tooth in the case of the two motors :

- the mean values of the tangential forces are equal ; this result agrees with the fact that the two motors have the same torque.
- the mean value of the radial force is three times lower in the case of the motor with radially magnetized magnets ; it is due to the fact that the mean value of the air gap flux density is one and half times lower in this motor than the motor with tangentially magnetized magnets.
- the amplitudes of the harmonics are higher when the motor has tangentially magnetized magnets except for the fourth one ; ferro-magnetic materials placed between magnets, in this motor, modulate strongly the air gap flux density and, consequently, the magnetic forces too.

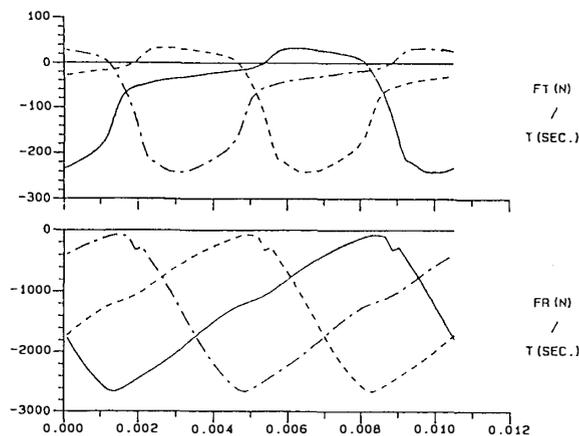


Figure 2 : Evolution of radial force (FR) and tangential force (FT) applied on teeth of the first motor  $f_c = 48$  Hz.

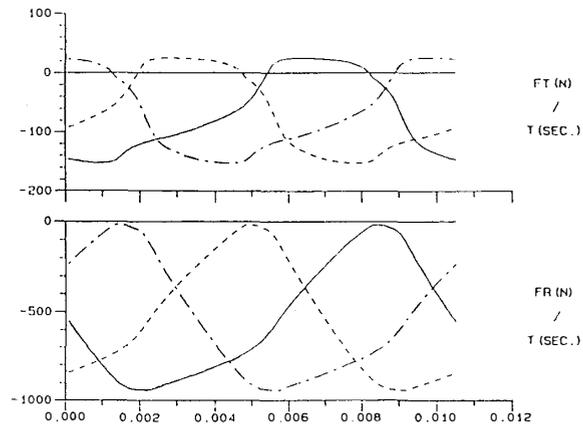


Figure 3 : Evolution of radial force (FR) and tangential force (FT) applied on teeth of the second motor  $f_c = 48$  Hz.

Motor	Tangentially oriented magnets		Radially oriented magnets	
	FT*	FR*	FT*	FR*
Harmonic 1	116N	1108N	88N	433N
Harmonic 2	62N	334N	22N	88N
Harmonic 3	15N	46N	6N	7N
Harmonic 4	7N	9.5N	7N	18N
Harmonic 5	13N	29N	4N	9N
Mean Value	-69N	-1280N	-68N	-541N

FT\* Tangential      FR\* Radial

Table 1 : Amplitudes of harmonics forces

From these results we can conclude that the second motor produces lower vibrations than the first one. The same study can be done also to explore the vibratory effects of different stator windings or of other critical design parameters as far as these parameters do not change the mechanical and the geometrical features of the stator. If these features change, structural analysis is needed to study vibratory effects of design parameters.

#### STRUCTURAL ANALYSIS

The mechanical displacements caused by magnetic forces have weak value [3], therefore mechanical equations remain linear and the superposition principle can be used. Then it is possible to study the effect of each harmonic of magnetic forces on the vibrations of the stator.

The Lagrange equation of motion for a conservative system is [6] :

$$\frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{x}_m} \right) - \frac{\partial L}{\partial x_m} = 0 \quad (5)$$

where  $x_m$  is one of the generalized coordinates, and the Lagrangian  $L$  is the difference between the total kinetic energy of the system  $T$  and the total potential energy of the system  $V$ . When a finite element approximation is used the kinetic energy  $T$  and the total potential energy  $V$  can be expressed easily :

$$T = \frac{1}{2} [\dot{x}]^T [\eta] [\dot{x}] \quad (6)$$

$$V = \frac{1}{2} [x]^T [\eta] [x] - [F][x]$$

Where  $[x]$  is the column matrix of the generalized coordinates which can be the components of the vector displacement of each node of the equivalent discretized structure,  $[\dot{x}]$  is the derivative of  $[x]$  and contains the components of the vector velocity of each node,  $[M]$  is the mass matrix,  $[K]$  is the stiffness matrix and  $[F]$  is the column matrix of the generalized forces applied on the nodes and equivalent to all the actual forces applied on the mechanical system. Equations (6) are used in equations (5) and we obtain the mechanical equations of the discretized structure :

$$[\eta] [\ddot{x}] + [K] [x] = [F] \quad (7)$$

where  $[\ddot{x}]$  is the column matrix of the acceleration of each node. When the forces applied on the mechanical system are sinusoidal and have the same frequency, complex notation can be employed :

$$[\ddot{x}] = -\omega^2 [\bar{x}] \quad (8)$$

$$([K] - \omega^2 [\eta]) [\bar{x}] = [\bar{F}] \quad (9)$$

where  $\omega$  is the pulsation of the forces. These last equations are employed to calculate vibrations due to harmonics of magnetic forces. The problem of determining vibrations is then reduced to the problem of solving algebraic complex linear equations similar to equations (9). The solutions of these equations give the displacement of each node, from which acceleration can be obtained by relation (8).

To illustrate the mechanical calculation we have computed the vibrations produced by the first three harmonics of the magnetic forces in the case of the motor with tangentially oriented magnets. Table 2 gives for each harmonic the frequency, the amplitudes and the phase angle difference of the components of the forces applied on two successive teeth of the stator. The following remarks must be underlined :

- force amplitudes are equal for each harmonic,
- force phase angles are different for the first two harmonics,
- force phase angles are equal for the third harmonic.

Rank	Frequency	Tangential Force	Radial Force	Différence phase Angle
1	2 x fc	116N	1108N	120°
2	4 x fc	62N	334N	240°
3	6 x fc	15N	46N	0°

Table 2 : Characteristics of the first three forces harmonics.

The frequency of the third harmonic is equal to what is usually called slot frequency fs :

$$f_s = 2 nps \times f_c \quad (10)$$

In this relation nps is the number of slots per pole. To perform the mechanical calculation a mesh corresponding to the stator has been done in two dimensions. The accelerations calculated for the third harmonic are about one hundred times higher than those calculated for the first two harmonics. This result is astonishing when amplitudes of the magnetic forces

harmonics are compared (table 2). The force amplitudes for the third harmonic are much lower than for the first two harmonics. The only way to explain this mechanical result is the fact that the force phase angles corresponding to the third harmonic are equal, so vibratory effects of forces exerted on the teeth are added. This is not the case for the first two harmonics ; the phase angles are not equal, therefore vibratory effects of forces applied on teeth tend to cancel out.

These mechanical results are in agreement with the well known fact that for a synchronous motor the vibrations at slot frequency are much higher than the others. The whole procedure presented in this paper allows to explain this fact clearly.

#### CONCLUSION

A method for the computation of magnetically induced stator vibrations in synchronous motors has been presented. Application of this method to a permanent magnet synchronous motor has indicated significant changes in vibrations level between a situation with radially oriented magnets and one with tangentially oriented magnets. The results obtained allow one to understand the mechanism of generation of these vibrations.

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